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VENEZIANO MODELS FOR VIRTUAL COMPTON SCATTERING*

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SUMMARY

Recently proposed Veneziano-type models for Compton scattering of neutral and charged currents are studied. Current commutators, sum rules, large q^2 behavior and factorizability are investigated. A new model is proposed which has good behavior in all these respects. Electroproduction phenomenology and electromagnetic mass differences are discussed.

1. INTRODUCTION

A model for vector current amplitudes consistent with the N-point beta-function model (generalized Veneziano model) of the hadron bootstrap has recently been proposed (1). The amplitudes for one or two vector currents and N hadrons satisfy exactly the constraints of current conservation (CVC) and current algebra (CA). However, it is only a leading-order model, since the form factors have only single vector-meson poles and factorization holds only for leading trajectories. In this paper we study the two-current amplitudes of II in greater detail for the simple case of two hadrons (virtual Compton scattering).

The results of II are specialized to virtual Compton scattering off pions in section 2.1. The current commutators implied by these amplitudes are studied through the Bjorken limit (2) in section 2.2. The large q^2 behavior of the amplitudes is studied in section 2.3. Since the amplitudes have only single vector-meson poles in q^2, it should not be surprising that they are badly behaved for large q^2. Better large q^2 behavior obtains in models with many vector-meson poles recently proposed by several authors (3-6); however, they fail to satisfy all the CVC, CA, and leading trajectory factorization constraints. We discuss these models in section 3.1. In section 3.2 we give "hybrid amplitudes" which combine the good features of the models of II and Refs. (3-6). However, all present models have features which we do not expect to find in a complete solution, e.g., slowly falling form factors (e.g., $\frac{1}{q^2}$) and fixed poles in the current-pion channel (s-channel). Finally we discuss Pomeranchuk exchange in virtual Compton scattering in section 4.1, and suggest a modification of the amplitudes of II which yields the scaling property for both electroproduction structure functions. Mass differences for pions and kaons are discussed (section 4.2) in the model of II, but we find that at present there is too much ambiguity to allow a reliable calculation.

2. LEADING ORDER CURRENT ALGEBRA MODEL

2.1. The Amplitudes

We first state the amplitudes obtained in II for the scattering of a vector current of invariant mass q_1^2 from a pion to produce a current of mass q_2^2. Energy momentum conservation is given by

$$q_1 + p_1 + q_2 + p_2 = 0 \quad , \tag{2.1}$$

where q_i are the current momenta and p_i the pion momenta. Spin and isospin conventions and channels are defined in Fig. 1.

The amplitudes may be expanded in terms of amplitudes of definite t-channel isospin (7), yielding, for two isoscalar currents,

$$M^{\mu\nu}_{cd}(0,0) = \frac{1}{2}\,\delta_{cd}\, M^{\mu\nu}(\overline{0}) \quad , \tag{2.2}$$

and for two isovector currents,

$$\begin{aligned} M^{\mu\nu}_{abcd}(1,1) = {} & \frac{1}{3}\,\delta_{ab}\delta_{cd}\, M^{\mu\nu}(0) \quad , \\ & + \frac{1}{2}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})\, M^{\mu\nu}(1) \\ & + \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \frac{2}{3}\,\delta_{ab}\delta_{cd})\, M^{\mu\nu}(2) . \end{aligned} \tag{2.3}$$

The isoscalar-isovector transition vanishes due to G-parity conservation. From the internal symmetry factors of II (eq. II. 3.20) we easily obtain

(2.4) $$M^{\mu\nu\,(\bar{0})} = M^{\mu\nu}(st) + M^{\mu\nu}(tu) + M^{\mu\nu}(us) \equiv M^{\mu\nu\,(\Sigma)} \quad ,$$

(2.5) $$M^{\mu\nu\,(0)} = -2M^{\mu\nu}(us) + \frac{3}{2}\,M^{\mu\nu\,(\Sigma)} \quad ,$$

(2.6) $$M^{\mu\nu\,(1)} = M^{\mu\nu}(st) - M^{\mu\nu}(tu) \quad ,$$

(2.7) $$M^{\mu\nu\,(2)} = M^{\mu\nu}(us) \quad ,$$

where $M^{\mu\nu}(xy)$ has resonances in only the x and y channels. As an illustration, we note that (2.5) to (2.7) are obtained from a reduction of the expression

$$\begin{aligned} M^{\mu\nu\,(1,1)}_{abcd} = {} & \frac{1}{2}\,\mathrm{Tr}\left[\tau_c(\tfrac{1}{2}\,\tau_a)(\tfrac{1}{2}\,\tau_b)\tau_d + \tau_d(\tfrac{1}{2}\,\tau_b)(\tfrac{1}{2}\,\tau_a)\tau_c\right] M^{\mu\nu}(st) \\ & + \frac{1}{2}\,\mathrm{Tr}\left[\tau_c(\tfrac{1}{2}\,\tau_b)(\tfrac{1}{2}\,\tau_a)\tau_d + \tau_d(\tfrac{1}{2}\,\tau_a)(\tfrac{1}{2}\,\tau_b)\tau_c\right] M^{\mu\nu}(tu) \\ & + \frac{1}{2}\,\mathrm{Tr}\left[\tau_c(\tfrac{1}{2}\,\tau_a)\tau_d(\tfrac{1}{2}\,\tau_b) + \tau_c(\tfrac{1}{2}\,\tau_b)\tau_d(\tfrac{1}{2}\,\tau_a)\right] M^{\mu\nu}(us) \;. \end{aligned}$$

The absence of exotic resonances is exhibited clearly by (2.7).

It is also convenient to expand the $M^{\mu\nu}$ in terms of invariant amplitudes, as follows:

$$M^{\mu\nu} = M_0 g^{\mu\nu} + M_1 P^\mu P^\nu + M_2 q_2{}^\mu P^\nu + M_3 P^\mu q_1{}^\nu$$

$$+ M_4 q_2{}^\mu q_1{}^\nu + M_5 q_1{}^\mu P^\nu + M_6 q_1{}^\mu q_1{}^\nu + M_7 P^\mu q_2{}^\nu \tag{2.8}$$

$$+ M_8 q_2{}^\mu q_2{}^\nu + M_9 q_1{}^\mu q_2{}^\nu \quad ,$$

where $P = \frac{1}{2}(p_1 - p_2)$.

From the orbital factors of II (eqs. II. 3.3 and II. 3.19) we find, after converting to the notation of (2.8),

$$M_0(st) = 2F(q_1{}^2)F(q_2{}^2)\Big[\alpha_s B(-\alpha_s,\ 1-\alpha_t) + (1-\alpha_t)B(1-\alpha_s,\ -\alpha_t)\Big] - 2m_V{}^2\ B(2-\alpha_s,\ -\alpha_t) - F(t) \quad , \tag{2.9}$$

$$M_1(st) = -4F(q_1{}^2)F(q_2{}^2)\ B(-\alpha_s,\ 2-\alpha_t) + 4F(t)\Big[B(-\alpha_s,\ 2-\alpha_t) - B(-\alpha_s,\ 1)\Big] ,$$

$$M_2(st) = -\ M_3(st) = 4F(q_1{}^2)F(q_2{}^2)\Big[B(-\alpha_s,\ 1-\alpha_t) - \frac{1}{2}\ B(-\alpha_s,\ 2-\alpha_t)\Big] - 2F(t)\Big[B(-\alpha_s,\ 2-\alpha_t) - B(-\alpha_s,\ 1)\Big] \quad ,$$

$$M_4(st) = 4F(q_1{}^2)F(q_2{}^2)\Big[B(1-\alpha_s,\ -\alpha_t) + \frac{1}{4}\ B(-\alpha_s,\ 2-\alpha_t)\Big] - F(t)\Big[B(-\alpha_s,\ 2-\alpha_t) - B(-\alpha_s,\ 1)\Big] ,$$

(2.9 cont.)

