Title

Refined Finite Element Analysis of Elastic Plastic Thin Shells of Revolution

Authors

Sharifi, Parviz
Popov, Egor

Publication Date

1969-09-01

SESM 69-28

STRUCTURES AND MATERIALS RESEARCH
DEPARTMENT OF CIVIL ENGINEERING

REFINED FINITE ELEMENT ANALYSIS OF ELASTIC-PLASTIC THIN SHELLS OF REVOLUTION

P. SHARIFI
E. P. POPOV

Report to
Army Research Office, Durham
Contract No. DAHCO 4 69 C 0037

DECEMBER 1969

STRUCTURAL ENGINEERING LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY CALIFORNIA

REFINED FINITE ELEMENT ANALYSIS OF ELASTIC-PLASTIC THIN SHELLS OF REVOLUTION

by

E. P. Popov
P. Sharifi

Report to the
Army Research Office - Durham
Contract No. DAHC04 69 C 0037

December 1969

ABSTRACT

A refined axisymmetric curved finite element for the analysis of thin elastic plastic shells of revolution is described in the report. The improved element is obtained by employing cubic polynomials for the assumed in-plane and out-of-plane displacements in terms of local Cartesian coordinates. This introduces into the solution two internal degrees of freedom in the cord direction of each element. These internal degrees of freedom are removed by static condensation before assembling the individual element stiffness matrices, and are subsequently recovered after the nodal displacements are obtained. On comparison with the previous formulation, this procedure greatly improves the accuracy of the solution especially with regards to in-plane stress-resultants at discontinuities in the meridional curvature and interelement equilibrium of forces. The latter fact makes it possible to analyze shells with discontinuous meridional slope. In using this element, improvement in the convergence of the elastic-plastic solutions has been also observed.

An earlier computer program using curved elements has been modified to incorporate the refined element. Complete listing of the modified program written in Fortran IV language is given. Two examples illustrate applications of the new program.

The reported analysis is limited to situations of axisymmetric loading and boundary conditions.

TABLE OF CONTENTS

NOTATION

dA	- surface element of the reference surface of shell
dS	- surface element
dv	- volume element
E	- Young's modulus
h	- shell thickness
ℓ	- cord length of an element
M_s, M_θ	- meridional and circumferential bending moments per unit length, respectively
N	- total degrees of freedom of the discretized structure
N_s, N_θ	- meridional and circumferential in-plane forces per unit length, respectively
p^E	- elastic limit load
p^{EP}	- elastic-plastic load
r_1	- meridional radius of curvature
S_T	- part of boundary surface where stresses are specified
u_1, u_2	- displacements as shown in Fig. 1
U	- meridional displacement
W	- transverse (radial) displacement
u_i	- displacement components
α_i	- generalized coordinates
β	- an angle as shown in Fig. 1
$\epsilon_s^o, \epsilon_\theta^o$	- meridional and circumferential strains of the reference surface of shell, respectively
η	- local coordinate for an element, see Fig. 1
κ_s, κ_θ	- meridional and circumferential change of curvatures of shell, respectively

ν	- Poisson ratio
ξ	- local coordinate for an element, see Fig. 1
σ_y	- yield stress in tension
τ^{ij}, τ_{ij}	- stress tensor
φ	- latitude angle, see Fig. 1
χ	- meridional rotation
ψ	- as shown in Fig. 1
{ }	- vector; column matrix
[ ]	- matrix
[A]	- displacement transformation matrix, see (12)
[B]	- as defined in (6)
[C]	- matrix of elastic-plastic moduli
[K]	- stiffness matrix of the entire system
$[\underline{k}]$	- condensed element stiffness matrix in physical coordinate, see (14)
[k]	- element stiffness matrix
$[k_\alpha]$	- element stiffness matrix in generalized coordinates, see (8)
{p}	- as defined in (10)
{Q}	- equivalent nodal point force in physical coordinates, see (14)
$\{Q_\alpha\}$	- equivalent nodal point force in generalized coordinates
{q}	- nodal point displacement
{R}	- external nodal point load of the system in global coordinates
$\{\underline{R}\}$	- condensed element load vector
{r}	- nodal point displacement of the system in global coordinates

$\{r_1\}$ - element nodal displacement corresponding to external degrees of freedom

$\{r_2\}$ - element nodal displacement corresponding to internal degrees of freedom

$[T]$ - as defined in (16) and (17)

$\{\alpha\}$ - generalized coordinates

$\{\epsilon\}$ - strain tensor expressed in column matrix

$\{\varepsilon\}$ - as defined in (6)

$\{\tau\}$ - stress tensor expressed in column matrix

$[\varphi]$ - as defined in (9)

$[\varphi_p]$ - interpolating function for surface loads, see (10)

INTRODUCTION

During the preparation of a paper on the elastic-plastic analysis of pressure vessel heads [1]*, considerable inaccuracies in the in-plane stress-resultants were observed at junctures between the cylinders and toroidal knuckles as well as between knuckles and spherical closures. An expedient of using very small elements in such locations somewhat resolved this difficulty, however this was achieved at an increase in the computer time. Moreover, although not uncommon in the finite element work, the interelement equilibrium of forces was not altogether satisfactory. To obtain these solutions the previously developed formulation [2,3] for shells of revolution for axisymmetrical loadings utilizing curved elements has been employed. In terms of local Cartesian coordinates, in the earlier formulation a cubic expansion was used for the transverse displacements, and a linear one for in-plane deformation. Subsequent study showed that this formulation inadequately relates the interaction of flexural and in-plane forces.

A more consistent approach is to use the same order of polynomial expansions for both the in-plane and out-of-plane displacements. In this regard, whereas a cubic expansion for transverse displacements is necessary to comply with the requirements of continuity, rigid body nodes, and constant curvature, this is not the case for in-plane displacements. By adopting a cubic polynomial expansion for the in-plane displacements, two internal degrees of freedom are added to the six

---

* The bracketed numbers refer to the corresponding items in the list of References.

nodal degrees of freedom existing in the previous formulation. This necessitates a process of static condensation in order to remove these two additional degrees of freedom before assembling the individual element stiffness matrices, and recovering them after the nodal displacements are found.

After deriving the appropriate expressions for a refined element having eight degrees of freedom instead of six and programming the modified solution, several advantages of the new formulation became apparent. By using the refined elements, faster rate of convergence of elastic-plastic solutions is obtained, and the results are more accurate. This is achieved at the expense of but slightly lengthier process of solution than that used in the previous approach.

The basic method of solution of axisymmetrically loaded elastic-plastic thin shells of revolution has been presented previously [2,3,4] and will not be repeated. Here discussion will be confined principally with regards to the development of the refined element. A new elastic-plastic analysis of shells with a discontinuity in the meridional slope, made possible by the greater accuracy of the improved formulation, is also given. The developed solution is illustrated by two examples. In the one, its superiority to the earlier results is brought up; in the other, an elastic-plastic shell with a discontinuity in the meridional slope is solved.

GEOMETRY OF A CURVED ELEMENT

The geometry of an axisymmetric curved shell element is illustrated in Fig. 1. As has been shown in [2] and [4], its meridional curve in dimensionless coordinates ξ and η may be expressed as

$$\eta = \xi(1 - \xi)\ (a_1 + a_2\,\xi + a_3\,\xi^2 + a_4\,\xi^3) \qquad (1)$$

where

$$a_1 = \tan\beta_i$$

$$a_2 = \tan\beta_i + \tfrac{1}{2}\,\eta_i''$$

$$a_3 = -\ (5\tan\beta_i + 4\tan\beta_j) + \tfrac{1}{2}\,\eta_j'' - \eta_i''$$

$$a_4 = 3(\tan\beta_i + \tan\beta_j) + \tfrac{1}{2}\,(\eta_j'' - \eta_i'')$$

$$\eta'' = \frac{d^2\eta}{d\xi^2} = -\ \frac{\ell}{r_1\cos^3\beta}$$

ℓ = cord length

Note that the curve given by (1) satisfies the requirements of continuity of slopes and curvatures at the nodal circles.

DISPLACEMENTS PATTERN

In this development the following displacement model, expressed in terms of local Cartesian coordinates, Fig. 1, is assumed over each discrete element:

$$u_1 = \alpha_1 + \alpha_2 \xi + \alpha_3 \xi^2 + \alpha_4 \xi^3$$
$$u_2 = \alpha_5 + \alpha_6 \xi + \alpha_7 \xi^2 + \alpha_8 \xi^3 \qquad (2)$$

where α's are the generalized coordinates. The number of these generalized coordinates is equal to the total number of internal and external degrees of freedom of the element. The six degrees of freedom at the nodes i and j, Fig. 2, are the external D.O.F., and the two at the nodes m and n are the internal D.O.F. On assembling the elements into a representation of the overall shell, compatibility must be maintained for all the displacements degrees of freedom occurring at the interelement nodes, i.e., at i and j in Fig. 2. The two displacement degrees of freedom at the internal nodes m and n are not required in the assemblage. Thus they must be removed by a process of static condensation prior to assemblage of the total structural stiffness matrix.

The displacement model (2) must be specialized for the case of the central cap, Fig. 3. Here, if W and U are the radial and tangential components of displacements, because of symmetry, at the apex, the tengential component of displacement, U_i, and rotation, X_i, vanish. Hence,

$$U_i = u_1^i \sin\psi + u_2^i \cos\psi = 0$$

$$\chi_i = \frac{dW}{ds}\Big|_i - \frac{U_i}{r_1} = \frac{1}{\ell}\left(-\frac{du_1}{d\xi}\Big|_i \tan\beta + \frac{du_2}{d\xi}\Big|_i\right) = 0 \qquad (3)$$

Further, the following relation holds between (U, W) and (u_1, u_2):

$$\begin{Bmatrix} U \\ W \end{Bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \qquad (4)$$

where

$$\tan\beta = \frac{d\eta}{d\xi} \equiv \eta'$$

On substituting (2) into (3), one obtains:

$$\frac{\alpha_1}{\alpha_5} = -\frac{\cos\psi}{\sin\psi} \qquad (a)$$

$$\alpha_6 = \alpha_2 \tan\beta_i \qquad (b)$$

Defining $\alpha_1 \equiv -\alpha_3' \cos\psi$, (a) gives $\alpha_5 = \alpha_3' \sin\psi$.

Defining $\alpha_2 \equiv \alpha_4'$, (b) gives $\alpha_6 = \alpha_4' \tan\beta_i$.

Fulfillment of the symmetry conditions (3) reduces the number of generalized coordinates from eight to six for the cap element. The resulting displacement pattern in $(\xi - \eta)$ coordinates is:

$$u_1 = -\alpha_3' \cos\psi + \alpha_4' \xi + \alpha_5' \xi^2 + \alpha_6' \xi^3$$

$$u_2 = \alpha_3' \sin\psi + \alpha_4' \tan\beta_i \,\xi + \alpha_7' \xi^2 + \alpha_8' \xi^3 \qquad (5)$$

ELEMENT STIFFNESS MATRIX

The details of generating the element stiffness matrix for an axisymmetric curved element have been discussed elsewhere [2, 4]. Only the general outline of the procedure is indicated here, with the emphasis placed on the new features of the formulation that arise due to the modification of the old element.

By simple differentiation of (2) or (5) the strain-displacement relation may be expressed as follows:

$$\underset{4\times1}{\{\epsilon(\xi)\}} = \underset{4\times8}{[B(\xi)]}\ \underset{8\times1}{\{\alpha\}} \tag{6}$$

where the transpose of the strain vector $\{\epsilon\}$, as defined in [2], is given by

$$\{\epsilon\}^T = \langle \epsilon_s^o\ \epsilon_\theta^o\ \kappa_s\ \kappa_\theta \rangle$$

Here ϵ_s^o and ϵ_θ^o are the meridional and circumferential strains of the reference surface of the shell, respectively, and κ_s and κ_θ are the corresponding quantities of the changes in shell curvatures.

The B matrices for the cap and frustum elements are given in Appendix I.

As shown in [2] the relationship between stress, τ , and strain, ϵ , for an elastic-plastic material can be expressed as

$$\underset{4\times1}{\{\tau(\xi)\}} = \underset{4\times4}{[C(\xi)]}\ \underset{4\times1}{\{\epsilon(\xi)\}} \tag{7}$$

where C is a matrix of elastic-plastic moduli.

On applying the principle of virtual displacement, the element stiffness matrix in terms of generalized coordinates can be stated as

$$[k_\alpha]_{8\times 3} = \int_{\Delta V} [B]^T [C] [B] \, dV \tag{8}$$

where ΔV signifies the volume of the element.

The generalized load vector also can be obtained by utilizing the principle of virtual displacement in the following manner.

Let the relations (2) and/or (5) together with the expression for meridional rotation be represented in matrix form as:

$$\underset{3\times 1}{\{u(\xi)\}} = \underset{3\times 8}{[\varphi(\xi)]} \; \underset{8\times 1}{\{\alpha\}} \tag{9}$$

where

$$\{u\}^T = \langle u_1 \; u_2 \; \chi \rangle$$

and the matrix $[\varphi]$ is given in Appendix II.

Next, the surface loads $p(\xi)$ can be expressed in terms of interpolating functions φ_p as

$$\underset{3\times 1}{\{p(\xi)\}} = \underset{3\times 6}{[\varphi_p(\xi)]} \; \underset{6\times 1}{\{p_i\}} \tag{10}$$

where p_i is the intensity of the distributed loads at the external nodes.

Then, by applying the principle of virtual displacement, it can be shown that the equivalent generalized loads Q_α at the external and internal nodes are

$$\underset{8\times 1}{\{Q_\alpha\}} = \int_{\Delta S} [\varphi]^T \{p(\xi)\} \, dS \tag{11}$$

where ΔS is the surface area of the element.

To transform the above element stiffness matrix and the load vector to physical coordinates, one proceeds as follows.

By substituting the coordinates of the element nodal points into (9), one gets the nodal displacements $\{q\}$ in terms of generalized coordinates:

$$\underset{8\times 1}{\{q\}} \equiv \underset{\substack{3\times 1\\3\times 1\\2\times 1}}{\begin{Bmatrix} u_i \\ \hline u_j \\ \hline u_k \end{Bmatrix}} = \underset{8\times 8}{[A]} \ \underset{8\times 1}{\{\alpha\}} \tag{12}$$

and

$$\{\alpha\} = [A]^{-1} \{q\} \tag{13}$$

where

$$\{q\}^T = \langle u_1^i \ u_2^i \ \chi^i \ \vdots \ u_1^j \ u_2^j \ \chi^j \ \vdots \ u_1^m \ u_1^n \rangle$$

The matrices A and A^{-1} are given in Appendix III, both for the cap and frustum elements.

The stiffness matrix and the load vector in physical coordinates are obtained by the following transformation:

$$\begin{gathered} \underset{8\times 8}{[k]} = [A^{-1}]^T [k_\alpha] [A^{-1}] \\ \underset{8\times 1}{\{Q\}} \equiv \underset{\substack{6\times 1\\2\times 1}}{\begin{Bmatrix} Q_1 \\ \hline Q_2 \end{Bmatrix}} = [A^{-1}]^T \underset{\substack{6\times 1\\2\times 1}}{\begin{Bmatrix} Q_{1\alpha} \\ \hline Q_{2\alpha} \end{Bmatrix}} \end{gathered} \tag{14}$$

where

$\{Q_1\}_{6\times 1}$ are the equivalent generalized loads acting on the external nodes

and

$\{Q_2\}$ are the loads acting on the two internal nodes.

In (14) the matrices $[k]$ and $\{Q\}$ are defined in local Cartesian coordinates (ξ, η). In order to assemble the elements, these matrices must be expressed in global coordinates.

Two different sets of global coordinates are considered in this formulation:

(a) For the analysis of axisymmetric shells with discontinuous meridional slope, the (r,z) coordinates are taken as the global coordinates, Fig. 4(a). The required transformation can be stated as:

$$\underset{8\times 1}{\begin{Bmatrix} u_i \\ \hdashline u_j \\ \hdashline u_k \end{Bmatrix}} = \underset{8\times 8}{\begin{bmatrix} T_i & 0 & 0 \\ 0 & T_j & 0 \\ 0 & 0 & I \end{bmatrix}} \begin{Bmatrix} r_i \\ \hdashline r_j \\ \hdashline r_k \end{Bmatrix} \begin{matrix} 3\times 1 \\ 3\times 1 \\ 2\times 1 \end{matrix} \qquad (15)$$

where

$$\{r\}^T = \langle u_r^i \; u_z^i \; \chi^i \; \vdots \; u_r^j \; u_z^j \; \chi^j \; \vdots \; u_1^m \; u_1^n \rangle \quad ,$$

I is an identity matrix, and T has the following form:

$$T_i = T_j = \begin{bmatrix} \sin\psi & -\cos\psi & 0 \\ \cos\psi & \sin\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (16)$$

The angle ψ is defined in Fig. 4(a).

(b) For the analysis of shells with continuous meridional slope, the (U, W) coordinates are taken as the global coordinates, Fig. 4(b). The transformation is similar to (15) except that:

$$\{r\}^T = \langle U^i \; W^i \; \chi^i \; \vdots \; U^j \; W^j \; \chi^j \; \vdots \; u_1^m \; u_1^n \rangle ,$$

$$T_i = \begin{bmatrix} \cos \beta_i & -\sin \beta_i & 0 \\ \sin \beta_i & \cos \beta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (17)$$

and an expression similar to (17) applies for T_j , where the slope, $\tan \beta$, of the meridional curve is expressed in local (ξ, η) coordinates.

For the cap element, the top left corner sub-matrix, T_i , in (15) is replaced by an identity matrix in both cases (a) and (b).

Either in case (a) or (b), the element stiffness matrix and the load vector, Equations (14), are transformed into global coordinates using the following relations:

$$\underset{8\times8}{[k]_G} = \underset{8\times8}{[M]^T} \; \underset{8\times8}{[k]_L} \; \underset{8\times8}{[M]}$$

$$\{R\} \equiv \{Q\}_G = [M]^T \{Q\}_L \qquad (18)$$

where letters L and G signify the local and global coordinate, respectively, and M is the transformation matrix defined in (15).

Before proceeding with the actual assembly of the stiffness matrices, the terms in (18) corresponding to the internal degrees of freedom must be condensed out from $[K]_G$ and $\{R\}$. This operation is discussed in the following section.

STATIC CONDENSATION

The equilibrium equations for an element in global coordinates can be written as

$$\underset{8\times 8}{[k]} \; \underset{8\times 1}{\{r\}} = \underset{8\times 1}{\{R\}} \tag{19}$$

where for simplicity in subsequent discussion subscripts G have been deleted from all quantities.

Equation (19) can be partitioned to distinguish between the terms corresponding to the external and the internal degree of freedom. Thus, one has

$$\begin{bmatrix} k_{11} & \vdots & k_{12} \\ \hline k_{21} & \vdots & k_{22} \end{bmatrix} \begin{Bmatrix} r_1 \\ \hline r_2 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ \hline R_2 \end{Bmatrix} \tag{19a}$$

$$\begin{matrix} 6\times 6 & 6\times 2 & 6\times 1 & 6\times 1 \\ 2\times 6 & 2\times 2 & 2\times 1 & 2\times 1 \end{matrix}$$

where the subscript 2 designates the terms related to the two internal degrees of freedom. These terms can be removed as follows:

$$\{R_1\} = [k_{11}]\{r_1\} + [k_{12}]\{r_2\} \tag{20a}$$

$$\{R_2\} = [k_{21}]\{r_1\} + [k_{22}]\{r_2\} \tag{20b}$$

On solving (20b) for $\{r_2\}$, one has

$$\{r_2\} = [k_{22}]^{-1}\{R_2\} - [k_{22}]^{-1}[k_{21}]\{r_1\} \tag{20c}$$

whence on substituting into (20a) the expression for $\{r_2\}$ from (20c)

$$[k_{11} - k_{12}\,k_{22}^{-1}\,k_{21}]\{r_1\} = \{R_1 - k_{12}\,k_{22}^{-1}\,R_2\} \tag{21}$$

which can be rewritten as

$$[\underline{k}] \{r_1\} = \{\underline{R}\} \quad \text{(21a)}$$
$$6 \times 6 \quad 6 \times 1 \quad 6 \times 1$$

where $[\underline{k}]$ and $\{\underline{R}\}$ are the condensed stiffness matrix and load vector, respectively, and are defined in (21). The matrix multiplications and inversion involved in the process of condensation can be done directly since the only matrix to be inverted is only a 2×2 .

The condensed $\underline{k}$ and $\underline{R}$ must be found for all elements and assembled according to the well established procedures of the direct stiffness method. The resulting equilibrium equations for the assemblage of the elements are:

$$[K] \{r\} = \{R\} \quad \text{(22)}$$
$$N \times N \quad N \times 1 \quad N \times 1$$

where $N = 3 \times$ (total number of nodes), and the stiffness matrix K is a banded symmetric matrix having a width of 6 .

Equations (22), after being modified for the geometric boundary conditions, can be solved by a process of Gauss elimination and back substitution to obtain the displacement vector $\{r\}$. This vector $\{r\}$ contains the displacements occurring at the external degrees of freedom. The displacements at the internal degrees of freedom are needed to complete the solution. For each element, they are recovered, using (20c).

Using in sequence Eqs. (15), (13), (6), and (7), the stress field may be determined, and thus the solution of the problem becomes complete.

ANALYSIS OF ELASTIC-PLASTIC SHELLS OF REVOLUTION

The details of elastic-plastic analysis of shells of revolution, utilizing axisymmetric curved finite elements, has been fully described before [2, 4], and will not be elaborated upon here. Essentially, the theory discussed in this report is used to modify an existing computer program [3] into which the use of a refined element is incorporated. In addition, a second version of the program is so coded that the shells with discontinuous meridional slope can be analyzed. These programs are written in FORTRAN IV language, and their listings and user's guide are provided in Appendix IV, and V.

For the purpose of analysis, the shell must be subdivided into a number of elements. It is recommended to use smaller elements in the regions where high gradients of displacements are anticipated. Drastic changes in the size of neighboring elements should be avoided. For a closed shell, numbering of the nodal circles should be started from the point on the axis of symmetry.

The shell is assumed to be initially stress free. After the first increment of loading is applied, the magnitude of the load is so scaled that plastic deformation just sets-in in some region of the shell. This constitutes an elastic analysis of the system. In the remainder of the elastic-plastic analysis, the loading is continued in small increments.

EXAMPLES

1. Shells with Continuous Meridional Slope

A Pressure Vessel with a Torispherical Head

Consider a typical torispherical pressure vessel head attached to a long cylinder, as shown in the insert of Fig. 5. Let the diameter D of the head skirt be 100 in., the radius of the spherical crown $L = D$, the radius of the toroidal part $r = 0.06\ D$, and the shell thickness $h = 0.008$. These proportions of the head conform to the 1965 ASME Code for Unfired Pressure Vessels. The material of the shell is assumed to be elastic-perfectly plastic, with $E = 30 \times 10^6$ psi, $\nu = 0.3$, and yield stress, $\sigma_y = 30{,}000$ psi.

This vessel under a uniform internal pressure has been previously analyzed [1] using the old finite element. The elastic-plastic analysis is repeated here using the new refined finite-element. The same element layout and load increment sizes were used in both solutions. The results of the two analyses are shown in Figs. 5 through 8.

Figures 5 and 6 show the meridional in-plane force and moment, respectively. The curves are plotted for the elastic-limit pressure of 104 psi, and an elastic-plastic load of 204 psi.

Comparison of the two solutions in Figs. 5 and 6 clearly shows the improvements obtained by using the refined element. The dips in the curves of N_s, Fig. 5, at the junctures of torus with the sphere and cylinder are smoothed out. The refined element also gives smoother variation of M_s in the vicinity of the juncture of the torus and the cylinder, Fig. 6.

A comparison of the two load-deflection curves in Fig. 7 shows a better convergence with the refined elements.* The gradual spread of elastic-plastic zones with increasing internal pressure, as obtained in the two solutions, is shown in Fig. 8. Although the pattern of plastic propagation remains essentially alike in the two solutions, the new solution shows a somewhat faster rate of plastification. Stated alternatively, at the same load level, more material appears to be plastified according to the new solution. The first hinge circle forms at an internal pressure of 1.69 p^E and 1.79 p^E according to new and old solutions, respectively.

2. Shells with Discontinuous Meridional Slope

A Pressure Vessel with a Shallow Spherical Head

Consider a cylindrical pressure vessel with a shallow spherical cap of radius $L = D/(2 \sin \varphi_o) = 100$ in. as shown in the insert of Fig. 9. Let $\varphi_o = 45^\circ$ and the shell thickness $h = 0.02$ D, where D is the diameter of the cylindrical vessel. The material of the shell is assumed to be elastic-perfectly plastic, with $E = 30 \times 10^6$ psi, $\nu = 0.3$, and yield stress, $\sigma_y = 30{,}000$ psi.

For the purposes of analysis, the vessel was divided into 22 uneven elements - 12 in the spherical cap, and 10 in the cylindrical body. A total of 16 load increments were used, starting from the elastic limit load of 53.5 psi.

Some results of the elastic-plastic analysis are shown in Figs. 9 through 12. The meridional moment and the in-plane force distributions are plotted in Figs. 9 and 10, respectively. The curves

---

* The results of a convergence study [2,4] indicate that the new load-deflection curve, Fig. 7, could have been obtained if the old elements were used with much smaller load increments.

correspond to an elastic limit load of 53.5 psi, and an elastic-plastic load of 114 psi. At the latter pressure, according to present analysis, the first hinge circle is formed. The sharp peak in M_s , Fig. 19, and the high compressive circumferential forces N_θ , Fig. 10(b), at and near the juncture of the head with the cylinder show the undesirable features of this type of an attachment.

Due to the discontinuity in the slope of the meridian at the juncture between the head and the cylinder, there is a slight discontinuity in N_s at the same point, Fig. 10(a). Here equilibrium is satisfied by the presence of transverse shear forces.

