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SYNTHESIS OF AN NR ROBOT WITH FOUR-BAR CONSTRAINING MODULES

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ABSTRACT

This paper uses coupler-path synthesis to design four-bar linkage modules that constrains the movement of links in an nR serial chain. The goal is to formulate a general procedure for path-synthesis of a robotic system using four-bar linkages. A desired end-effector trajectory is transformed into a secondary trajectory that is used for nine-point synthesis of a constraining four-bar linkage. This procedure constrains an nR chain to become a 4n-2 bar linkage. An example presents the constraint of a 2R chain to a six-bar that has a prescribed trajectory for an end-effector point.

INTRODUCTION

This paper presents a method for the synthesis of constraining four-bar linkage modules in order constrain the trajectory of a point in the end-effector of an nR serial chain. This is a procedure that constrains an nR chain to be a 4n-2 bar linkage.

The goal of the research is to obtain mechanical modules that support repetitive motion for robotic rehabilitation devices and exoskeletons. Recent designs for devices for gait rehabilitation include of a primary serial chain that parallels the human limb, which is combined with actuators and support structure to facilitate movement in treadmill training for gait rehabilitation for stroke or spinal cord injury subjects [1], [2], [3], and [4].

In this work the designer specifies the movement of the primary serial chain and designs the supporting four-bar linkage for nine task points of an end-effector trajectory using the homotopy solver, Bertini, [8].

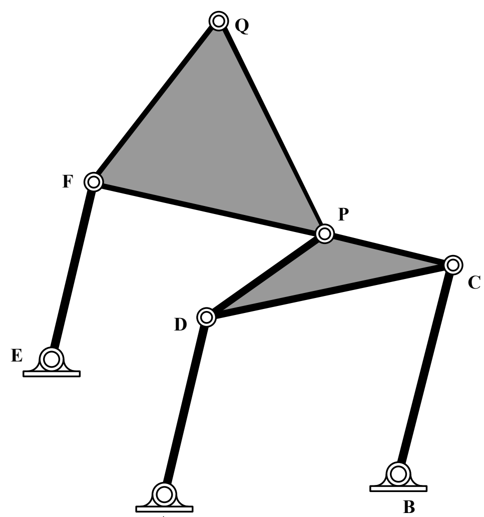

FIGURE 1. STEPHENSON III SIX-BAR LINKAGE.

LITERATURE REVIEW

The nine-position synthesis of a four-bar linkage was first completely solved by Wampler, Morgan, and Sommese [5]. See Huang et al. (2008) [10] for recent work on path synthesis for four-bar linkages. The use of four-bar linkage modules to

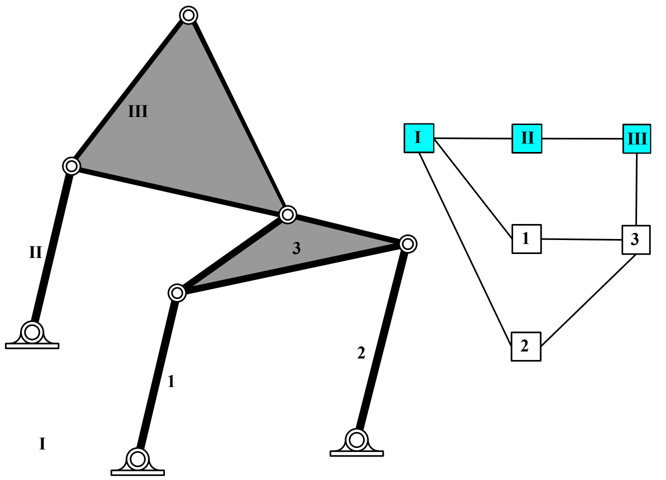

FIGURE 2. Graph of a 2R chain constrained by one four-bar linkage module.

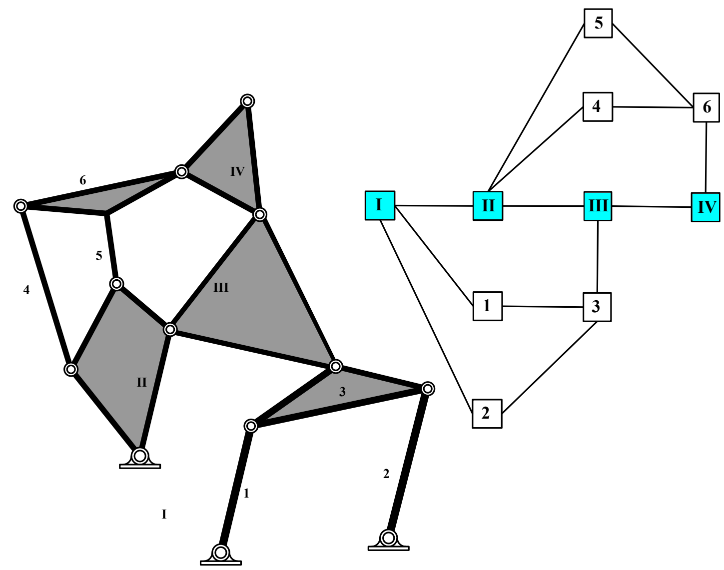

FIGURE 3. Graph of a 3R chain constrained by two four-bar linkage modules.

