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Design and Theory of Diverse Forms of Participation in Mathematics through Geometris, a Collaborative, Body-scale Game

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by

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Committee in charge:

Professor Dor Abrahamson, Chair
Professor Kimiko Ryokai
Professor Michelle Wilkerson

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Abstract

Design and Theory of Diverse Forms of Participation in Mathematics through Geometris, a Collaborative, Body-scale Game

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Mathematics education, especially in school settings, is too often a disempowering experience for learners, who come away with the educationally and economically limiting impression that mathematics is not for them. I seek to broaden participation in mathematical practices by exploring the theoretical implications and design heuristics that can be gleaned from contextualizing geometric and spatial reasoning within play in the form of a collaborative, body-scale game.

My work is informed by theories of learning as materially and socially situated and of cognition as inextricably rooted in embodied, physical experience. The analyses presented herein use both close case analysis and qualitative coding techniques to identify and characterize interaction behaviors.

This dissertation contains three papers. The first paper focuses on the interpersonal dynamics of parent-child player groups and characterizes *participatory facilitation*, a pattern of flexible adoption, adaptation, casting-off, and redeployment of scaffolding behaviors by adults toward alternating pedagogical and performance goals. The second paper zooms out to examine the learning opportunities available through observation of gameplay. I identify range of participation roles – from player to spectator and some in between – that constitute a rich learning ecology and document the ways participants fluidly within it. Finally, my third paper zooms out again to consider how the social relationships that players bring to the body-scale game shape their collaborative movement patterns and resultant learning opportunities.

Throughout, I propose theoretical constructs about the role of material and social forces in informal learning and re-assert the role of these forces in mathematics learning. I also inform the design of other embodied mathematics learning activities by implicating particular design decisions as enabling or constraining the documented interactions.

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Acknowledgements

I wish to thank all the people who played *Geometris* as part of these studies. Your play renewed my respect and appreciation for the complexities of learning in interaction and for the diverse means by which people constantly and fluidly navigate those complexities. I wish to acknowledge the contributions of the research assistants – Japleen Kaur, Jake Cha, and SreeVidya Ganga – whose enthusiasm and time made possible the analyses presented in this work.

Geometris, the game at the heart of this work, was truly a collaborative project between myself, Elena Durán-López, and Ganesh V. Iyer. The game's ideation, prototyping, and iteration reflect the special union of our interests and passions, and it would not have been possible without them. In this development work, we received significant guidance from Professor Kimiko Ryokai and Dr. Noura Howell as well as financial support from the Barbara White Bequest and the CITRIS Tech for Social Good initiative.

For the intellectual development both of *Geometris* as a project and of this dissertation's argument, I am indebted to current and former members of Professor Dor Abrahamson's Embodied Design Research Laboratory (EDRL). From listening to me gush over 30-second video clips to entertaining seemingly out-there ideas (the Spectatorship paper began with the question "What if learning math was like watching a football game?"), Dor and the EDRL group were endlessly generous with their time and intellectual energy, engaging with this project every step of the way.

The actual writing of this dissertation was made so much more fulfilling by contributions and support from Professor Michelle Wilkerson's Academic Writing Support Group. Michelle and my fellow writing group members embraced all the twists and turns as these papers developed, providing invaluable feedback and guidance on everything from big picture argument to sentence level organization to version on versions of diagrams. My writing would not be the same without you all.

Finally, I want to thank my family who made this work possible in so many ways. My interests and skills are indelibly formed by the upbringing my parents provided. They always encouraged me to play around with materials, to explore their properties, and to express my ideas through them. These experiences fostered a persistent interest in crafts and tinkering and, I suspect, shaped my general approach to thinking and learning. And finally finally, to my husband whose support made this graduate school process possible. This project would not have happened without your willingness to help me develop ideas, to give me a real gamer's perspective, and to always give me a hug when I need it.

