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MORE STUFF ON ORTHOTROPIC CYLINDERS

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MORE STUFF ON ORTHOTROPIC CYLINDERS				

Stress-strain equations.

$$\left. \begin{aligned} \epsilon_r &= \frac{\sigma_r}{E_r} - \nu_{rt} \frac{\sigma_t}{E_t} \\ \epsilon_t &= \frac{\sigma_t}{E_t} - \nu_{tr} \frac{\sigma_r}{E_r} \end{aligned} \right\} (1)$$

Case I: Start with stress-free body. Apply stress σ_r first, then apply stress σ_t .

Apply stress σ_r . The strains are

$$\epsilon_r = \frac{\sigma_r}{E_r}, \quad \epsilon_t = -\nu_{tr} \frac{\sigma_r}{E_r}.$$

Work done on body and stored energy in body, per unit volume, are

$$W_1 = \frac{1}{2} \sigma_r \epsilon_r = \frac{1}{2} \frac{\sigma_r^2}{E_r}$$

Apply stress σ_t . The changes in the strains are

$$\Delta \epsilon_r = -\nu_{rt} \frac{\sigma_t}{E_t}, \quad \Delta \epsilon_t = \frac{\sigma_t}{E_t}.$$

The work done on the body by σ_t is

$$W_2 = \frac{1}{2} \sigma_t \Delta \epsilon_t = \frac{1}{2} \frac{\sigma_t^2}{E_t}$$

The work done on the body by σ_r is

$$W_3 = \sigma_r \Delta \epsilon_r = -\nu_{rt} \frac{\sigma_r \sigma_t}{E_t}$$

and so the total work done on the body, and its internal energy, are

$$U_I = W_1 + W_2 + W_3 = \frac{1}{2} \frac{\sigma_r^2}{E_r} + \frac{1}{2} \frac{\sigma_t^2}{E_t} - \nu_{rt} \frac{\sigma_r \sigma_t}{E_t} \quad (2)$$

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Case II: Start with stress-free body. Apply stress σ_t first, then apply stress σ_r .

The internal-energy formula is the same as that for Case I with all subscripts switched:

$$U_{II} = \frac{1}{2} \frac{\sigma_t^2}{E_t} + \frac{1}{2} \frac{\sigma_r^2}{E_r} - \nu_{tr} \frac{\sigma_r \sigma_t}{E_t} \quad (3)$$

For the internal energy to be independent of the stress-strain path

$$U_I = U_{II} \quad \text{which gives}$$

$$\boxed{\frac{\nu_{rt}}{E_t} = \frac{\nu_{tr}}{E_r}} \quad (4)$$

The change in cross-sectional area of a unit square is

$$\Delta V = \epsilon_r + \epsilon_t \quad \text{which, from Eqs 1 and 4, gives}$$

$$\Delta V = \frac{1}{E_t} \left[\left(\frac{E_t}{E_r} - \nu_{rt} \right) \sigma_r + (1 - \nu_{tr}) \sigma_t \right]$$

For ΔV to have the same sign as σ_r or σ_t *

$$\boxed{\begin{aligned} \nu_{rt} &\leq 1 \quad \text{and} \quad \nu_{rt} \leq \frac{E_t}{E_r} \\ \nu_{tr} &\leq 1 \quad \text{and} \quad \nu_{tr} \leq \frac{E_r}{E_t} \end{aligned}}$$

* I don't know for sure why this should be necessary - probably has something to do with entropy - but it is equivalent to the condition for a 3-dimensional isotropic material requiring that $\nu \leq .5$

See also Eng. Notes M 5104, M 5153, M 5537

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