Title
Elliptic Flow at Large Viscosity

Journal
Nuclear Physics A, 830(1-4)

ISSN
0375-9474

Author
Koch, Volker

Publication Date
2009-11-01

DOI
10.1016/j.nuclphysa.2009.10.042

Peer reviewed

Elliptic Flow at Large Viscosity

Volker Koch

Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA

Abstract

In this contribution we present an alternative scenario for the large elliptic flow observed in relativistic heavy ion collisions. Motivated by recent results from Lattice QCD on flavor off-diagonal susceptibilities we argue that the matter right above T_c can be described by single-particle dynamics in a repulsive single-particle potential, which in turn gives rise to elliptic flow. These ideas can be tested experimentally by measuring elliptic flow of heavy quarks, preferably via the measurement of J/Ψ elliptic flow.

1. Introduction

One of the big surprises of the RHIC experimental program has been the large elliptic flow [1, 2], which, contrary to SPS energies, agreed more or less with predictions from ideal hydrodynamics [3, 4]. This surprising agreement let to the conjecture that the matter at RHIC is a strongly coupled, nearly perfect fluid, with very small shear viscosity. Indeed using the AdS-CFT correspondence, is was shown that a large class of strongly coupled theories seem to have a universal minimal shear viscosity of $\eta/s = 1/4\pi$ [5]. Meanwhile, more refined calculations based on relativistic viscous hydrodynamic [6, 7] seem to indicate that a finite but small value for the shear viscosity is required in order to reproduce the p_t dependence of the measured transverse flow. On the theoretical side very little is known about the shear viscosity of high temperature QCD. Perturbative calculations lead to a considerably larger value than the conjectured lower bound. Extracting a value for the shear viscosity from Lattice QCD (LQCD), on the other hand, requires analytic continuation to real time, thus leading to substantial uncertainties [8]. It was also found that the elliptic flow of the observed hadrons scales with the number of quarks [2] as predicted by a quark coalescence picture of hadronization [9]. This in turn implies that the scaled v_2 may be interpreted as that of the quarks prior to hadronization, and we will use this interpretation in the following where we always refer to the elliptic anisotropy of quarks.

In this contribution we want to entertain an entirely different view and interpretation of the observed elliptic flow. First, we note that Lattice QCD results [10] suggest a quasi-particle picture, at least for the quarks. Both flavor-off-diagonal susceptibilities [11] and higher order baryon number susceptibilities [12] are consistent with vanishing correlations for temperatures right above the transition, $T \gtrsim 1.2T_c$. Estimating the strength of correlations in the gluon sector is not so straightforward, due to the lack of any additional quantum numbers, such as flavor, which one can use to study correlations. Therefore, let us conjecture, that gluons behave like quasi-particles as well. In this case we have a single-particle description of the QGP right above T_c.

Next we need to address the equation of state above T_c, where LQCD finds the pressure to be considerably ($\approx$ 15%) below that of a free gas of massless quarks and gluons. Since LQCD calculations are carried out in the grand-canonical ensemble, i.e. at fixed chemical potential rather than particle number, a reduction of the pressure in a single-particle picture implies a *repulsive*, density dependent single-particle potential, i.e,

$$p \sim \int d^3p \, \exp\left[-\frac{E_0 + U}{T}\right] < \int d^3p \, \exp\left[-\frac{E_0}{T}\right] \sim p_0 \tag{1}$$

for $U > 0$. Here p_0 denotes the pressure of a free, non-interacting gas of partons.

The presence of a *repulsive* single-particle potential has interesting consequences, especially for the elliptic anisotropy, v_2. Given the almond shaped initial distribution of matter in the transverse plane in a semi-central heavy-ion collision, the (negative) gradient of the potential, and thus the force, is larger in the in-plane than in the out-of-plane direction. As a consequence the momentum kick due to the repulsive single-particle potential is larger in plane than out of plane, resulting in a deformation of the momentum distribution in qualitative agreement with the observed elliptic anisotropy, $v_2(p_t)$ (see also [13]). We note that this effect does *not* require a short mean free path or, equivalently, a small viscosity.