1 Introduction

In the U.S. and elsewhere, there exists a cultural narrative that mathematics is a solitary and abstract domain of formulas and number-crunching. This narrative misrepresents the work of many professional mathematicians, and, more importantly, often discourages young mathematics learners. In fact, expert mathematics work is often richly imaginative (Hadamard, 1945), collaboratively constructed through gesture (McNeill, 1992), and suffused with experiences of emotion and beauty (Zeki et al., 2014). However, research suggests that many mathematics learners develop a considerably less enthusiastic view of the field. In *Mindstorms*, Seymour Papert recounts the story of Bill, a student who describes his experience learning multiplication tables, “You learn stuff like that by making your mind a blank and saying it over and over until you know it” (1980, p.65). Decades of research on mathematics anxiety indicates that many mathematics learners share Bill’s perspective and, worse, can develop highly negative, even phobic reactions to basic mathematical work (Ashcraft, 2002; Faust, 1994), leading to strong avoidance of the subject. Despite decades of reform-based mathematics pedagogies, many mathematics learners’ experiences remain unsatisfactory, often reinforcing cultural narratives that discourage meaningful engagement with mathematics.

There are many approaches to support students in developing positive impressions of mathematics and of themselves as mathematicians, from facilitation practices as entrained in teacher education and professional development to extracurricular opportunities for math enrichment. My work takes up this challenge through the creation and evaluation of mathematics-learning activities. I pursue a design based research approach (Brown, 1992; Collins, 1992) to integrate novel interaction technologies into learning activities with the goal of enabling more intuitive and meaningful engagement with mathematics concepts.

One largely untapped resource to nurture children’s spatial-mathematical reasoning is the rich, informal knowledge they already possess from their daily, lived experience (Sheets-Johnstone, 1999). Pre-dating and in tandem with formal school systems, socially situated work and activity have been the primary means through which children learn about their world (Cole & Scribner, 1974; Vygotsky, 1978; Wertsch, 1979) and have served as productive contexts for developing children’s reasoning skills (Keifert & Stevens, 2019). The knowledge generated in such activity is often considered to be latent, habituated in the body as felt experience or muscle memory. And modern conceptions of education, especially within mathematics, largely compartmentalize this lived, experiential knowledge from academic learning (e.g. homework time vs. playtime). This dissertation is motivated by a desire to leverage children’s expertise at exploring and operating within their physical world as a resource for their mathematical learning.

One of the main ways that children explore their physical worlds is through play, an informal resource that scholars have long recognized as valuable for children's learning and development (Dewey, 1913; Froebel, 1899; Huizinga, 1955; Montessori, 1967; Vygotsky, 1978). Research has also illustrated an array of mathematics concepts, such as classification, patterning, and equal partitioning, latent within children's play (Ginsburg, 2006; Wager & Parks, 2014). Yet, with the exception of building blocks that serve the youngest geometry learners, informal play is seldom leveraged for learners' mathematical development. Empirical work and theory-building is needed around how play could contextualize mathematics reasoning practices and how to design for such productive interactions.

In an effort to build theory around body-based mathematical play, this dissertation studies mathematical reasoning practices in the context of *Geometris*, a collaborative math game that I co-developed. Guided by principles of *embodied design* (Abrahamson, 2009a), *Geometris* instantiates geometric and spatial reasoning concepts within body-scale movements, similar to the games *Twister*® or *DanceDanceRevolution*®. My work is based on the perspective of learning as a process that is situated both within social practice (Rogoff, 1990; Vygotsky, 1978) and in interaction with material resources (Kirsh, 2010; Piaget, 1968).

As a design-based research project, I aim to address research questions of both a theoretical and design nature such as:

- Theory: What forms of participation emerge in an embodied collaborative mathematical game? What forms of teaching and learning are enabled by those forms of participation?
- Design: In what ways do the game's material configurations and collaborative participation structures create opportunities or present challenges for informal teaching and learning?

Across the following papers, I take three levels of focus[1] to characterize participants' interactions with their social and material environments in relation to their resulting learning opportunities (Figure 1).

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[1] I recognize the similarity to Rogoff's (2008) three levels for observing sociocultural practice, however I do not consider the personal level in this analysis and instead consider two levels of community.

