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I. INTRODUCTION

The use of pair spectrometers to measure γ-ray energy spectra with high resolution is well known. We have built two such spectrometers to measure the characteristics of neutral-pion production in proton-proton collisions at 735 MeV.[1] Because of the many refinements employed in our spectrometers we thought it worth while to publish a detailed description of these instruments.

In addition the present article includes a discussion of the methods of data collection, storage, and analysis. Also expressions are derived for the absolute efficiency of a pair spectrometer as a function of the geometrical arrangement, magnetic field, and pair-production cross section. Corrections are shown for electron scattering and for radiation straggling.

II. DESCRIPTION OF SPECTROMETERS

A. 180-deg Spectrometer

We built a conventional 180-deg spectrometer (shown in Fig. 1) to measure γ-ray energies from 20 to 240 MeV. Six scintillators on each side—1/2-in. thick, 1-in. wide, and 4-in. high—detect electrons with radii of curvature between 3 and 8 in. The scintillators and converter are placed in a magnet with 24-by 36-in. pole pieces, capable of a maximum uniform field of 20 000 G. Lucite light pipes attached to the scintillators extend upward through 2-in.-diam holes drilled in the upper pole piece and yoke. These holes perturb the magnetic field only in

the vicinity of the scintillators, and thus have a negligible effect on the orbits of the impinging electrons. The light pipes are viewed by 6810A photomultipliers which are surrounded by iron magnetic-shields, and are placed atop the magnet.

With this arrangement only the scintillators themselves, plus the converter, lie in the median horizontal plane. This permits maximum flexibility in shielding the counters from stray radiation. Blocks of 1-in.-thick aluminum, backed up by several inches of lead, are placed in back of the counters to absorb electrons after they pass through the scintillators. This prevents them from circling back and entering an adjacent counter, thus causing a multiple coincidence.

B. Circular Spectrometer

For higher energies, up to 650 MeV, we at first used a spectrometer with scintillators arrayed on two straight lines diverging from the converter center, the two lines making an angle of 74 deg. With this instrument we found that as much as 10% of the time, two or more counters on the same side--usually adjacent--produced a pulse simultaneously, as though a single electron had traversed two or more scintillators. A study of this problem indicated that the best way to suppress this effect was to design a spectrometer in which adjacent scintillators lie along a curve everywhere perpendicular to the electron direction. This makes it much more difficult for an electron to scatter or circle back into an adjacent counter.

We were thus led to the circular geometry shown in Fig. 2. The converter and all scintillators lie on the circumference of a circle of 16-in. radius. Eight scintillators on a side are each 1/2-in. thick and 4-in. high, but have variable widths. These widths were chosen in order to keep the percent energy resolution a constant for each detector (see Sec. III for analysis of this problem). By varying the widths from 0.94 in. to 1.50 in., a uniform resolution of 9.4%

(full width) is attained. In contrast, the electron energy resolutions of the detectors in the 180-deg spectrometer vary from 6 to 17%. As with the 180-deg spectrometer, light pipes extend vertically through holes in the magnet.

The operating characteristics of the circular spectrometer have been very satisfactory. No multiple coincidences are observed. The overall counting efficiency is significantly larger than either the 180-deg spectrometer or the original 74-deg spectrometer. This is due both to the larger number of detectors on each side and to the large widths of the detectors. Under typical running conditions in our experiment on neutral pion production, as many as 4000 γ rays per minute were analyzed.

The geometrical characteristics of these two spectrometers are summarized in Table I.

C. Converters

The converter must be kept quite thin in order to minimize multiple scattering of the electrons vertically out of the plane of the detectors. Since the average scattering angle is inversely proportional to the electron energy, one must use thinner converters when low-energy γ rays are being analyzed (see Sec. III for more detailed analysis of electron scattering). We used a 0.010-in.-thick tantalum converter with the circular spectrometer, corresponding to 6% of a radiation length. In the 180-deg spectrometer, 0.004- and 0.002-in.-thick tantalum converters were used. In no case are losses in overall efficiency due to vertical scattering greater than 50%, and over most of the energy region scattering losses are less than 20%. The effects of horizontal scattering, which broaden the energy resolution function, are negligible over most of the energy region.

D. Electronics

A schematic diagram of the electronics is shown in Fig. 3. In addition to the positron and electron detectors, a counter using a 0.020-in.-thick plastic scintillator is placed in the γ-ray beam. It functions in anticoincidence with the spectrometer counters in order to eliminate events due to charged particles in the γ-ray beam. The signals from a given side of the spectrometer are first added together. The summed signals are put into a Wenzel-type coincidence circuit[2] to determine the twofold coincidences, along with the signal from the anti-coincidence counter. The resolving time of this circuit is approximately 10 nsec (full width at half-maximum). The fast output triggers a gate unit, which informs the binary coder that an event has taken place. The binary coder then records the signals from the particular counters that have produced a signal over the past 20 nsec. Validity circuits test to ensure that one and one counter only on each side has fired. A record is kept of the number and type of invalid events. If the event is valid, pulses in binary code are transmitted to the core storage matrix, which adds one to the number stored in the channel corresponding to the pair of counters that fired. The core storage unit is a transistorized 256-channel pulse-height analyzer manufactured by Nuclear Data Corporation (Model ND-101); it was modified for our use. On command, the core storage unit reads out the number of counts in each channel by punching a series of eight IBM cards, each prefaced by a code indicating the run number and running conditions. These cards are later analyzed by an IBM 709 computer, which uses a program described in Sec. III.

