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Author
Nagarajan, M.A.

Publication Date
1973-10-01

Submitted to Nuclear Physics

LBL-2300 c.2
Preprint

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M. A. Nagarajan

October 1973

Prepared for the U. S. Atomic Energy Commission
under Contract 7405-ENG-48

LBL-2300 c.2

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A THREE BODY MODEL FOR NUCLEON TRANSFER REACTIONS*

M. A. Nagarajan

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

October 1973

Abstract

The model of Vainshtein, Presnyakov, and Sobelman is applied to the study of nucleon transfer reactions. It is shown that the method is applicable to a few cases of nuclear transfer in collisions between heavy ions, and greatly simplifies the inclusion of recoil corrections. It is argued that the method should also be applicable to deuteron stripping reactions at high energies providing a simpler method to estimate finite range effects.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

1. Introduction

The distorted wave Born approximation (DWBA) has been commonly used in the analysis of deuteron stripping reactions[1]. Three body models have been suggested for (d,p) reactions on heavy nuclei[2], and the theory of Pearson and Coz[3] has been able to make interesting predictions on the spin polarization of the outgoing particles and also on the experimentally observed j dependence in the stripping reactions[4]. DWBA has been applied to the analysis of nuclear transfer in heavy ion reactions by Buttle and Goldfarb[5] and improved upon by Schmittroth, Tobocman and Golestaneh[6]. The above theories of heavy ion induced transfer reactions have introduced a no-recoil approximation, which ignores the momentum carried by the transferred nucleon, thus effecting a decoupling between the dynamics of the transfer process and the relative motion of the ion pairs. Recent experiments[7] have shown the no-recoil approximation to be inadequate in reactions above the Coulomb barrier. The importance of recoil in high energy heavy ion reactions had been originally pointed out by Dodd and Greider[8], and recently, semiclassical methods[9] and approximate methods[10] have been suggested for the inclusion of recoil effects. Utilizing the formalism for finite range calculations by Austern *et al.*[11] exact DWBA calculations[12] have been performed which exhibit the necessity of including recoil. The exact DWBA calculations, however, involve large computational time because of the large numbers of partial waves necessary for the analysis of the elastic scattering of heavy ions.

Vainshtein, Presnyakov and Sobelman[13]) had proposed an approximate method* for the computation of electron impact excitation cross-sections. Their approximation, has been applied to the analysis of charge transfer in collisions with atoms by Salin[14]), with remarkable success. Recently, Gayet[15]) has been able to rederive the VPS approximation, utilizing the approach of Dodd and Greider[16]), as the first term of a modified distorted wave series.

We suggest that the VPS approximation would be a suitable approximation for the study of nucleon transfer in heavy ion collisions at energies above the Coulomb barrier, as well as in high energy deuteron stripping reactions.

In sec. 2, we briefly rederive the VPS approximation and in sec. 3, we consider its application to specific reactions.

*This approximation will henceforth be referred to as the VPS approximation.

2. The Model

We shall use the notation of Buttle and Goldfarb[5]) throughout this paper. The reaction we consider is

$$\underbrace{(c_1 + n)}_{a_1} + c_2 \longrightarrow c_1 + \underbrace{(c_2 + n)}_{a_1} \quad .$$

We shall treat c_1 and c_2 as inert cores. We thereby neglect the coupled channel effects. The Hamiltonian of the system is

$$H = K + V_{nc_1}(r_1) + V_{nc_2}(r_2) + V_{c_1c_2}(r) \quad . \tag{2.1}$$

The co-ordinate vectors are shown in fig. 1. The kinetic energy operator, K, can be expressed as a sum of the kinetic energy operator for the center-of-mass of the system and an internal energy operator. Since the motion of the center-of-mass is not relevant, we shall represent K by the internal kinetic energy operator. One can choose as the linearly independent vectors, the pair $(\vec{r}_1, \vec{r}_\alpha)$ or $(\vec{r}_2, \vec{r}_\beta)$. Then K is expressed as