The inelastic moment and in-plane force distribution retain in general their elastic trends. However, due to plastic flow, there is some stress redistribution. The drastic change in the magnitude of N_θ at the juncture of the head and the cylinder, Fig. 10(b), is due to the formation of a narrow plastic band in this zone. As the width of this plastic region increases, the negative peaks of N_θ are shifted further apart into the cylinder and the spherical head. This phenomenon can be explained by noting the fact that the state of stress of the points outside the plastic region falls inside the yield surface, and these stresses can increase at an elastic rate, whereas the stress paths of the points in the plastic region are constrained to follow the boundary of the yield surface, resulting in a slower rate of increase and decrease in the magnitude of the forces in the meridional and circumferential directions, respectively.

Figure 11 shows the load deflection characteristics of the analyzed pressure vessel. The progressive yielding within the wall thickness of the vessel is illustrated in Figure 12. The yielding starts at the inner face of the juncture, and with increasing internal pressure propagates across the thickness and along the shell. The first hinge circle is formed at an internal pressure of 114 psi or 2.13 p^E.

CONCLUSIONS

A modified version of the Khojasteh-Bakht axisymmetric shell element is developed for elastic-plastic analysis of shells of revolution. With a limited amount of extra work involved, the modified element proves to be superior to the old element in the following respects.

(a) The internal forces are computed more accurately. This is especially true in the case of in-plane forces. The inter-element force equilibrium is greatly improved so that the analysis of shells with discontinuous meridional slope is possible without the danger of getting inaccurate results at the discontinuities.

(b) The rate of convergence is also improved, making it possible to choose a smaller number of elements and/or fewer load increments to obtain comparable results with those one might get using the old element.

REFERENCES

[1] Popov, E. P., Khojasteh-Bakht, M., Sharifi, P., "Elastic-Plastic Analysis of Some Pressure Vessel Heads," ASME paper No. 69-WA/PVP-7, presented at the Winter Annual Meeting in Los Angeles, Nov. 1969.

[2] Khojasteh-Bakht, M., "Analysis of Elastic-Plastic Shells of Revolution Under Axisymmetric Loading by the Finite Element Method," Report SESM 67-8, Structural Eng. Lab., Univ. of Calif., Berkeley, April 1967. Also available as NASA Report CR-85735.

[3] Khojasteh-Bakht, M., "Computer Program for Elastic-Plastic Analysis of Axisymmetrically Loaded Shells of Revolution," Report SESM 68-3, Struct. Eng. Lab., Univ. of Calif., Berkeley, April 1968.

[4] Khojasteh-Bakht, M., Popov, E. P., "Analysis of Elastic-Plastic Shells of Revolution," ASCE J. of Eng. Mech. Div. (in press; scheduled for publication April 1970).

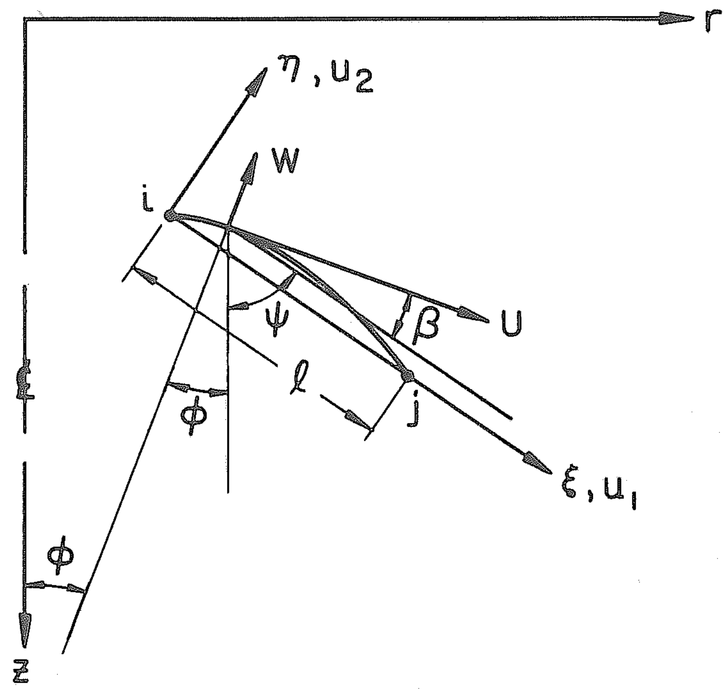


FIG. 1 GEOMETRY OF DISPLACEMENTS OF A CURVED ELEMENT

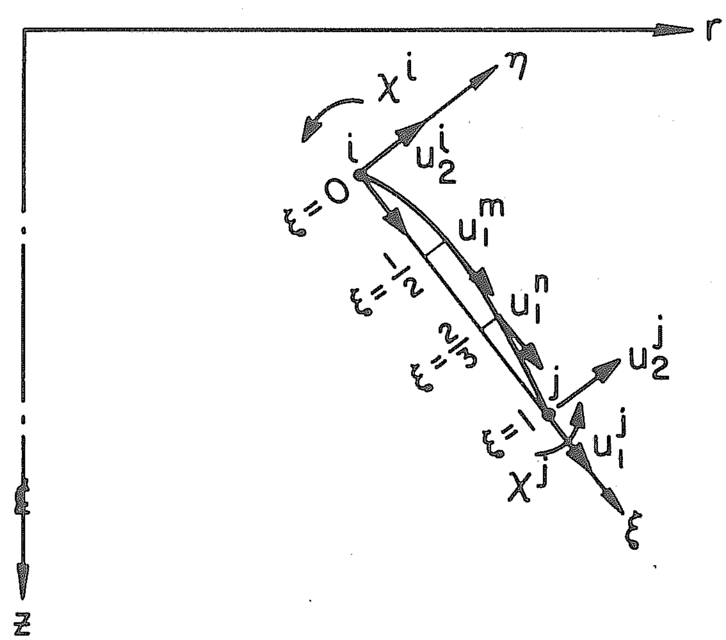


FIG. 2 INTERNAL AND EXTERNAL DEGREES OF FREEDOM

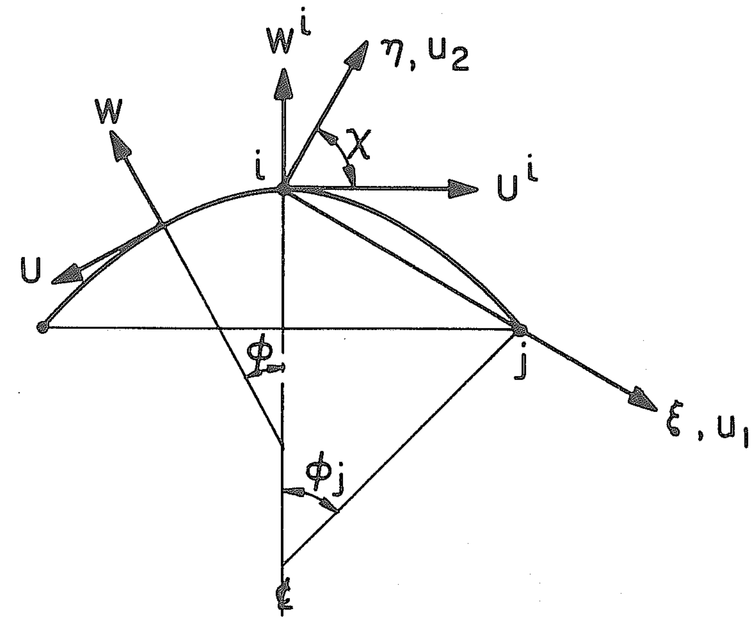


FIG. 3 CENTRAL CAP ELEMENT

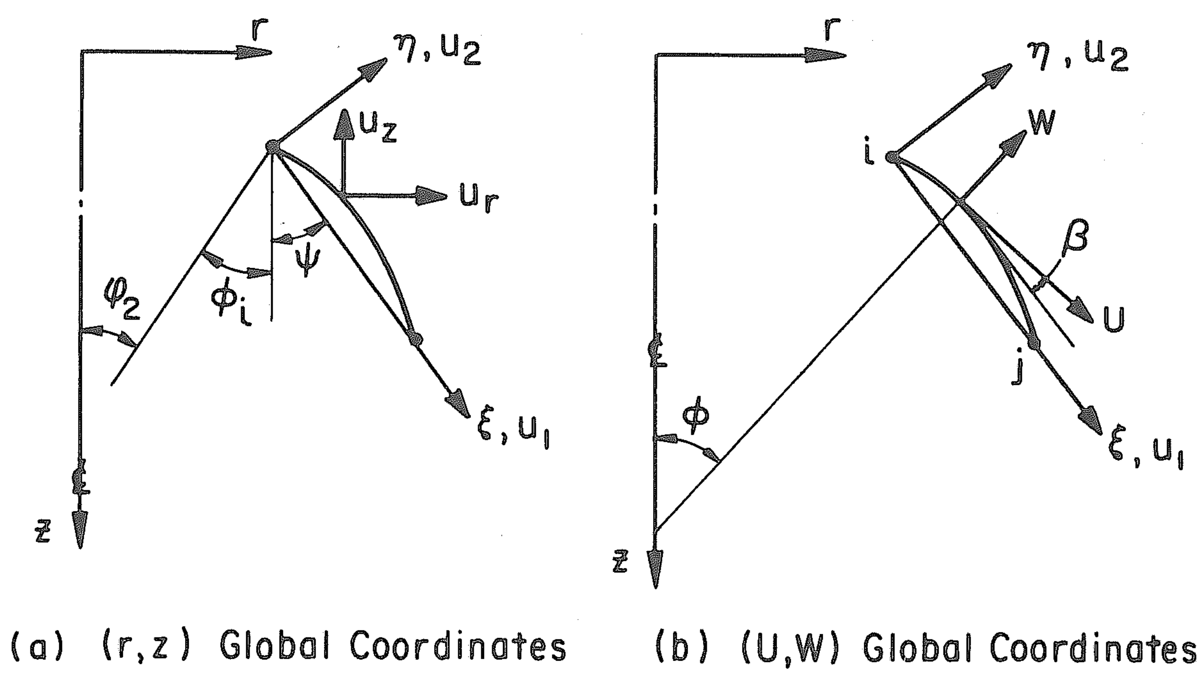


FIG. 4 GLOBAL AND LOCAL COORDINATES

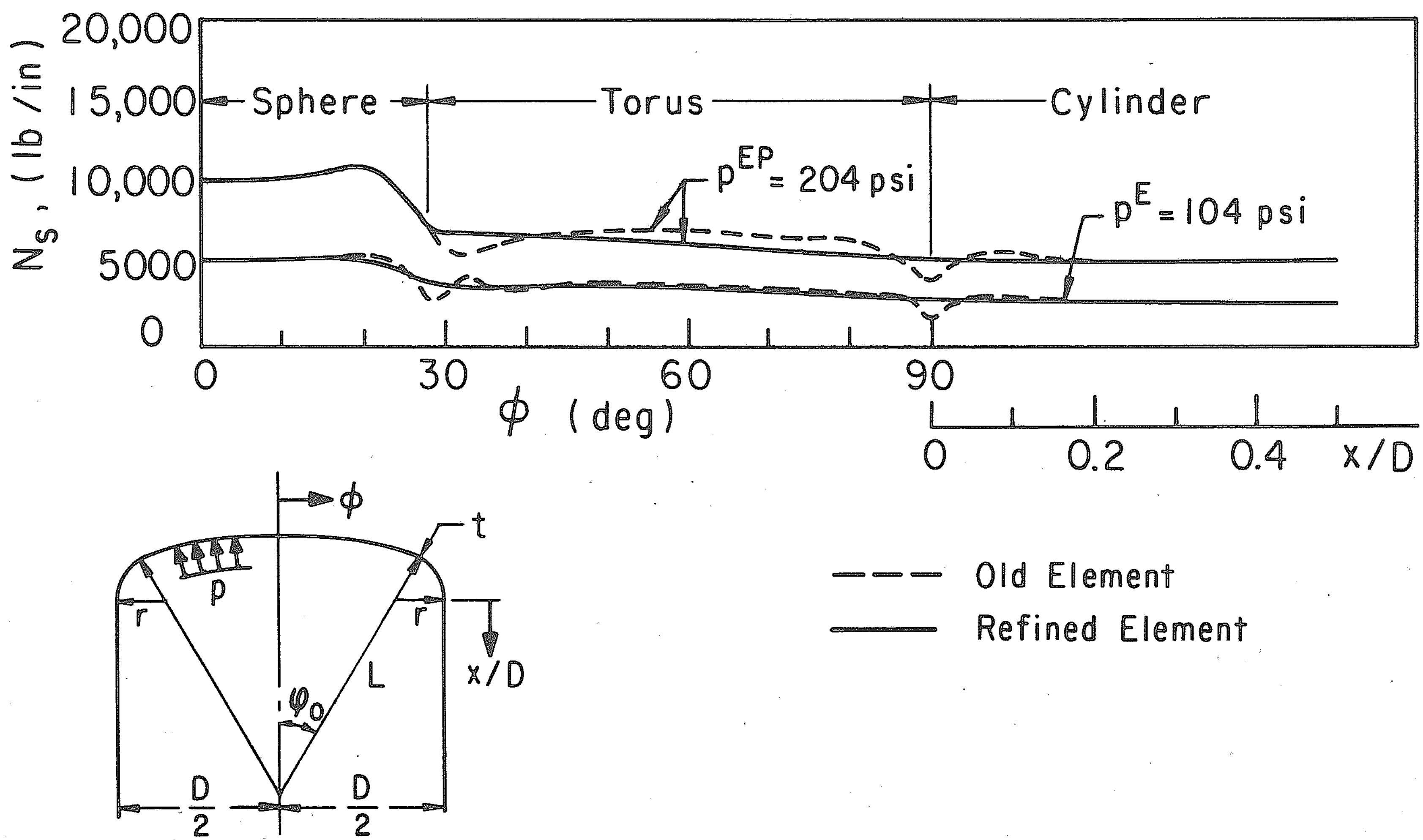


FIG. 5 MERIDIONAL IN-PLANE FORCE IN A TORISPHERICAL HEAD

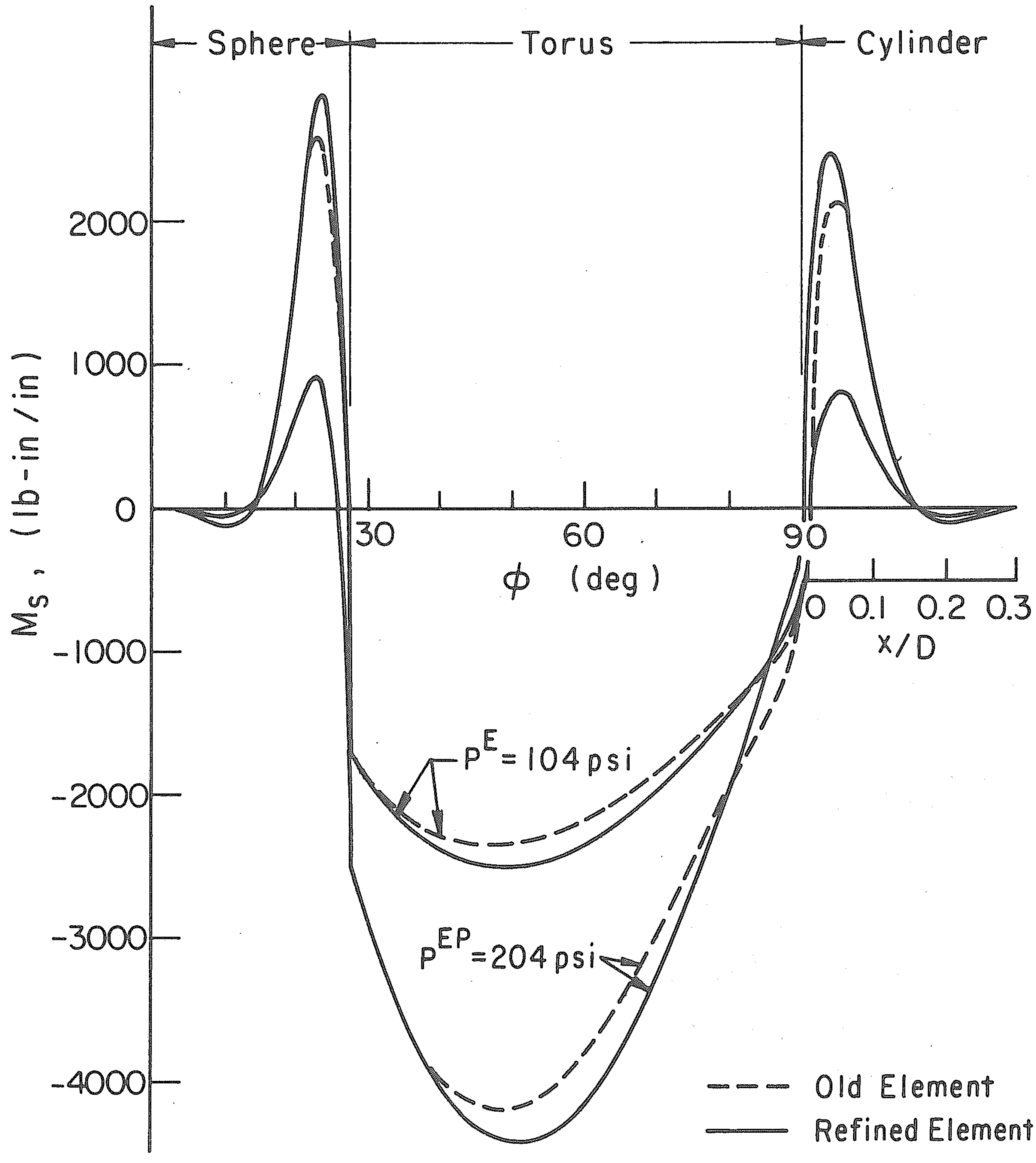


FIG. 6 MERIDIONAL MOMENT IN A TORI SPHERICAL HEAD WITH D = L = 100 in., r/L = 0.06 , AND t/L = 0.008

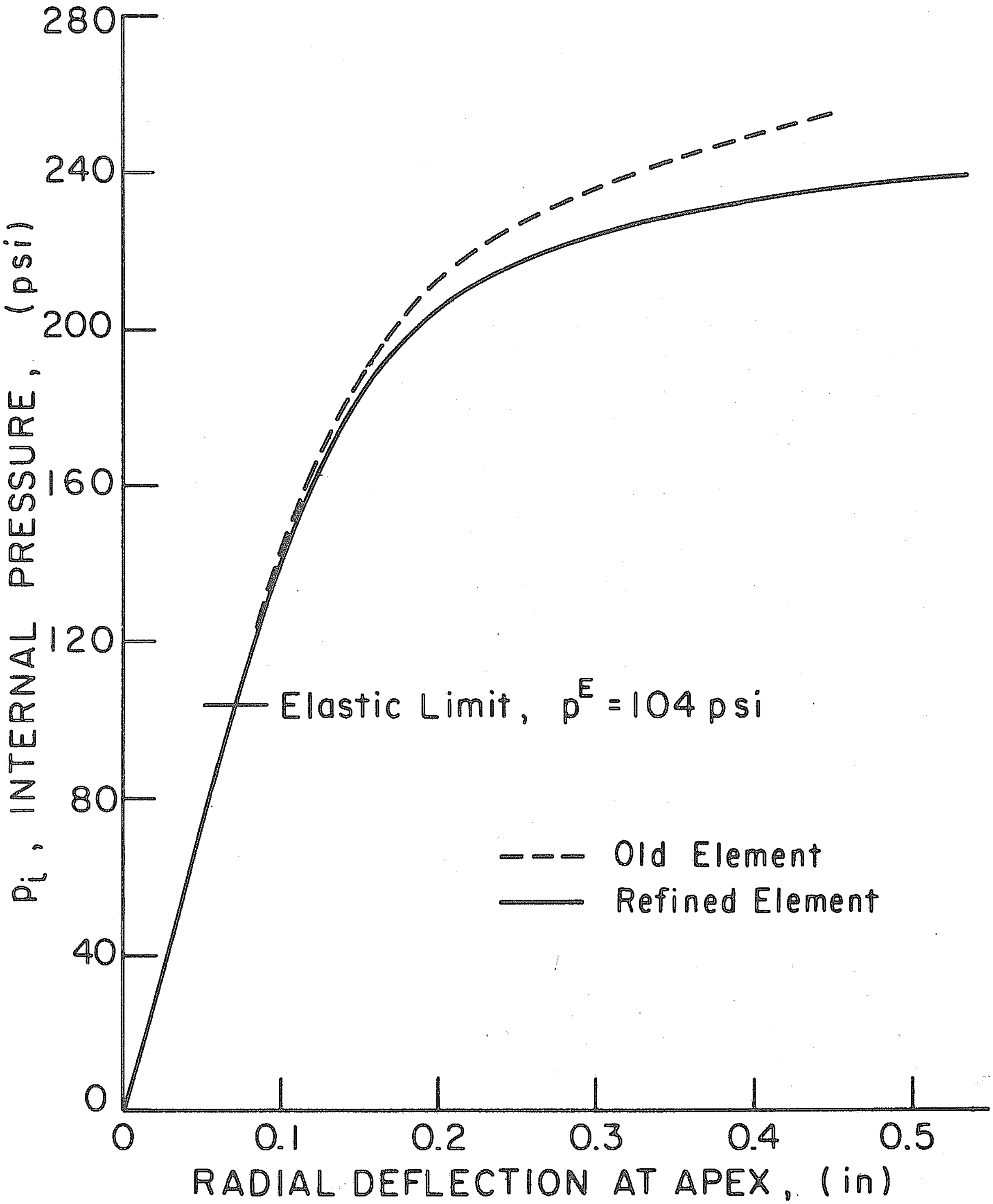
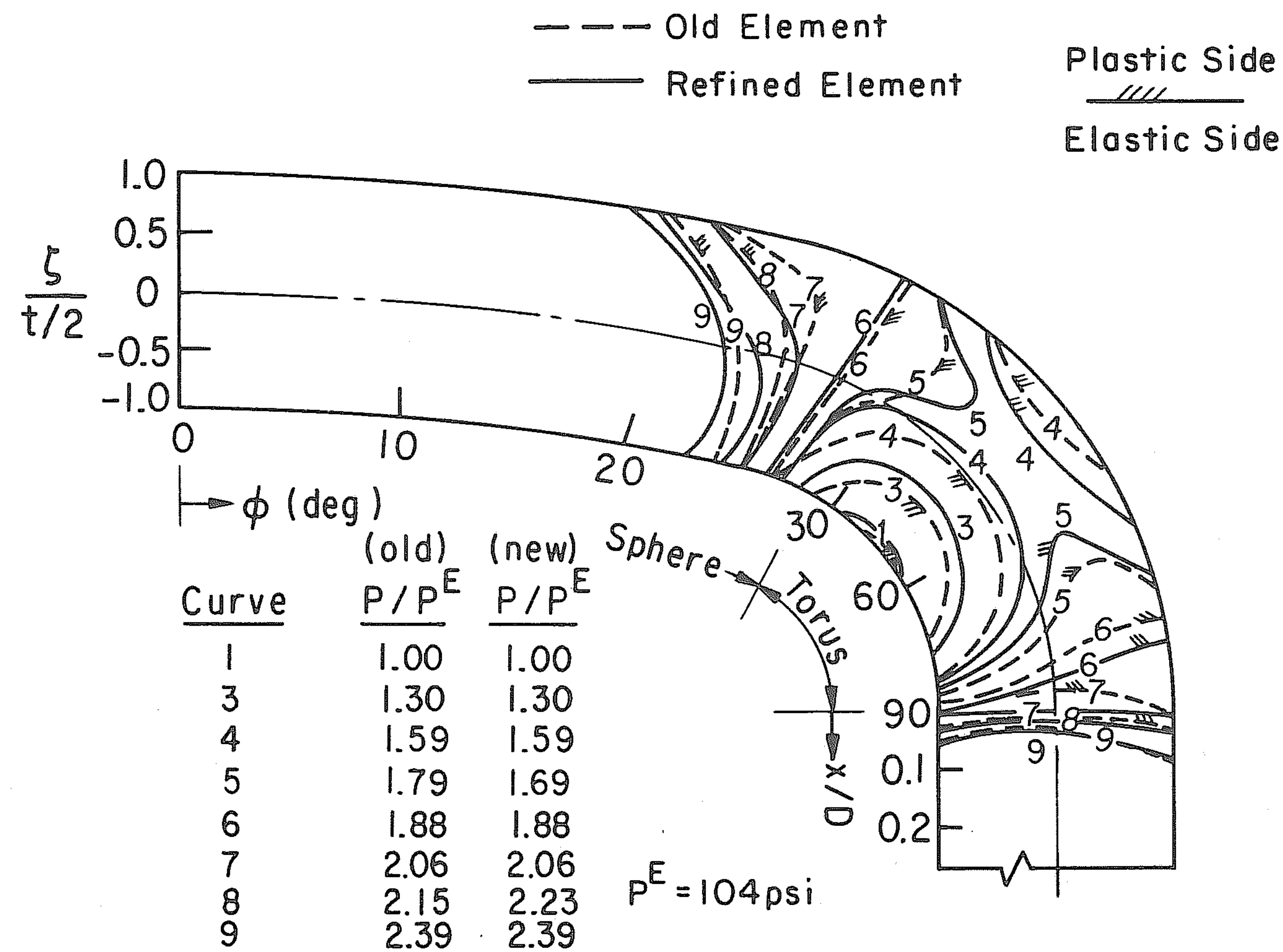


FIG. 7 LOAD DEFLECTION CHARACTERISTICS TORISPHERICAL HEAD



Curve	(old) P/P^E	(new) P/P^E
1	1.00	1.00
3	1.30	1.30
4	1.59	1.59
5	1.79	1.69
6	1.88	1.88
7	2.06	2.06
8	2.15	2.23
9	2.39	2.39

