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ISBN
9781479936854

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Publication Date
2014-05-01

DOI
10.1109/icra.2014.6907784

Peer reviewed

A Hybrid Feedback Controller for Robust Global Trajectory Tracking of Quadrotor-like Vehicles with Minimized Attitude Error

Pedro Casau, Ricardo G. Sanfelice, Rita Cunha, David Cabecinhas, Carlos Silvestre

***Abstract*— In this paper, we tackle the problem of trajectory tracking for a particular class of underactuated vehicles with full torque actuation and a single force direction (thrust) that is fixed relative to a body attached frame. Additionally, we consider that thrust reversal is not available. We present the design of a hybrid controller that, under some given assumptions, is able to globally asymptotically stabilize the vehicle to a reference position trajectory while minimizing the angle to the desired attitude trajectory. This objective is achieved robustly and globally, in the sense that small perturbations do not lead to instability and it is achieved regardless of the initial state of the vehicle. The algorithm is tested in a experimental setup, using a small scale quadrotor vehicle and the VICON motion capture system.**

I. INTRODUCTION

There exist several vehicles that are usually modeled as rigid-body vehicles for controller design purposes, namely, aircraft, spacecraft, marine vessels, among others. New sensor and processor technology have enabled their miniaturization and, ultimately, the development of automatic control and planning algorithms, which, in turn, have increased their autonomy, allowing for complex tasks to be carried out with little human intervention. These vehicles can be used for targeting, surveillance, inspection, among a plethora of other applications [14], [19].

More often than not, autonomous vehicles need to move within cluttered and unstructured environments. In order to avoid collisions or any other hazardous situation, several path planning algorithms have been proposed such as the ones given in [25], [2] and [5]. However, these strategies require a reference tracking controller to work as intended. To this end, trajectory tracking controllers were developed for fully-actuated vehicles (see e.g. [28], [27], [7], [6]) and also for underactuated vehicles (see e.g. [12], [1], [16], [8], [31]). Since the rotation of a rigid-body vehicle is represented by an element of $SO(3)$, the application of these controllers to such vehicles is hindered by the topological obstructions to global stabilization in $SO(3)$ by means of continuous feedback. In fact, given a vector field defined in $SO(3)$, there exists no equilibrium point that is *globally asymptotically stable* (see [4] for an enlightening discussion on this topic). In other words, if a continuous feedback law is used in order to stabilize the attitude of a rigid body to a given set-point (or trajectory), there are certain initial states that do not allow the accomplishment of such objective. The controller presented in [11] works around this issue by globally stabilizing a given set point by means of discontinuous feedback. However, it has been proved in [21] that a given point in a compact manifold cannot be globally robustly stabilized by means of continuous nor discontinuous feedback. In this context, the design of hybrid controllers has been shown to be able to overcome the topological hindrances of attitude stabilization for fully-actuated rigid-bodies, by means of hybrid quaternion feedback [22] and rotation matrix feedback [22].

In this paper, we address the trajectory tracking problem for an underactuated vehicle with a single thrust force direction and full torque maneuverability. This class of vehicles encompasses most UAVs and AUVs, as long as the externals disturbances are within reasonable bounds. Similarly to [16], [8], to deal with the problem of unmodeled dynamics, we propose a saturated controller with disturbance rejection properties. Also, due to the underactuation of the class of vehicles that we are considering, it is not possible to follow arbitrary trajectories. Therefore, similarly to [12], we propose a controller that performs position tracking with asymptotically stable position error, while keeping a minimum distance to an arbitrary attitude trajectory. Moreover, we achieve this objective globally by resorting to hybrid quaternion feedback strategies, as in [23]. The work presented in this paper expands that of [10] by adding robustness to unmodeled dynamics, achieving optimal tracking of a reference attitude trajectory and providing both simulation and experimental results. The experimental results are obtained using a motion capture system which, over the last few years, has become instrumental in the test of novel control methodologies (see e.g. [25]).

The remainder of this paper is organized as follows. In Section II, we present notational conventions that are used throughout the paper. Section III describes the problem setup that is addressed in the subsequent sections. In Section IV, we describe the controller design for the position subsystem and in Section V we present the control law that achieves the desired goal. Experimental and simulation results are provided in Section VI considering a quadrotor vehicle. Finally, some concluding remarks are given in Section VII. Proofs of the results presented in this paper will appear

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The work of Pedro Casau and David Cabecinhas was supported with grants SFRH/BD/70656/2010 and SFRH/BD/31439/2006, respectively, from Fundação para a Ciência e a Tecnologia.