$$K = -\frac{\hbar^2}{2\mu_1} \nabla_1{}^2 - \frac{\hbar^2}{2\mu_\alpha} \nabla_\alpha{}^2 \tag{2.2a}$$

$$= -\frac{\hbar^2}{2\mu_2} \nabla_2{}^2 - \frac{\hbar^2}{2\mu_\beta} \nabla_\beta{}^2 \tag{2.2b}$$

where μ_1, μ_2, μ_α and μ_β, respectively, represent the reduced masses of the pairs $(n + c_1)$, $(n + c_2)$, $(a_1 + c_2)$ and $(a_2 + c_1)$. In the DWBA, the post version of the transition amplitude is approximated by

$$T_{\beta\alpha}\ (\vec{K}_\beta,\ \vec{K}_\alpha) = \langle \Phi_\beta^{(-)}(\vec{r}_2,\ \vec{r}_\beta)\,|V_{nc_1}(r_1)|\,\psi_\alpha^{(+)}(\vec{r}_1,\vec{r}_\alpha)\ \rangle \tag{2.3}$$

where $\langle \Phi_\beta^{(-)}(\vec{r}_2,\vec{r}_\beta)$ is a product of the bound state of the nucleon in the nucleus a_2, and the elastic scattering wave function of the pair a_2 and c_1. $\psi_\alpha^{(+)}(\vec{r}_1,\vec{r}_\alpha)$ is an eigenstate of the total Hamiltonian of the system satisfying the initial asymptotic boundary conditions.

We seek a solution $\psi_\alpha^{(+)}(\vec{r}_1,\vec{r}_\alpha)$ of the form

$$\psi_\alpha^{(+)}(\vec{r}_1,\vec{r}_\alpha) \quad = \quad \phi_\alpha(r_1)\,F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) \tag{2.4a}$$

where $\phi_\alpha(r_1)$ is the bound state of the nucleon n in the nucleus a_1 and satisfies the equation

$$\left(-\frac{\hbar^2}{2\mu_1}\ \nabla_1{}^2 + V_{nc_1}(r_1)\right)\phi_\alpha(r_1) = -\ \varepsilon_\alpha\ \phi_\alpha(r_1) \quad , \tag{2.4b}$$

ε_α being its binding energy. $F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta)$ satisfies the equation

$$\left(-\frac{\hbar^2}{2\mu_2}\nabla_2{}^2 - \frac{\hbar^2}{2\mu_\beta}\nabla_\beta{}^2 + V_{nc_2}(r_2) + V\ (r_\beta)-E-\varepsilon_\alpha\right) F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) = \mathcal{L}\, F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) \tag{2.5a}$$

where the operator $\mathcal{L}$ is given by

$$\mathcal{L} \;=\; V(r_\beta) - V_{c_1c_2}(r) + \frac{\hbar^2}{2\mu_1}\ \nabla_1(\ell n\phi_\alpha)\ \ \nabla_1\ \ell n\ (F_\alpha{}^+) \quad . \tag{2.5b}$$

The VPS approximation consists in ignoring the right hand side of eq. (2.5a) and obtaining

$$F_{\alpha 0}^{(+)}(\vec{r}_1,\vec{r}_\beta) \cong A_\alpha\ f_\alpha^{(+)}(\vec{k}_2,\vec{r}_2)\ \ f_\alpha^{(+)}(\vec{k}_\beta,\vec{r}_\beta) \tag{2.6a}$$

where A_α is a normalization constant and the functions $f_\alpha^{(+)}(\vec{k}_2,\vec{r}_2)$ and $f_\alpha^{(+)}(\vec{k}_\beta,\vec{r}_\beta)$ are solutions of the differential equations

$$\left(-\frac{\hbar^2}{2\mu_2}\nabla_2^{\,2} + V_{nc_2}(r_2) - \frac{\hbar^2 k_2^{\,2}}{2\mu_2}\right) f_\alpha^{(+)}(k_2,r_2) = 0 \quad , \tag{2.6b}$$