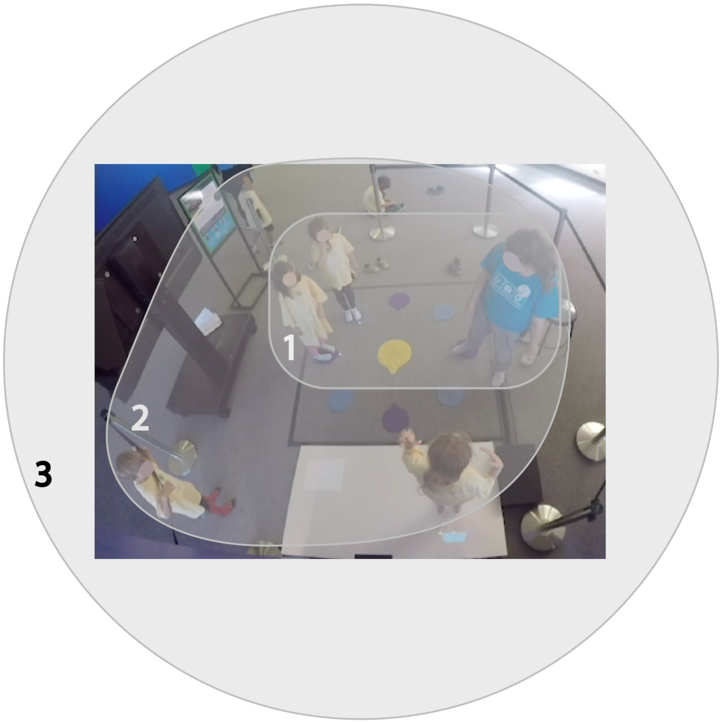

Figure 1. Three levels of focus across these dissertation papers.

In my first paper, I perform a close case analysis of 3 parent-child groups in which the adults took up consistent behaviors seemingly intended to help children learn the spatial relationships on which the game is built. These patterns of interaction, which I call *participatory facilitation*, seemed to change in consistent ways in response to both children's developing skill and elements of the designed environment. I describe participatory facilitation in terms of scaffolding techniques employed by adults and identify design features that enabled participatory facilitation. I suggest that informal STEM learning activities that enable or encourage participatory facilitation could better align with the intergenerational, mixed skill-level groups in which, per historical and socio-cultural perspectives on learning, novices learn new skills and practices. This paper was published in a special issue of DEME focused on the intertwined contributions of physical and digital tools for the teaching and learning of mathematics (Rosenbaum et al., 2020).

My second paper takes a slightly wider focus and considers the variety of observing, coaching, and other peripheral forms of participation that emerged around *Geometris* play. From this spectator position, many participants offered correct recommendations to players, suggesting that these spectators learned about the game's spatial relationships without playing it themselves. Additionally, people seemed to move somewhat fluidly between participation roles. Not only were the forms of participation with *Geometris* more varied than designed for, but people also moved through and between them in unanticipated ways, likely shaping each other's experiences of the interaction and their informal learning from it. In this paper, I consider the affordances for informal teaching and learning of various spectator roles and of fluidity between spectator and player roles. Using both close case analysis and qualitative video coding, I characterize these new forms of participation and explore whether they serve a pedagogically valuable role in this learning activity. I evaluate design and contextual features that enable and encourage spectator-like forms of participation and learning, drawing out design heuristics that could inform other learning activities.

In my third paper, I zoom out further to consider how the social relationships between participants might shape their willingness to engage in a collaborative, body-scale task such as *Geometris* in the first place. Considering that moving with another person can feel very different depending on who that person is (e.g., navigating a sidewalk with strangers vs. a small kitchen with a partner), I evaluate how social relationships among participants – be they strangers, friends, or family members – may shape and constrain their movement and learning opportunities. I develop a framework to characterize participants' physical proximity as they play *Geometris* and apply it to video recordings of 41 visitor dyads. These dyads were composed of family members, friends, and strangers and demonstrated markedly different collaborative movement patterns. Considering these patterns in the design of other collaborative embodied learning activities could create a more comfortable and supportive environment for learners to move and learn together.

2 Design and Data Sources

All three papers presented herein analyze data from the same installation of *Geometris* at a family science museum. To avoid repetition across the papers, I present *Geometris*'s design and the details of its installation here. I present the motivation, guiding theory, and results of each analysis in the following chapters.

Inspired by theories of embodied cognition (Wilson, 2002) and practices of embodied design (Abrahamson, 2009, 2014), *Geometris* (Figure 2) is an interactive, movement-based game in which learners collaboratively explore geometric and spatial relationships at body-scale. As a full-body interaction learning environment (Malinverni & Pares, 2014), *Geometris*'s scale and interactions are similar to games such as *Twister*®, *DanceDanceRevolution*®, art installations like *Boundary Functions* (Snibbe, 1998), and educational design like FUTUREGYM (Takahashi, Oki, Bourreau, Kitahara, & Suzuki, 2018), which take body-scale movement as inputs. Along with Elena Durán–López and Ganesh Iyer, I co-developed *Geometris* to support youths' learning and exploration of geometry and to offer a rich platform for research on embodied learning environments (e.g., Antle, 2013; Desai, Blackler, & Popovic, 2019). *Geometris* has won two international awards for educational design (CHI 2017, IDC 2017).

