Title
AN ELECTRONIC ANALOG SYSTEM FOR KINEMATICAL ANALYSIS OF INTERACTION VERTICES

Author
Murray, Joseph J.

Publication Date
1962-06-06

UCRL-10278

University of California

Ernest O. Lawrence Radiation Laboratory

Berkeley, California

UCRL-10278

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

AN ELECTRONIC ANALOG SYSTEM FOR KINEMATICAL ANALYSIS OF INTERACTION VERTICES

Joseph J. Murray

June 6, 1962

AN ELECTRONIC ANALOG SYSTEM FOR KINEMATICAL ANALYSIS OF INTERACTION VERTICES*

Joseph J. Murray

Lawrence Radiation Laboratory
University of California
Berkeley, California

June 6, 1962

Typically, the majority of bubble chamber events that have been selected visually by simple topological criteria "fit" one or more of the common hypotheses associated with the topology. Usually, however, there is also a residue of events, amounting to perhaps 10% of the total number of events analyzed, which for more than trivial reasons do not fit any of the common hypotheses. These consist of rare reactions not considered among the common types, misinterpretations of the actual topology, and errors in calculation.

Existing analysis machinery is well adapted for processing large numbers of "successful events," but for reprocessing the "failures" it is cumbersome because of the relatively large delay in obtaining results of revised calculations. It would be a great advantage in treating the failures to have immediate feedback, so that a physicist could acquaint himself with a problem event and lay it to rest in a single operation.

An electronic analog of the KICK-GUTS part of the analysis system was conceived about a year ago as a means of providing such a facility, and a prototype is now nearing completion. It was named PEBL (momentum, energy balance). Inputs to the analog—momenta, angles and errors—are now obtained from the geometrical reconstruction programs. The inputs are inserted digitally in a decimal system by manual switching at present, with provision for semi-automatic insertion by a punched card or tape-reading machine, if such a procedure should prove desirable in the future. Output is obtained digitally in either visual or printed form, one quantity at a time or in groups, by automatic scanning of programmed formats. The system is a fully automatic steady-state analog requiring a few seconds to reach equilibrium. Very little specialized technical know-how is required to use the machine.

*This work was done under the auspices of the U. S. Atomic Energy Commission.

In principle the basis for the analog may be illustrated as follows. A set of measured values χ_i^m of variables with uncorrelated errors δ_i (a convenient but unnecessary restriction) is adjusted in a servo-system with integral feedback to produce a set of fitted values χ_i^f which satisfy the constraints of energy and momentum conservation. That is

$$\chi_i^f - \chi_i^m = \sum_\lambda C_{i\lambda} \mathcal{E}_\lambda , \quad (1)$$

where $C_{i\lambda}$ are feedback coefficients to be determined, and the $\mathcal{E}_\lambda$ are integrated errors, one corresponding to each constraint;

$$\mathcal{E}_\lambda = \int_0^t F_\lambda (\chi_1^f \cdots \chi_n^f) \, dt/\tau_\lambda , \quad (2)$$

where $F_\lambda(\chi_i^f)$ are the constraint functions, equal to zero when the system is in equilibrium. The τ_λ are the time constants of the integration.

Differentiate Eq. (2) and expand F to first order about the measured values χ_i^m

$$\tau_\lambda \dot{\mathcal{E}}_\lambda = F_\lambda(\chi_i^m) + \sum_i (\chi_i^f - \chi_i^m) F_{\lambda i}, \qquad F_{\lambda i} = \partial F_\lambda / \partial \chi_i ,$$

and substitute for $\chi_i^f - \chi_i^m$ from Eq. (1),

$$\tau_\lambda \dot{\mathcal{E}}_\lambda = F_\lambda(\chi_i^m) + \sum_\Lambda \sum_i C_{i\Lambda} \mathcal{E}_\Lambda F_{\lambda i} . \quad (3)$$

