DESCRIPTION OF AN EXACT RECURSIVE METHOD TO SIMPLIFY SHADING CALCULATIONS

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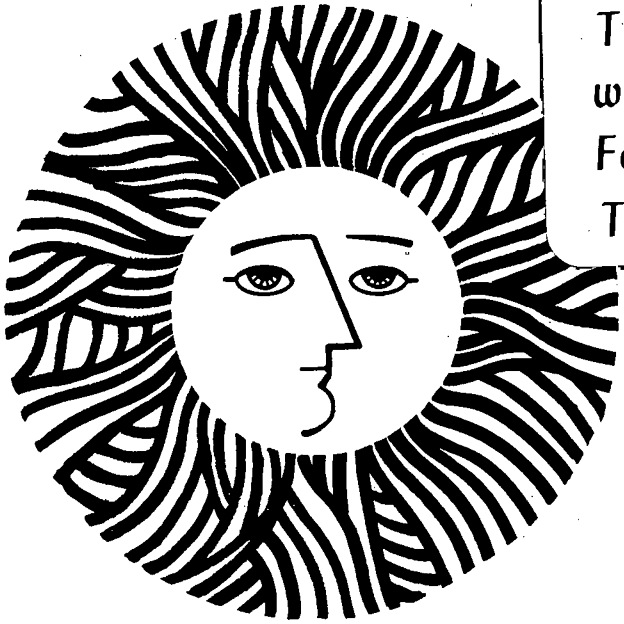

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DESCRIPTION OF AN EXACT RECURSIVE METHOD TO SIMPLIFY SHADING CALCULATIONS*

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ABSTRACT

This paper: (i) describes an exact, recursive method called SHADE which attempts to simplify shading calculations as performed by a programmable calculator or microcomputer; and (ii) outlines preliminary applications of SHADE using a Hewlett Packard HP-41C programmable calculator. In particular, for a given solar hour, SHADE is used to compute the following quantities for overhang and side fin combinations which shade various openings: the percentage of the total area of the opening which is shaded; the shaded area itself; the cosine of the angle of incidence between sun and glazing surface; the direct insolation at this surface, with and without shading; and the direct solar power at this surface, with and without shading.

Hence, in its present HP-41C application, SHADE can be used in preliminary design and comparative analyses of shading devices on an hourly, daily, or seasonal basis, provided that: (i) the fins and overhangs be square or rectangular, and lie in planes perpendicular to the plane of the opening; and (ii) the opening itself be vertical and rectangular, with arbitrary building azimuths. Design candidates include conventional overhangs and side fins, porches, and reveals. In principle, SHADE can be extended to awnings, slatted sun screens, and bevelled recesses; in addition, its HP-41C application can be extended to calculations of direct solar gain through vertical and non-vertical glazings, thereby providing a more useful tool in building heating and cooling load calculations.

BACKGROUND

A considerable body of literature and software for calculating the amount of shading of apertures by fixed shading devices already exists (1,2,3,4,5,6). However, these approaches were developed primarily as manual and graphic methods and/or for applications to digital computers. Here the motivation is quite different; this work investigates the applications of programmable calculators--devices which are accurate, but relatively slow and have small memories--to shading calculations. Such an approach requires the development of a method in which conserving calculator storage space is more important than calculation time. On the other hand, the calculation time ought not to be too long if it is to be useful to the solar practitioner.

An exact, numerical method called SHADE appears to meet these requirements. In terms of software inputs and constraints, SHADE bears some resemblance to the FORTRAN code "SHADOW1"(4), although a FORTRAN version of SHADE suggests that fewer steps are involved. This simplification is made possible by treating vertical fins as special cases of an overhang; by recursively calculating overlapping areas of shadow projections from corner points of the shading device; and by requiring that (i) shading geometries be composed of triangular, square, or rectangular figures and (ii) the aperture be square or rectangular. Examples of possible SHADE applications include overhangs, side fins, awnings, porches, reveals, and slatted solar screens.

SHADE has also been incorporated into a FORTRAN code whose output provides graphic overlays for use in the Libby-Owens-Ford "Sun Angle Calculator".[1] This approach--described in a companion paper (7)--is quite different, since it is intended primarily for less precise, graphic evaluation of shading devices. In this paper, the mathematical basis for SHADE will be explicated, followed by a brief discussion of preliminary results obtained by applying it to three software versions using a Hewlett-Packard HP-41C programmable calculator.

Finally, the question of "product fit" arises: what is it we hope to accomplish

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[1] Copyright 1974, Libbey-Owens-Ford Company.

with this approach and who are intended users? Clearly there is a need to reduce fossil fuel consumption through energy conscious building design, but the conventional approach to shading problems among practicing engineers and architects has been largely manual and graphic. Yet, it appears that "early adapters" in various architectural and engineering schools, as well as within the field, are beginning to employ microelectronics as design aids. Thus it is possible that the role of small computer systems in the design process will become increasingly important. Tools such as SHADE, alone or in combination with other computation techniques, will be needed. In the short term, the usefulness of this particular effort will be determined through interactions between interested practicing professionals, the Regional Solar Energy Centers, and other national laboratories.