This research has been partially supported by the National Science Foundation under CAREER Grant no. ECS-1150306 and by the Air Force Office of Scientific Research under Grant no. FA9550-12-1-0366.

elsewhere.

II. Preliminaries

Let $SO(n)$ denote the set of $n \times n$ matrices that satisfy $\det(R) = 1$ and $R^\top R = RR^\top = I_n$. Furthermore, let $SE(n) := \mathbb{R}^n \times SO(n)$, and $\mathbb{S}^n := \{x \in \mathbb{R}^{n+1} \,|\, x^\top x = 1\}$. Two very common representations of the attitude of a rigid-body are the rotation matrices and the unit quaternions, given by $R \in SO(3)$ and $q = \begin{bmatrix}\eta & \epsilon^\top\end{bmatrix}^\top \in \mathbb{S}^3$, respectively. The mapping $R : \mathbb{S}^3 \to SO(3)$, given by

$$R(q) := I_3 + 2\eta S(\epsilon) + 2S(\epsilon)^2,$$

with

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix},$$

is known as the *Rodrigues formula* and it maps a given quaternion to a rotation matrix. This mapping is a local diffeomorphism but many-to-one globally, since $R(q) = R(-q)$. Quaternion multiplication is given by the mapping

$$q_1 q_2 = \begin{bmatrix} \eta_1\eta_2 - \epsilon_1^\top \epsilon_2 \\ \eta_1\epsilon_2 + \eta_2\epsilon_1 + S(\epsilon_1)\epsilon_2 \end{bmatrix}.$$

The inverse of the unit quaternion is given by $q^{-1} = [\eta \ -\epsilon^\top]^\top$ and is such that $qq^{-1} = q^{-1}q = [1\ 0\ 0\ 0]^\top$. Moreover, the following relationship holds: $\nu(R(q)v) = q\nu(v)q^{-1}$ for any $v \in \mathbb{R}^3$, with $\nu(v) := [0\ v^\top]^\top$. For more information on quaternion algebra, the reader is referred to [30] or [17].

Another representation of the attitude of a rigid-body is the angle-axis representation, described as follows: given a angle-axis pair $(\theta, v) \in [0, \pi] \times \mathbb{S}^2$, the rotation matrix associated with a rotation of θ around the unit vector v is given by

$$R(\theta, v) := I_3 + \sin(\theta)S(v) + (1 - \cos(\theta))S(v)^2.$$

The following notation is also used in the sequel: the canonical basis for $\mathbb{R}^n$ is the set $\bigcup_{i=1}^{n} \{e_i\}$, where $e_i \in \mathbb{R}^n$ is a vector whose entries are zeros, except for the i-th entry which is 1; the inner product between two matrices $A, B \in \mathbb{R}^{m\times n}$ is given by $\langle A, B\rangle := \operatorname{trace}\left(A^\top B\right)$. If $n = 1$, then the previous inner product reduces to the standard vector inner product $\langle A, B\rangle := A^\top B$; the closed unit ball $\mathbb{B} \subset \mathbb{R}^n$ is given by $\mathbb{B} := \{x \in \mathbb{R}^n \,|\, \|x\| \le 1\}$. Given a set valued mapping $M : \mathbb{R}^m \rightrightarrows \mathbb{R}^n$, the range of M is the set

$$\operatorname{rge}(M) = \{y \in \mathbb{R}^n \,|\, \exists x \in \mathbb{R}^m \text{ such that } y \in M(x)\}.$$

In this paper, we follow the same notation in [20] to represent the derivatives of differentiable functions. Let $F : \mathbb{R}^{m\times n} \to \mathbb{R}^{p\times q}$ be a differentiable function, then

$$\mathcal{D}_X(F) := \frac{\partial \operatorname{vec}(F)}{\partial \operatorname{vec}(X)^\top},$$

where $\operatorname{vec}(A) := \begin{bmatrix} e_1^\top A^\top & \dots & e_n^\top A^\top \end{bmatrix}^\top$, for any $A \in \mathbb{R}^{m\times n}$. In some particular cases, we use a more conventional notation. Given a differentiable function $V : \mathbb{R}^n \to \mathbb{R}$, the gradient is defined as $[\nabla V]_i := \frac{\partial V}{\partial x_i}$, for each $i \in \{1, 2, \dots, n\}$. Let $v \in \mathbb{R}^m$, then we define

$$\nabla_v V := \begin{bmatrix} \frac{\partial V}{\partial v_1} & \frac{\partial V}{\partial v_2} & \dots & \frac{\partial V}{\partial v_m} \end{bmatrix}^\top.$$

Given a function $H : \mathbb{R} \to \mathbb{R}^{m\times n}$ we define

$$\left[\frac{dH}{dt}\right]_{ij} := \frac{dH_{ij}}{dt},$$

for each $i \in \{1, 2, \dots, m\}$ and for each $j \in \{1, 2, \dots, n\}$.