E. Typical Experimental Procedures

The magnetic field is varied between 1.92 and 19.7 kG in 16 steps. Successive values differ from one another by a constant factor of 1.168. By running at so many different fields, small variations in efficiencies of individual detectors or detector pairs can be averaged out. Each experimental point in the final spectrum receives a contribution from many different detector pairs at many different fields. The data analysis becomes more complex, but this causes no difficulty if the data processing is done by computers.

Time is allocated between converter-in and converter-out approximately as the square root of the ratio of counting rates, in order to minimize the statistical error for a fixed amount of running time. The observed converter in-out ratios vary considerably, and depend on such factors as magnet current, collimation of the γ-ray beam, and converter used. Under some conditions it is as high as 15:1. More typically, the ratio is between 5:1 and 10:1, and extends down to 1.5:1 and less at the lowest magnetic fields.

III. PAIR-SPECTROMETER THEORY

A. Efficiency of a Spectrometer

The efficiency, or counting rate, of a spectrometer is a function of a number of variables: the position, widths, and number of individual detectors; the weight of the converter and the material out of which it is made; the magnetic field; and the pair-production cross section as a function of energy.

To simplify the analysis, we consider only one pair of detectors. The expression for the efficiency, which we will derive, can then be applied to each of the possible combinations of detector pairs (of which there are 64 for our circular spectrometer at each magnetic field setting). The counting rate in this channel

can thus be properly normalized and combined with results from several other channels to give one experimental point. Since the expression we will derive is an absolute one, with no arbitrary proportionality factors, the counting rate can be directly related to experimental cross sections, with no need to calibrate the spectrometer.

Let us assume we have a uniform parallel beam of γ rays whose differential energy flux, i.e., the number of gammas per cm^2 per sec whose energy lies between k and k+dk, is given by F(k) dk. Into the beam we place a converter of area S cm^2 and thickness t_0 g/cm^2, (therefore weight $W = S \times t_0$ grams), made of material of atomic number Z and atomic weight A. Let $\Phi(k, E^+)\,dE^+$ represent the differential cross section in this material for the creation of a positron of total energy between E^+ and $E^+ + dE^+$ by a γ ray of energy k.

Then the double differential

$$P(k, E^+)\,dE^+\,dk = F(k)\,\Phi(k, E^+)\,\frac{W\,N_0}{A}\,dE^+\,dk$$

represents the rate of creation of positrons of energy between E^+ and $E^+ + dE^+$ from γ rays of energy between k and k+dk. Here N_0 is Avogadro's number.

We now place a positron detector in a magnetic field H such that it will detect all positrons whose radii of curvature are within the range $\Delta r^+ = r^+_{max} - r^+_{min}$, and none outside this range. The average radius we denote by r^+. In order to have this idealized response, of course, the converter would have to be infinitely narrow. A finite-width converter will introduce some spectral broadening, but will not affect the efficiency. Similarly, we insert an electron detector whose average radius and range is r^- and Δr^-, respectively.

We denote corresponding energies as follows:

$$E^+_{max} = C\,H\,r^+_{max},$$

$$E^+ = C\,H\,r^+,$$

$$\Delta E^+ = C\,H\,\Delta r^+ = C\,H\,(r^+_{max} - r^+_{min}),$$

$$k = E^+ + E^- = C\,H\,(r^+ + r^-),$$

and similarly for the electron. Here C is a proportionality factor that converts gauss-cm into energy units, where we assume that the electrons are highly relativistic. If r is in cm, H is in gauss, and E is in MeV, then $C = 3\times10^{-4}$. Thus the positron detector will detect all positrons with energies between E^+_{max} and E^+_{min}, and so will the electron detector.

The coincidence rate is then given by

$$R = \int\int S(k, E^+)\, P(k, E^+)\, dE^+\, dk.$$

That is, we must perform a double integration: one over the range of γ-ray energy k, and the second over the range of positron energies E^+ for a given k. The step function $S(k, E^+)$ is either 1 or 0, depending on whether or not <u>both</u> the electron and positron are within the energy range to be detected by the appropriate detector for a given k and E^+. In performing the integration, the effect of the step function is to impose upper and lower energy limits on the two integrals. The integral may be carried out if we assume that the spectrum F(k) and the pair-production cross section $\Phi(k, E^+)$ are not rapidly varying over the region of integration, which will be the case if the detectors are not too wide. We simply substitute average values and remove these factors from the integrand. The reader is referred to the unpublished report in reference 1 for details of the integration. The result is

$$R = \frac{W\,N_0}{A}\,F(k)\,\left[k\,\Phi(k, E^+)\right]\,\frac{C\,H\,\Delta r^+\,\Delta r^-}{r^+ + r^-}.$$

The factor in the first bracket is the differential pair-production cross section times k, which is a constant to within a factor of two over the range of γ-ray energies and pair splittings covered by our spectrometers. See Bethe and Heitler,[3] Bethe and Ashkin,[4] and Davies et al.[5] for calculated expressions for

this cross section, including corrections to account for deviations from the Born approximation.