Besides the positive v_2 the single-particle dynamics leads to two additional, qualitative predictions. First, the elliptic anisotropy should vanish for large transverse momenta, since the additional momentum kick due to the potential becomes negligible. Thus, contrary to ideal hydrodynamics we predict a maximum of the transverse momentum dependent elliptic anisotropy, $v_2(p_t)$, which is observed in experiment. At large p_t, of course, v_2 will be dominated by the attenuation of fast partons in the matter, and it needs to be determined at what momentum this transition will take place [14]. Second, since for a given temperature the momentum distribution for heavy quarks is titled towards higher momenta, $T \sim \frac{p^2}{m}$, the resulting elliptic anisotropy for heavy quarks should be considerably smaller than that for light quarks.

2. Schematic Model

In order to study the qualitative features of the proposed single-particle dynamics in a transparent fashion let us start with a simple schematic model. If we ignore collisions among the partons, the dynamics of the phasespace distribution follows as Vlasov equation. To allow for an analytical treatment of the expansion, we assume a Gaussian distribution in configuration space and non-relativistic kinematics, resulting in a Gaussian momentum space distribution,

$$f(x, y, v_x, v_y, t = 0) = \frac{N}{2\pi^2 \sigma_x \sigma_y (T/m)} \exp\left(\frac{-x^2}{\sigma_x^2}\right) \exp\left(-\frac{m v_x^2}{2T}\right) \exp\left(\frac{-y^2}{\sigma_y^2}\right) \exp\left(-\frac{m v_y^2}{2T}\right). \tag{2}$$

Here, v_x, v_y are the velocities and σ_x, σ_y denote the widths of the distribution in the transverse x, y direction, respectively. Assuming, for simplicity, that the single particle potential U is proportional to the density of the light degrees of freedom,

$$U(\vec{x}, t) = g \, \rho(\vec{x}, t) = g \int d\vec{v} \, f(\vec{x}, \vec{v}, t),$$

the Vlasov equation leads to the following expression for the velocity space distribution, $n(\vec{v})$,

$$\frac{\partial}{\partial t} n(\vec{v}) = \frac{g}{m} \int d^2x \, \vec{\nabla}_v f(\vec{x}, \vec{v}, t) \vec{\nabla}_x \rho(\vec{x}, t) \tag{3}$$

which can be solved analytically under the assumption that the time dependence of the density follows free streaming, i.e. we ignore the effect of the potential on the density distribution. A fully consistent solution will require a numerical treatment, which will be briefly discussed below. Since the heavy quarks are rare, we ignore their contribution to the potential, and propagate them in the potential generated by the light degrees of freedom.

To leading order in the initial spatial eccentricity $\epsilon \equiv \frac{\sigma_y^2-\sigma_x^2}{\sigma_x^2+\sigma_y^2}$ we then obtain the following result for the elliptic anisotropy of the light quarks

$$v_{2,light}\,(u) = \frac{1}{2}\epsilon\frac{U_0}{T}\frac{1}{u^2}\left[1-\exp\left(-u^2\right)\left(u^2+1\right)\right] \tag{4}$$

which only depends on the kinetic energy $u = \frac{mv^2}{2T}$. Here, $U_0 = U(\vec{r} = 0, t = 0)$ is the initial strength of the potential at the center. We note that v_2 is (a) proportional to the initial spatial eccentricity, (b) proportional to the initial transverse density (via U_0) and (c) a function of the kinetic energy only. This is precisely what is seen in experiment.

At a given velocity the heavy quark elliptic anisotropy, $v_{2,heavy}$, is related to that of the light quarks by $v_{2,heavy}(v) = \frac{1+\gamma}{2}v_{2.light}(v)$. Both, $v_{2,light}$ and $v_{2,heavy}$ are plotted as a function of the kinetic energy in the left panel of Fig.1 for an initial temperature of $T = 300$ MeV. As expected v_2 exhibits a maximum, the position of which depends on the choice of initial temperature and is shifted to larger values of the kinetic energy for heavy quarks. As a result the momentum averaged, or integrated anisotropy, $\overline{v_2}$, for the heavy quarks is much smaller. This is shown in left panel of Fig.1, where we plot the ratio of heavy over light quark v_2 as a function of the ratio of the quark masses, $\gamma = m_{light}/M_{heavy}$. Thus, a unique prediction of the single-particle dynamics is that the integrate v_2 for heavy quarks, specifically the J/Ψ should be considerably smaller than that for the pions, contrary to hydrodynamics where they should be about equal.

