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Fair rate assignment in interference limited multi-hop networks

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Fair Rate Assignment in Interference Limited Multi-hop Networks

A dissertation submitted in partial satisfaction of the
requirements for the degree
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in

Electrical Engineering
(Communication Theory and Systems)

by

Mustafa Arisoylu

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The dissertation of Mustafa Arisoylu is approved, and it is acceptable in quality and form for publication on microfilm:

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PUBLICATIONS

Mustafa Arisoylu, Tara Javidi and Rene L. Cruz. *End-to-End and Mac Layer Fair Rate Assignment in Interference Limited Wireless Access Networks.*, IEEE International Conference on Communications (ICC 2006), Istanbul, June 11, 2006 (to appear).

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ABSTRACT OF THE DISSERTATION

Fair Rate Assignment in Interference Limited Multi-hop Networks

by

Mustafa Arisoylu
Doctor of Philosophy in Electrical Engineering
(Communication Theory and Systems)
University of California San Diego, 2006
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In this study, we consider two types of interference limited multi-hop networks. The first type is a micro-buffered multi-hop networks, whereas the second type is a CDMA based multi-hop wireless access network.

Micro-buffered networks are high speed packet switched networks, and consist of either nodes with no buffering or nodes with limited buffering such that there is no specific collision resolution mechanism inside the core. Packet losses inside the core are dealt with using end-to-end or edge-to-edge coding techniques. In this study, we discuss the rate allocation problem for a micro-buffered high speed network.

First the classical slotted aloha type protocols with exponential backoff are examined and it is observed that such mechanisms may result in unfair rate allocations.

Next, we consider weighted α-proportional , weighted max-min and hierarchical max-min fair rate assignments. Simple distributed algorithms achieving these notions of fairness by exchanging local information are discussed. It is found that weighted max-min fair information rate allocation assigns information rates to each

flow in the same group inversely proportional to their weights. Furthermore, we also show that hierarchical max-min fairness can be achieved in a micro-buffered network if and only if weighted max-min fairness among the flows is ensured.

Secondly, the problem of end-to-end weighted max-min fair rate assignment in a two-channel multi-hop CDMA wireless access network is discussed. We show that end-to-end weighted global max-min fairness (hierarchical as well as flow-based) can be achieved by a simple extension of mac-layer fairness. In particular, we show that weighted end-to-end flow-based as well as hierarchical global max-min fairness can be simply insured if and only if weighted mac-layer max-min and weighted transport-layer max-min fair rates are achieved. The same results can easily be shown to be valid for more general wireless networks, which will be briefly discussed in this study as well.

In addition, we discuss a mac-layer algorithm, $MAC - \alpha - G$ algorithm, that, with careful choice of parameters, not only provides weighted α-proportional fairness at the mac layer, but also leads to end-to-end weighted global max-min fairness (both flow-based and hierarchical) with an appropriate higher-layer protocol (i.e. weighted transport-layer max-min fair protocol).

1

Background on Rate Assignment

In this chapter, the basics on the problem of rate assignment are discussed. In particular, in a multi-hop network, max-min and weighted α-proportional fair rate assignments are defined and examined.

A network (V, L) is assumed to consist of sets of nodes, V, and directed links, L. A link $l \in L$ is an ordered pair of nodes (i.e. $l = (t, r)$) and is said to be directed from the node t (transmitter node) to node r (receiver node). In other words, a link $l = (t, r)$ is a direct logical connection from node t to node r. Each link l is assumed to have capacity C_l which defines the information rate it can carry directly from the transmitter node to the receiver node.

A flow f, on the other hand, can also be considered as an ordered pair of nodes such that $f = (s, d)$ where s is called the source node and d is called the destination node. Unlike a link, a flow can traverse multiple links and nodes from the source node to the destination node. The rate of a flow $f = (s, d)$, R_f, is the information rate received by the destination node d which is originated from node s. A path setup algorithm (e.g. routing or bridging) assigns each flow a path beginning from the source node to the destination node.

Let T_l be the aggregate flow rate on link l such that

$$T_l = \sum_{\text{all sessions f traversing link l}} R_f \tag{1.1}$$

and let C_l denote the capacity of link l.

A general constraint on the flow rates is that

$$C_l \geq T_l \quad \forall \quad l \in L \qquad\qquad R_f \geq 0 \tag{1.2}$$

This set of equations also define the feasibility region of the flow rates, that is any flow rate vector (a vector whose elements are the rate of the flows in the network) that satisfies the above equations is called a feasible rate vector.

In general, the problem is to assign rates to certain entities in the network such as flows or links such that a well defined notion of fair policy is ensured. In this chapter, as an introduction to the fair rate allocation concept, we mainly consider networks where the link capacities are independent of each other and fixed and the information rate of the flows are conserved at each link as the flow rate vector satisfies equation (1.2). A good example for such networks is classical wired networks with sufficient buffering at the nodes. However, as we will examine in the following chapters, one or both of these assumptions may not hold for interference limited multi-hop networks. For instance, in chapter 2 even though the link rates are assumed to be fixed, the information rates of each flow are not conserved at each link, in fact the information rate may multiplicatively decrease with the number of links traversed. In addition to that, in chapter 3, link rates are not independent of each other in interference limited wireless networks, and they are related through a capacity formula.

The details for such interference limited networks and the related capacity formulations are discussed in the related chapters.

In the following sections, classical definitions of the Max-Min and Alpha-Proportional fair rate assignments are defined and some examples are presented.

1.1 Max-Min Fair Rate Assignment

First, let us give definitions for weighted max-min fairness.

Definition 1: A vector of rates R is **weighted max-min fair** with weight vector, W, if it is feasible and for each flow i, rate of the flow (e.g. flow or link) i, R_i, can not be increased while maintaining feasibility without decreasing R_j for some flow j for which $R_j W_j \leq R_i W_i$. As a special case, when $W_i = 1\ \forall i$, the vector R is said to be max-min fair [2].

Definition 2: Given a feasible rate vector R, link l is said to be a weighted bottleneck link with respect to R for a flow f traversing link l if $T_l = C_l$ and $W_f R_f \geq W_p R_p$ for all sessions p traversing link l.

In [2], a simple centralized algorithm is given to compute the max-min fair rates in a wired network. On the other hand, there are several studies in the literature designing distributed algorithms to compute the max-min fair rate allocation [7, 10, 11]. In all these studies, the link rates are assumed to be independent of each other and fixed.

The weighted max-min fair rate assignment that we have discussed above ensures the fair rate assignment on individual flows. However, in real life various other types of fair rate assignments may be necessary. For instance, in a typical residential area community network application, each resident accesses the Internet over a multi-hop access network. In such scenarios each resident would like to utilize a fair share of the available network bandwidth regardless of the number of flows they utilize. In that sense a more general fairness policy can be necessary such that first the fair rate allocation is ensured among the residents (that is among some subset of flows) and then for each resident (that is within the same subset of flows) a certain fairness can be ensured among the flows belonging to that resident.

Considering such scenarios, as in [21], we define a hierarchical mac-min fair rate allocation as follows.

Definition 6: Let M_a be the set of flows belonging to subgroup a such that $\bigcup_a M_a = F$ and $M_a \bigcap M_b = \emptyset\ \forall a, b : a \neq b$. Let D be the rate vector where the ath element denotes the aggregate information rate of subgroup a : $D_a = \sum_{i \in M_a} R_i$. Lastly, let H_a denote the vector of rates of the individual flows belonging to M_a.

A vector of flow rates, R, is said to be **weighted hierarchical max-min fair** with a weight vector Z, and a vector of vectors $V = (V_1, V_2, ...V_T)$, where T is the number of subgroups, and V_a is the weight vector for subgroup a such that $V_a = (V_{a,1}, V_{a,2}...V_{a,L_a})$ (where L_a is the number of flows in subgroup a), if first the vector, D, is weighted max-min fair with weight vector Z and the rate vector for flows in each subgroup a, H_a, is weighted max-min fair with weight vector V_a.

More formally, a vector, R, is said to be **weighted hierarchical max-min fair** with a weight vector Z, and a vector of vectors $V = (V_1, V_2, ...V_T)$, if it is feasible and if for each flow, $i \in M_a$, the rate, R_i, can not be increased, while maintaining feasibility without decreasing R_j for some flow $j \in M_b$ for which $V_{b,j}R_j \leq V_{a,i}R_i$ when $a = b$ or $Z_bD_b \leq Z_aD_a$ when $a \neq b$. When all the weights are all equal to 1, the rate vector R is called Hierarchical Max-Min fair as similarly defined in [21].

1.2 Utility Based Fair Rate Assignment

In this section, we examine a fair rate allocation policy where each flow has a utility function of its rate. The policy in question maximizes the sum of all the individual utility functions in the system.

Let R be the rate vector such that R_i denotes the rate of flow i and $U(x)$ be the utility function.

The utility based fair rate assignment policy (maximizing the sum of individual utility functions) targets to solve the following optimization problem:

$$\textbf{Maximize:} \quad \sum_i U(R_i) \quad \text{s.t.} \quad \text{the feasibility constraint on flows} \tag{1.3}$$

Based on the utility function various fairness policies can be ensured. In the next section, we discuss weighted alpha proportional fair rate assignment where by appropriate choice of the parameters of the utility function, different fairness policies can be ensured.

1.2.1 Weighted Alpha-Proportional Fairness

The optimization problem for weighted Alpha-Proportional fairness is

$$\textbf{Maximize} \quad \sum_i U(R_i) = \sum_i -\frac{1}{g_i}(-ln(g_i R_i))^{\alpha} \tag{1.4}$$

$$\textbf{Subject To:} \quad \text{Feasibility Constraints}$$

where g_i is a weight associated to the flow (e.g. flow, link) (i).

If all the weights (i.e g_is) are equal to 1 then the fairness policy is called Alpha-Proportional fairness, in addition if $\alpha = 1$, then as defined in [8] the policy is called Proportional fairness. As can easily be seen, for proportional fairness the utility function for each flow is just a logarithmic function.

In the following chapters, we examine two types of multi-hop interference limited networks. The first type is a collision based multi-hop micro-buffered high speed network, while the second type is a CDMA based multi-hop wireless access network.

2

Rate Assignment in Micro-Buffered High-Speed Networks

As the transmission speed in the core network increase substantially, the cost of transmitting a bit in the core is effectively decreasing. However, the cost of buffering and processing a bit inside the core does not follow the same trend, due to the electronic bottleneck in buffering and processing.

In the literature, there are various studies on buffer sizing for the routers in the core networks. For instance, in [36]-[39], the buffer size is proposed to be set to a small value (e.g. 10 to 20 packets), at the cost of a small amount of bandwidth utilization. Authors claim that this decrease in link utilization is worth considering for future all-optical routers due to the very high transmission rates in optical lines.

The speed of current buffers implemented in the electronic domain are not able to match the transmission bandwidth offered by optical transmission. Moreover, it is not clear that commercial optical buffers will be available in the foreseeable future.

The end-to-end argument for communication networks is one of the main design criterions in the current and possibly future network architectures [12]. The main idea in this argument is that the functions in the network should be performed at the possible highest layer, that is , any task that can be performed in an end-to-end (or edge-to-edge) manner should be performed at the end hosts (or edges). In today's networks, due to the growing asymmetry in the speed of transmission and the processing in the core, electronic buffering at the network layer makes it difficult to utilize the transmission capacity.

In this study, we study micro-buffered networks [1], where instead of resolving packet contention by using buffering at intermediate nodes, forward error correction is used at the end nodes. In particular, we study the problem of rate assignment in micro-buffered networks.

The problem of rate assignment has been studied in many contexts so far. For classical packet switching networks with buffers, the max-min fair rate assignment policy is defined in [2] as a fair rate allocation scheme. A well known centralized algorithm to compute the max-min fair rate allocation is given in [2]. Many distributed algorithms have been proposed to achieve the max-min fair rate assignment [7, 10, 11]. In each of these studies, the nodes in the network are assumed to have enough buffer space to avoid packet drops as long as the total arrival rate is less than or equal to the link rate. The notion of proportional fairness is introduced in [8] where each user has a logarithmic utility as a function of its rate. This fairness policy maximizes the sum of individual utility functions.

In this study, we focus on the problem of rate assignment in a micro-buffered network. We first discuss classical slotted aloha protocols with exponential back-off. It is observed that some flows may be assigned zero rates due to the asymmetry in collision sets. Next, we consider several fairness criterions, such as weighted α-proportional fairness, weighted max-min fairness, and hierarchical max-min fairness. In our micro-buffered network, a max-min fair information rate allocation assigns any two users in the same contention group the same information rate, although these two flows may not contend with each other. A contention group corresponds to a connected component in the contention graph between flows. More generally, in the weighted max-min fair case, each flow in the same group is assigned a rate that is inversely proportional to the related weight. Furthermore, we show that a hierarchical max-min fair rate information rate assignment is achieved if and only if weighted max-min fairness with a certain weight vector among the flows is ensured. Related distributed algorithms using only local information are proposed to achieve the various notions of fairness we study. For instance, an MIMD-$\alpha - G$ algorithm is proposed to ensure not only weighted α-proportional fairness, but also to ensure weighted max-min and hierarchical max-min fair rate assignments with the appropriate choice of parameters.

Simulation results of the distributed algorithms on example networks are demonstrated. The results indicate that max-min fair information rate assignment can be significantly under-utilize the network resources. On the other hand, while slotted aloha utilizes the network resources efficiently, it has a potential to exhibit unfair rate allocation. Unlike these two schemes, the proportional and α- proportional fair rate assignment policies exhibit better performance by achieving higher network throughput than max-min fair case, according to a well-defined fairness criterion. Moreover, a hierarchical/weighted max-min fair rate allocation is also shown to be achieved by the related algorithms.

The rest of the chapter is organized as follows. In Section 2, sample optical infrastructures for micro-buffered networks are discussed. Various rate assignment policies are examined in Section 3. Some discussions and example networks are presented in Section 4, and finally Section 5 concludes the chapter.

2.1 Network Models

In this section, we discuss physical and analytical models for micro-buffered optical networks. We describe first the underlying optical network architecture and then give some examples that have already been proposed in previous studies.

In this study, we consider the case of bufferless nodes in the core such that if two packets arrive at a node for the same output link simultaneously, the packets will be simply added to each other without further processing. The potential collisions are allowed and no action is taken to avoid collisions at the node. However the flow control mechanism can adjust the overall transmission rate (both coding rate and transmission rate) with respect to collision measures in the network. As can be seen in the Figure 2.1, packets A and B destined to the same outgoing link are just added to each other without any collision avoidance mechanism.

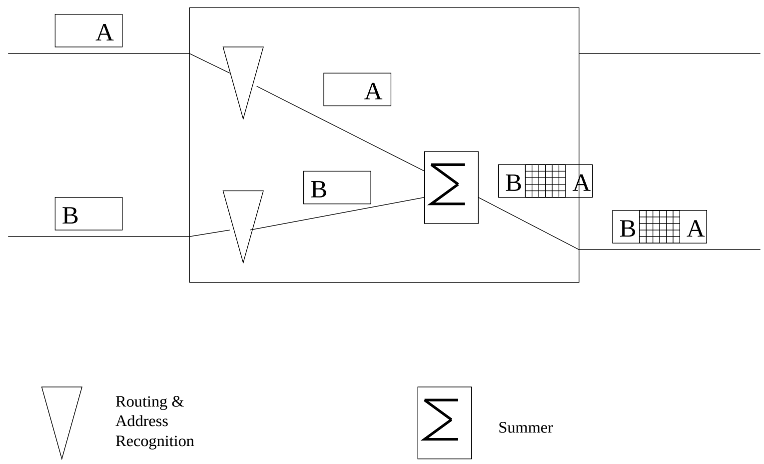

Figure 2.1 General structure of a switch in micro-buffered high speed optical networks

In this section, we discuss three sample network models that can be considered in the context of micro-buffered networks. The first model is simply Wavelength Division Multiplexing (WDM) based, the second model is Optical Burst Switching (OBS) based and in the third model a packet synchronized all optical network model will be described.

2.1.1 Model 1:

In this section, we consider WDM based wavelength routed networks. In WDM networks, bandwidth is allocated by one wavelength at a time leading to a coarse granularity. No two flows passing through the same link can be assigned the same wavelength. However when we approach the same architecture in the context of micro-buffered WDM networks, for any destination, multiple flows sharing the same links can be assigned the same wavelength.As can be seen in the Figure 2.2, all three flows may use the same wavelength. There is no change in the network elements and all the switching and transmission equipment is the same as in classical WDM networks. Any packet simultaneously entering the same outgoing port are simply added and the collisions are compensated with appropriate end to end coding techniques.

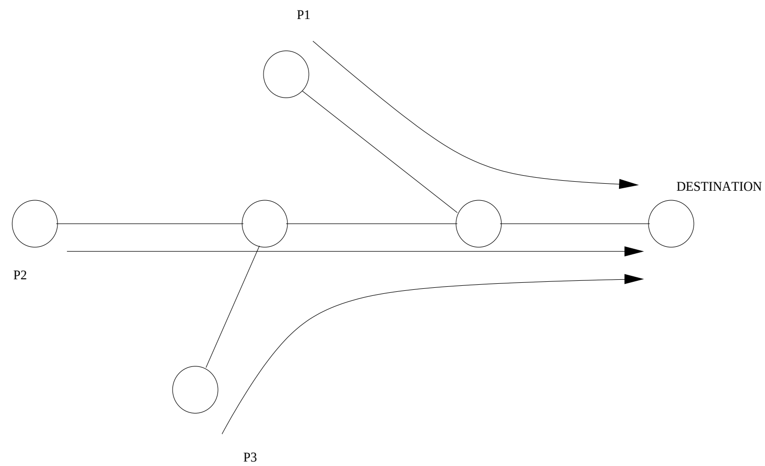

Figure 2.2: A sample topology for wdm-based micro-buffered networks

2.1.2 Model 2:

This model is based on Optical Burst Switching (OBS). In an OBS network, the data and the control signals are separated within the core network. A control packet precedes a data burst and is processed electronically to configure the optical switches on the path. There is an offset time between the control packet and the data burst which is long enough to let the control packet configure the switches along the path before the data burst arrives. The data burst is then switched in an all optical manner without any electronic processing. For this architecture, many burst reservation (JET) and scheduling schemes (LAUC, LAUC-VF)and contention resolution algorithms (Wavelength conversion, deflection routing...etc) have been proposed.

In the context of micro-buffered networks, these mechanisms (scheduling, resolution..etc.) within the core network are avoided. In this technique, the control packets configure the switches without any scheduling and contention resolution. The colliding bursts (or packets) are just added in the switch.

The switch in question just changes its state with respect to the configuration data obtained from the control packets for each burst. A simple two hop example can be seen in Figure 2.3 (a). As can be seen in the figure, burst A and burst B arrive at switch i (i=1,2) at times $t(i)_1$ and $t(i)_2$ respectively for which the switches were configured by the data packets beforehand. And assume that burst

A will leave the switch at time $t(i)_3$ where burst B leaves at time $t(i)_4$ where $t(i)_1 < t(i)_2 < t(i)_3 < t(i)_4$.

Figure 2.3 (b) and (c) indicates the states of switches throughout the time. With such a technique, each packet traverses through its own path even though the information on the packet may be lost due to a collision.

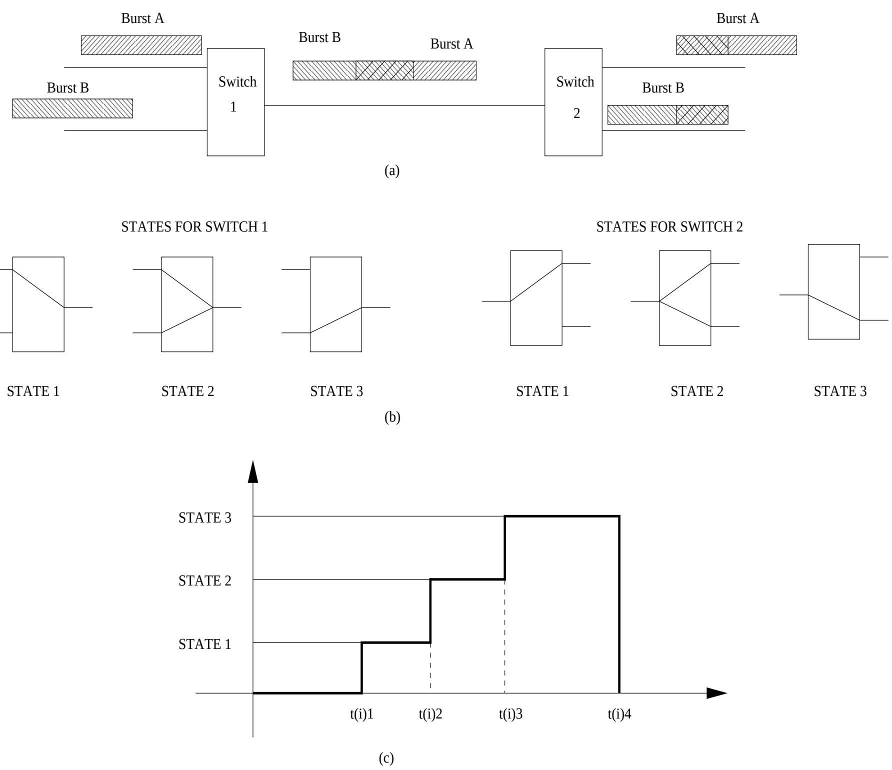

Figure 2.3: A sample topology for obs-based micro-buffered networks

In this model, we assume a routing protocol that ensures that any two flows throughout their entire path can contend not more than once. For instance, shortest path routing ensures this condition.

2.1.3 Model 3:

This model is based on an all optical self routing scheme which requires packet level synchronization at each switch. In this section, we first describe a sample routing scheme and then an example compatible optical switch.

A Sample All Optical Routing and Addressing Scheme:

One of the challenging functions to be designed is the routing in all optical domain. We are interested in a self routing scheme, which is also capable of multicast and multi-path routing.

In [15], authors discuss a novel self routing and addressing scheme for all optical packet switched networks with arbitrary topologies. This addressing scheme leads to a source routing algorithm where the header of a packet includes the information on the end to end path (link by link) that the packet is supposed to traverse. A single bit in the address field indicates whether the packet traverses a single link in the network or not.

Assuming a network with N nodes and L links, each address field of a packet is composed of $2L$ bits such that each output link (port) of a node in the network has a one to one correspondence with a bit in the address field. The address field of a packet has N sub-fields (a sub-field for each node), such that in the ith subfield, there are $n(i) - 1$ bits where $n(i) - 1$ is the number of outgoing links from node i.

The addressing scheme in question is as follows. When node j receives a packet destined to node i, it only checks the jth subfield of the address field. If all the bits in the jth subfield are zero then the packet is destined to node j, $(i = j)$. On the other hand, if there exist a single bit location, xth bit, set to 1 then node j forwards the packet to the xth outgoing link.That is, when a node receives a packet, it just checks the related field in the header and sends the packet to an outgoing link (port) whose bit location is set to 1. The details of the routing protocol can be seen in [15]. So in this case only single bit processing is necessary and the authors claim that such a scheme is deployable with the current all optical technology.

Extended All Optical Routing and Addressing Scheme:

The above routing scheme is discussed for unicast flow routing, however it can easily be modified to support a micro-buffered network architecture as well as multi-path and multi-cast routing.