The factor in the second bracket is the geometric efficiency of a pair of detectors. Note that Δr^+ and Δr^- are not the physical widths of the detectors, but refer to the range of electron radii that can be detected. This range is a function both of the detector widths and their position in the spectrometer. Note also that R is proportional to H.

This analysis does not consider either losses in counting rate due to vertical scattering or changes in the observed spectrum due to radiation straggling. Both these effects are considered below.

B. Scattering Correction

The spectrometer efficiency, as calculated above, does not take into consideration the fact that some of the electron-positron pairs will be missed, owing to the possibility that one or both particles will scatter vertically out of the plane of the scintillators. To correct for this effect, we must multiply the final expression above by a scattering correction factor $T(k, E^+)$, which represents the probability that both the electron and positron will be detected.

To calculate this probability, we assume that a γ ray of energy k enters a converter of thickness t_0 and height h at a distance x above the center of the converter (see Fig. 4). At a point which is t g/cm^2 from the other side of the converter, an electron-positron pair with energies E^- and E^+ is formed. The positron, as it continues through the remainder of the converter, will be scattered through a projected angle ϕ. It then travels along an arc of length D^+ before arriving at the detector of height a. The probability of the positron being detected is then

$$P^+ = \frac{1}{2} \left[P(\phi_1^+, t, E^+) + P(\phi_2^+, t, E^+) \right] ,$$

where

$$\phi_1^+ = \tan^{-1}\left(\frac{\frac{a}{2} - x}{D^+}\right),$$

$$\phi_2^+ = \tan^{-1}\left(\frac{\frac{a}{2} + x}{D^+}\right),$$

and $P(\phi, t, E)$ is the probability that the projected scattering angle will be less than ϕ when an electron or positron of total energy E passes through a thickness t of converter material. Thus

$$P(\phi, t, E) = \int_0^{\phi} f(\alpha, t, E)d\alpha,$$

where $f(\alpha, t, E)d\alpha$ is the probability that the projected scattering angle is between α and $\alpha + d\alpha$. In our calculations, we used the Moliere scattering theory, as summarized by Bethe and Ashkin,[4] to determine scattering-angle distributions.

A similar expression may be written for P^-, the probability of the electron being detected, with appropriate changes in superscripts. The total probability for an electron-positron coincidence, averaged over both the thickness and height of converter, is then

$$\Gamma(k, E^+) = \frac{2}{h\, t_0} \int_{x=0}^{h/2} dx \int_{t=0}^{t_0} dt\, (P^+)(P^-).$$

This factor must be evaluated for each pair of detectors at each magnetic field setting. With the converters that we use, scattering losses are never greater than 50% even at the lowest field settings, and in most cases are less than 20%.

C. Resolving Power

The resolving power of a spectrometer depends upon several factors. The two most important are the geometric resolving power due to finite detector widths and the radiation straggling of electrons in the converter. Other factors

are horizontal scattering of electrons, effects due to the finite width of the converter, and pair production at finite angles with respect to the incoming beam of γ rays. Each of these factors will be discussed briefly.

1. Geometric Resolving Power

This depends only upon the widths of the detectors and their position in the spectrometer. The total width of the resolution function is related to the maximum and minimum detectable electron energies by

$$\Delta k = k_{max} - k_{min}$$

$$= (E^{+}_{max} + E^{-}_{max}) - (E^{+}_{min} + E^{-}_{min})$$

$$= C H (\Delta r^{+} + \Delta r^{-}) .$$

The fractional resolution is then

$$\frac{\Delta k}{k} = \frac{\Delta r^{+} + \Delta r^{-}}{r^{+} + r^{-}} .$$

This ranges from 6 to 17% (full width) for the 180-deg spectrometer, and is a constant at 9.4% for the circular spectrometer.

The shape of this resolution function is either triangular or trapezoidal, depending upon the geometry. The width at the top of the trapezoid is given by

$$\Delta k_c = C H | \Delta r^{+} - \Delta r^{-} | .$$

Since Δr is a constant for the 180-deg spectrometer (see Table I), $\Delta k_c = 0$, and its resolution function is triangular for all detector pairs. The resolution function for the circular spectrometer is trapezoidal, except for symmetric pairs.

