Title
AMIPROTON EXPERIMENTS IN A HYDROGEN BUBBLE CHAMBER

Author
Solmitz, F.

Publication Date
1960-08-01

UCRL-8738

UNIVERSITY OF
CALIFORNIA

Ernest O. Lawrence
Radiation
Laboratory

BERKELEY, CALIFORNIA

DISCLAIMER

UCRL-8738

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

NUCLEAR SPIN, HYPERFINE-STRUCTURE SEPARATION AND MAGNETIC MOMENT OF 22-HOUR POTASSIUM-43

F. Russell Petersen, Vernon J. Ehlers, W. Bruce Ewbank
Lawrence L. Marino and Howard A. Shugart

April 23, 1959

Printed for the U. S. Atomic Energy Commission

NUCLEAR SPIN, HYPERFINE-STRUCTURE SEPARATION AND MAGNETIC MOMENT OF 22-HOUR POTASSIUM-43

F. Russell Petersen, Vernon J. Ehlers, W. Bruce Ewbank
Lawrence L. Marino and Howard A. Shugart

Department of Physics and Lawrence Radiation Laboratory
University of California
Berkeley, California

April 23, 1959

ABSTRACT

With the atomic-beam magnetic-resonance method, the nuclear spin and hyperfine-structure separation have been measured for 22-hour K^{43}. The results are: $I = 3/2$, $\Delta\nu\,(^2S_{1/2}) = 192.64 \pm .05$ Mc/sec. The nuclear magnetic moment calculated from these measurements is: $|\mu| = 0.163 \pm .002$ nuclear magnetons.

NUCLEAR SPIN, HYPERFINE-STRUCTURE SEPARATION AND MAGNETIC MOMENT OF 22-HOUR POTASSIUM-43*

F. Russell Petersen, Vernon J. Ehlers, W. Bruce Ewbank
Lawrence L. Marino and Howard A. Shugart

Department of Physics and Lawrence Radiation Laboratory
University of California
Berkeley, California

April 23, 1959

I. INTRODUCTION

The atomic-beam flop-in technique has been used to measure the nuclear spin and hyperfine-structure separation of 22-hr potassium-43 in the $^2S_{1/2}$ electronic state.[1] Since the apparatus and procedure employed in making measurements of these quantities with radionuclides has been described in detail elsewhere,[2] only a brief summary of the method is included here. The convenient 22-hr half life of K^{43} and its β^- decay made beams of this isotope suitable for radioactive detection with high efficiency. The experimental results extend the evidence for a general trend in the magnetic moments of the odd-mass-number isotopes of potassium.

II. THEORY OF THE EXPERIMENT

A free atom of potassium in the $^2S_{1/2}$ electronic ground state may be represented in an external magnetic field $\underset{\sim}{H}$ by the Hamiltonian,

$$\mathcal{H} = -\mu_0 g_J \underset{\sim}{J} \cdot \underset{\sim}{H} - \mu_0 g_I \underset{\sim}{I} \cdot \underset{\sim}{H} + \frac{h\Delta\nu}{I + \frac{1}{2}} \underset{\sim}{I} \cdot \underset{\sim}{J}, \qquad (1)$$

---

* This research was supported in part by the U. S. Air Force Office of Scientific Research and the Atomic Energy Commission.

where μ_0 is the absolute value of the Bohr magneton, $\underset{\sim}{I}$ and $\underset{\sim}{J}$ are the nuclear and electronic angular momenta in units of $\hbar$, g_I is the nuclear g factor $\left[\mu_I/(\mu_0 I)\right]$, g_J is the electronic g factor $\left[\mu_J/(\mu_0 J)\right]$, and $\Delta\nu$ is the zero-field hyperfine-structure separation between the F = I+$\frac{1}{2}$ and F = I-$\frac{1}{2}$ levels, in cycles per second. The energy levels of this Hamiltonian are given by the Breit-Rabi formula,[3]

$$W(F, m_F) = -\frac{h\Delta\nu}{2(2I+1)} - g_I\mu_0 H m_F \pm \frac{h\Delta\nu}{2}\left(1 + \frac{4mx}{2I+1} + x^2\right)^{1/2}, \qquad (2)$$

where

$$x = (-g_J + g_I)\frac{\mu_0 H}{h\Delta\nu}.$$

The positive sign is taken with the F = I+$\frac{1}{2}$ levels and the negative sign with the F = I-$\frac{1}{2}$ levels. The qualitative variation of the hyperfine energy levels for the case of J = 1/2, I = 3/2 and a positive nuclear moment is shown in Fig. 1.