Let again N and L be the number of nodes and links in the network. Assuming bidirectional links there are $2L$ input output ports needs to be defined. As can be seen in Figure 2.4, each packet has an address field of $2L + N$ bits where there are still N subfields each with $b(i) + 1 = n(i)$ bits. In this scheme, instead of

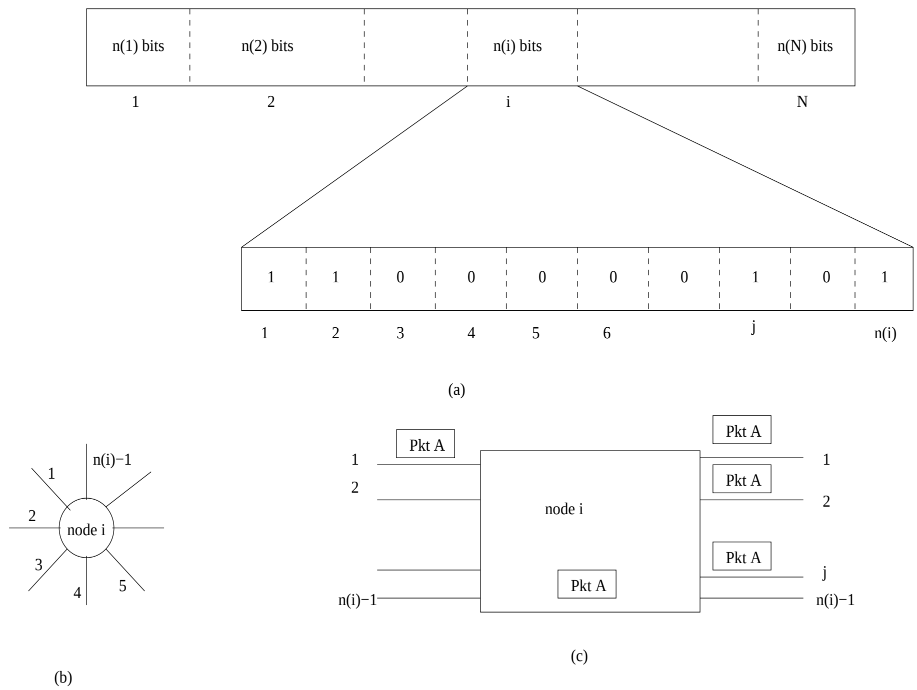

Figure 2.4: An example addressing structure

having only one bit set to 1 in a subfield, there can be multiple 1s. So each node receiving the packet will check all the $b(i) + 1 = n(i)$ bits in the related subfield and then send the packet to the outgoing links of which bit locations are set to 1 simultaneously. There is an extra bit location in each subfield, (n_ith bit) which is for the node itself. In other words if that bit is set, the node decides that the packet is also destined to it. In Figure 2.4, the packet not only is destined to node i but also is forwarded to the 1st, 2nd and jth outgoing links simultaneously.

A source node can set multiple bits in each subfield such that multicast or multipath routing can easily be supported.

A Sample All Optical Self Routing Switch Architecture:

In the following paragraphs, to implement this routing scheme, a sample switching structure requiring 2-bit processing in each parallel stage is considered. There are many potential switching structure that can be considered to implement the above routing scheme. In [15], authors give examples of the switching architectures described in [18] and [19] to implement their routing scheme. On the other hand, in [16] a self routing switching infrastructure has been proposed such that the pulse interval addressing scheme has been adapted in their switching infrastructure. In this chapter, we will discuss the one in [16] to implement such a routing scheme. Basically, the switching structures discussed in [16] and in [15] are similar to each other.

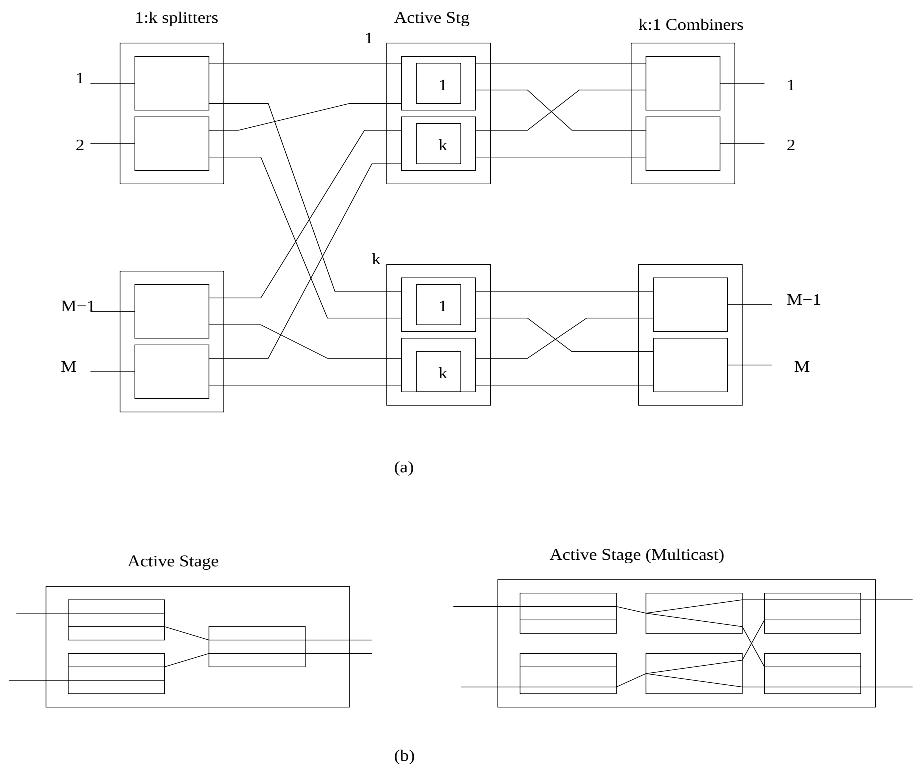

Figure 2.5: A sample switching architecture for micro-buffered optical networks

In summary, the switch in question [16] consists of three main stages (see Figure 2.5) , a passive splitter and combiner at the input and output respectively and an active stage implemented in the middle by directional couplers. This switching structure is nonblocking in the strict sense, so there is no collisions of two packets destined to different outgoing links. (e.g. a simple banyan network structure suffers from the internal blocking problem.) In the same paper, the active stage is proposed in detail for the case of the pulse interval addressing scheme of [17]. This switch structure can also be designed for multicast and multipath routing purposes which is also briefly discussed in the paper.

In either case (unicast, multicast) each active stage has two input and two output ports. Receiving a packet header an active stage checks the related address field and decides whether the bits corresponding to the two outgoing links are set or not. If only one of them is set then the packet will be switched to the correct outgoing link, however if both of the bits are set then the packet is sent to both of the output ports. So in this case, all we need is at most 2-bit processing at each active stage for multicast and multipath routing.

A Micro-buffered Approach:

In this section, the extended all optical routing scheme and the sample optical switch are assumed in the context of a micro-buffered network architecture.

As we have mentioned previously, we do not propose any collision resolution algorithm within the network. All the colliding packets are assumed to be combined (i.e summed) at the collision point. In case of a time slotted system such that the packet headers arrive only at the boundaries of a time slot, collisions generate multicast or multi-path like packet headers having potentially multiple 1 entries in the address field for each outgoing port (see Figure 2.6). This is basically due to the simple summer (combiner) structure at the switch discussed above.

Thus in case of a collision, the packet header includes the union of the all the routing information available in the colliding packets and seems like a multipath or multicast routing packet header. Therefore, the collisions do not affect the routing of the packet and do not lead to unpredictable routing of the packets throughout the network.

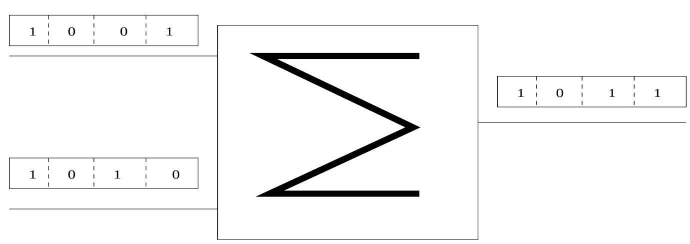

Figure 2.6 Two colliding packet header, generating a new header containing both path information

Figure 2.7, represents a case where packets A and B collide at node i and leave the outgoing link number 1 simultaneously. Path of packet A traverses outgoing link 1 of node j, whereas path of packet B goes through the outgoing link 2 of node

j. Since the packet header includes both path information, when the summed version of the packet headers arrives at node j, the incoming packet is sent to both outgoing links using the switching described in Figure 2.5.

Such a scheme requires packet-level synchronization which requires a careful design of the fiber lengths in the network design level. In the literature there are many studies for packet level synchronization in many different contexts such as deflection routing [13]. Deflection routing functions properly if packets are aligned since the packet headers should be compared "on the fly" assuming no optical buffering to correct asynchronous arrivals of packets. On the other hand, asynchronous models are also considered [40].

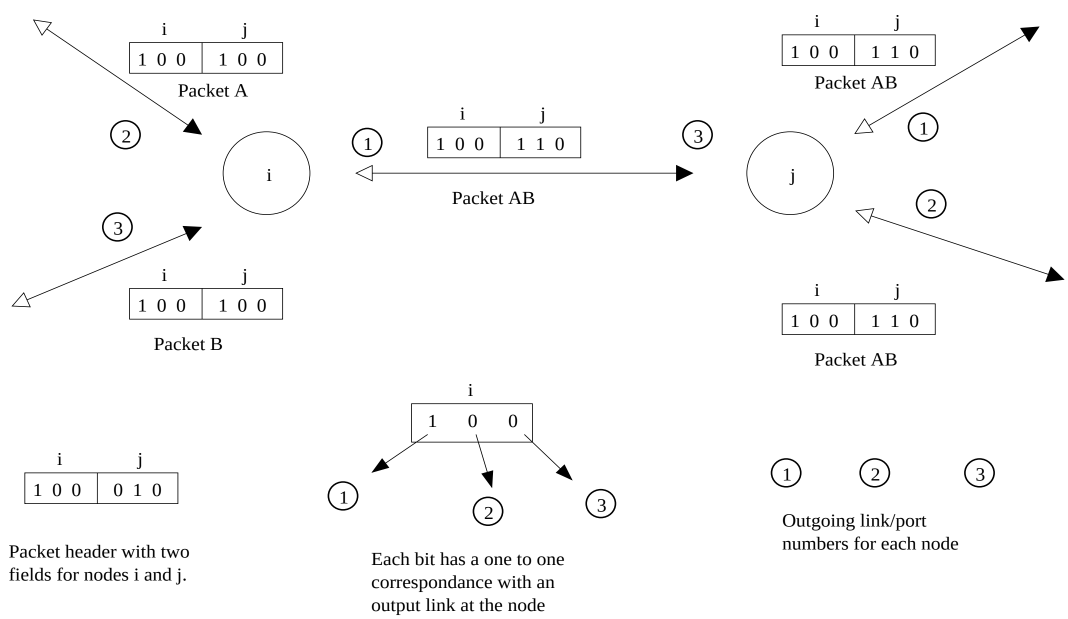

Figure 2.7 Two colliding packet header, generating a new header containing both path information

The extended self routing scheme that we have discussed has also a similar constraint to that discussed above such that any two flows i and j should contend no more than once with each other throughout their paths to the destination. In addition to that, any two contending flows are assumed not to traverse the same nodes after they are separated. As an example, shortest path routing satisfies both these conditions. Our constraint is closely related to the constraint in [15].

In the next section, the rate assignment problem for these models is discussed.

Some well known rate assignment policies will be described, analyzed, simulated and the corresponding results are compared.

2.2 Rate Assignment Problem

We consider both synchronous and asynchronous systems depending on the network model used (Model 1 and 2 are asynchronous and Model 3 is synchronous). Time is assumed to be slotted in both cases and each user transmits within the boundaries of its own slots. The slot lengths (or correspondingly packet lengths) are assumed to be the same for all users, but the relative phase may be random in models 1 and 2. Moreover, we assume that each user i has always a packet to send (i.e. fully backlogged case) and transmits a packet in a slot with a probability P_i independent of other slots. In fact assuming Bernoulli sources is not necessary in this context, considering the capacity discussions in [4]. Each packet is assumed to be routed as described in the previous sections. Whenever two or more packets contend for the same output link, the information in the overlapped regions of the packets are assumed to be lost. Any capture effect is not considered in this model.

Let I_i be the information rate of flow i (or the capacity of the channel that flow i experiences) and $X(i)$ be the set of flows that are directly contending with flow i from source node to destination node. The relationship between the information rates and the transmission rates defining the capacity region is as follows

$$I_i = P_i \prod_{j \in X(i)} (1 - P_j) \tag{2.1}$$

The above result is a multi-hop extension of the result presented in [4] where all the flows contend with each other (that is $X(i) = F \ \forall i$, where F is the set of all flows), where for any flow i, $I_i = P_i \prod_{j \neq i, j \in F} (1 - P_j)$.

In [4], the capacity of a single hop contention channel (Figure 2.8 (a)) without feedback is considered. The capacity region for both synchronous and asynchronous case is explicitly defined and shown that the capacity regions for both unsynchronized and synchronized case are shown to coincide. In the network model, each flow is assumed to have arbitrary delay to a single destination (e.g. a base station).

Our model can be considered as a cascaded channel model of [4] where as can be seen in Figure 2.8, at step k, flow i contends with the flows in the set $X(i, k)$. Flow i sees an equivalent contention channel where each flow it contends with has some arbitrary delay to the destination node which is the same channel model described in [4].

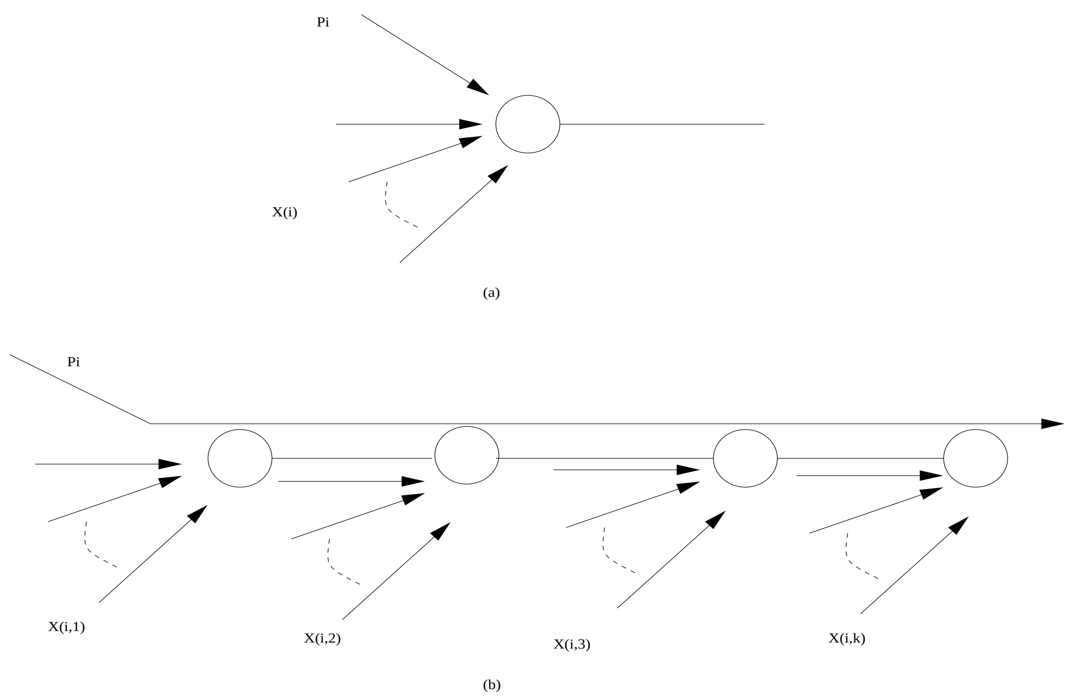

Figure 2.8 Channel Models (a) Single Hop; Each flow contends with each other (b) Path of Flow i, and at each hop it contends with a different set of flows.

So if $X(i) = \bigcup_{z=1}^{k} X(i, z)$, then the two channels seen by flow i in Figure 2.8 (a) and (b) are the same. Overall, the difference is that, in [4], each flow sees the same contention channel whereas in our case, each flow may experience a different contention channel with respect to the path it takes.

The network model 1 (i.e. the WDM model) has the same channel model as in [4], since each flow contends with all other flows to the destination. However, the network models 2 and 3 have exactly the same channel model described in equation (2.1).

In this study, we are interested in developing local rules where user i uses $P_i^{n+1} = H_i(P_i^n, g_i^n)$ where $g_i^n = (P_j^n | \forall j \in X(i))$. Here n denotes the iteration number.

In the following sections, certain rate assignment policies (aloha with exponential backoff, weighted α-proportional fair, weighted max-min fair) are examined. Algorithms, achieving related fairness policies are proposed and examined via analysis and simulations.

2.2.1 Classical Slotted Aloha with Exponential Backoff

In this section, we examine the classical slotted aloha techniques with exponential backoff in the context of our model. We first describe the classical slotted aloha with exponential backoff where each user contends with all other users.

We assume that each user is always backlogged, i.e. the full buffer case. In other words, each user has always something to transmit. Next, we assume ternary feedback (Idle, Success, Collision) where after each slot, each user is informed about the current state of the network. A collision occurs if there exist multiple packet transmissions at a time slot, and a successful transmission is achieved only if one of the users transmits a packet. Moreover, if no packet is transmitted in a time slot then the time slot is considered as being an idle slot. Since each user is full buffered and is able to get the ternary feedback, each user can be assumed to have the same transmission rate for each slot (assuming the initial rates are all the same).

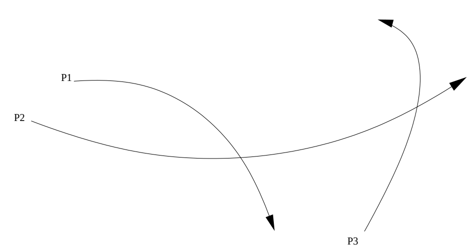

Figure 2.9 Example network in flow domain. Each curve represents a flow and any two intersecting curves represents contention

Let $P(n)$ be the transmission rate (i.e probability of transmission) of a user in the system for time slot n, and let P_{max} and P_{min} denote the maximum and

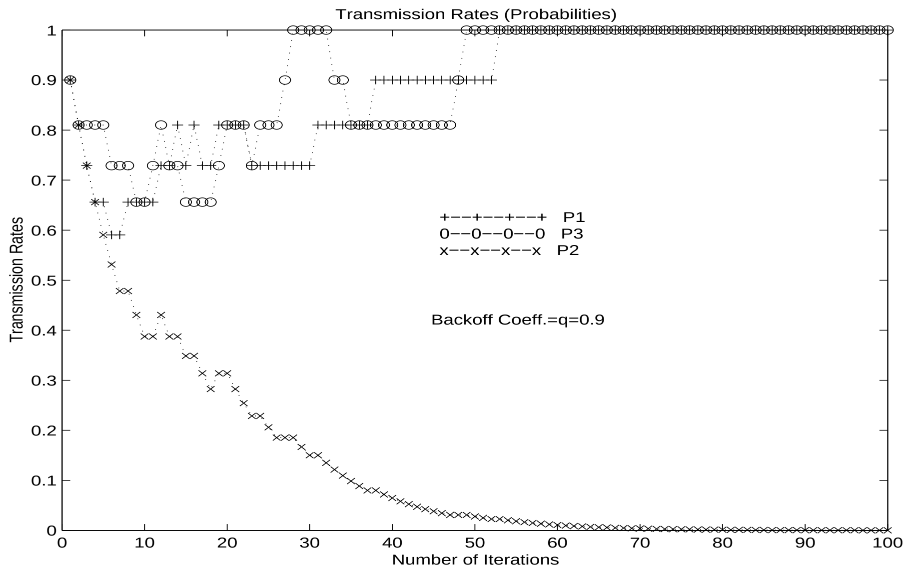

Figure 2.10 Transmission rates of each flow in slotted aloha with exponential backoff

minimum transmission rates that are allowed by the system such that the entire system is a truncated system in terms of the allowable transmission rates. Moreover each user is assumed to start the transmission with a rate P_{max} at the first time slot. Each user adjusts its transmission rate for the next slot with respect to feedback from the previous slot as follows. Unless otherwise specified, we assume that $P_{max} = 1$ and $P_{min} = 0$.

Here $P(n+1)$ is updated such that

$P(n+1) = Max(P_{min}, P(n) \times q_1)$ If collision at slot n

$P(n+1) = Min(P_{max}, P(n)/q_2)$ If slot n is idle

$P(n+1) = P(n)$ If success at slot n

where q_1, q_2 are called the collision and idle coefficients

and are less than 1.

The whole system can be modelled using a discrete Markov model; however, in this study we only would like to focus on the overall behavior of the aloha system in question. As can be understood from the above model, the entire system (when

$q_1 = q_2 = q$) reaches equilibrium where the probability of idle, $(1-P(n))^N \approx e^{-G}$, equals the probability of collision, $1-(1-P(n))^N - NP(n)(1-P(n))^{N-1} \approx 1-e^{-G}-Ge^{-G}$, where $G \approx NP(n)$ is the offered load and N is the number of users in the system (see appendix A.1). Equilibrium is reached at $G = G^* \approx 1.14$ or $P(n) = \frac{G^*}{N}$ in the system above where $q_1 = q_2 = q$. In other words, each user in such a system tries to keep the offered load in the system at G^* (see appendix A.1). In [20], authors engineer the collision and idle coefficients such that the overall throughput is maximized (i.e. $G^* = 1$).

We would like to consider the performance of slotted aloha systems in micro-buffered network model. In this model, as described in equation (2.1), each user i sees a possibly different set of flows as the contenders, $X(i)$.

First, we still can assume the ternary feedback. Each user i gets a feedback about the channel which is formed by the flows in $X(i)$ and the flow i itself. Let $Xh(i)$ be the set which is equal to the union of $X(i)$ and i. Briefly there exist a collision for user i if multiple transmissions occur in $Xh(i)$, and a success occurs if only one of the flows in $Xh(i)$ transmits. Finally, an idle slot corresponds to the case that none of the flows in $Xh(i)$ transmits in that slot. Again the system reaches equilibrium (for $q_1 = q_2 = q$) when for each user i, the probability of idle $\prod_{j\in Xh(i)}(1-P_j) \approx e^{-\sum_{j\in Xh(i)} P_j} = e^{-G_i}$ equals the probability of collision $1-\prod_{j\in Xh(i)}(1-P_j)-\sum_{j\in Xh(i)} P_j(\prod_{k\in Xh(i),k\neq j}(1-P_k)) \approx 1-e^{-G_i}-G_ie^{-G_i}$ where $G_i = \sum_{j\in Xh(i)} P_j$. Having slotted aloha with exponential backoff mechanism, each user will again try to keep the offered load on the channel it experiences around G^* (see the appendix A.1).

Using the above idea, a distributed rate allocation algorithm targeting the same equilibrium point in a multi-hop environment can be considered where each user is assumed to get feedback from the intermediary nodes on its path. Instead of a ternary feedback, only the load, G, on the entire path could be provided. The overall load on the channel (path) seen by user i is simply the sum of all loads on the outgoing links on the path. And assume that each user is imitating the slotted aloha model described above such that if the overall load exceeds G^* then the user cuts its transmission rates exponentially otherwise if the load is less than G^* then increase the transmission rates. Finally, if the load is close to G^* then it just keeps the same rate.

Potential Unfairness:

Such schemes (either with ternary feedback or not) have a great potential of cutting off the rates of some users in certain possible situations. They roughly try to achieve the following conditions for steady state behavior $\sum_{i \in Xh(i)} P_i = G^* \;\; \forall i$. For instance, consider a simple case where there are three users $1, 2, 3$ in the network as in Figure 2.9. The contention sets are such that $X(1) = 2$, $X(2) = 1, 3$ and $X(3) = 2$. The objectives of each user is to achieve $P_1 + P_2 = G^* \approx 1.14$, $P_1 + P_2 + P_3 = G^*$ and $P_2 + P_3 = G^*$. Assume that the initial rates are P_{max} for all users and as time elapses each user tend to decrease their rates to satisfy the above conditions. User 1 and 3 tend to stop decreasing their rates roughly until the conditions $P_1 + P_2 = G^*$ and $P_2 + P_3 = G^*$ are satisfied respectively. However, user 2 will still see a load equals $P_1 + P_2 + P_3$ which is greater than G^*. Therefore, user 2 will continue to decrease its rate, P_2. As P_2 decreases, this time user 1 and 3 will increase their rates in order to keep their channel load around G^*. Eventually, P_1 and P_3 both converge to 1, (P_{max}), and P_2 converges to 0, (P_{min}). We performed a simulation for the network in Figure 2.9 using the ternary feedback available to each user after each time slot. The simulation result can be seen in Figure 2.10. The same conclusion can be reached for the case where the load is fed back, instead of ternary feedback per slot.