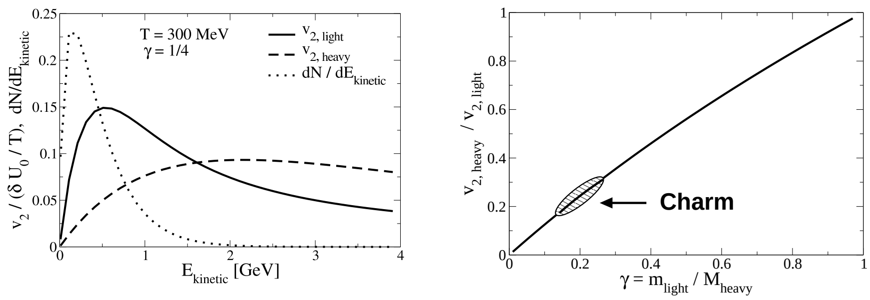

Figure 1: Left panel: Schematic model results for the elliptic anisotropy as a function of the kinetic energy, $v_2(E_{kinetic})$ for light quarks (full line) and heavy quarks (dashed line). The dotted line represents the kinetic energy spectrum for both heavy and light quarks. Right panel: Ratio of integrated v_2 for heavy over light quarks as a function of the ratio of heavy and light quark mass.

3. More realistic (transport) model

A realistic treatment of the proposed single-particle dynamics requires relativistic kinematics as well as a fully consistent treatment of the Vlasov (Boltzmann) equation. In order to ensure energy momentum conservation it is best to start from an effective energy functional which is

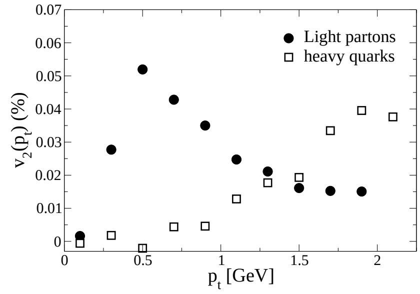

Figure 2: Result for elliptic anisotropy from transport calculation without parton scattering.

tuned to reproduce the Lattice equation of state and from which one derives the single-particle Vlasov equation. The results from such an exercise (for details see [15]) is shown in Fig. 2. Again we see the same features as in the schematic model, and we find that the intergrated v_2 for heavy quarks is considerably smaller than that of light quarks.

4. Conclusions

In this contribution we have shown that the qualitative features of the observed elliptic anisotropy can be obtained from single-particle dynamics motivated by recent Lattice QCD results. In this description there is no need for a very short mean free path or, equivalently, small viscosities. This model can be tested in experiment by measuring the elliptic anisotropy of heavy quarkonium, which is predicted to be considerably smaller than that of pions, in contradistinction to the predictions from hydrodynamics.

Acknowledgments

This work is supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Divisions of Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

References

[1] K. H. Ackermann, et al., *Phys. Rev. Lett.* **86** (2001) 402.
[2] S. A. Voloshin, A. M. Poskanzer, R. Snellings, [arXiv:0809.2949]
[3] D. Teaney, J. Lauret, E. V. Shuryak, *Phys. Rev. Lett.* **86** (2001) 4783.
[4] P. F. Kolb, U. W. Heinz, Quark gluon plasma 3, 634, Hwa, R.C. (ed.), World Scientific, 2004.
[5] D. T. Son, M. A. Stephanov, *Phys. Rev.* **D70** (2004) 056001.
[6] M. Luzum and P. Romatschke, *Phys. Rev.* **C78** (2008) 034915.
[7] H. Song and U. W. Heinz, *Phys. Rev.* **C77** (2008) 064901.
[8] H.B. Meyer, these proceedings.
[9] S. A. Voloshin, *Nucl. Phys.* **A715** (2003) 379.
[10] M. Cheng, et al., *Phys. Rev.* **D79** (2009) 074505.
[11] V. Koch, A. Majumder, J. Randrup, *Phys. Rev. Lett.* **95** (2005) 182301.
[12] S. Ejiri, F. Karsch, K. Redlich, *Phys. Lett.* **B633** (2006) 275–282.
[13] W. Cassing et al, [arXiv:0808.0202]
[14] J. Liao, V. Koch, *Phys. Rev. Lett.* **103** (2009) 042302.
[15] V. Koch, in preparation.