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Evaluating the Impacts of Start-Up and Clearance Behaviors in a Signalized Network: A Network Fundamental Diagram Approach

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Evaluating the Impacts of Start-Up and Clearance Behaviors in a Signalized Network: A Network Fundamental Diagram Approach

A Research Report from the University of California Institute of Transportation Studies

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16. Abstract

Numerical simulations have shown that the network fundamental diagram (NFD) of a signalized network is significantly affected by the green ratio. An analytical approximation of the NFD has been derived from the link transmission model. However, the consistency between these approaches has not been established, and the impacts of other factors are still unrevealed. This research evalutes the impacts of start-up and clearance behaviors in a signalized network from a network fundamental diagram approach. Microscopic simulations based on Newell's car-following model are used for testing the bounded acceleration (start-up) and aggressiveness (clearance) effects on the shape of the NFD in a signalized ring road. This new approach is shown to be consistent with theoretical results from the link transmission model, when the acceleration is unbounded and vehicles have the most aggressive clearance behaviors. This consistency validates both approaches; but the link transmission model cannot be easily extended to incorporate more realistic start-up or clearance behaviors. With the new approach, this project demonstrates that both bounded acceleration and different aggressiveness lead to distinct network capacities and fundamental diagrams. In particular, they lead to start-up and clearance lost times of several seconds; and these lost times are additive. Therefore, the important role that these behaviors play in the NFD shape is studied to reach a better understanding of how the NFD responds to changes. This will help with designing better start-up and clearance behaviors for connected and autonomous vehicles.

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Evaluating the Impacts of Start-Up and Clearance Behaviors in a Signalized Network: A Network Fundamental Diagram Approach

UNIVERSITY OF CALIFORNIA INSTITUTE OF TRANSPORTATION STUDIES

April 2019

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Introduction

Motivation

A well-founded knowledge of the Fundamental Diagram (FD) is essential for its importance as a network performance indicator. Particularly, for stationary urban road networks, we will refer to it as the Network Fundamental Diagram (NFD) which is the relationship between the average flow rate and the average density of the network. Over the years, many studies have been exploring such relationship to better understand how it is affected by certain changes in the network or in vehicles operation. Nevertheless, most previous investigations were conducted with a macroscopic approach and not considering the signalized network scenarios. Fundamental research combining microscopic models and signalized networks is limited to date. Therefore, we want to fill this gap with a new simulation microscopic approach in a signalized network.

This study aims to shed light on how the NFD can be influenced in a signalized ring road by different start-up and clearance behaviors. Before this, we also want to check with the theory that our microscopic model can generate an accurate NFD for the signalized network. This is the foundation to ensure that the method is valid to then extend it to study different start-up and clearance behaviors. On the one hand, for start-up behaviors (beginning of the green interval) we consider the application of a bounded acceleration (BA) to vehicles. On the other hand, we set different levels of aggressiveness to vehicles approaching the traffic light as clearance behaviors (end of green interval). By microscopically controlling these individual vehicle trajectories, we can describe how the NFD is impacted by every possible configuration.

In a signalized network, it is known that the maxim flow rate is limited by the theoretical green ratio ($\pi = g/T$). However, this study shows how the effective green ratio (π_i, with $i = 1, 2, 3$, for every study case) differs (is actually reduced) from π when different start-up and clearance behaviors are considered. This just suggests that bounded acceleration and aggressiveness can play an important role that cannot be neglected in urban signalized networks.

The obtained insights might be fundamental to help to clarify some of the questions regarding connected or autonomous vehicles implementation. Which start-up and clearance behaviors should be implemented to obtain a better performance (in terms of the NFD approach) for a signalized urban network? Of course, these essential decisions are influenced for many other elements (security, morality, mechanical limitations, …) and autonomous vehicles will face different situations. However, this analysis is not distracted by secondary effects and it gives us specific and meaningful insights, that may open new horizons.

Literature Review

In stationary urban networks, the observation that a relation between network-average flow and density exists is quite old [1]. Such a relationship is called the NFD and has shown to be unique in homogeneous networks.

Since [2], the LWR model has been applied to study the impacts of traffic signals on traffic dynamics in an urban network. In [3], it was mathematically proved that asymptotic periodic traffic patterns, which can be defined as stationary states, exists in a ring road with a pre-timed signal with two phases within the framework of a network kinematic wave theory. Moreover, the impacts of signal settings on the macroscopic FD in stationary states were simulated with the Cell Transmission Model [4]. LWR model was also used in [5], which studied in a signalized road the large time behavior of the solutions of a Hamilton-Jacobi equation with an x-periodic Hamiltonian and what can be interpreted as a flux limiter depending periodically on time.

Some efforts have been devoted to derive the NFD in simple signalized networks from various traffic flow models. A simulation study using cellular automatons for traffic on a ring road was conducted by [6], in which they examined the relationship between flow and density. It was found that offsets can have drastic impacts on the maximum saturation flow rate and, therefore, on the NFD shape. In [7], empirical evidences were provided for the existence of an urban-scale macroscopic FD; and later in [8] a variational method was proposed to estimate the macroscopic FD, for a network with many intersections but it did not provide an explicit formulation for the macroscopic FD. Another study was conducted by [9] to evaluate the impacts of green ratio, signal coordination and turning movements using field data to reproduce the NFD. Results showed that the capacity is not only affected by the green ratio (as it is already known) but also affected by signal coordination and turning movements. It was also noticed that the appearance of the diagram suggests a trapezoidal form (under-saturated, saturated and over-saturated parts), which is consistent with the traditional understanding of the NFD. However, no general formulation was deduced due to the lack of data in congested regimes.