Motivated by the problems with this approach, we attempt to develop rate allocation schemes based on certain fairness criteria.

2.2.2 Weighted Alpha-Proportional Fairness

The proportional fair rate allocation [8] aims to maximize the sum of utilities of all the users in the network. The utility function for each user is assumed to be a logarithmic function of its rate.

We now generalize this model, to incorporate weights associated with each flow. Let $U(I_i) = -\frac{1}{g_i}(-ln(g_i I_i))^{\alpha}$ be the utility of user i as a function of the information rate where g_i is the weight associated to flow i. We have

$$U(I_i) = -\frac{1}{g_i}(-log(g_i I_i))^{\alpha} = -\frac{1}{g_i}(-log(g_i P_i \prod_{j \in X(i)} (1 - P_j)))^{\alpha} \tag{2.2}$$

A rate assignment $\{I_i\}$ is said to be weighted α-Proportional fair if it maximizes the aggregate utility function $U = \sum_i U(I_i)$. Note that $\alpha = 1$ corresponds to the proportional fair case, whereas as α approaches infinity the system converges to the weighted max-min fair rate assignment (see proof in appendix A.4). Therefore, a general weighted α-proportional fair rate assignment can be considered as a compromise between proportional-fairness and weighted max-min fairness.

Fortunately, the negative utility function above for an arbitrary α can be shown to be a convex function of the transmission rates (e.g. P_i) where $0 \leq P_i \leq 1$. The proof can be found in the appendix A.2. Therefore, the problem of maximization of the the sum of individual utilities is a convex programming problem.

We use the gradient projection method [3] to solve the above optimization problem, with the updates

$$P_k^{n+1} = H_k^{n+1}(P_k^n, g_k^n) = [P_k^n + \theta^n \frac{\partial U}{\partial P_k}]^+ \tag{2.3}$$

where n denotes the iteration number and $[f]^+$ denotes projection on the set $0 \leq P_i \leq 1 \forall i$ which is equal to max(0,f) in this case and

$$\frac{\partial U}{\partial P_k} = \frac{\alpha}{g_k P_k}(-log(g_k I_k))^{\alpha-1} - \frac{\alpha}{1 - P_k} \sum_{j \in X(k)} (\frac{1}{g_j})[-log(g_j I_j)]^{\alpha-1} \tag{2.4}$$

Equation (2.4) becomes negative (positive, zero) when P_k is less than (greater than, equal to) $f_k(\vec{P})$ where $\vec{P} = (P_1, P_2, \ldots, P_{|F|})$ and

$$f_k(\vec{P}) = (1 + \sum_{m \in X(k)} (\frac{g_k}{g_m})(\frac{log(g_m I_m)}{log(g_k I_k)})^{\alpha-1})^{-1} \tag{2.5}$$

Given any arbitrary $\vec{P}$ vector initially, at each iteration n each user k computes the value of $f_k(\vec{P})$ and compares this value with the current transmission rate

P_k^n. Thus each user increases its rate P_k^n if $f_k^n > P_k^n$ or decreases if otherwise. In order to implement the algorithm in a distributed way, each user k must learn the value of $g_j I_j$ for $j \in X(k)$. This can be done by having each user (flow) j send its transmission rates P_j to all other users it contends with. Each user j can then calculate its information rate I_j, and subsequently send the value of $g_j I_j$ to all other users it contends with. We assume the existence of a control channel to facilitate these information exchanges.

The whole rate control scheme, which we call MIMD-$\alpha - G$, can be written as follows for user k:

MIMD-$\alpha - G$ Algorithm

STEP1: Initially start with an arbitrary transmission rate.

STEP2: At iteration n

IF $f_k^n > P_k^n$ then

Increase P_k (e.g. $P_k^{n+1} = \min\{P_k^n \times \beta, 1\}$)

ELSE

Decrease P_k (e.g. $P_k^{n+1} = P_k^n \times q$)

where $\beta > 1$ and $q < 1$ are appropriate step size constants.

It is interesting to note that in [6], the number of neighbor's neighbors (second hop neighbor information) is required for the distributed algorithm to calculate rates for mac layer fairness. In our case, we need only the direct contender information to calculate the rates for end to end fairness.

Here α can be set by the system designer such that as the value of α increases the smaller flows are getting higher priority and the system is converging to weighted max-min fair state (see the discussion on the convergence in the appendix A.4). One of the advantages of this scheme is that, as we will also discuss later, by adjusting α one can trade off the total network utilization and the fairness among the flows.

THEOREM: For the case of $\alpha = 1$ and all weights equal to 1, (i.e. Proportional Fair Case) the optimum value of P_k is given by

$$P_k = (X_k + 1)^{-1} \tag{2.6}$$

where X_k is the cardinality of $X(k)$.

Remark: It is interesting to note that for the case where all flows contend with each other (as in classical aloha models), we have $X_k = N - 1$. Thus the proportional fair transmission probability for all users is $\frac{1}{N}$, which is the same value known to maximize system throughput.

In the next section, we will discuss rate allocation for weighted max-min fairness.

2.2.3 Weighted Max-Min Fairness

First, let us give a formal definition of max-min fairness.

Definition 1: A vector of rates R is **weighted max-min fair** with weight vector, W, if it is feasible and for each flow i, the rate of flow i, R_i, can not be increased while maintaining feasibility without decreasing R_j for some flow j for which $R_j W_j \leq R_i W_i$. As a special case, when $W_i = 1\ \forall i$, the vector R is said to be max-min fair [2].

Definition 2: Let F be the set of flows (which is called a group) with the property that if a flow $i \in F$ then any flow $j \in X(i)$, $j \in F$. In other words, if any flow i is in a group then all the flows contending with this flow are also in the same group. However, in order to be in the same group any two flows do not have to contend directly at the same link, for instance if flow i is contending with flow j at a node in the network and if flow j is contending with flow k at any other node then all the three flows are in the same group even though flows i and k do not directly contend at any node in the network.

Fact 1: A weighted max-min fair rate assignment with weight vector W and rate vector R assigns the rates to each flow in the same group such that $R_i W_i = R_j W_j\ \forall i, j \in F$. Note that as a special case, the max-min fairness rate assignment ($W_i = 1$, $\forall i$) assigns the same rate to all the flows in the same group.

PROOF:. Assume by contradiction that there exists a vector of information rates, $I\prime$, which is weighted max-min fair in the information rate with weight vector W, and there exists some flows i and j in the same group that do not satisfy the condition $I\prime_i W_i = I\prime_j W_j \ i,j \in F$. Without loss of generality assume that $I\prime_i W_i < I\prime_j W_j$. Even though flow i and j may not contend directly (i.e $i \notin X(j)$), since they are in the same group there exists a set of flows $f_{j,i} = (f_1, ...f_M)$ such that $f_1 \in X(j)$ and $f_M \in X(i)$ and $f_a \in X(f_{a+1})$ ($\forall a : a = 1, 2..., M$), otherwise flows i and j would not be in the same group. Decreasing the transmission rate of flow j, P_j, will increase the information rate of flow f_1. Then decreasing the transmission rate of f_1 till the information rate of f_1 is equal to $I\prime_{f_1}$ (original value) will increase the information rate of f_2. And one can continue this process until the transmission rate of flow f_M is decreased and the information rate of flow i is increased. By this way, one is able to increase the information rate of a flow i, $I\prime_i$, by only decreasing $I\prime_j$ for some flow j for which $I\prime_i W_i < I\prime_j W_j$ which is a clear contradiction to the definition of weighted max-min fairness.

□

This result agrees with the result in [14] where the authors claim that max-min fair rate assignment results in equal rates in a multi-hop adhoc wireless network where all the nodes are able to hear each other and there is no clustering. The model in [14] assumes that all wireless links contend (interfere) with each other. However in our case, max-min fair rate assignment still results in equal information rates even any two flows in the same group do not directly contend with each other.

Fact 2: A vector of flow information rates, I, is weighted max-min fair with weight vector W, if it is achievable (i.e. in the capacity region) and it is the maximal among vectors E such that $W_i E_i = W_j E_j$ for all $\forall i, j \in F$. (Similarly, $E_i = E_j \ \forall i, j \in F$ for max-min fair case.) (A vector, V, is maximal (or on the boundary of the capacity region) when there is no other vector, D, of which elements are not less than those of V and at least one is strictly greater).

The proof of this fact is a direct result of Fact 1 and the definition of weighted max-min fairness.

2.2.4 Weighted Max-Min Fair Rate Assignment

In this section, we discuss two approaches to compute the max-min fair information rates in a distributed manner.

Approach 1 (Weighted α-Proportional Fairness) :

The $MIMD - \alpha - G$ algorithm described in the previous section is proposed to achieve weighted max-min fair information rate allocation where the value of α is assumed to be sufficiently high.

Approach 2:

In this approach, we use the method described in [5] which is based on a duality method. The capacity model in [5] for a multi-hop adhoc network is similar to our model. We simply extend the formulation for weighted max-min fairness as in [5], where only max-min fairness is examined.

Basically, in this method, the original weighted max-min fair problem (which is a non-convex problem) is converted into a convex programming problem. Using convex duality and again the gradient projection method, a distributed algorithm for computing max-min fair information rates is described.

Using Fact 2, the original problem for max-min fair rate computation is reduced to the problem of maximization of the minimum weighted information rate. That is,

$$\textbf{Maximize X ; s.t.} \quad X < g_i P_i \prod_{j \in X(i)} (1 - P_j) \quad \forall i \tag{2.7}$$

This problem is equivalent to the following problem, the details of which can be found in the appendix A.3.

$$\textbf{Minimize} \frac{1}{2} \sum_{i \in F} n_i^2 \tag{2.8}$$

$$\textbf{Subject to} \quad n_i - ln(g_i) - ln(P_i) - \sum_{j \in x(i)} ln(1 - P_j) < 0 \quad \forall i \in F \tag{2.9}$$

$$n_i \leq n_j \quad n_j \leq n_i \quad \forall i \in F \text{ and } \forall j \in X(i) \ . \tag{2.10}$$

The objective function is convex and the functions in the constraints form a convex set. So the problem is a convex programming problem.

The Lagrangian for the above problem is

$$L(P, n, \lambda, \mu) = 1/2 \sum_{i \in F} n_i^2 + \sum_{i \in F} \lambda_i (n_i - ln(g_i) - ln(P_i) - \sum_{j \in X(i)} (1 - P_j)) + \sum_{i \in F} \sum_{j \in X(i)} (n_i - n_j) \mu_{i,j} \tag{2.11}$$

We can apply convex duality, which implies that there is no duality gap.

The dual function is $D(P, n) = Min_{P,n} L(P, n, \lambda, \mu)$ and the dual problem is $max_{\mu,\lambda} D(P, n)$

After minimizing the Lagrangian function we have the following results.

$$P_k^* = (1 + \lambda_k^{-1} \sum_{m \in X(k)} \lambda_m)^{-1} \qquad n_k^* = [-\lambda_k - \sum_{m \in X(k)} (\mu_{k,m} - \mu_{m,k})]^- \tag{2.12}$$

where $[x]^- = min(0, x)$

The dual problem can be solved using the gradient projection method, i.e.

$$\lambda_i^{n+1} = [\lambda_i^n + \theta^n \frac{\partial D}{\partial \lambda_i}]^+, \quad \mu_{i,j}^{n+1} = [\mu_{i,j}^n + \theta^n \frac{\partial D}{\partial \mu_{i,j}}]^+ \tag{2.13}$$

where $[f]^+$ is again turns out to be $max(0, f)$ and

$$\frac{\partial D}{\partial \lambda_i} = n_i - ln(g_i) - ln(P_i) - \sum_{j \in X(i)} ln(1 - P_j), \quad \frac{\partial D}{\partial \mu_{i,j}} = n_i - n_j. \tag{2.14}$$

The control plane of the previous approach, where transmission probabilities and weights of direct contenders are exchanged between users, is also necessary for this approach.

2.2.5 Weighted Hierarchical Max-Min Fair Rate Assignment

In this section, we discuss a fairness policy that ensures weighted max-min fairness first among some subgroups of flows and then among the individual flows in each subgroup. There may be several occasions where such a policy is required. For instance, this may arise in a network where there exists entities (e.g. corporations) owning some set of flows in the network and each entity would like to have a fair share of the overall network bandwidth. As a special case, in the case of multi-path routing between each source destination (S-D) pair, fairness can be required first among the S-D pairs and then among the individual flows of each S-D pair.

For simplicity, throughout this section we assume that all the flows in F are in the same contention group. If this is not the case, the results in this section apply to each contention group.

Definition 3: Let M_a be the set of flows belonging to subgroup a such that $\bigcup_a M_a = F$ and $M_a \bigcap M_b = \emptyset \; \forall a, b : a \neq b$. Let D be the rate vector where the ath element denotes the aggregate information rate of subgroup a : $D_a = \sum_{i \in M_a} I_i$. Lastly, let H_a denote the vector of rates of the individual flows belonging to M_a. A vector of rates, I, is said to be **weighted hierarchical max-min fair** with a weight vector Z, and a vector of vectors $W = (W_1, W_2, ...W_T)$, where T is the number of subgroups, and W_i is the weight vector for subgroup a such that $W_a = (W_{a,1}, W_{a,2}...W_{a,L_a})$ (where L_a is the number of flows in subgroup a), if first the vector, D, is weighted max-min fair with weight vector Z and the rate vector for flows in each subgroup a, H_a, is weighted max-min fair with weight vector W_a.

More formally, a vector, I, is said to be **weighted hierarchical max-min fair** with a weight vector Z, and a vector of vectors $W = (W_1, W_2, ...W_T)$, if it is feasible and if for each flow, $i \in M_a$, the rate, I_i, can not be increased, while maintaining feasibility without decreasing I_j for some flow $j \in M_b$ for which $W_{b,j} I_j \leq W_{a,i} I_i$ when $a = b$ or $Z_b D_b \leq Z_a D_a$ when $a \neq b$. When all the weights are all equal to 1, the rate vector I is called Hierarchical Max-Min fair as similarly defined in [21].

Fact 3: A weighted hierarchical max-min fair rate assignment with Z and W as described above assigns aggregate rates to each subgroup such that $Z_a D_a = Z_b D_b$ $\forall a, b$, and assigns rates to each individual flow belonging to the same subgroup such that $W_{a,i} I_i = W_{a,j} I_j \; \forall i, j \in M_a \; \forall a$.

Proof. Assume by contradiction that a vector information rates, I, is weighted hierarchical max-min fair with Z and W, but there exists two subgroups M_a and

M_b such that $Z_a D_a < Z_b D_b$. Assume there exist two flows $f_a \in M_a$ and $f_b \in M_b$. Considering the proof for Fact 1, one can increase I_{f_a} as well as D_a with only decreasing I_{f_b} as well as D_b which contradicts with the definition of hierarchical fairness.

In addition to this, again by contradiction assume that I is weighted hierarchical max-min fair with Z and W, but in a subgroup k there exist at least two flows $i \in M_k$ and $j \in M_k$ such that $W_{k,j} I_i < W_{k,j} I_j$. The situation results in the same contradiction as in the proof of Fact 1.

□

Fact 4: A vector of flow information rates, I is weighted hierarchical max-min fair with Z and W as defined above, if it is achievable (i.e. in the capacity region) and corresponding D vector is the maximal among vectors, E, such that $Z_a E_a = Z_b E_b \; \forall a, b$ and given the D vector, for each subgroup a, the related vector H_a is the maximal among the vectors, B^a, such that $W_{a,i} B^a_i = W_{a,j} B^a_j \; \forall i, j \in M_a$. (It is a direct result from the previous fact on hierarchical fairness.)

Let Q be the weighted rate of each subgroup utilizes such that $Q = Z_a D_a \; \forall a$ where the weighted hierarchical fairness policy with Z and W is enforced. Then the aggregate rate that the subgroup a utilizes will be $\frac{Q}{Z_a}$, whereas the rate of flow i, I_i in the same subgroup will be $\frac{C}{W_{a,i}}$ where C is the weighted rate of each flow in subgroup a that is $C = W_{a,i} I_i$. From the definition of D_a, $D_a = \sum_{i \in M_a} I_i$, $\sum_i \frac{C}{W_{a,i}} = \frac{Q}{Z_a}$.

Therefore, the rate of each flow $i \in M_a$ can be written as $I_i = \frac{Q}{Z_a W_{a,i} (\sum_i \frac{1}{W_{a,i}})}$

Let $s(i)$ denote the subgroup to which ith flow belongs and let

$K_{a,i} = Z_a W_{a,i} (\sum_i \frac{1}{W_{a,i}})$.

Let G be the vector such that

$G = ((K_{s(1),1}), (K_{s(2),2}), (K_{s(3),3}), ...(K_{s(|F|),|F|}))$ where $|F|$ is the number of all flows.

(Considering the above discussion, we can safely say that any weighted hierarchical max-min fair vector of flow rates, I, satisfies the condition $I_i G_i = I_j G_j$.)

Theorem: A vector of flow rates, R, is weighted hierarchical max-min fair with Z and W, if and only if it is weighted max-min fair with weight vector G.

Outline of The Proof: Assume by contradiction I is weighted hierarchical max-min (then $I_i G_i = I_j G_j$) but not weighted max-min with weight vector G, which implies that I is not maximal (otherwise it would be weighted max-min) which contradicts the definition of weighted hierarchical max-min. Conversely, assume I is weighted max-min with G, but not weighted hierarchical max-min with Z and W, then either the conditions described in Fact 4 on element of vectors D and H_as do not hold, or the vectors are not maximal. In either case there is a contradiction to the definition of weighted max-min fair rates with vector G (e.g. Fact 4). $\square$

2.3 Discussions and Examples

We simulated various different scenarios and compared the performances of slotted aloha, α-proportional and max-min fair rate allocation policies.

2.3.1 Example 1

Our first example network can be seen in Figure 2.11 which represents an asymmetric example in terms of the contention channels such that flow k contends with N flows each of which does not contend with any other flow.

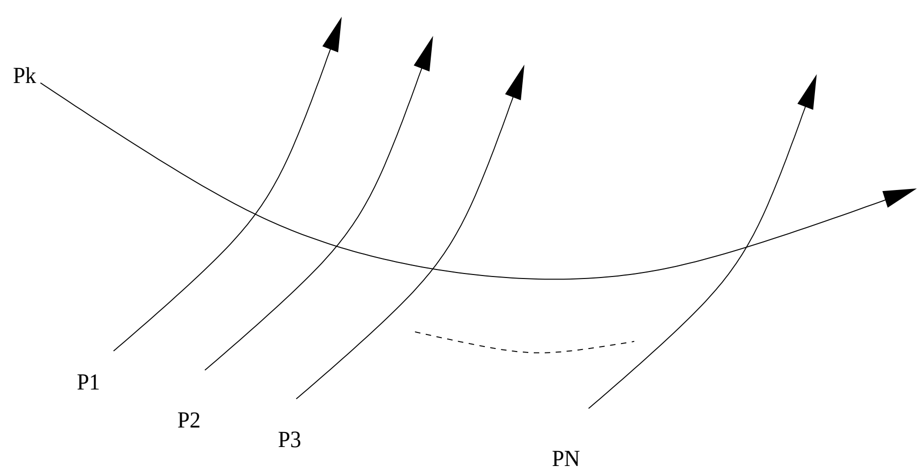

Figure 2.11 Example network in flow domain. Each curve Represents a flow and any two intersecting curves represents contention

We simulate the network with $N = 5$ using both our MIMD-$\alpha - G$ algorithm and approach 2 (in both cases, the weights are all equal to 1). Figures 2.12 and 2.13 show the resulting information rates of the flows along time for $MIMD - \alpha - G$ and approach 2 respectively. As can be seen in both figures, in agreement with the max-min fair discussion in the previous sections, the information rates converge to the same equilibrium point.

Next, we perform simulations using slotted aloha model and as expected slotted aloha eventually assigns $P_k = 0$ and $P_i = 1\ \forall i = 1...N$. Figure 2.10 represents the result for $N = 2$.

Next, we compute the total information rate (i.e. sum of the individual information rates) of the system for the slotted aloha, proportional and max-min fair cases. Figure 2.14 indicates the total information rates versus N. As can be seen in the figure, as N increases, total capacity for slotted aloha increases linearly since I_k always converges to zero and all other information rates converge to one (except $N = 1$). On the other hand, max-min fairness allocates the same information rate to all the flows, and as N increases the total information rate increases slightly. Proportional fairness in this example not only gives non-zero information rates to each flow with respect to a well-defined fairness measure but also keeps the utilization of the network much higher than max-min fair case does.

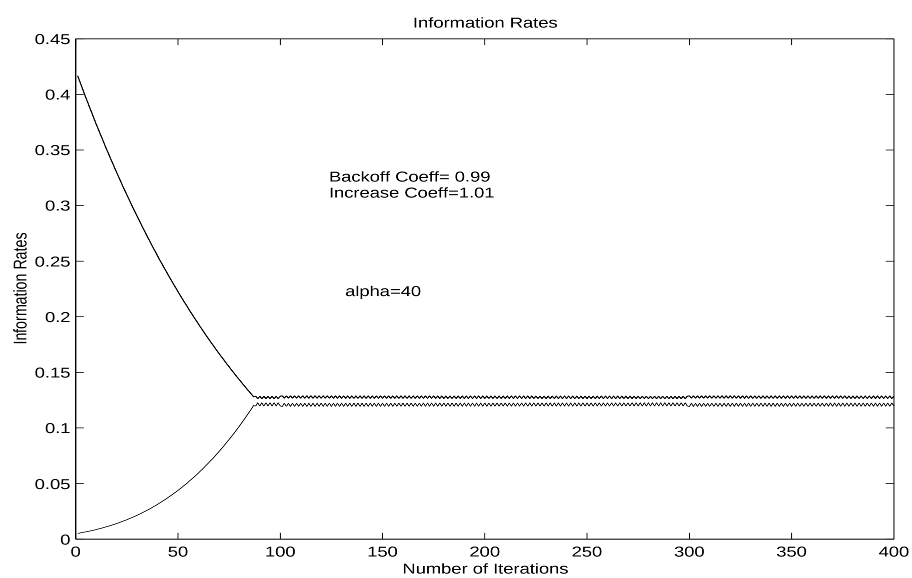

Figure 2.12: First example network, (MIMD-α)

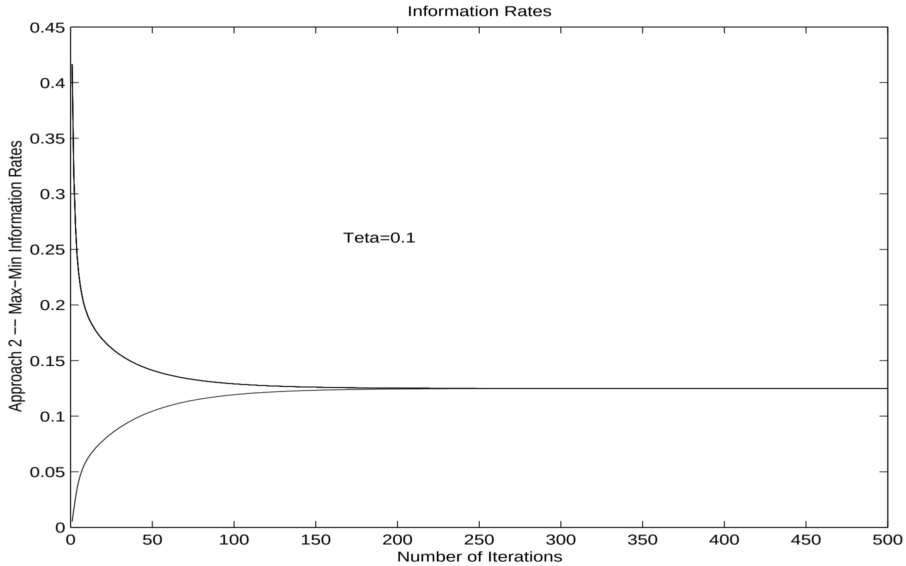

Figure 2.13: First example network, (Approach 2 —- Max-Min Fair)

In order to see the further total capacity performance of slotted aloha, α-proportional fair algorithm, (i.e. the MIMD-$\alpha - G$ algorithm with equal weights) , and the max-min fair rate computation algorithm (using approach 2 with equal weights), we generate random collision based networks for increasing number of flows from 10 to 50 and we compare the total information rates. As can be seen in Figure 2.15, slotted aloha has the highest total information rate for all the scenarios; however, at least 60% of the flows end up with zero transmission rates. Another interesting result to observe is that as α increases MIMD-$\alpha - G$ algorithm generates smaller total capacity for the same network. For all the cases as α increases, the results of both $MIMD - \alpha - G$ and Approach 2 for max-min fair case gets very close to each other. So α in MIMD-$\alpha - G$ can be set by the system administrator considering the tradeoff between the fairness and the total network utilization.