In this paper, we also make use of the function $\Gamma : SO(3) \to \mathbb{R}^9$, given by

$$\Gamma(R) := -\begin{bmatrix} S(Re_1) & S(Re_2) & S(Re_3) \end{bmatrix}^\top,$$

with the following property

$$\operatorname{vec}(RS(\omega)) = -\Gamma(R)R\omega, \tag{1}$$

and of the K-saturation function, defined as follows.

Definition 1: A *K-saturation function* is a smooth non-decreasing function $\sigma_K : \mathbb{R} \to \mathbb{R}$ that satisfies the following properties:

1) $\sigma_K(0) = 0$,
2) $s\sigma_K(s) > 0$ for all $s \neq 0$,
3) $\lim_{s\to\pm\infty} \sigma_K(s) = \pm K$, for some $K > 0$.

Moreover, for each $x \in \mathbb{R}^n$ we define

$$\Sigma_K(x) := \begin{bmatrix} \sigma_K(x_1) & \dots & \sigma_K(x_n) \end{bmatrix}^\top. \qquad \square$$

Also, we make use of recent developments on hybrid systems theory which are described in [13]. Under this framework, a hybrid system $\mathcal{H}$ is defined as

$$\mathcal{H} = \begin{cases} \dot{x} \in F(x) & x \in C \\ x^+ \in G(x) & x \in D \end{cases},$$

where: the set-valued map $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is the *flow map* and governs the continuous dynamics (also known as flows) of the hybrid system; the set $C \subset \mathbb{R}^n$ is the *flow set* and defines the set of points where the system is allowed to flow; the set-valued map $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is the *jump map* and defines the behavior of the system during jumps; the set $D \subset \mathbb{R}^n$ is the *jump set* and defines the set of points where the system is allowed to jump.

The definition of global asymptotic stability of a closed set $\mathcal{A} \subset \mathbb{R}^n$ for a hybrid system $\mathcal{H}$ is given in [13, Chapter 7]. In the next section, we proceed to establish the problem setup.

III. Problem Setup

In this paper, we consider the problem of designing a controller for a class of rigid bodies with a single thrust direction and full torque actuation. This includes, for example, helicopters, quadcopters and underactuated underwater vehicles. For controller design purposes, we consider the following equations of motion

$$\dot{p} = v, \tag{2a}$$

$$\dot{v} = -R(q)e_3\frac{T}{m} + ge_3 + b, \tag{2b}$$

$$\dot{q} = \frac{1}{2}q\nu(\omega), \tag{2c}$$

where $p \in \mathbb{R}^3$ denotes the position of the rigid body in the inertial reference frame, $v \in \mathbb{R}^3$ represents its linear velocity, $q \in \mathbb{S}^3$ denotes the unit quaternion that represents the orientation of the body fixed frame with respect to the inertial reference frame, $\omega \in \mathbb{R}^3$ denotes the angular velocity, $g \in \mathbb{R}$ denotes the acceleration of gravity, $b \in \mathbb{R}^3$ represents a constant unknown disturbance, $T \in \mathbb{R}$ is the thrust magnitude and $m \in \mathbb{R}$ denotes the mass of the rigid body. For more information on this model the reader is referred to [26] and [3]. Notice that we have not explicitly included aerodynamic/hydrodynamic effects in (2) because these are application dependent and typically difficult to model. In order to overcome modelling uncertainties to some degree, we design a controller that is robust both with respect to small measurement noise and constant force disturbances.

Let $(p_d(t), R_d(t)) \in SE(3)$ denote a reference position and attitude trajectory which is defined for all $t \geq 0$. In this paper, we consider only reference trajectories that satisfy the following assumption.