constrain an nR serial chain extends the work by Soh and McCarthy [6, 7], who use RR links to constrain nR serial chains.

Our goal is to extend the work of Nesbit and Elzinga (2007) [9], who applied path synthesis for four-bar linkage to generate cyclic human movement, to more complex linkage systems. Our approach yields six-bar and 10-bar linkages designed to guide leg movement. In the case of the six-bar linkage, specialized version of the approach to the 15-loin path synthesis presented by Kim et al. (1971) [11].

THE CONSTRAINED NR CHAIN

A serial nR chain consists of $n+1$ links including the ground link, which are connected in sequence by n revolute joints. For convenience denote the primary chain using roman numerals, such that the ground link is I, and the first moving link is II, and so on. Introduce a four-bar linkage module that constrains the trajectory of a point in link III relative to link I of a 3R chain, Fig. 2, to obtain a six-bar linkage.

A 3R serial chain can be constrained by introducing one four-bar module to constrain the a trajectory in link III relative to link I, and a second four-bar module to constrain a trajectory in link IV relative to II. The result is a 10-bar chain, Fig.3. This process can be continued to obtain increasingly 14-bar and $4n+2$-bar linkages.

In what follows, we focus on constraining a 2R chain by designing a four-bar linkage that guides a point-path in the end-effector. The result is a Stephenson III six-bar linkage.

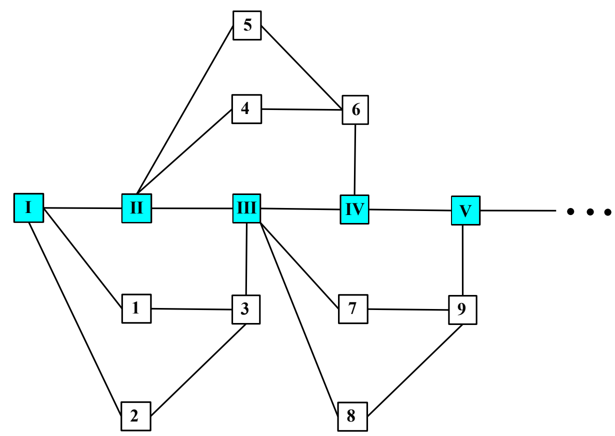

FIGURE 4. Graph of n nR chain (I II III IV V ...) constrained by four-bar linkage modules.

THE PRESCRIBED END-EFFECTOR TRAJECTORY

The first step to designing this Stephenson six-bar linkage, as shown in Figure (1), is to specify the nine precision points, the fixed pivot at point **E**, and the link lengths EF and FQ. Figure (5) illustrates the values that are to be defined by the user. Inverse kinematics can then be used to determine the location of the moving pivot **F**, the angle that link EF makes with the x-axis θ_1, and the angle that link FQ makes with the line collinear to link EF, θ_2, in all nine positions along the trajectory [12]. The angle ψ is the angle between link EF and the imaginary line from point **E**

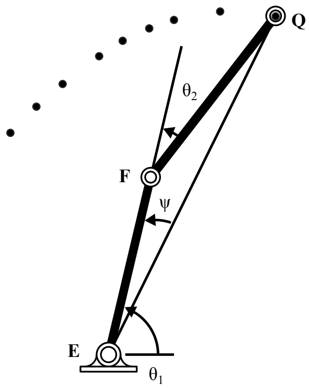

FIGURE 5. RR SERIAL CHAIN WITH 9 PRECISION POINTS

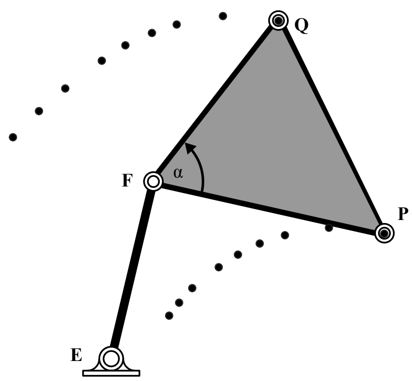

FIGURE 6. RR SERIAL CHAIN WITH SECONDARY TRAJECTORY

to point **Q**.The equation for θ_2 is given by

$$\theta_2 = \pm \arccos\left[\frac{Q_x^2 + Q_y^2 - EF^2 - FQ^2}{2(EF)(FQ)}\right] \quad (1)$$

where Q_x and Q_y or the x and y components of point **Q** respectively. The value of ψ is calculated with the equation

$$\psi = \arctan\left[\frac{FQ\sin(\theta_2)}{EF + FQ\cos(\theta_2)}\right]. \quad (2)$$

