Title
MORE STUFF ON ORTHOTROPIC CYLINDERS

Author
Meuser, R.

Publication Date
1981-12-01

UC-38
LBID-464

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Engineering & Technical Services Division

DISCLAIMER

LBID 464

LAWRENCE RADIATION LABORATORY - UNIVERSITY OF CALIFORNIA

ENGINEERING NOTE

CODE	SERIAL	PAGE
MD1111	M5832	1 of 2

AUTHOR	DEPARTMENT	LOCATION	DATE
R. MEUSER	MECH. ENG	BERK.	DEC 11, 1981

PROGRAM — PROJECT — JOB

HIGH-FIELD MAGNET DEVELOPMENT

ANALYSIS

TITLE

MORE STUFF ON ORTHOTROPIC CYLINDERS

Stress-strain equations.

$$\left.\begin{aligned} \epsilon_r &= \frac{\sigma_r}{E_r} - \nu_{rt}\frac{\sigma_t}{E_t} \\ \epsilon_t &= \frac{\sigma_t}{E_t} - \nu_{tr}\frac{\sigma_r}{E_r} \end{aligned}\right\} (1)$$

Case I: Start with stress-free body. Apply stress σ_r first, then apply stress σ_t.

Apply stress σ_r. The strains are

$$\epsilon_r = \frac{\sigma_r}{E_r}, \quad \epsilon_t = -\nu_{tr}\frac{\sigma_r}{E_r}.$$

Work done on body and stored energy in body, per unit volume, are

$$W_1 = \frac{1}{2}\sigma_r\epsilon_r = \frac{1}{2}\frac{\sigma_r^2}{E_r}$$

Apply stress σ_t. The changes in the strains are

$$\Delta\epsilon_r = -\nu_{rt}\frac{\sigma_t}{E_t}, \quad \Delta\epsilon_t = \frac{\sigma_t}{E_t}.$$

The work done on the body by σ_t is

$$W_2 = \frac{1}{2}\sigma_t\,\Delta\epsilon_t = \frac{1}{2}\frac{\sigma_t^2}{E_t}$$

The work done on the body by σ_r is

$$W_3 = \sigma_r\,\Delta\epsilon_r = -\nu_{rt}\frac{\sigma_r\sigma_t}{E_t}$$

and so the total work done on the body, and its internal energy, are

$$U_I = W_1 + W_2 + W_3 = \frac{1}{2}\frac{\sigma_r^2}{E_r} + \frac{1}{2}\frac{\sigma_t^2}{E_t} - \nu_{rt}\frac{\sigma_r\sigma_t}{E_t} \quad (2)$$

AUTHOR R. MEUSER | DEPARTMENT MECH. ENG. | LOCATION BERK. | DATE DEC 11 1981

Case II: Start with stress-free body. Apply stress σ_t first, then apply stress σ_r.

The internal-energy formula is the same as that fore Case I with all subscripts switched:

$$U_{II} = \frac{1}{2}\frac{\sigma_t^2}{E_t} + \frac{1}{2}\frac{\sigma_r^2}{E_r} - \nu_{tr}\frac{\sigma_r \sigma_t}{E_t} \qquad (3)$$

For the internal energy to be independent of the stress-strain path

$U_I = U_{II}$ which gives

$$\boxed{\frac{\nu_{rt}}{E_t} = \frac{\nu_{tr}}{E_r}} \qquad (4)$$

The change in cross-sectional area of a unit square is

$\Delta V = \epsilon_r + \epsilon_t$ which, from Eqs 1 and 4, gives

$$\Delta V = \frac{1}{E_t}\left[\left(\frac{E_t}{E_r} - \nu_{rt}\right)\sigma_r + (1-\nu_{rt})\sigma_t\right]$$

For ΔV to have the same sign as σ_r or σ_t*

$$\boxed{\begin{array}{ll} \nu_{rt} \le 1 \text{ and } & \nu_{rt} \le \frac{E_t}{E_r} \\ \nu_{tr} \le 1 \text{ and } & \nu_{tr} \le \frac{E_r}{E_t} \end{array}}$$

* I don't know for sure why this should be necessary — probably has something to do with entropy — but it is equivalent to the condition for a 3-dimensional isotropic material requiring that $\nu \le .5$

See also Eng. Notes M 5104, M 5153, M 5537

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720