Assumption 1: A given reference trajectory is a function $t \mapsto (p_d(t), R_d(t)) \in SE(3)$ defined for all $t \geq 0$, satisfying: the reference position trajectory $p_d(t)$ and its derivatives up to $p_d^{(4)}(t)$ are bounded, and $p_d^{(4)}(t) \in M_p\mathbb{B}$ for some $M_p > 0$; the reference attitude trajectory is generated by the system

$$\dot{R}_d = R_d S\left(\omega_d\right), \quad \dot{\omega}_d \in M_\omega \mathbb{B}, \tag{3}$$

for some $M_\omega > 0$. □

Given Assumption 1 and (1), we say that

$$r(t) := [p_d(t)^\top, \, p_d^{(1)}(t)^\top, \; p_d^{(2)}(t)^\top, \; p_d^{(3)}(t)^\top, \\ \operatorname{vec}\left(R_d(t)\right)^\top, \; \omega_d(t)^\top]^\top \in \mathcal{R}$$

is a solution to the differential inclusion

$$\dot{r} \in F_d(r) := \{p_d^{(1)}\} \times \{p_d^{(2)}\} \times \{p_d^{(3)}\} \times M_p\mathbb{B} \\ \times \{-\Gamma(R_d)R_d\omega_d\} \times M_\omega\mathbb{B},$$

and $\mathcal{R} := \mathbb{R}^{12} \times SO(3) \times \mathbb{R}^3$.

In this paper, our main goal is to design a controller that tracks the desired position trajectory, while steering the orientation q to q_0, which is defined to be the closest unit quaternion to one of the unit quaternions corresponding to $R_d \in SO(3)$. Moreover, it satisfies the rotation kinematics $\dot{q}_0 = q_0\nu(\omega_0)/2$ for some function $\omega_0 : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$. To this end, let us define the error variables

$$p_0 := p - p_d, \quad v_0 := v - \dot{p}_d, \quad q_1 := qq_0^{-1}.$$

Differentiating the error variables and using (3), (2c) and the properties of quaternion algebra described in Section II, we obtain

$$\begin{aligned} \dot{p}_0 &= v_0, \\ \dot{v}_0 &= -R(q_1)R(q_0)e_3\frac{T}{m} + ge_3 + b - p_d^{(2)}, \\ \dot{q}_1 &= \frac{1}{2}q_1\nu(R(q_0)(\omega - \omega_0)). \end{aligned}$$

With these definitions, we are now able to precisely state the problem to solve in this work.

Problem 1: Given a reference trajectory satisfying Assumption 1, and a desired orientation $q_0(t)$ which minimizes

$$\operatorname{trace}\left(I_3 - R(q_0(t))R_d(t)^\top\right)$$

for each $t \geq 0$, design a hybrid controller

$$\dot{x}_c \in F_c(r, p, v, q, x_c) \qquad (r, p, v, q, x_c) \in C_c, \tag{4a}$$
$$x_c^+ \in G_c(r, p, v, q, x_c) \qquad (r, p, v, q, x_c) \in D_c, \tag{4b}$$

and $(T, \omega) = \kappa(r, p, v, q, x_c)$ such that the set $\mathcal{A} \subset \mathcal{X} := \mathcal{R} \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{S}^3 \times \mathbb{S}^3 \times \mathcal{X}_c$, given by

$$\mathcal{A} := \left\{(r, p_0, v_0, q_1, q_0, x_c) \in \mathcal{X} \,|\, p_0 = v_0 = 0, \; q_1 = [\pm 1 \quad 0^\top]^\top\right\},$$

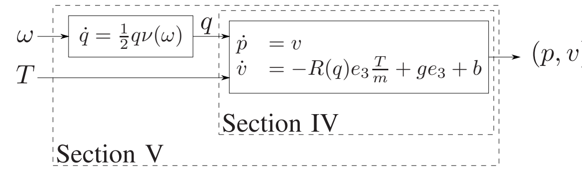


Fig. 1: Structure of system (2).

is *globally asymptotically stable* for the hybrid system given by

$$\dot{r} \in F_d(r), \tag{5a}$$
$$\dot{p}_0 = v_0, \tag{5b}$$
$$\dot{v}_0 = -R(q_1)R(q_0)e_3\frac{T}{m} + ge_3 + b - p_d^{(2)}, \tag{5c}$$
$$\dot{q}_1 = \frac{1}{2}q_1\nu(R(q_0)(\omega - \omega_0)), \tag{5d}$$
$$\dot{q}_0 = \frac{1}{2}q_0\nu(\omega_0), \tag{5e}$$
$$\dot{x}_c \in F_c(r, p, v, q, x_c) \;\; (r, p, v, q, x_c) \in C_c, \tag{5f}$$
$$x_c^+ \in G_c(r, p, v, q, x_c) \;\; (r, p, v, q, x_c) \in D_c, \tag{5g}$$

In Section IV, we design a controller for the position subsystem and then, in Section V, we design a control law for the whole system using backstepping techniques, taking advantage of the structure of the system (5) depicted in Figure 1.