2. Radiation Straggling

After an electron-positron pair has been produced by a γ ray in the converter, the electrons and positrons can undergo radiative collisions, or

bremsstrahlung, during their passage through the remainder of the converter. These collisions cause them to lose a certain fraction of their initial energy. For electron energies above 10 MeV, this energy loss in Ta (Z = 73) is large compared to the loss from inelastic atomic collisions, and therefore the latter can be ignored. At these energies, the average radiative loss is proportional to the thickness of converter traversed by the electron and to its initial energy E. Specifically,

$$\overline{\Delta E} = \overline{\frac{dE}{dx}}\,\Delta x \approx \frac{E}{X_0}\,\Delta x$$

or

$$\frac{\overline{\Delta E}}{E} \approx \frac{\Delta x}{X_0} \quad ,$$

where X_0 is one radiation length of the converter material. Thus if an electron on the average passes through one-half of a 10-mil Ta converter (total thickness = 0.41 g/cm^2, X_0 = 6.4 g/cm^2), it will lose on the average about 3% of its energy. As a first approximation, therefore, we may correct for radiative losses by shifting the energy scale of the observed γ-ray spectra by 3%, when using a 10-mil Ta converter.

However, there is a large amount of straggling in the energy loss. In particular, the most probable energy loss is zero, yet some of the electrons will lose almost all their energy. The exact calculation of the electron distribution cannot be done analytically. We have performed machine calculations where the correct bremsstrahlung formulae are used, and the proper integrations through the converter are made. The resulting radiation-straggling resolution functions are then folded into the geometric resolution functions, and these folded functions are used to correct the observed γ-ray spectra.

3. Other factors

Several other factors can broaden the resolution function, but in our case these were all small in comparison with the two effects already discussed. Electrons scattered horizontally by the converter will contribute some broadening. To first order, this effect is zero for a 180-deg spectrometer, due to the refocusing of rays with the same radius of curvature at the 180-deg point. With the circular spectrometer, the broadening effect is largest at low energies. The effect becomes negligible at the higher energies at which this spectrometer was normally operated.

There will be some broadening with the circular spectrometer owing to the finite width of the converter. If a gamma ray enters the converter at a distance δ from the center line of the converter, the fractional error in the observed γ-ray energy is given to first order by

$$\frac{\Delta k}{k} = \frac{\delta}{2R^2} \left| r^+ - r^- \right| ,$$

where r^+ and r^- are the positron and electron radii of curvature, and R is the radius of circle on which the detectors lie (R = 16 in.). Since our converter was 2-in. wide, the maximum error is 3.4%, and is usually less than 1%. This effect is zero for the 180-deg spectrometer, since the apparent change in energy of the electron is compensated exactly by a corresponding change in the apparent energy of the positron.

In the conversion process, the electrons are produced at a finite angle with respect to the incoming γ ray. This angle is of order $\theta_0 = mc^2/E$, where E is the total electron or positron energy. This angle is always small compared with the average scattering angle except for very thin converters, and therefore effects due to it can be ignored.

IV. DATA ANALYSIS

In contrast with most other workers using pair spectrometers we take data at many different magnetic field values. This greatly adds to the complexity of the data analysis. For this reason it is imperative to use a high-speed computer to analyze the data. A program was written for the IBM 709 that (a) divides the γ-ray spectrum into energy increments, (b) places the events from each energy channel of the pair spectrometer (576 for the circular spectrometer) into its appropriate energy increment, (c) applies the above described corrections which have been previously calculated by other programs, (d) performs the target in-out and converter in-out subtractions, and (e) calculates the errors.

Figure 5 shows a spectrum taken at a laboratory angle of 60 deg with respect to a 735-MeV proton beam bombarding a hydrogen target, after all corrections are made.[1] The spectra from the circular spectrometer and 180-deg spectrometer are plotted separately. A few percent adjustment of the normalization of one spectrometer to the other has been made. Except for this normalization, the absolute efficiencies as calculated from the theory described above have been used to obtain the differential cross sections. The close similarity of the spectral shapes obtained with two spectrometers of such widely differing geometries, as well as the closeness of the overall normalization, gives us considerable confidence in both the method of data analysis and the correctness of the underlying theory.

Figure 6 shows the spectra obtained at three different laboratory angles, after the data from the two spectrometers have been combined. The error on most points is less than 2%, except at the lowest energies, where the counting rate is greatly reduced. The interpretation of these spectra is the substance of reference 1.

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FOOTNOTES AND REFERENCES

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† Present address: Goddard Space Flight Center, NASA, Greenbelt, Maryland.

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2. W. Wenzel, Lawrence Radiation Laboratory Report UCRL-8000, October 1957 (unpublished).
3. H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London), Ser. A, 146, 83 (1934).
4. H. A. Bethe and J. Ashkin, Passage of Radiations through Matter, in Experimental Nuclear Physics, ed. by E. Segrè (John Wiley & Sons, Inc., New York, 1953), Vol. 1.
5. H. Davies, H. A. Bethe, and L. C. Maximon, Phys. Rev. 93, 788 (1954).