This coupled set of differential equations determines the temporal behavior of the analog system if the τ_λ are large enough so the contribution from subloops in the feedback are negligible. In equilibrium we have $\dot{\mathcal{E}}_\lambda = 0$. Solving the corresponding set of algebraic equations (3) for the $\mathcal{E}_\lambda$, and inserting the result in Eq. (1), gives

$$\chi_i^f - \chi_i^m = - \sum_\lambda C_{i\lambda} \left\{ \sum_\Lambda F_\Lambda (\chi_i^m) \frac{A^{\Lambda\lambda}}{|A|} \right\} , \quad (4)$$

where $A_{jK} = \sum_i C_{ij} F_{Ki}$, and A^{jK} is the corresponding cofactor. If the constants are given the values $\delta_i^2 F_{\lambda i}$, Eq. (4) becomes the solution that minimizes

$$\chi^2 = \sum \frac{(\chi_i^f - \chi_i^m)^2}{\delta_i^2} ,$$

for the case of uncorrelated errors, hence the basis for the analog system. The $\mathcal{E}_\lambda$ are analogs of the Lagrangian multipliers sometimes introduced in solving the problem of minimizing χ^2.

From a design standpoint, the system consists of the following major sections:

(a) Input switching logic to generate the χ_i^m and the variance factors in convenient units;

(b) Subsystems duplicated for each track, which compute (1) the derivatives of the constraint functions $F_{\lambda i}$ (one per variable), (2) the fitted values of the variables, and (3) the energy.

(c) Subsystems duplicated for each vertex which (1) generate the constraint functions F_λ, and (2) compute the integral feedback $\mathcal{E}_\lambda$.

(d) Read-out devices to measure, scan, and display system variables in convenient units.

It has been found generally convenient to use two-dimensional vector technique in a steady-state AC(f = 1 kc) analog in which momentum components are represented by magnitudes, and azimuth and dip by phase angles of AC variables. With this technique <u>all</u> required computations (aside from linear addition and multiplication by a constant) have been reduced to two basic two-dimensional vector operations: (a) resolution of a vector into components with respect to an arbitrary reference, and (b) rotations, usually to obtain a reference phase. Both operations are performed in closed subloops in which the basic nonlinearity is always a rectification. A typical example is the calculation of total energy E, given the total momentum p and mass m. By the manner in which it is obtained, the signal representing p has an arbitrary phase. It is therefore first rotated or rephased into quadrature with the signal representing m. Addition of the two signals so prepared then gives a signal of magnitude E and phase angle with respect to p equal to $\cos^{-1}\beta$. The latter condition is useful in another connection. That is, one of the factors in the $F_{\lambda i}$ associated with the integral feedback from the energy constraint, $\mathcal{E}_E$, is β. The $\mathcal{E}_E$ is therefore generated in quadrature with m and its component with respect to E is resolved, the result having a magnitude $\beta\,\mathcal{E}_E$. Operational techniques are used extensively. Both the rephasing and resolving operations require four operational amplifiers. About 65 amplifiers are needed for each track. The system is completely transistorized, however, including all choppers. An amplifier occupies 1/4 or 1/6 of a 5 × 8 in. plug-in board, depending on whether it is chopper-stabilized or not. The circuitry exclusive of amplifiers associated with a resolver or rephaser occupies one such board. Exclusive of read-in decades, a track requires a total of 25 boards. The prototype, a 4-track 1-vertex system, occupies three standard 5-ft-high racks, including one rack reserved exclusively for readout equipment such as a digital voltmeter, scope, or scanning device. All subsystems have been reduced to printed circuitry, so that expansion to a system with more tracks and vertices will mainly involve copywork. Provision has been made to accommodate simultaneous 2-vertex fits.

To date the system has been operated under realistic conditions successfully, so far as basic performance is concerned; that is, it finds correct solutions, is stable, etc. Evolution and modification are at present so rapid, however, that neither exhaustive tests nor application to real problems in physics have been undertaken. The effective level of inaccuracies inherent in the analog is suggested by the fact that spurious χ^2 values obtained with test problems (exact solutions), with errors set to ≈ 2 or 3 MeV on all variables, have been $\lesssim 0.1$ or 0.2 in typical situations. It has been the goal to achieve reliable accuracy of one part in 10^3 of full scale for all significant variables. Accuracy of perhaps a half or third of this has been achieved to date, but is being improved.

ACKNOWLEDGMENTS

The importance of the extensive contribution of my coworker, Seth Shepard, in the development and construction of this device cannot be overemphasized. We very much appreciate the assistance received from members of our data analysis section in the preparation of input data.