FIG. 8 ELASTIC PLASTIC BOUNDARIES

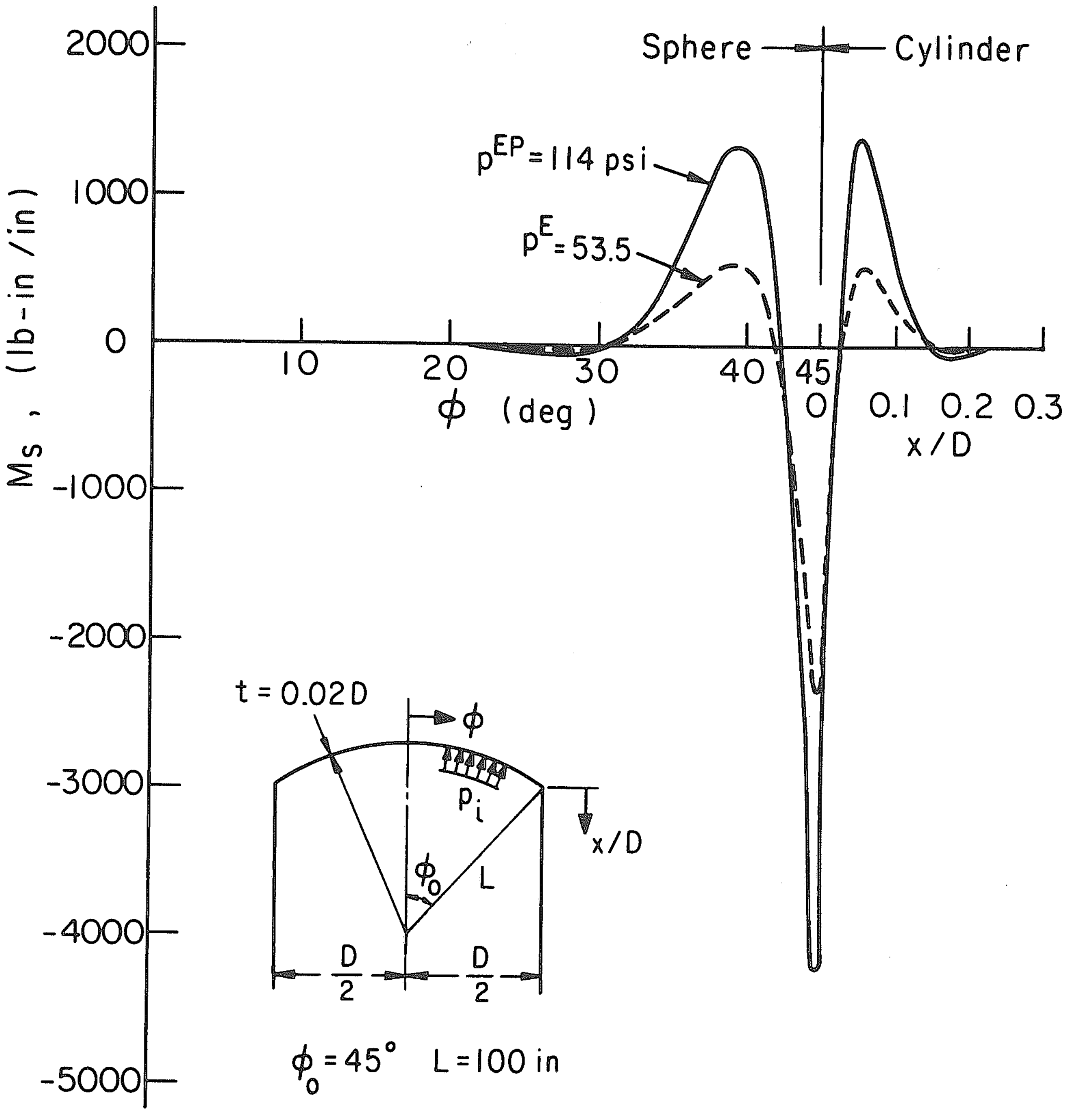


FIG. 9 MERIDIONAL MOMENTS

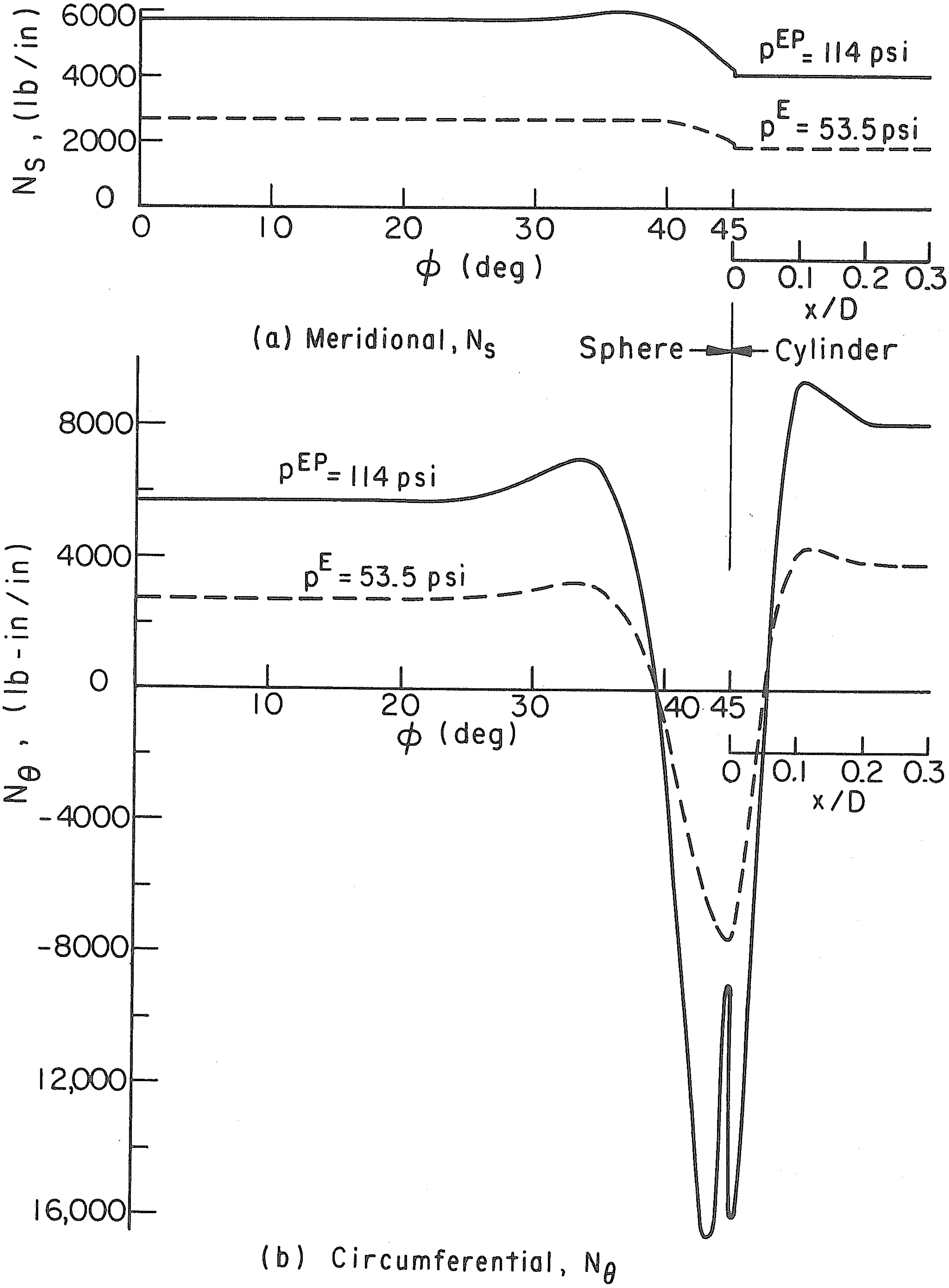


FIG. 10 IN-PLANE FORCES

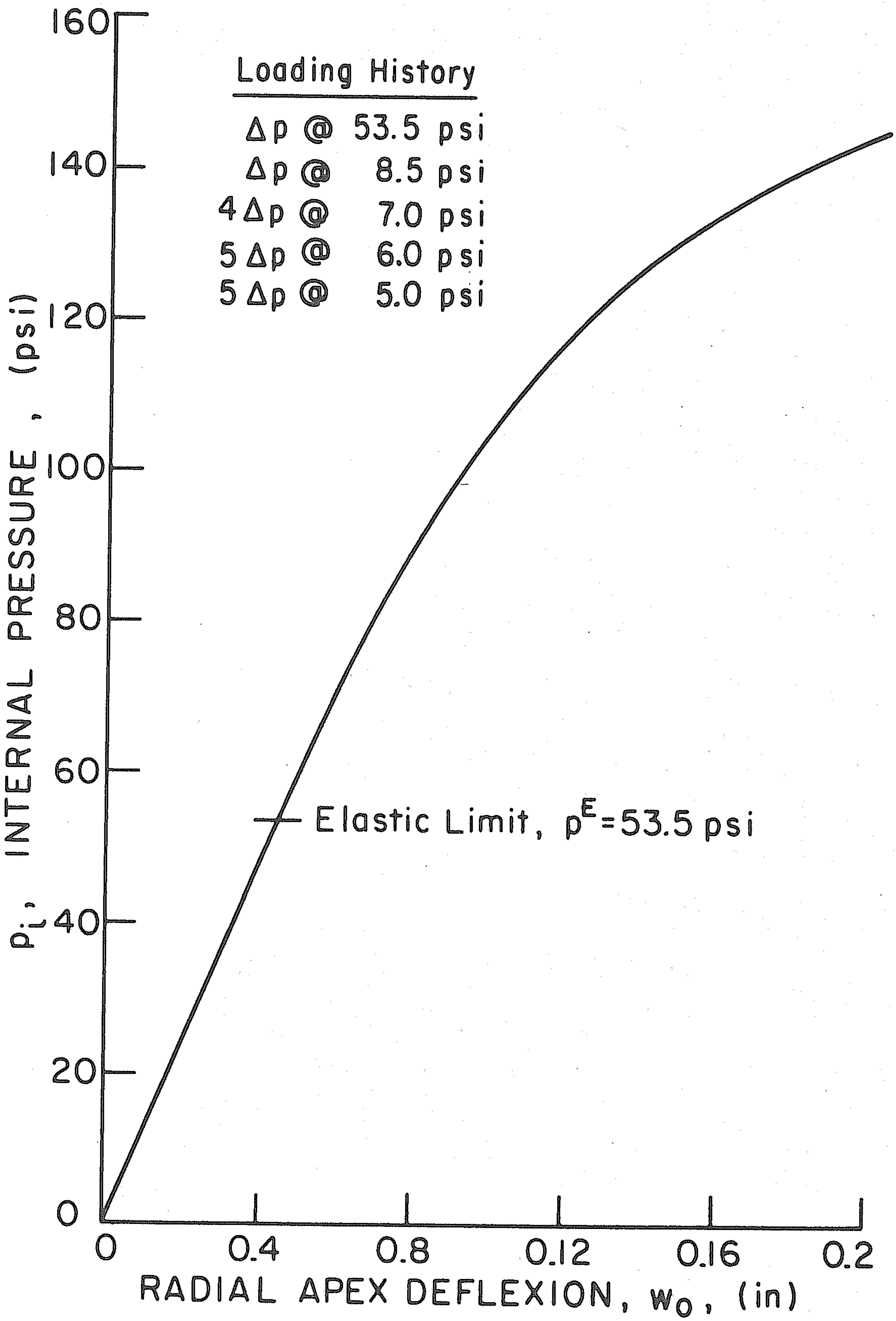


FIG. 11 LOAD DEFLECTION CHARACTERISTICS

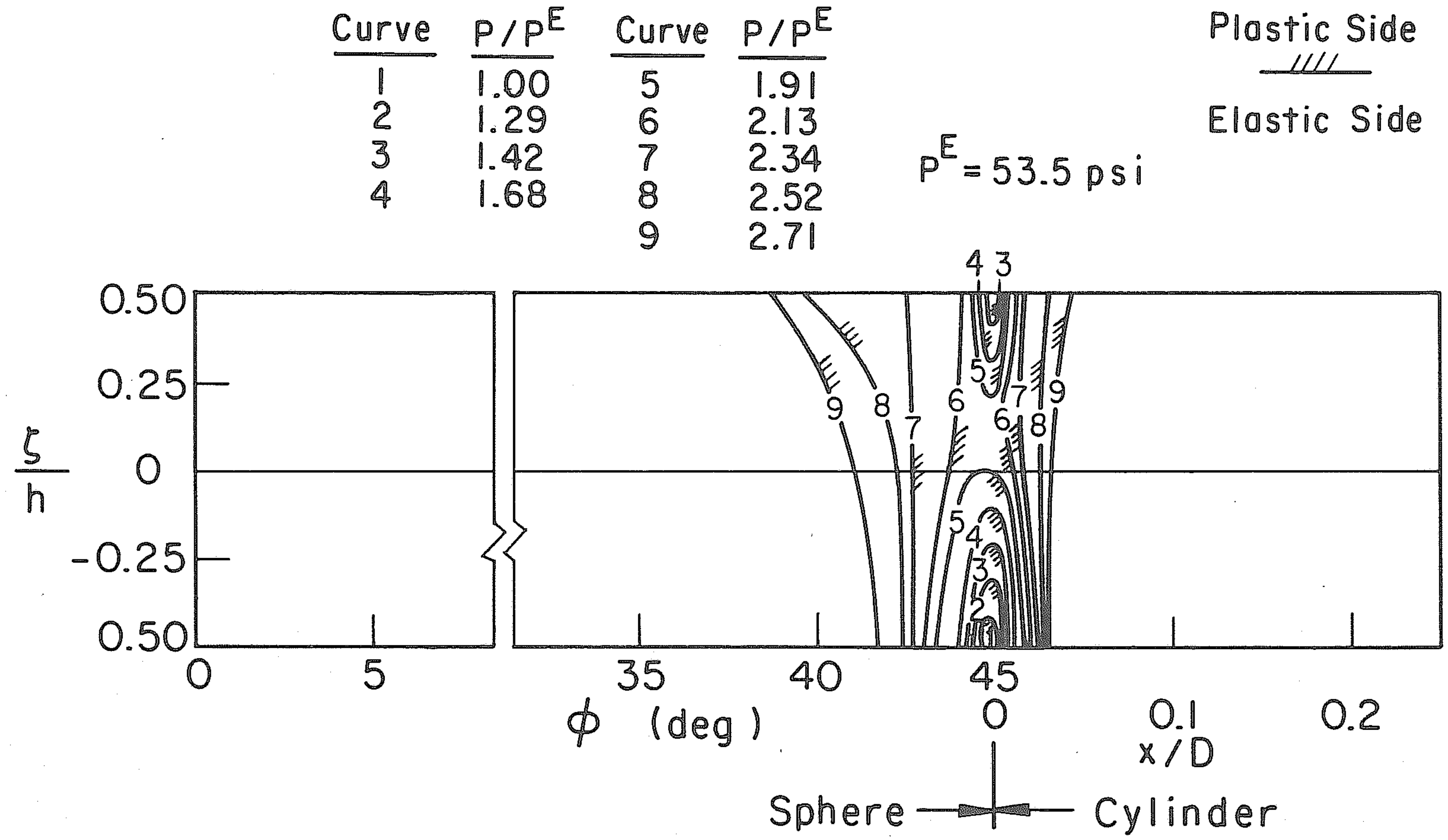


FIG. 12 ELASTIC PLASTIC BOUNDARIES

APPENDIX I

MATRIX [B], Eq. (6)

1 - CAP ELEMENT

$$\begin{bmatrix} 0 & 0 & 0 & \rho(1+\eta'\tan\beta_i) & 2\xi\rho & 3\xi^2\rho & 2\xi\eta'\rho & 3\xi^2\eta'\rho \\ 0 & 0 & 0 & \dfrac{\cos\varphi_i}{\bar{r}\cos\beta_i} & \dfrac{\xi\sin\psi}{\bar{r}} & \dfrac{\xi^2\sin\psi}{\bar{r}} & \dfrac{\xi\cos\psi}{\bar{r}} & \dfrac{\xi^2\cos\psi}{\bar{r}} \\ 0 & 0 & 0 & \Phi(1+2\eta'\tan\beta_i-\eta'^2) & \Omega+2(1-\eta'^2)\xi\Phi & 3\xi[\Omega+\xi(1-\eta'^2)\Phi] & 4\xi\eta'\Phi-\mu & 3\xi(2\xi\eta'\Phi-\mu) \\ 0 & 0 & 0 & \dfrac{\eta'-\tan\beta_i}{\xi}\Psi & 2\eta'\Psi & 3\xi\eta'\Psi & -2\Psi & -3\xi\Psi \end{bmatrix}$$

where $\bar{r} = \frac{r}{\xi}$, Note $r(0) = 0$

$$\rho = \frac{1}{\ell(1+\eta'^2)} \quad ; \quad \mu = \frac{2}{\ell^2(1+\eta'^2)^{3/2}}$$

$$\Phi = \frac{\eta''}{\ell^2(1+\eta'^2)^{5/2}} \quad ; \quad \Psi = \frac{\sin\psi+\eta'\cos\psi}{\ell\bar{r}(1+\eta'^2)^{3/2}} \quad ; \quad \Omega = \frac{2\eta'}{\ell^2(1+\eta'^2)^{3/2}}$$

Continued/

2 - FRUSTUM ELEMENT, [B]

$$
\begin{bmatrix}
0 & \rho & 2\xi\rho & 3\xi^2\rho & 0 & \eta'\rho & 2\xi\eta'\rho & 3\xi^2\eta'\rho \\
\frac{\sin\psi}{r} & \frac{\sin\psi}{r}\xi & \frac{\sin\psi}{r}\xi^2 & \frac{\sin\psi}{r}\xi^3 & \frac{\cos\psi}{r} & \frac{\cos\psi}{r}\xi & \frac{\cos\psi}{r}\xi^2 & \frac{\cos\psi}{r}\xi^3 \\
0 & (1-\eta'^2)\Phi & 2\xi(1-\eta'^2)\Phi+\Omega & 3\xi[\xi(1-\eta'^2)\Phi+\Omega] & 0 & 2\eta'\Phi & 4\xi\eta'\Phi-\mu & 3\xi(2\xi\eta'\Phi-\mu) \\
0 & \eta'\Psi & 2\xi\eta'\Psi & 3\xi^2\eta'\Psi & 0 & -\Psi & -2\xi\Psi & -3\xi^2\Psi
\end{bmatrix}
$$

where

$$\Psi = \frac{\sin\psi+\eta'\cos\psi}{\ell r(1+\eta'^2)^{3/2}}$$; The rest are defined in previous page.

APPENDIX II

MATRIX $[\varphi]$, Eq. (9)

1 - CAP ELEMENT

$$\begin{bmatrix} 0 & 0 & -\cos\psi & \xi & \xi^2 & \xi^3 & 0 & 0 \\ 0 & 0 & \sin\psi & \xi \tan\beta_i & 0 & 0 & \xi^2 & \xi^3 \\ 0 & 0 & 0 & \rho(\tan\beta_i - \eta') & -2\xi\eta'\rho & -3\xi^2\eta'\rho & 2\xi\rho & 3\xi^2\rho \end{bmatrix}$$

2 - FRUSTUM ELEMENT

$$\begin{bmatrix} 1 & \xi & \xi^2 & \xi^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \xi & \xi^2 & \xi^3 \\ 0 & -\eta'\rho & -2\xi\eta'\rho & -3\xi^2\eta'\rho & 0 & \rho & 2\xi\rho & 3\xi^2\rho \end{bmatrix}$$

where

$$\rho = \frac{1}{\ell(1 + \eta'^2)}$$

APPENDIX III

MATRIX [A], Eq. (12)

1 - CAP ELEMENT

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\cos\psi & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & \sin\psi & \tan\beta_i & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & \left(\frac{\tan\beta_i - \tan\beta_j}{\ell}\right)\cos^2\beta_j & \frac{-2\sin\beta_j\cos\beta_j}{\ell} & \frac{-3\sin\beta_j\cos\beta_j}{\ell} & \frac{2\cos^2\beta_j}{\ell} & \frac{3\cos^2\beta_j}{\ell} \\ 0 & 0 & -\cos\psi & 1/3 & 1/9 & 1/27 & 0 & 0 \\ 0 & 0 & -\cos\psi & 2/3 & 4/9 & 8/27 & 0 & 0 \end{bmatrix}$$

NOTE that for cap element $\{u_i\}^T = \langle U^i \; W^i \; \chi^i \rangle$ in (U,W) coordinates.

Continued/

2 - FRUSTUM ELEMENT, [A]

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & \frac{-\sin\beta_i \cos\beta_i}{\ell} & 0 & 0 & 0 & \frac{\cos^2\beta_i}{\ell} & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & \frac{-\sin\beta_j \cos\beta_j}{\ell} & \frac{-2\sin\beta_j \cos\beta_j}{\ell} & \frac{-3\sin\beta_j \cos\beta_j}{\ell} & 0 & \frac{\cos^2\beta_j}{\ell} & \frac{2\cos^2\beta_j}{\ell} & \frac{3\cos^2\beta_j}{\ell} \\
1 & 1/3 & 1/9 & 1/27 & 0 & 0 & 0 & 0 \\
1 & 2/3 & 4/9 & 8/27 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Continued/

MATRIX $[A]^{-1}$, Eq. (13)

1 - CAP ELEMENT

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.5 \cos\psi & 0 & 1 & 0 & 0 & 9 & -4.5 \\ 0 & -9.0 \cos\psi & 0 & -4.5 & 0 & 0 & -22.5 & 18. \\ 0 & 4.5 \cos\psi & 0 & 4.5 & 0 & 0 & 13.5 & -13.5 \\ 0 & \begin{array}{l} -\cos\psi(11 \tan\beta_i + \tan\beta_j) \\ -3 \sin\psi \end{array} & 0 & \begin{array}{r} -2 \tan\beta_i \\ -5.5 \tan\beta_j \end{array} & 3 & -\ell(1+\tan^2\beta_j) & \begin{array}{r} -18 \tan\beta_i \\ -4.5 \tan\beta_j \end{array} & \begin{array}{r} 9(\tan\beta_i + \\ \tan\beta_j) \end{array} \\ 0 & \begin{array}{l} \cos\psi(5.5 \tan\beta_i + \tan\beta_j) \\ +2 \sin\psi \end{array} & 0 & \begin{array}{r} \tan\beta_i \\ +5.5 \tan\beta_j \end{array} & -2 & \ell(1+\tan^2\beta_j) & \begin{array}{r} 9 \tan\beta_i \\ +4.5 \tan\beta_j \end{array} & \begin{array}{r} -4.5 \tan\beta_i \\ -9 \tan\beta_j \end{array} \end{bmatrix}$$

Continued/

2 - FRUSTUM ELEMENT, $[A^{-1}]$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-5.5 & 0 & 0 & 1 & 0 & 0 & 9 & -4.5 \\
9 & 0 & 0 & -4.5 & 0 & 0 & -22.5 & 18 \\
-4.5 & 0 & 0 & 4.5 & 0 & 0 & 13.5 & -13.5 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-5.5 \tan \beta_i & 0 & \ell(1+\tan^2 \beta_i) & \tan \beta_i & 0 & 0 & 9 \tan \beta_i & -4.5 \tan \beta_i \\
11 \tan \beta_i + \tan \beta_j & -3 & -2\ell(1+\tan^2 \beta_i) & -2 \tan \beta_i - 5.5 \tan \beta_j & 3 & -\ell(1+\tan^2 \beta_j) & -18 \tan \beta_i - 4.5 \tan \beta_j & 9(\tan \beta_i + \tan \beta_j) \\
-5.5 \tan \beta_i - \tan \beta_j & 2 & \ell(1+\tan^2 \beta_i) & \tan \beta_i + 5.5 \tan \beta_j & -2 & \ell(1+\tan^2 \beta_j) & 9 \tan \beta_i + 4.5 \tan \beta_j & -4.5 \tan \beta_i - 9 \tan \beta_j
\end{bmatrix}
$$

Continued/

APPENDIX IV

USER'S GUIDE

A. Elastic-Plastic Analysis of Axisymmetric Shells with Continuous Meridional Slope

Input Data Instructions

1. Number of Input Control Card (I5)

Col. 1 to 5 - number of structural systems to be analyzed at the same time in one run

Note: Only one card is necessary for any number of input structural systems

2. Title Card (12A6)

Col. 1 to 72 - alphameric information to be printed in the output

3. Control Card (4I5)

Col. 1 to 5 - NN, number of nodal circles (max. 50)

Col. 6 to 10 - NL, number of layers in shell thickness h (max. 20)

Col. 11 to 15 - NLI, number of load increments

Col. 16 to 20 - NUMBC, number of constrained nodes (max. 5)

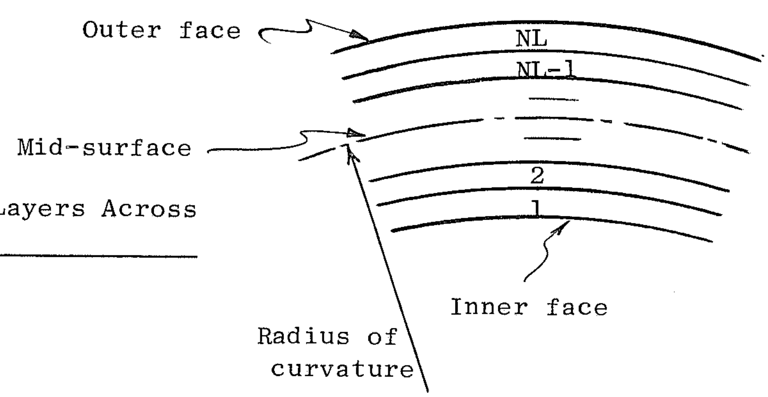


Fig. (A-1) - Numbering of Layers Across Thickness, NL

4. Nodal Coordinate Cards (4F15.0)

One card for each nodal point (total of NN)

4. (Cont'd)

Col. 1 to 15 - r_i , radical coordinate of the node

Col. 16 to 30 - z_i , vertical coordinate of the node

Col. 31 to 45 - PHI_i , latitude angle φ_i of node

Col. 46 to 60 - H_i , shell thickness at node i

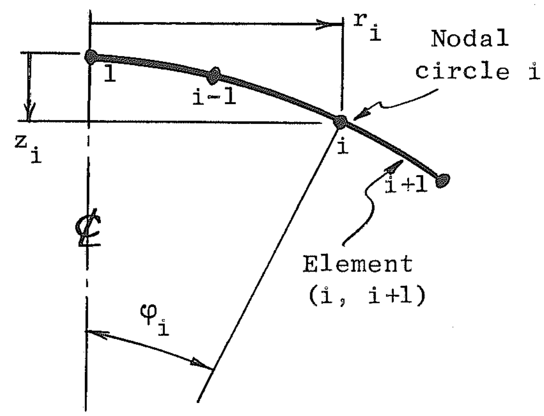


Fig. (A-2) - Nodal Coordinate Sign Convention

5. Element Curvature Cards (2F15.0)

One for each element (total of NN-1)

Col. 1 to 15 - CRV(m,i), Meridional Curvature at i of element (i,i+1)

Col. 16 to 30 - CRV(m,i+1), Meridional Curvature at i+1 of element (i,i+1)

6. Material Index Card (I5)

Col. 1 to 5 - NP, number of points which describe the $\sigma - \epsilon$ relation ($NP \geq 2$)

7. Material Property Card (3E15.0)

(Total of NP cards) describing $\epsilon_i, \sigma_i, E_i$ at each station starting from $\epsilon_1 = 0, \sigma_1, E_1^t$, (elastic limit)

Col. 1 to 15 - ϵ_i , strain at i

Col. 16 to 30 - σ_i , stress at i

Col. 31 to 45 - E_i^t , tangent modulus at i

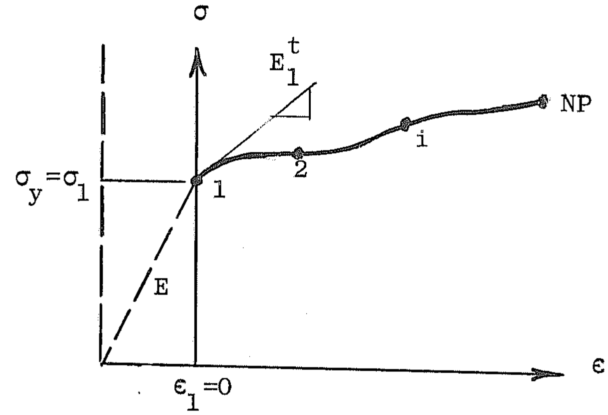


Fig. (A-3) - Stress-strain Diagram

8. Elastic Constants Card (E10.3, F5.0)

   Col. 1 to 10 - E, Elastic Young's Modulus

   Col. 11 to 15 - ν, Poisson ratio

9. Boundary Conditions Cards (I4, 1X, 3I1)

   (Total of NUMBC cards.) For closed top shells, the boundary condition at node 1 is not required.