Table I. Spectrometer parameters.*

Spectrometer	Counter No.	r(in.)	w(in.)	Δr(in.)	$\frac{\Delta r}{r}$	Max energy (MeV)
180-deg	1	3.0	1.00	0.50	0.167	45
	2	4.0	1.00	0.50	0.125	60
	3	5.0	1.00	0.50	0.100	75
	4	6.0	1.00	0.50	0.083	90
	5	7.0	1.00	0.50	0.071	105
	6	8.0	1.00	0.50	0.063	120
Circular	1	5.6	0.94	0.53	0.094	84
	2	7.3	1.13	0.69	0.094	109
	3	9.1	1.28	0.96	0.094	136
	4	11.1	1.41	1.04	0.094	166
	5	13.3	1.47	1.26	0.094	200
	6	15.9	1.50	1.51	0.094	239
	7	19.1	1.47	1.80	0.094	286
	8	22.9	1.41	2.15	0.094	343

*Here r is the radius of curvature for an electron entering the midpoint of a detector, w is the physical width of a detector, Δr is the range of electron radii which can be detected, Δr/r is the energy resolution (full width), and the last column is the electron energy at maximum field, 19 700 gauss. The γ-ray energy is the sum of the electron and positron energies for a given pair of counters.

FIGURE CAPTIONS

Fig. 1. The 180-deg spectrometer.

Fig. 2. Plan view of the circular spectrometer.

Fig. 3. Block diagram of the electronics.

Fig. 4. Scattering correction diagram.

Fig. 5. Spectra obtained with the circular spectrometer and the 180-deg spectrometer, plotted separately.

Fig. 6. Gamma-ray spectra as measured in the laboratory. Data from the two spectrometers have been combined.

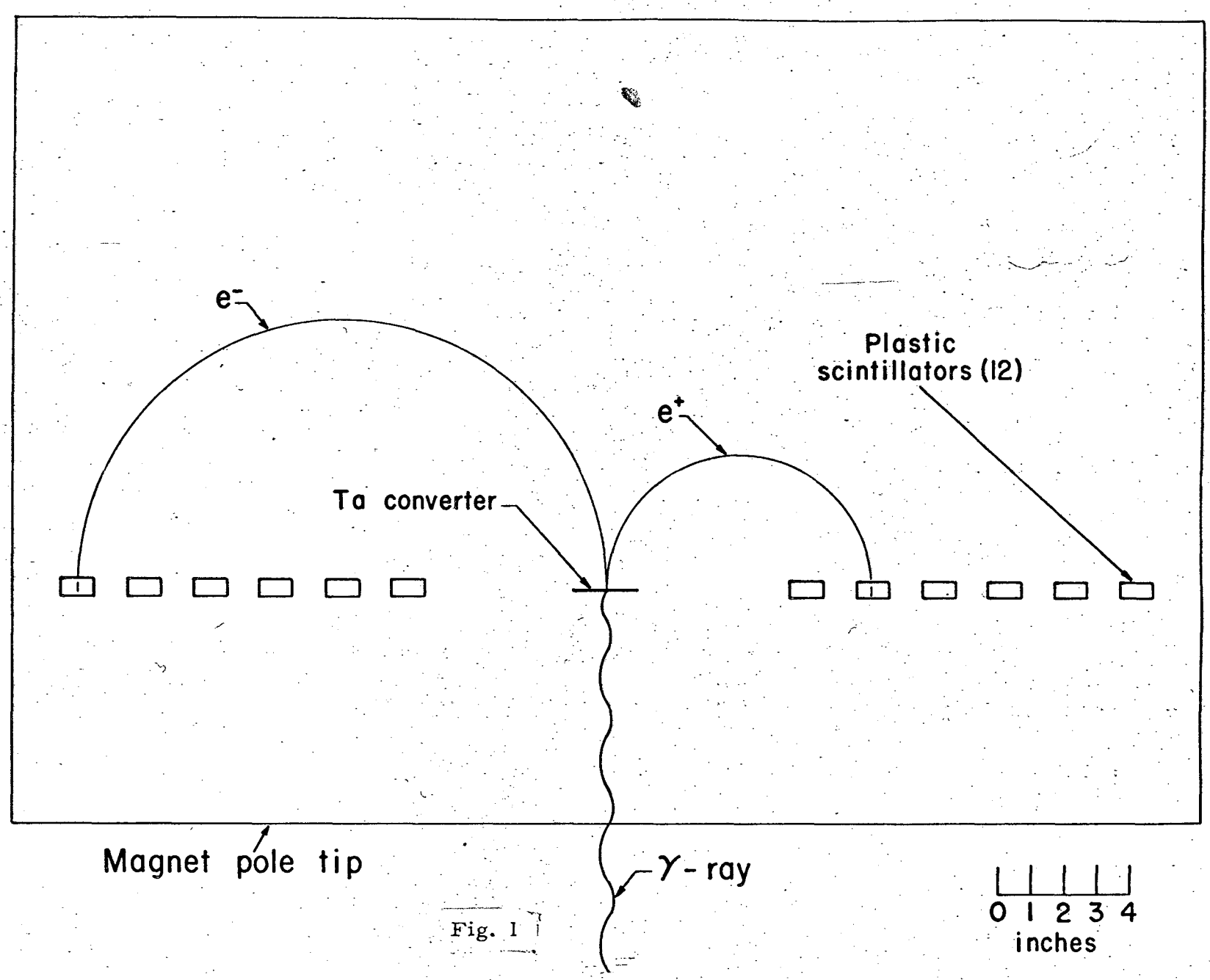
e⁻
Plastic
scintillators (12)
e⁺
Ta converter
Magnet pole tip
γ-ray
0 1 2 3 4
inches
MUB-883

