Design of one-dimensional Lambertian diffusers of light

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Abstract

We describe a method for designing a one-dimensional random surface that acts as a Lambertian diffuser. The method is tested by means of rigorous computer simulations and is shown to yield the desired scattering pattern.

Optical devices that give rise to a scattered intensity that is proportional to the cosine of the scattering angle are frequently used in the optical industry, e.g. for calibrating scatterometers [1]. Such diffusers have the property that their radiance or luminance is the same in *all* scattering directions. Due to this angular dependence such devices are often referred to as *Lambertian diffusers*. In the visible region of the optical spectrum volume disordered media, e.g. compacted powdered barium sulphate, and freshly smoked magnesium oxide [2] are used as Lambertian diffusers. However, this type of diffuser is inapplicable in the infrared region due to its strong absorption and the presence of a specular component in the scattered light, in this frequency range.

The design of a random surface that acts as a Lambertian diffuser, especially in the infrared region of the optical spectrum, is therefore a desirable goal, and one that has been regarded as difficult to achieve [3]. In this paper we present a solution to this problem that is based on an approach used in several recent papers to design one-dimensional random surfaces with specified scattering properties [4-6], and to fabricate them in the laboratory [5,7]. The design of a two-dimensional random surface that acts as a Lambertian diffuser will be described elsewhere [8].

To motivate the calculations that follow we begin by considering the scattering of s-polarized light of frequency ω from a one-dimensional, randomly rough, perfectly conducting surface defined by $x_3 = \zeta(x_1)$. The region $x_3 > \zeta(x_1)$ is vacuum, the region $x_3 < \zeta(x_1)$ is the perfect conductor (Fig. 1). The plane of incidence is the x_1x_3-plane. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable, and to constitute a random process.

The mean differential reflection coefficient $\langle \partial R/\partial\theta_s\rangle$, where the angle brackets denote an average over the ensemble of realizations of the surface profile function, is defined such that $\langle \partial R/\partial\theta_s\rangle d\theta_s$ is the fraction of the total time-averaged flux incident on the surface that is scattered into the angular interval $(\theta_s, \theta_s + d\theta_s)$ in the limit as $d\theta_s \to 0$. In the geometrical optics limit of the Kirchhoff approximation it is given by [5]

$$\begin{aligned}\left\langle \frac{\partial R}{\partial\theta_s}\right\rangle &= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{1}{\cos\theta_0}\left[\frac{1+\cos(\theta_0+\theta_s)}{\cos\theta_0+\cos\theta_s}\right]^2\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}du\,\exp[i(q-k)u] \\ &\qquad \times\langle\exp[iau\zeta'(x_1)]\rangle. \qquad (1)\end{aligned}$$

In this expression L_1 is the length of the x_1-axis covered by the random surface, θ_0 and θ_s are the angles of incidence and scattering, respectively, $a = (\omega/c)(\cos\theta_0 + \cos\theta_s)$, and $q = (\omega/c)\sin\theta_s, k = (\omega/c)\sin\theta_0$. In what follows, we will restrict ourselves to the case of normal incidence ($\theta_0 = 0°$), in which case Eq. (1) simplifies to

$$\left\langle \frac{\partial R}{\partial\theta_s}\right\rangle = \frac{1}{L_1}\frac{\omega}{2\pi c}\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}du\,\exp iqu\langle\exp[iau\zeta'(x_1)]\rangle, \qquad (2)$$

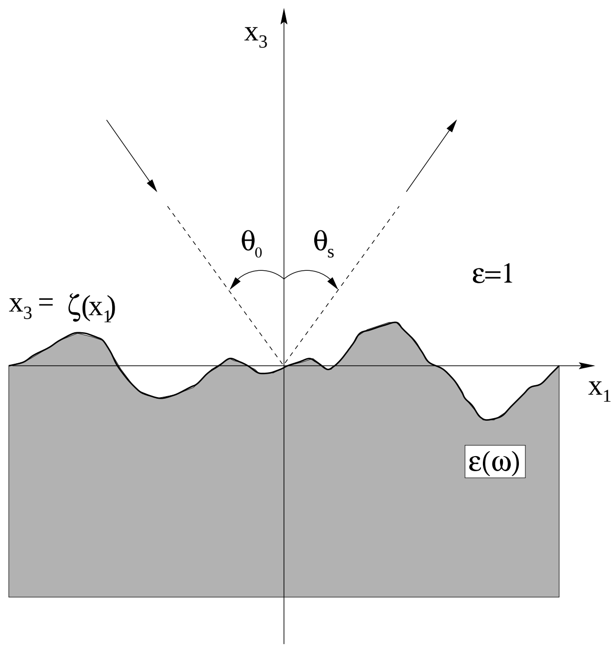

Figure 1: The scattering geometry assumed in this paper.