$$M_5(st) = 2F(q_1^2)\left[F(q_2^2) -1\right]B(1-\alpha_s, 1-\alpha_t) \quad ,$$

$$M_6(st) = 2F(q_1^2)\left[F(q_2^2) -1\right]\left[B(1-\alpha_s, -\alpha_t) - \tfrac{1}{2} B(1-\alpha_s, 1-\alpha_t)\right] \quad ,$$

$$M_7(st) = -2F(q_2^2)\left[F(q_1^2) -1\right] B(1-\alpha_s, 1-\alpha_t) \quad ,$$

$$M_8(st) = 2F(q_2^2)\left[F(q_1^2) -1\right]\left[B(1-\alpha_s, -\alpha_t) - \tfrac{1}{2} B(1-\alpha_s, 1-\alpha_t)\right] \quad ,$$

$$M_9(st) = F(q_1^2)F(q_2^2)\, B(2-\alpha_s, -\alpha_t) + \frac{1}{m_V^2} F(q_1^2)F(q_2^2)\left[\alpha_s B(-\alpha_s, 1-\alpha_t) + (1-\alpha_t) B(1-\alpha_s, -\alpha_t)\right] - \left[F(q_1^2) + F(q_2^2)\right] B(2-\alpha_s, -\alpha_t) \quad ;$$

(2.10)
$$M_0(us) = 2F(q_1^2)F(q_2^2)\, B(1-\alpha_u, 1-\alpha_s) \quad ,$$

$$M_1(us) = 4F(q_1^2)F(q_2^2)\, B(-\alpha_u, -\alpha_s) \quad ,$$

$$M_2(us) = - M_3(us) = 2F(q_1^2)F(q_2^2)\left[B(-\alpha_u, 1-\alpha_s) - B(1-\alpha_u, -\alpha_s)\right]$$

$$M_4(us) = 4F(q_1^2)F(q_2^2)\left[B(1-\alpha_u, 1-\alpha_s) - \tfrac{1}{4} B(-\alpha_u, -\alpha_s)\right] \quad ,$$

$$M_5(us) = M_6(us) = M_7(us) = M_8(us) = M_9(us) = 0 \quad ;$$

where

(2.11)
$$F(x) = \frac{1}{1 - \frac{x}{m_V^2}} \quad ,$$

m_V is the common $I = 0$ (ω) and $I = 1$ (ρ) vector-meson mass, and B is the usual beta function. The units have been chosen so that the trajectory slope b in

(2.12) $$\alpha_i = a_i + b\, s_i$$

is equal to unity. The $M_i(tu)$ are obtained from (2.9) by the replacement $s \leftrightarrow u$ and an overall sign change in M_2, M_3, M_5, and M_7; this corresponds to the substitution $p_1 \leftrightarrow p_2$ in (2.8) and (2.9).

We mention briefly the following features of the amplitudes $M_i(xy)$:

(i) They have the correct kinematic behavior, e.g., no ancestor trajectories, correct helicity-flip factors, etc. This is assured by the method of construction of II and the corresponding properties of the N-point beta functions.

(ii) There are simple poles corresponding to physical particles in the variables s, t, and u. The corresponding leading Regge trajectories are, for the t-channel,

even signature - $f_0 \left[I^G(J^P) = 0^+(2^+)\right]$,

odd signature - $\rho \left[1^+(1^-)\right]$,

i.e., $\alpha_t = 1 + (t - m_V^2)$; and for the s-channel and u-channel,

even signature - $\pi \left[1^-(0^-)\right]$,

odd signature - ? $\left[0^-(1^+)\right]$,

i.e., $\alpha_s = s - m^2$, $\alpha_u = u - m^2$, where m is the pion mass.

(iii) At $q_i^2 = m_V^2$ they reduce to the corresponding amplitudes for vector mesons in the simple N-point beta-function model discussed in II.

(iv) The current algebra divergence conditions,

$$q_{1\mu} M^{\mu\nu\,(1)} = 4F(t)\, P^{\nu} \quad , \tag{2.13}$$

$$q_{1\mu} M^{\mu\nu\,(i)} = 0 \;, \; i = \bar{0},\, 0,\, 2 \;,$$

are satisfied exactly.

(v) The sum rule of Adler (8), Dashen and Gell-Mann (9), and Fubini (10),

$$\int_{-\infty}^{\infty} \mathrm{Im}\, M_1^{(1)}\,(s,t;\, q_1^2,\, q_2^2)\, ds = -\,8\pi\, F(t) \quad , \tag{2.14}$$

is satisfied exactly. In addition to the fixed pole at $J = 1$ in the t-channel angular momentum plane implied by (2.14) (11), we have a Kronecker-delta singularity at $J = 1$ (12). The symmetric amplitudes have corresponding singularities at $J = 0$, but these may well change when nonleading trajectories are properly treated. Finally, there are fixed poles in the s-channel angular momentum plane at $J = -1, -2, \ldots$, as noted by Brower and Halpern (13). All of these are due to the term $F(t)\, B(-\alpha_s, 1)$ in $M_i(st)$ $\left[\text{or, similarly, in } M_i(tu)\right]$.

(vi) There are the following pathologies, as can be seen from (ii) and (iii). A spin-zero ghost of imaginary mass in the t-channel, the absence of the A_1, ω, and A_2 trajectories and the presence of a spurious $0^-(1^+)$ trajectory in the s and u channels, various ghosts with imaginary coupling constants on nonleading trajectories, and the lack of factorization on nonleading trajectories.

2.2. Current Commutators

We now study the current commutation relations implied by the amplitudes given above. The time-time and time-space commutators can be defined phenomenologically by the divergences $q_{1\mu}M^{\mu\nu}$, as was done in I and II (see, e.g., I.2.12, I.2.13). Here we define the commutators through the Bjorken limit (2). This allows as to study the space-space commutators as well.

If local current operators $V_a^{\ \mu}(x)$ exist, the covariant current correlation tensor is given by

$$(2.15)\quad M^{\mu\nu}_{abcd}(1,1) = i\int d^4x\, e^{-iq\cdot x}\langle 0|T^*\left\{V_a^{\ \mu}(\tfrac{1}{2}x),\ V_b^{\ \nu}(-\tfrac{1}{2}x)\right\}|(p_1,c)(p_2,d)\rangle,$$

and similarly for isoscalar currents (14). In (2.15) we have taken

$$(2.16)\quad q = \tfrac{1}{2}(q_1-q_2),\ Q = (q_1+q_2) = -(p_1+p_2)\quad .$$

Taking the limit $|q_o| \to \infty$ with $\vec{q}, Q^\mu$, and P^μ fixed, we find (2)

$$(2.17)\quad M^{\mu\nu}_{abcd}(1,1) \xrightarrow[|q_0|\to\infty]{} \frac{1}{q_0}\int d^3x\, e^{i\vec{q}\cdot\vec{x}}$$

$$\langle 0|[V_a^{\mu}(0,\tfrac{1}{2}\vec{x}),\, V_b^{\nu}(0,-\tfrac{1}{2}\vec{x})]|(p_1,c)(p_2,d)\rangle$$

$$-\frac{i}{q_0^2}\int d^3(x)\, e^{i\vec{q}\cdot\vec{x}}\langle 0|[\dot{V}_a^{\mu}(0,\tfrac{1}{2}\vec{x}),\, V_b^{\nu}(0,-\tfrac{1}{2}\vec{x})]|(p_1,c)(p_2,d)\rangle$$

$$+ \ldots + \text{ polynomial in } q_0 .$$

We now compare (2.17) with (2.3) to (2.10). From the relations

$$\alpha_s = s-m^2 = q_0^2 + 2q_0P_0 - \frac{1}{4}t - \vec{q}^{\,2} - 2\vec{P}\cdot\vec{q} ,$$

$$\alpha_u = u-m^2 = q_0^2 - 2q_0P_0 - \frac{1}{4}t - \vec{q}^{\,2} + 2\vec{P}\cdot\vec{q} ,$$

it is clear that the terms $(q_0)^{-k}$ in (2.17) can arise only from the fixed pole contributions in (2.9). For the moment we therefore neglect the Regge terms in (2.9) and $M^{\mu\nu}(us)$; the behavior of these terms in the Bjorken limit is discussed in the following subsection.