From this value of ψ, the value of θ_1 can be determined:

$$\theta_1 = \arctan\left[\frac{FQ}{EF}\right] - \psi \quad (3)$$

The designer also chooses the link lengths of FP and QP, which locates the point **P** in all nine positions. This creates a secondary trajectory **P** that is used for the path-synthesis of the constraining four-bar linkage, Figure (6). The parameter α which is given by the law of cosines as,

$$\alpha = \pm \arccos\left[\frac{-(QP^2 - FQ^2 - FP^2)}{2(FQ)(FP)}\right]. \quad (4)$$

With all of the joint angles known, the locations of point **F** and point **P** can be determined. Completing the process for all nine positions will yield the nine positions needed for point **P**.

FOUR-BAR SYNTHESIS FOR NINE POINTS

The problem formulation used in this paper, for the four-bar linkage, is same that was formulated by Wampler, Morgan, and Sommese [5]. The formulation begins by setting up the four-bar linkage as a collection of vectors as shown in Figure (7). Points **A** and **B** are the fixed pivots, and points **C** and **D** are the moving pivots. Point $\mathbf{P_0}$ is the location of the first precision point. It can be seen that the summation of vectors around the left and right loops, which yield the equations,

$$u = x - a, \quad v = y - b, \quad (5)$$

where u is a complex vector $u = u_1 + iu_2$, and similarly for the complex vectors v, x, y, a and b. Figure (8) shows the four-bar linkage after it has been displaced to the new precision point $\mathbf{P_j}$ where $j = 1,...,8$. δ_j is the vector from the first precision point $\mathbf{P_0}$ to the next precision point $\mathbf{P_j}$. Figure (8) also shows that the angular displacements of link AD, BC, and the coupler are represented by λ_j, μ_j, and θ_j. After the linkage is displaced to the new precision point, the new loop equations of the linkage are

$$\begin{aligned} ue^{i\lambda_j} &= xe^{i\theta_j} + \delta_j - a, \\ ve^{i\mu_j} &= ye^{i\theta_j} + \delta_j - b, \quad j = 1,\ldots,8. \end{aligned} \quad (6)$$

Substitute this into of equations (6) to obtain,

$$\begin{aligned} (x-a)e^{i\lambda_j} &= xe^{i\theta_j} + \delta_j - a, \\ (y-b)e^{i\mu_j} &= ye^{i\theta_j} + \delta_j - b, \quad j = 1,\ldots,8, \end{aligned} \quad (7)$$

which are the vector loop equations for the four-bar linkage. Introduce the variable γ_j given by,

$$\gamma_j = e^{i\theta_j} - 1, \quad j = 1,\ldots,8, \quad (8)$$

and multiply the loop equations (7) and their respective complex conjugates—note the complex conjugate is denoted by $\hat{}$. The result is

$$(\hat{a}-\hat{\delta}_j)x\gamma_j+(a-\delta_j)\hat{x}\hat{\gamma}_j+\delta_j(\hat{a}-\hat{x})+\hat{\delta}_j(a-x)-\delta_j\hat{\delta}_j=0,$$
$$(\hat{b}-\hat{\delta}_j)y\gamma_j+(b-\delta_j)\hat{y}\hat{\gamma}_j+\delta_j(\hat{b}-\hat{y})+\hat{\delta}_j(b-y)-\delta_j\hat{\delta}_j=0,$$
$$j=1,\ldots,8. \quad (9)$$

The variables γ_j and $\hat{\gamma}_j$ have the property

$$\gamma_j\hat{\gamma}_j+\gamma_j+\hat{\gamma}_j=0. \quad (10)$$

Continuing to follow Wampler, we introduce the variables,

$$n=a\hat{x}, \quad \hat{n}=\hat{a}x, \quad m=b\hat{y}, \quad \hat{m}=\hat{b}y. \quad (11)$$

Introduce the homogeneous coordinates, x^0, y^0, and γ_j^0, so the equations (11) become,

$$nx^0=a\hat{x}, \quad \hat{n}x^0=\hat{a}x, \quad my^0=b\hat{y}, \quad \hat{m}y^0=\hat{b}y. \quad (12)$$