Furthermore, Figures 2.16 and 2.17 show the rate of each flow when hierarchical fairness (or Weighted Max-Min Fairness) is enforced via $MIMD - \alpha - G$ algorithm (i.e. Approach 1) and Approach 2 respectively. All the weight elements in vectors Z and W are set to 1. The network topology in Figure 2.11 is considered where $N = 5$. Flow k and 1 are in the first subgroup while flow 2 is the only flow in the

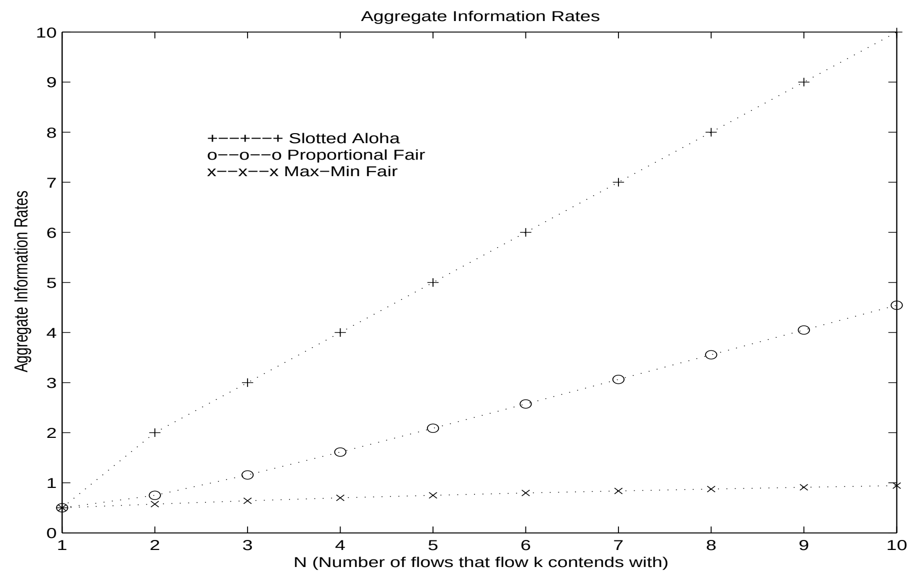

Figure 2.14: Total capacities for first example network

second subgroup and finally flows 3,4,and 5 are in the third subgroup. The weight vector, G, is as follows $G = [2, 2, 1, 3, 3, 3]$ and the resulting rate vectors (from both approach) is approximately as follows: $I = [0.12, 0.12, 0.24, 0.08, 0.08, 0.08]$ and the corresponding D and H vectors are as follows: $D = [0.24, 0.24, 0.24]$, $H_1 = [0.12, 0.12]$, $H_2 = [0.24]$, $H_3 = [0.08, 0.08, 0.08]$. As can be seen, by this policy, the max-min fairness is first ensured among the subgroups and then within each subgroup.

Next, we set the weights in the vectors, Z and W_is to some values other than all equal to 1, such as $Z = [3, 2, 1]$ and $W_1 = [2, 1]$ $W_3 = [1, 2, 3]$. Since the second subgroup has only one flow, W_2 is skipped. The corresponding weight vector, G, becomes as follows, $G = [9, 9/2, 2, 11/6, 11/3, 11/2]$. Figures 2.18 and 2.19 indicate the weighted hierarchical max-min fair flow rates via $MIMD - \alpha - G$ algorithm (i.e. Approach 1) and Approach 2 respectively. The resulting rate vectors (from both approach) is approximately as follows: $I = [0.06, 0.12, 0.27, 0.30, 0.15, 0.10]$ and the corresponding D and H vectors are as follows: $D = [0.18, 0.27, 0.55]$, $H_1 = [0.06, 0.12]$, $H_2 = [0.27]$, $H_3 = [0.30, 0.15, 0.10]$.

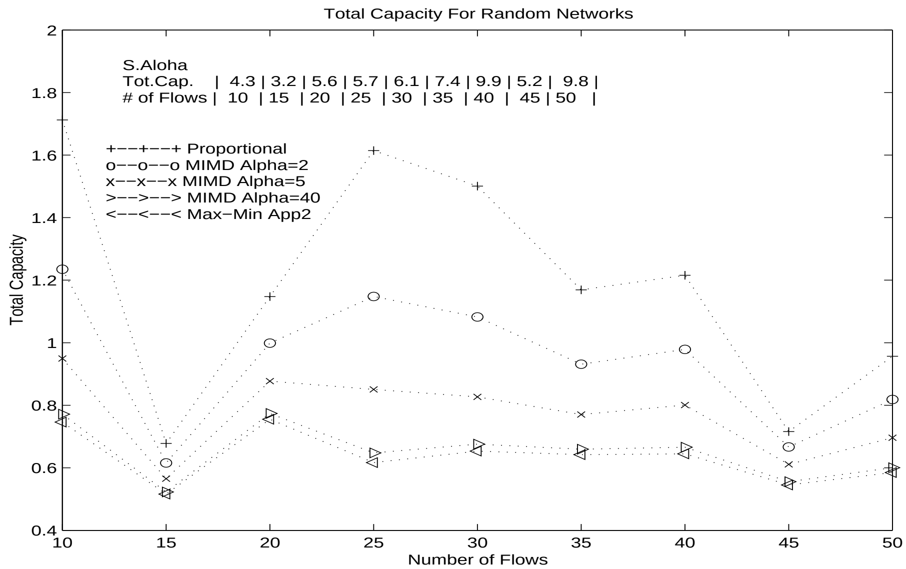

Figure 2.15: Total capacities for random networks

2.3.2 Example 2

Let H be the contention matrix representing which flow contends to which flow. The matrix H is symmetric and a 1 at location i, j indicates that flow i and flow j contends with each other somewhere throughout their paths.

As the next step, we have generated a random scenario for a 10 flow network. The randomly generated H matrix with 10 flows is as follows.

$(Flow1)$	0	1	0	1	1	1	1	0	1	1
$(Flow2)$	1	0	1	1	0	1	0	1	1	1
$(Flow3)$	0	1	0	1	1	1	1	1	0	1
$(Flow4)$	1	1	1	0	0	1	0	0	0	0
$(Flow5)$	1	0	1	0	0	0	0	0	0	0
$(Flow6)$	1	1	1	1	0	0	0	1	1	0
$(Flow7)$	1	0	1	0	0	0	0	1	0	1
$(Flow8)$	0	1	1	0	0	1	1	0	1	0
$(Flow9)$	1	1	0	0	0	1	0	1	0	1
$(Flow10)$	1	1	1	0	0	0	1	0	1	0

Figures 2.21 2.22 2.23 2.24 indicates the information rates of the 10 flows obtained by simulating the MIMD-α algorithm (i.e. $MIMD-\alpha-G : G_i = 1 \forall i$) for

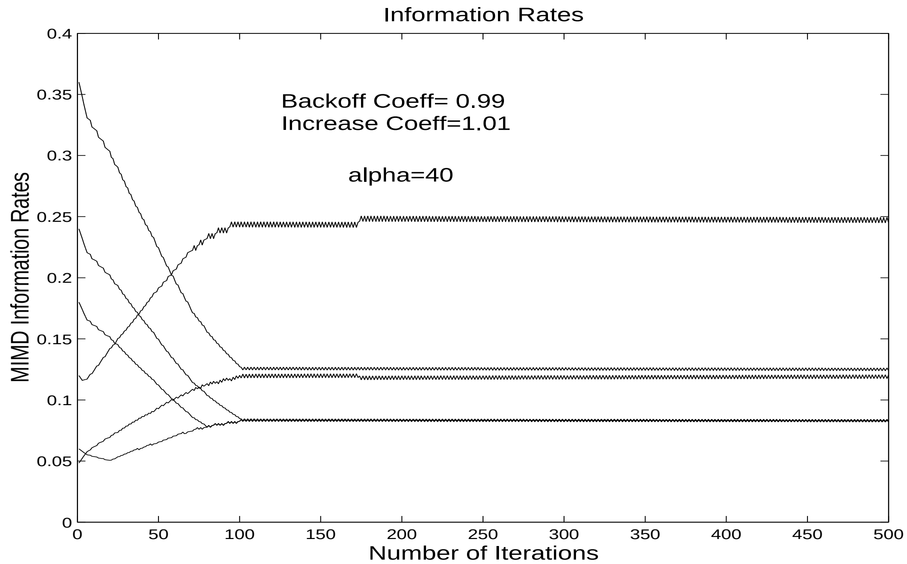

Figure 2.16: Hierarchical Max-Min Fair Rates Approach 1

different values of α from 1 to 40. In Figure 2.25, the information rates assigned with respect to Approach 2 of Max-Min fair rate computation are available. Initially all the flows are assigned their proportional fair rates. As can be seen in Figure 2.21, α is set to 1 which corresponds to the proportional fair case.

In Figures 2.22-2.24, it can easily be seen that as α increases from 1 to 40 the information rates are getting closer to each other which is consistent with the discussion in the previous sections such that max-min fairness (as α increases) gives equal rates to all the flows in the same group.

Moreover, the 10 flow network is also simulated with the slotted aloha scheme and the rate assignment over time is plotted in Figure 2.20. Six of the flows (with flow numbers 1,2,3,6,7,9) are assigned zero transmission rates, while other flows transmission rates converge to one. By examining the H matrix above, one can easily interprets the resulting rate assignment considering the discussion in the slotted aloha section.

Furthermore, Figures 2.26 and 2.27 exhibit the information rates of each flow when hierarchical fairness (or Weighted Max-Min Fairness) is enforced via $MIMD-\alpha-G$ algorithm (i.e. Approach 1) and Approach 2 respectively. All the weight elements in vectors Z and W are set to 1. The weight vector, G, is as follows

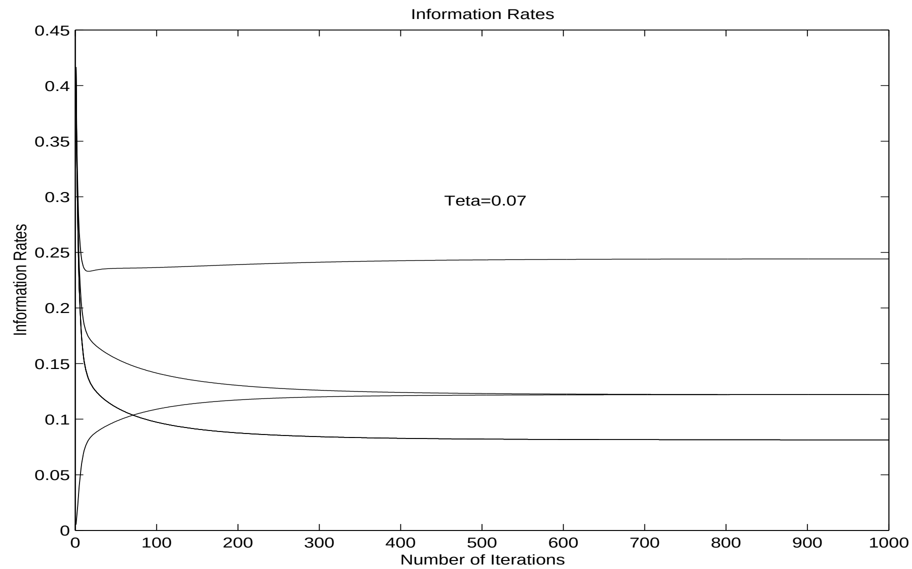

Figure 2.17: Hierarchical Max-Min Fair Rates Approach 2

$G = [4, 4, 4, 4, 2, 2, 3, 3, 3, 1]$, that is the first 4 flows belongs to subgroup 1, the next 2 flows belongs to the second subgroup, and flows 7,8,9 belongs to the subgroup 3, and last flow forms the subgroup 4 by itself. The resulting rate vectors (from both approach) is approximately as follows:

$I = [0.04, 0.04, 0.04, 0.04, 0.08, 0.08, 0.053, 0.053, 0.053, 0.16]$ and the corresponding D and H vectors are as follows: $D = [0.16, 0.16, 0.16, 0.16]$, $H_1 = [0.04,\ 0.04,\ 0.04,\ 0.04]$, $H_2 = [0.08, 0.08]$, $H_3 = [0.053, 0.053, 0.053]$ and $H_4 = [0.16]$. As can be seen, by this policy, the fairness is first ensured among the subgroups and then within each subgroup.

Next, we set the weights in the vectors, Z and W_is to some values other than all equal to 1, such as $Z = [2, 1, 4, 3]$ and $W_1 = [1, 2, 2, 3]$ $W_2 = [2, 3]$ $W_3 = [1, 2, 2]$. Since the second subgroup has only one flow, W_4 is skipped. The corresponding weight vector, G, becomes as follows, $G = [14/3\ ,28/3,28/3,14,\ 5/3,\ 5/2,8,16,16,3]$.

Figures 2.28 and 2.29 indicate the weighted hierarchical max-min fair flow rates via $MIMD\text{-}\alpha - G$ algorithm (i.e. Approach 1) and Approach 2 respectively. The resulting rate vectors (from both approach) is approximately as follows:

$I = [0.072, 0.036, 0.036, 0.024, 0.2, 0.137, 0.042, 0.021, 0.021, 0.111]$

and the corresponding D and H vectors are as follows:

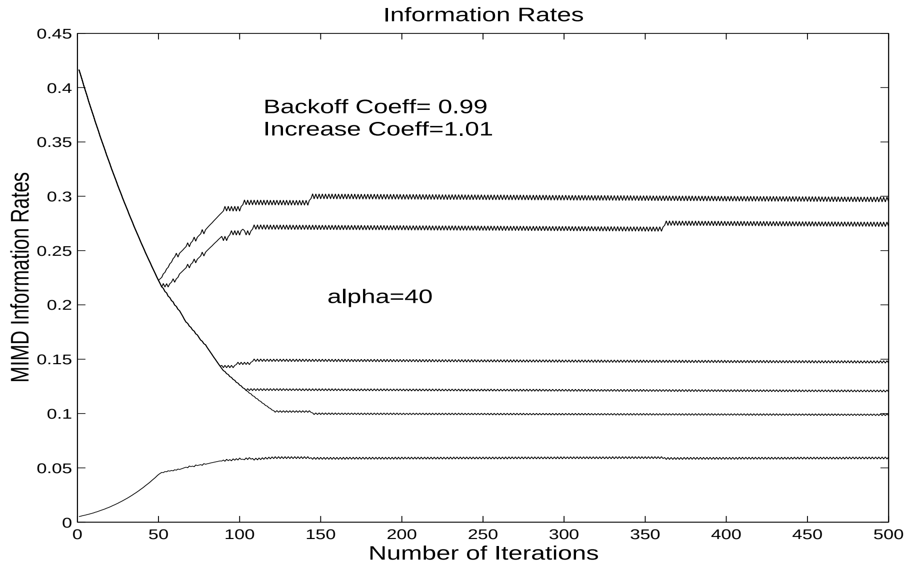

Figure 2.18: Weighted Hierarchical Max-Min Fair Rates Approach 1

$D = [0.168, 0.337, 0.084, 0.111]$,
$H_1 = [0.072, 0.036, 0.036, 0.024]$,
$H_2 = [0.2, 0.137]$,
$H_3 = [0.042, 0.021, 0.021]$,
$H_4 = [0.111]$.

2.4 Conclusion (Micro-Buffered Networks)

In this chapter, we first discussed an architecture, micro-buffered networks, for future high-speed networks. In this architecture, there is little or no buffering at switching nodes, and packet contentions result in collisions which are resolved by use of end-to-end forward error (erasure) codes. Related routing and addressing schemes were discussed and compatible switch architectures were examined.

We focused on the rate assignment problem in micro-buffered networks. First, we examined a distributed rate assignment scheme based on slotted aloha with exponential back-off. We found that in the case of asymmetric network examples, slotted aloha with exponential backoff may lead to highly undesirable rate allocations such that some flows may end up with zero information rates. This motivated

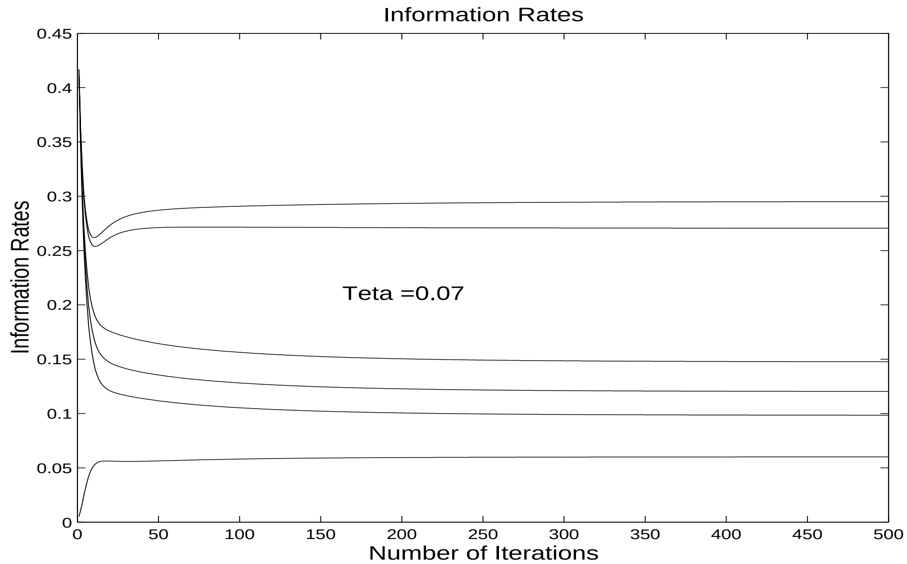

Figure 2.19: Weighted Hierarchical Max-Min Fair Rates Approach 2

us to consider various fairness criteria, namely weighted α-proportional fairness, weighted max-min fairness (which is a limiting form of weighted α-proportional fairness), and hierarchical max-min fairness. We found that max-min fairness may lead to inefficient rate allocations, and that α-proportional fairness can provide a good compromise between efficiency and fairness. We examined simple distributed algorithms that result in a α-proportional fair rate assignment. Finally, we showed that hierarchical max-min fairness can be achieved by a weighted max-min fair rate assignment.

The text of this section is in part a reprint of the material as it appears in The Proceedings of the 43th Annual Allerton Conference on Communication, Control, and Computing, pp. 124–126, Sept. 28–30 2005. The dissertation author was the primary researcher and the author, and the co-authors, Professor Rene L. Cruz and Professor Tara Javidi listed in this publication supervised the research which forms the basis for this section.

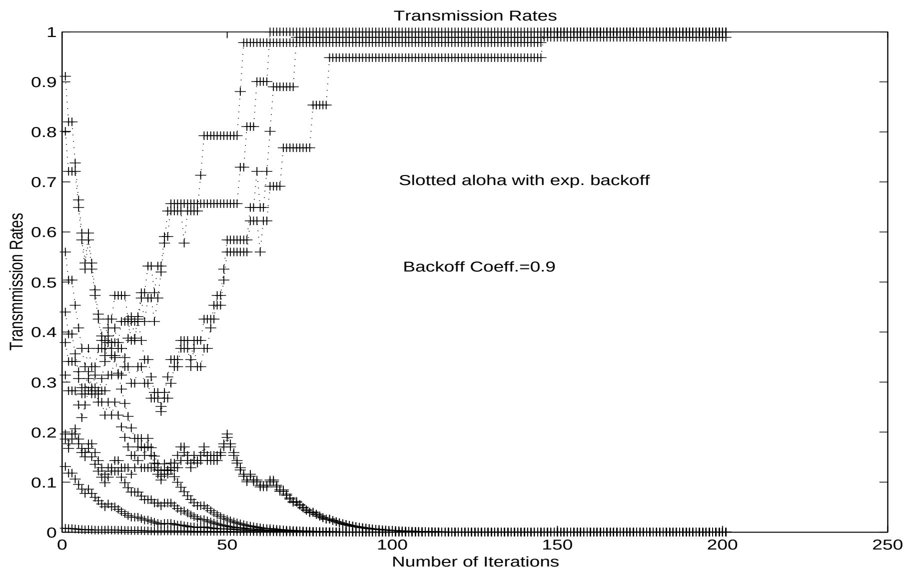

Figure 2.20: Second example network, (Slotted Aloha)

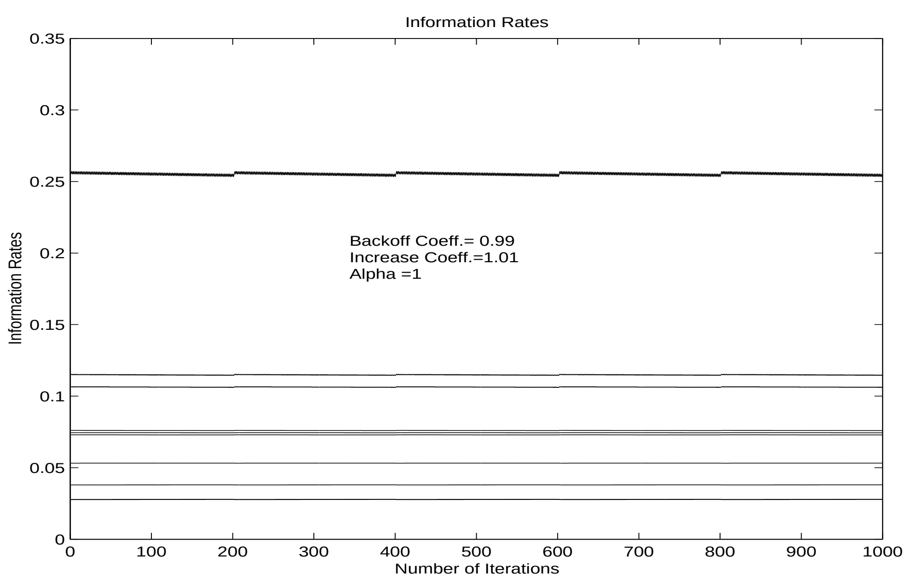

Figure 2.21: Second example network, (MIMD-α)

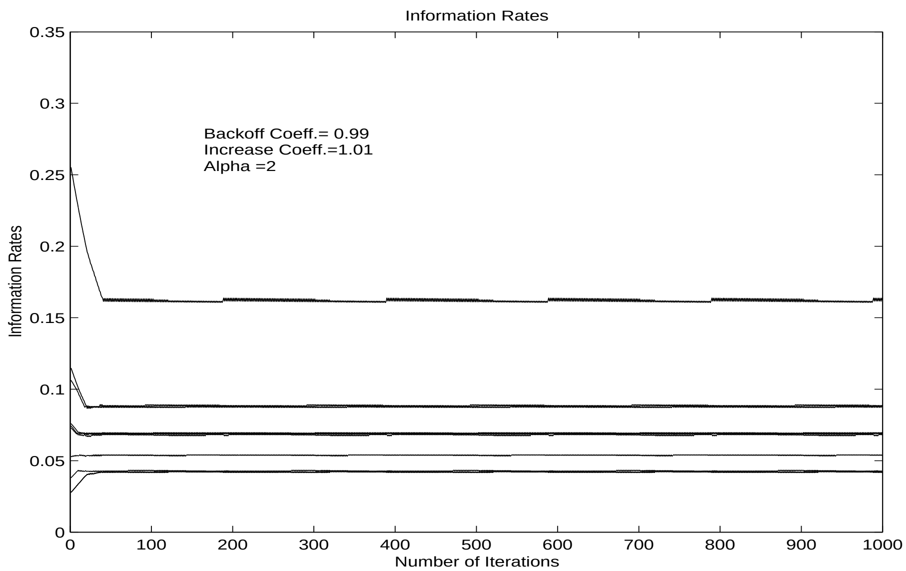

Figure 2.22: Second example network, (MIMD-α)