For $M^{\mu\nu(1)} = M^{\mu\nu}(st) - M^{\mu\nu}(tu)$, the only amplitude antisymmetric in a and b, we obtain

$$(2.18)\quad M^{\mu\nu}(st) - M^{\mu\nu}(tu) \xrightarrow[|q_0|\to\infty]{} \frac{1}{q_0}\, 4F(t)[\eta^{\mu}P^{\nu} + P^{\mu}\eta^{\nu} - \eta\cdot P\eta^{\mu}\eta^{\nu}] ,$$

where

$$(2.19)\quad \eta^{\mu} = (1,0,0,0) .$$

From (2.3), (2.17), and the relation

$$\langle 0|V_e^{\nu}(0)|(p_1,c)(p_2,d)\rangle = -2i\epsilon_{cde}\, F(t)\, P^{\nu} ,$$

one easily sees that (2.18) is a matrix element of the commutation relations

$$\delta(x_0)\left[V_a^{\,0}(x),\ V_b^{\,\nu}(0)\right] = i\epsilon_{abe}\, V_e^{\,\nu}(0)\delta^4(x) \quad , \tag{2.20}$$

$$\delta(x_0)\left[V_a^{\,r}(x),\ V_b^{\,s}(0)\right] = 0,\quad r,s=1,2,3 \quad , \tag{2.21}$$

to within Schwinger terms that do not contribute to the antisymmetric amplitude. The time-time and time-space commutators (2.20) have already been obtained in II. From (2.21) we see that the amplitudes (2.9) yield the commutators of field algebra, and we therefore call them the Field Algebra (FA) amplitudes.

Any term proportional to $(q_2^{\,\mu} q_1^{\,\nu} - q_1{\cdot}q_2 g^{\mu\nu})$ can be added to $M^{\mu\nu}$ without changing its divergence or (2.20). With the particular choice

$$(q_2^{\,\mu} q_1^{\,\nu} - q_1{\cdot}q_2 g^{\mu\nu})\ F(t)\left[B(-\alpha_s,\ 2-\alpha_t) - B(-\alpha_s, 1)\right],$$

$M_0(st)$ and $M_4(st)$ are changed to

$$M_0(st) = 2\ F(q_1^{\,2})F(q_2^{\,2})\left[\alpha_s\ B(-\alpha_s,\ 1-\alpha_t) + (1-\alpha_t)\ B(1-\alpha_s,\ -\alpha_t)\right]$$

$$- 2m_V^{\,2}\ B(2-\alpha_s,-\alpha_t) - F(t) - q_1{\cdot}q_2 F(t)\left[B(-\alpha_s,2-\alpha_t) - B(-\alpha_s,\ 1\right] , \tag{2.9'}$$

$$M_4(st) = F(q_1^{\,2})F(q_2^{\,2})\left[B(1-\alpha_s,\ -\alpha_t) + \frac{1}{4}\ B(-\alpha_s,\ 2-\alpha_t)\right] .$$

This gives

(2.18')

$$M^{\mu\nu}(st) - M^{\mu\nu}(tu) \xrightarrow[|q_0| \to \infty]{} \frac{1}{q_0} 4F(t)\left[\eta^{\mu}P^{\nu} + P^{\mu}\eta^{\nu} - \eta\cdot P\, q^{\mu\nu}\right] .$$

Since the pion matrix elements of axial currents $A_a^{\ \mu}(x)$ vanish, eq. (2.18') is consistent with the quark algebra commutation relations

(2.21')

$$\delta(x_0)\left[V_a^{\ r}(x),\ V_b^{\ s}(0)\right] = i\left[\delta^{rs}\epsilon_{abe}\ V_e^{\ 0}(0) + \left(\frac{2}{3}\right)^{1/2} \epsilon^{rst}\ \delta_{ab}A_0^{\ t}(0)\right]\delta^4(x) .$$

We therefore call (2.9') the Quark Algebra (QA) amplitudes.

The commutators of time derivatives of currents with currents can be obtained from amplitudes symmetric in a and b. However, since these correspond to lower singularities in the angular momentum plane (e.g., J = 0 fixed poles compared to J = 1 fixed poles in the antisymmetric amplitudes), the results may well change when factorization on nonleading trajectories is imposed. We remark that both the FA and QA amplitudes imply the existence of nonvanishing commutators,

$$\delta(x_0)\left[\dot{V}_a^{\ r}(x),\ V_b^{\ s}(0)\right],\quad r,s = 1,2,3 \quad .$$

The FA commutators are more singular than the QA commutators; the FA has q-number Schwinger terms in the time-space commutators (2.20) and constant behavior has $|q_0| \to \infty$, whereas the QA has c-number Schwinger terms (at least for pion matrix elements).

2.3. Large q^2 Behavior

Since the above amplitudes have only single vector-meson poles in form factors, we expect them to be a good representation only for small q^2. However, it is interesting to investigate their behavior for large q^2 in order to better understand their possible deficiencies.

For simplicity we consider $t=0$ and $q_1^2 = q_2^2 = q^2$ (i.e., $Q^\mu = 0$). This is the point of interest for electroproduction and electromagnetic mass differences (see section 4). We define

$$\alpha_s = s - m^2 = q^2(1-\rho) \quad , \tag{2.24}$$

$$\alpha_u = u - m^2 = q^2(1+\rho) \quad ,$$

where

$$\rho = -\frac{2m\nu}{q^2} = -\frac{s-u}{2q^2} \quad . \tag{2.25}$$

As examples of typical terms in (2.9) and (2.10), we have

$$B(-\alpha_s, -\alpha_t) \xrightarrow[q^2 \to \infty]{} \Gamma(-a_t)\left[-q^2(1-\rho)\right]^{a_t} \tag{2.26}$$

and

$$B(-\alpha_u, -\alpha_s) \xrightarrow[q^2 \to \infty]{} \left[\frac{4\pi}{q^2(\rho^2-1)}\right]^{1/2}$$

$$\times \exp\left\{q^2\left[2\log 2 - (1+\rho)\log(1+\rho) - (1-\rho)\log(1-\rho)\right]\right\} \quad .$$

The power behavior in q^2 of the (st) and (tu) terms (2.26) and the exponential behavior in q^2 of the (us) term (2.27) are quite undesirable. For example, (2.27) leads to exponentially divergent mass differences and exponential increase of electroproduction amplitudes in the physical region ($q^2 \to -\infty$, $\rho \geq 1$). Both (2.26) and (2.27) give electroproduction structure functions which are either zero or infinite in the limit $q^2 \to -\infty$ with ρ fixed, in violation of the commonly accepted scaling law of Bjorken (15).

We also note that for $t = 0$ the Bjorken limit of the preceding subsection corresponds to $\rho \to 0$ in (2.26) and (2.27). Thus the Regge terms in $M^{\mu\nu}(st)$ and $M^{\mu\nu}(tu)$ behave like $(q^2)^{\alpha_t}$ and may dominate the fixed-pole terms for sufficiently large t. These correspond to determinable subtractions in (2.17) and cause no basic difficulty. However, the $M^{\mu\nu}(us)$ terms are seen to grow exponentially for $\mathrm{Re}(q^2) \to +\infty$. Since we shall see in the following section that it is possible to obtain amplitudes without the undersirable behaviors (2.26) and (2.27), we shall not regard these difficulties as necessarily fundamental to Veneziano parameterizations of virtual Compton scattering.

3. OTHER MODELS

Several other models for the virtual Compton scattering amplitude have been proposed recently. Bander (3) has given an expression for $M_1(st)$ which satisfies exactly the current algebra sum rule (2.14) and the requirements of current conservation (see section II. B of I). His amplitude does not factorize on leading trajectories in the s channel, however. Sugawara (4), Ohba (5), and Ademollo and Del Giudice (6) have obtained very similar models based on the six-point beta function. Factorization on leading trajectories is assured by the factorizability of the six-point beta function (16). On the other hand, the requirements of current algebra and current conservation and the sum rule are not satisfied.