IV. Controller Design for the Position Subsystem

In this section, we address the problem of designing a controller that solves Problem 1 when (T, q) are considered as inputs. The results presented in this section are pivotal in the design of the controller that solves Problem 1, which is presented in Section V.

We start the controller design process by giving the following assumption.

Assumption 2: There exists a locally Lipschitz control law $\overline{u}_0 : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$ such that $(p_0, v_0) = 0$ is globally asymptotically stable for the system

$$\dot{p}_0 = v_0, \quad \dot{v}_0 = \overline{u}_0(p_0, v_0).$$ □

If Assumption 2 holds, then, by [18, Theorem 4.17], there exists a smooth, positive definite and radially unbounded function $\overline{V}_0$ and positive definite function W_0 such that, for each $(p_0, v_0) \in \mathbb{R}^3 \times \mathbb{R}^3$,

$$\left\langle \nabla\overline{V}_0(p_0, v_0), \begin{bmatrix} v_0 \\ \overline{u}_0(p_0, v_0) \end{bmatrix} \right\rangle \leq -W_0(p_0, v_0).$$

Notice that, in general, the control law $(p_0, v_0) \mapsto \overline{u}_0(p_0, v_0)$ is not robust with respect to the unknown disturbance b. In order to cancel out this disturbance, we redesign $\overline{u}_0$ as a new control law $u_0(p_0, v_0, z)$, given by

$$u_0(p_0, v_0, z) := \overline{u}_0(p_0, v_0) - \Sigma_K(z), \tag{6}$$

where $z \in \mathbb{R}^3$ is an integral state. Imposing bounds on the inputs limits the perfomance of the controller, in the sense that, there is a limit on the disturbances that it is able to overcome. As a consequence, the controller that we devise in this paper must also satisfy the following assumption.

Assumption 3: Given $K > 0$, $|b_i| < K$, for each $i \in \{1,2,3\}$. $\square$

In Lemma 1, we show that if the given assumptions are satisfied, then it is possible to redesign $\overline{u}_0$ so as to achieve global tracking of the position trajectory.

Lemma 1: For any $k_z > 0$ and $K > 0$, if Assumptions 2 and 3 hold, then the control law (6) renders the set

$$\mathcal{A}_0 := \{(p_0, v_0, z) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \,|\, p_0 = v_0 = 0, z = \Sigma_K^{-1}(b)\}$$

globally asymptotically stable for the system

$$\begin{aligned} \dot{p}_0 &= v_0, \\ \dot{v}_0 &= u_0(p_0, v_0, z) + b, \\ \dot{z} &= k_z \nabla_{v_0} \overline{V}_0(p_0, v_0). \end{aligned}$$

Remark 1: Notice that $u_0(p_0, v_0, z)|_{(p_0,v_0,z)\in\mathcal{A}_0} = -b$, thus the integrative state z modifies the control law for the unperturbed system $\overline{u}_0$ so as to cancel the unknown bias b.

From Lemma 1, we conclude that global asymptotic stabilization of the set $\mathcal{A}_0$ when (T, q) are used as inputs depends on the existence of solutions to

$$R(q)e_3 \frac{T}{m} := -\mu, \tag{7}$$

where $\mu := u_0(p_0, v_0, z) - ge_3 + p_d^{(2)}$.

In the same spirit as [12], we define $T_0 := m\,\|\mu\|$ and q_0 as the solution to the optimization problem

$$\begin{aligned} &\text{minimize} && \tfrac{1}{2}\text{trace}\left(I_3 - R(q)R_d^\top\right) \\ &\text{subject to} && R(q)e_3 \tfrac{T_0}{m} = -\mu \\ &&& q \in \mathbb{S}^3 \end{aligned} \tag{8}$$

which minimizes the angle ψ between $R(q)$ and R_d. The solution to this optimization problem is such that

$$R(q_0) = \left(I_3 + S(\gamma) + \left(1 + e_3^\top R_d^\top \frac{\mu}{\|\mu\|}\right) S\left(\frac{\gamma}{\|\gamma\|}\right)^2\right) R_d,$$

where $\gamma := -S(R_d e_3)\frac{\mu}{\|\mu\|}$. Notice that both q_0 and $-q_0$ are solutions to this optimization problem, thus we must resort to robust path-lifting techniques, such as the one described in [24], in order to ensure that the sign of the quaternion is consistently selected. Moreover, notice that the optimization problem (8) is infeasible if $\|\mu\| = 0$ or $e_3^\top R_d^\top \mu / \|\mu\| = 1$. To prevent these issues we will need to make further assumptions on the reference trajectory and control law.