So far, the most relevant work to that we propose is explained in [10]. The investigation derived an approximate NFD based on the LWR model and using the continuous formulation of Link Transmission Model [11]. The impacts of cycle length on the NFD were also discussed and optimal cycle lengths were obtained. Nevertheless, the most important insight is that the formulation for an approximate three-pieced NFD was obtained for simple stationary states on a signalized ring road with one traffic light. This is completely relevant for our study because the NFD obtained through simulations using a microscopic model can be compared with the theoretical equations derived from the macroscopic approach. And if the microscopic simulation matches the theory, it will give consistency to our results and the theoretical formulation will be trustworthy for microscopic simulations.

The differences between the mentioned literature and this particular research are basically four-fold. First, the microscopic simulation approach can be considered independent from the previous studies. Second, bounded acceleration is implemented in the car-following model as a start-up behavior. Third, the clearance behaviors related to different vehicles’ aggressiveness when the traffic light changes are included in the simulations. And finally, our goal is not only to test the impacts on the shape of the NFD but also quantify the lost time per signal cycle for each configuration.

The rest of the paper is organized as follows. In Section 2, the microscopic model and the network configuration for the simulations are defined. In Section 3, a first simulation is done to compare the theoretical formulation of the NFD and our microscopic approach. This is essential to ensure the reliability of the following sections. Section 4 shows the impacts on the NFD when we apply the BA as a start-up behavior. In a similar way, in Section 5 different aggressiveness are applied to the simulation and the impacts on the NFD are quantified. In Section 6, lost time additive property is tested. Finally, in Section 7 we summarize the conclusions and discuss future research.

Model Definition and Simulation Setup

Newell's Car-Following Model

In order to test different individual vehicles behaviors in a signalized intersection, a microscopic model is needed. This study adopts Newell's car-following model derived from the macroscopic LWR model [2]. It was shown that this microscopic model is exactly equivalent to the LWR model [12]. In particular, we use the time- and vehicle-discrete car following formulation derived from the continuous car-following model in [13]. For simplicity, the triangular fundamental diagram [14] [15] [16] is considered:

$$\phi(k) = min\{\, uk, w(k_j - k)\}$$

where u is the free flow speed, $-w$ the shock wave speed in congested traffic and k_j the jam density. Thus, the critical density is $k_c = \frac{w}{u+w} k_j$, and the capacity $C = uk_c$.

Every time step in the simulation, the car-following model updates the vehicle's velocity, position, and acceleration:

$$X_t(t + \Delta t, n) = min\left\{\, u, \frac{X(t, n - \Delta n) - X(t, n) - \rho\Delta n}{\tau\Delta n}\right\} \quad (1)$$

$$X(t + \Delta t, n) = min\left\{X(t, n) + \Delta t \cdot u, X(t, n) + \Delta t \cdot \frac{X(t, n - \Delta n) - X(t, n) - \rho\Delta n}{\tau\Delta n}\right\} \quad (2)$$

where the time gap is $\tau = 1/(wk_j)$, and the jam spacing is $\rho = 1/k_j$.

Signalized Network

We want to reproduce vehicles' trajectories inside a city's signalized network grid. To achieve this network scenario, the road configuration used is a signalized ring road with one lane and one traffic light. For a ring road of a length of L, the x-axis increases in the traffic direction, and we place a signal at x = 0, L, 2L, 3L, … and so on for every lap. It is a closed system equivalent to an infinite one-way street without turning movements, where all the intersections are the same. Furthermore, the ring road can also be used to model a homogeneous, symmetric one-way road grid network, where there are no turning movements, all signals have the same settings and all links have the same characteristics and traffic conditions [10].

A typical three-color traffic light system that switches between green, yellow and red is selected. The traffic light is configurated with a pre-timed cycle time of T = 60 s: 23 s green light, 5 s of yellow light, 2 s of all red and 30 s of red light. It is assumed that all red interval can be used to clear the intersection, therefore, a green ratio of $\pi = (g+Y)/C = 0.5$ is theoretically achieved (where Y = yellow + all red). Then, vehicles could use up to 30 s of the cycle time to cross the intersection.

Comparison Between Macroscopic and Microscopic Models

As a starting point, we are willing to reproduce the theoretical results obtained with the LTM in [10]. To do so, first, some ideas need to be explained in the following subsections.

Equivalence Configuration

To test the car-following model consistency with the macroscopic results, an equivalent configuration needs to be defined for the simulation. In this case, vehicles need to use 30 s of the cycle time to cross the intersection, so a highly aggressive behavior needs to be applied (which is not realistic, only useful for the theoretical verification). In this case, vehicles ignore yellow light and wait until the red light to make a decision. During the green, yellow and all red phase ($g+Y$ = 30 s), vehicles can cross the traffic light but when it turns red, they are not allowed to cross it. Then, the closest vehicle approaching the intersection (which will be called the *signal leader*) must stop at the red light and wait until the next green. Thus, any car before the intersection must be able to stop at any given velocity and no matter how close it is from the intersection if it is selected as the *signal leader*. This is because the deceleration rate can be big enough (it is not bounded) to make the car stop in one time step. Admittedly, this a simple and not very realistic logic but it will let us reproduce the results obtained with the macroscopic model and confirm that the microscopic model is valid to test start-up and clearance behaviors in a ring road with one traffic light.

Microscopic Model Logic for a Traffic Light

Due to the existence of the signalized intersection, the car-following model needs some modifications to capture traffic light effects. This is a special case of car-following where vehicles can either follow the previous vehicle or the traffic light. Vehicles will mostly follow the previous vehicle but when the traffic light turns red, the *signal leader* is selected and it must stop at the intersection and wait until the next green. This vehicle will follow the intersection constraint and forget about the previous vehicle. To do so, a virtual vehicle called the *control* *vehicle* is introduced in the ring road every time the light turns red and it is located accurately after the traffic light. It is static and the *signal leader* stops behind it as if it was obeying the traffic light. This control vehicle is the equivalence of the red traffic light in the car-following model. However, it doesn't change the network's density since it is just the car following representation of the red traffic light. Once the light turns green, the *control vehicle* disappears and the *signal leader* becomes a regular vehicle following its predecessor. For every time the signal turns red again, a new *signal leader* will be selected and the *control vehicle* will appear in the simulation.