Now substitute (12) into (9) to obtain,

$$(\hat{n}-\hat{\delta}_jx)\gamma_j+(n-\delta_j\hat{x})\hat{\gamma}_j+\left(\delta_j(\hat{a}-\hat{x})+\hat{\delta}_j(a-x)-\delta_j\hat{\delta}_jx^0\right)\gamma_j^0=0,$$
$$(\hat{m}-\hat{\delta}_jy)\gamma_j+(m-\delta_j\hat{y})\hat{\gamma}_j+\left(\delta_j(\hat{b}-\hat{y})+\hat{\delta}_j(b-y)-\delta_j\hat{\delta}_jy^0\right)\gamma_j^0=0,$$
$$\gamma\hat{\gamma}_j+\gamma_j\gamma_j^0+\hat{\gamma}_j\gamma_j^0=0$$
$$j=1,\ldots,8. \quad (13)$$

which we use to design the four-bar linkage. Wampler shows that these equations can be simplified by eliminating $\gamma_j, \hat{\gamma}_j$, and γ_j^0 using Cramer's rule to compute,

$$\gamma_j=\begin{vmatrix} n-\delta_j\hat{x} & \delta_j(\hat{a}-\hat{x})+\hat{\delta}_j(a-x)-\delta_j\hat{\delta}_jx^0 \\ m-\delta_j\hat{y} & \delta_j(\hat{b}-\hat{y})+\hat{\delta}_j(b-y)-\delta_j\hat{\delta}_jy^0 \end{vmatrix},$$
$$\hat{\gamma}_j=\begin{vmatrix} \delta_j(\hat{a}-\hat{x})+\hat{\delta}_j(a-x)-\delta_j\hat{\delta}_jx^0 & \hat{n}-\hat{\delta}_jx \\ \delta_j(\hat{b}-\hat{y})+\hat{\delta}_j(b-y)-\delta_j\hat{\delta}_jy^0 & \hat{m}-\hat{\delta}_jy \end{vmatrix},$$
$$\gamma_j^0=\begin{vmatrix} \hat{n}-\hat{\delta}_jx & n-\delta_j\hat{x} \\ \hat{m}-\hat{\delta}_jy & m-\delta_j\hat{y} \end{vmatrix}$$
$$j=1,\ldots,8. \quad (14)$$

Equation (12) and the equations resulting from substituting equations (14) into equation $\gamma\hat{\gamma}_j+\gamma_j\gamma_j^0+\gamma_j^0\gamma_j^0=0$ result in a system of 12 equations with 12 unknown variables. The calculation of these 12 variables are the parameters needed to construct a four bar linkage that guides point **P** from figure (1) through the desired nine precision points.

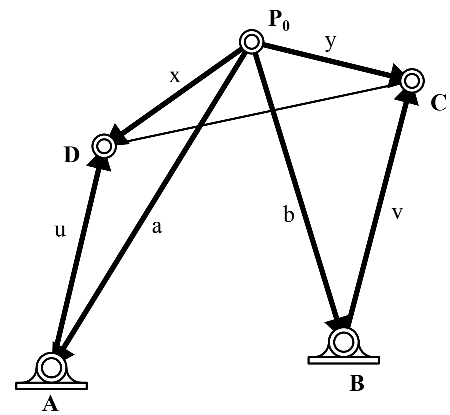

FIGURE 7. FOUR-BAR LINKAGE REPRESENTED AS VECTORS

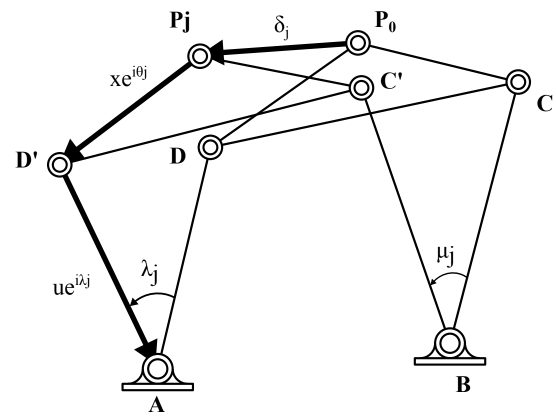

FIGURE 8. FOUR-BAR LINKAGE DISPLACED TO A NEW PRECISION POINT

NUMERICAL EXAMPLE

The four-bar linkage synthesis problem was solved using, the homotopy solver, Bertini [8]; the computations were carried out on the high performance computing, (HPC), cluster at UC Irvine. The calculations were completed using parallel processing with one of the nodes that had 64 cores.