DESIGN STRATEGY

SHADE is based upon certain design assumptions and considerations. First, it is assumed that in the preliminary stage of design and specification of a shading device, designers need to interact with software inputs and outputs so that design parameters can be altered quickly and easily. For this purpose, they must be able to change the solar-surface azimuth and solar altitude angles for perhaps a few select cases; or at the other extreme, on a seasonal basis using certain design days over a range of solar angles. In addition, they must be able to change the geometry of the shading device, as well as the shaded aperture, so that the significance of increasing or decreasing a particular dimension can be discerned readily.

Second, it is assumed that in the majority of cases, designers will specify off-the-shelf building components whose geometries tend to be rectangular in nature; similarly it is assumed that in most cases, shading devices will be installed in a rectangular frame, so that shadows cast by them will lie either perpendicular to or parallel with the edges of the shaded aperture. Although many intriguing and aesthetically pleasing shade/aperture structures are nonrectangular, the resultant geometries are rather complex to analyze. In principle, these more complex geometries can be approximated as the superimposed result of many, small squares, but this approach is beyond the scope of this effort.

Third, although diffuse and reflected sunlight contributes significantly to the overall building thermal and electrical lighting loads, it is assumed that the primary purpose for a shading device is to seasonally encourage or discourage direct beam solar penetration of glazed surfaces. Hence, useful information can be gained in preliminary analysis by simply accounting for the direct beam influence at the shaded aperture, e.g. by obtaining hourly profiles of the cosine-corrected direct beam solar power incident upon the glazed surface both with and without the shading device in place.

In this fashion, designers can determine the relative merits of the shading device, using unshaded glazing as a point of reference. For convenience, designers need to be able to specify the month they wish to analyze, with prompts for solar angles, such that insolation levels are calculated automatically, using say the clear sky method described by ASHRAE (1). Alternatively, designers need to be able to input their own insolation values, based upon local data which they may prefer to use.

MATHEMATICAL DESCRIPTION OF SHADE

A Description of Principles

As a preliminary step to development of the mathematical equations, consider the case of a flat overhang which shades a single, vertical, rectangular glazed opening. For the sake of generality, this overhang may be inclined in position and/or trapezoidal in shape; it need not be centered over the opening and it may be arbitrarily long and wide. However, its front and back edges must be parallel to the horizontal edges of the glazing.

This case is depicted in Fig. 1. In clockwise direction, points P_1, Q_1, Q_2, and P_2 define the overhang plane and vertices; similarly, points A, G, F, E define the plane and vertices of the glazing. The plane of the glazing and the plane of the overhang intersect along line Q_1Q_2. The entire shadow cast by the overhang from the direct beam solar radiation is defined in the plane of the glazing by vertex points Q_1, P_1', P_2', and Q_2; but only the area $ABP_2'C$, defined by points A, B, P_2', C, shades the glazing.

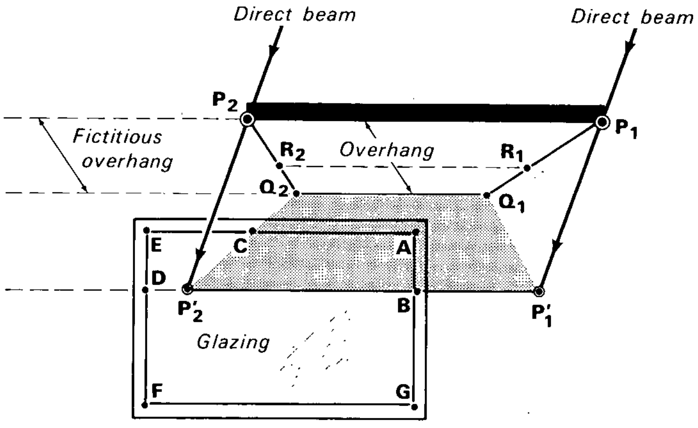

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FIGURE 1. Projection of direct beam rays from overhang to cast shadow onto glazing

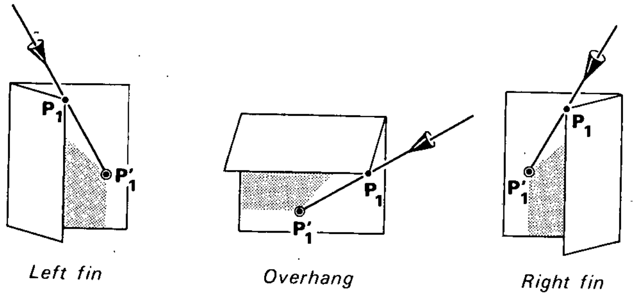
In SHADE, this shadow area is determined by two fictitious overhangs and shadows. The first such overhang is located with its right edge at P_1Q_1 and its left edge extending to infinity. By inspection, it is apparent that the shadow cast by this overhang extends toward the left from point P_1' to infinity, such that its area of overlap with the glazing is given by area ABDE. The second fictitious overhang is located with its right edge at P_2Q_2 and its left edge also extending to infinity; similarly it casts a shadow extending toward the left from point P_2' to infinity, such that its area of overlap with the glazing is given by area $CP_2'DE$. Clearly the actual overlap area $ABP_2'C$ is obtained by simply subtracting area $CP_2'DE$ from area ABDE.