Microscopic Simulation

We consider a ring road of L = 900 m where there is an intersection of L_{int} = 10 m (included in L). We also set τ = 1.5 s [17], u = 15 m/s and k_j = 1/7 veh/m. Therefore w =4.67 m/s. The green ratio is π = 0.5, so we are expecting to get the maximum average flow rate equal to πC to verify the theoretical results. This exclusively happens in the ideal case that matches the theory. Further, we will see that the maximum average flow rate will be $\pi_i C$, where π_i is the mentioned effective green ratio which is different from π.

Convergence of the maximum average flow rate

For different values of Δn and $\Delta t = \tau\Delta n$, the convergence of the maximum flow rate is tested in order to set an accurate Δn for the following simulations. For the convergence, we run one simulation only for a certain density (N = 20 vehicles in the ring road) that is expected to be in the saturated part of the NFD to capture the maximum average flow. Results are shown in Table 1.

We can see that as we decrease Δn and consequently Δt, the π_0 value converges to the specified green ratio of 0.5. This can be explained by the fact that the macroscopic model considers traffic flow as a fluid and it can only be equivalent to a continuum car-following model with a small enough Δn, in fact with a Δn = 0. However, for $\Delta n \leq 0.1$ and $\Delta t \leq 0.15$ s an error of less than 1% is already achieved, so we will use Δn = 0.1 from now on due to the computational constraints.

TABLE 1. Convergence analysis for the maximum average flow rate.

Δn	1	0.5	0.25	0.2	0.1	0.05	0.025	0.015
Δt (s)	1.5	0.75	0.375	0.3	0.15	0.075	0.0375	0.0225
π_0	0.5244	0.5081	0.5081	0.5048	0.5015	0.5015	0.5007	0.5001

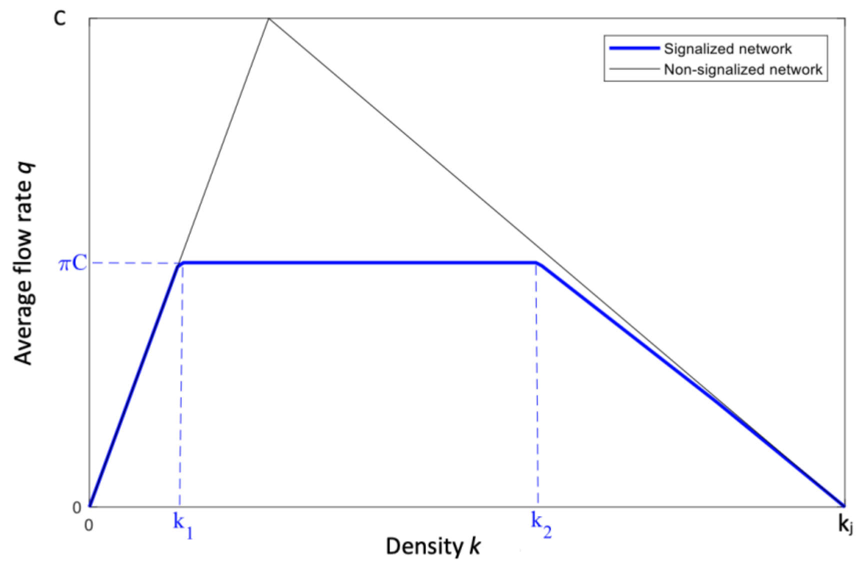


Figure 1. Theoretical three-pieced trapezoidal NFD defined by πC and k_1 and k_2

NFD approach

In [10] a macroscopic fundamental diagram is derived and it is approximated by a piecewise linear function. The obtained three-pieced trapezoidal NFD is mainly defined by the maximum average flow rate πC and the critical densities k_1 and k_2 (Figure 1). The maxim average flow rate was already obtained in the convergence study for $\Delta n = 0.1$. Nevertheless, we also need to test if the shape of the obtained NFD is consistent with the under-saturated, saturated and over-saturated branches defined by the critical densities k_1 and k_2 [10]:

$$\frac{L}{u} = \theta_1 T, \quad \theta_1 = j_1 + \alpha_1, \quad j_1 = \left\lfloor \frac{L}{uT} \right\rfloor = 0, 1, \ldots, \quad 0 \leq \alpha_1 < 1 \tag{3}$$

$$\frac{L}{w} = \theta_2 T, \quad \theta_2 = j_2 + \alpha_2, \quad j_2 = \left\lfloor \frac{L}{wT} \right\rfloor = 0, 1, \ldots, \quad 0 \leq \alpha_2 < 1 \tag{4}$$

$$k_1 \equiv \frac{j_1 + min\left\{\frac{\alpha_1}{\pi}, 1\right\}}{j_1 + \alpha_1} \pi k_c \tag{5}$$

$$k_2 \equiv k_j - \frac{j_2 + min\left\{\frac{\alpha_2}{\pi}, 1\right\}}{j_2 + \alpha_2} \pi \frac{C}{w} \tag{6}$$

Simulation results

We consider again a ring road of $L = 900$ m with one traffic signal with a green ratio of $\pi = 0.5$. The defined Δn and Δt in the convergence test are used. Unlike what it was done for the convergence analysis, now we want to obtain the whole NFD. Therefore, all possible densities from 0 to k_j will be simulated to get the shape of the NFD. Every dot in Figure 2 symbolizes one simulation with a certain number of cars on the ring road. The density is constant for every simulation as the number of cars is constant inside the ring road. The average flow rate is obtained as the density multiplied by the average speed of all vehicles. The first ten cycles (600 s) aren't considered for the average speed calculation. Every simulation runs vehicles for 10 hours.