For the numerical example, the base of the RR chain was set at $(0,0)$ and the link lengths of EF and FQ were both specified to a value of 3.00. The user defined precision points are defined in Table 1.The resulting RR chain with corresponding precision points is shown in Figure (9). Links FP and FQ were both set to be of length 2.00.

The angle between links FQ and FP can be calculated using the law of cosines. Then, inverse kinematics can once again be used to determine the secondary trajectory, which are the locations of point **P** in all nine positions. The resulting precision points of the secondary trajectory are listed in Table 2 and are illustrated in Figure (9). These new, nine precision points were then implemented into the four-bar synthesis algorithm mentioned above.

On the 64 core node, on UC Irvine's HPC, the computation took a total of 51 minutes and 21 seconds. From these particular precision points, there were at total of 8652 solutions. Of these 8652 solutions, there were 240 real solutions. Solutions were eliminated if the perimeter was greater than 8. Also, solutions were eliminated if the Grashof condition was not met, in order to verify that the linkage was cyclic. After undesirable solutions were deleted, 88 linkages remained. One of these solutions is illustrated in Figure (10).

CONCLUSION

In this paper, we present a design procedure for a Stephenson six-bar linkage that reaches nine user defined precision points. The user also specifies the parameters of an RR chain that can reach these points. A secondary set of precision points is derived from the end effector of this chain. The new precision points can then be driven by a four-bar linkage. This four-bar linkage is synthesized using the method presented by Wampler, Morgan, and Sommese [5]. This design process was expedited with the use of the homotopy solver, Bertini [8], and the high performance computing cluster at the University of California, Irvine. A numerical example yielded a total of 240 real solutions; these results were achieved in the relatively short amount of time. Lastly, this method has potential for allowing linkage designers to constrain nR serial chains with additional four-bar linkages. Additional work is needed to ensure that resulting linkage is cyclic, in order to accommodate rehabilitation mechanisms. Randomizing the user defined location of point P within a certain zone can be done a set number of times in order to increase the number of potential solutions.

TABLE 1. Nine User Defined Precision Points

Point	x	y
1	3.24308	4.87983
2	3.65837	4.64454
3	4.20336	4.18766
4	4.70158	3.43835
5	4.66366	3.56091
6	4.33759	3.57467
7	3.54971	4.05579
8	3.13476	4.01075
9	2.68975	4.33752

TABLE 2. NINE POINTS OF THE SECOND TRAJECTORY

Point	x	y
1	3.08147	2.88637
2	3.42765	2.65789
3	3.77918	2.23316
4	3.81556	1.64531
5	3.87315	1.72377
6	3.35037	1.8353
7	2.67774	2.5588
8	2.17855	2.25415
9	1.93687	2.48464

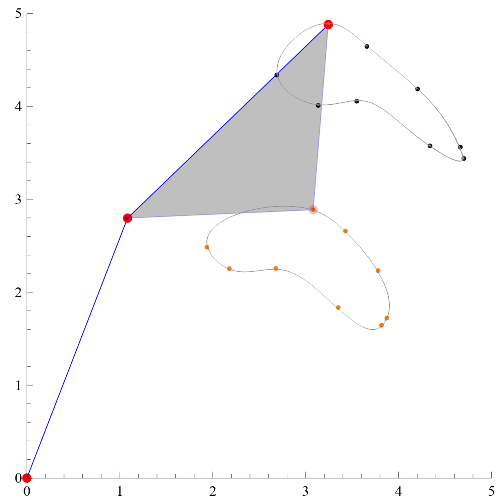

FIGURE 9. EXAMPLE OF AN RR CHAIN WITH SECONDARY TRAJECTORY

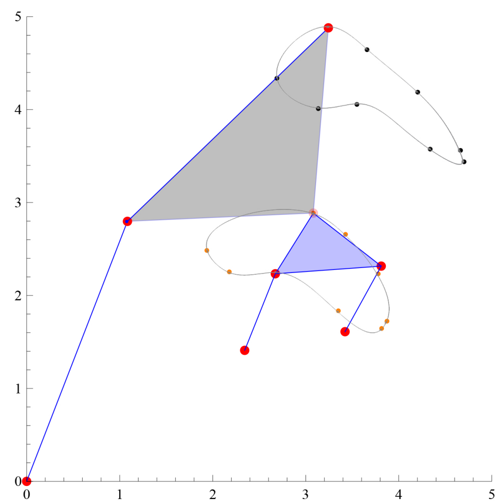

FIGURE 10. ONE EXAMPLE OF 240 SOLUTIONS

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