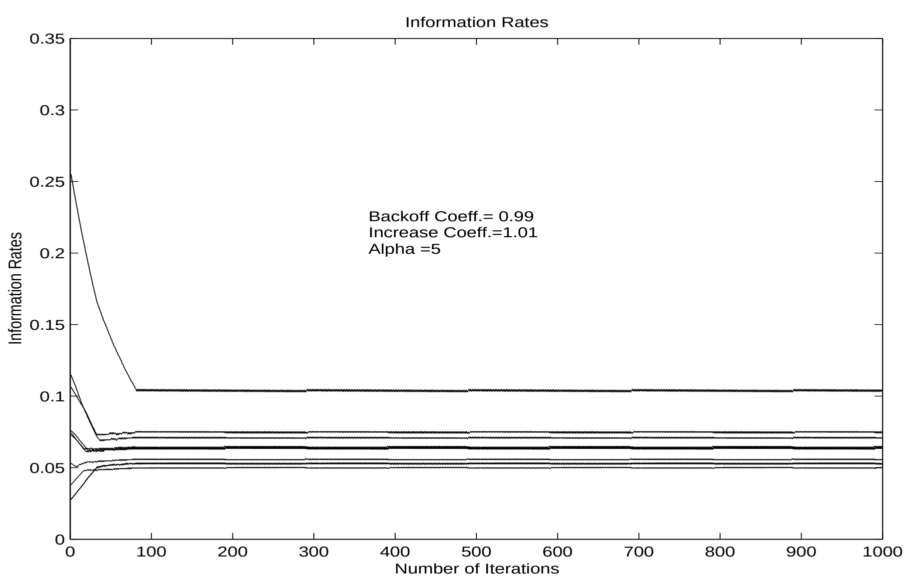

Figure 2.23: Second example network, (MIMD-α)

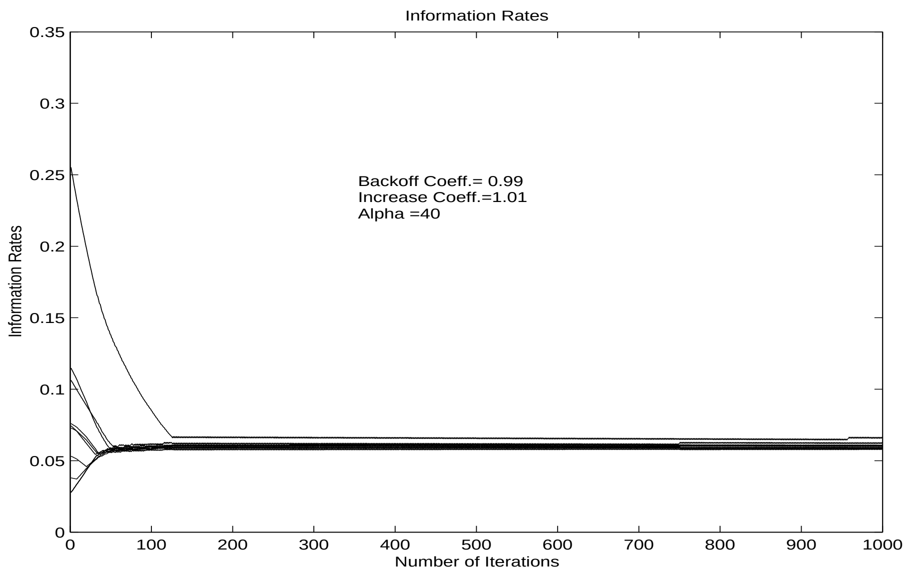

Figure 2.24: Second example network,(MIMD-α)

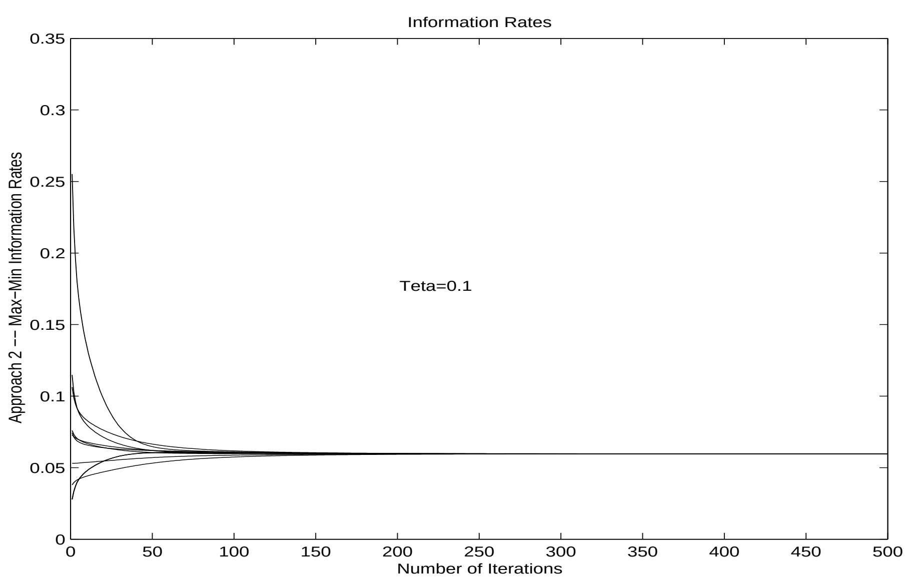

Figure 2.25: Second example network,Approach 2 – Max-Min Fair

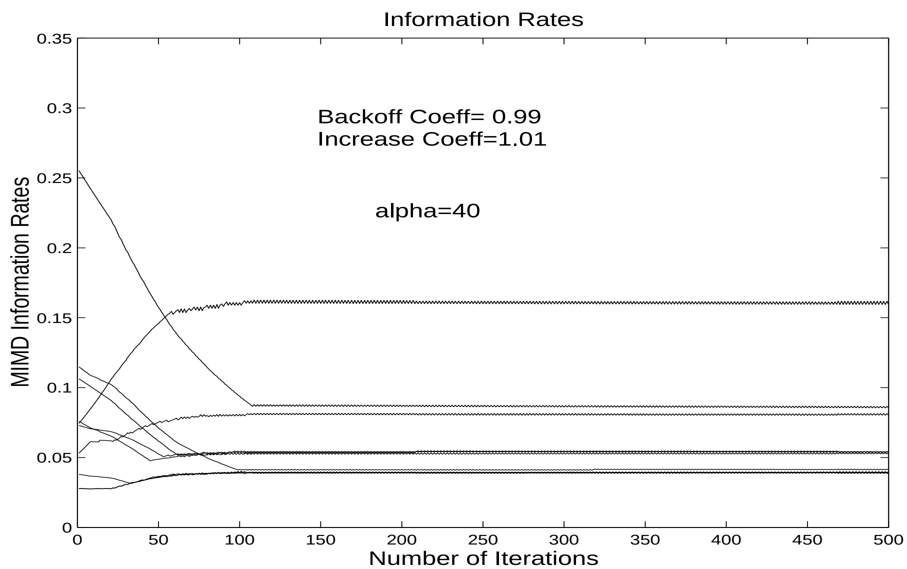

Figure 2.26: Hierarchical Max-Min Fair Information Rates Approach 1

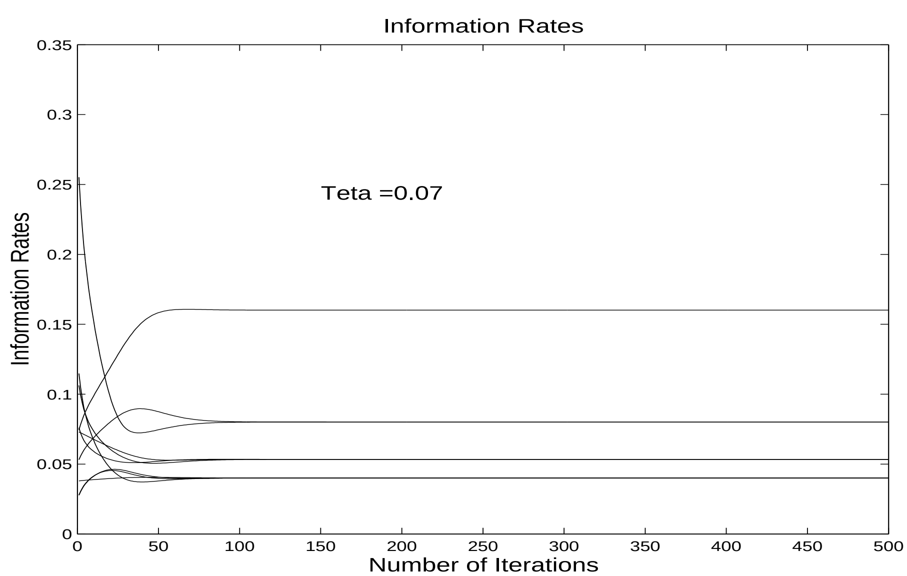

Figure 2.27: Hierarchical Max-Min Fair Information Rates Approach 2

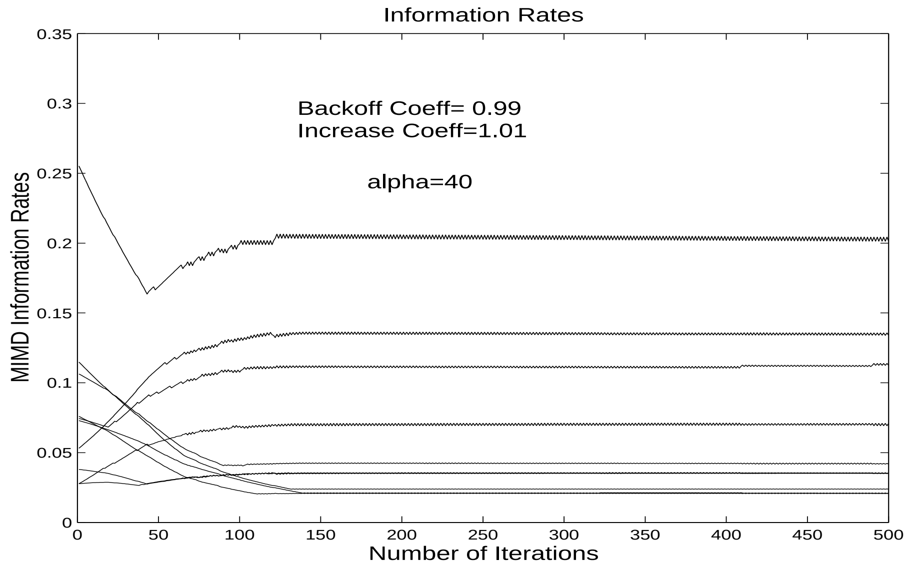

Figure 2.28: Weighted Hierarchical Max-Min Fair Rates Approach 1

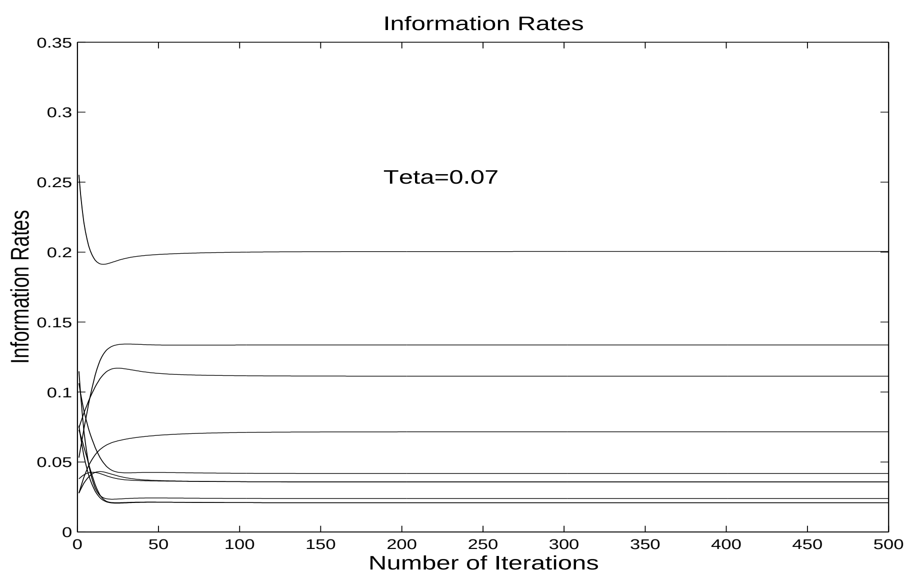

Figure 2.29: Weighted Hierarchical Max-Min Fair Rates Approach 2

3

End-to-End and Mac-Layer Fair Rate Assignment in Interference Limited Wireless Access Networks

Multi-hop wireless access networks, with their easy and cost effective deployment and reconfigurability features, are getting attention for many potential applications as the last mile solution. The potential applications of multi-hop wireless access networks include public safety, military and community access networks. In community network projects, a high capacity gateway providing Internet connection is located in the neighborhood and the residents are able to reach internet over a multi-hop wireless access network. Similarly, in public safety and military applications, in order to cooperate and coordinate the operations, the first responders and military personnel use wireless access networks.

Multi-hop wireless access mesh networking technology is still to overcome many important challenges to be widely deployed. These challenges range from range and capacity limitation of the wireless links to secure and fair resource allocation. These issues are being addressed and studied by researchers both at industry [32] and academia [35, 21].

Furthermore, leveraging high statistical multiplexing gains in a residential environment, multi-hop wireless and wired access networks are recently introduced for peer-to-peer resource sharing to Internet access such that each individual is able to utilize the fair amount of the peak bandwidth available to the entire community (e.g. neighborhood) [33]. For example, each resident having a broadband access (i.e. DSL) is able to utilize not only his/her own connection bandwidth but also those of his/her neighbors connections over a multihop wireless or wired access network [33].

In [35], authors propose a multi-hop wireless mesh architecture using 802.11 protocol utilizing two wireless cards on each node in the network. In contrast in this chapter, we focus on multi-hop CDMA or UWB based wireless access networks. Via power/interference management, we seek to provide fair rate assignment over wireless (multi-access) channels. The literature on mac-layer fairness is rich (see [34, 6, 5, 27, 28, 25]). While [34, 6, 5, 27, 28]) address the fairness issues at the mac layer over a single-hop network, [27] extends this to multi-hop scenario ignoring flow-based end-to-end fairness. Our study complements these works as it concretizes the relationship between mac-layer fairness and end-to-end fairness. Furthermore, [25] discusses the joint rate control and scheduling problem, and [26] examines joint congestion and medium access control both for multi-hop wireless networks in the context of aggregate utility maximization. In both papers, the joint problem is shown to be decomposed into two protocol layers and can be solved individually. Our work, on the other hand, specifically discusses that global max-min fair rate assignment problem (via joint transport rate and mac-layer control) decomposes such that it can be solved as independent fair rate assignment problems in each layer.

The main contributions of this study can be summarized as follows:

End-to-end (flow-based and hierarchical) global weighted max-min fairness can be achieved if and only if both weighted transport-layer and weighted mac-layer max-min fairness are ensured with appropriate weights. (The weight for each link for mac-layer fairness is a function of the weights associated to each flow.)

The remainder of this chapter is organized in the following manner: In section

2, the network model is discussed. Section 3 discusses the rate assignment problem and provides the main result of the study on mac layer vs. end-to-end fairness. In section 4, a weighted mac-layer fair algorithm is discussed. Section 5 includes discussions and examples. Finally, Section 6 concludes the chapter.

3.1 Network Models

In this section, we mainly describe a 2-channel CDMA type network model for which the main results of the chapter is presented. Furthermore, briefly, we discuss a general single-channel network model for which the main results of the study still apply.

3.1.1 Network Model 1

We consider a multi-hop access network which is formed by wireless client devices and the Access Points (APs). Each client associates to one of the APs and the APs form a mesh network together. There is also a gateway access point which provides internet connectivity. The clients are able to communicate with each other and access the Internet through this access network. We consider a Infrastructure Basic Service Set (IBSS) type architecture which is comprised of an AP and the client devices that associate to that AP. Clients are not able to communicate with each other over a direct link. They first need to send the information to the AP that they associate as in the case of 802.11 IBSS.

Each AP is considered to be a wireless bridge such that the packets are forwarded in layer 2 throughout the entire access network. The spanning tree protocol is used [23] to form a loop free topology where the learning bridge algorithm works well [23]. There are many recent papers considering loop free topologies [35] for multi-hop access networks.

In the spanning tree protocol, participating nodes first choose a root node and find the shortest path to the root using a distance vector type algorithm. Then each node includes some of its ports (links) in the spanning tree while the others are blocked in order to prevent loops in the network. In our case the gateway is assumed to be the root of the tree topology. The details of the algorithm can be found in [23].

The optimum spanning tree protocol for wireless and wired networks are also considered in many research papers [31, 29]. These issues are beyond the scope of this thesis. However, these problems can also be coupled with the rate assignment problem and are considered as future work.

Each node in the network (APs and clients) is able to utilize a single transmitter and a receiver. Both transmitter and the receiver can be tuned to 2 non-overlapping channels. Thus each node is assumed to transmit and receive simultaneously over these 2 non-overlapping channels where the inter-channel interference is neglected (Figure 3.2). The logical connections between a node and its AP or between two adjacent APs is called a link. Let $L = L_1 \bigcup L_2$ denote the set of directional links in the network where L_1 and L_2 are the set of links tuned to channel 1 and 2 respectively. Any link $l \in L$ can also be represented by the transmitter node i and the receiver node j such that $l = (i, j)$. As the case with CDMA networks, each link is given a code. In other words, the links tuned to the same channel have the ability to be active simultaneously.

It is assumed that the root node in the spanning tree allocates one of the channels to downlink and the other to uplink and advertises it to its neighbors and each AP can easily decide on the channel assignment on the up and downlinks. The spanning tree protocol can also be modified for the channel setup. On the other hand, each client device associating to one of these APs learns the channel allocation from the corresponding AP and sets the channels accordingly.

A DHCP server is available in each AP such that it is able to assign a unique address to each wireless client. In a LAN environment with multiple DHCP servers, there are mechanisms that ensure the unique address assignment [41]. We assume that the spanning tree protocol is used to form the loop free topology. The learning bridge concept and spanning tree protocol can be found in [23].

In this work, we assume that all nodes in the network are able to hear each other.

We also consider the capacity of each wireless link as a linear function of the related signal to interference noise ratio (SINR)(e.g. the low signal to noise ratio regime).

$$X_{i,j} = B\frac{P_{i,j}G_{i,j}}{\sum_{m,n\neq i,j} P_{m,n}G_{m,j} + \gamma} \tag{3.1}$$

where $X_{i,j}$ is the capacity of link i, j and i and j are the end nodes of the link. B is the bandwidth allocated for the related channel, $P_{i,j}$ is the power transmitted on link i, j and $G_{i,j}$ is the attenuation constant such that $P_{i,j}G_{i,j}$ is the received power at the receiving end of link i, j, and γ is the ambient noise power.

Each link i, j has a power budget such that

$$0 \leq P_{i,j} \leq Pmax_{i,j} \tag{3.2}$$

On the other hand, a flow is defined to be a logical connection between any mobile client device and the gateway, or between any two mobile client devices. Let F denote the set of all flows in the network. We assume that the routing determines a unique matrix $\Psi = [\Psi_{p,l}]_{|F|\times|L|}$, $\Psi_{p,l} = 1$ if $p \in F_{l=(i,j)}$ otherwise it is 0 where $F_{i,j}$ denotes the set of flows traversing link (i, j).

The rate of flow p, R_p, is the information rate that the related source node conveys to the destination node.

The rate of link $l = (i, j)$ should be greater than or equal to the aggregate rate of flows that are traversing the link such that

$$T_{i,j} = \sum_{p\in F_{i,j}} R_p \leq X_{i,j}, \quad R_p \geq 0 \quad \forall p \in F \tag{3.3}$$

We assume a transport layer protocol, given the link rates, that assigns the rates among the end-to-end flows. It is assumed that each flow has infinite demand. Independent of the end-to-end flow rates, we also assume a mac-layer protocol that sets the link capacities with respect to equation (3.1.)

3.1.2 Network Model 2

In this model, we assume a multi-hop wireless access network where still each node is able to hear each other. In other words, each link interfere with all other links in the access network. There exist a single channel that is used by all the links in the network. Routing or bridging or any other path setup mechanism is assumed. The capacity of each link, $l = (i, j)$, $X_{l=(i,j)}$ is assumed to be a strictly increasing function of the average transmitted power $ATP_{i,j}$ which is equal to $P_{i,j} \times S_{i,j}$, where $S_{i,j}$ is roughly the fraction of time that the link i, j being used. The notion of average transmitted power can describe perfectly scheduled networks as in [24] or any other mechanism like 802.11 or a CDMA type multi-hop network. On the other hand, the link rate $X_{i,j}$ is assumed to be a strictly decreasing function of the average transmitted powers of all other links (e.g. $ATP_{m,n} \; \forall \; (m, n) \neq (i, j)$).

The feasibility region for flow rates is the same as in network model 1.

3.2 Rate Assignment Problem

In this section, end-to-end global (flow based as well as hierarchical) and mac-layer rate assignment policies and the relationship between them are discussed.

3.2.1 Definitions

First, we introduce the following definitions.

Definition 1: A vector of rates R is **weighted max-min fair** with weight vector, W,if it is feasible and for each flow i, the rate of flow i, R_i, can not be increased while maintaining feasibility without decreasing R_j for some flow j for which $R_j W_j \leq R_i W_i$. As a special case, for $W_i = 1 \; \forall i$, vector R is said to be max-min fair [2].

Definition 2: A feasible vector of link rates, X is said to be **(weighted) mac-layer max-min fair** if the link rate vector, X, belongs to Y defined as:

$$Y = \{(.., X_{i,j}, ..) : X_{i,j} = B\frac{P_{i,j}G_{i,j}}{\sum_{m,n\neq i,j} P_{m,n}G_{m,j} + \gamma},$$
$$0 \leq P_{i,j} \leq Pmax_{i,j}\} \tag{3.4}$$

and is (weighted) max-min fair.

Definition 3: A vector of end-to-end flow rates, R, is said to be **(weighted) transport layer max-min fair**, given fixed routing matrix Ψ and link rate vector, X,, if R belongs to V_X as defined:

$$V_X = \{(.., R_p, ..) : \sum_{p\in F_{i,j}} R_p \leq X_{i,j} \quad \forall(i, j) \in L,$$
$$R_p \geq 0 \quad \forall p \in F\} \tag{3.5}$$

and is (weighted) max-min fair.

Definition 4: Given a routing matrix, Ψ, a vector of end-to-end flow rates is said to be **end-to-end global flow-based max-min fair**, if the flow rates are chosen from set S: $S = \bigcup_{\forall X\in Y} V_X$ and is max-min fair.