Geometris consists of a 6x6 ft. interactive mat and a projection screen, both on the floor (Figure 2). Each gameplay level begins with a shape projected onscreen (Figure 2, the blue rectangle). As the shape descends on the screen, players must recreate it on the mat, coordinating their limbs to activate the corresponding pressure-sensitive fabric pads before the shape reaches the horizontal red line and time runs out. The pressure pads are agnostic to individual players, meaning multiple people can activate the same pad simultaneously. As visual feedback, activated vertices are depicted on-screen as red points and lines (Figure 2). As a rule, the central pad on the mat corresponds to the yellow circle in the projection, orienting the shape relative to the mat. The game also emits a celebratory sound upon shape completion, offering auditory feedback. Completed shapes are overlaid into an abstract design that becomes the players' score (Figure 2, screen bottom right).

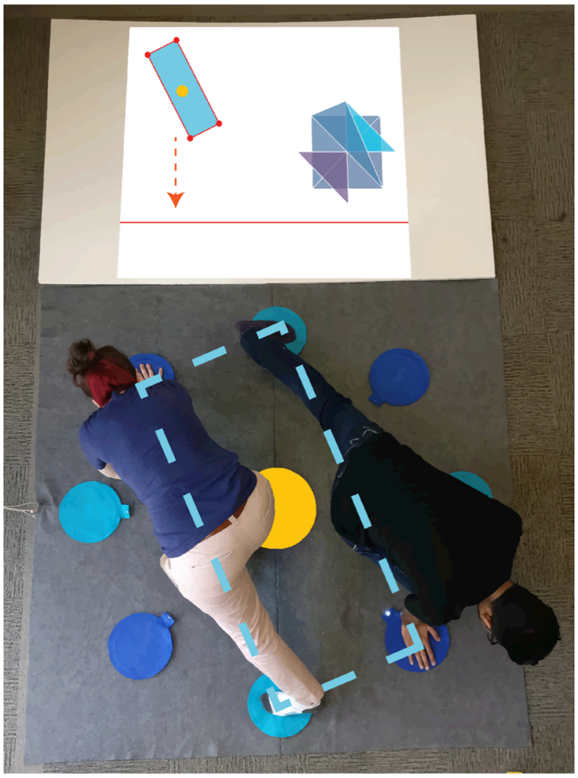

Figure 2. Geometris. Shapes descend on the screen (top) toward the horizontal red line. The mat (bottom) contains 9 pressure-sensitive pads. The central yellow pad maps to a yellow circle on each target shape.

Geometris's three timed levels each contain 8 shapes and vary in difficulty by the number of vertices in the prompted shapes. Level 1 contains triangles; Level 2 includes quadrilaterals; and Level 3 includes pentagons and hexagons (Figure 3). *Geometris* also includes a Practice level in which players can explore the mat's input/output functionality without a time constraint or specified target shape. Shapes with more than four vertices were intentionally included in the harder levels to promote collaboration. In these levels, two or more players must coordinate their positioning, through verbal or gestural utterances, to activate the appropriate sensors. For more details on *Geometris* hardware and software, see Appendix A as well as Durán–López, Iyer, and Rosenbaum (2017) and Durán–López, Rosenbaum, and Iyer (2017).

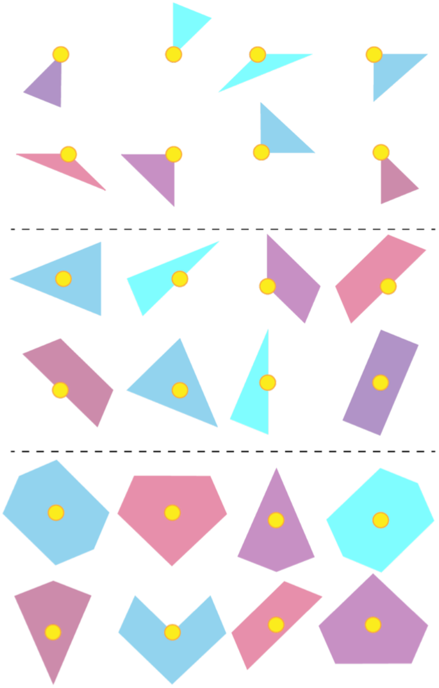

Figure 3. Geometris levels. Level 1 (top) includes triangles. Level 2 (middle) includes quadrilaterals. Level 3 (bottom) includes higher-order polygons.