$$\left(-\frac{\hbar}{2\mu_\beta}\nabla_\beta^{\,2} + V(r_\beta) - \frac{\hbar^2 k_\beta^{\,2}}{2\mu_\beta}\right) f_\alpha^{(+)}(\vec{k}_\beta,\vec{r}_\beta) = 0 \tag{2.6c}$$

with

$$\frac{\hbar^2 k_2^{\,2}}{2\mu_2} + \frac{\hbar^2 k_\beta^{\,2}}{2\mu_\beta} = \frac{\hbar^2 K_\alpha^{\,2}}{2\mu_\alpha} \tag{2.6d}$$

$\hbar\vec{K}_\alpha$ is the asymptotic momentum of the projectile in the incident channel. The above approximation thus takes into account the effect of the interaction of the transferred nucleon and the target nucleus exactly. Comparing the above approximation with the standard DWBA, one can see that the effect of the polarization of the projectile, which is absent in the DWBA is contained in the VPS approximation.

The wave numbers $\vec{k}_\beta$ and $\vec{k}_2$ can be determined by comparing the asymptotic forms of $F_{\alpha 0}^{(+)}$ with that of the exact wave function. One obtains

$$\vec{k}_\beta = \frac{Mc_1}{Ma_1}\vec{k}_\alpha \tag{2.7a}$$

and

$$\vec{k}_2 = -\frac{M M_n}{Ma_1 Ma_2}\vec{k}_\alpha \tag{2.7b}$$

where M is the total mass of the system.

At low energies, where Coulomb distortion is important, one must also consider the long range Coulomb force and its effect on the asymptotic boundary condition. Rewriting the total Hamiltonian

$$H = K+V_{nc_1}(r_1)+V_{nc_2}(r_2)+V_{c_1c_2}(r_2) - \frac{Z_nZ_{c2}e^2}{r_2} - \frac{Z_nZ_{c1}e^2}{r_1} - \frac{Z_{c1}Z_{c2}e^2}{r} \tag{2.8}$$

where V_{ab} refers only to the nuclear interaction between the pairs (a,b) and $Z_a e$ refers to the charge of the nucleus a, the initial wave function has the asymptotic form

$$\psi_\alpha^{(+)}(\vec{r}_1,\vec{r}_\alpha) \xrightarrow[r_\alpha \to \alpha]{} \phi_\alpha(r_1) \exp\left\{i\,\vec{k}_\alpha \cdot \vec{r}_\alpha + i\eta_\alpha \, \ell n\,(k_\alpha r_\alpha - \vec{k}_\alpha \cdot \vec{r}_\alpha)\right\} \tag{2.9a}$$

$$+ \text{ outgoing waves}$$

where η_α is the Sommerfeld parameter defined by

$$\eta_\alpha = \frac{\mu_\alpha Z_{a_1} Z_{c_2} e^2}{\hbar^2 K_\alpha} \quad . \tag{2.9b}$$

Equation (2.9a) is strictly true when $r_1 << r_\alpha$. We rewrite eq. (2.5a) as

$$\left(-\frac{\hbar^2}{2\mu_2}\nabla_2{}^2 - \frac{\hbar^2}{2\mu_\beta}\nabla_\beta{}^2 + V_{nc_2}(r_2) + V(r_\beta) - \frac{Z_nZ_{c_2}e^2}{r_2} - \frac{Z_{eff}Z_{c_1}e^2}{r_\beta} - E - \varepsilon_\alpha\right) \tag{2.10a}$$

$$\times \; F_\alpha^{(+)} = \mathcal{L}\, F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta)$$

where

$$\mathcal{L} = \Delta V(\vec{r}_2,\vec{r}_\beta) + \frac{\hbar^2}{2\mu_1} \nabla_1 (\ln\phi_\alpha) \nabla_1 \ln (F_\alpha^{(+)}) \quad . \tag{2.10b}$$