Assumption 4: Given a reference trajectory that satisfies Assumption 1, the following holds $\|u_0(p_0, v_0, z) + p_d^{(2)}\| < g$, for all $(p_0, v_0, z) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3$ and for all $r \in \text{rge}(F_d)$. $\square$

Assumption 5: Given a reference trajectory that satisfies Assumption 1, the following holds: $e_3^\top R_d(t) e_3 \geq 0$, for all $t \geq 0$. $\square$

Assumption 4 prevents the commanded thrust from becoming zero, i.e., the case $\|\mu\| = 0$, and a combination of Assumption 4 and Assumption 5 prevents the situation $e_3^\top R_d^\top \mu / \|\mu\| = 1$. An example of a controller that satisfies Assumption 4 for certain controller parameters is given in [10, Appendix A].

Since we are considering q as an input, setting $q(t) = q_0(t)$ for all $t \geq 0$ guarantees that, under the given assumptions, Problem 1 is solved. In the next section, we include the rotation kinematics using backstepping techniques.

V. Global Asymptotic Stabilization of the Attitude Kinematics by Hybrid Quaternion Feedback

In this section, we develop a controller that solves Problem 1 when the thrust T and the angular velocity ω are considered as inputs. To achieve this objective, we extend the hybrid quaternion feedback strategy that was introduced in [23] so as to deal with the presence of unknown disturbances and underactuation. To do so, we need to estimate ω_0.

Notice that ω_0 can be computed from $dR(q_0)/dt = R(q_0)S(\omega_0)$ using the relationship

$$\mathcal{D}_t\left(R(q_0)\right) = \text{vec}\left(R(q_0)S(\omega_0)\right), \tag{9}$$

and replacing (1) into (9), yielding $\mathcal{D}_t(R(q_0)) = -\Gamma(R(q_0))R(q_0)\omega_0$. Finally, it is easy to verify that $\Gamma(R(q_0))^\top\Gamma(R(q_0)) = 2I_3$, thus we obtain $\omega_0 = -\frac{1}{2}R(q_0)^\top\Gamma(R(q_0))^\top\mathcal{D}_t(R(q_0))$.

However, ω_0 depends on the variables (r, p_0, v_0, z) and their time derivatives, thus, in particular, it depends on $b \in \mathbb{R}^3$ which is an unknown constant. To overcome this, we use $b_1 \in \mathbb{R}^3$ which denotes the estimate of b. Also, let $\omega_{0,1}$ denote the estimate of ω_0 that depends on b_1 rather than b, i.e.,

$$\omega_{0,1} := -\frac{1}{2}R(q_0)^\top\Gamma(R(q_0))^\top\, \mathcal{D}_t\left(R(q_0)\right)\big|_{b=b_1}.$$

It is possible to verify that the difference between $\omega_{0,1}$ and ω_0 is given by

$$\omega_{0,1} - \omega_0 = -\frac{1}{2}R(q_0)^\top\Gamma(R(q_0))^\top\mathcal{D}_{v_0}\left(R(q_0)\right)\widetilde{b}_1,$$

where we have used the definition $\widetilde{b}_1 := b_1 - b$.