An important common feature of the models of Refs (3-6) is their good behavior as $q^2 \to \infty$ (e.g., nontrivial electroproduction limit) (17). In the following subsection we discuss the large q^2 behavior of these other models. We consider only amplitudes of the form given in Refs. (4-6), since Bander's amplitude does not factorize. Such amplitudes are particularly interesting, since we shall show in section 3.2 how to use them to obtain virtual Compton scattering amplitudes which satisfy current algebra <u>and</u> have good $q^2 \to \infty$ behavior. The "hybrid model" given there thus combines the good features of the models of II and Refs. (3-6).

3.1. Discussion

We consider for definiteness a typical example of an amplitude occurring in Refs. (4-6),

$$A_{k\ell}(\alpha_s,\alpha_t-2) \equiv \int_0^1 du\, u^{-\alpha_s-1}(1-u)^{1-\alpha_t} \int_0^1 du_1 du_2\, u_1^{-\alpha_1} u_2^{-\alpha_2}$$

(3.1)

$$(1-uu_1)^{\alpha_t-3-\ell}(1-uu_2)^{\alpha_t-3-\ell}(1-uu_1u_2)^{-\alpha_t+2+2\ell-k} ,$$

where

$$\alpha_i = 1 + (q_i^2 - m_V^2) \quad , \quad i = 1,2 \quad .$$

$A_{k\ell}(\alpha_s,\alpha_t-2)$ is appropriate for parameterizing the double-flip amplitude $M_1(st)$. It corresponds to the choice of trajectories in the six-point beta function shown in Fig. 2.

Using the "Veneziano Transform" technique (18), one may easily show that (3.1) has leading fixed power behavior in s of s^k and s^ℓ and leading Regge behavior s^{α_t-2}. In $M^{\mu\nu}$ this would correspond to fixed poles at $k+2, k+1, \ldots$ and $\ell+2, \ell+1, \ldots$. Thus, for example, $k = -1$ and $\ell = -2$ would give a fixed pole at the same point as required by current conservation (see section III. B of I). The residue is not $F(t)$ as required, however, although it is independent of q_1^2 and q_2^2.

The behavior in the limit $q^2 \to \infty$ $(t = 0, \rho$ fixed) is easily obtained. We observe that, if k_c and k_d are taken to be the incoming momenta in a multiperipheral diagram like Fig. 2, this limit has the same form as a Regge limit in the leftmost link of the chain (19). If the variables in (3.1) are changed to those corresponding to this multiperipheral diagram, the asymptotic limit may be easily calculated. We find

$$(3.2) \qquad A_{k\ell}(\alpha_s, a_t - 2) \xrightarrow[q^2 \to \infty]{} (q^2)^k f_{k\ell}(\rho) \quad ,$$

and

$$(3.3a) \qquad \mathrm{Re}\, f_{k\ell}(\rho) \xrightarrow[\rho \to \infty]{} \rho^{\max(k,\ell)} c_R \quad ,$$

$$(3.3b) \qquad \mathrm{Im}\, f_{k\ell}(\rho) \xrightarrow[\rho \to \infty]{} \rho^{a_t - 2} c_I \quad .$$

For $k = -1$, we see that, if $M_1 \propto A_{k\ell}$,

then

$$\nu \mathrm{Im} M_1 \xrightarrow[q^2 \to \infty]{} f(\rho) \quad ,$$

as obtained by Bjorken (15). Since we have consistency between Regge behavior and the electroproduction limit, it is not surprising that (3.3b) is just what Abarbarnel <u>et al</u>. (20) showed must be true when both hold.

The reduced t-channel Regge residues for the leading trajectory behave like

$$\gamma(t) \xrightarrow[q^2 \to \infty]{} \left(\frac{1}{q^2}\right)^{\alpha_t - 2 - k} , \tag{3.4}$$

as one expects (20) from (3.2) and (3.3b). It is now obvious why the single vector-meson pole model of II has the undesirable behavior (2.26)-- with single poles one cannot possibly obtain the rapid falloff (3.4) for high-mass t-channel resonances. Thus one can easily understand the large q^2 behavior on the basis of the leading t-channel trajectory alone in (st) [and (tu)] terms.

For (us) terms (see, for an example, Ref. 5) the electroproduction limit again corresponds to a Regge limit in the six-point beta function, and power behavior as in (3.2) obtains. However, since the leading s (or u) channel trajectories in both the simple beta function and the models of Refs. (4-6) give (21)

$$A(\alpha_u, \alpha_s) \underset{\substack{q^2 \to \infty \\ \rho \to \infty}}{\to} e^{\rho q^2} ,$$

the nonleading trajectories must account for this improvement.

We now make a few qualitative remarks on how sum rules are satisfied in various models. We note above that (3.1) has fixed power behavior s^{-1} for $k = -1$; it thus satisfies a sum rule of the form (2.14) -- with some function $G_\ell(t)$ [not equal to $F(t)$] on the right-hand side. Since the imaginary part in the integrand is a sum of delta functions, the sum rule becomes

$$(3.5) \qquad \sum_{n=0}^{\infty} \text{Res}_{\alpha_s = n} \left\{ A_{-1\ell}(s,t;\, q_1^2,\, q_2^2) \right\} = G_\ell(t) \quad .$$

We find that the spin J part of the residue at $\alpha_s = n$ in general behaves like $(q_1^2 q_2^2)^{-J-1}$ for large q_1^2 and q_2^2. It is therefore clear that the sum in (3.5) does <u>not</u> converge uniformly in q_1^2 and q_2^2 (17). The sum rule is therefore being satisfied in a subtle way, the decrease in any one resonance as $q^2 \to \infty$ being compensated by higher mass resonances. On the other hand, the model of II satisfies the sum rule in a rather trivial way. The contribution of the leading trajectory is proportional to $F(q_1^2)\, F(q_2^2)$ and vanishes as $q_i^2 \to \infty$, whereas the lower trajectories have a part independent of q_i^2 [coming from $F(t)\, B(-\alpha_s, 2-\alpha_t)$ in (2.9)] and this allows the sum like (3.5) to converge uniformly in q_i^2.

If a completely factorized set of amplitudes consistent with current algebra can be found, we expect all form factors to fall and thus the sum rule (3.5) must converge nonunimformly. We also expect that the form factors will fall faster than any power and the fixed poles in the s-channel of the Compton amplitude occurring in all present models will go away. Further, in a completely bootstrapped solution with infinitely composite particles, we expect no fixed poles in single-current amplitudes (22). The models of Refs. (4-6) have such fixed poles, whereas the model of II does not. Thus we see that all the models proposed so far have some features we do not expect to find in the final solution.

3.2. Hybrid Model

We now propose expressions for $M^{\mu\nu}(st)$ and $M^{\mu\nu}(tu)$ which satisfy all the properties (i) - (vi) of section 2.1 [(iii) holds only if $q_1^2=q_2^2=m_V^2$] and have the good behavior (3.2) as $q^2 \to \infty$. These are obtained by replacing the beta functions in (2.9) by amplitudes of the form (3.1).