It can be verified that if one chooses the Sommerfeld parameter $\eta_\beta = \frac{Z_{c_1}}{Z_{a_1}} \eta_\alpha$, the boundary condition (2.9a) is satisfied and one obtains

$$\Delta V(\vec{r}_2,\vec{r}_\beta) = V(r_\beta) - V_{c_1c_2}(r) + \frac{M_{c_2}}{M_{a_2}} \frac{Z_{c_1}Z_{c_2}e^2}{r_\beta} - \frac{Z_{c_1}Z_{c_2}e^2}{r} \tag{2.10c}$$

Thus, ΔV is of the order $\frac{M_n}{M_{c_2}}$ and will be small when the target nucleus is very massive. The function $F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta)$ is a product of scattering wave functions of transferred nucleon and the heavy ion in the final state and is given by

$$F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) = A_\alpha\, f_\alpha^{(+)}(\eta_2,\vec{k}_2,\vec{r}_2)\; f^{(+)}(\eta_\beta,\vec{k}_\beta,\vec{r}_\beta) \tag{2.11a}$$

where $f_\alpha^{(+)}$ satisfy the equations of motion

$$\left(-\frac{\hbar^2}{2\mu_2} \nabla_2^{\,2} + V_{nc_2}(r_2) - \frac{Z_nZ_{c_2}e^2}{r_2} - \frac{\hbar^2k_2^{\,2}}{2\mu_2}\right) f_\alpha^{(+)}(\eta_2,\vec{k}_2,\vec{r}_2) = 0 \tag{2.11b}$$

$$\left(-\frac{\hbar^2}{2\mu_\beta} \nabla_\beta^{\,2} + V(r_\beta) - \frac{M_{c_2}}{M_{a_2}} \frac{Z_{c_1}Z_{c_2}e^2}{r_\beta} - \frac{\hbar^2k_\beta^{\,2}}{2\mu_\beta}\right) f_\alpha^{(+)}(\eta_\beta,\vec{k}_\beta,\vec{r}_\beta) = 0 \quad . \tag{2.11c}$$

Since the interaction between the transferred nucleon and the target nucleus is treated exactly, we feel that a large part of the polarization of the projectile has been accounted for. The validity of the approximation will depend upon the importance of the term $\mathcal{L}F_{\alpha}^{(+)}$. The first term, being of the order $\frac{M_n}{M_{c_2}}$, is not likely to be important for very heavy target nuclei. Hence, the importance of the term $\mathcal{L}F_{\alpha}^{(+)}$ hinges upon the term coupling the bound state to the scattering wave function $F_{\alpha}^{(+)}$, i.e.;

$$\mathcal{L}F_{\alpha}^{(+)}(\vec{r}_2,\vec{r}_\beta) \simeq \frac{\hbar^2}{2\mu_1} \nabla_1 \ \ell n \ (\phi_\alpha(r_1)) \ \nabla_1 \ F_{\alpha}^{(+)}(\vec{r}_2,\vec{r}_\beta) \quad . \tag{2.12}$$

In the next section, we shall consider two cases where the term, eq. (2.12), is likely to be unimportant.

3. Nucleon Transfer Reactions

3.1. HIGH ENERGY DEUTERON STRIPPING

If we consider deuteron stripping reactions at high energies, where the Coulomb distortion plays a negligible role, then we have

$$\nabla_1 \left(F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta)\right) \cong 1/2\ (\nabla_2 - \nabla_\beta)\ F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta)$$

$$\simeq 0 \quad . \tag{3.1}$$

The vanishing of the gradient of $F_\alpha^{(+)}$ is due to the fact that in the absence of the Coulomb distortion, the neutron and proton each carry one-half of the deuteron energy, and if the target nucleus is very massive, their scattering wave functions would be identical.