In principle, the deflecting fields (A and B) of the flop-in apparatus focus on the detector only those atoms which change the signs of their effective magnetic moments while the atoms traverse the region between the A and B magnets. At high A and B fields ($x \gg 0.5$ for the case in Fig. 1), the refocusing condition is satisfied for the transitions, $\Delta m_J = \pm 1$. As shown in the figure, nine allowed ($\Delta m_F = \pm 1, 0$) transitions are readily observable.

To second order in H, the transition labeled i has a frequency dependence of

$$\nu = \frac{-g_J - 2Ig_I}{2I+1}\,\frac{\mu_0 H}{h} + 2I\,\frac{(-g_J + g_I)^2}{(2I+1)^2}\,\frac{\mu_0^2}{h^2}\,\frac{H^2}{\Delta\nu} + \cdots . \qquad (3)$$

In the "linear" Zeeman region, where the magnetic field is low, the first term of this expression is dominant and the higher-order terms may be neglected. Similarly, since g_I is about 1/2000 of g_J, its effect in the equation is small. Therefore, the transition frequency is dependent essentially upon H, I, and the known constants g_J, μ_0, and h. Observation of this transition at given low fields and frequencies thus establishes the nuclear spin I.

Initial estimates of the hyperfine-structure separation $\Delta\nu$ result when transition i is followed to higher fields where the second- and higher-order terms in Eq. (3) contribute. For this transition, the hyperfine-structure separation may be calculated exactly from the equation

$$\Delta\nu = \frac{\left(\nu + \frac{g_I\mu_0 H}{h}\right)\left(\frac{-g_J\mu_0 H}{h} - \nu\right)}{\nu + \frac{g_J\mu_0 H}{(2I+1)h} + \frac{2I}{2I+1}\frac{g_I\mu_0 H}{h}}, \qquad (4)$$

where ν is the resonant frequency of transition i and other symbols have been defined previously. In this equation, g_I is an unknown but may be estimated with the aid of the Fermi-Segrè formula,[4]

$$\frac{\Delta\nu}{\Delta\nu'} \cong \left|\frac{g_I}{g_I'}\right| \frac{2I+1}{2I'+1}, \qquad (5)$$

where the primed and umprimed quantities refer to two isotopes of the same element. Equations (4) and (5) may be solved simultaneously for $\Delta\nu$ and g_I by assuming first a positive and then a negative sign for g_I. Although the Fermi-Segrè formula involves certain simplifying assumptions, moments calculated from it are normally in error by less than 1%.

As seen in Fig. 1, eight of the observable transition frequencies approach $\Delta\nu$ as the external field approaches zero. The field dependence of these transitions may be computed from the Breit-Rabi equation. With the apparatus used in this work, the transitions b and c, as well as e and f, form unresolved doublets. The transition d exhibits only small field dependence at low fields. The resulting resonance is narrow and therefore provides the best measurements of the zero-field hyperfine-structure separation.

III. ISOTOPE PRODUCTION AND IDENTIFICATION

K^{43} was produced on the Berkeley 60-inch Crocker cyclotron by the reaction $A^{40}(\alpha, p)K^{43}$. The natural argon gas at 2 atmospheres absolute pressure was contained in a water-cooled aluminum cylinder of cross section 1-1/2 × 5 inches and of length 19 inches. One end of this container was provided with a "window assembly" to admit the bombarding particles. After a bombardment, potassium atoms were recovered from the walls of the target container by solution in distilled water containing about 30 mg of potassium chloride carrier. Three washings, each approximately 200 ml in volume, were adequate to remove the major portion of the activity. Then the solution was reduced in volume, pipetted into the atomic-beam oven, and evaporated to dryness. An excess of finely divided calcium metal was added to cause reduction of the potassium ions from the chloride when the oven was later heated in the atomic-beam apparatus.