Unlike the transport-layer fairness, in global fairness the link rates are not assumed to be given. In our wireless access network the capacity of a link is a function of the other link capacities, therefore in order to enforce the global fairness we need to compute both the link rates and the flow rates. In Figure 3.1, a sample network with two flows are shown. A max-min fair protocol is assumed to be running in the transport layer. It can be seen that different power allocations among the links leads to different transport rates. For instance, the transmit power allocation vector

$P = [100mw; 100mw; 100mw; 100mw; 100mw]$ with bandwidth, $B = 10MHz$ and $\gamma = 1mw$, and distance $d = 5meter$ leads to link rate vector

$X = [4.4Mb.s|8Mb.s|6.9Mb/s|7.35Mb/s|18Mb/s]$

and transport rate vector

$R = [2.2Mb/s|2.2Mb/s]$,

whereas the transmit power vector

$P = [100mw|100mw|20mw|100mw|100mw]$ leads to the link rate vector

$X = [6.9Mbs|8Mb/s|1.3Mb/s|7.3Mb/s|18Mb/s]$ lead to a transport rate vector

$R = [1.3Mb/s|5.6Mb/s]$. As can be seen in this example, having a transport-layer max-min fair protocol does not ensure the end to end global fair rate allocation. Therefore, both the link rates and the transport rates should be set jointly to achieve end-to-end global fairness.

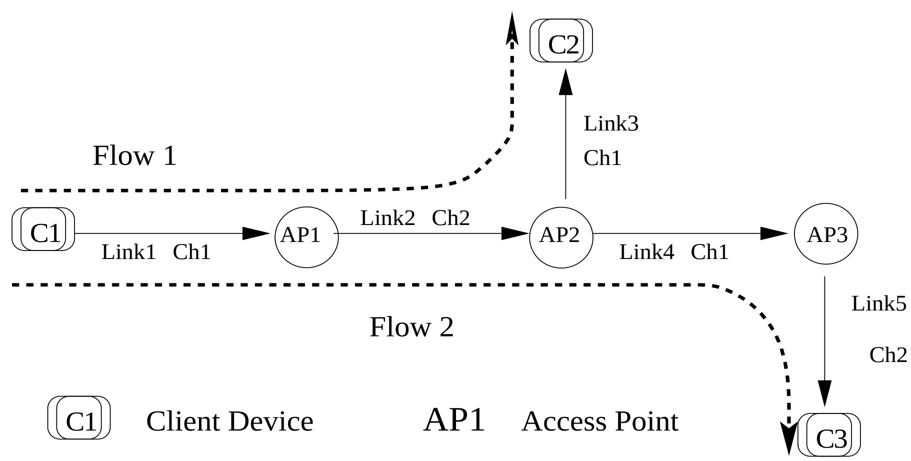

Figure 3.1: First example network

Definition 5: Given a feasible vector of flow rates, R, we say that link $l = (i, j)$ is a **weighted bottleneck link** with weight vector W with respect to R for a flow p traversing link l , if $T_{i,j} = X_{i,j}$ and $R_pW_p \geq R_qW_q$ for all the flows q traversing link l. If all the weights are equal to 1 then link l is said to be a **bottleneck link** as in [2].

3.2.2 Weighted End-to-End Flow-Based Global Max-Min Fairness: Main Result

In this section, we establish a relationship between mac-layer and end-to-end global fair rate assignments in the discussed access network. We show weighted fairness at the mac-layer and weighted max-min fairness in the transport layer ensure end-to-end global weighted max-min fairness per flow, and vice versa. To prove this we need the following facts.

Fact 1: Weighted mac-layer max-min fair rate assignment in our network with weight vector W and rate vector X assigns the rates to each link tuned to the same channel such that $X_l W_l = X_d W_d \ \forall l, d \in L_i : i = 1, 2$. Note that as a special case mac-layer max-min fairness ($W_l = 1, \forall l$) assigns the same rate to all the links [14].

Proof. Assume by contradiction that link rate vector X is weighted max-min fair with weight vector W. However, without loss of generality there exist two links in the same channel, l_1 and l_2 such that $W_{l_1} X_{l_1} > W_{l_2} X_{l_2}$. If we decrease the transmitted power over link l_1 then the capacity of l_2 will increase which will contradict by the definition of the (weighted) max-min fair rate allocation. □

Fact 2: A vector of link rates X is weighted mac-layer max-min fair with weight vector W if it is achievable (i.e. in the capacity region) and it is the maximal among vectors E such that $W_l E_l = W_d E_d$ for all $\forall l, d \in L_i : i = 1, 2$. (Similarly, $E_l = E_d$ for the max-min fair case.) (A vector, V, is maximal when there is no other vector,D, of which elements are not less than those of V and at least one is strictly greater). The proof of this fact is a direct result of Fact1 and the definition of (weighted) max-min fairness.

Fact 3: End-to-end flow-based global weighted max-min fair rate assignment with weight vector W and flow rate vector R assigns the flow rates such that $R_i W_i = R_j W_j \ \forall i, j \in F$. Note that if all the weights are the same then the rate assignment in question assigns the same rates to each flow in our access network. Also note that each flow in our network utilizes both the channels. The proof for all the weights equal to 1 has the same logic as in [14].

Proof. By contradiction assume that the vector of flow rates, R, is end-to-end flow-based global weighted max-min fair with weight W, but that, there exist two flows i and j such that $W_i R_i > W_j R_j$. Let Z_i and Z_j be the set of links that flow i and j traverses respectively. The following steps can be performed while maintaining feasibility for some value of δ and ϵ.

(1) Decrease R_i by δ

(2) $\forall l \in Z_i \setminus Z_j$; Decrease X_l by δ

(e.g. by decreasing P_l)

(3) Increase R_j by ϵ

(Second step increases the link rates, $X_l : l \in Z_j \setminus Z_i$)

By this way, the value of R_j is increased without decreasing any other flow rate of which weighted value is already smaller than or equal to $W_j R_j$ which is a clear contradiction to the end-to-end global flow-based weighted max-min fair rate allocation. □

Fact 4: If a feasible flow rate vector, R, is end-to-end global flow-based weighted max-min fair with weight vector W then there exists a channel, i, such that with respect to R, each flow has a weighted bottleneck link, l (with W), tuned to channel i and each such link l (i.e $\forall l \in Li$) is a weighted bottleneck link (with W)for some flow in the network and the link rate vector of channel i is maximal.

Proof. **Part a)** Assume by contradiction that the vector of rates, R, is end-to-end global flow-based weighted max-min fair with weight vector W but there exists at least one link on both channels (i.e. $l_1 \in L_1$ and $l_2 \in L_2$) which are not weighted bottleneck for any flow with respect to R, that is, $T_{l_i} < X_{l_i}$ $\forall i$. Let ll_b be the set of all weighted bottleneck links (for some flows). So decreasing the capacity of l_1 and l_2 (e.g. decreasing the transmitted power) will increase the capacity (dedicated by the mac layer) of all the links in ll_b. The rates of all the flows, for which the links in ll_b are bottleneck links, can be increased while maintaining feasibility which clearly contradicts the end-to-end global weighted max-min fair definition.

Part b) Furthermore, assume by contradiction again that R is end-to-end global flow-based weighted max-min fair with vector W but there exist two flows f_1 and f_2 such that the former one does not have any bottleneck on any link tuned to channel j (e.g. $j = 2$) and the latter one does not have any bottleneck link on channel c (e.g. $c = 1$). Let $Lnk_a^{f_k}$ be the set of links tuned to channel a that flow f_k traverses.

From Fact 3, we know that end-to-end global weighted max-min fairness assigns rates to all the flows inversely proportional to their weights in our access network. So for any link $l \in Lnk_j^{f_1}$, $X_l < T_l$ and for any link $l \in Lnk_c^{f_2}$, $X_l < T_l$ for otherwise those links would be bottleneck links for f_1 and f_2 respectively. The rest of the proof is the same as part (a).

The proof for maximality of the link rate is relatively easy, since if the link rate vector X is not maximal then there should be some other link rate vector $X\prime$ which has strictly larger link rates as its elements, and this latter link rate vector can result in a flow rate vector $R\prime$ which is greater than R in all elements which again contradicts with the global weighted max-min fair definition.

$\square$

Now we provide the main result of this section.

Let W be the weight vector with which R (the flow rate vector) is end-to-end global flow-based weighted max-min fair.

Let N denote a vector such that
$N = ((n_1)^{-1}, ..., (n_l)^{-1}, ...(n_{|L_b|})^{-1})$ where $n_l = \sum_{f \in F_{l=(i,j)}} (W_f)^{-1}$ and $F_{i,j}$ denotes the set of flows traversing link (i, j) and $|L_b|$ is the number of bottleneck links.

Theorem 1: In our access network, end-to-end flow-based global weighted max-min fair rate allocation, R, with weight vector W can be achieved if and only if transport-layer weighted max-min fairness with weight vector W and the weighted mac-layer max-min fairness with weight vector N (among the resulting bottleneck links with respect to R) are ensured.

Proof. We first show that end-to-end flow-based global weighted max-min fairness with weight vector W leads to weighted mac-layer max-min fairness with weight vector N and transport-layer weighted max-min fairness with weight vector W. Note that throughout the proof, the links discussed are the weighted bottleneck links with weight vector W as defined in Definition 5.

First, we can easily show that if a vector of flow rates R is end-to-end global flow-based weighted max-min fair with W then it is also transport-layer weighted max-min fair with W. The proof follows easily by contradiction. Assume that the vector R is global weighted max-min fair but not transport-layer weighted max-min fair with the same weight vector. Then there should exist a flow i, of which rate R_i can be increased without decreasing R_j for some flow j for which $W_iR_i \geq W_jR_j$, which is also a contradiction to the end-to-end global flow-based max-min fairness.

Next, by contradiction assume that the vector of end-to-end flow rates, R, is both end-to-end global weighted max-min fair and transport-layer weighted max-min fair with W but the corresponding link capacity vector, X, is not weighted max-min fair with weights N.

The end-to-end global flow-based weighted max-min fair rate assignment assigns rates to each flow inversely proportional to their weights,i.e. $W_qR_q = W_pR_p = r \quad \forall p, q \in F$ (see Fact3). In the presence of end-to-end weighted max-min fairness, the resulting aggregate flow rate traversing link $l = (i, j)$, T_l, will be

$T_l = \sum_{p \in F_l} R_p = \sum_{p \in F_l} (W_p)^{-1} r = r \times n_l$.

Since we are discussing the bottleneck links then

$T_l = X_l = r \times n_l \; \forall l \in B$.

It can easily be seen that the link capacities satisfy the condition in Fact 2 for weighted fairness such that $X_{i,j}N_{i,j} = X_{a,b}N_{a,b}$ where $N_{i,j}$ denotes the element of vector N corresponding to link i, j.

However, by contradiction we have assumed that the vector X is not weighted max-min fair with weight N, so according to Fact 2, vector X can not be maximal, for otherwise it would be the weighted max-min fair rate.

Therefore, one can increase all the link rates in the network and this increase can be easily mapped into an increase in all the end-to-end flow rates which contradicts with the end-to-end global weighted max-min fair rate assumption that we had in the beginning. Thus, we have shown so far that if a vector of flow rates is end-to-end flow-based global weighted max-min fair with W then it is also transport-layer weighted max-min fair with W, and the corresponding vector of link rates is weighted mac-layer max-min fair with weight vector N.

Conversely, if we assume that the link rate vector X is weighted max-min fair with weights N and we assume transport layer weighted max-min fairness with weight vector W, then all the flow rates will be assigned inversely proportional to their weights. By contradiction assume that the end-to-end flow rate vector R is not weighted max-min fair with W globally. Then it is possible increase all the R_is without decreasing any of them (since we know that end-to-end flow-based global max-min fairness indeed assigns rates to all the flows inversely proportional to the corresponding weights). This actually requires an increase in all the link rates inversely proportional to their link weights which contradicts with the weighted mac-layer max-min fair assumption. □

(This theorem is valid for both network model 1 and network model 2.)

This result is interesting in the sense that there is small interaction between the mac layer and the transport layer. The only information that the mac layer needs to know is the sum of the inverse weights of the flows that are passing through (For $W_i = 1 \forall i$, that is the max-min fair case, the only information to be passed to the mac-layer is the number of flows passing through). Then the weighted max-min mac-layer scheme with appropriate weights and with a weighted max-min fair transport protocol leads to a weighted fair flow rate allocation in the end-to-end and global manner.

In the next section, we discuss hierarchical global weighted max-min fairness which can be achieved by end-to-end flow-based global weighted max-min fairness with appropriate choice of the system parameters.

3.2.3 End-to-End Hierarchical Weighted Max-Min Fair Rate Assignment

Although flow based weighted max-min fairness is the classical way of studying the fairness problem, in the real world different fairness variations may appear.

Considering a community network application as in [33] where each IBSS belongs to a resident, each resident participating the access network would like to have a fair share of the overall bandwidth which is set according to what they pay for their Internet access speed. In an access network where end-to-end flow based weighted max-min fairness is enforced, a resident utilizing a higher number of connections will have a higher share of the overall bandwidth with respect to the ones having a smaller number of connections. By enforcing hierarchical fairness each resident is ensured a fair share of the network bandwidth first and then within the same IBSS the fairness among the flows can be enforced.

Therefore, in our network, one of the interesting fairness criterion may be a hierarchical fairness such that fairness is first ensured among the Infrastructure Basic Servive Sets (IBSS) (i.e among the set of flows utilized by different IBSSs) and then among the individual flows in the same IBSS.

Definition 6: Let M_a be the set of flows belonging to subgroup a such that $\bigcup_a M_a = F$ and $M_a \bigcap M_b = \emptyset \; \forall a, b : a \neq b$. Let D be the rate vector where the ath element denotes the aggregate information rate of subgroup a : $D_a = \sum_{i \in M_a} R_i$. Lastly, let H_a denote the vector of rates of the individual flows belonging to M_a.

A vector of flow rates, R, is said to be **weighted hierarchical max-min fair** with a weight vector Z, and a vector of vectors $V = (V_1, V_2, ...V_T)$, where T is the number of subgroups, and V_a is the weight vector for subgroup a such that $V_a = (V_{a,1}, V_{a,2}...V_{a,L_a})$ (where L_a is the number of flows in subgroup a), if first the vector, D, is weighted max-min fair with weight vector Z and the rate vector for flows in each subgroup a, H_a, is weighted max-min fair with weight vector V_a.

More formally, a vector, R, is said to be **weighted hierarchical max-min fair** with a weight vector Z, and a vector of vectors $V = (V_1, V_2, ...V_T)$, if it is feasible and if for each flow, $i \in M_a$, the rate, R_i, can not be increased, while maintaining feasibility without decreasing R_j for some flow $j \in M_b$ for which $V_{b,j} R_j \leq V_{a,i} R_i$ when $a = b$ or $Z_b D_b \leq Z_a D_a$ when $a \neq b$. When all the weights are all equal to 1, the rate vector R is called Hierarchical Max-Min fair as similarly defined in [21].

Fact 5: Using similar arguments for mac-layer and end-to-end flow based max-min fair rate assignment policies, it can be claimed that such a hierarchical fairness policy leads to the aggregate flow rates for each IBSS such that $Z_a D_a = Z_b D_b \; \forall a, b$. Again using the same argument, we can claim that each flow belonging to the same IBSS have rates as follows, $V_{a,i} R_i = V_{a,j} R_j \; \forall a$ and $\forall i \in M_a$.

Proof. By contradiction assume that a vector of flow rates R is hierarchical weighted max-min fair; however, there exits at least two groups of flows a and b of which total rates, D_a and D_b are such that $Z_a D_a > Z_b D_b$. Let there exist two flows $i \in M_a$ and $j \in M_b$. Considering the proof for Fact 3, one can increase R_j as well as D_b with only decreasing R_i as well as D_a which contradicts the definition of hierarchical fairness.

In addition to this, again by contradiction assume that R is Hierarchical weighted max-min fair, but in a group k there exist at least two flows $i \in M_k$ and $j \in M_k$ such that $V_{k,i} R_i < V_{k,j} R_j$. The situation results in the same contradiction as in Fact 3. □

Fact 6: A vector of flow rates R is hierarchical weighted max-min fair with Z and V as defined above, if it is achievable (i.e. in the capacity region) and corresponding D vector is the maximal among vectors, E, such that $Z_a E_a = Z_b E_b$ $\forall a, b$ and given the D vector, for each subgroup a, the related vector H_a is the maximal among the vectors, B^a, such that $V_{a,i} B_i^a = V_{a,j} B_j^a$ $\forall i, j \in M_a$. (It is a direct result from the previous fact on hierarchical fairness.)

Let Q be the weighted rate of each subgroup (e.g. resident) utilizes such that $Q = Z_a D_a$ $\forall a$ where the weighted hierarchical fairness policy with Z and V is enforced. Then the aggregate rate that the subgroup a utilizes will be $\frac{Q}{Z_a}$, whereas the rate of flow i, R_i, in the same subgroup will be $R_i = \frac{V_{a,j} R_j}{V_{a,i}} \forall i$. From the definition of D_a, $D_a = \sum_{j \in M_a} R_j = \sum_{i \in M_a} \frac{V_{a,j} R_j}{V_{a,i}} = \frac{Q}{Z_a}$.

Therefore, the rate of each flow $i \in M_a$ can be written as $R_i = \frac{Q}{Z_a V_{a,i} (\sum_i \frac{1}{V_{a,i}})}$

Let $s(i)$ denote the subgroup to which ith flow belongs and let

$K_{a,i} = Z_a V_{a,i} (\sum_i \frac{1}{V_{a,i}})$.

Let W be the vector such that

$W = ((K_{s(1),1}), (K_{s(2),2}), (K_{s(3),3}), ...(K_{s(|F|),|F|}))$ where $|F|$ is the number of all flows.

(Considering the above discussion, we can safely say that any weighted hierarchical max-min fair vector of flow rates, R, satisfies the condition $R_i W_i = R_j W_j$.)

Theorem 2: End-to-end hierarchical global weighted max-min fairness with Z and V can be achieved if and only if end-to-end flow-based global weighted max-min fairness with weight vector W is ensured.

Proof. Assume by contradiction R is weighted hierarchical max-min (then $R_i W_i = R_j W_j$) but not end-to-end global flow-based weighted max-min with weight vector W. This implies that R is not maximal (otherwise it would be end-to-end global flow-based weighted max-min) which contradicts the definition of weighted hierarchical max-min. Conversely, assume R is end-to-end global flow-based weighted max-min with W, but not weighted hierarchical max-min with Z and V, then either the conditions described in Fact 6 on element of vectors D and H_as do not hold, or the vectors are not maximal. In either case there is a contradiction to the definition of weighted max-min fair rates with vector W.

□

Corollary: Using the results above, it can easily be seen the a vector of flow rates R is hierarchical weighted max-min fair, if and only if it is transport layer weighted max-min fair with weight vector W (as defined above) and weighted mac-layer max-min fair with weight vector N, where the lth element of vector N, N_l, equals $(\sum_{f \in F_l}(W_f)^{-1})^{-1}$

We have seen that both hierarchical and flow-based global weighted max-min fair rate allocations can be achieved if and only if weighted max-min transport layer and weighted max-min mac layer fairness policies are enforced with appropriate weights.

In the next section, we would like to propose a mac-layer scheme that is able to achieve the weighted mac-layer max-min fairness we are looking for.

3.2.4 Mac-Layer Algorithm Ensuring End-to-End Global and Mac-Layer Fairness

In this section, we discuss a mac-layer algorithm that enforces mac-layer weighted α-proportional and mac-layer weighted max-min fair rate assignments.

As we discuss in the previous sections (i.e. Theorems 1 and 2), with appropriate weights, the mac-layer weighted max-min fair rate assignment results in link capacities such that with appropriate higher layer mechanisms (i.e. (weighted) max-min transport layer protocol), end-to-end (flow based and hierarchical) max-min fairness is achieved globally. In addition to this, considering Fact 4, that is, all the links tuned to a single channel are bottleneck links for all the flows in the network, all we need to have is a mac-layer scheme that ensures weighted mac-layer fairness among the links tuned to the same channel.

In this section, we begin with a discussion of a mac-layer weighted α-proportional fair rate assignment. Next, we introduce a mac-layer algorithm, $MAC - \alpha - G$, which is a general mac-layer algorithm where with appropriate choice of parameters α and G, not only mac-layer α-proportional fairness but also end-to-end max-min (flow based and hierarchical) fairness can be achieved.

Mac-layer Weighted Max-Min Fair Rate Assignment (End-to-End Global Fair Rate Assignment)

In this section, we discuss an approach to compute the weighted max-min fair link rates in a distributed manner.

Definition 8: Similar to [8], we consider a generalization of proportional fairness by considering a different utility function for each entity. A vector of rates, R, is **α-proportional fair** if it maximizes the sum of the utilities when the utility function for entity i is $-(-ln(R_i))^\alpha$

Weighted α-Proportional Fairness Approach:

The overall optimization problem for weighted max-min fairness is

$$\textbf{Maximize} \quad \sum_{i,j} U(X_{i,j}) = \sum_{i,j} -\frac{1}{g_{i,j}}(-ln(g_{i,j}X_{i,j}))^\alpha \tag{3.6}$$

$$\textbf{Subject To:} \quad 0 \leq P_{i,j} \leq P_{max} \tag{3.7}$$

where $X_{i,j}$ is defined in equation (3.1) and $g_{i,j}$ is a weight associated to the link (i, j) to ensure the weighted fairness.

CLAIM: The system trying to maximize the aggregate utility function $U = \sum_{i,j} U(X_{i,j})$ leads to a weighted α-Proportional fair rate assignment. Weights equal to 1 and $\alpha = 1$ corresponds to the mac-layer proportional fair case whereas as α goes to infinity the system converges to the weighted max-min fair rate assignment

Proof. The utility function for each link l, $U_l(X_l) = -\frac{1}{g_l}(-ln(g_l X_l))^{\alpha}$ for all $0 < x < 1$ and $\alpha \geq 1$ is an increasing function of X_l. The formulation above maximizes the sum of the individual utility functions.

Assume an arbitrary rate assignment to any two links l_1 and l_2 with weights g_1 and g_2 respectively such that $0 < g_1 X_{l_1} < g_2 X_{l_2} < 1$.

Then the ratio of the partial derivatives of each utility function with respect to the corresponding link rate goes to infinity as α goes to infinity.
$\frac{U\prime_{l_1}(X_{l_1})}{U\prime_{l_2}(X_{l_2})} = \frac{g_2 X_{l_2}}{g_1 X_{l_1}}(\frac{ln(g_1 X_{l_1})}{ln(g_2 X_{l_2})})^{\alpha-1} \rightarrow \infty$ as $\alpha \rightarrow \infty$

For sufficiently large α, any increase in X_{l_1} and decrease in X_{l_2} will increase the overall utility function. So the optimum value of this non-linear programming problem satisfies the condition $g_1 X_{l_1} = g_2 X_{l_2}$ for any two link l_1 and l_2, and since the utility function is an increasing function of each link rate, the optimum value should be a maximal rate vector. These two conditions are necessary and sufficient conditions for weighted mac-layer max-min fairness as described in Fact 2.

□

The above problem is not a convex programming problem (CPP). Using the change of variable $P_{a,b} = e^{Z_{a,b}}$, the problem can be converted into a CPP such that $Y_{i,j} = Z_{i,j} + ln(g_{i,j} W G_{i,j}) - ln(\sum_k (e^{Z_{m,n}} G(m,j)) + \gamma)$ is a concave function. (log-sum-exp function is a convex function [22] and an affine composition of any such function is also convex [22]). The constraint is also convex as $0 \leq e^{Z_{a,b}} \leq P_{max}$.

In addition to that $\frac{1}{g_{i,j}}(-Y_{i,j})^{\alpha}$ can be shown to be a convex function of the logarithmic transmitted power strengths (e.g. $Z_{i,j} = ln(P_{i,j})$) where $0 \leq g_{i,j} X_{i,j} \leq 1$ and $\alpha \geq 1$. The proof can be found in appendix B.1.