One can then obtain the transition amplitude as

$$T_{\beta\alpha}(\vec{K}_\beta,\vec{K}_\alpha) = \int d\vec{r}_\beta\ \chi_\beta^{(-)*}(\vec{K}_\beta,\vec{r}_\beta)\ F(\vec{r}_\beta)\ f_\alpha^{(+)}(\vec{k}_\beta,\vec{r}_\beta) \tag{3.2a}$$

where $\chi_\beta^{(-)*}(\vec{K}_\beta,\vec{r}_\beta)$ describes the elastic scattering of proton and the residual nucleus in the final state, and the form factor $F(\vec{r}_\beta)$ is given by

$$F(\vec{r}_\beta) = \int d\vec{r}_2\ \phi_\beta(\vec{r}_2)\ f_\alpha^{(+)}(\vec{k}_2,\vec{r}_2)\ V_{nc_1}(r_1)\ \phi_\alpha(\vec{r}_1) \tag{3.2b}$$

where $\phi_\beta(\vec{r}_2)$ is the bound state wave function of the neutron in the residual nucleus. The form factor can be computed using the technique of Sawaguri and Tobocman[17].

Comparing with the usual treatment of finite range DWBA[11]), one can see that eqs. (3.2) can be computed more easily because the two three dimensional integrals can be evaluated independently causing a considerable reduction in computing time. Equations (3.2) constitute a different approximation to the stripping amplitude than the DWBA. It should be interesting to compare the two approximations. The VPS approximation can be understood to be different from the model of Pearson and Coz[3]), being primarily a high energy approximation.

3.2. NUCLEON TRANSFER IN HEAVY ION REACTIONS

The semiclassical approach of Brink[9]) as well as the approximate treatment of the authors[10]) use an approximate factorizability of the distorted wave function for the elastic scattering of the pair of ions in the initial and final states. The argument is based on a belief that the nucleon transfer occurs in a localized region in configuration space about the distance of closest approach. The potentials in this region being slowly varying, one could use a WKB approximation for the relative motion of the ions, thus allowing for a separability of the motion of the core and the transferred nucleon. The VPS approximation provides a product representation for the distorted wave function of relative motion. It implies that the two constituents of the projectile scatter independently, their correlation being determined by their bound state wave function. The approximation is thus an adiabatic approximation which should strictly hold in the case of a weakly bound projectile.

If the VPS approximation were valid for heavy ion reactions, one obtains an alternate form for the transition amplitude, eqs. (3.2), which provide for a simpler evaluation of recoil effects.

There exists a special case where the VPS approximation is likely to be particularly valid in heavy ion induced transfer reactions. This is the case of a S-wave projectile. The gradient of the bound state wave function in this case will be oriented along the relative vector $\vec{r}_1$, i.e.;

$$\nabla_1 \ln \phi_\alpha(\vec{r}_1) = \vec{r}_1 f(r_1) \tag{3.3a}$$

whereas the gradient of the scattering wave function will be approximately along the direction of the classical Coulomb trajectory, i.e.;

$$\begin{aligned} \nabla_1 F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) &= i\left[\frac{M_n M}{M_{a_1} M_{a_2}} \nabla_\beta + \frac{M_{c_1} M_{c_2}}{M_{a_1} M_{a_2}} \nabla_2\right] F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) \\ &\cong i\left(\frac{M_n M}{M_{a_1} M_{a_2}} \vec{\kappa}_\beta + \frac{M_{c_1} M_{c_2}}{M_{a_1} M_{a_2}} \vec{\kappa}_2\right) F_\alpha^{(+)}(\vec{r}_2,\vec{r}_\beta) \end{aligned} \tag{3.3b}$$

where $\vec{\kappa}_\beta$ and $\vec{\kappa}_2$ are the local wave numbers for the scattering wave functions $f_\alpha^{(+)}(\vec{k}_\beta,\vec{r}_\beta)$ and $f_\alpha^{(+)}(\vec{k}_2,\vec{r}_2)$, respectively. If one makes the usual approximation of the localization of the transfer region, the vector $\vec{r}_1$ will be approximately normal to the Coulomb trajectories, and hence the coupling term on the right hand side of eq. (2.10a) should be negligible. Thus, the VPS approximation should be valid in this case.