   Col. 1 to 4 - $NBC_{(i)}$, boundary node number

   Col. 6 - NTAG(1), displacement u_i at boundary node i

   Col. 7 - NTAG(2), displacement w_i at boundary node i

   Col. 8 - NTAG(3), rotation χ_i at boundary node i

   NTAG(i) { = 1 constrained displacement or rotation
   = 0 free displacement or rotation

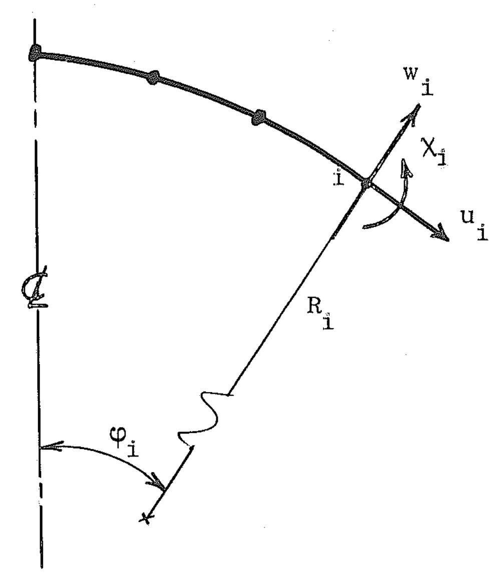


Fig. (A-4) - Sign Convention for Displacement and Rotation

10. Load Index Card (I5)

Col. 1 to 5 - INDEX, two cases may occur:

A. INDEX = 0, (or blank) when non uniform and/or concentrated load is acting on shell. In this case, one must specify the magnitude of loads on all the nodes of shell - see 11.

B. INDEX = number of uniformly loaded nodal points. This can occur when:

1) the entire shell is subjected to a uniformly distributed loading; or

2) the top part of shell (say node 1 to n) is under a uniformly distributed load and no load is acting on the remainder of the shell. (Here INDEX = n.)

When Option B is used only one load card is the input and program will generate the same loading for all the INDEX nodes.

Note: When a concentrated load is acting anywhere, Option B cannot be used.

11. Nodal Load Card(s) (6E12.0)

If INDEX = 0 (or blank), one node-load card per node. If INDEX $\neq 0$, only one load card is required.

Col. 1 to 12 - (meridional concentrated force, P_t) X r_i; where r_i is the radial coordinate of nodal circle (see Fig. 2)

Col. 13 to 24 - (radial concentrated force, P_r) X r_i

Col. 25 to 36 - (concentrated moment, M) X r_i

Col. 37 to 48 - meridional distributed force, p_t (force/area)

Col. 49 to 60 - radial distributed force, p_r (force/area)

Col. 61 to 72 - distributed moment, m (force-length/area)

Note: a. In step 11, the intensity of load at the nodal points are the input; the program will compute the equivalent nodal loads.

b. Steps 10 and 11 must be repeated NLI times (see 3).

c. A concentrated force cannot be applied at the apex.

d. The concentrated loads must be multiplied by their respective radial distances from shell axis.

If more input is wanted, i.e., different structural systems are to be analyzed, card in step 1 is set equal to the number of input cases and steps 2-11 are repeated that many times.

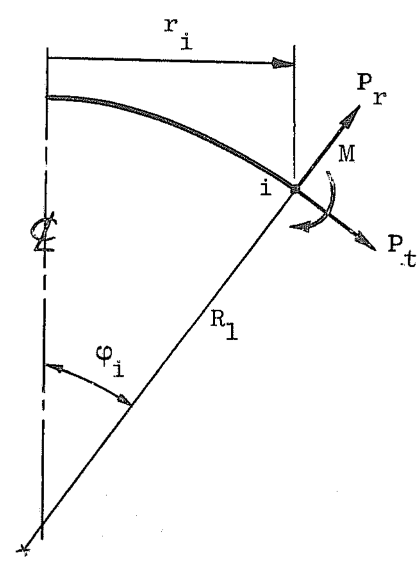


Fig. (A-5) - Nodal Load Sign Convention

Output Description

The following information is printed out on the basis of the above program:

1. Complete echo check of the input data

2. Computed nodal forces on system

3. Incremental and total magnitudes of:

   a. Nodal displacements in u-w-X coordinates (see Fig. 4 for sign convention)

   b. Nodal forces, N_θ, N_φ, M_θ, M_φ

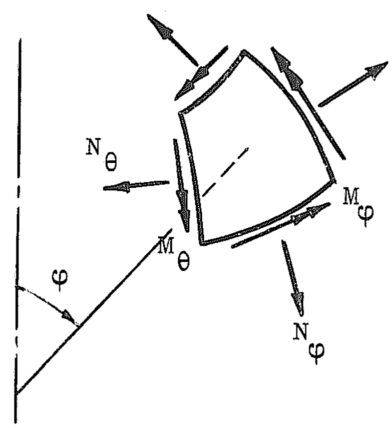


Fig. (A-6) - Sign Convention for Output Forces

4. For each nodal point and at each layer across the thickness, the following quantities are output:

   a. Increments of ε_θ and ε_φ

   b. Total σ_θ , σ_φ , σ_e (equivalent stress) and the modification factor.

$$M_F \begin{cases} = 1 \text{ if state of stress is inside the yield surface} \\ \neq 1 \text{ if state of stress is on the yield surface} \end{cases}$$

$$\sigma_e = (\sigma_\theta^2 + \sigma_\varphi^2 - \sigma_\theta\sigma_\varphi)^{\frac{1}{2}}$$

σ_θ - hoop stress

σ_φ - meridional stress

$\epsilon_\theta, \epsilon_\varphi$ - hoop and meridional strains

Strain and stress are positive when tension.

B. Elastic-Plastic Analysis of Axisymmetric Shells with Discontinuous Meridional Slope

Input Data Instructions

1. (as before)*

2. (as before)

3A. Control Card (5I5)

Col. 1 - 5 - NN, number of nodal circles (max 50)

Col. 6 - 10 - NL, number of layers in shell thickness (max 20), see Fig. (A-1)

Col. 11 - 15 - NLI, number of load increments

Col. 16 - 20 - NUMBC, number of constrained nodes (max 5)

Col. 21 - 25 - NDISC, number of nodes where meridional slope is discontinuous (max 5)

Note: More than one element should be laid out between any two consecutive discontinuities.

3B. Node Number Card (5I5)

The node numbers where slope is discontinuous (NDISC number).

4. One card for each element end (total of 2(NN-1) cards).

Col. 1 - 15 - r_i (or r_j), radial coordinate of node i, (or j)

Col. 16 - 30 - z_i (or z_j), vertical coordinate of node i, (or j)

Col. 31 - 45 - φ_i (or φ_j), latitude angle (deg.) of node i, (or j)

Col. 46 - 60 - H_i (or H_j), shell thickness at node i, (or j)

---

* Refer to the instructions given under the same item number in part A of Appendix IV.

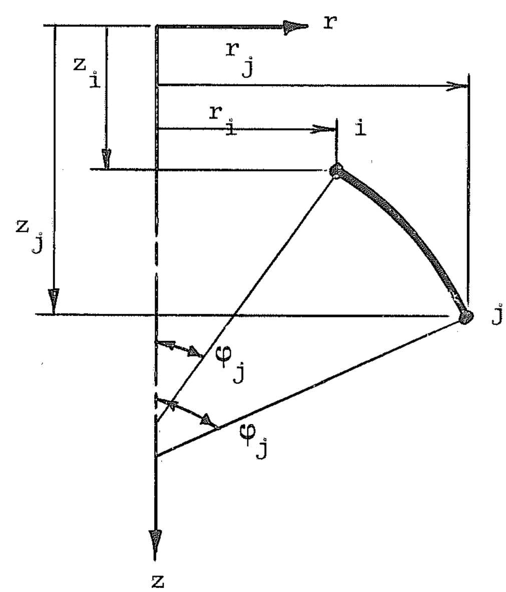


Fig. (B-1) - Nodal Coordinate Sign Convention

5, 6, 7, and 8. (as before)

9. Boundary Conditions Cards (I4, 1X, 3I1)

Total of NUMBC cards. For closed top shells, the boundary condition at node 1 is not required.

Col. 1 - 4 - NBC(i), constrained node number

Col. 6 - NTAG(1), displacement u_r^i

Col. 7 - NTAG(2), displacement u_z^i

Col. 8 - NTAG(3), rotation χ^i

NTAG(i) = { = 1 constrained displacement or rotation
= 0 free displacement or rotation }

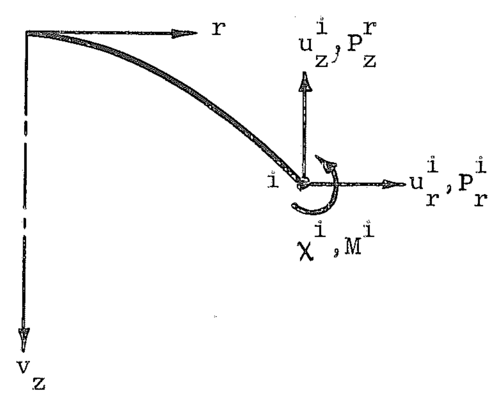


Fig. (B-2) - Sign Convention for Nodal Displacements and Concentrated Loads

10. (as before)

11. As before except that the sign convention for concentrated loads is changed to the one shown in Fig. (B-2). The sign convention for distributed loads remains the same as in Fig. (A-5).

Output Description

The computer output is similar to the one described in part A of this appendix except for the following items:

(a) The incremental and total nodal displacements are output in the (r,z,χ) coordinates (see Fig. B-2 for sign convention).

(b) The interelement forces and strains in the meridional direction are not averaged up at the nodes where meridional slope is discontinuous. Thus, the force and strain quantities at a discontinuity are output separately for the two elements meeting there.