Fig. 1

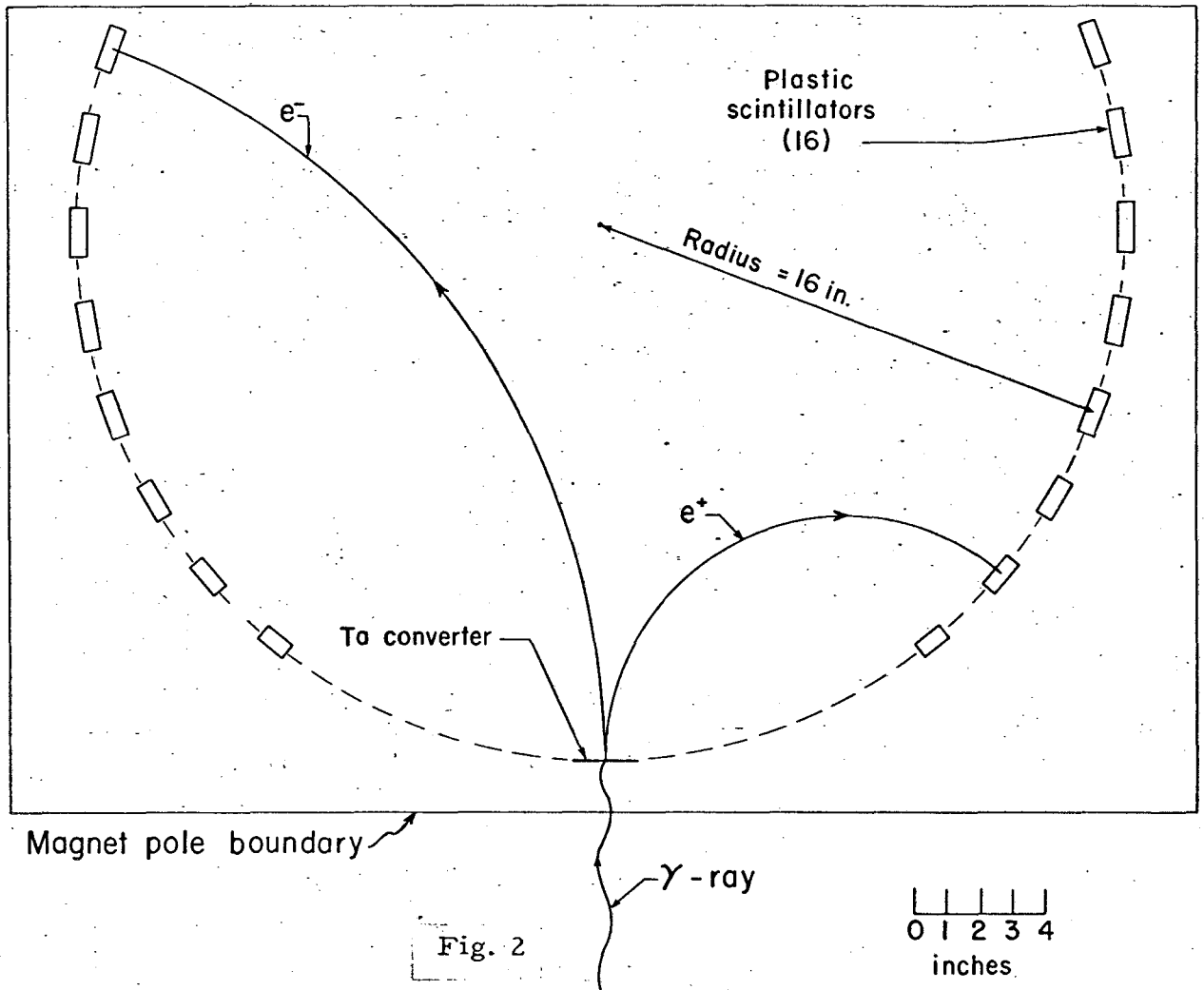
Plastic
scintillators
(16)
e⁻
Radius = 16 in.
e⁺
To converter
Magnet pole boundary
γ-ray
0 1 2 3 4
inches
MUB-881