where a is now given by $a = (\omega/c)(1 + \cos\theta_s)$.

We wish to find a surface profile function $\zeta(x_1)$ for which the mean differential reflection coefficient has the form

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{2}\cos\theta_s. \tag{3}$$

To this end we write $\zeta(x_1)$ in the form [5]

$$\zeta(x_1) = \sum_{\ell=-\infty}^{\infty} c_\ell \, s(x_1 - \ell 2b). \tag{4}$$

Here the $\{c_\ell\}$ are independent, positive, random deviates, b is a characteristic length, and the function $s(x_1)$ is defined by [5]

$$s(x_1) = \begin{cases} 0 & x_1 \le -(m+1)b \\ -(m+1)bh - hx_1 & -(m+1)b \le x_1 \le -mb \\ -bh & -mb \le x_1 \le mb \\ -(m+1)bh + hx_1 & mb \le x_1 \le (m+1)b \\ 0 & (m+1)b \le x_1 \end{cases}, \tag{5}$$

where m is a positive integer. Such trapezoidal grooves can be generated experimentally [5,7].

Since the $\{c_\ell\}$ are positive random deviates, their probability density function (pdf) $f(\gamma) = \langle \delta(\gamma - c_\ell) \rangle$ is nonzero only for positive values of γ.

It has been shown [5] that when the surface profile function is given by Eqs. (4) and (5), the expression (2) for the mean differential reflection coefficient becomes

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{4h} \left(1 + \tan^2 \frac{\theta_s}{2}\right) \left[f\left(-\frac{1}{h} \tan \frac{\theta_s}{2}\right) + f\left(\frac{1}{h} \tan \frac{\theta_s}{2}\right)\right]. \tag{6}$$

Thus, we find that in the geometrical optics limit of the Kirchhoff approximation the mean differential reflection coefficient is determined by the pdf of the coefficients $\{c_\ell\}$ entering the expansion (4), and is independent of the wavelength of the incident light. If we make the change of variable $\tan(\theta_s/2) = \gamma h, 0 \leq \gamma h \leq 1$, so that $\frac{1}{2}\cos\theta_s = \frac{1}{2}(1 - \gamma^2 h^2)/(1 + \gamma^2 h^2)$, on combining Eqs. (3) and (6) we find that the equation determining $f(\gamma)$ is

$$f(-\gamma) + f(\gamma) = 2h \frac{1 - \gamma^2 h^2}{(1 + \gamma^2 h^2)^2}. \tag{7}$$

It follows that

$$f(\gamma) = 2h \frac{1 - \gamma^2 h^2}{(1 + \gamma^2 h^2)^2} \theta(\frac{1}{h} - \gamma)\theta(\gamma). \tag{8}$$

The preceding results were obtained in the geometrical optics limit of the Kirchhoff approximation for a perfectly conducting surface. However, our earlier experience in designing surfaces with specified scattering properties [4-6] shows that when a surface designed on the basis of these assumptions is ruled on a lossy metal, the results of rigorous scattering calculations show that the resulting scattering pattern retains the form prescribed in the approximate, single-scattering calculations. We now demonstrate that such a result is obtained in the context of the present problem.

From the form of $f(\gamma)$ given in Eq. (8) a long sequence of $\{c_\ell\}$ was generated by applying the rejection method [9], and the resulting surface profile function $\zeta(x_1)$ was generated by the use of Eqs. (4) and (5). We found from numerical experiments that in order to have a surface that acts as a Lambertian diffuser in reflection the parameter b had to be large. Physically this means that the grooves $\zeta(x_1)$ have to be wide.