APPENDIX V

```
C     ELASTIC PLASTIC ANALYSIS OF AXISYMMETRIC SHELLS WITH CONTINUOUS
C     MERIDIONAL SLOPE
C
      PROGRAM MAIN (INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4,TAPE5=INPUT,
     1 TAPE6 = OUTPUT ,TAPE 8, TAPE9 )
C
      COMMON/ DIV  / NE , NL , NLI
C
      READ (5,22)NIPT
      DO 100 I=1, NIPT
    1 CALL INPUTD
C
      CALL GEOMTY
C
      DO 100 NLL = 1,NLI
C
      WRITE (6,1000)  NLL
C
      CALL STIFNS (NLL)
C
      CALL NODLOD
C
      CALL DISPL
C
      CALL STRESS(NLL)
C
      CALL MATPP
C
  100 CONTINUE
C
      STOP
C
   22 FORMAT (I5)
 1000 FORMAT(31H1NUMBER OF LOADING INCREMENT = ,I5)
C
      END
1
```

```
      SUBROUTINE INPUTD
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
      COMMON/STSS1 / T(2,20) , F(4,50 )
      COMMON/BNDRCN/ NEQBC,NEBC(15)
      COMMON/INPTDT/ NTAG(5,3) , NBC(5)
      COMMON/BANARG/ NUMEQ,NBNWD,A(150,6),B(150)
      COMMON/ DSPL1/ BT(150)
      COMMON/ NTAP / NTAP1 , NTAP2
C
      WRITE(6,2000)
      READ (5,1000)
      WRITE(6,1000)
C
C     NUMBER OF NODES , LAYERS , LOAD INCREMENTS , AND BOUNDRY CONDITIONS
C
      READ (5,1001)NN,NL, NLI, NUMBC
      WRITE(6,2001)NN,NL, NLI, NUMBC
C
      READ (5,1002) (  R(I),Z(I),PHI(I),H(I), I= 1,NN)
      WRITE(6,2002) (I,R(I),Z(I),PHI(I),H(I), I= 1,NN)
C
      NE = NN - 1
      READ (5,1003) (  (CRV(M,I) , M=1,2),I=1,NE)
      WRITE(6,2003) (I,(CRV(M,I) , M=1,2),I=1,NE)
C
      READ (5,1004) NP,(EP(I),SIGMA(I),ETAN(I),I=1,NP)
      WRITE(6,2004)  (I,EP(I),SIGMA(I),ETAN(I),I=1,NP)
C
      READ (5,1005) E,U
      WRITE(6,2005) E,U
C
      READ (5,1006) (NBC(N) , (NTAG(N,I) , I=1,3) , N=1,NUMBC)
      WRITE(6,2006) (NBC(N) , (NTAG(N,I) , I=1,3) , N=1,NUMBC)
C
C     BAND WIDTH , NUMBER OF SIMULTANEOUS EQUATIONS, NUMBER OF ELEMENTS
C
      NBNWD = 6
      NUMEQ = 3 * NN
      NE = NN - 1
C
C     ANGLE PHI IN TERMS OF RADIAN
C
      DO 10 I = 1,NN
   10 PHI(I) = PHI(I)/57.2957795130822
C
C     NORMALIZED TANGENT MODULII
C
      DO 150 I = 1,NP
  150 ETAN(I) = ETAN(I) / E
C
C     SET UP TOTAL DISPLACEMENTS
```

```
C
      DO 160  I=1,NUMEQ
  160 BT(I) = 0.0
C
C     ELASTIC MODULI IN CASE OF PLANE STRESS (ISOTROPIC)
C
      EE(1,1) = E / (1.-U) / (1.+U)
      EE(1,2) = U * EE(1,1)
      EE(2,1) = EE(1,2)
      FE(2,2) = EE(1,1)
C
C     SET UP STRESS RESULTANTS
C
      DO 220  I=1,NN
      DO 220  M=1,4
  220 F(M,I) = 0.0
C
C     SET UP EQUIVALENT STRESSS , EQUIVALENT PLASTIC STRAIN , STRESS
C     VECTOR , AND (AP) MATRIX
C
      NTAP1 = 8
      NTAP2 = 9
      REWIND NTAP1
      DO 240  I=1,NN
      DO 230  K=1,NL
      STI(K) = SIGMA(1)
      EPL(K) = 0.0
      DO 230  N=1,2
      T(N,K) = 0.0
      DO 230  M=1,2
  230 AP(M,N,K) = 0.0
      WRITE(NTAP1)  STI    ,EPL   , AP, T
  240 CONTINUE
      END FILE NTAP1
C
C
C     BOUNDRY CONDITIONS
C
      L = 0
      DO 250 N=1,NUMBC
      DO 250 I=1,3
      M = NTAG(N,I)
      IF(M .LE. 0)    GO TO 250
      L = L + 1
      NEBC(L) = 3 * (NBC(N) - 1) + I
  250 CONTINUE
      NEQBC = L
C
      RETURN
C
 1000 FORMAT(72H
     1                                                                    )
 1001 FORMAT(4I5)
 1002 FORMAT ( 4F15. )
 1003 FORMAT(2F15.0)
```

```
 1004 FORMAT(I5 / (3E15.0))
 1005 FORMAT(E10.3,F5.0)
 1006 FORMAT(I4,1X,3I1)
C
 2000 FORMAT(1H1)
 2001 FORMAT( //
     1 30H NUMBER OF NODAL CIRCLES . . .    I10 /
     2 30H NUMBER OF LAYERS  . . . . . .    I10 /
     3 30H NUMBER OF LOAD INCREMENTS . .    I10 /
     4 30H NUMBER OF BOUNDRY CONDITIONS     I10 ///)
 2002 FORMAT(2X,4HNODE,6X,11HABSCISSA-R-,8X,12H ORDINATE-Z-,7X,
     1 14HLATITUDE ANGLE , 9X,9HTHICKNESS / 15X,5H(IN.),15X,5H(IN.),
     2  13X,8H(DEGREE),14X,5H(IN.)//(I4,4E20.7))
 2003 FORMAT(30H-CURVATURES AT NODAL CIRCLES       /
     1 8H ELEMENT , 7X , 6HEND(1) , 14X , 6HEND(2)  //(I5,2E20.7))
 2004 FORMAT(1H-,14X ,26HEQUIVALENT PLASTIC STRAIN  ,8X,18HEQUIVALENT ST
     1RESS  ,13X,16HTANGENT MODULUS  //(I5,3E30.7))
 2005 FORMAT( 30H-MODULUS OF ELASTICITY (PSI)  ,E20.7/
     1         30H POISSON RATIO . . . . . . .    ,E20.7///)
 2006 FORMAT(43H BOUNDRY CONDITIONS (1=RESTRAINED,0=FREE)  //
     1 6H POINT , 3X , 3H U  , 2X , 3H W  , 2X , 3HROT //(I6,3I5))
C
      END
1
```

```
      SUBROUTINE GEOMTY
C
C     COMPUTE GEOMETRIC PARAMETERS FOR COPLETELY COMPATIBLE ELEMENT  (3)
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/GEOM4 /YBAR,YP,YPP,RW,XT,ARC,RV,CNP1,CNP2,B(4,8)
      COMMON /GEOM5/ A1,A2,A3,A4
      COMMON/BANARG/NUMEQ,NBNWD,BK(150,6),P(150)
      COMMON/INTEG1/ NUMIPD
      COMMON/INTEG2/YP1(10),FAC(10)
C
      DIMENSION X(12) , W(10) , SINF(50 ) , CSNF(50 )
C
      EQUIVALENCE  (SINF(1),BK(1)) , (CSNF(1),BK(151))
C
      DATA X / 0.0,0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586, 1.0 /
C
      DATA W /  0.066671344308688, 0.149451349150581,
     1 0.219086362515982, 0.269266719309996, 0.295524224714753,
     2 0.295524224714753, 0.269266719309996, 0.219086362515982,
     3 0.149451349150581, 0.066671344308688  /
C
      REWIND 1
      REWIND 2
      REWIND 3
      REWIND 4
C
      NUMIPD = 10
      WRITE(6,1006)
C
      NN = NE + 1
      DO  10  I = 1,NN
      SINF(I) = SIN(PHI(I))
   10 CSNF(I) = COS(PHI(I))
C
C     COMPUTE GEOMETRICAL PARAMETERS
C
      DO 100  I = 1,NE
      DR = R(I+1) - R(I)
      DZ = Z(I+1) - Z(I)
      CORD = SQRT(DR**2 + DZ**2)
      SNT = DR / CORD
      CNT = DZ / CORD
C
      SNB1 = CSNF(I) * CNT - SINF(I) * SNT
      CNB1 = SINF(I) * CNT + CSNF(I) * SNT
      TNB1 = SNB1 / CNB1
      SNB2 = CSNF(I+1) * CNT - SINF(I+1) * SNT
      CNB2 = SINF(I+1) * CNT + CSNF(I+1) * SNT
```

```
      TNB2 = SNB2 / CNB2
C
      CNP1 = CSNF(I) / CNB1
      CNP2 = CSNF(I+1) / CNB2
C
      YPP1 = -CORD * CRV(1,I) / CNB1**3
      YPP2 = -CORD * CRV(2,I) / CNB2**3
C
      A1 = TNB1
      A2 = TNB1 + 0.50 * YPP1
      A3 =-(5.*TNB1 + 4.*TNB2) + (0.5*YPP2 - YPP1)
      A4 = 3. * (TNB1 + TNB2) + 0.50 * (YPP1 - YPP2)
C
      SNP = SIN(PHI(I+1) - PHI(I))
      CNP = COS(PHI(I+1) - PHI(I))
C
C     ESTABLISH STRAIN-DISPLACEMENT MATRICES
C
      NIPD2 = NUMIPD + 2
      DO  70  K = 1,NIPD2
      XT = X(K)
      YBAR = (1.-XT) * (A1 + XT * (A2 + XT * (A3 + XT * A4)))
      YP   = A1*(1.-2.*XT) + XT*(A2*(2.-3.*XT) + XT*(A3*(3.-4.*XT) +
     1 A4 * XT * (4.-5.*XT)))
      YPP  = 2.*(-A1 ^ A2 * (1.-3.*XT)) + XT * (A3 * (6.-12.*XT) +
     1 A4 * XT * (12.-20.*XT))
      RV   = CORD * (SNT + YBAR * CNT)
      RW   = R(I) + XT * RV
      ARC  = SQRT(1. + YP**2)
C
      IF(R(I) .NE. 0.0)   GO TO   20
C
C     STRAIN - DISPLACEMENT MATRIX FOR CENTRAL CAP
C
      CALL B1MATX
      GO TO  30
C
C     STRAIN - DISPLACEMENT MATRIX FOR A FRUSTRUM
C
   20 CALL BMATX
C
   30 IF(K.EQ. 1 .OR. K .EQ. NIPD2)   GO TO 50
      KK = K - 1
      YP1(KK) = YP
      FAC(KK) = 0.50 * CORD * RW * W(KK)
      COF(KK) = ARC * FAC(KK)
C
      DO  40  N = 1,8
      DO  40  M = 1,4
   40 B1(M,N,KK) = B(M,N)
      GO TO  70
C
C     TO BE USED IN STRESS SUBROUTINE
C
   50 KK = (NUMIPD + K) / (NUMIPD + 1)
```

```
      DO  60  N = 1,8
      DO  60  M = 1,4
   60 B2(M,N,KK) = B(M,N)
   70 CONTINUE
C
C     DISPLACEMENT TRANSFORMATION MATRIX
C
      IF(R(I) .NE. 0.0)   GO TO  80
C
C     DISPLACEMENT TRANSFORMATION MATRIX FOR CENTRAL CAP
C
      CALL C1MATX
      GO TO  90
C
C     DISPLACEMENT TRANSFORMATION MATRIX FOR A FRUSTRUM
C
   80 CALL CMATX
C
   90 WRITE (1) B1, COF
      WRITE (2) B2
      WRITE (3) C
      WRITE (4) YP1, FAC, TNB1, SNT, CNT
      WRITE(6,1007)  I,CORD,SNT,CNT,TNB1,TNB2
C
  100 CONTINUE
C
      END FILE 1
      END FILE 2
      END FILE 3
      END FILE 4
C
      RETURN
C
 1006 FORMAT(8H-ELEMENT,5X,4HCORD,14X,4HSINT,14X,4HCSNT,14X,4HTNB1,
     1 14X,4HTNB2 //)
 1007 FORMAT(I4,5E18.7)
C
      END
1
```

```
      SUBROUTINE STIFNS(NLL)
C
C     SUBROUTINE TO SET UP SYSTEM STIFFNESS MATRIX
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/ GEOM4/YBAR,YP, YPP , RW , XT , ARC , RV , CNP1,CNP2,B(4,8)
      COMMON/ MAT3 / SUMA(2,50 ,3), SUMB(2,50 ,3), TB(20), RAT(20)
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON/BANARG/NUMEQ,NBNWD,BK(150,6),P(150)
      COMMON/INTEG1/ NUMIPD
      COMMON DD(2,8,50), DB(6,2,50), RB(2,50)
C
      DIMENSION SKG(8,8) , TEMP(8,8)
      EQUIVALENCE (SKG(1),SK(1)) , (TEMP(1),SKI(1))
C
      REWIND 1
      REWIND 3
C
      DO 1   N=1,6
      DO 1  M=1,NUMEQ
    1 BK(M,N) = 0.0
C
      DO 100 I=1,NE
      II = I
C
      READ (1)  B1, COF
      READ (3)  C
C
C     ELEMENT STIFFNESS MATRIX IN GENERALIZED COORDINATES
C
      DO 5   N=1,8
      DO 5   M=1,8
    5 SKG(M,N) = 0.0
C
      DO 50  L=1,NUMIPD
      LL = L + 1
      IF (NLL .NE. 1)   GO TO  7
      CALL D1MATX(LL,II)
      GO TO  8
C
    7 CALL DMATX(LL,II)
C
    8 DO 10  N=1,8
      DO 10  M=1,4
   10 B(M,N) = 0.0
C
      DO 20  N=1,8
      DO 20  M=1,4
      DO 20  K=1,4
   20 B(M,N) = B(M,N) + D(M,K) * B1(K,N,L)
C
      DO 30  N=1,8
      DO 30  M=1,8
```

```
   30 SKI(M,N) = 0.0
C
      DO 40  N=1,8
      DO 40  M=1,8
      DO 40  K=1,4
   40 SKI(M,N) = SKI(M,N) + B1(K,M,L) * B(K,N)
C
      DO 50  N=1,8
      DO 50  M=1,8
   50 SKG(M,N) = SKG(M,N) + COF(L) * SKI(M,N)
C
C     ELEMENT STIFFNESS MATRIX IN SYSTEM CO-ORDINATES
C
      DO 60  N=1,8
      DO 60  M=1,8
   60 TEMP(M,N) = 0.0
C
      DO 70  N=1,8
      DO 70  M=1,8
      DO 70  K=1,8
   70 TEMP(M,N) = TEMP(M,N) + SKG(M,K) * C(K,N)
C
      DO 80 N =1, 8
      DO 80 M =1, 8
   80 SK(M,N) = 0.
C
      DO 90  N=1,8
      DO 90  M=1,8
      DO 90  K=1,8
   90 SK(M,N) = SK(M,N) + C(K,M) * TEMP(K,N)
      IF(R(I) .NE. 0.0)   GO TO  95
      SK(1,1) = 1.0
      SK(3,3) = 1.0
C
   95 CONTINUE
C
C     CONDENSATION
C
      DET = SK(7,7) * SK(8,8) - SK(7,8) * SK(8,7)
      DD(1,1,I) = SK(8,8) / DET
      DD(2,2,I) = SK(7,7) / DET
      DD(1,2,I) =-SK(7,8) / DET
      DD(2,1,I) =-SK(8,7) / DET
C
      DO 105 N =1, 6
      K = N + 2
      DO 105 M = 1,2
      DD(M,K,I) = 0.
  105 DB(N,M,I) = 0.
C
      DO 115  N =1, 6
      DO 110 M =1,2
      DO 110 K =1, 2
      J = N + 2
      DB(N,M,I) = DB(N,M,I) + SK(N,K+6) * DD(K,M,I)
```

```
  110 DD(M,J,I) = DD(M,J,I) + DD(M,K,I) * SK(K+6,N)
      DO 115 JJ =1, 6
      DO 115 KK = 1, 2
  115 SK(JJ,N) = SK(JJ,N) - SK(JJ,KK+6) * DD(KK,N+2,I)
C
      DO 100 M=1,6
      DO 100 N=M,6
      II = 3*(I-1) + M
      JJ = N-M+1
  100 BK(II,JJ) = BK(II,JJ) + SK(M,N)
      END FILE 9
C
      RETURN
      END
1
```

```
      SUBROUTINE NODLOD
C
C     SUBROUTINE TO GENERATE CONSISTENT NODAL LOADS
C
      COMMON/ DIV  /NE,NL,NLI
      COMMON/BANARG/ NUMEQ,NBNWD,A(150,6),B(150)
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/GEOM2 / COF(10) , B0(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/INTEG1/NUMIPD
      COMMON/INTEG2/YP1(10),FAC(10)
      COMMON DD(2,8,50), DB(6,2,50), RB(2,50)
C
      DIMENSION B1(150),DP(8),PSTAR(8),TEMP(8),X(10)
C
      DATA X /     0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586        /
C
      REWIND 3
      REWIND 4
C
      READ (5,900) INDX
      IF (INDX .EQ. 0) GO TO 5
      READ (5,1000) B(1), B(2), B(3), B1(1), B1(2), B1(3)
      NE3 = 3* INDX
      DO 3 I= 4,NE3, 3
      B (I) = B (1)
      B (I+1) = B (2)
      B (I+2) = B (3)
      B1(I) = B1(1)
      B1(I+1) = B1(2)
    3 B1(I+2) = B1(3)
      NN = NE +1
      IF (INDX .EQ. NN ) GO TO 7
      NEX = NE3 + 1
      NE3 = 3 * NN
      DO 4 I = NEX, NE3
      B (I) = 0.0
    4 B1(I) = 0.0
      GO TO 7
C
    5 NE3 = 3 * (NE + 1)
      READ (5,1000) (2(I),B(I+1),B(I+2),B1(I),B1(I+1),B1(I+2),I=1,NE3,3)
    7 WRITE(6,2000) (B(I),B(I+1),B(I+2),B1(I),B1(I+1),B1(I+2),I=1,NE3,3)
C
      DO 100  I=1,NE
      L = 3 * (I-1)
C
      READ  (3)  C
      READ  (4)  YP1, FAC, TNB1, SNT, CNT
C
      DO 10   J=1,8
   10 PSTAR(J) = 0.0
C
```

```
      DO 20   K=1,NUMIPD
      PU = B1(L+1) + X(K) * (B1(L+4) - B1(L+1))
      PW = B1(L+2) + X(K) * (B1(L+5) - B1(L+2))
C
      IF(R(I) .NE. 0.0)   GO TO  15
      SUM1 = (PU - PW * YP1(K)) * FAC(K)
      SUM2 = (PU * YP1(K) + PW) * FAC(K)
      DP(1) = 0.0
      DP(2) = 0.0
      DP(3) = -SUM1 * CNT + SUM2 * SNT
      DP(4) = (SUM1 + SUM2 * TNB1) * X(K)
      DP(5) = SUM1 * X(K) * X(K)
      DP(6) = X(K) * DP(5)
      DP(7) = SUM2 * X(K) * X(K)
      DP(8) = DP(7) * X(K)
      GO TO  16
C
   15 DP(1) = (PU - PW * YP1(K)) * FAC(K)
      DP(2) = X(K) * DP(1)
      DP(3) = X(K) *DP(2)
      DP(4) = X(K) *DP(3)
      DP(5) = (PU * YP1(K) + PW) * FAC(K)
      DP(6) = X(K) *DP(5)
      DP(7) = X(K) * DP(6)
      DP(8) = X(K) * DP(7)
C
   16 DO 20   J=1,8
   20 PSTAR(J) = PSTAR(J) + DP(J)
C
      DO 30   J=1,8
   30 TEMP(J) = 0.0
C
      DO 40   N=1,8
      DO 40   M=1,8
   40 TEMP(N) = TEMP(N) + C(M,N) * PSTAR(M)
C
C     CONDENSATION
C
      RB(1,I) = TEMP(7)
      RB(2,I) = TEMP(8)
      DO   45 N =1, 6
      DO   45 K =1, 2
   45 TEMP(N) = TEMP(N) - DB(N,K,I) * RB(K,I)
C
      DO 50   N=1,6
      LL = L + N
   50 B(LL) = B(LL) + TEMP(N)
C
  100 CONTINUE
C
      WRITE(6,2003) (B(I) , I=1,NUMEQ)
C
      RETURN
C
  900 FORMAT (I5)
```

```
 1000 FORMAT (6E12.0)
 2000 FORMAT(1H1,20X,18HCONCENTRATED LOADS ,42X,17HDISTIBUTED LOADS //
     1 8X,10HMERIDIONAL ,12X,6HRADIAL ,14X,6HMOMENT ,12X,10HMERIDIONAL ,
     2 12X, 6HRADIAL ,14X,6HMOMENT /(6E20.7))
 2003 FORMAT(44H1CONSISTENT EXTERNAL LOADS AT NODAL CIRCLES      //
     1 8X,10HMERIDIONAL ,12X,6HRADIAL ,14X,6HMOMENT /(3E20.7))
C
      END
1
```

```
      SUBROUTINE DISPL
C
      COMMON /BANARG/  NN, MM,A(150, 6),B(150)
      COMMON/BNDRCN/ NEQBC,NEBC(15)
      COMMON/ DSPL1/ BT(150)
C
      DO 100  N=1,NEQBC
      I = NEBC(N)
      A(I,1) = 1.0
      B(I) = 0.0
      DO 100  J=2,MM
      A(I,J) = 0.0
      L = I-J+1
      IF(L .LE. 0)   GO TO 100
      A(L,J) = 0.0
  100 CONTINUE
C
      CALL BANSOL
C
      DO 200  I=1,NN
  200 BT(I) = BT(I) + B(I)
C
      WRITE(6,2001) (B(I) , I=1,NN)
      WRITE(6,2002) (BT(I), I=1,NN)
C
      RETURN
C
 2001 FORMAT(50H1DISPLACEMENT INCREMENTS AT NODAL CIRCLES              //
     1 8X,10HMERIDIONAL ,12X,6HRADIAL ,13X,8HROTATION /(3E20.7))
 2002 FORMAT(50H1TOTAL DISPLACEMENTS AT NODAL CIRCLES                  //
     1 8X,10HMERIDIONAL ,12X,6HRADIAL ,13X,8HROTATION /(3E20.7))
C
      END
1
```

```
      SUBROUTINE  STRESS(NLL)
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/STSS1 / T(2,20) , F(4,50 )
      COMMON/STSS2 / ALPHA(8),DSTR(2) ,BE(2,8),DFRC(4)
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
      COMMON/BANARG/ NUMEQ,NBNWD,A(150,6),B(150)
      COMMON/INTEG1/ NUMIPD
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON DD(2,8,50), DB(6,2,50), RB(2,50)
C
      DIMENSION  TEMP(4,8) , DF(4,50), Q(8)
      EQUIVALENCE  (TEMP(1),SKI(1)) , (DF(1),A(1))
C
      REWIND  2
      REWIND  3
C
      NN = NE + 1
      ANL = NL
      DO  10  I=1,NN
      DO   5  K=1,NL
      DO   5  M=1,2
    5 DST(M,K,I) = 0.0
      DO  10  N=1,4
   10 DF(N,I) = 0.0
C
      WRITE (6,1000)
C
      DO 140  I=1,NE
      READ (2)  B2
      READ (3)  C
C
      DO  20  M=1,8
   20 ALPHA(M) = 0.0
C
C     RECOVERING CONDENSED DEG OF FREEDOM
C
      Q(7) = 0.
      Q(8) = 0.
      IX = 3 * (I-1)
C
      DO 15 M =1, 2
      DO 12 K =1, 2
      L = M + 6
   12 Q(L) = Q(L) + DD(M,K,I) * RB(K,I)
      DO 15 J = 1, 6
      JJ = J + IX
   15 Q(L) = Q(L) - DD(M,J+2, I) * B(JJ)
C
      DO 17 JJ = 1,6
      J = IX + JJ
   17 Q(JJ) = B(J)
C
```

```
      DO  30  M=1,8
      DO  30  N=1,8
   30 ALPHA(M) = ALPHA(M) + C(M,N) * Q(N)
C
      DO 140  L=1,2
      II = I+L-1
C
C     DETERMINATION OF STRAIN ALONG THE THICKNESS
C
      DO  70  K=1,NL
      AK = K
      ZBAR = ((AK - 0.50)/ANL - 0.50) * H(II)
      DO  40  N=1,8
      DO  40  M=1,2
   40 BE(M,N) = B2(M,N,L) + ZBAR * B2(M+2,N,L)
C
      DO  50  M=1,2
   50 DSTR(M) = 0.0
C
      DO  60  M=1,2
      DO  60  N=1,8
   60 DSTR(M) = DSTR(M) + BE(M,N) * ALPHA(N)
C
      DO  70  M=1,2