One can verify that the following substitutions yield the above properties:

(3.6)
$$B(-\alpha_s,\ 2-\alpha_t) \to \overline{B}(-\alpha_s,\ 2-\alpha_t) \equiv (\alpha_1-1)(\alpha_2-1)\ A_{-3,-2}(\alpha_s,\ \alpha_t-2)\ ,$$

$$B(-\alpha_s,\ 1-\alpha_t) \to \overline{B}(-\alpha_s,\ 1-\alpha_t) \equiv \frac{\alpha_s+\alpha_t-1}{\alpha_{t-1}}\ \overline{B}(-\alpha_s,\ 2-\alpha_t)\ ,$$

$$B(1-\alpha_s,\ 1-\alpha_t) \to \overline{B}(1-\alpha_s,\ 1-\alpha_t) \equiv \frac{\alpha_s}{\alpha_t-1}\ \overline{B}(-\alpha_s,\ 2-\alpha_t)\ ,$$

$$B(1-\alpha_s,\ -\alpha_t) \to \overline{B}(1-\alpha_s,\ -\alpha_t) \equiv (\alpha_1-1)(\alpha_2-1)A_{-3,-2}(\alpha_s-1,\ \alpha_t)$$

$$B(2-\alpha_s,\ -\alpha_t) \to \overline{B}(2-\alpha_s,\ -\alpha_t) \equiv \overline{B}(1-\alpha_s,\ -\alpha_t) - \overline{B}(1-\alpha_s,\ 1-\alpha_t)$$

$$B(-\alpha_s,\ -\alpha_t) \to \overline{B}(-\alpha_s,\ -\alpha_t) \equiv \overline{B}(1-\alpha_s,\ -\alpha_t) + \overline{B}(1-\alpha_s,\ 1-\alpha_t) + \overline{B}(-\alpha_s,\ 2-\alpha_t).$$

Note that

$$\overline{B}(-\alpha_s,\ 1) = B(-\alpha_s,\ 1) = -\frac{1}{\alpha_s}$$

guarantees the absence of a pole at $\alpha_t = 1$ in M_1.

Since we have $k = -3$ and $\ell = -2$ in (3.6), the entire fixed pole at $J = 1$ is exhibited explicitly by $F(t)\, B(-\alpha_s, 1)$ in (2.9). Although the model of Bander, for example, appears to contain the fixed pole in a much more subtle way within the integrals, the difference between it and the hybrid model is only superficial. One may in fact rewrite Bander's expression III.1 as

$$M_1^{\text{Bander}} = -4\, F(q_1^2)\, F(q_2^2)\, \hat{G}_2 + 4F(t)\hat{G}_1 + 4F(t)\, \frac{1}{\alpha_s}$$

$$+ \frac{4F(q_1^2)F(q_2^2)}{m_V^2}\, (2q_1 \cdot q_2 + m_V^2)\, [\hat{G}_2 - \hat{G}_1] \quad ,$$

where $\hat{G}_k = (\alpha_1 - 1)(\alpha_2 - 1)\, G_k$ and G_k, which is defined by II.1 of Ref. (3), has fixed power behavior s^{k-4}. The similarity to (2.9) plus (3.6) is now manifest. We note that with the hybrid amplitudes the sum rule is satisfied by nonuniform convergence, since all form factors fall as described above.

The Bjorken limit of the basic amplitudes (2.9) with the substitution (3.6) differ from (2.18) by a term of the form $\frac{1}{q_0}(g^{\mu\nu} - \eta^\mu \eta^\nu)$. To obtain the FA and QA, terms proportional to

$(q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu})$ must be added.

We have not been able to construct hybrid amplitudes for the (us) terms (2.10). We do not know if the hybrid amplitudes can be generalized to N hadrons so as to give a solution with leading trajectory factorization and consistency with single-current amplitudes. Nevertheless, they are interesting examples of functions satisfying many of the required properties.

4. APPLICATIONS

The obvious deficiencies of our amplitudes [see comment (vi) of section 2.1] and the lack of data for Compton scattering off meson targets rule out any detailed phenomenological analysis (23). However, we consider here two applications, which are especially relevant to the issues raised in section 3.

First (section 4.1) we consider the Pomeranchuk exchange in Compton scattering. The parameterization of II gives a nice example of a Pomeranchon that contributes to forward elastic Compton scattering for $\alpha_p(0) = 1$, as suggested in Abarbanel et al. (24). Moreover, consistent with recent electroproduction data, this Pomeranchon does not fall off rapidly at large q^2. We also show how the scaling property (15) for both structure functions can be naturally obtained in vector-meson dominance models.

Then (section 4.2) we discuss electromagnetic mass differences. After enumerating the various divergent contributions of the amplitudes of II, we calculate the finite contributions for pions and kaons. Our numerical results are in worse disagreement with experiment than those calculated from the Born terms alone.

4.1. Pomeranchon in Compton Scattering

We consider here several of the important features of the Pomeranchuk solution of II in the special case N=2 as they relate to earlier models and, through the optical theorem to total electroproduction cross sections. The contribution of the Pomeranchon (to symmetric amplitudes) takes the simple form

$$M^{\mu\nu}_{Pom} = -\left[q_1\cdot q_2 P^\mu P^\nu + m\nu(P^\mu q_1{}^\nu - q_2{}^\mu P^\nu) - m^2\nu^2 g^{\mu\nu}\right] t_2$$

(4.1)
$$- \left[q_1 \cdot q_2\, g^{\mu\nu} - q_2{}^\mu q_1{}^\nu\right] t_1 \quad ,$$

The two independent amplitudes may be given in terms of M_1 and M_0,

$$M_1{}^{Pom} \equiv - q_1 \cdot q_2 \; t_2$$

(4.2)
$$= -\left\{\frac{2q_1\cdot q_2}{\alpha_P-1} B(2-\alpha_P, -\alpha_s) + \frac{D}{\alpha_P-1} B(1, -\alpha_s) + (s \leftrightarrow u)\right\}$$

$$+ B(-\alpha_s, -\alpha_u)$$

$$M_0{}^{Pom} \equiv -q_1 \cdot q_2\, t_1 + m^2\nu^2 t_2$$

(4.3)
$$= \frac{1}{2}\left\{B(-\alpha_P, 1-\alpha_s) + D\, B(-\alpha_P, 2-\alpha_s) - \frac{D}{2(\alpha_P-1)} + (s \leftrightarrow u)\right\}$$

$$+ \frac{1}{2} B(1-\alpha_s, 1-\alpha_u) \quad ,$$

where $D = \alpha_P(t) - 1 + \alpha_s(s) + \alpha_u(u) = q_1{}^2 + q_2{}^2 + \alpha_P(0) - 1$.

For $q_1{}^2 = q_2{}^2 = \alpha_p(0) - 1 = 0$, the parameterization of M_1 is the same as that of Ref. (13).

Because these amplitudes are nonsingular, at $\alpha_s = 0$ or $\alpha_u = 0$, this Pomeranchuk contribution may be added to symmetric amplitudes with <u>arbitrary</u> strength without disturbing the normalization of the external line insertion poles (Born terms). Another important feature is the existence of a right-signature fixed pole located at $J = 0$, as the reader may quickly verify from the t-channel helicity amplitudes,

$$H_{11}^{t} = - M_0 - \frac{\phi}{2\gamma^2} M_1 \ , \ H_{1-1}^{t} = \frac{\phi}{2\gamma^2} M_1 \ .$$

It is curious to note that on mass shell($q_1^2 = q_2^2 = 0$), the fixed pole is absent only if $\alpha_P(0) = 1$ (i.e., $D = 0$), and that this model for Compton scattering closely resembles that of Abarbanel <u>et</u> <u>al</u>.(24). Our parameterization of $H_{1-1}^{t}(I_t = 0)$ for $\gamma\pi^{\pm} \to \gamma\pi^{\pm}$ is

$$(4.4) \quad H_{1-1}^{t} = 4e^2 \frac{\phi}{-t} \left\{ \frac{C_0}{1-\alpha_P} B(2-\alpha_P, -\alpha_s) + \frac{C_0}{1-\alpha_P} B(2-\alpha_P, -\alpha_u) + (1 + C_0) \frac{1}{t} B(-\alpha_s, -\alpha_u) \right\} .$$

The first two terms give asymptotically a Pomeranchon with a singular residue,

$$(4.5) \quad H^{t}_{1-1} \sim 4e^2 \frac{\phi}{-t} C_0 \Gamma(1-\alpha_P(t))(1+e^{-i\pi\alpha_P}) s^{\alpha_P - 2} .$$

The third term has no asymptotic contribution, but corresponds to a wrong-signature $(J = 1)$ fixed pole with a singular residue. The partial-wave projection (25) of the third term is

$$b^+(J,t) \propto \frac{1+C_0}{J-1} \frac{1}{t} 2^{-t} \quad .$$

This is precisely the J plane structure suggested in Ref. (24,) on the basis of an N/D model for t-channel unitarity.