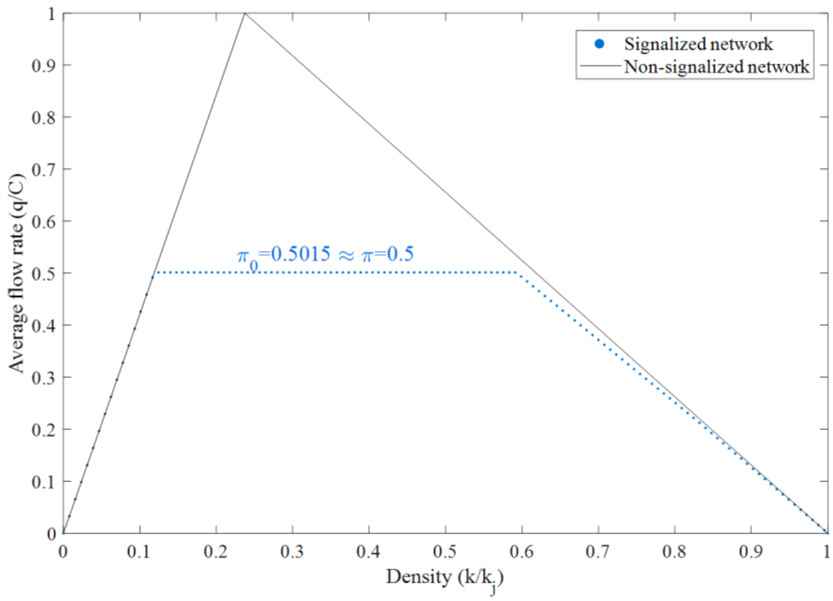


Figure 2. Microscopic simulation results to show the equivalence between the theoretical definition of the NFD and the obtained shape through simulations.

As we expected, the theoretical k_1 and k_2 match the ones calculated in the obtained through simulations NDF. So, it has already been proved that the three-pieced trapezoidal NFD's formulation in [10] is completely consistent with NFD obtained with the Newell's car-following model. These results give us the confidence to rely on the microscopic car-following model to test the proposed start-up and clearance behaviors impacts on the NFD.

We also represented individual's vehicles trajectories in Figure 3a and 3b. Vehicles obey the traffic light whenever it is required and it is ensured that the simulation configuration is a solid foundation for our study.

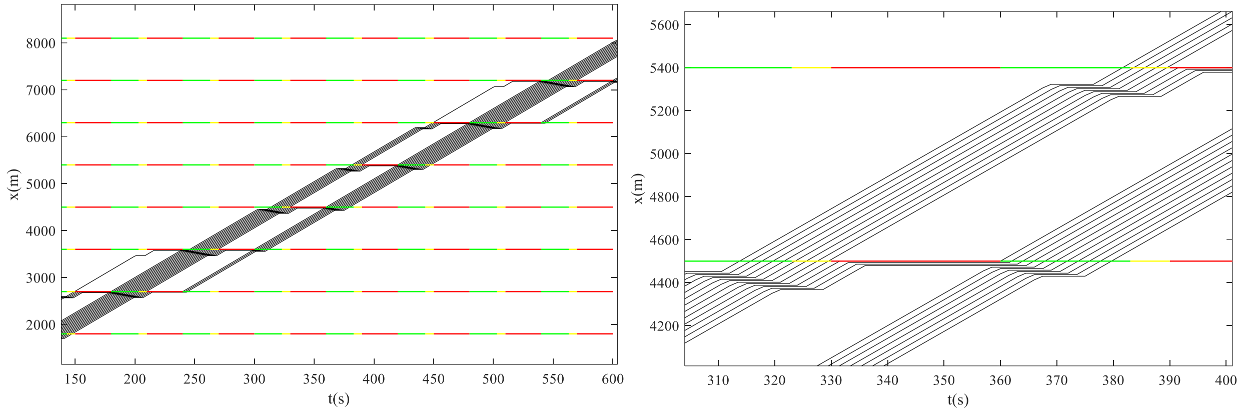


Figure 3. Vehicles' trajectories obtained for the simulation 0. General overview of the performance and detailed zoom-in of phase changes.

Impacts of Start-Up Behaviors

Start-up lost time is the additional time consumed for queued vehicles after the red light turns green to accelerate to the steady flow condition. It is due to the reaction time and the bounded acceleration. In real scenarios, vehicles can't immediately accelerate to reach the equilibrium speed so there is some lost green time due to that at the beginning of every green phase. Our purpose in this section is to focus on the effects of bounded acceleration as a start-up behavior to quantify the impacts on the NFD.

Newell's Car-Following Model with Bounded Acceleration

Bounded acceleration is added to the microscopic model (equations 1 & 2). With this modification, the bounded acceleration car-following formulation is [13],

$$X_t(t+\Delta t,n) = min\left\{X_t(t,n) + \Delta t \cdot A(X_t), u, \frac{X(t,n-\Delta n) - X(t,n) - \rho\Delta n}{\tau\Delta n}\right\} \tag{7}$$

$$\begin{aligned} X(t+\Delta t,n) = min\Big\{&X(t,n) + \Delta t \cdot X_t(t,n) + \Delta t^2 \cdot A(X_t), \\ &X(t,n) + \Delta t \cdot u, X(t,n) + \Delta t \cdot \frac{X(t,n-\Delta n) - X(t,n) - \rho\Delta n}{\tau\Delta n}\Big\} \end{aligned} \tag{8}$$

Even if in the general version the bounded acceleration term is a function of velocity ($A(X_t)$), we are going to assume that it is just a constant parameter *a*. For simplicity, it will also be the same bound for all the vehicles.

Simulation 1

We consider again a ring road of L = 900 m. We also set τ = 1.5 s [17], u = 15 m/s and k_j = 1/7 veh/m. Therefore w =4.67 m/s. We set Δn = 0.1 and then Δt = 0.15. The simulation will run for 10 hours. The green ratio is π = 0.5 theoretically, but we expect that it will be reduced by the impact of the start-up lost time induced by bounded acceleration. Therefore, the maximum flow rate will be defined by the effective green ratio π_1 times the capacity, as we mentioned before.