   70 DST(M,K,II) = DST(M,K,II) + 0.50 * DSTR(M)
C
C     DETERMINATION OF STRESS RESULTANTS
C
      III = I
      LL = 1 + (NUMIPD+1) * (L-1)
      IF (NLL .NE. 1)   GO TO 80
      CALL  D1MATX(LL,III)
      GO TO 90
C
   80 CALL  DMATX(LL,III)
C
   90 DO 100  N=1,8
      DO 100  M=1,4
  100 TEMP(M,N) = 0.0
C
      DO 110  N=1,8
      DO 110  M=1,4
      DO 110  K=1,4
  110 TEMP(M,N) = TEMP(M,N) + D(M,K) * B2(K,N,L)
C
      DO 120  M=1,4
  120 DFRC(M) = 0.0
C
      DO 130  M=1,4
      DO 130  K=1,8
  130 DFRC(M) = DFRC(M) + TEMP(M,K) * ALPHA(K)
C
      WRITE (6,1001)  I,L,(DFRC(M) , M=1,4)
C
      DO 140  M=1,4
```

```
  140 DF(M,II) = DF(M,II) + 0.50 * DFRC(M)
C
      DO 150  M=1,2
      DO 150  K=1,NL
      DST(M,K, 1) = 2.0 * DST(M,K, 1)
  150 DST(M,K,NN) = 2.0 * DST(M,K,NN)
C
      DO 160  M=1,4
      DF(M, 1) = 2.0 * DF(M, 1)
  160 DF(M,NN) = 2.0 * DF(M,NN)
C
C     TOTAL STRESS RESULTANTS
C
      DO 170  I=1,NN
      DO 170  M=1,4
  170 F(M,I) = F(M,I) + DF(M,I)
C
      WRITE (6,1002)  (I,(DF(M,I),M=1,4),I=1,NN)
      WRITE (6,1003)  (I,( F(M,I),M=1,4),I=1,NN)
      DO 180  I=1,NN,2
      I1 = I + 1
  180 WRITE (6,1004)  (I,K, (DST(M,K,I),M=1,2),I1 ,K,(DST(M,K,I1 ),
     1 M=1,2),K=1,NL)
C
      RETURN
 1000 FORMAT(55H1INCREMENTS OF STRESS RESULTANTS AT THE END OF ELEMENTS/
     1 8H ELEMENT,2X,7HEND NO.,7X, 16HMERIDIONAL FORCE ,9X,16HCIRCUMFER.
     2 FORCE ,8X,18HMERIDIONAL MOMENT  ,7X,18HCIRCUMFER. MOMENT  //)
 1001 FORMAT(I5,4X,I5,4E25.7)
 1002 FORMAT(50H1INCREMENTS OF STRESS RESULTANTS AT NODAL CIRCLES    /
     1 6H  NODE,10X,16HMERIDIONAL FORCE ,9X,16HCIRCUMFER. FORCE ,8X,
     2 18HMERIDIONAL MOMENT ,7X,18HCIRCUMFER. MOMENT //(I5,4E25.7))
 1003 FORMAT(50H1TOTAL STRESS RESULTANTS AT NODAL CIRCLES           /
     1 6H  NODE,10X,16HMERIDIONAL FORCE ,9X,16HCIRCUMFER. FORCE ,8X,
     2 18HMERIDIONAL MOMENT ,7X,18HCIRCUMFER. MOMENT //(I5,4E25.7))
 1004 FORMAT(30H2INCREMENTS OF STRAINS                               /
     15H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER.,21X,
     25H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER.          //
     3(I4,2X,I4,2E20.5,20X,I4,2X,I4,2E20.5))
C
      END
1
```

```
      SUBROUTINE  MATPP
C
C     SUBROUTINE TO ESTABLISH MATERIAL PROPERTIES
C
      COMMON/ DIV   / NE , NL , NLI
      COMMON/ MAT1  / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT2  / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
      COMMON/ MAT3  / SUMA(2,50 ,3), SUMB(2,50 ,3), TB(20), RAT(20)
      COMMON/ MAT4  / DSTP(2) , DSTE(2) , DT(2) , TT(2)
      COMMON/STSS1  / T(2,20) , F(4,50 )
      COMMON/ NTAP  / NTAP1 , NTAP2
C
      REWIND NTAP1
      REWIND NTAP2
C
      NN = NE + 1
      DO   5  J=1,3
      DO   5  I=1,NN
      DO   5  N=1,2
      SUMA( N,I,J) = 0.0
    5 SUMB( N,I,J) = 0.0
C
      DO 200  I=1,NN
C
      READ (NTAP1)  STI, EPL, AP, T
C
      DO 150  K=1,NL
      DO  10  M=1,2
   10 DSTP(M) = 0.0
C
      DO  20  M=1,2
      DO  20  N=1,2
   20 DSTP(M) = DSTP(M) + AP(M,N,K) * DST(N,K,I)
C
      DO  30  M=1,2
   30 DSTE(M) = DST(M,K,I) - DSTP(M)
C
      DO  40  M=1,2
   40 DT(M) = 0.0
C
      DO  50  M=1,2
      DO  50  N=1,2
   50 DT(M) = DT(M) + EE(M,N) * DSTE(N)
C
      DO  60  M=1,2
   60 TT(M) = T(M,K) + DT(M)
C
      S1 = TT(1) - 0.50 * TT(2)
      S2 = TT(2) - 0.50 * TT(1)
      TBAR = SQRT(TT(1)*S1 + TT(2)*S2)
      IF (DSTP(1) .EQ. 0.0 .AND. DSTP(2) .EQ. 0.0)   GO TO 70
C
      S1 = T(1,K) - 0.50 * T(2,K)
      S2 = T(2,K) - 0.50 * T(1,K)
      FLAG = S1 * DT(1) + S2 * DT(2)
```

```
      IF (FLAG .LT. 0.0)   GO TO 70
C
      EP1 = DSTP(1) + 0.50 * DSTP(2)
      EP2 = DSTP(2) + 0.50 * DSTP(1)
      DEP = SQRT(4./3. * (DSTP(1)*EP1 + DSTP(2)*EP2))
      EPL(K) = EPL(K) + DEP
      GO TO 80
C
   70 STM = 0.999 * STI(K)
      IF (TBAR .LT. STM)   GO TO 90
C
   80 KK = K
      II = I
      CALL  INTERP(KK,II)
      RAT(K) = STN / TBAR
      GO TO 100
C
   90 ZT = 1.0
      STN = STI(K)
      RAT(K) = 1.0
C
  100 DO 110  M=1,2
  110 T(M,K) = RAT(K) * TT(M)
C
      S1 = T(1,K) - 0.50 * T(2,K)
      S2 = T(2,K) - 0.50 * T(1,K)
      TB(K) = SQRT(S1 * T(1,K) + S2 * T(2,K))
      IF (TB(K) .EQ. 0.0)  GO TO 120
      S1 = S1 / TB(K)
      S2 = S2 / TB(K)
      SS1 = (S1 + U*S2) * (1. - ZT)
      SS2 = (S2 + U*S1) * (1. - ZT)
      DNOM = (1.-U)*(1.+U)*ZT + S1*SS1 + S2*SS2
      S1 = S1 / DNOM
      S2 = S2 / DNOM
      AP(1,1,K) = S1 * SS1
      AP(1,2,K) = S1 * SS2
      AP(2,1,K) = S2 * SS1
      AP(2,2,K) = S2 * SS2
      GO TO 135
C
  120 DO 130  M=1,2
      DO 130  N=1,2
  130 AP(M,N,K) = 0.0
C
  135 AK = K
      DO 140  N=1,2
      SUMA( N,I,1) = SUMA( N,I,1) + AP(1,N,K)
      SUMB( N,I,1) = SUMB( N,I,1) + AP(2,N,K)
      SUMA( N,I,2) = SUMA( N,I,2) + AP(1,N,K) * (AK-0.50)
      SUMB( N,I,2) = SUMB( N,I,2) + AP(2,N,K) * (AK-0.50)
      SUMA( N,I,3) = SUMA( N,I,3) + AP(1,N,K) * (AK*(AK-1.) + .33333333)
  140 SUMB( N,I,3) = SUMB( N,I,3) + AP(2,N,K) * (AK*(AK-1.) + .33333333)
C
  150 STI(K) = STN
```

```
C
      WRITE(NTAP2)  STI, EPL, AP, T
      WRITE (6,1000) (I,(K,(T(M,K  ),M=1,2),TB(K),RAT(K)), K=1, NL)
C
  200 CONTINUE
      END FILE NTAP2
C
      NTAPT = NTAP1
      NTAP1 = NTAP2
      NTAP2 = NTAPT
C
      RETURN
C
 1000 FORMAT(30H2STRESS DISTRIBUTION                                  /
     1 8H  NODE   , 2X , 5HLAYER , 11X , 10HMERIDIONAL , 15X , 10HCIRCUM
     2FER.  ,  11X , 18HEQUIVALENT STRESS  , 4X , 20HMODIFICATION FACTOR
     3  // (I5 , 3X , I5 , 3E25.7 , F20.5))
C
      END
1
```

```
      SUBROUTINE  INTERP(K,I)
C
C     SUBROUTINE FOR LINEAR INTERPOLATION OF MATERIAL PROPERTIES
C
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
C
      IF (EPL(K) .GT. EP(NP))   GO TO 100
C
      DO  10  IP=2,NP
      IF (EPL(K) .LE. EP(IP))   GO TO  50
   10 CONTINUE
C
   50 RHO = (EPL(K) - EP(IP-1)) / (EP(IP) - EP(IP-1))
      STN = SIGMA(IP-1) + RHO * (SIGMA(IP) - SIGMA(IP-1))
      ZT  =  ETAN(IP-1) + RHO * ( ETAN(IP) -  ETAN(IP-1))
      RETURN
C
  100 WRITE (6,1000)  K,I,EPL(K)
      STOP
C
 1000 FORMAT(I5,I5,E20.5 / 40H-MATERIAL PROP. DATA IS EXCEEDED         )
      END
1
```

```
      SUBROUTINE B1MATX
C
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/GEOM4 /YBAR,YP,YPP,RW,XT,ARC,RV,CNP1,CNP2,B(4,8)
      COMMON /GEOM5/ A1,A2,A3,A4
C
      DO 10   N=1,3
      DO 10   M=1,4
   10 B(M,N) = 0.0
C
      RHO = 1. / (CORD * ARC**2)
      AMU = 1. / RV
      ALPHA = RHO / ARC
      PHI =  ALPHA * RHO * YPP
      PSI = (SNT + YP * CNT) * ALPHA * AMU
      GAMA = 2.0*(A2-A1) + XT*(3.0*(A3-A2) + XT*(4.0*(A4-A3) -
     1 XT*5.0*A4))
      OMG = 2. * YP * ALPHA / CORD
      TET = (1. - YP * YP ) * PHI
C
      B(1,4) = RHO * (1. + YP * TNB1)
      B(2,4) = AMU * CNP1
      B(3,4) = (1. + YP * (2. * TNB1 - YP)) * PHI
      B(4,4) = GAMA * PSI
C
      B(1,5) = 2. * RHO * XT
      B(2,5) = AMU * XT * SNT
      B(3,5) = 2. * XT * TET + OMG
      B(4,5) = 2. * YP * PSI
C
      B(1,6) = 1.5 * XT * B(1,5)
      B(2,6) = XT * B(2,5)
      B(3,6) = 3. * XT * (XT * TET + OMG )
      B(4,6) = 1.5 * XT * B(4,5)
C
      B(1,7) = YP * B(1,5)
      B(2,7) = AMU * XT * CNT
      B(3,7) = 2. * (2.*PHI * XT * YP - ALPHA/CORD)
      B(4,7) = -2. * PSI
C
      B(1,8) = 1.50 * B(1,7) * XT
      B(2,8) = B(2,7) * XT
      B(3,8) = 6. * (PHI * XT * YP - ALPHA/CORD) * XT
      B(4,8) = -3. * PSI * XT
C
      RETURN
      END
1
```

```
      SUBROUTINE BMATX
C
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/GEOM4 /YBAR,YP,YPP,RW,XT,ARC,RV,CNP1,CNP2,B(4,8)
C
      RHO = 1./(CORD * ARC**2)
      AU  = 2./(CORD**2 * ARC**3)
      PSI = (SNT + YP * CNT)/(CORD * RW * ARC**3)
      PHI = YPP / (CORD**2 * ARC**5)
      OMG = YP * AU
C
      B(1,1) = 0.0
      B(2,1) = SNT / RW
      B(3,1) = 0.0
      B(4,1) = 0.0
C
      B(1,2) = RHO
      B(2,2) = B(2,1) * XT
      B(3,2) = (1.-YP**2) * PHI
      B(4,2) = YP * PSI
C
      B(1,3) = 2. * XT * RHO
      B(2,3) = B(2,2) * XT
      B(3,3) = B(3,2) * 2. * XT + OMG
      B(4,3) = B(4,2) * 2. * XT
C
      B(1,4) = B(1,3) * 1.5 * XT
      B(2,4) = B(2,3) * XT
      B(3,4) = 3. * XT * ( XT * B(3,2) + OMG )
      B(4,4) = B(4,3) * 1.5 * XT
C
      B(1,5) = 0.0
      B(2,5) = CNT / RW
      B(3,5) = 0.0
      B(4,5) = 0.0
C
      B(1,6) = YP * RHO
      B(2,6) = B(2,5) * XT
      B(3,6) = 2. * YP * PHI
      B(4,6) = -PSI
C
      B(1,7) = 2. * B(1,6) * XT
      B(2,7) = B(2,6) * XT
      B(3,7) = 2. * B(3,6) * XT - AU
      B(4,7) = -2. * PSI * XT
C
      B(1,8) = 1.5 * B(1,7) * XT
      B(2,8) = B(2,7) * XT
      B(3,8) = 3. * (B(3,6) * XT - AU) * XT
      B(4,8) = 1.5 * B(4,7) * XT
C
      RETURN
      END
1
```

```
      SUBROUTINE C1MATX
C
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/ GEOM4/YBAR,YP, YPP , RW , XT , ARC , RV , CNP1,CNP2,B(4,8)
C
      DO 10   N=1,8
      DO 10   M=1,8
   10 C(M,N) = 0.0
      TICJ = TNB1 * CNB2
C
      C(3,2) = 1.0
      C(4,2) = CNT * 5.5
      C(5,2) = -9. * CNT
      C(6,2) = 4.5* CNT
      C(7,2) = -CNT * (11. * TNB1 + TNB2) - 3. * SNT
      C(8,2) =  CNT * (5.5 * TNB1 + TNB2) + 2. * SNT
C
      C(4,4) = CNB2
      C(5,4) = -4.5 * CNB2
      C(6,4) = -C(5,4)
      C(7,4) = -2. * TICJ - 2.5 * SNB2
      C(8,4) =       TICJ + 3.5 * SNB2
C
      C(4,5) = -SNB2
      C(5,5) = 4.5 * SNB2
      C(6,5) = -C(5,5)
      C(7,5) = SNB2 * (2. * TNB1 + 5.5 * TNB2) + 3. * CNB2
      C(8,5) =-SNB2 * (      TNB1 + 5.5 * TNB2) - 2. * CNB2
C
      C(7,6) = -CORD / CNB2 **2
      C(8,6) = -C(7,6)
C
      C(4,7) = 9.
      C(5,7) = -22.5
      C(6,7) = 13.5
      C(7,7) = -18. * TNB1 - 4.5 * TNB2
      C(8,7) =  9.  * TNB1 + 4.5 * TNB2
C
      C(4,8) = -4.5
      C(5,8) = 18.
      C(6,8) = -13.5
      C(7,8) = 9. * (TNB1 + TNB2)
      C(8,8) = -4.5 * TNB1 - 9.0 * TNB2
C
      RETURN
      END
1
```

```
      SUBROUTINE CMATX
C
C     SUBROUTINE TO CONSTRUCT DISPL. TRANS. MATRIX IN U-W CO-ORDINATES
C
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
C
      TJCI = TNB2 * CNB1
      TICJ = TNB1 * CNB2
C
      C(1,1) =  CNB1
      C(2,1) = - 5.5 * CNB1
      C(3,1) =   9. * CNB1
      C(4,1) = -4.5 * CNB1
      C(5,1) = SNB1
      C(6,1) = -5.5 * SNB1
      C(7,1) = 8. * SNB1 + TJCI
      C(8,1) = -3.5 * SNB1 - TJCI
C
      C(1,2) = -SNB1
      C(2,2) = 5.5 * SNB1
      C(3,2) = -9. * SNB1
      C(4,2) = 4.5 * SNB1
      C(5,2) = CNB1
      C(6,2) = 5.5 * TNB1 * SNB1
      C(7,2) = -SNB1 * (11. * TNB1 + TNB2 ) - 3. * CNB1
      C(8,2) = SNB1 * (5.5 * TNB1 + TNB2 ) + 2. * CNB1
C
      C(1,3) = 0.0
      C(2,3) = 0.0
      C(3,3) = 0.0
      C(4,3) = 0.
      C(5,3) = 0.
      C(6,3) = CORD / CNB1**2
      C(7,3) = -2.*C(6,3)
      C(8,3) = C(6,3)
C
      C(1,4) = 0.0
      C(2,4) = CNB2
      C(3,4) = -4.5 * CNB2
      C(4,4) = -C(3,4)
      C(5,4) = 0.0
      C(6,4) = TICJ
      C(7,4) = -2. * TICJ - 2.5 * SNB2
      C(8,4) = TICJ + 3.5 * SNB2
C
      C(1,5) = 0.0
      C(2,5) = -SNB2
      C(3,5) = 4.5 * SNB2
      C(4,5) = -C(3,5)
      C(5,5) = 0.0
      C(6,5) = -TNB1 * SNB2
      C(7,5) =  SNB2 * (2. * TNB1 + 5.5 * TNB2) + 3. * CNB2
      C(8,5) = -SNB2 * (      TNB1 + 5.5 * TNB2) - 2. * CNB2
C
```

```
      C(1,6) = 0.0
      C(2,6) = 0.0
      C(3,6) = 0.0
      C(4,6) = 0.0
      C(5,6) = 0.0
      C(6,6) = 0.0
      C(7,6) = -CORD / CNB2**2
      C(8,6) = -C(7,6)
C
      C(1,7) = 0.
      C(2,7) = 9.
      C(3,7) = -22.5
      C(4,7) = 13.5
      C(5,7) = 0.
      C(6,7) = 9. * TNB1
      C(7,7) = -18. * TNB1 - 4.5 * TNB2
      C(8,7) =  9.0 * TNB1 + 4.5 * TNB2
C
      C(1,8) = 0.
      C(2,8) = -4.5
      C(3,8) = 18.
      C(4,8) = -13.5
      C(5,8) = 0.
      C(6,8) = -4.5 * TNB1
      C(7,8) = 9. * (TNB1 + TNB2)
      C(8,8) = -4.5 * TNB1 - 9.0 * TNB2
C
      RETURN
      END
1
```

```
      SUBROUTINE  D1MATX(L,I)
C
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
C
      DIMENSION  X(12)
C
      DATA X / 0.0,0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586, 1.0 /
C
      HI = H(I) + X(L) * (H(I+1) - H(I))
      DO  10  N=1,2
      DO  10  M=1,2
   10 D(M,N) = EE(M,N) * HI
C
      DO  20  N=3,4
      DO  20  M=1,2
   20 D(M,N) = 0.0
C
      DO  30  N=1,2
      DO  30  M=3,4
   30 D(M,N) = 0.0
C
      DO  40  N=3,4
      DO  40  M=3,4
   40 D(M,N) = D(M-2,N-2) * HI**2 / 12.
C
      RETURN
      END
1
```

```
      SUBROUTINE  DMATX(L,I)
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(50 ),Z(50 ),PHI(50 ),H(50 ) , CRV(2,50 )
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT3 / SUMA(2,50 ,3), SUMB(2,50 ,3), TB(20), RAT(20)
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
C
      DIMENSION   X(12) , TEMP(2,2,3) , TMP(2,2)
C
      DATA X / 0.0,0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586, 1.0 /
C
      DO  10  N=1,4
      DO  10  M=1,4
   10 D(M,N) = 0.0
C
      ANL = NL
      HI = H(I) + X(L) * (H(I+1) - H(I))
      FAC = HI / ANL
      DO  20  J=1,3
      DO  20  N=1,2
      TEMP(1,N,J) = (SUMA( N,I,J) + X(L)*(SUMA( N,I+1,J)-SUMA( N,I,J)))
     1 * FAC**J
   20 TEMP(2,N,J) = (SUMB( N,I,J) + X(L)*(SUMB( N,I+1,J)-SUMB( N,I,J)))
     1 * FAC**J
C
      TMP(1,1) = -TEMP(1,1,1) + HI
      TMP(1,2) = -TEMP(1,2,1)
      TMP(2,1) = -TEMP(2,1,1)
      TMP(2,2) = -TEMP(2,2,1) + HI
C
      DO  30  N=1,2
      DO  30  M=1,2
      DO  30  K=1,2
   30 D(M,N) = D(M,N) + EE(M,K) * TMP(K,N)
C
      DO  40  N=1,2
      DO  40  M=1,2
   40 TMP(M,N) = 0.50 * HI * TEMP(M,N,1) - TEMP(M,N,2)
C
      DO  50  N=3,4
      DO  50  M=1,2
      DO  50  K=1,2
   50 D(M,N) = D(M,N) + EE(M,K) * TMP(K,N-2)
C
      DO  60  N=1,2
      DO  60  M=3,4
   60 D(M,N) = D(N,M)
C
      DO  70  N=1,2
      DO  70  M=1,2
   70 TMP(M,N) = (-0.25 * HI*TEMP(M,N,1) + TEMP(M,N,2))*HI - TEMP(M,N,3)
```

```
C
      TMP(1,1) = TMP(1,1) + HI**3 / 12.
      TMP(2,2) = TMP(2,2) + HI**3 / 12.
C
      DO  80  N=3,4
      DO  80  M=3,4
      DO  80  K=1,2
   80 D(M,N) = D(M,N) + EE(M-2,K) * TMP(K,N-2)
C
      RETURN
      END
1
```

```
      SUBROUTINE BANSOL
C
C     ******************************************************************
C     IN-CORE LINEAR EQUATION SOLVER FOR SYMMETRIC BAND MATRICES
C
C     ******************************************************************
C
      COMMON /BANARG/  NN, MM,A(150, 6),B(150)
      DIMENSION S(1)
      EQUIVALENCE (S,A)
      NCOL = 150
      NR = NN
      NRS = NR - 1
      MMR = MM - 1
C     DECOMPOSE MATRIX A
      DO 120  N = 1,NRS
      M = N - 1
      MR = MIN0 (MM,NR-M)
      PIVOT = S(N)
      J = N
      DO 120  L = 2,MR
      J = J + NCOL
      C = S(J)/PIVOT
      I1 = M + L
      I2 = I1 + (MR-L)*NCOL
      II = J
      DO 110  I = I1,I2,NCOL
      S(I) = S(I) - C*S(II)
  110 II = II + NCOL
  120 S(J) = C
C     REDUCE AND BACKSUBSTITUTE VECTOR B
      DO 220  N = 1,NRS
      MR = MIN0 (MMR,NR-N)
      C = B(N)
      B(N) = C/S(N)
      K = N
      L1 = N + 1
      L2 = N + MR
      DO 220  L = L1,L2
      K = K + NCOL
  220 B(L) = B(L) - S(K)*C
      B(NR) = B(NR)/S(NR)
  300 DO 320  I = 1,NRS
      N = NR - I
      MR = MIN0 (MMR,I)
      J = N
      L1 = N + 1
      L2 = N + MR
      DO 320  L = L1,L2
      J = J + NCOL
  320 B(N) = B(N) - S(J)*B(L)
      RETURN
      END
```

```
C      ELASTIC PLASTIC ANALYSIS OF AXISYMMETRIC SHELLS WITH DISCONTINUOUS
C      MERIDIONAL SLOPE
C
       PROGRAM MAIN (INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4,TAPE5=INPUT,
     1 TAPE6 = OUTPUT ,TAPE 8, TAPE9 )
C
       COMMON/ DIV  / NE , NL , NLI
C
       READ (5,22)NIPT
       DO 100 I=1, NIPT
     1 CALL INPUTD
C
       CALL GEOMTY
C
       DO 100 NLL = 1,NLI
C
       WRITE (6,1000)  NLL
C
       CALL STIFNS (NLL)
C
       CALL NODLOD
C
       CALL DISPL
C
       CALL STRESS(NLL)
C
       CALL MATPP
C
   100 CONTINUE
C
       STOP
C
    22 FORMAT (I5)
  1000 FORMAT(31H1NUMBER OF LOADING INCREMENT = ,I5)
C
       END
1
```