Geometris was installed on the exhibit floor of a science museum for 1 month in Summer 2018. While on display, Geometris was facilitated by a rotating set of high-school aged volunteers, who wore teal t-shirts and had a designated stool in situated just off the mat but within the stanchions that delineated the exhibit area (Figure 4). These facilitators were trained by museum educators but were not coached by researchers on how to scaffold visitors' experiences. Collected data includes 6 hours of audio-video recordings, filmed by GoPro camera mounted at 45^{o} to the floor, which captured the game screen, mat, and some surrounding floorspace (Figure 4).

Figure 4. *Geometris* as installed at a science museum.

3 Shaping Perception: Designing for Participatory Facilitation of Collaborative Geometry

This first paper was co-authored by myself, Japleen Kaur, and Professor Dor Abrahamson and draws on theoretical frameworks from ecological dynamics, genetic epistemology, and sociocultural semiotics to study play and informal teaching behaviors among parents and children. Micro-analysis of 3 parent–child groups playing *Geometris* implicates two design features as supporting mediated development of geometry skills: (1) spatial distribution across two displays—the screen and the mat—poses cross-display figural mapping as a tactical problem whose perceptual solution constitutes the game's learning objective; and (2) a multi-sensor input interface—the mat's "vertices"—enables flexible divisions of group labor for scaffolding solution enactment. We put forth the construct of *participatory facilitation*—an emergent interaction pattern in groups with inter-personal differences in content-domain knowledge and sensorimotor coordination. We tentatively generalize principles for designing informal educational activity architecture that create opportunities for relative experts to enculturate content learning via participatory facilitation.

3.1 Introduction

3.1.1 Let's Start with the Design

Consider the following scenario.

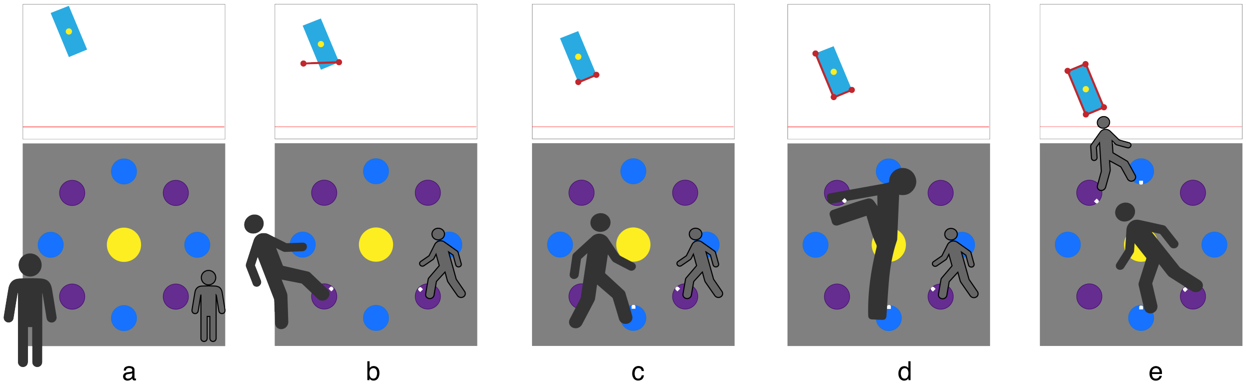

Figure 5. Geometris gameplay. a) The problem scenario. b) An initial attempt. c) An adjusted attempt. d) Testing a mapping. e) Completing the target shape. (Icons by Bradley Avison, James Keuning from Noun Project.)