In Fig. 2 we present the results of rigorous numerical Monte Carlo simulations [10] for the angular dependence of the mean differential reflection coefficient $\langle \partial R/\partial \theta_s \rangle$ for s-polarized incident light of wavelength $\lambda = 612.7$nm scattered from a randomly rough silver surface of the type described above (noisy curve). The value of the dielectric constant of silver at this wavelength is $\epsilon(\omega) = -17.2 + i0.5$. The surface was characterized by the parameters $b = 80\lambda = 49\mu$m, $h = 0.2$,

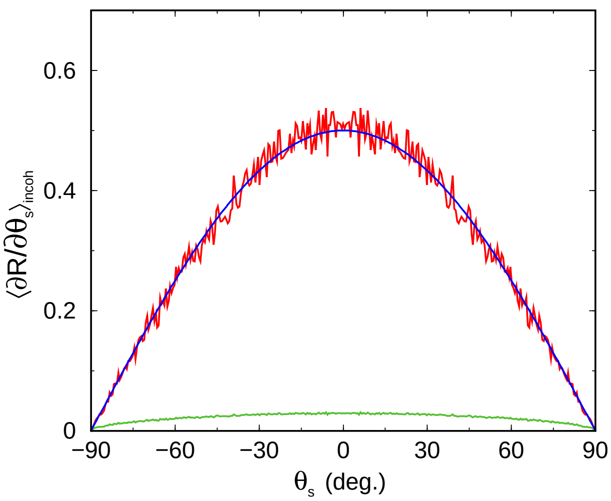

Figure 2: The noisy curve is $\langle \partial R/\partial\theta_s\rangle$ calculated by a numerical simulation approach for a random silver surface defined by Eqs. (4) and (5) with $b = 80\lambda$, $h = 0.2$, $m = 1$, and the pdf (8), when s-polarized light of wavelength $\lambda = 612.7\text{nm}$ ($\epsilon(\omega) = -17.2 + i0.5$) is incident normally on it. The upper solid curve is $\langle \partial R/\partial\theta_s\rangle$ given by Eq. (3). The lower solid curve is the error in the calulated mean differential reflection coefficient as measured by its standard deviation.

and $m = 1$, and its length used in the simulation was $L_1 = 164\lambda = 100\mu\text{m}$. Furthermore, the plot in Fig. 2 was obtained by averaging the results for $N_\zeta = 35,000$ realizations of the surface profile function $\zeta(x_1)$. Such a large number of surface realizations was needed in order to reduce the noise level sufficiently. The reason for the slow convergence of the mean DRC with increasing N_ζ we believe is due to the large value of b used in the simulations. Without compromising the spatial discretization used in the numerical calculation ($\Delta x_1 = 0.164\lambda$) needed in order to resolve the oscillations of the incident field, only a few grooves $s(x_1)$ could be included for each realization in the sum (4) defining the surface.

The lower smooth curve represents an estimate of the error in the calculated $\langle \partial R/\partial\theta_s\rangle$ due to the use of an finite number of surface realizations for its calculation. This error is obtained as the standard deviation of the mean differential reflection coefficient (see Ref. [10] for details).

The upper smooth solid curve in Fig. 2 represents the geometrical optics limit of the Kirchhoff approximation, Eq. (3). As can be readily observed from this figure, the agreement between the geometrical optics limit of the Kirchhoff approximation for a random perfectly conducting surface and the result of rigorous numerical simulations for a real random silver surface is excellent within the noise level. This is indeed the case for all scattering angles θ_s, which we find somewhat surprising, since one might have expected the geometrical optics approximation to break down for the largest scattering angles. That this is not observed in our simulation results is probably an indication that multiple scattering processes are of minor importance in the scattering taking place at the random surface even for the largest scattering angles.

Simulations (results not shown) were also performed where the wavelength of the incident light was changed by plus and minus 10% from its original value of $\lambda = 612.7$nm. Such changes did not affect the Lambertian nature of the scattered light in any significant way. This weak wavelength sensitivity is consistent with our earlier experience in designing surfaces with specified scattering properties [4-6]. Surfaces generated on the basis of different b parameters have also been considered. We found that the scattered intensity showed little sensitivity to this parameter as long as it is large.

Acknowledgments

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