If we make the assumption of Ref. (24) that there is no fixed pole $(C_0 = -1)$; we arrive at the same asymptotic cross section [Ref. (24), eq. (4.18)]

$$\sigma_{Tot}(s) \sim 16\pi^2 \frac{e^2}{4\pi} \alpha_P'(0) \frac{1}{3} \quad .$$

In our case, the slope of the Pomeranchon must be canonical $(\alpha_P'(0) = b \cong 1 \text{ GeV}^{-2})$, and the resultant cross section is too large by a factor of three (26). We see no reason in favor of this special value of C_0, particularly since fixed poles at wrong-signature and nonsense points are a typical feature of the Veneziano model for both strong and non strong processes.

The contribution of our Pomeranchon [neglecting (us) terms] to the electroproduction structure functions,

$W_1 = -\frac{1}{\pi} \text{Im } M_0$ and $W_2 = \frac{m^2}{\pi} \text{Im } M_1$ at $q_1^{\mu} = -q_2^{\mu} = q^{\mu}$, is easily calculated. In the limit of large q^2 and fixed $\rho = -2m\nu/q^2$, the structure functions are

$$W_1(q_1^2,\nu) \sim \frac{1}{\pi} q^4 (\rho-1)X, \quad \nu W_2 \sim -\frac{m^2}{\pi} q^2(\rho-1)^{-1}X ,$$

where

$X = \Gamma(2-\alpha_P(0)) [q^2(1-\rho)]^{\alpha_P(0)-1}$. The Bjorken scaling law (15) is not obeyed, but the linear increase of νW_2 for $\alpha_P(0) = 1$ (i.e., X = 1) may be consistent with experiment for moderate q^2. On the other hand, if we suppose that the J=0 fixed pole can be removed by adding terms of lower order in ν, one might multiply our Pomeranchuk amplitude by $F(q_1^2)\, F(q_2^2)$. In this approximation one is led to scaling for W_1, but not for νW_2. However, if we recall that for $\rho\pi \to \rho\pi$ one should have a nonsense factor $[\alpha_p(t) -1] \propto t$ in $H_{1-1}{}^{t}$, we see that νW_2 should not have a double pole at $q^2 = m_V^2$. In this case, we must multiply t_2 by $C_2[F(q_1^2) + F(q_2^2)]$ and t_1 by $C_1F(q_1^2)\, F(q_2^2)$ (so that $M^{\mu\nu}_{Pom}$ is still conserved), and both W_1 and νW_2 obey the scaling law

$$(4.6) \quad W_1 \sim \frac{C_2}{2} m_V^2\, \rho^2(\rho-1)^{-1} + C_1 m_V^4(\rho-1) ,$$

$$\nu W_2 \sim C_2 m^2 m_V^2 \rho(\rho-1)^{-1} .$$

These arguments are, of course, independent of the Veneziano model and they show that scaling of the Pomeranchuk contribution to both W_1 and νW_2 is not necessarily in contradiction with simple rho-dominance models(27).

Recently Sakurai (28) has proposed a rho-dominance model for the Pomeranchon which gives a similar result for νW_2, although he does not get scaling for W_1. Using the relationship

$$\nu W_2 = \frac{\nu(-q^2)}{\nu^2 - q^2} \text{ Im } [-H_{00}{}^s + H_{11}{}^s]$$

and parameterizing the s-channel helicity amplitudes,
$\text{Im } H_{00}{}^s \sim \frac{q^2}{m_V{}^2} F^2(q^2)\, s^{\alpha_P(0)}$, $\text{Im } H_{11}{}^s \sim \xi(\infty)\, F^2(q^2)\, s^{\alpha_P(0)}$, Sakurai arrives at scaling for νW_2. [Actually Sakurai parameterizes transverse and longitudinal cross sections (σ_T, σ_s), but the result is the same up to functions of ρ.] The important point is the kinematical factor of q^2 in $H_{00}{}^s$, which causes $H_{00}{}^s$ to dominate as $q^2 \to \infty$. If one imposes the condition that $H_{1-1}{}^t$ have a nonsense factor at $q^2 = m_V{}^2$ (i.e., for $\rho\pi \to \rho\pi$), one has, by Hara's theorem (29), $H_{00}{}^s/H_{11}{}^s \xrightarrow[s \to \infty]{} 1$ or $\xi(\infty) = 1$. The result is to introduce a zero at $q^2 = m_V{}^2$ which converts the double pole in W_2[from $F^2(q^2)$] into a single pole. Consequently, using Hara's theorem, we see that Sakurai gets the scaling law for νW_2 in the same manner as we do.

Sakurai's approach avoids the question of fixed poles since they do not contribute to W_1 and W_2. However, if they cannot be removed from the residue of the rho poles (at $q_i{}^2 = m_V{}^2$), they invalidate the use of a rho-dominance model. A careful analysis of the electroproduction data at high energy and low q^2 should be made to determine if the singular part of the Pomeranchuk residue for virtual Compton scattering has rho poles.

4.2. Mass Differences

To order e^2, electromagnetic self masses are given by (14)

$$\Delta m^2 = -\frac{ie^2}{2(2\pi)^4}\int \frac{d^4q}{q^2+i\epsilon}\, g_{\mu\nu}M^{\mu\nu} \quad , \tag{4.7}$$

where, from (2.8) (with $q_1 = -q_2 = q$) ,

$$g_{\mu\nu}M^{\mu\nu} = 3M_0 + m^2\left(\frac{q^2-\nu^2}{q^2}\right) M_1 \quad . \tag{4.8}$$

It is instructive to calculate separately the contributions of M_0 and M_1,

$$\Delta m^2 = \Delta m_0^{\ 2} + \Delta m_1^{\ 2} \quad , \tag{4.9}$$

where, after Wick rotation (30),

$$\Delta m_0^{\ 2} = \frac{3e^2}{(2\pi)^3}\int_{-\infty}^{0} \frac{dq^2}{q^2}\int_0^{(-q^2)^{\frac{1}{2}}} d\nu(-\nu^2-q^2)^{\frac{1}{2}} M_0(q^2,\ i\nu) \tag{4.10}$$

and

$$\Delta m_1^{\ 2} = \frac{e^2m^2}{(2\pi)^3}\int_{-\infty}^{0} \frac{dq^2}{q^2}\int_0^{(-q^2)^{\frac{1}{2}}} d\nu(-\nu^2-q^2)^{\frac{1}{2}}\left(\frac{q^2+\nu^2}{q^2}\right) M_1(q^2,i\nu) \quad . \tag{4.11}$$

For pion mass differences $(m_{\pi^+}{}^2 - m_{\pi^0}{}^2)$ which are $I = 2$ we have, from (2.7),

$$M_i = - M_i(us) \qquad \text{(pions)} \quad , \tag{4.12}$$

and for kaons $(m_{K^+}{}^2 - m_{K^0}{}^2)$ which are $I = 1$ we have from the internal symmetry factors of II

$$M_i = \frac{1}{3}\,[M_i(st) + M_i(tu) - 2M_i(us)] \qquad \text{(kaons)} \quad . \tag{4.13}$$

The Wick rotated expressions (4.10) and (4.11) follow from (4.7) only if the semicircle at $|q_0| = \infty$ gives no contribution. The (us) terms are clearly not Wick rotatable [see eq. (2.27)]. However, if they are inserted into (4.10) and (4.11), they may give a reasonable approximation, since they should be good for small q^2, and the small q^2 range of integration is emphasized due to their rapid decrease for $q^2 \to - \infty$. We shall therefore use the Wick rotated expressions.