4. Summary and Acknowledgments

We have considered the approximation of Vainshtein, Presnyakov and Sobelman[13]) and its application to nucleon transfer. We have shown that it should provide a good approximation in heavy ion induced reactions above the Coulomb barrier as well as in high energy deuteron stripping reactions. In the application of the VPS approximation to charge transfer in atomic collisions, Salin[14]) has found that it is valid over a wide energy region. One has to make numerical computation based on this approximation before one can test the range of its validity. In the case of heavy ion induced reactions, it provides a considerable simplication in numerical computation, and tends to the correct high energy limit. Numerical results based on the approximation shall be presented in a future publication.

The author is grateful to Dr. M. Kleber, who familiarized him with the theory of charge transfer in atomic collisions, and to Dr. N. K. Glendenning for many critical comments.

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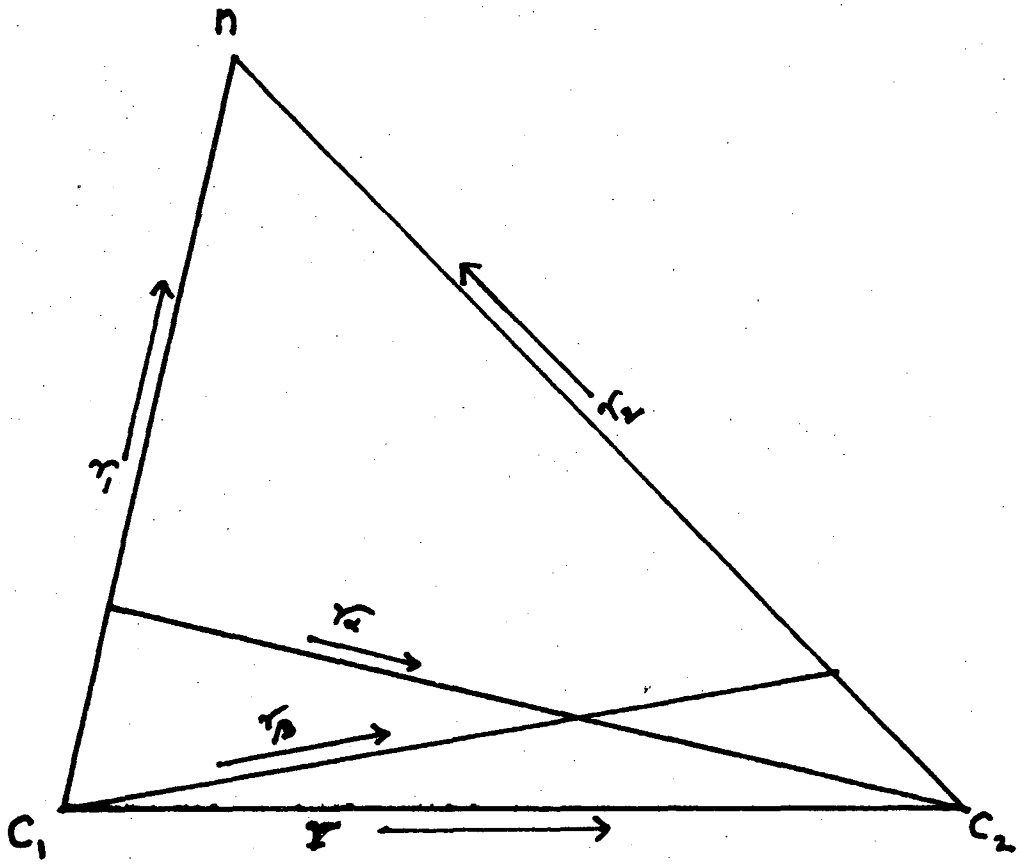
Figure Captions

Fig. 1. The system of co-ordinate vectors.



XBL 7310-1279

Fig. 1