Since K^{42} (12.5-hr) is also produced by the reaction $A^{40}(\alpha, pn)K^{42}$ during a bombardment, this isotope forms an unwanted background in these experiments. For bombardments with 40-Mev alpha particles, continuous-flow proportional counters showed the initial activity of K^{42} to be 60 times that of K^{43}. As a result, an experiment was conducted to determine roughly the relative yield of K^{43} to K^{42} as a function of the beam energy.

At about 20 Mev, the activity ratio of K^{42}/K^{43} was reduced to 4. Subsequently, the preferential decay of K^{42} further decreased this ratio before use of the sample. At 20 Mev, 100 to 140 microampere-hours of bombardment produced adequate K^{43} activity for 15 hours of running time, with resonance signals of 3 to 30 counts per minute (10-minute collecting time) above a 2-cpm counter background.

Samples of the transmitted potassium beam were collected on sulfur surfaces and counted in continuous-flow proportional counters. Because the samples were inserted directly into the sensitive volume of the counters, radiation into 2π radians of solid angle was counted. Each resonance exposure was decayed for 3 or 4 days to verify the presence of 22-hour K^{43}. The half life and identity of this isotope have been well established by previous investigators.[5, 6, 7]

IV. EXPERIMENTAL PROCEDURE

For the work on K^{43}, the resistance-heated oven was inserted into the atomic beam apparatus by means of an oven-loader assembly. This assembly, containing electrical, thermocouple, and water-cooling connections, could be introduced into the apparatus without disturbing the high vacuum within.

The easily detected potassium carrier, which was added during the chemistry, facilitated initial alignment of the oven. Also, observation of the low-field flop-in resonance ($F, m_F = 2, -1 \longleftrightarrow 2, -2$) of stable potassium before and after each radioactive exposure served to calibrate the transition magnetic field (C field) and to indicate the beam intensity for normalization.

Radiofrequencies for the $\Delta F = 0$ transitions were generated by a Tektronix Type 190 oscillator. For the $\Delta F = \pm 1$ transitions, a Hewlett-Packard Model 608A oscillator and two Intruments For Industries wide-band amplifiers were used. All frequencies were monitored with a Hewlett-Packard Model 524B frequency counter, whose 100-Kc/sec internal-reference frequency was compared weekly with an Atomichron.

V. RESULTS

From Eq. (3), the spins and frequencies of two detectable low-frequency resonances at a given field are related approximately by

$$\nu_1 = \frac{2I_2 + 1}{2I_1 + 1} \nu_2,$$

where the subscripts 1 and 2 may refer to radioactive K^{43} and stable K^{39} respectively. When a search was made at frequencies corresponding to spins of 5/2, 2, 3/2, and 1/2, the buttons corresponding to $I = 2$ and $I = 3/2$ gave definite indications of resonances. Subsequent decay of the activity collected on the $I = 2$ button confirmed its identity as K^{42}, which has a known spin of two.[8] Similarly, the decay of the $I = 3/2$ sample showed an enrichment of K^{43} over the normal composition of the beam. Because these resonances were quite broad, a portion of the tail of the spin-2 resonance contributed to the $I = 3/2$ signal. However, decay analysis distinguished the contribution of each isotope to the resonance.

On a subsequent run, four resonances of K^{43} were resolved at progressively higher values of the C field. These confirmed the spin ($I = 3/2$) and roughly determined the hyperfine-structure separation, $\Delta\nu$, of K^{43}. A summary of these results appears in Table I. The relatively large uncertainties in the frequencies of these resonances resulted from broad resonance lines. It should be noted that the usual consistency argument[9] for determining the sign of the moment cannot be used for K^{43} in this experiment. Owing to a small magnetic moment, the hyperfine-structure

Table I. Values of $\Delta\nu$ predicted from low-frequency resonances.

K^{39} (Mc/sec)	K^{43} (Mc/sec)	hfs separation (for either positive or negative magnetic moment) (Mc/sec)
10.89 ± .05	12.15 ± .50	177 ± 41
15.36 ± .05	17.70 ± .75	184 ± 33
24.30 ± .05	29.90 ± .50	186 ± 9
39.25 ± .30	52.60 ± .50	188 ± 6

separations calculated for a positive magnetic moment and for a negative magnetic moment, respectively, lie well within the errors of the measurements. As a result, the data presented here cannot determine the sign of the moment.