In a similar manner, we can determine the amount of shading which occurs when the overhang is displaced away from the building. In this case, the problem reduces to treating the overhang as if it were two "real" overhangs, both connected along Q_1Q_2 as in the above example: the first overhang has an outside edge defined by P_1P_2, the second by outside edge R_1R_2. Clearly, the resultant shaded area is obtained by subtracting the shaded area created by overhang $Q_1Q_2R_1R_2$ from that created by $Q_1Q_2P_1P_2$. By extension, an overhang consisting of a series of thin, parallel slats can be analyzed as the superposition of shadows cast by each slat, treated as an individual overhang. Note that in all cases, the same method is applied: projecting a single overhang corner point, as if the overhang were infinite in the opposite direction and then computing and summing resultant positive and negative "running" areas.

Extension to Combinations of Overhangs and Fins

SHADE treats vertical fins as if they were rotated overhangs, thus allowing the same computational principles to be applied to both cases. For example, as shown in Fig. 2 below, a left fin with rays incident from the left can be considered as an overhang with rays incident from the right (with the rays and overhang rotated counterclockwise by 90°). Similarly, a right fin with rays incident from the right can be considered as an overhang with rays incident from the left (with the rays and overhang rotated clockwise by 90°).

Hence, the above method can be applied to any combination of overhangs and fins, provided that certain precautions be taken so that shadows cast by overlapping fins and overhangs are not counted twice when the running areas from fin(s) and overhang are summed. These precautions are discussed in greater detail below.



Left fin
Overhang
Right fin

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FIGURE 2. Equivalent projections of direct beam rays from fin and overhang corner points

Description of Equations

The equations to project the corner point P_1 to the shadow point P_1' are determined as follows. First we define an xyz "body" coordinate system located at the interior point Q_1 of the overhang in the plane of the glazing as shown in Fig. 3.

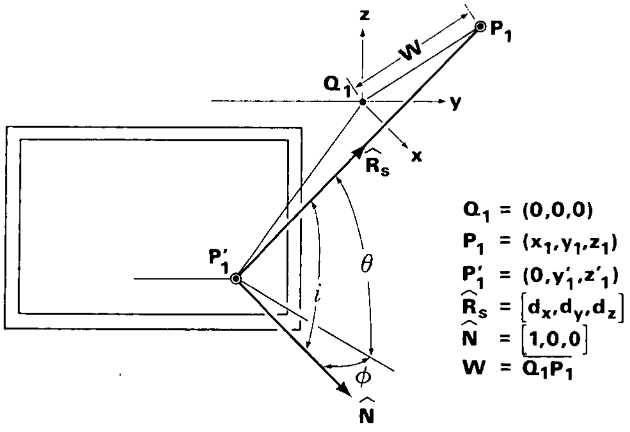

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FIGURE 3. Geometric relations for projecting direct beam ray from cornerpoint to glazing plane.

The yz plane contains the glazed area, as well as the shadow line Q_1P_1' created by the projection P_1P_1'. Unit vector $-\hat{R}_S$ defines the direction of the incoming sun ray from P_1 and is given by the negative value of the position vector for the sun $\hat{R}_S$ whose x,y,z components are d_x, d_y, d_z respectively.

To determine values of $P_1' = (0, y_1', z_1')$ from given values of $P_1 = (x_1, y_1, z_1)$ and of $\hat{R}_S = [d_x, d_y, d_z]$, we use the equation of the line between points P_1 and P_1':

$$\frac{0 - x_1}{-d_x} = \frac{y_1' - y_1}{-d_y} = \frac{z_1' - z_1}{-d_z} = S_1$$

This leads to the computationally simple result:

$$S_1 = x_1/d_x$$

$$y_1' = y_1 - S_1 d_y$$

$$z_1' = z_1 - S_1 d_z \qquad (1)$$

For perpendicular, rectangular shades of width W, Eqs.(1) reduce to $S_1 = W/d_x$, $y_1' = -S_1 d_y$, and $z_1' = -S_1 d_z$. In the present version of SHADE, only vertical surfaces are of interest, so that

$$d_x = \cos\theta \cos\phi$$

$$d_y = \cos\theta \sin\phi$$

$$d_z = \sin\theta$$

Here θ and ϕ are respectively the solar altitude angle and the solar-surface azimuth angle (contained in the xy plane) subject to conditions that $0 < \theta < 90^{\circ}$ and $-90^{\circ} < \phi < 90^{\circ}$; we adopt the convention that $\phi = 0$ when the sun is in the x-z plane and is positive when rotated into the positive xy quadrant. If the surface normal vector $\hat{N}$ faces south, ϕ reduces to the solar azimuth angle.

Running Area Calculation for Semi-Infinite Overhang or Fin

For notational convenience in discussion later, Fig. 4 provides a drawing of additional coordinates which are needed to calculate the area A of a shadow cast across the glazing by a semi-infinite overhang or fin as described above. We assume below that $(0, y_1', z_1')$ has been calculated from Eqs. (1) and (2) above. Coordinates y_{ur}, y_{ll}, z_{ur}, z_{ll} determine the location of vertices which define the glazed area in relation to the shade and coordinate system as shown. Coordinates y_u, z_u, y_l, z_l determine the vertical and/or horizontal intersection of the shadow line Q_1P_1, with lines defining the glazing edges.