We use the gradient projection method [3] to solve the above optimization problem such that

$$Z_{i,j}^{n+1} = [Z_{i,j}^{n} + \theta^{n} \frac{\partial U}{\partial Z_{i,j}}]^{+} \tag{3.8}$$

where n denotes the iteration number and U is the sum of all utility functions and $[f]^+$ denotes projection on the set $0 \leq P_{i,j} \leq P_{i,j}^{max} \forall i$ which is equal to $min(max(0,f), P_{i,j}^{max})$ and

$$\frac{\partial U}{\partial Z_{a,b}} = \alpha(\frac{1}{g_{a,b}})(-ln(g_{a,b}X(a,b)))^{\alpha-1} - \sum_{k,l \neq a,b} \alpha \frac{e^{Z_{a,b}} G_{a,l}}{\sum_{q,r \neq k,l} e^{Z_{q,r}} G_{q,l} + \gamma} (\frac{1}{g_{k,l}})(-ln(g_{k,l}X_{k,l}))^{\alpha-1} \tag{3.9}$$

$$f_{a,b} = 1 - \sum_{k,l \neq a,b} \frac{e^{Z_{a,b}} G_{a,l}}{\sum_{q,r \neq k,l} e^{Z_{q,r}} G_{q,l} + \gamma} (\frac{g_{a,b}}{g_{k,l}})(\frac{ln(g_{k,l}X_{k,l})}{ln(g_{a,b}X_{a,b})})^{\alpha-1} \tag{3.10}$$

we can say that the equation (3.9) becomes negative when $f_{a,b} < 0$ or vice versa.

The above iterative algorithm based on gradient projection method corresponds to a distributed algorithm that targets the weighted α-proportional fair rates (e.g. asymptotically weighted max-min fair rates). Here we present a mac-layer algorithm where the increase and decrease coefficients are assumed to be small enough for convergence. The decision whether to increase or decrease the power is based on the gradient projection method. Having the coefficients small enough, the gradient projection method is shown to converge to one of the stationary points (the point where the gradient is zero) [3]. Here since the problem is a CPP it has only a single stationary point.

As a feedback mechanism we assume that each link in the network advertises its weight, $g_{i,j}$, its current capacity, and the power it receives from all other links that it is able to hear.

However, in case of a static network where the wireless nodes know the exact location of each other, the only information that is to be advertised by each link is the associated weight value. Each wireless node is assumed to measure the received signal strength from other links in the network and estimate easily all the received signals at other nodes and the capacities of all the links in the network using equation (3.1). In other words, each link, in a distributed manner, can compute equation (3.10) by only knowing the weights of the other links and measuring the received signals from all other links and using the equation (3.1). Thus a distributed algorithm only advertising the weight values can be designed based on the above discussion in static multi-hop wireless networks.

Given any arbitrary initial power vector, P, at each iteration n, each link (a,b) computes the value of $f_{a,b}(P)^n$ and increases its transmit power, P_k^n, if $f_{a,b}(P)^n$ is positive or decrease if it is negative.

The whole power control scheme which we call MAC-$\alpha - G$ for link a, b can be written as follows.

MAC-$\alpha - G$ Algorithm

STEP1: Initially start with a random transmission power at each link.

STEP2: At iteration n

IF $f_{a,b}^n > 0$ (or a positive threshold) then

Increase $P_{a,b}$ (e.g. $P_{a,b}^{n+1} = P_{a,b}^n + \beta$)

ELSE IF $f_{a,b}^n < 0$ (or a negative threshold) then

Decrease $P_{a,b}$ (e.g. $P_{a,b}^{n+1} = P_{a,b}^n - q$)

ELSE Do not change $P_{a,b}$ (e.g. $P_{a,b}^{n+1} = P_{a,b}^n$)

where β and q are appropriate constants.

As mentioned previously, the mac-layer algorithm (i.e MAC-$\alpha - G$) (providing mac-layer max-min fairness with weight vector G) can be used with a weighted transport layer protocol to ensure end to end global max-min fairness.

An alternative problem formulation and algorithm is discussed in the next section.

Duality Approach:

The method used here is the same as in [5], where authors assume a simple non-capture capacity model for multi-hop adhoc aloha networks and formulate the mac-layer max-min fairness problem.

Basically, in this method, the original (weighted) max-min fair problem is converted into a convex programming problem. Using convex duality and again the gradient projection method a distributed algorithm for computing weighted mac-layer max-min fair rates is described.

Using Fact 2, the original problem for (weighted) max-min fair rate computation is reduced to the problem of maximization of the minimum link rate. That is,

$$\textbf{Maximize E} \tag{3.11}$$

$$\textbf{s.t.} \quad E < g_{i,j} X_{i,j} \quad \forall (i,j) \in L \tag{3.12}$$

$$0 \leq P_{i,j} \leq P_{max} \quad \forall (i,j) \in L \tag{3.13}$$

This problem is equivalent to the following problem and the details can be found in appendix B.2.

$$\textbf{Minimize} \quad \frac{1}{2} \sum_{a,b} (Y_{a,b})^2 \tag{3.14}$$

$$\textbf{s.t.} \quad Y_{a,b} - ln(g_{a,b}) - ln(W) - Z_{a,b} - ln(G_{a,b})$$

$$+ ln(\sum_{m,n \neq a,b} e^{Z_{m,n}} G(m,b) + \gamma) \leq 0 \quad \forall (a,b) \in L \tag{3.15}$$

$$Y_{a,b} \geq Y_{c,d} \quad \text{and} \quad Y_{a,b} \leq Y_{c,d} \quad \forall (c,d)(a,b) \in L \tag{3.16}$$

The objective function is convex and the functions in the constraints form a convex set. Thus the problem is a convex programming problem.

The Lagrange function for the above problem is

$$L(Z,y,\lambda) = \frac{1}{2} \sum_{i,j} (Y_{i,j})^2 + \sum_{i,j} \lambda_{i,j} (Y_{i,j} - ln(g_{i,j})$$

$$-ln(W) - Z_{i,j} - ln(G_{i,j}) + ln(\sum_{m,n \neq i,j} e^{Z_{m,n}} G_{m,j} + \gamma))$$

$$+ \sum_{(a,b)} \sum_{(c,d) \neq (a,b)} (Y_{a,b} - Y_{c,d}) \mu_{(a,b),(c,d)} \tag{3.17}$$

Convex duality applies, which implies that there is no duality gap; that is the solution to the primal problem is the same as the solution to the dual problem.

The dual function is $D(\lambda, \mu) = Min_{Z,Y} L(Z, Y, \lambda, \mu)$ and the dual problem is $max_{\lambda,\mu} D(\lambda, \mu)$

After minimizing the Lagrange function we have the following results.

$$\frac{\partial L}{\partial Y_{k,l}} = 0 \Longrightarrow$$

$$Y_{k,l} = [-\lambda_{k,l} - \sum_{(m,n)\neq(k,l)} \mu_{(k,l),(m,n)} - \mu_{(m,n),(k,l)}]^{-} \tag{3.18}$$

$$\frac{\partial L}{\partial Z_{k,l}} = 0 \Longrightarrow$$

$$-\lambda_{k,l} + \sum_{a,b\neq k,l} \frac{\lambda_{a,b} e^{Z_{k,l}} G_{k,b}}{\sum_{m,n\neq a,b} e^{Z_{m,n}} G_{m,b} + \gamma} = 0 \quad \forall (k,l) \in L \tag{3.19}$$

The dual problem can be solved using the gradient projection method.

$$\lambda_{i,j}^{n+1} = [\lambda_{i,j}^{n} + \theta^{n} \frac{\partial D}{\partial \lambda_{i,j}}]^{+} \tag{3.20}$$

$$\mu_{(a,b),(c,d)}^{n+1} = [\mu_{(a,b),(c,d)}^{n} + \theta^{n} \frac{\partial D}{\partial \mu_{(a,b),(c,d)}}]^{+} \tag{3.21}$$

where $[f]^{+}$ is again turns out to be $max(0, f)$ and

$$\frac{\partial D}{\partial \lambda_{i,j}} = Y_{i,j} - ln(g_{i,j}) - ln(W) - Z_{i,j} - ln(G_{i,j})$$

$$+ln(\sum_{m,n\neq i,j} e^{Z_{m,n}} G_{m,j} + \gamma) \tag{3.22}$$

$$\frac{\partial D}{\partial \mu_{(a,b),(c,d)}} = Y_{a,b} - Y_{c,d} \tag{3.23}$$

3.3 Discussions and Examples

In this section, we consider a single gateway access network where there are 3 Access Points and 5 client devices, as in Figure 3.2.

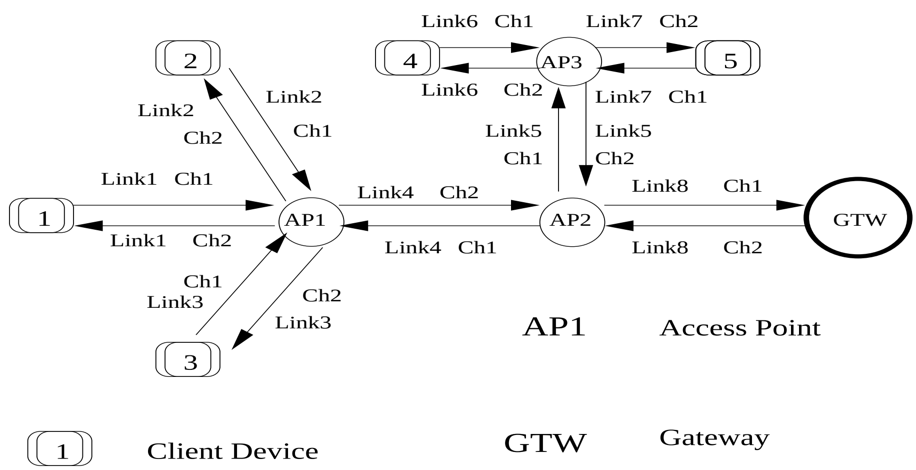

Figure 3.2: First example network

The attenuation constant is modelled as $G_{i,j} = d_{i,j}^{-n}$ where $d_{i,j}$ is the distance between nodes i and j and n is assumed to be 2. Link capacities presented are normalized between zero and one unit of capacity.

We first consider the following scenario where each link ($l = (i, j)$) is assumed to have an infinite amount of traffic to transmit from the transmitter node i to the receiver node j all the time. Each of the links are assumed to run the MIMD-$\alpha - G$ algorithm where all the weights and α are all equal to 1 (mac-layer fairness scenario).

The resulting rates of each link tuned to channel 1 are illustrated as the value of α increases from 1 to 60 in Figures 3.3 and 3.4 respectively. As can be seen in the figures as the value of α increases the capacities on each link converges to the same value which is consistent with the max-min fair rate assignment fact (i.e. Fact 1).

Next, we consider a traffic scenario where the wireless clients number 1,2,3 and 5 have infinite traffic demand to the outside world through the gateway node. Here the end-to-end flow-based global max-min fair rate allocation and the end-to-end hierarchical global fairness are examined.

Consider the end-to-end flow-based case: The link weight, $g_{l=(i,j)}$ for a link l is as follows (1 over the number of flows traversing): For Channel 1, $g_1 = 1$; $g_2 = 1$; $g_3 = 1$; $g_8 = 1/4$; $g_7 = 1$; and for Channel 2, $g_4 = 1/3$; $g_5 = 1$;. Figure 3.5 and

3.6 show the MAC-$\alpha - G$ allocations for large α ($\alpha = 60$) such that link rates converge to the weighted max-min fair rates in the mac layer with the weights given above. The resulting link capacities are available in Figure 3.5 for channel 1 and in Figure 3.6 for channel 2 respectively. As can be seen, the capacities assigned on each link are inversely proportional to the weights assigned to that link (i.e. for channel 1 X=[0.0192,0.0196,0.0197,0.0216,0.0848] and for channel 2 X=[0.0992,0.0317]) which is consistent with Fact 1. In this case, channel 1 includes the bottleneck links which assigns lower rate (0.02 unit capacity) to a single flow than channel 2 does (0.03 unit capacity) in the presence of a max-min fair transport protocol.

Here we see the need for hierarchical fairness. If such a network is deployed in a community network where each AP is located in a residence, then a flow based max-min fair rate assignment assigns to residence 1, 3 times the bandwidth (0.06 unit capacity) it assigns to residence 3 (0.02 unit capacity). This is not desirable for such applications.

Instead, the hierarchical fairness model can be more appropriate to consider, where each residence or AP has the fair share of the network capacity.

In the case of a hierarchical max-min fair rate assignment,we have the following link weights. For Channel 1, $g_1 = 3$; $g_2 = 3$; $g_3 = 3$; $g_8 = 1/2$; $g_7 = 1$;. For Channel 2, $g_4 = 1$; $g_5 = 1$; In Figures 3.7 and 3.8 the corresponding rates for links tuned to channel 1 and channel 2 are available respectively. Each IBSS (or AP) is given around 0.0450 unit capacity, where each flow of IBSS1 (or AP1) gets around 0.0150 unit capacity while single flow of IBSS3 (or AP3) gets 0.0450 unit capacity by itself. Channel 1 has all the weighted bottleneck links also in this case, whereas channel 2 offers slightly higher link rates (i.e. for channel 1 X=[0.0153,0.0147,0.0148,0.0477,0.0938] and for channel 2 X=[0.0544,0.0545]).

Next, we consider two examples with different weight vectors. We consider again the traffic scenario where the wireless clients number 1,2,3 and 5 have infinite traffic demand to the outside world through the gateway node. The flows belonging to each wireless client are numbered in ascending order with the number of each wireless client. Here the end-to-end flow-based global weighted max-min fair rate

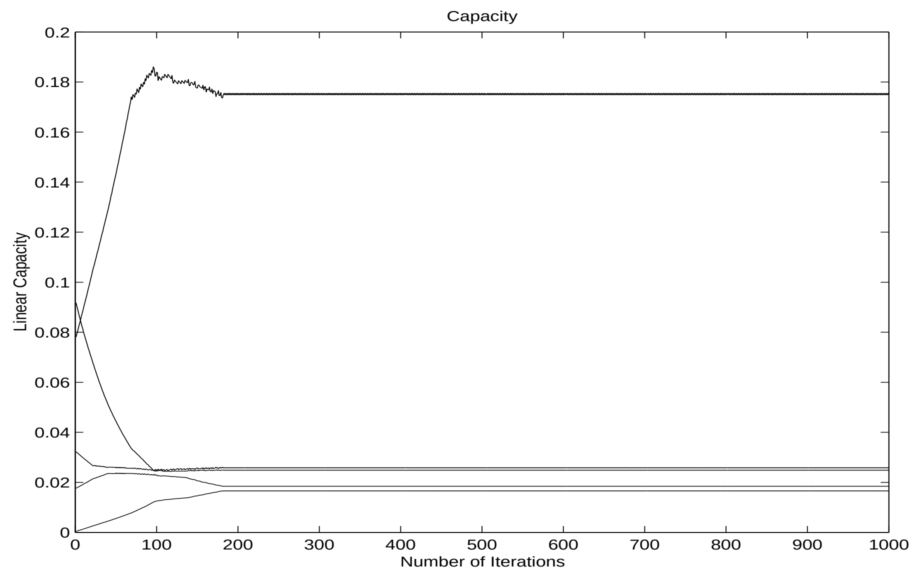

Figure 3.3: Capacity, mac layer, alpha =1, channel 1

allocation and the end-to-end hierarchical global weighted max-min fairness are examined.

In the next example, the flow rate vector, R, is required to be weighted hierarchical max-min fair with weight vectors $Z = [2, 3]$ and $V_1 = [1, 2, 3]$. V_2 can take any value since there is only one flow in that residence. Using Theorem 2, R is also end-to-end weighted max-min fair with weight vector W which is equal to $W = [11/3, 22/3, 11, 3]$. Considering Theorem 1, the link rate vector, X, is weighted mac-layer max-min fair with weight vector N. In this example, $N = [11/3, 22/3, 11, 3, 6/5]$ for the links tuned to channel 1 and $N = [2, 3]$ for the links tuned to channel 2. Figures 3.9 and 3.10 indicate the resulting rates of each link tuned to channel 1 and 2 respectively (when each link run the MAC-$\alpha - G$ algorithm). The link rate vector, X, turns out to be $X = [0.0288, 0.0144, 0.0096, 0.0379, 0.0908]$ for the links on channel 1 and $X = [0.0652$, $0.0412]$ for the links on channel 2. Assuming a weighted transport layer max-min fair protocol with weight vector W, the links on channel 1 become the bottleneck link as described in Fact 4. Then the flow rate vector R becomes $R = [0.0288$, 0.0144 , 0.0096 , $0.0379]$. Furthermore, the rate vector, D, denoting the aggregate rate utilized by each IBSS (or residence) becomes $D = [0.0528, 0.0379]$. As can be

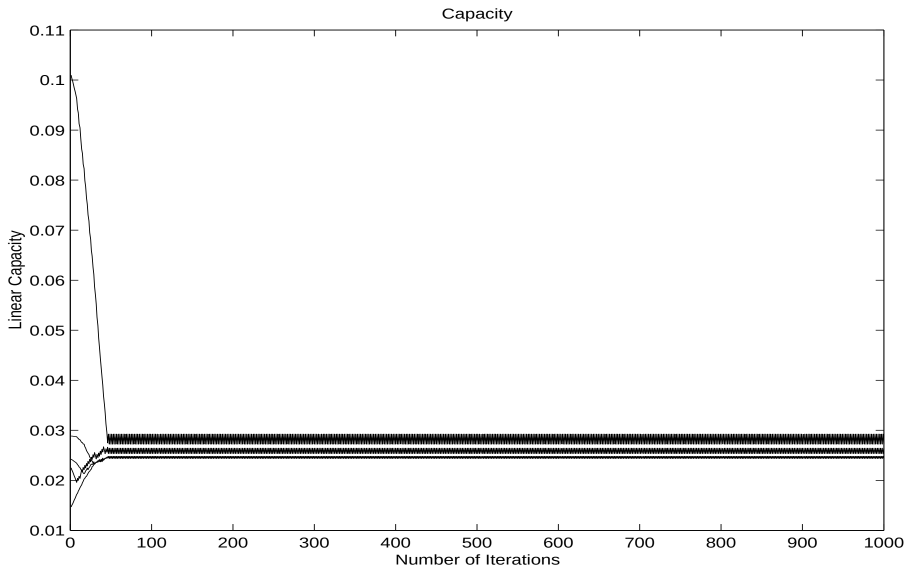

Figure 3.4: Capacity, mac layer, alpha = 60, channel 1

seen the elements of both vectors X, D and R are almost inversely proportional to the elements of the vectors N, Z and W respectively as stated in Facts 1, 3 and 5.

As an additional example, for the same traffic demand, the values of the vectors Z and V are changed as follows. $Z = [1, 2]$ and $V_1 = [1, 4, 3]$. V_2 can take any value since there is only one flow in that residence. Accordingly, the W vector turns out to be equal to $W = [19/12, 19/3, 19/4, 2]$. The link weight vector N for channel 1 is $N = [19/12, 19/3, 19/4, 2, 2/3]$ and for channel 2 is $N = [1, 2]$. Again Figures 3.11 and 3.12 shows the corresponding link rates achieved via the Mac-$\alpha - G$ algorithm for both channel 1 and 2 respectively. The link rate vector, X, turns out to be $X = [0.038, 0.0092, 0.0123, 0.0308, 0.0950]$ for the links on channel 1 and $X = [0.078, 0.038]$ for the links on channel 2. Assuming a weighted transport layer max-min fair protocol with weight vector W, the links on channel 1 become the bottleneck link as described in Fact 4. Then the flow rate vector R becomes $R = [0.038, 0.0092, 0.0123, 0.0308]$ Furthermore, the rate vector, D, denoting the aggregate rate utilized by each IBSS (or residence) becomes $D = [0.0595, 0.0308]$. As can be seen the elements of both vectors X, D and R are almost inversely proportional to the elements of the vectors N, Z and W respectively as stated in Facts 1, 3 and 5.

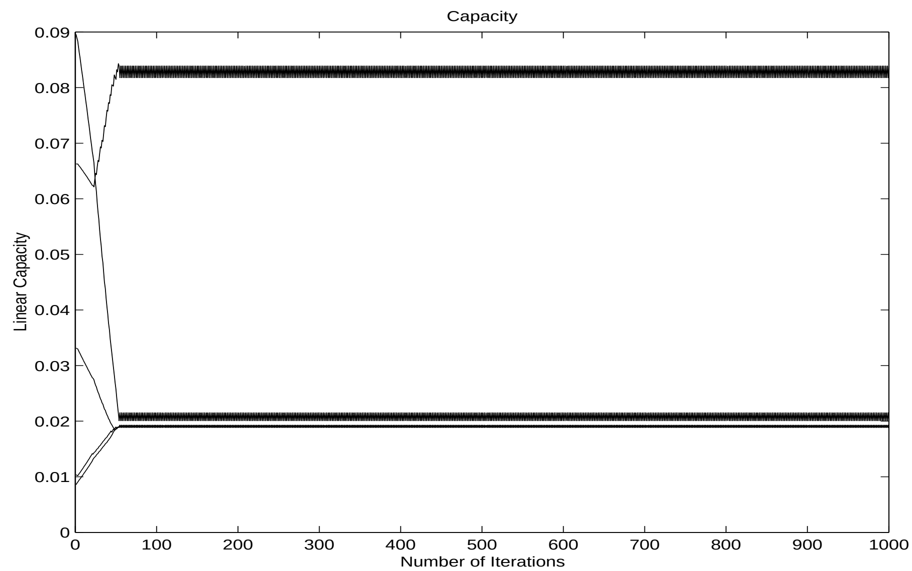

Figure 3.5: Capacity, e2e flow based, alpha =60, channel 1

As can be seen in both examples, using appropriate transport layer (weighted max-min) protocols and the mac-layer (weighted max-min) protocols, fair resource allocation (weighted max-min fair) can be ensured globally not only among individual flows but also among the residents that may utilize several number of flows.

3.4 Conclusion (Wireless Access Networks)

In this chapter, we show that in our wireless access network, end-to-end global fairness can be achieved via enforcing weighted mac-layer fairness. Particularly, we show that end-to-end global (flow-based and hierarchical)weighted flow-based max-min fairness is achieved if and only if transport-layer weighted max-min and weighted mac-layer max-min fair rate assignments are ensured. This result suggests that by designing intelligent mac-layer schemes, one can ensure end-to-end global fairness while requiring small interaction among layers. Furthermore, we propose a mac-layer algorithm to achieve weighted mac-layer fairness. Needless to say that such mac-layer algorithms in conjunction with Theorem 1 and 2 can be used to achieve end-to-end global fairness.

The text of this section is in part a reprint of the material as it appears in

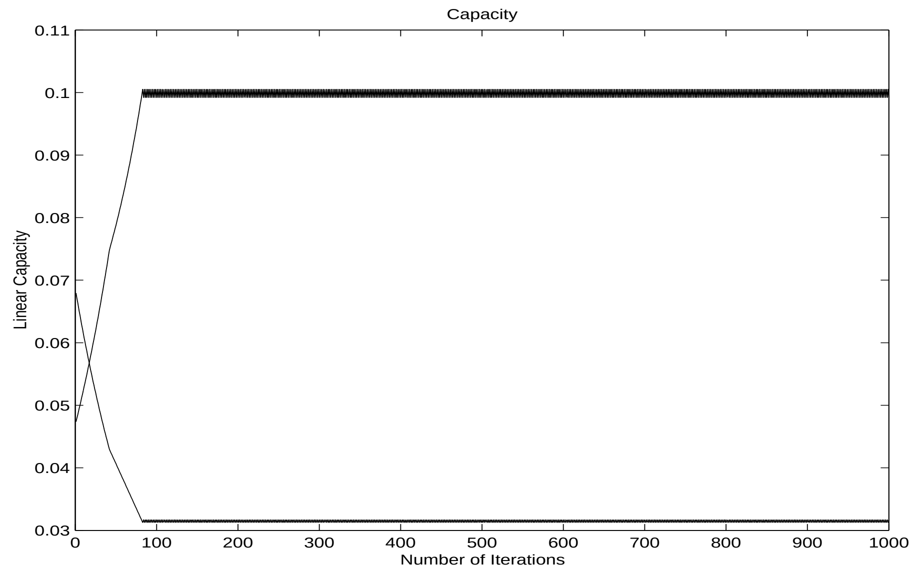

Figure 3.6: Capacity, e2e flow based, alpha = 60, channel 2

The Proceedings of the IEEE International Conference on Communications, ICC 2006. The dissertation author was the primary researcher and the author, and the co-authors, Professor Tara Javidi and Professor Rene L. Cruz listed in this publication supervised the research which forms the basis for this section.