After the wide line width was reduced by changing the radio-frequency hairpin and by repositioning it in the C field, a search for the direct transitions ($\Delta F = \pm 1$) was begun. Table II summarizes the results. The field-independent line ($F, m_F = 2, 0 \longleftrightarrow 1, 0$, transition d in Fig. 1) was observed seven times in fields from ~ 2.3 to ~ 8 gauss. Proper dependence of this line upon the magnetic field established its identity. The two unresolved doublet frequencies which occur above and below that of the field-independent line were also measured.

For each resonance, all data were corrected for counter background, for fluctuations in beam intensity, and for radioactive decay. Each resonance peak button was also decayed to establish the enrichment of K^{43}. Next, a bell-shaped curve was fitted to the data of each resonance by a least-squares procedure. An example of a fitted curve of one of the field-independent resonances is shown in Fig. 2. From the curve-fitting procedure, the peak frequency, the width at half-maximum, and the uncertainty in peak frequency due to the uncertainties of input points were obtained. The uncertainty of the peak frequency was taken as a combination of one-eighth of the full width at half-maximum and of the uncertainty of the peak due to the statistical uncertainty of input data points. The uncertainty in the calibration frequency was estimated from consideration of the reproducibility of the calibration resonance.

The final value of $\Delta\nu$ is taken as the weighted average of all

Table II. Results of $\Delta F = \pm 1$ transitions. $\Delta\nu^{+}$ and $\Delta\nu^{-}$ are hyperfine-structure separations predicted by the Breit-Rabi equation by assuming a positive and a negative magnetic moment respectively.

$F, m_F \longleftrightarrow F', m_{F'}$	ν_{cal} (Mc/sec)	ν_{res} (Mc/sec)	$\Delta\nu^{+}$ and $\Delta\nu^{-}$ (Mc/sec)
2, 0 ↔ 1, 0	5.845 ± .020	193.995 ± .049	192.681 ± .050
2, 0 ↔ 1, 0	4.650 ± .020	193.515 ± .038	192.670 ± .038
2, 0 ↔ 1, 0	4.290 ± .020	193.335 ± .036	192.613 ± .037
2, 0 ↔ 1, 0	3.515 ± .020	193.120 ± .059	192.630 ± .060
2, 0 ↔ 1, 0	2.550 ± .020	192.897 ± .018	192.636 ± .018
2, 0 ↔ 1, 0	2.040 ± .020	192.787 ± .020	192.618 ± .021
2, 0 ↔ 1, 0	1.595 ± .020	192.764 ± .020	192.661 ± .020
Unresolved 2, 0 ↔ 1, -1 doublet 2, -1 ↔ 1, 0	2.580 ± .020	190.407 ± .068	192.709 ± .070
Unresolved 2, 0 ↔ 1, 1 doublet 2, 1 ↔ 1, 0	2.575 ± .020	195.364 ± .057	192.600 ± .062
Weighted average with estimated uncertainty			192.64 ± .05

hyperfine-structure separation measurements listed in Table II. The measurements are plotted in Fig. 3, which also shows the weighted average and stated uncertainty by the full and dashed lines respectively. The best value of $\Delta\nu$, with estimated uncertainty, is therefore

$$\Delta\nu = 192.64(5) \text{ Mc/sec.}$$

In conjunction with the known constants of K^{39} or K^{41}, the Fermi-Segrè formula was used to obtain the absolute value of the nuclear magnetic dipole moment of K^{43} to within about 1%. The result is

$$|\mu| = 0.163(2) \text{ nuclear magnetons.}$$

Some of the ground-state properties of three odd isotopes of potassium are now known (K^{39}, K^{41}, and K^{43}). The nuclear spins of all are $I = 3/2$ which, on the basis of the simple shell model, arises from one missing proton in the $d_{3/2}$ shell. The nuclear magnetic moments of this series show a monotonic decrease, with values of +0.391 nm for K^{39}, +0.215 nm for K^{41}, and ±0.163 nm for K^{43}. Although the resolution in this experiment was insufficient to establish the sign of K^{43}, it should be noted that the positive sign choice would make the measurement lie within the lower Schmidt limit of +0.124 nm.

The authors wish to thank Professors W. A. Nierenberg and H. B. Silsbee for their interest in this research.