Fig. 2

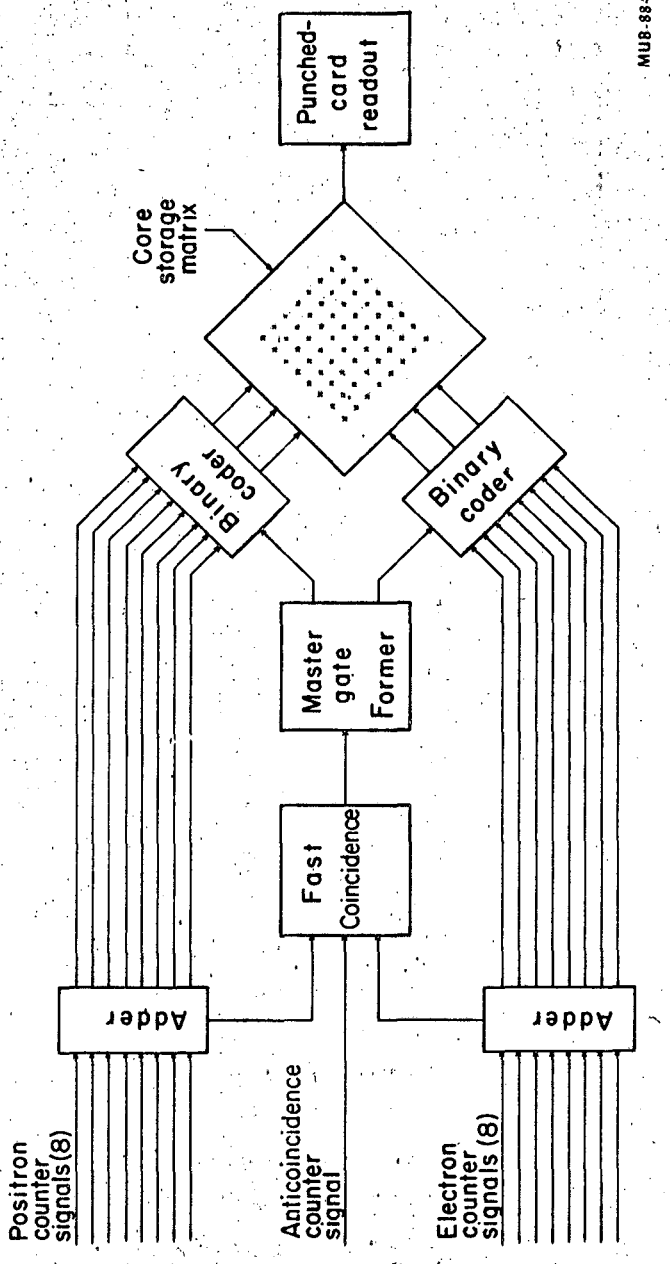
Punched-card readout
Core storage matrix
Binary coder
Binary coder
Master gate Former
Fast Coincidence
Adder
Adder
Positron counter signals (8)
Anticoincidence counter signal
Electron counter signals (8)
MUB-884

Fig. 3

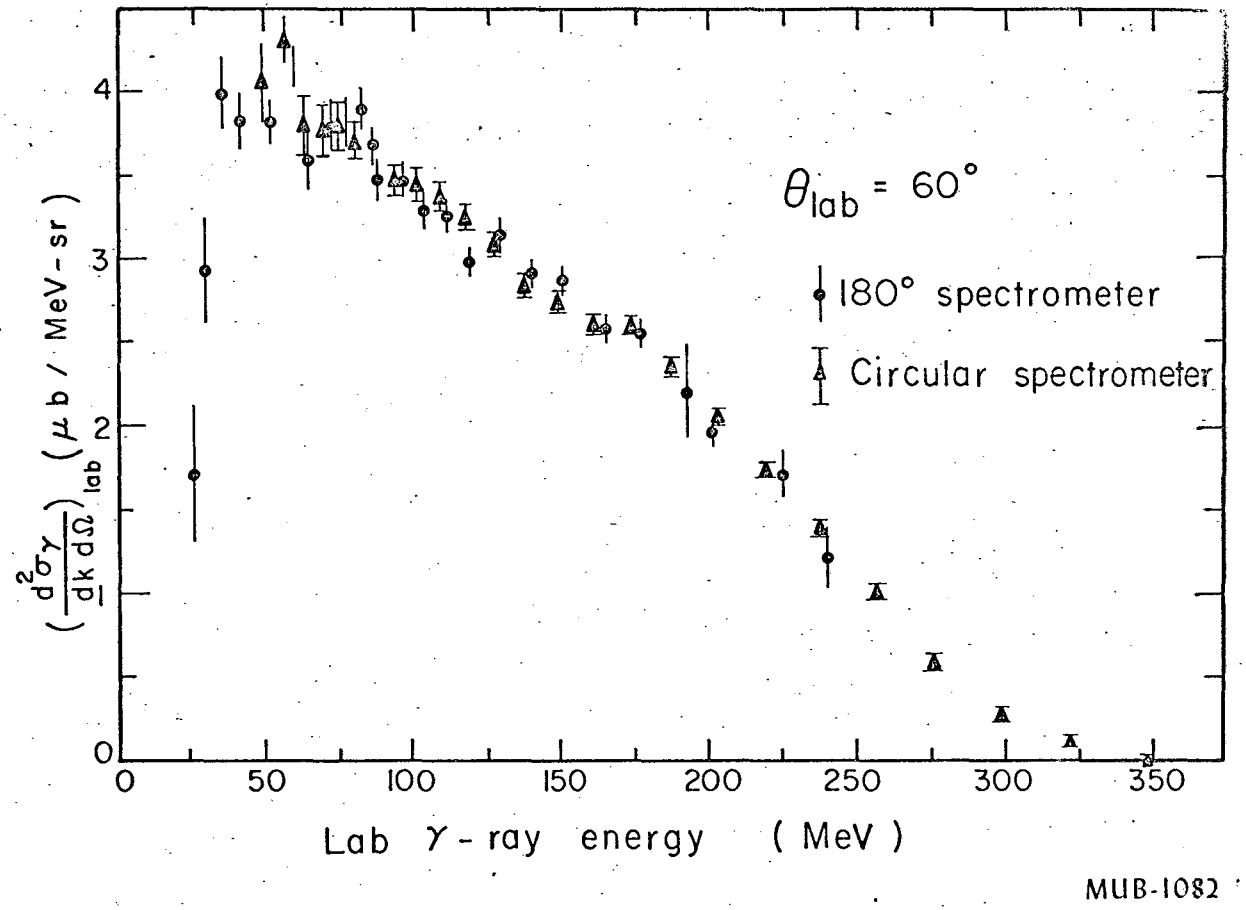
θ_{lab} = 60°
180° spectrometer
Circular spectrometer
$\left(\frac{d^2\sigma_\gamma}{dk\,d\Omega}\right)_{lab}$ (μ b / MeV - sr)
Lab γ - ray energy (MeV)
0
1
2
3
4
0
50
100
150
200
250
300
350
MUB-1082