Using the definitions $H := \{-1, 1\}$, $\mathcal{X}_c := \mathbb{R}^3 \times H \times \mathbb{R}^3$, $x_c := (z, h, \widetilde{b}_1) \in \mathcal{X}_c$ and

$$\dot{b}_1 := \frac{1}{2}k_{b_1}kh\mathcal{D}_{v_0}\left(R(q_0)\right)^\top\Gamma(q_0)\epsilon_1,$$

the hybrid system $\mathcal{H}_1 := (C_1, F_1, D_1, G_1)$ with $x = (r, p_0, v_0, q_1, q_0, z, h, \widetilde{b}_1) \in \mathcal{X}$ as follows

$$F_1(x) := \begin{pmatrix} F_d(r) \\ v_0 \\ -R(q_1)R(q_0)e_3\frac{T}{m} + ge_3 - f_{\dot{p}_d}(\dot{p}_d) + b \\ \frac{1}{2}q_1\nu(R(q_0)(\omega - \omega_0)) \\ \frac{1}{2}q_0\nu(\omega_0) \\ k_z\nabla_{v_0}\overline{V}_0(p_0, v_0) \\ 0 \\ \frac{1}{2}k_{b_1}kh\mathcal{D}_{v_0}\left(R(q_0)\right)^\top\Gamma(q_0)\epsilon_1 \end{pmatrix} \tag{10a}$$

$$x \in C_1 := \{x \in \mathcal{X} \,|\, h\eta_1 \geq -\delta\}, \tag{10b}$$

$$G_1(x) = (r, p_0, v_0, q_1, q_0, z, -h, \widetilde{b}_1) \tag{10c}$$

$$x \in D_1 := \{x \in \mathcal{X} \,|\, h\eta_1 \leq -\delta\}, \tag{10d}$$

for some $\delta \in (0, 1)$, where (T, ω) are given by

$$T := m\|\mu\|, \tag{11a}$$

$$\omega := \omega_{0,1} + R(q_0)^\top\left(-\omega_1^\star - k_q h\epsilon_1\right), \tag{11b}$$

with $\omega_1^\star := \frac{2k_z k_{V_0}}{kh}\left(\eta_1 S\left(\mu\right) - S\left(\mu\right) S\left(\epsilon_1\right)\right) \nabla_{v_0} V_0$, μ given in (7), and $h \in H = \{-1, 1\}$ is a logic variable that enables controller switching.[1]

Theorem 1: Let Assumptions 1-4 hold. Then, for any $k_{V_0}, k_q, k, k_z, k_{b_1} > 0$, the set

$$\mathcal{A}_1 := \left\{ x \in \mathcal{X} \,|\, p_0 = 0, v_0 = 0, q_1 = \begin{bmatrix} h & 0^\top \end{bmatrix}^\top \right\},$$

is *globally asymptotically stable* for the hybrid system $\mathcal{H}_1$, using the control law (11). $\square$

At this stage it should be clear that the proposed controller solves Problem 1 with $x_c = (z, h, \widetilde{b}_1)$ and $\kappa(r, p, v, q_1, q_0, x_c)$ as given in (11) . Notice that the controller is robust to constant forces disturbances and small perturbations because the closed-loop hybrid system (10) satisfies the hybrid basic conditions (see [13] or [23]). In the next section, we present both simulation and experimental results.

VI. Simulation/Experimental Results

To experimentally evaluate the performance of the controller described in Section V, we make use of the following components:1) *Blade mQX quadrotor* [15], 2) VICON Bonita motion capture system [29], 3) MATLAB/Simulink software, and 4) custom-made RF interface. The *Blade mQX quadrotor* weighs 80 g and has a radius of approximately 11 cm. The overall control architecture is depicted in Figure 4. For more details on the system architecture and identification, the reader is referred to [9].

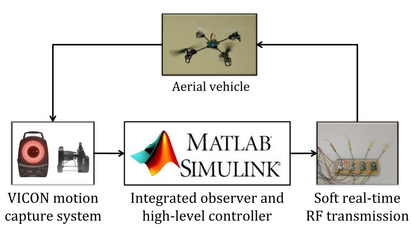


Fig. 4: Quadrotor control architecture.

Since the VICON motion capture system outputs the rotation matrix of the vehicle we resort to the path lifting strategy outlined in [24] so as to obtain consistent quaternion representations of attitude.

To assess whether the hybrid controller was working as intended, we carried out the following experiment: set the desired position trajectory to a circular trajectory with a radius of 1 m and an angular frequency of 20 deg/s; set the initial yaw of the quadrotor to be approximately 180 degrees away from the desired orientation; run the experiment for $h(0,0) = 1$ and $h(0,0) = -1$.

In this experiment we test specifically the hybrid nature of the proposed controller since different values of the logic variable produce different outcomes when the quadrotor is near a rotation error of 180 degrees. The experimental results were also compared with simulation results using the same controller parameters, which are: $k_p = 3$, $k_v = 5$, $k_{V_0} = 0.01$, $k_z = 0.3$, $k_q = 3$, $k_{b_1} = 1$, $K = 1$ and $\delta = 0.5$.