REFERENCES

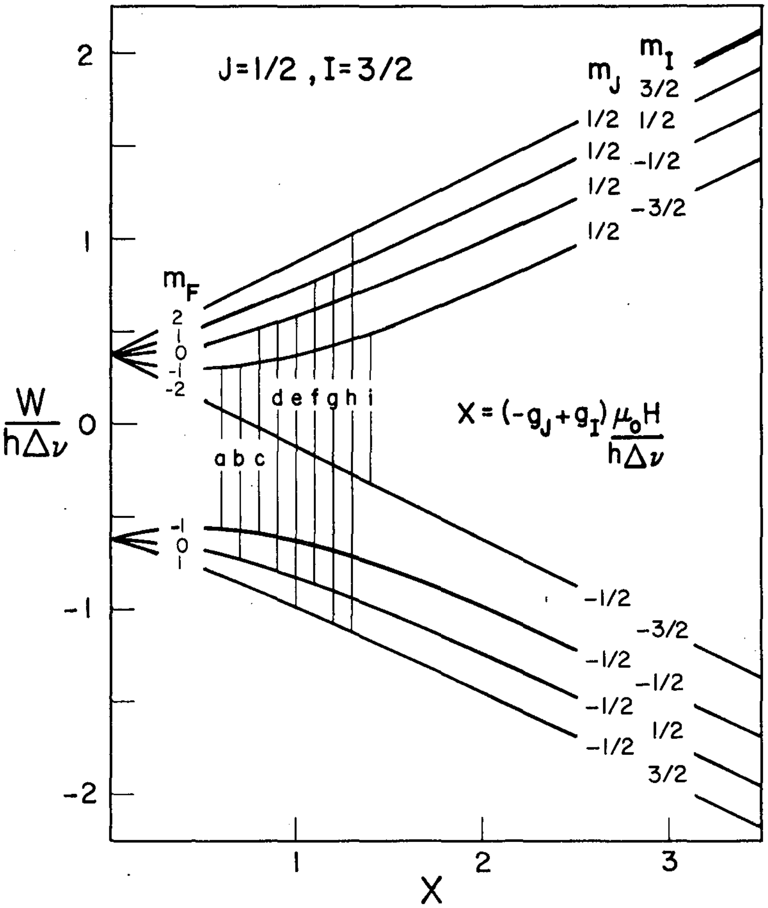
1. Petersen, Ehlers, Ewbank, Marino, and Shugart, Bull. Am. Phys. Soc. II, **3**, 415 (1958).
2. Hobson, Hubbs, Nierenberg, Silsbee, and Sunderland, Phys. Rev. **104**, 101 (1956).
3. G. Breit and I. I. Rabi, Phys. Rev. **38**, 2082 (1931), as extended by Millman, Rabi, and Zacharias, Phys. Rev. **53**, 384 (1938).
4. E. Fermi and E. Segrè, Z. Physik **82**, 729 (1933).
5. Overstreet, Jacobson, and Stout, Phys. Rev **75**, 231 (1949).
6. G. Anderson, Phil. Mag. **45**, 621 (1954).
7. T. Lindqvist and A. C. G. Mitchell, Phys. Rev. **95**, 444 (1954).
8. E. H. Bellamy and K. F. Smith, Phil. Mag. **44**, 33 (1953).
9. J. R. Zacharias, Phys. Rev. **61**, 270 (1942).

FIGURE LEGENDS

Fig. 1. The Breit-Rabi diagram for potassium-43 with an assumed positive nuclear magnetic moment.