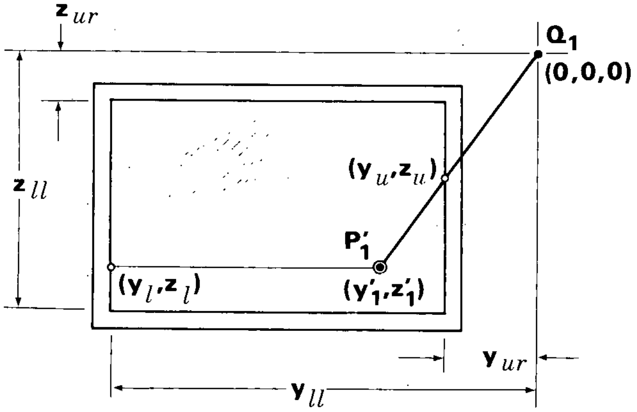

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FIGURE 4. Geometric relations in plane of glazing for calculating fictitious overhang created by shaded area.

Since the shadow area ΔA can assume various shapes (triangular, square, rectangular, pentagonal) the following generalized expression is used to calculate this area:

$$\Delta A = (y_{ur} - y_{ll})(z_{ur} - z_u) + (z_u - z_l)[0.5(y_u + y_l) - y_{ll}] \quad (3)$$

The logic and arithmetic expressions shown in Fig. 5, which are convenient for computing, describe the manner in which this equation, and its special cases, are solved in the current software version of SHADE. Note that y_u, z_u, y_l, and z_l assume different values depending upon the location of y_1' and z_1'. For example, in the case depicted in Fig. 4, the equations in Fig. 5 would determine that point P_1' lies within the opening and hence that y_u, z_u, y_l, and z_l should be located as shown. Due to the recursive nature of this area calculation, it is convenient to place the equations in Fig. 5 into a subroutine.

```
     Assign $\Delta y = y_{ur} - y_{ll}$
       ⋮
     Input present $y_1'$, $z_1'$
     IF($z_1' \geq z_{ur}$) GO TO 8
     IF($y_1' = 0$) GO TO 1
     IF($y_1' < y_{ur}$) GO TO 2
1    IF($z_1' \leq z_{ll}$) GO TO 5
     $z_u = z_1'$
     GO TO 6
2    $m = z_1'/y_1'$
     $y_u = y_{ur}$
     $z_u = m y_u$
     IF($z_u \leq z_{ll}$) GO TO 5
     $y_l = y_{ll}$
     $z_l = m y_{ll}$
     IF($z_l \geq z_{ur}$) GO TO 8
     IF($z_{ur} \geq z_u$) GO TO 3

     [continued next column]

     $z_u = z_{ur}$
     $y_u = z_u/m$
3    IF($z_l \geq z_{ll}$) GO TO 4
     $z_l = z_{ll}$
     $y_l = z_l/m$
4    IF($z_1' \leq z_l$) GO TO 7
     $z_l = z_1'$
     $y_l = y_1'$
     GO TO 7
5    $z_u = z_{ll}$
6    $z_l = z_u$
7    $\Delta A = \Delta y(z_{ur} - z_u)$
       $+ (z_u - z_l) \times [0.5$
       $\times (y_u + y_l) - y_{ll}]$
     RETURN
8    $\Delta A = 0$
     RETURN
```

FIGURE 5. Logic and Arithmetic Expressions to Compute Glazing Shaded Area

Geometric Parameters For Glazing-Overhang-Fin Combinations

The logic and equations in Fig. 5 are capable of calculating the running areas of discrete fins and overhangs, but they ignore certain geometric relationships which exist when fins and overhangs are combined to form a single shading device. In particular, the relative positions of fin and overhang terminations must be taken into account, as shown in Fig. 6, since fins and overhangs can overlap in such a way that overlapping shadows are cast. Under these conditions, the potential exists for counting these shadows twice when the running areas for overhang + fins are summed.

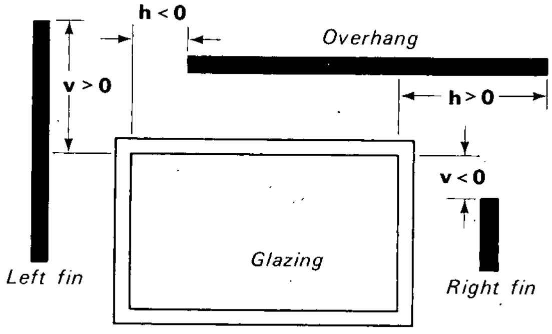

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FIGURE 6. Geometric relations to define relative position of fins, overhang, and glazing in arbitrary combination.

In SHADE, this problem is solved as follows. First, for a given fin-overhang termination, the magnitude and sign of directed distances h and v are determined. These parameters provide useful software gauges as to the type of termination which the designer has chosen. For example, non-overlapping fins and overhangs can occur only if $h \leq 0$ and $v \leq 0$. In the case shown in Fig. 6, the right fin is overlapped by the overhang ($h > 0$, $v < 0$) and the overhang itself is overlapped by the left fin ($h < 0$, $v > 0$).