The highly aggressive behavior of vehicles adopted in Section 3 is still being used. We will run many identical simulations (changing the density of the network) with two configurations: one is modified with a bounded acceleration term in the car-following model and the other is not (as in simulation 0). This way we can focus on the bounded acceleration effects on the NFD without being distracted by other phenomena.

Impacts of Bounded Acceleration

The numerical solutions of the Newell's car-following model with a bounded acceleration of 2 m/s^2 and without BA are presented in Figure 4, in which flow rate and density are normalized with respect to C and k_j, respectively. What stands out in this figure is the significant reduction of the maximum average flow rate on the NFD when BA is applied.

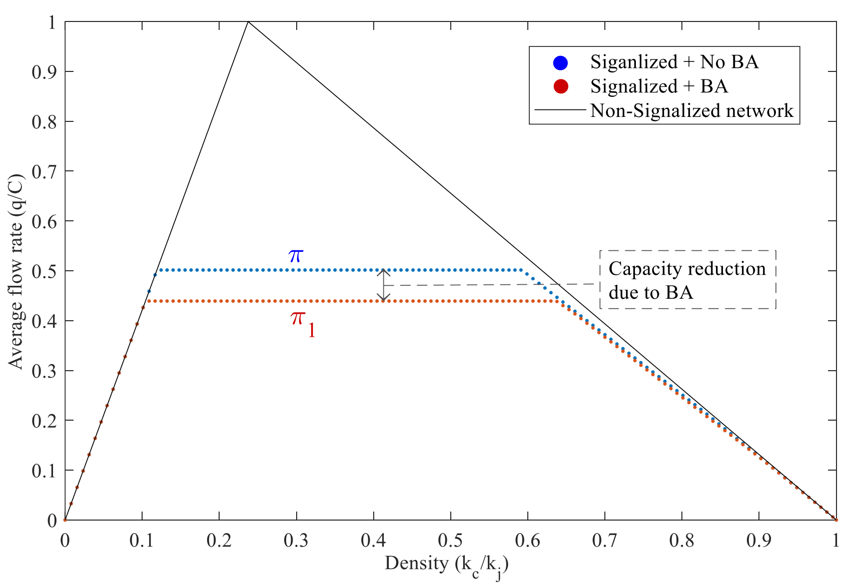
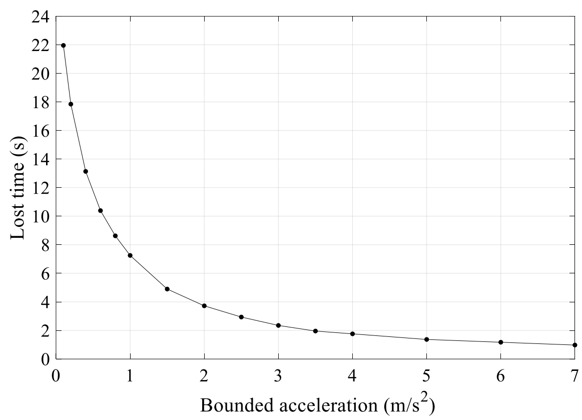


Figure 4. NFD obtained for simulation 1 that incorporates BA of 2 m/s2 to the car-following model

The maximum flow rate is 0.4392C which means that the effective green ratio π_1 = 0.4392 is achieved for the BA model. This produces flow rate losses of 12.42% with respect to the green ratio π of the no-BA case. As we detailed before, we call π_1 'effective' because the theoretical green ratio π doesn't change, and what we obtain is a reduction due to the clearance behavior. Notice then, that a reduction on the maxim average flow rate means a reduction of the effective green ratio which leads to some lost time. Results basically show that only 26.27 s of 30 s are effective green time when BA is applied. Therefore, we can already say that there is a start-up lost time of 3.72 s due to a BA of 2 m/s^2. It can be concluded then that the start-up behavior affects significantly the performance of the network and consequently the associated NFD.



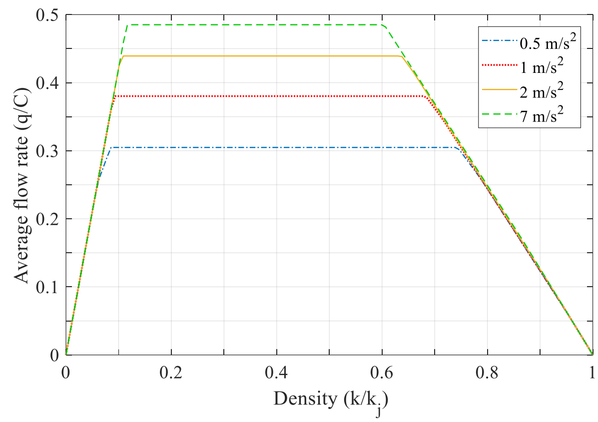


Figure 5. (a) Relation between bounded acceleration and start-up lost time and (b) the NFD for different BA rates

It is also important to know how different acceleration bounds affect the start-up lost time and therefore the NFD shape. The start-up lost time per cycle (Figure 5a) and the obtained NFD (Figure 5b) are represented for different BA values. It can be clearly seen that there is a significant variability of the magnitude of the lost time (also of the NFD shape) for different values of the BA. Lost times vary from 22 s to less than 1 s for acceleration bounds of 0.1 and 7 m/s^2, respectively. This demonstrates that the lost time is very sensitive to changes in BA, because small variations lead to big differences of lost time (especially for BA < 2 m/s^2). Therefore, it can be concluded that BA as a start-up behavior not only produces changes to the NFD and start-up lost time; but also, these effects are shown to be significantly sensitive to BA changes.

