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VOLUME VERSUS SURFACE SAMPLING OF MAXWELLIAN DISTRIBUTIONS IN NUCLEAR REACTIONS

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ABSTRACT

The pre-exponential factor in a thermal distribution of classical particles depends on whether the particles have been emitted from the surface of a hot body or are contained in a volume of gas of given temperature. Precisely the appropriate factors describe nuclear evaporation of very low energy neutrons (volume sampling) or moderate energy fragments (surface), as well as the high energy processes of projectile fragmentation and "fireball" breakup (volume).

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Some time ago, Weisskopf[1] described the spectral distribution to be expected for neutrons emitted by an excited nucleus. The principle of detailed balance implies that the distribution must contain a factor σv, where σ is the cross section for the inverse process of neutron absorption, and v is the neutron velocity. The remaining factors arise from the neutron phase space volume, giving $d^3p = d\Omega \; p(M+E/c^2)dE$, with E the neutron's kinetic energy and M its mass, and from statistical weights for the many nuclear states, giving the Boltzmann function $e^{-E/kT}$, with kT the temperature in energy units.

At very low energy the cross section σ exhibits a 1/v divergence, so that the product σv is nearly constant. However, over a significant range of (nonrelativistic) energies σ itself is slowly varying, and may be treated as constant.[2] Thus, at extremely low energies the pre-exponential factor in the neutron spectral distribution is proportional to $\sqrt{E}$, but at higher energies it is proportional to E. The factor E(or $E-E_c$, with E_c the Coulomb barrier for a charged particle[3]) has been used for a long time in phenomenological analyses of nuclear evaporation spectra. In the case of charged particle emission, it is doubtful whether E and $\sqrt{E}$ could be distinguished experimentally, since the Coulomb barrier effects are strongly energy dependent. Therefore the success of fits using the one or the other dependence on E does not constitute evidence that this is the proper choice.

In high-energy (> 20 MeV/nucleon) collisions of nuclear projectiles with nuclear targets, slow fragments in the projectile rest frame exhibit an energy distribution which is roughly of the thermal form, with a factor $\sqrt{E}$ before the exponential.[4] This is consistent with, but certainly no proof of, the assumption that all or part of the projectile came to equilibrium at a definite temperature following the collision.[5]

Recently spectra for products of nuclear collisions involving projectile energies of hundreds of MeV/nucleon were fit roughly in a "fireball" model, in which overlapping projectile and target matter were supposed to convert all their relative kinetic energy to thermal excitation.[6] The spectra were fit by a thermal distribution in the fireball center-of-mass frame, with the pre-exponential factor taken as $\sqrt{E}$. Changing to the factor E would have destroyed the agreement.[7]

Of course, one would like to have a unified physical picture of these various "thermal" processes. The key to such a picture is in Weisskopf's paper,[1] where he notes that the form linear in E corresponds to a nonrelativistic Maxwell-Boltzmann distribution for particles emitted from the surface of a hot object. The extra velocity factor arises because the faster particles are relatively more likely to come out in any given time interval. This factor, then, corresponds to surface sampling of a thermal distribution, which is an appropriate description of the evaporation process if the evaporated particles may be treated classically (that is, for small de Broglie wavelengths). On the other hand, if one were able to sample all of the particles in some volume of gas at thermal equilibrium, one would expect to see just the factor $\sqrt{E}$ before the exponential. This is quite reasonable for the fireball, which is supposed to break up into many fragments instead of simply emitting a few particles from its surface, and is also plausible for projectile fragmentation. Volume sampling is a good description of very low energy neutron emission for a different reason. In this regime the excited nucleus certainly doesn't break up, but the de Broglie wavelength λ of the neutron becomes large compared to nuclear dimensions. Consequently, the notion of surface emission of these neutrons becomes absurd, since they cannot be localized to a volume much smaller than $(\lambda/2)^3$. One is really sampling the neutrons in such a volume, in thermal equilibrium with the excited nucleus.

A moral of this discussion is that the pre-exponential, energy-dependent factor in a supposedly thermal spectrum contains significant information about the detailed process leading to that spectrum. Since one often has independent experimental or theoretical knowledge about the process, the pre-exponential factor may supply an important consistency check. In the nuclear reactions discussed here, that consistency appears to obtain in all cases where the factor is well determined.[8] A second moral is that the term "Maxwellian" applied to an energy distribution is ambiguous, since it may be used for either choice of pre-exponential factor.[9] Since the volume distribution is really the primary concept, it might be helpful to refer to a distribution with a linear E factor explicitly as "surface Maxwellian", "evaporative Maxwellian", or[9] "effusive Maxwellian."

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REFERENCES

1. V. F. Weisskopf, Phys. Rev. 52, 295 (1937).
2. J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (Wiley, New York, 1952).
3. For example, I. Dostrovsky, Z. Fraenkel and G. Friedlander [Phys. Rev. 116, 683 (1959)] multiply a factor equivalent to E by another factor $(1-E_c/E)$, yielding finally $E-E_c$.
4. D. E. Greiner, P. J. Lindstrom, H. H. Heckman, B. Cork and F. S. Bieser, Phys. Rev. Lett. 35, 152 (1975).
5. A. S. Goldhaber, Phys. Lett. 53B, 306 (1974).
6. G. D. Westfall, J. Gosset, P. J. Johansen, A. M. Poskanzer, W. G. Meyer, H. H. Gutbrod, A. Sandoval and R. Stock, Phys. Rev. Lett. 37, 1202 (1976).
7. G. D. Westfall, private communication.
8. G. D. Westfall, R. G. Sextro, A. M. Poskanzer, A. M. Zebelman, G. W. Butler and E. K. Hyde, Berkeley report LBL-6558, November, 1977.
9. See, for example, E. H. Kennard, Kinetic Theory of Gases (McGraw-Hill, New York, 1938) pp. 46-47, 62-63.

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