Second, SHADE tests for the signs (-,0,+) of h and v, as well as the slope and termination of the incoming ray in the plane of the glazing at (y_1', z_1'). If these tests indicate that the resultant shadows overlap each other, the magnitude of h and/or v is corrected to remove this overlap for a particular set of solar angles θ and ϕ.

Calculation of Total Shaded Area

In present applications of SHADE to the HP-41C, only the geometric arrangements depicted in Fig. 6 are permitted, i.e., a square or rectangular opening shaded by a combination of a square or rectangular right fin, left fin, or overhang. Thus the total area A_s of shadow cast across the glazing is obtained by summing the shaded area from each component:

$$A_s = A_l + A_o + A_r, \qquad (4)$$

where A_l, A_o, and A_r represent the individual shaded areas ΔA of the left fin, overhang, and right fin respectively [obtained by Eq. (3)]. However, more complex combinations, such as a deep overhang which shades three windows with shallow side fins, can be analyzed by applying SHADE to each window separately.

Additional Design Information Calculated by SHADE

As discussed in the last section, it is assumed that in preliminary analysis and design of shading devices, a simple accounting of the direct beam influence at the shaded aperture would be useful to solar designers. Hence, as a minimum requirement, it is necessary to obtain hourly, daily, or monthly profiles of the cosine-corrected direct beam solar power incident upon a given glazed surface, with and without a particular shading device. In addition, for a particular set of solar angles θ and ϕ, it is instructive to know the cosine correction factor, angle of incidence between sun and glazed surface, the percent of total glazed surface which is shaded, and the shaded area itself. These parameters are determined as follows.

First, we note that the percentage p of the total glazed area A_g shaded by the overhang-fin combination is given by:

$$p = 100A_s/A_g$$

where A_s is the total shaded area as determined by Eq. (3).

Next, we calculate the angle of incidence and cosine factor for the sunlight and glazing surface. Returning to Fig. 3 above, we find at point P_1' a unit vector $\hat{N}$ which denotes the glazing surface normal at this point. The angle of incidence i between the sun and the glazing at this point is given by:

$$i = \cos^{-1}(\hat{N} \cdot \hat{R}_s),$$

where $\hat{N} \cdot \hat{R}_s$ denotes a vector dot product; cos(i) is the cosine factor which converts the direct normal insolation I_{DN} to direct insolation I_g:

$$I_g = I_{DN}\cos(i).$$

Then the total direct beam solar power $\dot{E}_g$ incident upon the opening is given by:

$$E_{go} = (1 - p/100)I_gA_g. \qquad (5)$$

For comparison during design, the total solar power incident upon the glazing with shading devices removed $\dot{E}_{go}$ is given by:

$$\dot{E}_{go} = \dot{E}_g(p=0) = I_gA_g. \qquad (6)$$

In the case of vertical glazings, $\hat{N}$ is defined by $\hat{N} = [1,0,0]$ so that:

$$\begin{aligned} \cos(i) &= \hat{N} \cdot \hat{R}_s \\ &= [1,0,0] \cdot [d_x,d_y,d_z] = d_x. \qquad (7) \end{aligned}$$

The value of I_{DN} for a given solar hour can be read in from local data, or calculated from analytic expressions such as in the procedure outlined by ASHRAE (1).

CALCULATOR APPLICATIONS

To date, three programmable calculator applications have been developed, using a Hewlett-Packard HP-41C, and (optional) card reader, printer, and three memory modules. A fourth application is under development. Table 1 below summarizes the various characteristics of these versions. In addition, sample outputs from the HP-41C printer for the second and third versions are listed in Appendix I for the case of a window shaded by an overhang centered above a window and joined on either side by identical right and left fins.

The first version consisted of rudimentary prompts; no printer or card reader was required; and it was capable of analyzing only a centered overhang whose width (parallel to the upper edge of the glazing) was always greater or equal to the (horizontal) width of the glazing. Output, returned in 2-3 seconds, consisted of the percentage of the total area of the opening which was shaded, the shaded area itself, the cosine of the angle of incidence between sun and surface, the direct normal intensity, and the direct beam solar power incident upon the surface. The recursive equations basic to SHADE (see Fig. 5) were written as a subroutine, requiring 105 steps in Reverse Polish Notation, and subsequently employed in the other two versions. One memory module was required for the entire program.

The second version of software began with the premise that to be a useful design tool, calculators with alphanumeric display capabilities ought to provide visually attractive, easy-to-understand prompts. Also, all printed output should be so formatted that after some period of time, the user could refer to the printer tape without encountering difficulty in interpreting or searching for particular results.

Although this objective appeared to be met, approximately one third of the resultant 555 step program was devoted to input and output. Sufficient storage was available to calculate the total shaded/unshaded areas of glazing for an arbitrary fin/overhang combination and a given set of solar angles; however, there was insufficient storage to compute the incident solar power functions (Eqs. 4 - 5), despite the use of all three memory modules. In addition, there was insufficient space to calculate shading effects created by overlapping fins and/or overhangs (see Fig. 6).