Impacts of Clearance Behaviors

Different clearance lost time will be achieved considering possible reactions of drivers when at time *t* signal turns from green to yellow. Until now, we were considering the theoretical highly aggressive driver which waits until the moment when the light turns red to decide (based on its position) if it stops or not. In this section, however, we need a more realistic behavior to test different aggressiveness.

Possible Clearance Decisions

Every time the light turns yellow, some vehicles face the following decision: whether they can stop or they can go through the intersection during the remaining time before red. Both possibilities are calculated and checked for every single car in the ring road. Thus, if there is a possible decision, we may want to settle how the vehicle will respond to it. This will be completely related to the different clearance behaviors represented by the aggressiveness. First of all, we are going to discuss the logic mechanism of the possible decisions and, after that, we will focus on how vehicles can respond to that uncertain scenario.

The stopping condition

The total distance to bring a vehicle located at x_{veh} to full stop is the sum of the reaction distance D_{RE} and the breaking distance D_B.

$$D_B = \frac{v_n^2(t)}{2b} \tag{9a}$$

$$D_{STOP} = D_{RE} + D_B = t_{RE} \cdot v_n(t) + \frac{v_n^2(t)}{2b} \tag{9b}$$

$$D_{veh-int} = x_{int} - x_{veh} \tag{9c}$$

$$if \;\; D_{STOP} \leq D_{veh-int} \;\Rightarrow\; The\ vehicle\ can\ stop \tag{9d}$$

The breaking distance D_B is a function of the current car velocity $v_n(t)$ and the deceleration rate b. The maximum deceleration rate is set as b = 4 m/s^2. Nevertheless, in this study, deceleration is not bounded in the car following model and it can be infinite. Then this maximum deceleration b will only affect the *can stop decision* but it doesn't mean that higher decelerations can't be achieved in other situations during the simulation.

The stopping distance D_{STOP} for every car can be calculated with the equations in 9a 9b and 9c. Therefore, if the stopping distance is smaller or equal than the distance to the beginning intersection $D_{veh-int}$, the car will be able to safely stop with a maximum deceleration rate of b. Otherwise, the car can't stop before the intersection at the given situation.

The go-through condition

At a determined speed $v_n(t)$ (when the signal turns yellow) the vehicle can or cannot go through the intersection before the light turns red (which means using the yellow and all red intervals) depending on the running distance D_{RUN}. It is the distance that the vehicle is going to run during the yellow and all red phase at the given speed and it can be calculated for every car. Also, the distance from the car to the end of the intersection (we consider the intersection length L_{INT}) is calculated.

$$D_{RUN} = v_n(t) \cdot (yellow + all\ red) \tag{10a}$$

$$D_{veh-int} = x_{int} + L_{INT} - x_{veh} \tag{10b}$$

$$if\ D_{RUN} \geq D_{veh-int} \Rightarrow The\ vehicle\ can\ go\ through\ the\ intersection \tag{10c}$$

If at its current speed the vehicle can pass the intersection before the all red phase expires, the vehicle will be able to go through. On the contrary, a vehicle with a running distance smaller than the distance needed to cross the intersection won't satisfy the "can go through" condition.

Dilemma zone

Vehicles driving towards a yellow light in an intersection can face the situation where some of them can't either safely stop before the stop line nor proceed through the intersection during the amber interval. This is widely known as the yellow light dilemma zone [18].

On the current research, given the speed limit u, the reaction time t_{RE} and the maximum breaking deceleration b, the dilemma zone can be removed when the condition in (11) is satisfied.

$$Y \geq \frac{L_{INT}}{u} + t_{RE} + \frac{u}{2b} \tag{11}$$

Then, considering a yellow time of 5s + 2s of all red (Y = 7 s) the condition is satisfied so dilemma zone is eliminated and it is no longer a design concern. Therefore, our scenario will have the following features: a *can go* zone, a *stopping* zone and a zone where both things can be safely done are presented before the traffic light.

Clearance Behaviors

Non-aggressive behavior

This clearance behavior stands for a conservative driver that will always decide to stop at the intersection if the stopping condition is satisfied even if it has other options. There are only two possible situations since the dilemma zone is eliminated:

- If the vehicle satisfies both conditions, it will decide to stop.
- If the vehicle does not satisfy the stopping condition it must satisfy the "go through" condition. So, that will do.

The logic implemented in the code to reproduce this non-aggressive behavior is the following. At the time that the light turns yellow, all the vehicles' stopping condition will be calculated (equation 9d). So vehicles are organized from the farther location to the intersection to the closest. Starting from the first vehicle (the farthest) the stopping condition will be tested and if the vehicle can stop it will do that and we move to test the following vehicle (which is closer to the intersection). We will keep doing that until we find a vehicle that doesn't satisfy the stopping condition. Once we find it, this vehicle will go through the intersection. Then, the previous vehicle (the one that is further to the intersection) is the first one that can safely stop at the intersection. This vehicle is the *signal leader* which will follow the *control vehicle* that will be virtually placed after the intersection.

Aggressive behavior

The aggressive clearance behavior reproduces a driver that will always decide to cross the intersection if the go through condition is satisfied. In this case, there are only two different scenarios that the vehicle can experience when the traffic light turns yellow.

- If the vehicle satisfies the condition to go through, it will do that. This will be the decision even if the vehicle can also safely stop at that situation.
- If the vehicle is not able to safely go through, it will satisfy the condition to stop. Then it will stop.

The logic to be implemented in the code for this aggressive decision is in some way different than the explained for the non-aggressive behavior. When the light turns yellow, the go through condition (equation 10c) will be tested in every vehicle, starting from the closest vehicle to the intersection. If the vehicle can safely go through, it will do that and we will move to test the following one (which is farther to the intersection). We will keep doing this until we find a vehicle that does not fulfill the go through condition. This vehicle will be the *signal leader* so the first one to stop at the traffic light due to the *control vehicle*.