TAR

```
      SUBROUTINE INPUTD
C
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV   / NE , NL , NLI
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/STSSD2/FA(4, 5), FB(4,5)
      COMMON/STSSD3/NDIS( 5), NDISC
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
      COMMON/MATD2/ STIX(2,20, 5), EPLX(2,20, 5), ZTX, STNX
      COMMON/STSS1 / T(2,20) , F(4,50 )
      COMMON/BNDRCN/ NEQBC,NEBC(15)
      COMMON/INPTDT/ NTAG(5,3) , NBC(5)
      COMMON/BANARG/ NUMEQ,NBNWD,A(150,6),B(150)
      COMMON/ DSPL1/ BT(150)
      COMMON/ NTAP / NTAP1 , NTAP2
      COMMON APA1(2,20, 5), APB1(2,20, 5), APA2(2,20, 5), APB2(2,20, 5),
     A TAX(2,20, 5), TBX(2,20, 5)
C
      WRITE(6,2000)
      READ (5,1000)
      WRITE(6,1000)
C
C     NUMBER OF NODES , LAYERS , LOAD INCREMENTS , AND BOUNDRY CONDITIONS
C
      READ (5,1001)NN,NL, NLI, NUMBC, NDISC
      WRITE(6,2001)NN,NL, NLI, NUMBC, NDISC
C
      READ (5,1102) (NDIS(I), I= 1,NDISC)
      WRITE(6,2102) (NDIS(I), I= 1,NDISC)
C
      NE = NN - 1
      READ (5,1002) (( R(M,I),Z(M,I),PHI(M,I),H(M,I), M=1,2), I=1,NE)
      WRITE(6,2002)(I,(M,R(M,I),Z(M,I),PHI(M,I),H(M,I), M=1,2), I=1,NE)
C
      READ (5,1003) (  (CRV(M,I)  , M=1,2),I=1,NE)
      WRITE(6,2003) (I,(CRV(M,I)  , M=1,2),I=1,NE)
C
      READ (5,1004) NP,(EP(I),SIGMA(I),ETAN(I),I=1,NP)
      WRITE(6,2004)  (I,EP(I),SIGMA(I),ETAN(I),I=1,NP)
C
      READ (5,1005) E,U
      WRITE(6,2005) E,U
C
      READ (5,1006) (NBC(N) , (NTAG(N,I) , I=1,3) , N=1,NUMBC)
      WRITE(6,2006) (NBC(N) , (NTAG(N,I) , I=1,3) , N=1,NUMBC)
C
C     BAND WIDTH , NUMBER OF SIMULTANEOUS EQUATIONS, NUMBER OF ELEMENTS
C
      NBNWD = 6
      NUMEQ = 3 * NN
C
C     ANGLE PHI IN TERMS OF RADIAN
C
```

```
      DO 10 I = 1,NE
      DO 10 M= 1,2
   10 PHI(M,I) = PHI(M,I) / 57.2957795130823
C
C     NORMALIZED TANGENT MODULII
C
      DO 150 I = 1,NP
  150 ETAN(I) = ETAN(I) / E
C
C     SET UP TOTAL DISPLACEMENTS
C
      DO 160  I=1,NUMEQ
  160 BT(I) = 0.0
C
C     ELASTIC MODULI IN CASE OF PLANE STRESS (ISOTROPIC)
C
      EE(1,1) = E / (1.-U) / (1.+U)
      EE(1,2) = U * EE(1,1)
      EE(2,1) = EE(1,2)
      EE(2,2) = EE(1,1)
C
C     SET UP STRESS RESULTANTS
C
      DO 220  I=1,NN
      DO 220  M=1,4
  220 F(M,I) = 0.0
C
C     ADDITIONAL INITIALIZATION IN THE CASE OF SLOPE DISCONTINUITY.
C
      DO 225 I= 1,NDISC
      DO 222 M = 1,4
      FA(M,I) = .0
  222 FB(M,I) = .0
      DO 225 K =1, NL
      DO 223 JX = 1, 2
      STIX(JX,K,I) = SIGMA(1)
      EPLX(JX,K,I) = 0.
      TAX(JX,K,I) = .0
  223 TBX(JX,K,I) = .0
      DO 225 N=1,2
      APA1(N,K,I) = .0
      APA2(N,K,I) = .0
      APB2(N,K,I) = .0
  225 APB1(N,K,I) = .0
C
C     SET UP EQUIVALENT STRESSS , EQUIVALENT PLASTIC STRAIN , STRESS
C     VECTOR , AND (AP) MATRIX
C
      NTAP1 = 8
      NTAP2 = 9
      REWIND NTAP1
      DO 240  I=1,NN
      DO 230  K=1,NL
      STI(K) = SIGMA(1)
      EPL(K) = 0.0
```

```
      DO 230  N=1,2
      T(N,K) = 0.0
      DO 230  M=1,2
  230 AP(M,N,K) = 0.0
      WRITE(NTAP1)  STI   ,EPL   , AP, T
  240 CONTINUE
      END FILE NTAP1
C
C     BOUNDRY CONDITIONS
C
      L = 0
      DO 250 N=1,NUMBC
      DO 250 I=1,3
      M = NTAG(N,I)
      IF(M .LE. 0)    GO TO 250
      L = L + 1
      NEBC(L) = 3 * (NBC(N) - 1) + I
  250 CONTINUE
      NEQBC = L
C
      RETURN
C
 1000 FORMAT(72H
     1                                                 )
 1001 FORMAT(5I5)
 1002 FORMAT ( 4F15. )
 1003 FORMAT(2F15.0)
 1004 FORMAT(I5 / (3E15.0))
 1005 FORMAT(E10.3,F5.0)
 1006 FORMAT(I4,1X,3I1)
C
 2000 FORMAT(1H1)
 2001 FORMAT( //
     1 30H NUMBER OF NODAL CIRCLES . . .   I10 /
     2 30H NUMBER OF LAYERS   . . . . . .  I10 /
     3 30H NUMBER OF LOAD INCREMENTS . .   I10 /
     4 30H NUMBER OF BOUNDRY CONDITIONS    I10 /
     5* NUMBER OF NODES WITH DISCONT. SLOPE* I4 /// )
 1102 FORMAT (10I5)
 2102 FORMAT ( * NODES WHERE SLOPE IS DISCONTINUOUS...*10I10///)
 2002 FORMAT (//
     A2X,*ELEMENT* 2X,4H END 7X,11HABSCISSA-R-,8X,12H ORDINATE-Z-,7X,
     1 14HLATITUDE ANGLE , 9X,9HTHICKNESS / 25X,5H(IN.),15X,5H(IN.),
     2  13X,8H(DEGREE),14X,5H(IN.)//(  2I7,4E20.9/7X, I7, 4E20.9))
 2003 FORMAT(30H-CURVATURES AT NODAL CIRCLES      /
     1 8H ELEMENT , 7X , 6HEND(1) , 14X , 6HEND(2)  //(I5,2E20.7))
 2004 FORMAT(1H-,14X ,26HEQUIVALENT PLASTIC STRAIN  ,8X,18HEQUIVALENT ST
     1RESS  ,13X,16HTANGENT MODULUS  //(I5,3E30.7))
 2005 FORMAT( 30H-MODULUS OF ELASTICITY (PSI)  ,E20.7/
     1          30H POISSON RATIO . . . . . . .   ,E20.7///)
 2006 FORMAT(43H BOUNDRY CONDITIONS (1=RESTRAINED,0=FREE)  //
     1 6H POINT , 3X , 3H R  , 2X , 3H Z  , 2X , 3HROT //(I6,3I5))
C
      END
1
```

```
      SUBROUTINE GEOMTY
C
C     COMPUTE GEOMETRIC PARAMETERS FOR COPLETELY COMPATIBLE ELEMENT  (3)
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/GEOM4 /YBAR,YP,YPP,RW,XT,ARC,RV,CNP1,CNP2,B(4,8)
      COMMON /GEOM5/ A1,A2,A3,A4
      COMMON/BANARG/NUMEQ,NBNWD,BK(150,6),P(150)
      COMMON/INTEG1/ NUMIPD
      COMMON/INTEG2/YP1(10),FAC(10)
C
      DIMENSION X(12) , W(10) , SINF(2,50), CSNF(2,50)
C
      EQUIVALENCE  (SINF(1),BK(1)) , (CSNF(1),BK(151))
C
      DATA X / 0.0,0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586, 1.0 /
C
      DATA W /  0.066671344308688, 0.149451349150581,
     1 0.219086362515982, 0.269266719309996, 0.295524224714753,
     2 0.295524224714753, 0.269266719309996, 0.219086362515982,
     3 0.149451349150581, 0.066671344308688  /
C
      REWIND 1
      REWIND 2
      REWIND 3
      REWIND 4
C
      NUMIPD = 10
      WRITE(6,1006)
C
      NN = NE + 1
      DO  10  I = 1,NE
      DO 10 M = 1,2
      SINF (M,I) = SIN(PHI(M,I))
   10 CSNF (M,I) = COS(PHI(M,I))
C
C     COMPUTE GEOMETRICAL PARAMETERS
C
      DO 100  I = 1,NE
      DR = R(2,I) - R(1,I)
      DZ = Z(2,I) - Z(1,I)
      CORD = SQRT(DR**2 + DZ**2)
      SNT = DR / CORD
      CNT = DZ / CORD
C
      SNB1 = CSNF(1,I) * CNT - SINF(1,I) * SNT
      CNB1 = SINF(1,I) * CNT + CSNF(1,I) * SNT
      TNB1 = SNB1 / CNB1
```

```
      SNB2 = CSNF(2,I) * CNT - SINF(2,I) * SNT
      CNB2 = SINF(2,I) * CNT + CSNF(2,I) * SNT
      TNB2 = SNB2 / CNB2
C
      CNP1 = CSNF(1,I) / CNB1
      CNP2 = CSNF(2,I) / CNB2
C
      YPP1 = -CORD * CRV(1,I) / CNB1**3
      YPP2 = -CORD * CRV(2,I) / CNB2**3
C
      A1 = TNB1
      A2 = TNB1 + 0.50 * YPP1
      A3 =-(5.*TNB1 + 4.*TNB2) + (0.5*YPP2 - YPP1)
      A4 = 3. * (TNB1 + TNB2) + 0.50 * (YPP1 - YPP2)
C
      DPHI = PHI(2,I) - PHI(1,I)
      SNP = SIN(DPHI)
      CNP = COS(DPHI)
C
C     ESTABLISH STRAIN-DISPLACEMENT MATRICES
C
      NIPD2 = NUMIPD + 2
      DO 70 K = 1,NIPD2
      XT = X(K)
      YBAR = (1.-XT) * (A1 + XT * (A2 + XT * (A3 + XT * A4)))
      YP   = A1*(1.-2.*XT) + XT*(A2*(2.-3.*XT) + XT*(A3*(3.-4.*XT) +
     1 A4 * XT * (4.-5.*XT)))
      YPP  = 2.*(-A1 + A2 * (1.-3.*XT)) + XT * (A3 * (6.-12.*XT) +
     1 A4 * XT * (12.-20.*XT))
      RV   = CORD * (SNT + YBAR * CNT)
      RW   = R(1,I) + XT * RV
      ARC  = SQRT(1. + YP**2)
C
      IF(R(1,I) .NE. .0)  GO TO  20
C
C     STRAIN - DISPLACEMENT MATRIX FOR CENTRAL CAP
C
      CALL B1MATX
      GO TO  30
C
C     STRAIN - DISPLACEMENT MATRIX FOR A FRUSTRUM
C
   20 CALL BMATX
C
   30 IF(K.EQ. 1 .OR. K .EQ. NIPD2)   GO TO 50
      KK = K - 1
      YP1(KK) = YP
      FAC(KK) = 0.50 * CORD * RW * W(KK)
      COF(KK) = ARC * FAC(KK)
C
      DO  40  N = 1,8
      DO  40  M = 1,4
   40 B1(M,N,KK) = B(M,N)
      GO TO  70
C
```

```
C     TO BE USED IN STRESS SUBROUTINE
C
   50 KK = (NUMIPD + K) / (NUMIPD + 1)
      DO  60  N = 1,8
      DO  60  M = 1,4
   60 B2(M,N,KK) = B(M,N)
   70 CONTINUE
C
C     DISPLACEMENT TRANSFORMATION MATRIX
C
      IF(R(1,I) .NE. .0)  GO TO   80
C
C     DISPLACEMENT TRANSFORMATION MATRIX FOR CENTRAL CAP
C
      CALL C1MATX
      GO TO  90
C
C     DISPLACEMENT TRANSFORMATION MATRIX FOR A FRUSTRUM
C
   80 CALL CMATX
C
   90 WRITE (1) B1, COF
      WRITE (2) B2
      WRITE (3) C
      WRITE (4) YP1, FAC, TNB1, SNT, CNT
      WRITE(6,1007)  I,CORD,SNT,CNT,TNB1,TNB2
C
  100 CONTINUE
C
      END FILE 1
      END FILE 2
      END FILE 3
      END FILE 4
C
      RETURN
C
 1006 FORMAT(8H-ELEMENT,5X,4HCORD,14X,4HSINT,14X,4HCSNT,14X,4HTNB1,
     1 14X,4HTNB2 //)
 1007 FORMAT(I4,5E18.7)
C
      END
1
```

```
      SUBROUTINE STIFNS(NLL)
C
C     SUBROUTINE TO SET UP SYSTEM STIFFNESS MATRIX
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV   / NE , NL , NLI
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/MATD1 /SUMA1(2,5 ,3), SUMA2(2,5 ,3), SUMB1(2,5 ,3)
     A ,SUMB2(2,5 ,3)
      COMMON/STSSD3/NDIS( 5), NDISC
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/ GEOM4/YBAR,YP, YPP , RW , XT , ARC , RV , CNP1,CNP2,B(4,8)
      COMMON/ MAT3 / SUMA(2,50 ,3), SUMB(2,50 ,3), TB(20), RAT(20)
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON/BANARG/NUMEQ,NBNWD,BK(150,6),P(150)
      COMMON/INTEG1/ NUMIPD
      COMMON/CONDNS/ DD(2,8,50), DB(6,2,50), RB(2,50)
C
      DIMENSION SKG(8,8) , TEMP(8,8)
      EQUIVALENCE (SKG(1),SK(1)) , (TEMP(1),SKI(1))
C
      REWIND 1
      REWIND 3
C
      NX = 1
      MA = 0
      MB = 0
      DO 1    N=1,6
      DO 1   M=1,NUMEQ
    1 BK(M,N) = 0.0
C
      DO 100 I=1,NE
      IF (NX .GT. NDISC) GO TO 2
      IF ( I+1 .EQ. NDIS(NX))  MB =1
      IF ( I    .EQ. NDIS(NX))  MA =1
    2 II = I
C
      READ (1)  B1, COF
      READ (3)  C
C
C     ELEMENT STIFFNESS MATRIX IN GENERALIZED COORDINATES
C
      DO 5   N=1,8
      DO 5   M=1,8
    5 SKG(M,N) = 0.0
C
      DO 50  L=1,NUMIPD
      LL = L + 1
      IF (NLL .NE. 1)   GO TO  7
      CALL D1MATX(LL,II)
      GO TO  8
C
    7 CALL DMATX(LL,II,MA,MB,NX)
C
    8 DO 10  N=1,8
```

```
      DO 10   M=1,4
   10 B(M,N) = 0.0
C
      DO 20   N=1,8
      DO 20   M=1,4
      DO 20   K=1,4
   20 B(M,N) = B(M,N) + D(M,K) * B1(K,N,L)
C
      DO 30   N=1,8
      DO 30   M=1,8
   30 SKI(M,N) = 0.0
C
      DO 40   N=1,8
      DO 40   M=1,8
      DO 40   K=1,4
   40 SKI(M,N) = SKI(M,N) + B1(K,M,L) * B(K,N)
C
      DO 50   N=1,8
      DO 50   M=1,8
   50 SKG(M,N) = SKG(M,N) + COF(L) * SKI(M,N)
C
      IF (MA .EQ. 1)   NX = NX + 1
      MA = 0
      MB = 0
C
C     ELEMENT STIFFNESS MATRIX IN SYSTEM CO-ORDINATES
C
      DO 60   N=1,8
      DO 60   M=1,8
   60 TEMP(M,N) = 0.0
C
      DO 70   N=1,8
      DO 70   M=1,8
      DO 70   K=1,8
   70 TEMP(M,N) = TEMP(M,N) + SKG(M,K) * C(K,N)
C
      DO 80 N =1, 8
      DO 80 M =1, 8
   80 SK(M,N) = 0.
C
      DO 90   N=1,8
      DO 90   M=1,8
      DO 90   K=1,8
   90 SK(M,N) = SK(M,N) + C(K,M) * TEMP(K,N)
      IF(R(1,I) .NE. .0)  GO TO  95
      SK(1,1) = 1.0
      SK(3,3) = 1.0
C
   95 CONTINUE
C
C     CONDENSATION
C
      DET = SK(7,7) * SK(8,8) - SK(7,8) * SK(8,7)
      DD(1,1,I) = SK(8,8) / DET
      DD(2,2,I) = SK(7,7) / DET
```

```
      DD(1,2,I) =-SK(7,8) / DET
      DD(2,1,I) =-SK(8,7) / DET
C
      DO 105 N =1, 6
      K = N + 2
      DO 105 M = 1,2
      DD(M,K,I) = 0.
  105 DB(N,M,I) = 0.
C
      DO 115  N =1, 6
      DO 110 M =1,2
      DO 110 K =1, 2
      J = N + 2
      DB(N,M,I) = DB(N,M,I) + SK(N,K+6) * DD(K,M,I)
  110 DD(M,J,I) = DD(M,J,I) + DD(M,K,I) * SK(K+6,N)
      DO 115 JJ =1, 6
      DO 115 KK = 1, 2
  115 SK(JJ,N) = SK(JJ,N) - SK(JJ,KK+6) * DD(KK,N+2,I)
C
      DO 100 M=1,6
      DO 100 N=M,6
      II = 3*(I-1) + M
      JJ = N-M+1
  100 BK(II,JJ) = BK(II,JJ) + SK(M,N)
      END FILE 9
C
      RETURN
      END
1
```

```
      SUBROUTINE NODLOD
C
C     SUBROUTINE TO GENERATE CONSISTENT NODAL LOADS
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV   /NE,NL,NLI
      COMMON/BANARG/ NUMEQ,NBNWD,A(150,6),B(150)
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/GEOM2 / COF(10) , B0(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/INTEG1/NUMIPD
      COMMON/INTEG2/YP1(10),FAC(10)
      COMMON/CONDNS/ DD(2,8,50), DB(6,2,50), RB(2,50)
C
      DIMENSION B1(150),DP(8),PSTAR(8),TEMP(8),X(10)
C
      DATA X /      0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586       /
C
      REWIND 3
      REWIND 4
C
      READ (5,900) INDX
      IF (INDX .EQ. 0) GO TO 5
      READ (5,1000) B(1), B(2), B(3), B1(1), B1(2), B1(3)
      NE3 = 3* INDX
      DO 3 I= 4,NE3, 3
      B (I) = B (1)
      B (I+1) = B (2)
      B (I+2) = B (3)
      B1(I) = B1(1)
      B1(I+1) = B1(2)
    3 B1(I+2) = B1(3)
      NN = NE +1
      IF (INDX .EQ. NN ) GO TO 7
      NEX = NE3 + 1
      NE3 = 3 * NN
      DO 4 I = NEX, NE3
      B (I) = 0.0
    4 B1(I) = 0.0
      GO TO 7
C
    5 NE3 = 3 * (NE + 1)
      READ (5,1000) (B(I),B(I+1),B(I+2),B1(I),B1(I+1),B1(I+2),I=1,NE3,3)
    7 WRITE(6,2000) (2(I),B(I+1),B(I+2),B1(I),B1(I+1),B1(I+2),I=1,NE3,3)
C
      DO 100  I=1,NE
      L = 3 * (I-1)
C
      READ  (3)  C
      READ  (4)  YP1, FAC, TNB1, SNT, CNT
C
      DO 10   J=1,8
   10 PSTAR(J) = 0.0
```

```
C
      DO 20   K=1,NUMIPD
      PU = B1(L+1) + X(K) * (B1(L+4) - B1(L+1))
      PW = B1(L+2) + X(K) * (B1(L+5) - B1(L+2))
C
      IF (R(1,I) .NE. .0) GO TO  15
      SUM1 = (PU - PW * YP1(K)) * FAC(K)
      SUM2 = (PU * YP1(K) + PW) * FAC(K)
      DP(1) = 0.0
      DP(2) = 0.0
      DP(3) = -SUM1 * CNT + SUM2 * SNT
      DP(4) = (SUM1 + SUM2 * TNB1) * X(K)
      DP(5) = SUM1 * X(K) * X(K)
      DP(6) = X(K) * DP(5)
      DP(7) = SUM2 * X(K) * X(K)
      DP(8) = DP(7) * X(K)
      GO TO  16
C
   15 DP(1) = (PU - PW * YP1(K)) * FAC(K)
      DP(2) = X(K) * DP(1)
      DP(3) = X(K) *DP(2)
      DP(4) = X(K) *DP(3)
      DP(5) = (PU * YP1(K) + PW) * FAC(K)
      DP(6) = X(K) *DP(5)
      DP(7) = X(K) * DP(6)
      DP(8) = X(K) * DP(7)
C
   16 DO 20   J=1,8
   20 PSTAR(J) = PSTAR(J) + DP(J)
C
      DO 30   J=1,8
   30 TEMP(J) = 0.0
C
      DO 40   N=1,8
      DO 40   M=1,8
   40 TEMP(N) = TEMP(N) + C(M,N) * PSTAR(M)
C
C     CONDENSATION
C
      RB(1,I) = TEMP(7)
      RB(2,I) = TEMP(8)
      DO  45 N =1, 6
      DO  45 K =1, 2
   45 TEMP(N) = TEMP(N) - DB(N,K,I) * RB(K,I)
C
      DO 50   N=1,6
      LL = L + N
   50 B(LL) = B(LL) + TEMP(N)
C
  100 CONTINUE
C
      WRITE(6,2003) (B(I) , I=1,NUMEQ)
C
      RETURN
C
```

```
 900 FORMAT (I5)
1000 FORMAT (6E12.0)
2000 FORMAT(/// 20X,18HCONCENTRATED LOADS ,42X,17HDISTIBUTED LOADS //
    1 8X,10H  R-FORCE  ,11X,7HZ-FORCE ,14X,6HMOMENT ,12X,10HMERIDIONAL,
    2 12X, 6HRADIAL ,14X,6HMOMENT /(6E20.7))
2003 FORMAT(44H1CONSISTENT EXTERNAL LOADS AT NODAL CIRCLES      //
    1 8X,10H R-FORCE  ,10X,8H Z-FORCE,14X,6HMOMENT /(3E20.7))
C
      END
1
```

```
      SUBROUTINE DISPL
C
      COMMON /BANARG/  NN, MM,A(150, 6),B(150)
      COMMON/BNDRCN/ NEQBC,NEBC(15)
      COMMON/ DSPL1/ BT(150)
C
      DO 100  N=1,NEQBC
      I = NEBC(N)
      A(I,1) = 1.0
      B(I) = 0.0
      DO 100  J=2,MM
      A(I,J) = 0.0
      L = I-J+1
      IF(L .LE. 0)   GO TO 100
      A(L,J) = 0.0
  100 CONTINUE
C
      CALL BANSOL
C
      DO 200  I=1,NN
  200 BT(I) = BT(I) + B(I)
C
      WRITE(6,2001) (B(I) , I=1,NN)
      WRITE(6,2002) (BT(I), I=1,NN)
C
      RETURN
C
 2001 FORMAT(50H1DISPLACEMENT INCREMENTS AT NODAL CIRCLES          //
     1 7X,11H HORIZONTAL,10X,8HVERTICAL,13X,8HROTATION /(3E20.7))
 2002 FORMAT(50H1TOTAL DISPLACEMENTS AT NODAL CIRCLES              //
     1 7X,11H HORIZONTAL,10X,8HVERTICAL,13X,8HROTATION /(3E20.7))
C
      END
1
```

```
      SUBROUTINE  STRESS(NLL)
C
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/STSS1 / T(2,20) , F(4,50 )
      COMMON/STSS2 / ALPHA(8),DSTR(2) ,BE(2,8),DFRC(4)
      COMMON/STSSD1/DSTA(2,20, 5), DSTB(2,20,5)
      COMMON/STSSD2/FA(4, 5), FB(4,5)
      COMMON/STSSD3/NDIS( 5), NDISC
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
      COMMON/BANARG/ NUMEQ,NBNWD,A(150,6),B(150)
      COMMON/INTEG1/ NUMIPD
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON/CONDNS/ DD(2,8,50), DB(6,2,50), RB(2,50)
C
      DIMENSION DFA(4,5 ), DFB(4,5)
      DIMENSION  TEMP(4,8) , DF(4,50), Q(8)
      EQUIVALENCE  (TEMP(1),SKI(1)) , (DF(1),A(1))
C
      REWIND  2
      REWIND  3
C
      NN = NE + 1
      ANL = NL
      DO  10  I=1,NN
      DO   5  K=1,NL
      DO   5  M=1,2
    5 DST(M,K,I) = 0.0
      DO  10  N=1,4
   10 DF(N,I) = 0.0
C
      DO 15 I= 1,NDISC
      DO 12 K = 1,NL
      DO 12 M = 1,2
      DSTA(M,K,I) = 0.0
   12 DSTB(M,K,I) = 0.0
      DO 15 N=1,4
      DFA(N,I) = .0
   15 DFB(N,I) = .0
      WRITE (6,1000)
C
      NX = 1
      MA = 0
      MB = 0
      DO 148  I=1,NE
C
      READ (2)  B2
      READ (3)  C
C
      DO  20  M=1,8
   20 ALPHA(M) = 0.0
C
```

```
C     RECOVERING CONDENSED DEG OF FREEDOM
C
      Q(7) = 0.
      Q(8) = 0.
      IX = 3 * (I-1)
C
      DO 25 M =1, 2
      DO 22 K =1, 2
      L = M + 6
   22 Q(L) = Q(L) + DD(M,K,I) * RB(K,I)
      DO 25 J = 1, 6
      JJ = J + IX
   25 Q(L) = Q(L) - DD(M,J+2, I) * B(JJ)
C
      DO 27 JJ = 1,6
      J = IX + JJ
   27 Q(JJ) = B(J)
C
      DO  30  M=1,8
      DO  30  N=1,8
   30 ALPHA(M) = ALPHA(M) + C(M,N) * Q(N)
C
      DO 148  L=1,2
      II = I+L-1
      IF (NDIS(NX) .EQ. II .AND. NDIS(NX) .EQ. I+1) MB=1
      IF (NDIS(NX) .EQ. II .AND. NDIS(NX) .EQ. I) MA=1
C
C     DETERMINATION OF STRAIN ALONG THE THICKNESS
C
      DO  78  K=1,NL
      AK = K
      ZBAR = ((AK - 0.50)/ANL - 0.50) * H(L,I)
      DO  40  N=1,8
      DO  40  M=1,2
   40 BE(M,N) = B2(M,N,L) + ZBAR * B2(M+2,N,L)
C
      DO  50  M=1,2
   50 DSTR(M) = 0.0
C
      DO  60  M=1,2
      DO  60  N=1,8
   60 DSTR(M) = DSTR(M) + BE(M,N) * ALPHA(N)
      IF (MA .EQ. 1) GO TO 72
      IF (MB .EQ. 1) GO TO 75
C
      DO  70  M=1,2
   70 DST(M,K,II) = DST(M,K,II) + 0.50 * DSTR(M)
      GO TO 78
   72 DO 73 M= 1,2
   73 DSTA(M,K,NX) = DSTR(M)
      GO TO 78
   75 DO 76 M= 1,2
   76 DSTB(M,K,NX) = DSTR(M)
   78 CONTINUE
C
```

```
C     DETERMINATION OF STRESS RESULTANTS
C
      III = I
      LL = 1 + (NUMIPD+1) * (L-1)
      IF (NLL .NE. 1)   GO TO 80
      CALL  D1MATX(LL,III)
      GO TO 90
C
   80 MX = MA
      MY = MB
      MZ = NX
      CALL  DMATX(LL,III,MX,MY,MZ)
C
   90 DO 100   N=1,8
      DO 100   M=1,4
  100 TEMP(M,N) = 0.0
C
      DO 110   N=1,8
      DO 110   M=1,4
      DO 110   K=1,4
  110 TEMP(M,N) = TEMP(M,N) + D(M,K) * B2(K,N,L)
C
      DO 120   M=1,4
  120 DFRC(M) = 0.0
C
      DO 130   M=1,4
      DO 130   K=1,8
  130 DFRC(M) = DFRC(M) + TEMP(M,K) * ALPHA(K)
C
      WRITE (6,1001)   I,L,(DFRC(M) , M=1,4)
C
  139 IF (MA .EQ. 1) GO TO 142
      IF (MB .EQ. 1) GO TO 145
C
      DO 140  M=1,4
  140 DF(M,II) = DF(M,II) + 0.50 * DFRC(M)
      GO TO 148
  142 DO 143 M=1,4
  143 DFA(M,NX) = DFRC(M)
      MA = 0
      IF (NDISC .GT. NX) NX = NX +1
      GO TO 148
  145 DO 146 M=1,4
  146 DFB(M,NX) = DFRC(M)
      MB = 0
  148 CONTINUE
C
      DO 150   M=1,2
      DO 150   K=1,NL
      DST(M,K, 1) = 2.0 * DST(M,K, 1)
  150 DST(M,K,NN) = 2.0 * DST(M,K,NN)
C
      DO 160   M=1,4
      DF(M, 1) = 2.0 * DF(M, 1)
  160 DF(M,NN) = 2.0 * DF(M,NN)
```

```
C
      DO  220  I = 1, NDISC
      DO 215 N =2,4
      DFA(N,I) = 0.5 * (DFA(N,I) + DFB(N,I))
  215 DFB(N,I) = DFA(N,I)
C
      DO  220  K = 1, NL
      DSTA(2,K,I) = 0.5 * (DSTA(2,K,I) + DSTB(2,K,I))
  220 DSTB(2,K,I) = DSTA(2,K,I)
C
      WRITE (6,1002)
      IX = 1
      IF (NDISC .EQ. 0) GO TO  200
      NX = 1
  190 JX = NDIS(NX) -1
      WRITE(6,1100)(I, (DF(M,I), M=1,4), I= IX, JX)
      JN = JX +1
      WRITE (6,1110) JN,JX, (DFB(M,NX), M=1,4)
      WRITE (6,1110) JN,JN, (DFA(M,NX), M=1,4)
      IX = JX+2
      IF (NX .EQ. NDISC) GO TO 200
      NX = NX + 1
      GO TO 190
  200 WRITE(6,1100)(I, (DF(M,I), M=1,4), I= IX, NN)
C
C     TOTAL STRESS RESULTANTS
C
      WRITE (6,2000)
      NX = 1
      DO 170  I=1,NN
      IF ( I   .EQ. NDIS(NX))  GO TO 164
      DO 162 M= 1,4
  162 F(M,I) = F(M,I) + DF(M,I)
      WRITE (6,2001) I, (F(M,I), M=1,4)
      GO TO 170
  164 DO 165 M=1,4
      FA(M,NX) = FA(M,NX) + DFA(M,NX)
  165 FB(M,NX) = FB(M,NX) + DFB(M,NX)
      NEL = I-1
      WRITE (6,2002) I, NEL, (FB(M,NX), M=1,4)
      WRITE (6,2002) I,  I , (FA(M,NX), M=1,4)
      IF (NDISC .GT. NX) NX = NX +1
  170 CONTINUE
C
      IX = 1
      IF (NDISC .EQ. 0) GO TO  179
      NX = 1
  172 JX = NDIS(NX) -1
      DO 174 I = IX, JX, 2
      II = I+1
      IF ( II .GT. JX)  GO TO 173
      WRITE (6,1004)  (I,K, (DST(M,K,I),M=1,2),II ,K,(DST(M,K,II ),
     1 M=1,2),K=1,NL)
      GO TO 174
  173 WRITE (6,1005) (I,K, (DST(M,K,I) , M=1,2), K=1, NL)
```

```
  174 CONTINUE
      JN = JX +1
      WRITE(6,1006) JX,JN, (JN,K,(DSTB(M,K,NX), M=1,2),
     1JN, K, (DSTA(M,K,NX), M=1,2), K=1,NL)
      IX = JX+2
      IF (NX .EQ. NDISC) GO TO 179
      NX = NX + 1
      GO TO 172
  179 DO 180 I= IX, NN, 2
      I1 = I + 1
  180 WRITE (6,1004)  (I,K, (DST(M,K,I),M=1,2),I1 ,K,(DST(M,K,I1 ),
     1 M=1,2),K=1,NL)
C
      RETURN
 1000 FORMAT(55H1INCREMENTS OF STRESS RESULTANTS AT THE END OF ELEMENTS/
     1 8H ELEMENT,2X,7HEND NO.,4X, 16HMERIDIONAL FORCE ,5X,16HCIRCUMFER.
     2 FORCE ,5X,18HMERIDIONAL MOMENT  ,4X,18HCIRCUMFER. MOMENT //)
 1001 FORMAT(I5,4X,I5,4E21.7)