The third version, containing approximately 580 steps, met all computational objectives, but at the sacrifice of self-contained prompts. At the present it performs much like other HP programs, requiring a separate sheet for certain input and output

HP-41C Version #	Input/Output Characteristics	Run Times	Program Length/Storage Requirements
1	• Limited to centered overhang of finite length. • No printer used • Primitive use of alphanumeric prompts and data displays. • Output includes shaded area, cosine factor, direct solar radiation at outside glazing surface.	• Output returned in 2-3 seconds after last building datum entered	• Less than one memory module required • 105 steps in RPN used to code equations and logic in Fig. 5
2	• Overhang/fin combinations of arbitrary dimensions. • Printer required • Full use of graphically descriptive prompts for input and outputs • Output, listed in Fig. 7 below, is essentially the same as Version 1.	• Output returned in 9-13 seconds after last building datum entered (using 2 fins plus overhang)	• Three memory modules required • Total program length = 555 steps in RPN
3	• Overhang/fin combinations of arbitrary dimensions. • Printer optional. • Coded prompts for inputs and outputs. • Output, listed in Fig. 8 below, is essentially same as Version 1, except design month can be pre-selected.	• Output returned in 9-13 seconds after last building datum entered (using 2 fins plus overhang)	• Less than 3 memory modules required. • Total program length = 580 steps in RPN

TABLE 1. Summary of three HP-41C software versions of SHADE under development.

prompts. During one case study, consisting of an overhang plus two fins (see Appendix I), it was found that all output was returned after 9-13 seconds, depending upon the choice of solar angles. The third version also included a prompt for input as to the month of year to be studied. The program then selected the appropriate monthly values of H and K as required in the ASHRAE formula (4) for computing direct normal insolation on a clear day:

$$I_{DN} = H/\exp\left[K/\sin(\theta)\right]$$

These values were read into the calculator from a magnetic data card at the beginning of the program. Thus the user could quickly assess the relative seasonal merits, by simply changing the month and entering appropriate sun angles from existing tables or suncharts for various periods of the day (1).

The fourth version is being developed with the following performance objectives in mind: (a) more efficient programming to further reduce storage requirements and calculation time; (b) extension of solar angle calculations so that hourly values of θ, ϕ can be computed from given values of earth latitude, declination and solar time, provided that sufficient memory still remains; and (c) extension of incident solar power to include solar transmission through typical single and double glazings, provided that sufficient memory still remains.

CONCLUSIONS

The present applications of SHADE to the HP-41C calculator system appear to provide useful, exact information to solar designers. However, applications to microcomputers may ultimately prove to be more useful; for at no sacrifice in calculation speed, the larger amount of memory storage would permit integration of interactive, self-contained, narrative prompts for input with a larger set of calculations, such as solar transmission to interior spaces. Clearly from the professional engineer's point of view, the present versions of SHADE needs to go further so that the specific effects of shading devices upon building thermal and electrical lighting loads can be calculated. Additional work is needed to meet these needs, using either a programmable calculator or microcomputer system. The present effort may be one step in this direction.

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NOMENCLATURE

ΔA	shadow area cast by semi-infinite overhang to glazing, ft^2 (m^2)
A_g	total area of glazing ft^2 (m^2)
A_s, A_l, A_o, A_r	total and individual (left fin, overhang, right fin) areas of shadow cast across glazing respectively, ft^2 (m^2)
A,B,D,E	vertices of shadow cast on glazing by semi-infinite overhang
A,B,C,P_2'	vertices of shadow cast on glazing by actual overhang
A,E,F,G	vertices of glazed area

d_x, d_y, d_z	direction cosines for solar position vector $\hat{R}_s$ in x,y,z directions respectively
$\dot{E}_g$	direct beam solar power incident upon glazing, Btu/hr (W)
$\dot{E}_{go}$	direct beam solar power incident upon glazing with shading devices removed, Btu/hr (W)
h	directed distance defining relative position of overhang end-points to glazing vertices, ft (m)
H,K	constants used to calculate I_{DN}
i	angle of incidence between direct beam solar radiation and glazing surface normal, degrees (rad)
I_{DN}	direct normal insolation, Btu/ft^{2}hr (W/m^2)
I_g	direct insolation at exterior glazing surface, Btu/ft^{2}hr (W/m^2)
$\hat{N}$	normal vector at glazed surface
p	percentage of total glazed area shaded by an overhang-fin combination
P_1, P_2	exterior vertices of overhang defining the projected edge of the overhang parallel to its wall connection
P_1', P_2'	Projection points of P_1, P_2 respectively by sun rays into plane of glazing
Q_1, Q_2	interior vertices of overhang where plane of glazing intersects plane of overhang
R_1, R_2	interior vertices of overhang as if it were offset from glazing plane and wall.
$\hat{R}_s$	position vector for sun
S_1	directed distance along shadow line between P_1 and P_1', ft (m)
v	directed distance defining relative position of right and left end-points of glazing as viewed from the outside, ft (m)
W	depth of overhang as measured between P_1 and Q_1, ft (m)
x,y,z	coordinates used to define location of glazing, overhang, fins, and shadows, ft (m)
x_1, y_1, z_1	coordinates of P_1, ft (m)
y_1', z_1'	coordinates of P_1' in yz plane, ft (m)
y_l, z_l, y_u, z_u	intersection points between shadow lines and glazing edges in yz plane, ft (m)
y_{ll}, z_{ll}	lower left glazing vertex point in yz plane as viewed from outside, ft (m)
y_{ur}, z_{ur}	upper right glazing vertex point in yz plane as viewed from outside, ft (m)
θ	solar-building azimuth angle
ϕ	solar altitude angle