Simulation 2 and Clearance Impacts

The ring road parameters are the same as in simulation 1 in section 4.2. The green ratio is π = 0.5 theoretically, but we expect that it will be again reduced by the impact of the clearance lost time induced by the different aggressiveness.

Three different configurations of the simulation will run for all the possible densities. Our reference NFD will be the one obtained in Section 2 which is the highly aggressive form. The other two configurations will be the aggressive and non-aggressive behaviors described before. Aggressiveness is considered homogeneous for every configuration, which means that all vehicles are aggressive or all are non-aggressive. No BA is considered for any of this cases. Since we are only changing the clearance behaviors from the reference simulation, we can focus on the clearance effects on the NFD. The results of these simulations are provided in Figure 6.

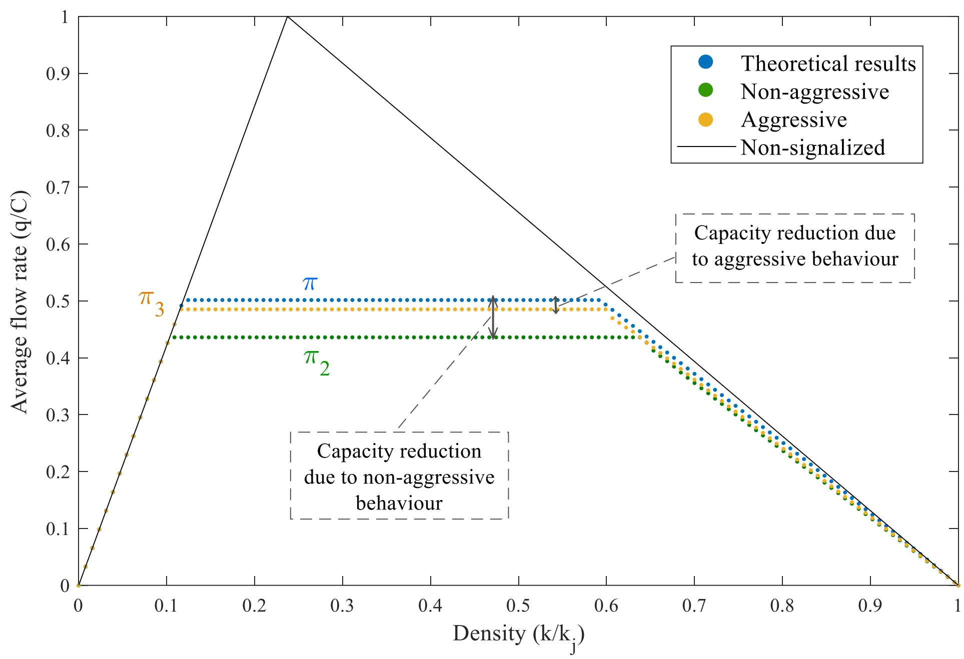


Figure 6. Simulation results of the NFD for different clearance behaviors

The maximum flow rate achieved for the non-aggressive case is 0.4359C which leads to an effective green ratio of $\pi_2 = 0.4359$. This produces flow rate losses of 13.08% with respect to the green ratio π of the reference case. This shows that only 26.07 s of 30 s are effective green time when the non-aggressive behavior is set.

For its part, the aggressive behavior configuration reaches a maximum flow rate of 0.4851C. The effective green ratio is then $\pi_3 = 0.4851$ and the flow rate is reduced by 3.27% with respect to π. Therefore, up to 29.02 s of 30 s are effective green time used in the aggressive clearance configuration.

An interesting insight is obtained comparing both aggressiveness. We can either lose about 4 seconds per cycle or just 1 second with the opposed clearance configurations. This 3 seconds' difference of lost clearance time per cycle may look insignificant but it can produce major impacts to a whole urban network performance. Therefore, the aggressiveness plays an important role in the NFD maximum average flow rate.

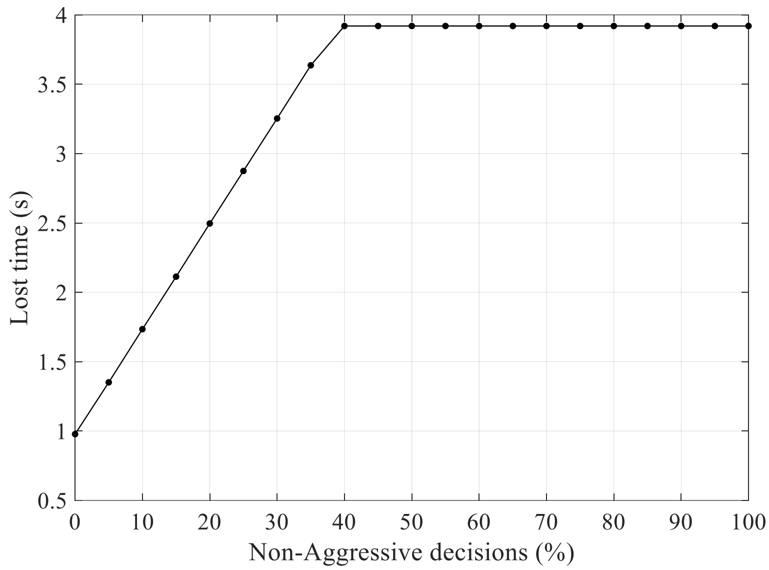


Figure 7. Lost time - Aggressiveness relation

Nevertheless, in reality, there is some percentage of aggressive drivers and some of non-aggressive and they have to coexist in the same space. To reproduce that, we obtain the lost time per cycle for different combinations of aggressive and non-aggressive decisions. Unexpected results are shown in Figure 7: when we reach a percentage of 40% of non-aggressive decisions, lost time reaches a maximum value that remains constant until the 100% of non-aggressiveness. This finding suggests that when the number of non-aggressive vehicles is big enough they might constraint all the others. However, this interpretation highlights the need of further studies to better analyze this phenomenon.