 1002 FORMAT(50H1INCREMENTS OF STRESS RESULTANTS AT NODAL CIRCLES    /
     1 6H   NODE,18X,16HMERIDIONAL FORCE ,6X,16HCIRCUMFER. FORCE ,6X,
     2 18HMERIDIONAL MOMENT ,4X,18HCIRCUMFER. MOMENT // )
 2000 FORMAT(50H1TOTAL STRESS RESULTANTS AT NODAL CIRCLES             /
     1 6H   NODE,18X,16HMERIDIONAL FORCE ,6X,16HCIRCUMFER. FORCE ,6X,
     2 18HMERIDIONAL MOMENT ,4X,18HCIRCUMFER. MOMENT // )
 1004 FORMAT(30H2INCREMENTS OF STRAINS                                /
     15H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER.,21X,
     25H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER.  //
     3(I4,2X,I4,2E20.6,20X,I4,2X,I4,2E20.6))
 1005 FORMAT(30H2INCREMENTS OF STRAINS                                /
     15H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER. //
     2(I4, 2X, I4, 2E20.6 ))
 1006 FORMAT
     A(1H2   * INCREMENTS OF STRAINS AT THE NODE WHERE SLOPE IS DISCONTI
     INUOUS. *   //17X,*ELEMENT*   70X, *ELEMENT* / 18X,I3, 74X, I3//
     15H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER.,21X,
     25H NODE,2X,5HLAYER,7X,10HMERIDIONAL,10X,10HCIRCUMFER.  //
     3(I4,2X,I4,2E20.6,20X,I4,2X,I4,2E20.6))
 2001 FORMAT (I5, 11X, 4E22.7)
 2002 FORMAT (I5,    * ELEMENT* I3, 4E22.7 )
 1100 FORMAT (I5, 11X, 4E22.7)
 1110 FORMAT (I5,    * ELEMENT* I3, 4E22.7 )
C
      END
1
```

```
      SUBROUTINE  MATPP
C
C     SUBROUTINE TO ESTABLISH MATERIAL PROPERTIES
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
      COMMON/ MAT3 / SUMA(2,50 ,3), SUMB(2,50 ,3), TB(20), RAT(20)
      COMMON/ MAT4 / DSTP(2) , DSTE(2) , DT(2) , TT(2)
      COMMON/STSS1 / T(2,20) , F(4,50 )
      COMMON/MATD1 /SUMA1(2, 5,3), SUMA2(2, 5,3), SUMB1(2, 5,3)
     A ,SUMB2(2, 5,3)
      COMMON/STSSD3/NDIS( 5), NDISC
      COMMON/ NTAP / NTAP1 , NTAP2
C
      REWIND NTAP1
      REWIND NTAP2
C
      NN = NE + 1
      DO   6  J=1,3
      DO   5  I=1,NN
      DO   5  N=1,2
      SUMA( N,I,J) = 0.0
    5 SUMB( N,I,J) = 0.0
      DO   6  I=1,NDISC
      DO   6  N=1,2
      SUMA1(N,I,J) = .0
      SUMA2(N,I,J) = .0
      SUMB2(N,I,J) = .0
    6 SUMB1(N,I,J) = .0
      NX = 1
C
      DO 200  I=1,NN
C
      READ (NTAP1)  STI, EPL, AP, T
C
      IF ( I  .NE.  NDIS(NX))  GO TO 8
      CALL MATPD(I,NX,RAT)
C
      GO TO 160
    8 DO 150  K=1,NL
      DO  10  M=1,2
   10 DSTP(M) = 0.0
C
      DO  20  M=1,2
      DO  20  N=1,2
   20 DSTP(M) = DSTP(M) + AP(M,N,K) * DST(N,K,I)
C
      DO  30  M=1,2
   30 DSTE(M) = DST(M,K,I) - DSTP(M)
C
      DO  40  M=1,2
   40 DT(M) = 0.0
C
```

```
      DO  50  M=1,2
      DO  50  N=1,2
   50 DT(M) = DT(M) + EE(M,N) * DSTE(N)
C
      DO  60  M=1,2
   60 TT(M) = T(M,K) + DT(M)
C
      S1 = TT(1) - 0.50 * TT(2)
      S2 = TT(2) - 0.50 * TT(1)
      TBAR = SQRT(TT(1)*S1 + TT(2)*S2)
      IF (DSTP(1) .EQ. 0.0 .AND. DSTP(2) .EQ. 0.0)   GO TO 70
C
      S1 = T(1,K) - 0.50 * T(2,K)
      S2 = T(2,K) - 0.50 * T(1,K)
      FLAG = S1 * DT(1) + S2 * DT(2)
      IF (FLAG .LT. 0.0)   GO TO 70
C
      EP1 = DSTP(1) + 0.50 * DSTP(2)
      EP2 = DSTP(2) + 0.50 * DSTP(1)
      DEP = SQRT(4./3. * (DSTP(1)*EP1 + DSTP(2)*EP2))
      EPL(K) = EPL(K) + DEP
      GO TO 80
C
   70 STM = 0.999 * STI(K)
      IF (TBAR .LT. STM)   GO TO 90
C
   80 KK = K
      II = I
      CALL  INTERP(KK,II)
      RAT(K) = STN / TBAR
      GO TO 100
C
   90 ZT = 1.0
      STN = STI(K)
      RAT(K) = 1.0
C
  100 DO 110  M=1,2
  110 T(M,K) = RAT(K) * TT(M)
C
      S1 = T(1,K) - 0.50 * T(2,K)
      S2 = T(2,K) - 0.50 * T(1,K)
      TB(K) = SQRT(S1 * T(1,K) + S2 * T(2,K))
      IF (TB(K) .EQ. 0.0)  GO TO 120
      S1 = S1 / TB(K)
      S2 = S2 / TB(K)
      SS1 = (S1 + U*S2) * (1. - ZT)
      SS2 = (S2 + U*S1) * (1. - ZT)
      DNOM = (1.-U)*(1.+U)*ZT + S1*SS1 + S2*SS2
      S1 = S1 / DNOM
      S2 = S2 / DNOM
      AP(1,1,K) = S1 * SS1
      AP(1,2,K) = S1 * SS2
      AP(2,1,K) = S2 * SS1
      AP(2,2,K) = S2 * SS2
      GO TO 135
```

```
C
  120 DO 130  M=1,2
      DO 130  N=1,2
  130 AP(M,N,K) = 0.0
C
  135 AK = K
      DO 140  N=1,2
      SUMA( N,I,1) = SUMA( N,I,1) + AP(1,N,K)
      SUMB( N,I,1) = SUMB( N,I,1) + AP(2,N,K)
      SUMA( N,I,2) = SUMA( N,I,2) + AP(1,N,K) * (AK-0.50)
      SUMB( N,I,2) = SUMB( N,I,2) + AP(2,N,K) * (AK-0.50)
      SUMA( N,I,3) = SUMA( N,I,3) + AP(1,N,K) * (AK*(AK-1.) + .33333333)
  140 SUMB( N,I,3) = SUMB( N,I,3) + AP(2,N,K) * (AK*(AK-1.) + .33333333)
C
  150 STI(K) = STN
C
      WRITE (6,1000) (I,(K,(T(M,K  ),M=1,2),TB(K),RAT(K)), K=1, NL)
  160 CONTINUE
      WRITE(NTAP2)  STI, EPL, AP, T
C
  200 CONTINUE
      END FILE NTAP2
C
      NTAPT = NTAP1
      NTAP1 = NTAP2
      NTAP2 = NTAPT
C
      RETURN
C
 1000 FORMAT(30H2STRESS DISTRIBUTION                                  /
     1 8H   NODE    , 2X , 5HLAYER , 11X , 10HMERIDIONAL , 15X , 10HCIRCUM
     2FER.   ,  11X , 18HEQUIVALENT STRESS   , 4X , 20HMODIFICATION FACTOR
     3  // (I5 , 3X , I5 , 3E25.7 , F20.5))
C
      END
1
```

```
      SUBROUTINE MATPD(I,NX,RAT)
C
C     SUPPLEMENT TO MATPP SUB. IN CASE OF SLOPE DISCON.
C
      COMMON/ DIV  / NE , NL , NLI
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/MATD2/ STIX(2,20, 5), EPLX(2,20, 5), ZTX, STNX
      COMMON/STSSD3/NDIS( 5), NDISC
      COMMON/STSSD1/DSTA(2,20, 5), DSTB(2,20,5)
      COMMON/MATD1 /SUMA1(2, 5,3), SUMA2(2, 5,3), SUMB1(2, 5,3)
     A ,SUMB2(2, 5,3)
      COMMON APA1(2,20, 5), APB1(2,20, 5), APA2(2,20, 5), APB2(2,20, 5),
     A TAX(2,20, 5), TBX(2,20, 5)
C
      DIMENSION DSTEX(2), DSTPX(2), TTX(2), DTX(2), TBB(20), RAT(1)
C
      DO 200 JX =  1,2
      DO 150  K=1,NL
      DO  10  M=1,2
   10 DSTPX(M) = 0.
C
      IF (JX .EQ. 1) GO TO 25
      DO 15 N =1,2
      DSTPX(1) = DSTPX(1) + APA1(N,K,NX)*DSTA(N,K,NX)
   15 DSTPX(2) = DSTPX(2) + APB1(N,K,NX)*DSTA(N,K,NX)
      DO 20 M =1,2
   20 DSTEX(M) = DSTA(M,K,NX)  - DSTPX(M)
      GO TO 35
C
   25 DO 30 N =1,2
      DSTPX(1) = DSTPX(1) + APA2(N,K,NX)*DSTB(N,K,NX)
   30 DSTPX(2)  = DSTPX(2) + APB2(N,K,NX)*DSTB(N,K,NX)
      DO 32 M =1,2
   32 DSTEX(M) = DSTB(M,K,NX)  - DSTPX(M)
C
   35 DO  40  M=1,2
   40 DTX(M) = .0
C
      DO  50  M=1,2
      DO  50  N=1,2
   50 DTX(M) = DTX(M) + EE(M,N) * DSTEX(N)
C
      TTX(1) = TAX(JX,K,NX) + DTX(1)
      TTX(2) = TBX(JX,K,NX) + DTX(2)
C
      S1 = TTX(1) - .50 * TTX(2)
      S2 = TTX(2) - .50 * TTX(1)
      TBAR = SQRT(TTX(1) * S1 + TTX(2) * S2 )
      IF(DSTPX(1)  .EQ. .0 .AND. DSTPX(2)  .EQ. .0) GO TO 70
C
      S1 = TAX(JX,K,NX) - .50 * TBX(JX,K,NX)
      S2 = TBX(JX,K,NX) - .50 * TAX(JX,K,NX)
      FLAG = S1 * DTX(1) + S2 * DTX(2)
      IF (FLAG .LT. 0.0)   GO TO 70
C
```

```
      EP1 = DSTPX(1) + .50 * DSTPX(2)
      EP2 = DSTPX(2) + .50 * DSTPX(1)
      DEP = SQRT(4./3. * (DSTPX(1) *EP1 + DSTPX(2) * EP2))
      EPLX(JX,K,NX) = EPLX(JX,K,NX) + DEP
      GO TO 80
C
   70 STM = 0.999 * STIX(JX,K,NX)
      IF (TBAR .LT. STM)   GO TO 90
C
   80 KK = K
      II = NX
      CALL INTERD(KK,II,JX)
      RAT(K) = STNX / TBAR
      GO TO 100
C
   90 ZTX = 1.
      STNX = STIX(JX,K,NX)
      RAT(K) = 1.0
C
  100 TAX(JX,K,NX) = RAT(K) * TTX(1)
      TBX(JX,K,NX) = RAT(K) * TTX(2)
C
      S1 = TAX(JX,K,NX) - .50 * TBX(JX,K,NX)
      S2 = TBX(JX,K,NX) - .50 * TAX(JX,K,NX)
      TBB(K) = SQRT(S1 * TAX(JX,K,NX)+ S2* TBX(JX,K,NX) )
      IF(TBB(K) .EQ. 0.0)  GO TO 120
      S1 = S1 / TBB(K)
      S2 = S2 / TBB(K)
      SS1 = (S1 + U*S2) * (1. - ZTX)
      SS2 = (S2 + U*S1) * (1. - ZTX)
      DNOM = (1.-U)*(1.+U)*ZTX + S1*SS1 + S2*SS2
      S1 = S1 / DNOM
      S2 = S2 / DNOM
C
      IF (JX .EQ. 1) GO TO 117
      APA1(1,K,NX) = S1 * SS1
      APA1(2,K,NX) = S1 * SS2
      APB1(1,K,NX) = S2 * SS1
      APB1(2,K,NX) = S2 * SS2
      GO TO 135
C
  117 APA2(1,K,NX) = S1 * SS1
      APA2(2,K,NX) = S1 * SS2
      APB2(1,K,NX) = S2 * SS1
      APB2(2,K,NX) = S2 * SS2
      GO TO 135
  120 CONTINUE
      IF (JX .EQ. 1) GO TO 127
      DO 124 N =1,2
      APA1(N,K,NX) = .0
  124 APB1(N,K,NX) = .0
      GO TO 135
  127 DO 128 N=1,2
      APA2(N,K,NX) = .0
  128 APB2(N,K,NX) = .0
```

```
C
  135 AK = K
      IF (JX .EQ. 1) GO TO 145
      DO 140  N=1,2
      SUMA1(N,NX,1) = SUMA1(N,NX,1) + APA1(N,K,NX)
      SUMB1(N,NX,1) = SUMB1(N,NX,1) + APB1(N,K,NX)
      SUMA1(N,NX,2) = SUMA1(N,NX,2) + APA1(N,K,NX) * (AK-.50)
      SUMB1(N,NX,2) = SUMB1(N,NX,2) + APB1(N,K,NX) * (AK-.50)
      SUMA1(N,NX,3) = SUMA1(N,NX,3) + APA1(N,K,NX) * (AK*(AK-1.) +
     1.33333333)
  140 SUMB1(N,NX,3) = SUMB1(N,NX,3) + APB1(N,K,NX) * (AK*(AK-1.) +
     1.33333333)
      GO TO 150
  145 DO 147 N=1,2
      SUMA2(N,NX,1) = SUMA2(N,NX,1) + APA2(N,K,NX)
      SUMB2(N,NX,1) = SUMB2(N,NX,1) + APB2(N,K,NX)
      SUMA2(N,NX,2) = SUMA2(N,NX,2) + APA2(N,K,NX) * (AK-.50)
      SUMB2(N,NX,2) = SUMB2(N,NX,2) + APB2(N,K,NX) * (AK-.50)
      SUMA2(N,NX,3) = SUMA2(N,NX,3) + APA2(N,K,NX) * (AK*(AK-1.) +
     1.33333333)
  147 SUMB2(N,NX,3) = SUMB2(N,NX,3) + APB2(N,K,NX) * (AK*(AK-1.) +
     1.33333333)
C
  150 STIX(JX,K,NX) = STNX
C
      IE = I - 2 + JX
      WRITE (6,1200) IE,(I, (K,  TAX(JX,K,NX), TBX(JX,K,NX), TBB(K),
     1 RAT(K)),  K = 1, NL)
C
  200 CONTINUE
      IF (NX .LT. NDISC)  NX = NX + 1
C
      RETURN
C
 1200 FORMAT
     A(1H2,  * STRESS DISTRIBUTION AT THE NODE WHERE SLOPE IS DISCONT.
     AELEMENT NO.* I5 //
     1 8H  NODE   , 2X , 5HLAYER , 11X , 10HMERIDIONAL , 15X , 10HCIRCUM
     2FER.  ,  11X , 18HEQUIVALENT STRESS  , 4X , 20HMODIFICATION FACTOR
     3  // (I5 , 3X , I5 , 3E25.7 , F20.5))
C
      END
1
```

```
      SUBROUTINE  INTERP(K,I)
C
C     SUBROUTINE FOR LINEAR INTERPOLATION OF MATERIAL PROPERTIES
C
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT2 / STI(20),EPL(20),AP(2,2,20),DST(2,20,50 ),ZT,STN
C
      IF (EPL(K) .GT. EP(NP))   GO TO 100
C
      DO  10  IP=2,NP
      IF (EPL(K) .LE. EP(IP))   GO TO  50
   10 CONTINUE
C
   50 RHO = (EPL(K) - EP(IP-1)) / (EP(IP) - EP(IP-1))
      STN = SIGMA(IP-1) + RHO * (SIGMA(IP) - SIGMA(IP-1))
      ZT  =  ETAN(IP-1) + RHO * ( ETAN(IP) -  ETAN(IP-1))
      RETURN
C
  100 WRITE (6,1000)  K,I,EPL(K)
      STOP
C
 1000 FORMAT(I5,I5,E20.5 / 40H-MATERIAL PROP. DATA IS EXCEEDED         )
      END
1
```

```
      SUBROUTINE   INTERD(K,I,J)
C
C     SUBROUTINE FOR LINEAR INTERPOLATION OF MATERIAL PROPERTIES
C     AT THE NODES WHERE SLOPE IS DISCONT.
C
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/MATD2/ STIX(2,20, 5), EPLX(2,20, 5), ZTX, STNX
C
      IF (EPLX(J,K,I) .GT. EP(NP))  GO TO 100
C
      DO  10  IP=2,NP
      IF (EPLX(J,K,I) .LE. EP(IP))  GO TO 50
   10 CONTINUE
   50 RHO = (EPLX(J,K,I) - EP(IP-1)) / (EP(IP) - EP(IP-1))
      STNX = SIGMA(IP-1) + RHO * (SIGMA(IP) - SIGMA(IP-1))
      ZTX  =  ETAN(IP-1) + RHO * ( ETAN(IP) -  ETAN(IP-1))
      RETURN
C
  100 WRITE (6,1000)  K,I,EPLX(J,K,I)
      STOP
C
 1000 FORMAT(I5,I5,E20.5 /   *-MATERIAL PROP. DATA IS EXCEEDED AT THE NO
     ADE WHERE THE SLOPE IS DISCONT.* )
      END
1
```

```
      SUBROUTINE B1MATX
C
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/GEOM4 /YBAR,YP,YPP,RW,XT,ARC,RV,CNP1,CNP2,B(4,8)
      COMMON /GEOM5/ A1,A2,A3,A4
C
      DO 10   N=1,3
      DO 10   M=1,4
   10 B(M,N) = 0.0
C
      RHO = 1. / (CORD * ARC**2)
      AMU = 1. / RV
      ALPHA = RHO / ARC
      PHI =  ALPHA * RHO * YPP
      PSI = (SNT + YP * CNT) * ALPHA * AMU
      GAMA = 2.0*(A2-A1) + XT*(3.0*(A3-A2) + XT*(4.0*(A4-A3) -
     1 XT*5.0*A4))
      OMG = 2. * YP * ALPHA / CORD
      TET = (1. - YP * YP ) * PHI
C
      B(1,4) = RHO * (1. + YP * TNB1)
      B(2,4) = AMU * CNP1
      B(3,4) = (1. + YP * (2. * TNB1 - YP)) * PHI
      B(4,4) = GAMA * PSI
C
      B(1,5) = 2. * RHO * XT
      B(2,5) = AMU * XT * SNT
      B(3,5) = 2. * XT * TET + OMG
      B(4,5) = 2. * YP * PSI
C
      B(1,6) = 1.5 * XT * B(1,5)
      B(2,6) = XT * B(2,5)
      B(3,6) = 3. * XT * (XT * TET + OMG )
      B(4,6) = 1.5 * XT * B(4,5)
C
      B(1,7) = YP * B(1,5)
      B(2,7) = AMU * XT * CNT
      B(3,7) = 2. * (2.*PHI * XT * YP - ALPHA/CORD)
      B(4,7) = -2. * PSI
C
      B(1,8) = 1.50 * B(1,7) * XT
      B(2,8) = B(2,7) * XT
      B(3,8) = 6. * (PHI * XT * YP - ALPHA/CORD) * XT
      B(4,8) = -3. * PSI * XT
C
      RETURN
      END
1
```

```
      SUBROUTINE BMATX
C
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/GEOM4 /YBAR,YP,YPP,RW,XT,ARC,RV,CNP1,CNP2,B(4,8)
C
      RHO = 1./(CORD * ARC**2)
      AU  = 2./(CORD**2 * ARC**3)
      PSI = (SNT + YP * CNT)/(CORD * RW * ARC**3)
      PHI = YPP / (CORD**2 * ARC**5)
      OMG = YP * AU
C
      B(1,1) = 0.0
      B(2,1) = SNT / RW
      B(3,1) = 0.0
      B(4,1) = 0.0
C
      B(1,2) = RHO
      B(2,2) = B(2,1) * XT
      B(3,2) = (1.-YP**2) * PHI
      B(4,2) = YP * PSI
C
      B(1,3) = 2. * XT * RHO
      B(2,3) = B(2,2) * XT
      B(3,3) = B(3,2) * 2. * XT + OMG
      B(4,3) = B(4,2) * 2. * XT
C
      B(1,4) = B(1,3) * 1.5 * XT
      B(2,4) = B(2,3) * XT
      B(3,4) = 3. * XT * ( XT * B(3,2) + OMG )
      B(4,4) = B(4,3) * 1.5 * XT
C
      B(1,5) = 0.0
      B(2,5) = CNT / RW
      B(3,5) = 0.0
      B(4,5) = 0.0
C
      B(1,6) = YP * RHO
      B(2,6) = B(2,5) * XT
      B(3,6) = 2. * YP * PHI
      B(4,6) = -PSI
C
      B(1,7) = 2. * B(1,6) * XT
      B(2,7) = B(2,6) * XT
      B(3,7) = 2. * B(3,6) * XT - AU
      B(4,7) = -2. * PSI * XT
C
      B(1,8) = 1.5 * B(1,7) * XT
      B(2,8) = B(2,7) * XT
      B(3,8) = 3. * (B(3,6) * XT - AU) * XT
      B(4,8) = 1.5 * B(4,7) * XT
C
      RETURN
      END
1
```

```
      SUBROUTINE C1MATX
C
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
      COMMON/ GEOM4/YBAR,YP, YPP , RW , XT , ARC , RV , CNP1,CNP2,B(4,8)
C
      DO 10    N=1,8
      DO 10    M=1,8
   10 C(M,N) = 0.0
      TJ = 5.5 * TNB2
      TITJ = 2. * TNB1 + TJ
C
      C(3,2) = 1.0
      C(4,2) = CNT * 5.5
      C(5,2) = -9. * CNT
      C(6,2) = 4.5* CNT
      C(7,2) = -CNT * (11. * TNB1 + TNB2) - 3. * SNT
      C(8,2) =  CNT * (5.5 * TNB1 + TNB2) + 2. * SNT
C
      C(4,4) = SNT
      C(5,4) = -4.5 * SNT
      C(6,4) = -C(5,4)
      C(7,4) = -SNT * TITJ + 3. * CNT
      C(8,4) = SNT * (TNB1 + TJ) - 2. * CNT
C
      C(4,5) = -CNT
      C(5,5) = C(6,2)
      C(6,5) = -C(5,5)
      C(7,5) = CNT * TITJ + 3. * SNT
      C(8,5) = -CNT * (TNB1 + TJ) - 2. * SNT
C
      C(7,6) = -CORD / CNB2 **2
      C(8,6) = -C(7,6)
C
      C(4,7) = 9.
      C(5,7) = -22.5
      C(6,7) = 13.5
      C(7,7) = -18. * TNB1 - 4.5 * TNB2
      C(8,7) =  9.  * TNB1 + 4.5 * TNB2
C
      C(4,8) = -4.5
      C(5,8) = 18.
      C(6,8) = -13.5
      C(7,8) = 9. * (TNB1 + TNB2)
      C(8,8) = -4.5 * TNB1 - 9.0 * TNB2
C
      RETURN
      END
1
```

```
      SUBROUTINE CMATX
C
C     SUBROUTINE TO CONSTRUCT DISPL. TRANS. MATRIX IN Ur-Uz CO-ORDINATES
C
      COMMON/GEOM2 / COF(10) , B1(4,8,10) , B2(4,8,2) , C(8,8)
      COMMON/GEOM3 /CORD,SNT,CNT,SNB1,CNB1,TNB1,SNB2,CNB2,TNB2,SNP,CNP
C
      TI = 5.5 * TNB1
      TA = 11. * TNB1
      TJ = 5.5 * TNB2
      TITJ = 2. * TNB1 + TJ
C
      C(1,1) = SNT
      C(2,1) = - 5.5 * SNT
      C(3,1) =   9. * SNT
      C(4,1) = -4.5 * SNT
      C(5,1) = CNT
      C(6,1) = -TI * SNT
      C(7,1) = SNT * (TA + TNB2) - 3. * CNT
      C(8,1) = -SNT * (TI + TNB2) + 2. * CNT
C
      C(1,2) = -CNT
      C(2,2) = 5.5 * CNT
      C(3,2) = -9. * CNT
      C(4,2) = 4.5 * CNT
      C(5,2) = SNT
      C(6,2) = CNT * TI
      C(7,2) = -CNT * (TA + TNB2) - 3. * SNT
      C(8,2) = CNT * (TI + TNB2) + 2. * SNT
C
      C(1,3) = 0.0
      C(2,3) = 0.0
      C(3,3) = 0.0
      C(4,3) = 0.
      C(5,3) = 0.
      C(6,3) = CORD / CNB1**2
      C(7,3) = -2.*C(6,3)
      C(8,3) = C(6,3)
C
      C(1,4) = 0.0
      C(2,4) = SNT
      C(3,4) = C(4,1)
      C(4,4) = -C(3,4)
      C(5,4) = 0.0
      C(6,4) = SNT * TNB1
      C(7,4) = -SNT * TITJ + 3. * CNT
      C(8,4) = SNT * (TNB1 + TJ) - 2. * CNT
C
      C(1,5) = 0.0
      C(2,5) = -CNT
      C(3,5) = C(4,2)
      C(4,5) = -C(3,5)
      C(5,5) = 0.0
      C(6,5) = -TNB1 * CNT
      C(7,5) = CNT * TITJ + 3. * SNT
```

```
      C(8,5) = -CNT * (TNB1 + TJ) - 2. *SNT
C
      C(1,6) = 0.0
      C(2,6) = 0.0
      C(3,6) = 0.0
      C(4,6) = 0.0
      C(5,6) = 0.0
      C(6,6) = 0.0
      C(7,6) = -CORD / CNB2**2
      C(8,6) = -C(7,6)
C
      C(1,7) = 0.
      C(2,7) = 9.
      C(3,7) = -22.5
      C(4,7) = 13.5
      C(5,7) = 0.
      C(6,7) = 9. * TNB1
      C(7,7) = -18. * TNB1 - 4.5 * TNB2
      C(8,7) =  9.0 * TNB1 + 4.5 * TNB2
C
      C(1,8) = 0.
      C(2,8) = -4.5
      C(3,8) = 18.
      C(4,8) = -13.5
      C(5,8) = 0.
      C(6,8) = -4.5 * TNB1
      C(7,8) = 9. * (TNB1 + TNB2)
      C(8,8) = -4.5 * TNB1 - 9.0 * TNB2
C
      RETURN
      END
1
```

```
      SUBROUTINE  DIMATX(L,I)
C
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
C
      DIMENSION  X(12)
C
      DATA X / 0.0,0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586, 1.0 /
C
      HI = H(1,I) + X(L) * (H(2,I) - H(1,I))
      DO  10  N=1,2
      DO  10  M=1,2
   10 D(M,N) = EE(M,N) * HI
C
      DO  20  N=3,4
      DO  20  M=1,2
   20 D(M,N) = 0.0
C
      DO  30  N=1,2
      DO  30  M=3,4
   30 D(M,N) = 0.0
C
      DO  40  N=3,4
      DO  40  M=3,4
   40 D(M,N) = D(M-2,N-2) * HI**2 / 12.
C
      RETURN
      END
1
```

```
      SUBROUTINE  DMATX(L,I,MA,MB,NX)
C
C     MODIFIED SUBROUTINE TO  HANDLE THE SLOPE DISCONTIN.
C
      COMMON/ DIV   / NE , NL , NLI
      COMMON/GEOM1 /R(2, 50),Z(2, 50),PHI(2, 50),H(2, 50),CRV(2,50)
      COMMON/ MAT1 / E,U,NP,SIGMA(15),ETAN(15),EP(15),EE(2,2)
      COMMON/ MAT3 / SUMA(2,50 ,3), SUMB(2,50 ,3), TB(20), RAT(20)
      COMMON/STF1/ D(4,4) , SKI(8,8) , SK(8,8)
      COMMON/MATD1 /SUMA1(2, 5,3), SUMA2(2, 5,3), SUMB1(2, 5,3)
     A ,SUMB2(2, 5,3)
C
      DIMENSION   X(12) , TEMP(2,2,3) , TMP(2,2)
C
      DATA X / 0.0,0.013046735741414 , 0.067468316655507,
     1 0.160295215850488, 0.283302302935376 , 0.425562830509184,
     2 0.574437169490816, 0.716697697064624, 0.839704784149512,
     3 0.932531683344493, 0.986953264258586, 1.0 /
C
      DO  10  N=1,4
      DO  10  M=1,4
   10 D(M,N) = 0.0
C
      ANL = NL
      HI = H(1,I) + X(L) * (H(2,I) - H(1,I))
      FAC = HI / ANL
      IF (MA .EQ. 1) GO TO 13
      IF (MB .EQ. 1) GO TO 11
C
      DO  20  J=1,3
      DO  20  N=1,2
      TEMP(1,N,J) = (SUMA( N,I,J) + X(L)*(SUMA( N,I+1,J)-SUMA( N,I,J)))
     1 * FAC**J
   20 TEMP(2,N,J) = (SUMB( N,I,J) + X(L)*(SUMB( N,I+1,J)-SUMB( N,I,J)))
     1 * FAC**J
      GO TO 25
C
   11 DO  12  J=1,3
      DO  12  N=1,2
      TEMP(1,N,J) = ( SUMA(N,I,J) + X(L) * ( SUMA2(N,NX,J) -
     D SUMA(N,I,J))) * FAC**J
   12 TEMP(2,N,J) = ( SUMB(N,I,J) + X(L) * ( SUMB2(N,NX,J) -
     D SUMB(N,I,J))) * FAC**J
      GO TO 25
   13 DO  15  J=1,3
      DO  15  N=1,2
      TEMP(1,N,J) = (SUMA1(N,NX,J)+ X(L) * ( SUMA(N,I+1,J) -
     E SUMA1(N,NX,J))) * FAC**J
   15 TEMP(2,N,J) = (SUMB1(N,NX,J)+ X(L) * ( SUMB(N,I+1,J) -
     E SUMB1(N,NX,J))) * FAC**J
   25 TMP(1,1) = -TEMP(1,1,1) + HI
      TMP(1,2) = -TEMP(1,2,1)
      TMP(2,1) = -TEMP(2,1,1)
      TMP(2,2) = -TEMP(2,2,1) + HI
C
```

```
      DO   30   N=1,2
      DO   30   M=1,2
      DO   30   K=1,2
   30 D(M,N) = D(M,N) + EE(M,K) * TMP(K,N)
C
      DO   40   N=1,2
      DO   40   M=1,2
   40 TMP(M,N) = 0.50 * HI * TEMP(M,N,1) - TEMP(M,N,2)
C
      DO   50   N=3,4
      DO   50   M=1,2
      DO   50   K=1,2
   50 D(M,N) = D(M,N) + EE(M,K) * TMP(K,N-2)
C
      DO   60   N=1,2
      DO   60   M=3,4
   60 D(M,N) = D(N,M)
C
      DO   70   N=1,2
      DO   70   M=1,2
   70 TMP(M,N) = (-0.25 * HI*TEMP(M,N,1) + TEMP(M,N,2))*HI - TEMP(M,N,3)
C
      TMP(1,1) = TMP(1,1) + HI**3 / 12.
      TMP(2,2) = TMP(2,2) + HI**3 / 12.
C
      DO   80   N=3,4
      DO   80   M=3,4
      DO   80   K=1,2
   80 D(M,N) = D(M,N) + EE(M-2,K) * TMP(K,N-2)
C
      RETURN
      END
1
```

```
      SUBROUTINE BANSOL
C
C     **************M*********************************************************
C     IN-CORE LINEAR EQUATION SOLVER FOR SYMMETRIC BAND MATRICES
C
C     ************************************************************************
C
      COMMON /BANARG/  NN, MM,A(150, 6),B(150)
      DIMENSION S(1)
      EQUIVALENCE (S,A)
      NCOL = 150
      NR = NN
      NRS = NR - 1
      MMR = MM - 1
C     DECOMPOSE MATRIX A
      DO 120  N = 1,NRS
      M = N - 1
      MR = MIN0 (MM,NR-M)
      PIVOT = S(N)
      J = N
      DO 120  L = 2,MR
      J = J + NCOL
      C = S(J)/PIVOT
      I1 = M + L
      I2 = I1 + (MR-L)*NCOL
      II = J
      DO 110  I = I1,I2,NCOL
      S(I) = S(I) - C*S(II)
  110 II = II + NCOL
  120 S(J) = C
C     REDUCE AND BACKSUBSTITUTE VECTOR B
      DO 220  N = 1,NRS
      MR = MIN0 (MMR,NR-N)
      C = B(N)
      B(N) = C/S(N)
      K = N
      L1 = N + 1
      L2 = N + MR
      DO 220  L = L1,L2
      K = K + NCOL
  220 B(L) = B(L) - S(K)*C
      B(NR) = B(NR)/S(NR)
  300 DO 320  I = 1,NRS
      N = NR - I
      MR = MIN0 (MMR,I)
      J = N
      L1 = N + 1
      L2 = N + MR
      DO 320  L = L1,L2
      J = J + NCOL
  320 B(N) = B(N) - S(J)*B(L)
      RETURN
      END
```