Subscripts

1,2	right and left overhang edges and shadows respectively
DN	direct normal
g,go	glazing with and without shading devices respectively
l,o,r	left fin, overhang, right fin respectively
ll,ur	lower left, upper right respectively
l,u	left or lower, right or upper glazing edges respectively

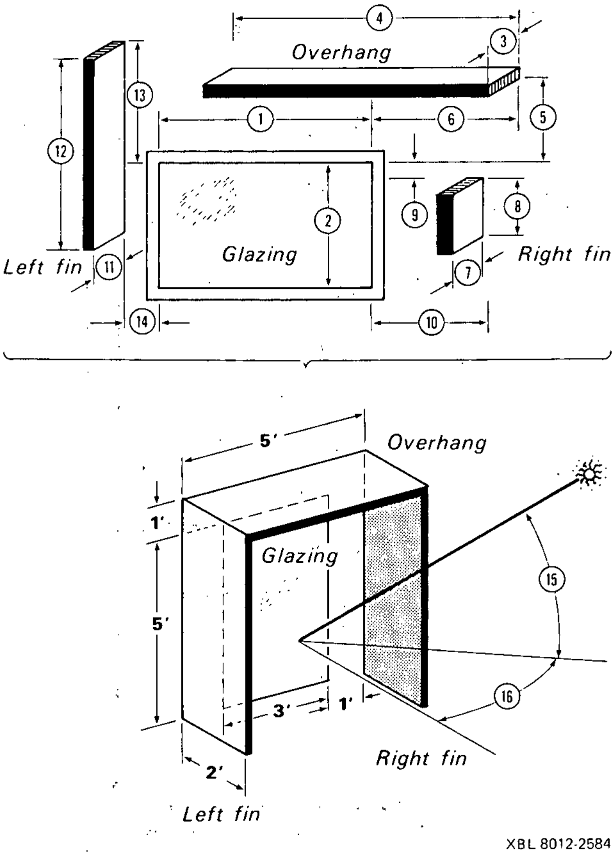
APPENDIX I

Sample Printer Outputs for HP-41C Versions of SHADE

In Figs. 7 and 8 below are printer tape output samples for the second and third HP-41C versions of SHADE, plus schematic drawings which illustrate the relationship between the nomenclature for data inputs appearing on the tape (left side of figures) and a particular case (right side of figures). This case consists of a 3 ft. by 5 ft. (0.914 m x 1.524 m) opening with a 2 ft. by 5 ft. (0.61 m x 1.524 m) centered overhang, joined on either side by 2 ft. x 6 ft. (0.61 m x 1.829 m) vertical fins as shown in the lower right drawings. The circled numbers, 1-16 in Fig. 7 and 1-17 in Fig. 8, indicate the geometric location and significance of the various data inputs, drawn over the tape with arrows along the left margins. Unnumbered arrows on the tape signify where decisions and program loops occur so as to provide design decision flexibility.

Note that the two versions have identical input requirements regarding dimensions and locations of the opening, overhang, and fins (Nos. 1-14). However, in the third version, the prompt to input the design month (No. 15) is inserted before solar angle inputs. In addition, another loop is created so that the design month can be changed conveniently. Finally, the two versions differ in outputs, as discussed in the last section:

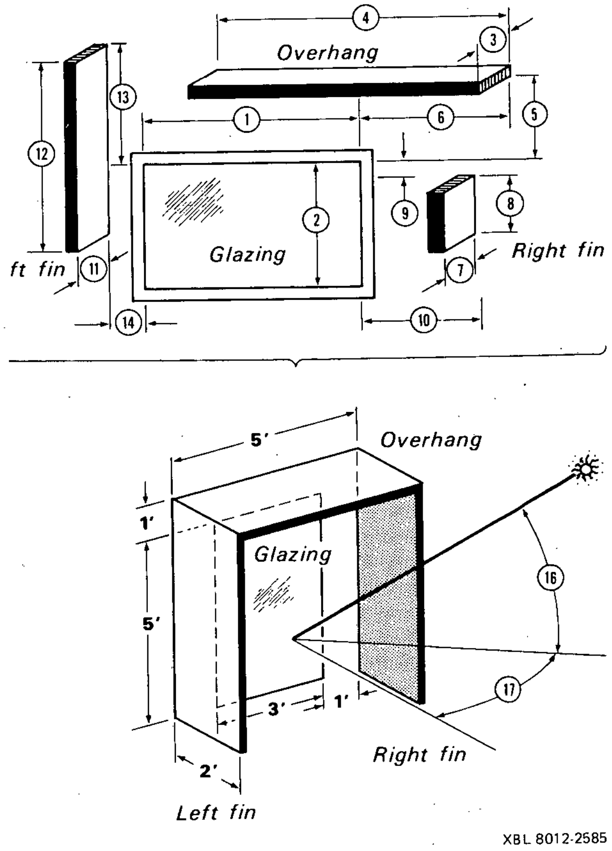
whereas the second version is limited to the cosine factor, percent of shaded area, and the shaded area itself in sq. ft. (m^2), the third version outputs IDN, the direct normal insolation in Btuh/sq.ft. (W/m^2); COS, the cosine factor (Eq. 7); E.GO, the direct beam solar power incident upon unshaded glazing (Eq. 6) in Btu/hr (W); E.G, the direct beam solar power incident upon shaded glazing (Eq. 5) in Btu/hr (W); PC, percent of shaded area; and A.S, the shaded area itself in sq. ft. (m^2).