Combination of Start-up and Clearance Behaviors

Start-up and clearance behavior effects have been studied separately and clear results have been obtained. However, reached this point it is especially interesting to see if these behaviors can be added together. We want to see if lost times calculated for a clearance behavior and for a start-up behavior separately and then summed are the same that if we apply both behaviors at the same time. For different BA, the total lost times obtained following these two possible procedures are shown in Figure 8.

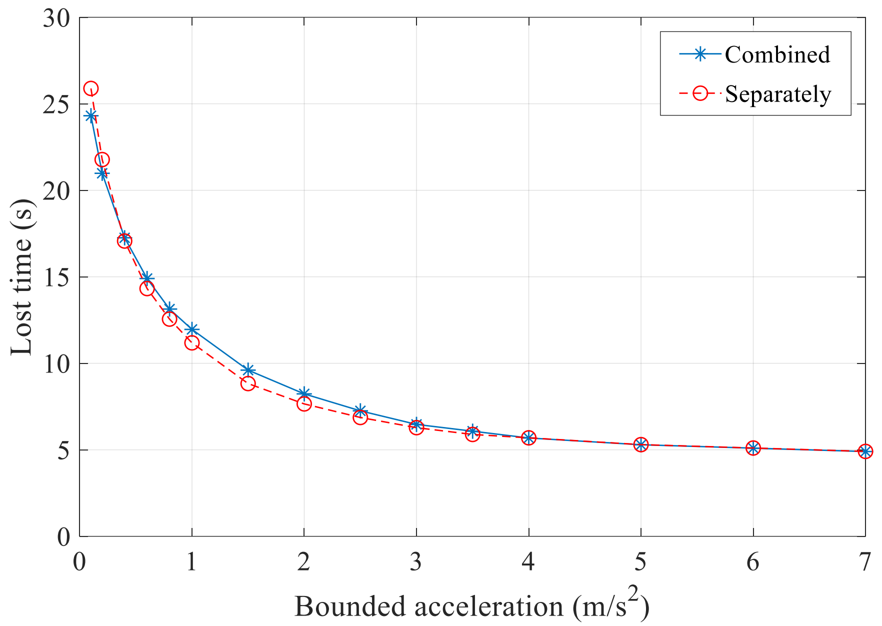


Figure 8. Total lost time comparison between combined and separate simulations

We can see that both lines follow the same trend even if they are not exactly the same, but still very close. There is an interval for the smallest BA where the combined addition curve is below the separate addition curve; however, for the middle points, it is the other way around. And for the biggest BA, both additions are equal. These small differences can be caused by the numerical error of the simulation or by the non-linear relationship effects. But in practice, we can just suggest that the obtained curves are the same. Thus, we should just add start-up and clearance behavior together in the simulation configuration because we will get the same lost time that if we do it separately. Then, it can be said that lost times are additive, which is an important finding.

Conclusions and Future Research

In this paper, microscopic simulations have been used to capture the impacts of start-up and clearance behaviors. The very first simulation results demonstrated that the Newell's car-following model configuration is accurate and reliable for reproducing the NFD for a signalized ring road. The results also showed that the effects of start-up and clearance behaviors are captured on the NFD and that they can be quantified. Therefore, a microscopic approach can be also useful for studying other phenomena. Moreover, it has been proved that the three-pieced trapezoidal NFD defined by the maximum average flow rate πC and the critical densities k_1 and k_2 is a robust formulation and can be used not only for the macroscopic approach.

Regarding the clearance behavior, the BA model has a relevant influence on the NFD, since it limits the maximum average flow rate and causes 3.72 s of start-up lost time with an acceleration bound of 2 m/s^2. This capacity reduction is clearly recognized in the shape of the NFD for several different values of the BA. The lost time per cycle varies from 22 s to less than 1 s for acceleration bounds of 0.1 and 7 m/s^2, respectively. Moreover, the start-up lost time is very sensitive to changes in low acceleration bounds. Since the BA is one of the explanations of the start-up lost time these findings suggest that it should be implemented in any signalized network study. Otherwise, the capacity will be overestimated.

With respect to the clearance behaviors, we have demonstrated that different aggressiveness' configurations to face the decision for the yellow phase impact the shape of the NFD. As it was expected, a conservative performance decreases more the capacity than an aggressive behavior: 3.93 s against 0.98 s of lost time, respectively. Surprisingly, it was found that more of 40% of non-aggressive drivers lead to the same lost time as 100% of them. Hence, clearance behaviors have unexpected and significant impacts on the NFD which just highlights its importance.

Taken together, these results provide an important insight pointing in a direction that some other authors haven't widely explored yet: the NFD is affected not only by the green ratio. In particular, here we demonstrated that the capacity of a signalized ring road can be also restricted due to the start-up and clearance lost times. Moreover, lost times are demonstrated to be additive.

This findings' usefulness is strengthened with the potential applications. As we suggested during the motivation lines, a fundamental understanding of the NFD performance under certain situations is essential to help decision-makers in the implementation of autonomous vehicles. These vehicles need to be pre-configurated or even constantly-configurated to respond to traffic challenges. And the best way to know how to respond to something is when you certainly know how your decisions will affect the NFD and consequently the network performance. Calculated lost times suggest that the major the acceleration bound the better and the more aggressive vehicles the better, but there are other costs and restrictions that can't be missed. So, in the end, there is a trade-off involving many factors but we have already shown one piece of the puzzle.

The results of this study encourage the authors to think about further research that can be done with the new microscopic approach. There is the possibility to explore other phenomena that can appear in a signalized network, like the impact of some green driving strategies or different levels of automatization of vehicles. It can also be useful from the control point of view: the study of optimal cycle lengths, offsets, and phase distribution could be done microscopically incorporating any desired condition to individual vehicles. Several questions still remain to be answered but some of them will be soon addressed in future investigations.

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