The Wick rotatability of amplitudes with Regge or fixed-pole behavior has been investigated by Rabl (31). Our amplitudes illustrate some of the conclusions drawn there. For example, the fixed-pole part of M_1 (2.9)

$$4F(t)\,\left(\frac{1}{\alpha_s} + \frac{1}{\alpha_u}\right) \quad ,$$

gives a logarithmically divergent contribution

$$(4.14) \qquad \Delta m_{1,\ \text{divergent}}^{\,2} = \left(\frac{e}{2\pi}\right)^2 \frac{3}{2} m^2 \log \nu_{max} \quad .$$

This is due to the nonvanishing of the commutator $\delta(x_0)[\dot{V}^{\mu}(x), V^{\nu}(0)]$ (2). Since the fundamental nature of the logarithmic divergence remains mysterious, we note that we may still compute a meaningful numerical result if we follow the recipe of the current algebraists (32): set the hadron mass (m^2) to zero and the divergence disappears.

The term $- F(t)$ in M_0 for the FA corresponds to a Schwinger term and gives a quadratic divergence in Δm_0. In the QA the quadratic divergence is reduced to a logarithmic one of the form (4.14) and thus we shall consider the QA from now on.

The Regge part of $M_1(st)$, $4[1-F^2(q^2)]\, B(-\alpha_s, 2-a_t)$, gives a contribution to the integrand of $\Delta m_1^{\,2}$ that behaves like ν^{a_t} for $\nu \to \infty$ at fixed q^2, like $(q^2)^{a_t-2}$ for $q^2 \to \infty$ at fixed ν, and like $(q_0^{\,2})^{a_t-4}$ for $q_0 = \nu \to \infty$ at fixed $\vec{q}$ [see eq. (2.26)]. Since $a_t < 1$, this contribution is finite and Wick rotatable. This is also true for the term $2F^2(q^2)\, B(1-\alpha_s, -a_t)$ in M_0. On the other hand, the term $-q_1 \cdot q_2\, F(t)\, B(-\alpha_s, 2-a_t)$ in the QA amplitudes behaves like $(q^2)^{a_t-1}$ for $\vec{q}$ or ν fixed, and gives a divergent and nonrotatable contribution for $a_t > 0$. The term $-2m_V^{\,2}\, B(2-\alpha_s, -a_t)$ behaves even worse, namely as $(q^2)^{a_t}$. These last two divergences could be canceled by adding terms like $(-q_1 \cdot q_2)^n\, (2m_V^{\,2})\, B(2-\alpha_s, n-a_t)$, but without further restrictions such a method is too arbitrary to give any confidence in a numerical result.

In spite of the above divergence difficulties, it is interesting to examine the numerical values of the convergent contributions to the Wick rotated expressions (4.10) and (4.11). These are roughly the contributions one obtains in a simple vector-dominance model. Our results are presented in Table I, where the contribution of a simple Feynman Born term is also given for comparison. We remark that the results are rather insensitive to the intercepts a_t used, and for the (st) and (tu) terms about half the contribution comes from $|q^2| < 1\ \mathrm{GeV}^2$. We see that our results are worse than those of simple Born terms (33). The lack of improvement in the kaon mass differences from the A_2 trajectory is consistent with the recent results of other authors (34).

As one can see, the above discussion was more a catalog of possible divergence difficulties than a calculation of actual mass differences. The hybrid amplitudes of section III may be used to give more convergent results [if (us) terms can be obtained]. However, before reliable calculations can be made one will need less arbitrary Veneziano parameterizations, and this depends upon a better understanding of the role of the factorization of nonleading trajectories and many vector-meson poles.

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Table 1. Numerical results for finite contributions to Δm^2, with $b = 0.9\ \text{GeV}^{-2}$, $m_\rho^{\ 2} = 0.59\ \text{GeV}^2$, $m_{A_2}^{\ 2} = 1.64\ \text{GeV}^2$.

	Pion Mass (MeV)	Kaon Mass (MeV)
Δm_0		
Born term (seagull)	+ 3.7	+ 0.9
(st) and (tu) Terms		
$2F^2(q^2)[B(1-\alpha_s,-a_t) + B(1-\alpha_u,-a_t)]$	-	+ 9.1
(us) Term		
$2F^2(q^2)\ B(1-\alpha_s,\ 1-\alpha_u)$	- 1.5	- 0.3
Total [see eqs. (4.12) and (4.13)]	+ 1.5	+ 3.3
Δm_1		
Born term		
$4F^2(q^2)(\frac{1}{\alpha_s} + \frac{1}{\alpha_u})$	+ 0.5	+ 0.9
(st) and (tu) Terms		
$4[1-F^2(q^2)][B(-\alpha_s,2-a_t) + B(-\alpha_u,2-a_t)]$	-	- 0.8
(us) Term		
$4F^2(q^2)\ B(-\alpha_s,\ -\alpha_u)$	- 0.5	- 0.8
Total [see eqs. (4.12) and (4.13)]	+ 0.5	+ 0.3
$m_+ - m_0$		
Feynman Born term	+ 4.2	+ 1.8
Veneziano Model of II	+ 2.0	+ 3.6
Experimental	+ 4.6	- 3.9

REFERENCES AND FOOTNOTES

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1. R. C. BROWER and J. H. WEIS: Lawrence Radiation Laboratory preprints UCRL-19221 and UCRL-19222, 1969. These are referred to as I and II, respectively, and equation numbers from them are preceded by I and II. The N-point beta-function model is discussed in II.

2. J. D. BJORKEN: Phys. Rev., 148, 1467 (1966).

3. M. BANDER: Weizmann preprint, 1969.

4. H. SUGAWARA: Tokyo University of Education preprint, 1969.

5. I. OHBA: Waséda University preprint, 1969.

6. M. ADEMOLLO and E. DEL GIUDICE: MIT preprint CTP-89, 1969. These authors propose a model for weak amplitudes to all orders in the coupling g. We discuss here only their results for the first-order current-current interaction term.

7. Pions of definite charge are given by
$$|\pi\pm\rangle = 2^{-\frac{1}{2}}(|1\rangle \mp i|2\rangle), \quad |\pi^0\rangle = |3\rangle.$$

8. S. L. ADLER: Phys. Rev. 143, 1144 (1966).

9. R. F. DASHEN and M. GELL-MANN: Phys. Rev. Letters, 17, 340 (1966).

10. S. FUBINI: Nuovo Cimento, 43A, 475 (1966).

11. J. B. BRONZAN, I. S. GERSTEIN, B. W. LEE, and F. E. LOW: Phys. Rev. Letters, 18, 32 (1967), V. SINGH: ibid, 36 (1967).

12. H. G. DOESCH and D. GORDON: Nuovo Cimento 57A, 82 (1968), D. J. GROSS and H. PAGELS: Phys. Rev. Letters 20, 961 (1968).

13. R. C. BROWER and M. B. HALPERN: Phys. Rev. (to be published).

14. Our amplitudes are normalized to (i) times the amplitudes obtained from the Feynman rules of J. D. BJORKEN and S. D. DRELL: Relativistic Quantum Fields (McGraw-Hill Book Co., New York, 1965).

15. J. D. BJORKEN: Phys. Rev. (to be published).

16. K. BARDAKCI and S. MANDELSTAM: University of California, Berkeley preprint, 1969; S. FUBINI and G. VENEZIANO: MIT preprint CTP 81, 1969.

17. M. BANDER: Ref.(3), has shown this for his model.

18. G. VENEZIANO: in Proceedings of the First Coral Gables Conference on Fundamental Interactions at High Energy, Jan. 1969, (to be published). Reference 6 shows the usefulness of this technique and discusses it in some detail.

19. This is because the overlapping subenergies, α_1 and α_s and the total energy, α_2, become proportional as $q^2 \to \infty$.

20. H. D. I. ABARBANEL, M. L. GOLDBERGER, and S. B. TREIMAN: Phys. Rev. Letters, **22**, 500 (1969).

21. Leading trajectories can be conveniently extracted by using the replacement $(1- u_i u_j \cdots u_k)^{\alpha_u + c} \to e^{-(u_i \cdots u_k)\alpha_u}$ for leading s trajectories.