XBL 8012-2584

FIGURE 7. Version 2.1 of SHADE with HP-41C

```
INPUTS
        GLAZING
 1 ->   width of glazing?
                   3.0000    ***
 2 ->   height of glazing?
                   5.0000    ***

        SHADING
   ->   is there an overhang?
        yes
   ->   right fin?
        yes
   ->   left fin?
        yes

 3 ->   overhang depth?
                   2.0000    ***
 4 ->   overhang lngth?
                   5.0000    ***
 5 ->   dist. above glazing?
                   1.0000    ***
 6 ->   dist. to right?
                   1.0000    ***

 7 ->   rt. fin depth?
                   2.0000    ***
 8 ->   rt. fin height?
                   6.0000    ***
 9 ->   dist. above glazing?
                   1.0000    ***
10 ->   dist. to right?
                   1.0000    ***

11 ->   lft. fin depth?
                   2.0000    ***
12 ->   lft. fin height?
                   6.0000    ***
13 ->   dist. above glazing?
                   1.0000    ***
14 ->   dist. to left?
                   1.0000    ***

        ANGLES
15 ->   sun alt. angle?
                  45.0000    ***
16 ->   bldg. asm. angle?
                  45.0000    ***

        RESULTS
        cosine factor=0.5000
        percnt=57.7%
        shaded area=8.66 sq. ft.   YES

        CHG. ANGLES?
        NO
        CHG. SHADES?
        NO
        START OVER?
        NO
        ----END----
```



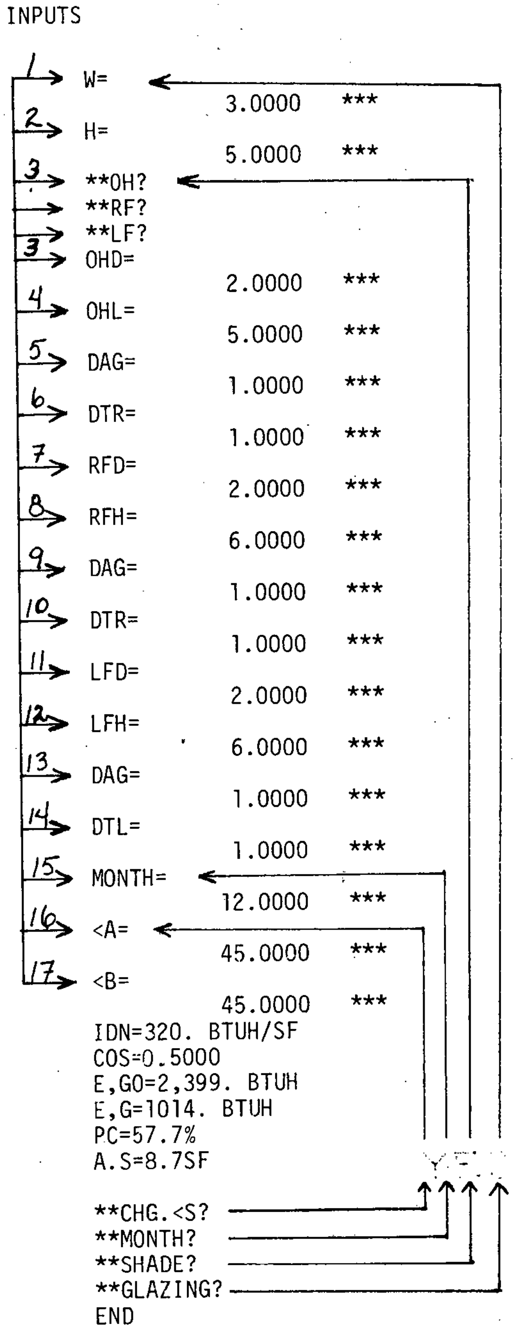

```
INPUTS
1 → W=
              3.0000    ***
2 → H=
              5.0000    ***
3 → **OH?
  → **RF?
  → **LF?
3 → OHD=
              2.0000    ***
4 → OHL=
              5.0000    ***
5 → DAG=
              1.0000    ***
6 → DTR=
              1.0000    ***
7 → RFD=
              2.0000    ***
8 → RFH=
              6.0000    ***
9 → DAG=
              1.0000    ***
10 → DTR=
              1.0000    ***
11 → LFD=
              2.0000    ***
12 → LFH=
              6.0000    ***
13 → DAG=
              1.0000    ***
14 → DTL=
              1.0000    ***
15 → MONTH=
             12.0000    ***
16 → <A=
             45.0000    ***
17 → <B=
             45.0000    ***
     IDN=320. BTUH/SF
     COS=0.5000
     E,GO=2,399. BTUH
     E,G=1014. BTUH
     PC=57.7%
     A.S=8.7SF

     **CHG.<S?
     **MONTH?
     **SHADE?
     **GLAZING?
     END
```

FIGURE 8. Version 3.1 of SHADE with HP-41C

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