Fig. 5

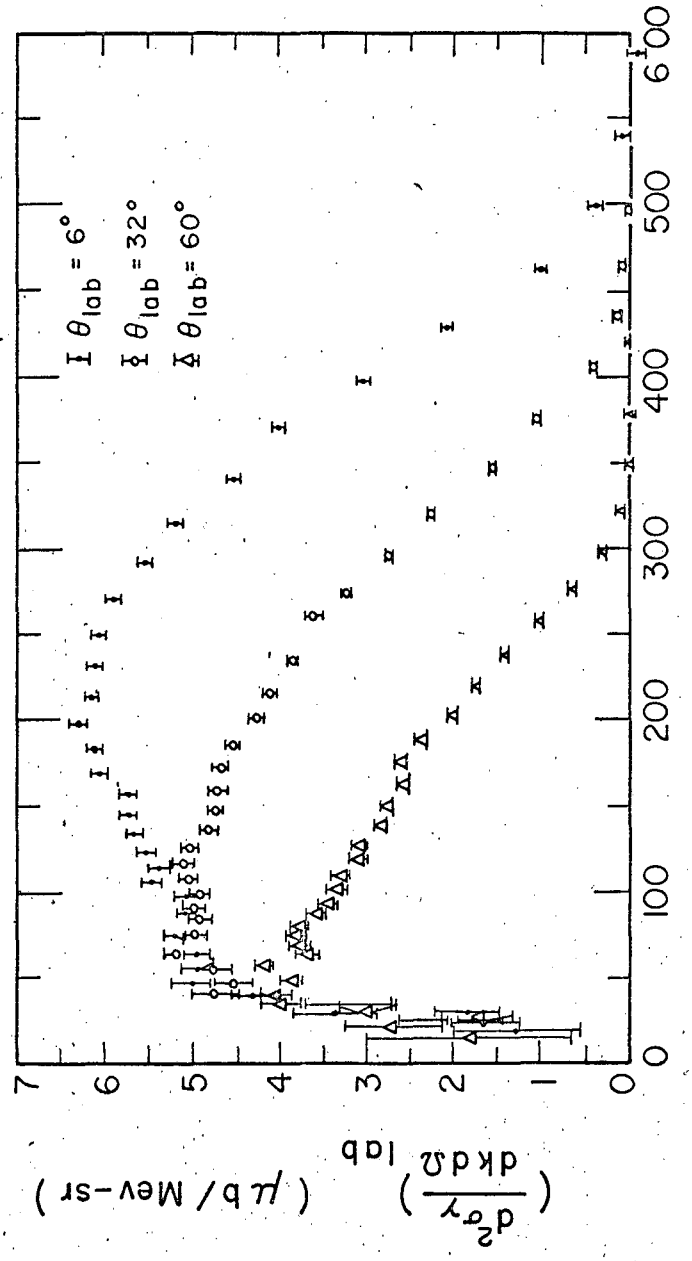
I θlab = 6°
θlab = 32°
θlab = 60°
Lab γ-ray energy (Mev)
(dσγ/dkdΩ)lab (μb/ Mev-sr)
MUB-1080

Fig. 6

[illegible]