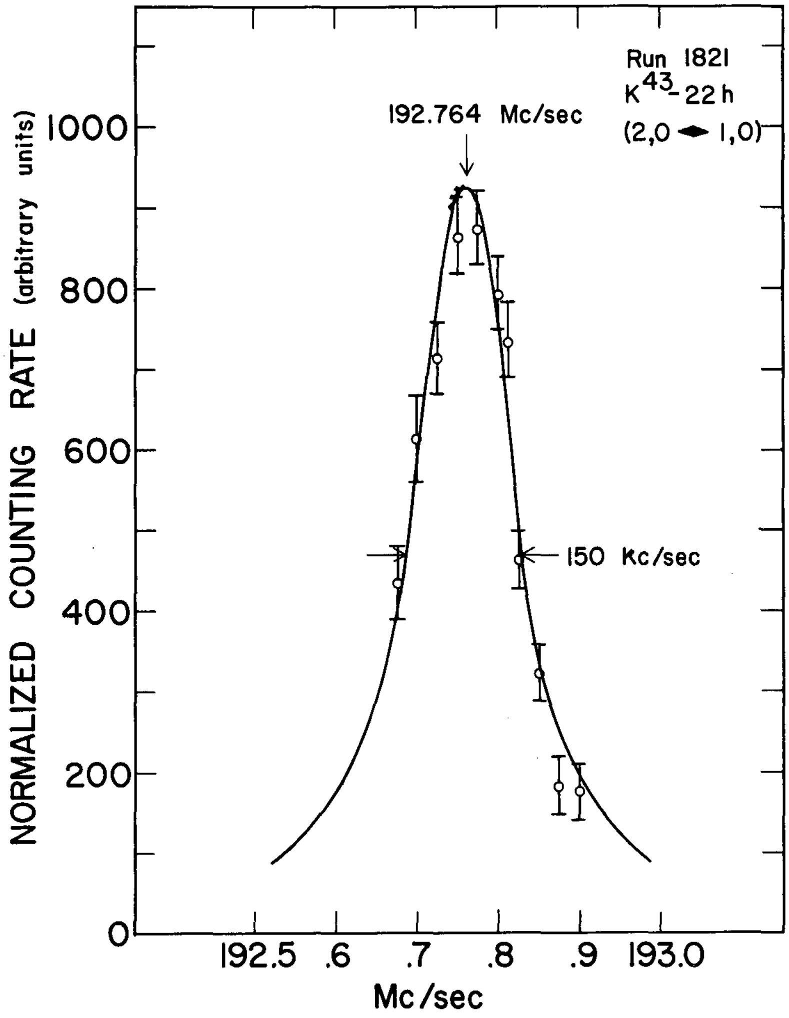
Fig. 2. A bell-shaped curve fitted to a field-independent resonance by a least-squares procedure.

Fig. 3. A plot of the calculated hyperfine-structure separations obtained from $\Delta F = \pm 1$ resonances. The heavy line indicates the weighted average of all observations, while the dashed lines indicate the quoted uncertainty in the final value.



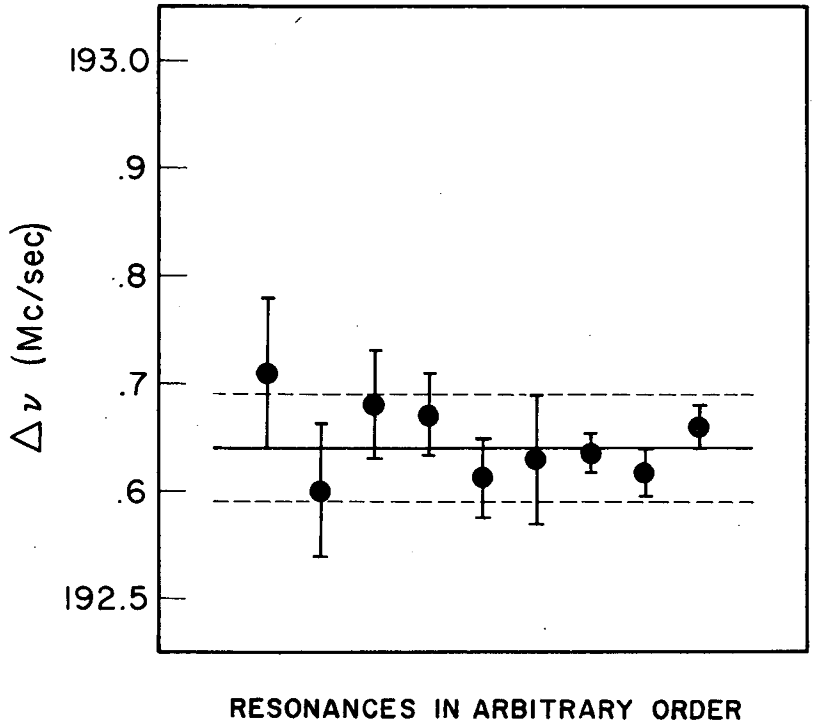
MU-17136

Fig. 1



MU-17137

Fig. 2



MU-17138

Fig. 3