22. For a discussion of form factors and fixed poles in potential and field theory see H. RUBENSTEIN, G. VENEZIANO, and M. VIRASORO: Phys. Letters, **27B**, 38 (1968), and R. DASHEN and S. Y. LEE: Phys. Rev. Letters, **22**, 366 (1969), and references contained therein.

23. E. L. BERGER and G. FOX: Lawrence Radiation Laboratory Report UCRL-18886, May, 1969, is a good example of the kind of analysis that could be carried out once a careful parameterization were made of Compton scattering off the nucleon target.

24. H. D. I. ABARBANEL, F. E. LOW, I. J. MUZINICH, S. NUSSINOV, J. H. SCHWARZ: Phys. Rev., 160, 1329 (1967).

25. Following the techniques of D. SIVERS and J. YELLIN: Lawrence Radiation Laboratory Report UCRL-18665, this partial-wave amplitude (defined in Ref. (24)) is easily calculated. To relate the J-plane structure to that of Ref. (24), one should notice that the so called "multiplicative" fixed pole is decomposed into fixed pole with singular residue minus a Regge pole with singular residue.

$$\frac{1}{J-1}\;\frac{1}{J-\alpha_P} = \frac{1}{1-\alpha_P}\;\frac{1}{J-1} - \frac{1}{1-\alpha_P}\;\frac{1}{J-\alpha_P} .$$

26. A. H. MUELLER and T. L. TRUEMAN: Phys. Rev., 160, 1306 (1967), have estimated this cross section.

27. The constants C_1 and C_2 may be obtained by comparison with physical Compton scattering data or from a fit of the electroproduction data itself. However, the value of C_2 obtained from rho photoproduction would undoubtedly be much smaller, since there H_{1-1}^{t} is very small. This lack of smoothness in invariant amplitudes in vector dominance has been emphasized by H. FRAAS and D. SCHILDKNECT: DESY preprint 69/18, 1969.

28. J. J. SAKURAI: Phys. Rev. Letters, 22, 981 (1969). See also Ref. (27).

29. Y. HARA: Phys. Letters, 23, 696 (1966).

30. W. N. COTTINGHAM: Ann. Phys., 25, 424 (1963).

31. A. RABL: Phys. Rev., 176, 2034 (1968).

32. T. DAS, G. S. GURALNIK, V. S. MATHUR, F. E. LOW, and J. E. YOUNG: Phys. Rev. Letters, **18**, 759 (1967).

33. "Better" results have been obtained by other authors: V. S. MATHUR, P. OLESEN, and M. A. RASHID: Rochester preprint, 1969, have calculated pion mass differences. They use (st) and (tu) terms rather than the no-exotic-resonances (us) terms. Further they obtain constraints on trajectory intercepts from kinematics. We regard both these features as unnatural. D. M. SCOTT: Ohio State preprint, 1969, has calculated kaon mass differences. Due to kinematic factors, his vector dominance amplitudes give divergent contributions whereas ours are convergent. His result is thus strongly cutoff dependent.

34. M. ELITZUR and H. HARARI: Weizmann preprint, 1969, and references therein.

FIGURE CAPTIONS

Fig. 1. Kinematics for Pion Compton Scattering.

Fig. 2. Choice of Trajectories for eq. (3.1).

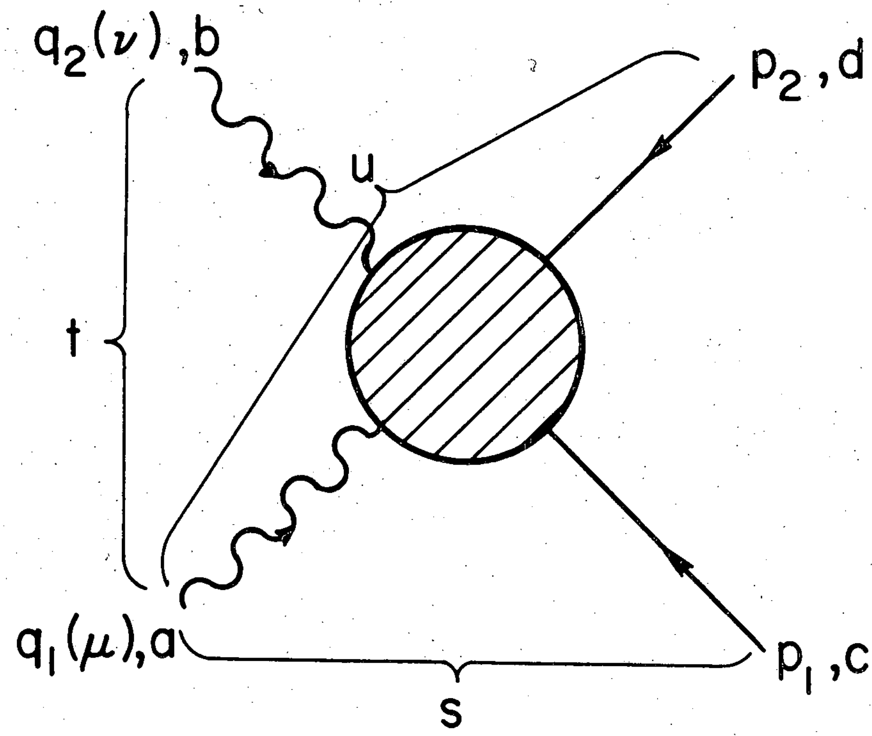
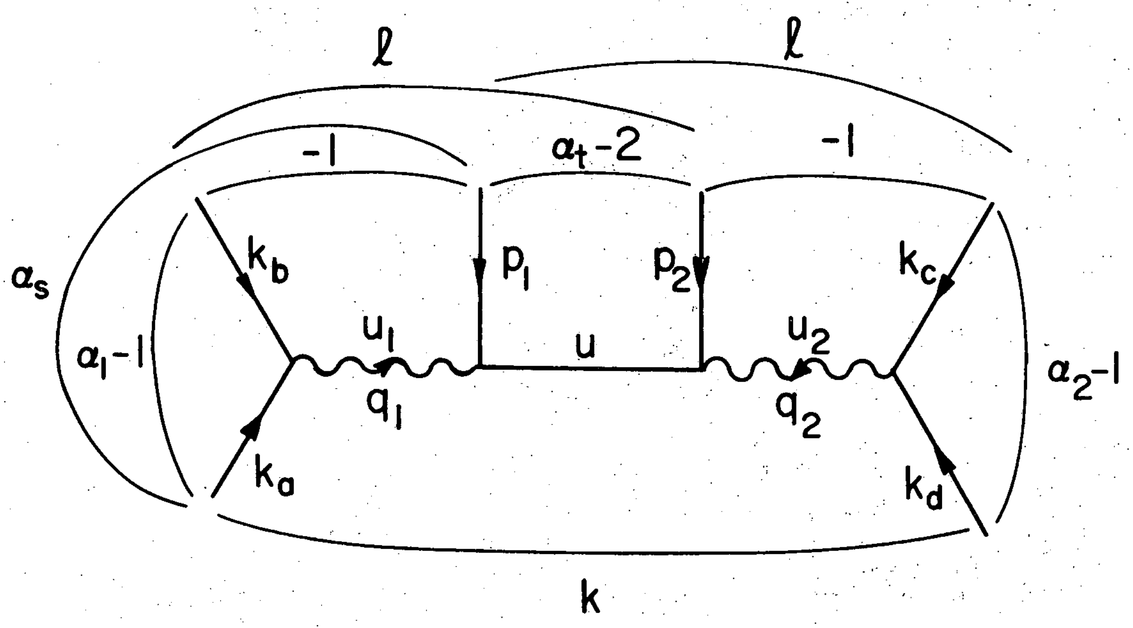


Fig. 1


ℓ
ℓ
-1
α_t-2
-1
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k_b
p_1
p_2
k_c
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u
u_2
q_1
q_2
α_2-1
k_a
k_d
k
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Fig. 2

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