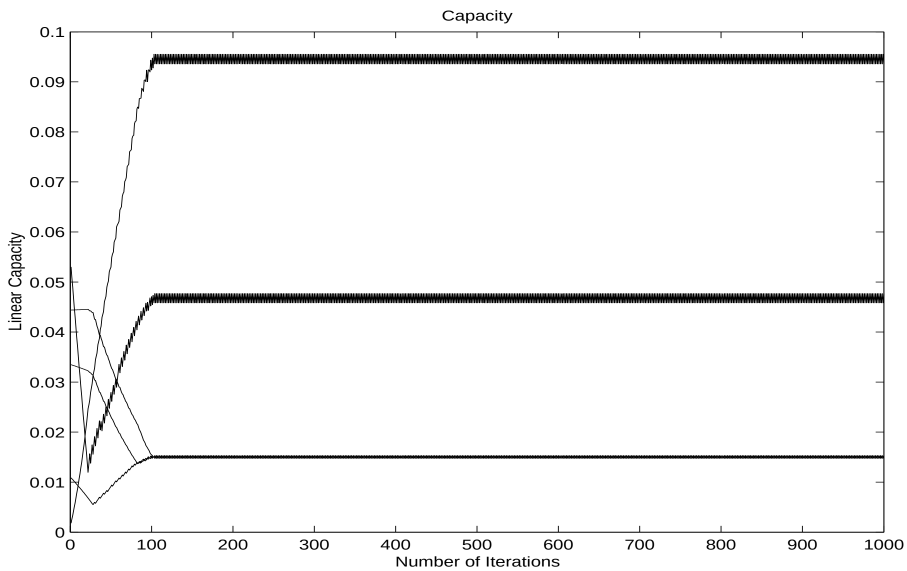

Figure 3.7: Capacity, hierarchical, alpha = 60, channel 1

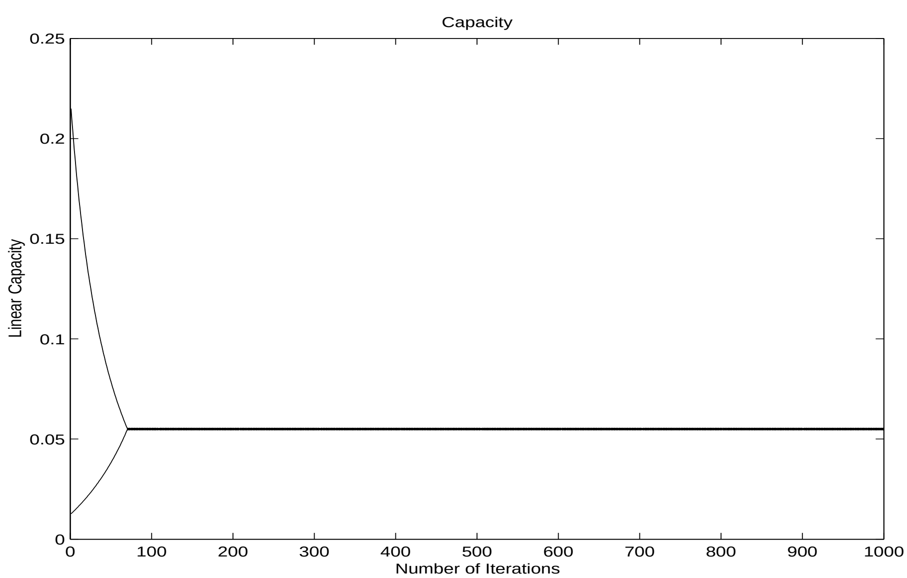

Figure 3.8: Capacity, hierarchical, alpha = 60, channel 2

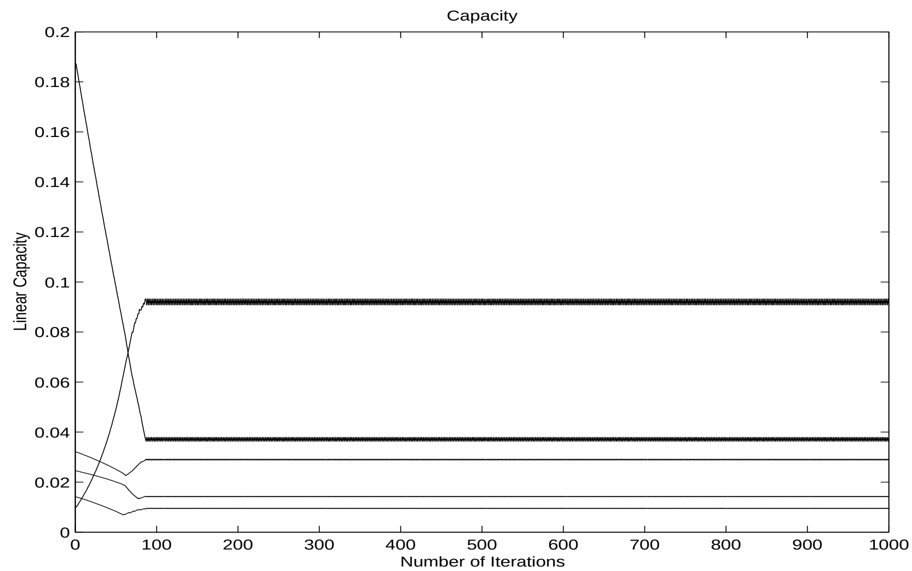

Figure 3.9: Capacity, e2e weighted hierarchical based, alpha =60, channel 1

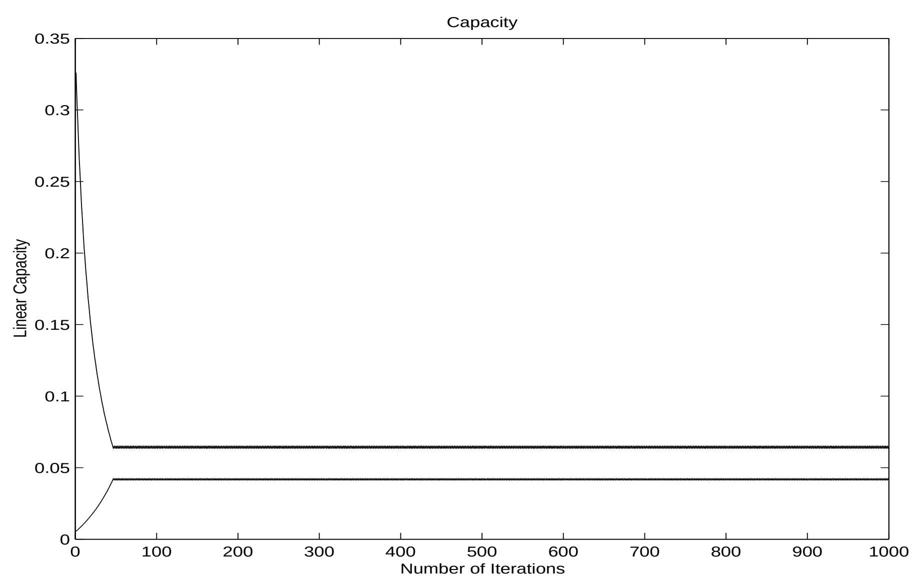

Figure 3.10: Capacity, e2e weighted hierarchical based, alpha =60, channel 2

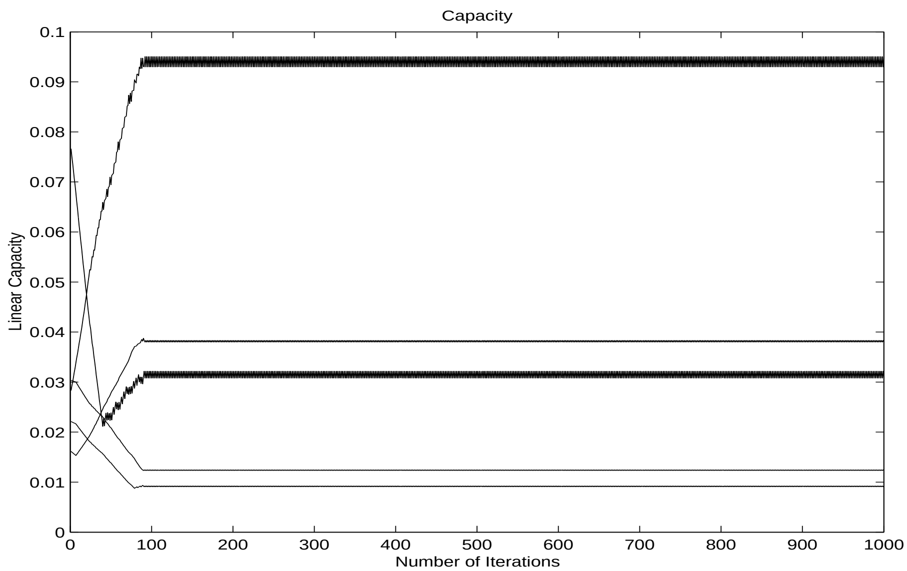

Figure 3.11: Capacity, e2e weighted hierarchical based, alpha =60, channel 1

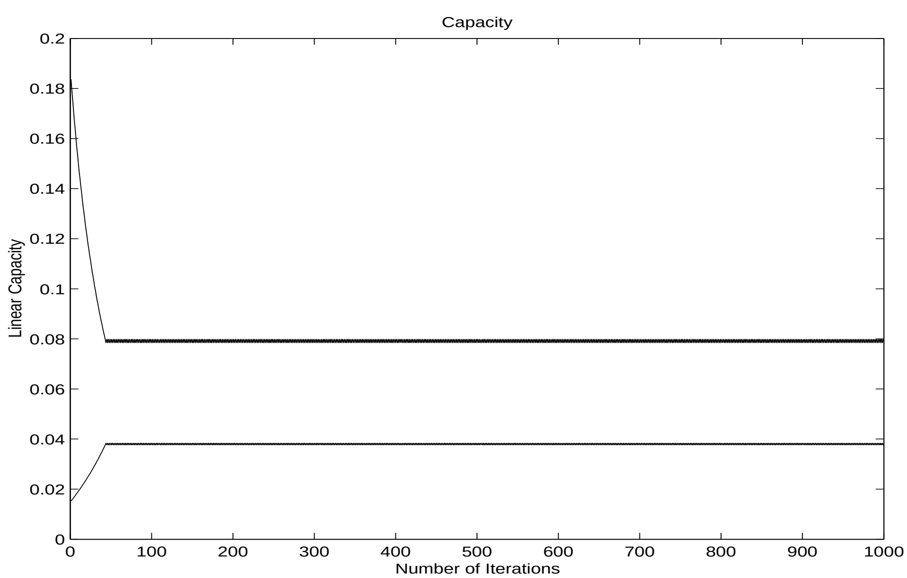

Figure 3.12: Capacity, e2e weighted hierarchical based, alpha =60, channel 2

4

Concluding Remarks

In this study, we consider fair rate assignment problem for two types of interference limited multi-hop networks.

First, a micro-buffered high-speed core network architecture is proposed. In this architecture, there is no buffering at the switching nodes. Packet losses are compensated by end-to-end or edge-to-edge forward error coding techniques. In this model, flows experience a lossy path from the source node to the destination node. A simple capacity formula relating the transmission rates to the information rates is considered.

It is found that in a micro-buffered network, classical slotted aloha techniques may end up with inefficient resource allocation such that some flows may end up with zero transmission and information rates. Thus, we examine various fairness policies for micro-buffered networks, such as weighted α-proportional, weighted (flow and hierarchical) max-min fair rate assignments. The fairness policies are compared with respect to total information capacity of the network, and fairness features. For instance, the weighted max-min fair rate assignment is found to assign information rates to each flow in the same group that is inversely proportional to the assigned weights. In particular, max-min fair rate assignment assigns the same information rates to the flows belonging to the same group, at the cost of lower network utilization (i.e. total network information rate). On the other hand, weighted α-proportional fairness result in higher total network information rate; however, does not assign the same information rates to the flows.

In addition, the relationship among these fairness policies are also examined. In particular, it is found that weighted α-proportional fairness converges to weighted max-min fairness as α increases. Moreover, weighted hierarchical max-min fair rate assignment is achieved if and only if weighted max-min fair rate allocation is ensured. Therefore, by tuning the policy parameters (e.g. weights and α in a weighted α-proportional fairness policy, or weights in a weighted max-min fair scheme), one can determine the trade off level between the fairness among the flows and the total network information rate.

Finally, several algorithms are proposed for setting the transmission rates in order to reach various fairness policies in the information rate domain.

Second, the problem of fair rate assignment in a two-channel CDMA based wireless multi-hop access networks is examined. Fairness is defined for different protocol layers. End-to-end global fair rate assignment problem is discussed. For this purpose, the interaction between the mac-layer and the transport layer protocols are studied. We found that the global max-min fair rate assignment problem can be decoupled into two independent max-min fair rate assignment problems in transport and mac layers. In particular, we show that end-to-end global (flow-based and hierarchical) weighted max-min fairness is achieved if and only if transport-layer weighted max-min and weighted mac-layer max-min fair rate assignments are ensured. In addition to that, it is found that end-to-end global weighted hierarchical max-min fair rate assignment can also be achieved if and only if end-to-end flow-based global weighted max-min fairness is ensured.

Thus, assuming an appropriate transport layer protocol (weighted max-min fair transport protocol), a weighted max-min fair mac layer protocol is required to achieve the global fairness objective. Therefore, a mac-layer algorithm is proposed such that with appropriate choice of parameters it achieves not only weighted α-proportional fairness in the mac-layer but also weighted mac-layer max-min fairness as α increases.

Appendix A

Micro-Buffered Networks

A.1 Equilibrium Discussion For Slotted Aloha

We let the system run for a sufficiently long amount of time (M slots) after the system reaches to equilibrium (if this is the case).So for any flow i, if the system reaches to equilibrium at a point P_i^*, then after M slots it should still be at that point, in other words

$$P_i^* \times q_1^{MP_I^i} \times q_2^{-MP_C^i} \approx P_i^* \tag{A.1}$$

where $P_I^i = e^{-G_i^*}$ and $P_C^i = 1 - e^{-G_i^*} - G_i^* e^{-G_i^*}$ are the idle and collision probabilities at equilibrium and G_i^* is the equilibrium load at channel seen by the ith flow.

If $q_1 = q_2 = q$ (as assumed in the related section) then the equilibrium is the solution of $P_I^i = P_C^i$ which is $G_i^* = 1.14$

In a single hop contention channel model where $P_i = P$ and $G_i = G$ for all i, the overall system is optimized when we set $G^* = 1$ since the total throughput $G^* e^{-G^*}$ is maximized. Then equation (A.1) becomes

$$\frac{ln(q_1)}{ln(q2)} = (e - 2) \tag{A.2}$$

Thus, optimization of the system gives us a freedom to choose the collision and idle coefficients with respect to a simple equation above. In this case, we assume that when there is a successful transmission in a slot, we do not change the rate, however in [20], a more general equation among all three coefficients (collision, idle, success) optimizing the system is found.

When we consider a micro-buffered network where the flows perform slotted aloha with exponential backoff with $q_1 = q_2 = q$ or any q_1, q_2 satisfying equation (A.2), according to the analysis above, each flow will try to set the aggregate offered load on the channel it sees, $G_i = \sum_{k \in X(i) \bigcup i} P_k$) to ($G_i^* = 1.14$ or 1 depending on the system parameters.

A.2 The convexity of the Negative Utility Function for Max-Min Fair Case

$$-U_i(P) = -U(I_i) = (-log(P_i \prod_{j \in X(i)} (1 - P_j)))^{\alpha} \tag{A.3}$$

where P is the vector of transmission probabilities.

Lets define the following functions

$$g(P) = -log(P_i \prod_{j \in X(i)} (1 - P_j)) \tag{A.4}$$

$$h(x) = x^{\alpha}; \quad f(P) = -U_i(P) = h(g(P)) \tag{A.5}$$

We are given that $0_v \leq P \leq 1_v$ where 0_v and 1_v are vectors of 0s and 1s. We can easily find the bound on $g(P)$ which is a convex function such that $0 \leq g(P) \leq \infty$. Furthermore, we know that $h(x)$ is both convex and non-decreasing for $x \in [0, \infty]$

First using the convexity of g and assuming a variable $0 \leq \theta \leq 1$

$$0 \leq g(\theta P_x + (1 - \theta)P_y) \leq \theta g(P_x) + (1 - \theta)g(P_y) \leq \infty. \tag{A.6}$$

Using the fact that h is non-decreasing on $[0, \infty]$

$$0 \le h(g(\theta P_x + (1-\theta)P_y)) \le h(\theta g(P_x) + (1-\theta)g(P_y)) \le \infty. \tag{A.7}$$

Since h is convex on $[0, \infty]$,

$$0 \le h(\theta g(P_x) + (1-\theta)g(P_y)) \le \theta h(g(P_x)) + (1-\theta)h(g(P_y)) \le \infty. \tag{A.8}$$

Combining last two equations

$$0 \le h(g(\theta P_x + (1-\theta)P_y)) \le \theta h(g(P_x)) + (1-\theta)h(g(P_y)) \le \infty \tag{A.9}$$

$$0 \le f(\theta P_x + (1-\theta)P_y) \le \theta f(P_x) + (1-\theta)f(P_y) \le \infty \tag{A.10}$$

which completes the proof.

A.3 Equivalence of the Problems

The idea here is the same as in [5], where you can find the same arguments for max-min fair rate assignment for multi-hop adhoc aloha networks. Since the capacity model for both networks are very similar, we have also added this approach in our study.

The problem described in equation (2.7) can also be written as follows.

$$\textbf{Maximize X} \tag{A.11}$$

$$\textbf{s.t.} \quad ln(X) < ln(P_i) + ln(\prod_{j \in X(i)} (1 - P_j)) \quad \forall i \tag{A.12}$$

Let $n = lnX$, then the problem turns out to be

$$\textbf{Maximize n} \tag{A.13}$$

$$\textbf{s.t.} \quad n < ln(P_i) + \sum_{j \in X(i)} ln((1 - P_j)) \quad \forall i. \tag{A.14}$$

Since $n < 0$, the objective function of the above problem can be translated to $1/2n^2|F|$ where $|F|$ is the cardinality of set of flows F.

Thus, it can easily be seen that the above problem with the minimizing objective is equivalent to the equation (2.10).

A.4 Convergence of Weighted Alpha-Prop Fairness to Weighted Max-min Fairness:

Proof: The utility function for each flow i is $U_i(I_i) = -\frac{1}{g_i}(-ln(g_i I_i))^\alpha$ for all $0 < g_i I_i < 1$, and $\alpha \geq 1$ is an increasing function of I_i. The problem formulation for weighted Alpha-Proportional Fairness maximizes the sum of the individual utility functions.

Assume an arbitrary rate assignment to any two flows f_1 and f_2 with weights g_1 and g_2 respectively such that $0 < g_1 I_{f_1} < g_2 I_{f_2} < 1$. Thus the ratio of the partial derivatives of each utility function with respect to the corresponding link rate goes to infinity as α goes to infinity, i.e.

$$\frac{U\prime_{f_1}(I_{f_1})}{U\prime_{f_2}(I_{f_2})} = \frac{g_2 I_{f_2}}{g_1 I_{f_1}}\left(\frac{ln(g_1 I_{f_1})}{ln(g_2 I_{f_2})}\right)^{\alpha-1} \rightarrow \infty \text{ as } \alpha \rightarrow \infty$$

For sufficiently large α, any increase in I_{f_1} and decrease in I_{f_2} will increase the overall utility function. Thus the optimum value of this non-linear programming problem satisfies the condition $g_1 I_{f_1} = g_2 I_{f_2}$ for any two flows, f_1 and f_2. Also since the utility function is an increasing function of each link rate, the optimum value should be a maximal rate vector. These two conditions are necessary and sufficient conditions for a weighted max-min fair rate allocation as described in the max-min fair section.

□

Appendix B

Multi-Hop Wireless Access Networks

B.1 The convexity of the Normalized Negative Utility Function

Proof. Note that

$$f(X_{i,j}(\vec{Z})) = (-ln(g_{i,j}X_{i,j}(\vec{Z})))^{\alpha} \tag{B.1}$$

where

$$X_{i,j}(\vec{Z}) = B\frac{e^{Z_{i,j}}G_{i,j}}{\sum_{m,n\neq i,j} e^{Z_{m,n}}G_{m,j} + \gamma}. \tag{B.2}$$

Let's define the following functions:

$$g(\vec{Z}) = -ln(g_{i,j}X_{i,j}(\vec{Z})) \tag{B.3}$$

$$h(x) = x^{\alpha}; \quad = f(\vec{Z}) = h(g(\vec{Z})). \tag{B.4}$$

Given that $0 \leq g_{i,j}X_{i,j} \leq 1 \; \forall(i,j)$, it can be easily be seen that $0 \leq g(P) \leq \infty$. Furthermore, we know that $g(P)$ is a convex function and $h(x)$ is both convex and non-decreasing for $x \in [0, \infty]$.

Assume that $Z_{i,j}$ and $Z_{k,l}$ are in **domain f** and $0 \leq \theta \leq 1$.
From the convexity of the function g, we have

$$\begin{aligned} 0 \leq g(\theta Z_{i,j} + &(1-\theta)Z_{k,l}) \leq \\ &\theta g(Z_{i,j}) + (1-\theta) g(Z_{k,l}) \leq \infty. \end{aligned} \tag{B.5}$$

Using the fact that h is non-decreasing on $[0, \infty]$, we have

$$\begin{aligned} 0 \leq h(g(\theta Z_{i,j} + &(1-\theta)Z_{k,l})) \leq \\ &h(\theta g(Z_{i,j}) + (1-\theta) g(Z_{k,l})) \leq \infty. \end{aligned} \tag{B.6}$$

From convexity of h on $[0, \infty]$, we have

$$\begin{aligned} 0 \leq h(\theta g(Z_{i,j}) + &(1-\theta) g(Z_{k,l})) \leq \\ &\theta h(g(Z_{i,j})) + (1-\theta) h(g(Z_{k,l})) \leq \infty. \end{aligned} \tag{B.7}$$

Combining last two equations

$$\begin{aligned} 0 \leq h(g(\theta Z_{i,j} + &(1-\theta)Z_{k,l})) \leq \\ &\theta h(g(Z_{i,j})) + (1-\theta) h(g(Z_{k,l})) \leq \infty \end{aligned} \tag{B.8}$$

$$\begin{aligned} 0 \leq f(\theta Z_{i,j} + &(1-\theta)Z_{k,l}) \leq \\ &\theta f(Z_{i,j}) + (1-\theta) f(Z_{k,l}) \leq \infty \end{aligned} \tag{B.9}$$

which completes the proof. □

B.2 Equivalence of the Problems

The problem described in equation (3.13) can also be written as follows:

$$\textbf{Maximize E} \tag{B.10}$$

$$\textbf{s.t.} \quad ln(E) < ln(g_{i,j} X_{i,j}) \quad \forall (i,j) \in L. \tag{B.11}$$

Letting $n = ln(E)$, the problem is equivalent to

$$\textbf{Maximize n} \tag{B.12}$$

$$\textbf{s.t.} \quad n < ln(g_{i,j}X_{i,j}) \quad \forall (i,j) \in L. \tag{B.13}$$

Assuming $0 < g_{i,j}X_{i,j} < 1 \; \forall i,j$, $n < 0$,
the problem becomes

$$\textbf{Minimize}\frac{1}{2}n^2 \tag{B.14}$$

$$\textbf{s.t.} \quad n < ln(g_{i,j}X_{i,j}) \quad \forall (i,j) \in L. \tag{B.15}$$

It can easily be seen that the above problem with the minimizing objective is equivalent to the equation (3.16).

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