[1]Recall that we are using the notation $q = [\eta \; \epsilon^\top]^\top$ to represent the unit quaternion. See Section II for more details.

From the analysis of Figure 5, it is possible to verify that the controller is working as intended, since the vehicle rotates around opposite directions depending on the initial value of the logic variable. Figure 6 depicts the tracking error for each of the experiments/simulations. In both figures, it is noticeable that the experiments have a small time delay before the initialization of the controller. If we discard this small time window, we argue that, even though the convergence time in the simulations is smaller, the overall behavior is very much alike in both simulations and experiments. Moreover, in each experiment and simulation the disturbances were small enough so as not to trigger any jump of the logic variable, therefore it remains constant throughout. Figures 2 and 3 show what the rotation around opposite directions looks like in the real-world. In these images, the direction of rotation can be tracked by following some features of the quadrotor, such as the white propellers.

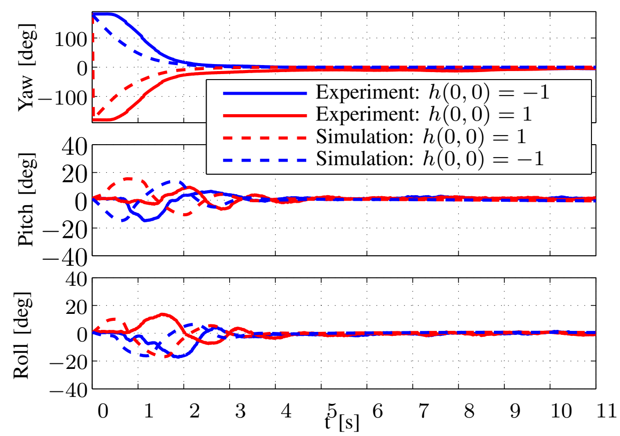


Fig. 5: Euler angles for two experiments/simulations, roughly with the same initial conditions but different values of the logic variable.

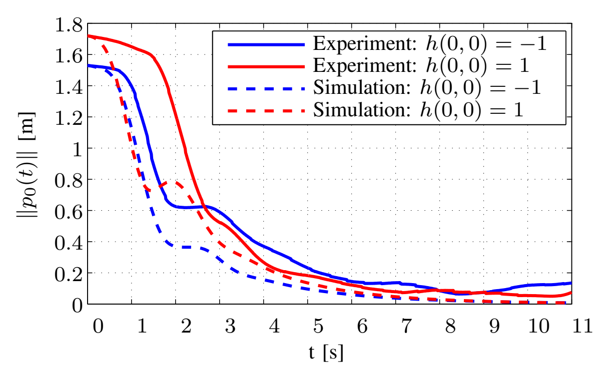
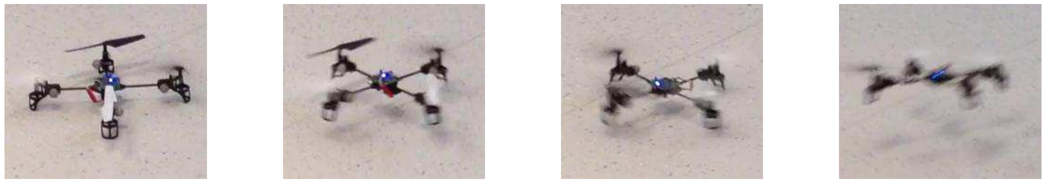


Fig. 6: Position error for two experiments/simulations with initial yaw error of approximately 180°.

VII. Conclusion

In this paper, we described a controller for a class of underactuated vechicles that globally performs the tracking of a reference position trajectory while minimizing the



(a) $t = t_0$ (b) $t = t_1 > t_0$ (c) $t = t_2 > t_1$ (d) $t = t_3 > t_2$

Fig. 2: First few seconds of the experiment with initial condition $h_0 = 1$.

(a) $t = t_0$ (b) $t = t_1 > t_0$ (c) $t = t_2 > t_1$ (d) $t = t_3 > t_2$

Fig. 3: First few seconds of the experiment with initial condition $h_0 = -1$.

orientation error to some desired attitude trajectory. The class of vehicles that we consider are characterized by full torque actuation and uni-directional thrust. This objective was achieved by means of novel hybrid control techniques and standard backstepping solutions. Experimental results that demonstrate the performance of the controller and its application to quadrotor vehicles were also provided.

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