You and a young person—perhaps your child or student—are standing on a gray carpet with a ring of colored pads (Figure 5a). On the floor in front of the mat, a screen displays a slowly descending rectangle. You are asked to make that rectangle by pressing the colored pads on the mat before the rectangle reaches the red line onscreen. You're confused: the rectangle is composed of 4 vertices and 4 edges. The carpet has an array of circles but no edges. Unsure, you step on the pad nearest you, and an LED on the pad turns on. Your young partner also steps on a pad, illuminating their LED. At the same time, two points appear onscreen, connected by a line. One point is on the rectangle's bottom-right vertex (bingo!), but the other point is to the left of the rectangle (Figure 5b). On a hunch, you step over one pad. Your onscreen point now appears on the rectangle's bottom-left corner (good!), and a line appears on its bottom edge (Figure 5c). Hmm, the pads on the floor seem to map to the rectangle's vertices on the screen: left to the left, right to the right, and perhaps the two yellow circles correspond—the little one on screen and the large on one the mat? There are still two vertices left. Lunging forward, you press a pad near the top of the mat with your hand. A third point appears on the rectangle along with one long edge (Figure 5d). Ah hah! So up on the mat is up on the screen, too. A warning tone sounds, as the rectangle nears the bottom of the screen... The last corner of your rectangle must be to your right, but you're precariously balanced. Can your young partner stretch that far? You decide to reposition, ask your partner to take your spot, and you return to the bottom of the mat. Having switched spots, the rectangle is complete (Figure 5e). Go team! A high-pitched "Yay!" sounds from the game console, and the rectangle is replaced by another falling shape.

What, if anything, have you learned from this scenario? What skills, assumptions, and understandings did you use to establish a figural mapping between the rectangle on the screen and the pads on the mat? What, if anything, has your young partner learned? As more shapes appear onscreen, how might you structure your play to help your young partner to develop those same figural mapping skills?

This paper reports on the empirical evaluation of *Geometris*, the game described above. *Geometris* is rooted in the assumption that geometric knowledge rests in visualization and imagination of 2D shapes and operations upon them (Initiative, 2017; NCTM, 2000). Research suggests that such spatial reasoning skills are highly correlated with children's mathematical learning and achievement (Gilligan et al., 2019; Okamoto et al., 2014; Wolfgang et al., 2003) and can be improved with training (Uttal et al., 2013). *Geometris* was designed to create playful opportunities for children to encounter and grapple with challenges of geometric and spatial reasoning. We argue that two qualities of the design support interactions conducive to pursuing this learning objective. First, we explain how *Geometris*'s spatial distribution across two displays—a physical floor interface and an adjacent digital screen, also on the floor—poses the perceptual mapping of geometric figures across the displays as an emergent problem of enacting the game mechanics, a problem whose solution constitutes the activity's learning objective. Second, we illustrate through analysis of three focal groups how the game's large-scale, multi-sensor floor interface affords unusual pedagogical opportunities for adult participants to enculturate young participants into the target mathematical practice.

Our analysis has led us to propose the construct of *participatory facilitation*, a behavioral pattern observed within groups with inter-personal differences in content-domain knowledge and sensorimotor coordination, such as between parents and children. This study looks to characterize how *Geometris*'s activity architecture creates opportunities for participants to scaffold collaborative achievement of the game objective through exercising participatory facilitation. We position this work within broader research efforts to understand how novel digital technologies enable new forms of participation in educational activities (e.g., Hegedus & Penuel, 2008). The remainder of this section situates *Geometris* in the context of related mathematics learning activities and explains its design rationale.

3.1.2 Game Style and Related Works

Geometris is a collaborative, body-scale, player-vs-environment game that challenges players to map shapes from a 6x3ft screen onto a 6x6ft sensor array (Figure 2). Similar to early work by Nemirovsky et al. (1998), *Geometris* occupies a middle ground between, on the one hand, sedentary activities at the desktop scale, such as traditional pencil-and-paper or Dynamic Geometry Software and its variants for individuals (Howison et al., 2011; Leung et al., 2013) or pairs (Nemirovsky et al., 2013), and, on the other hand, ambulatory activities at the city-block scale, such as *Walking Scale Geometry* (Ma, 2017) or *GPS Graphing* (Hall et al., 2015). *Geometris* is similar to these latter exemplar activities in that it requires players to tackle the posed challenges collaboratively. And as in technologically enabled collaborative simulations that use a large screen to display collective activity of iconized participants (e.g., Enyedy et al., 2015; Wilensky & Stroup, 2000), *Geometris* players must locate and monitor virtual traces of their individual actions in coordinating the enactment of a collective configuration—for example, distributing a rectangle among their bodies while recognizing oneself at a particular point. In addition, *Geometris* includes design features, such as a time limit, music, and the possibility for failure, that classify it as an educational game rather than a playful learning activity (e.g., Kelton & Ma, 2020; Price & Duffy, 2018).

Geometris was designed as a game through which learners could recognize and collaboratively enact geometric and spatial relationships. As the game's levels progress, the software challenges players to make polygons of increasingly higher order (Figure 3), even as the time limit remains constant. This design poses a trade-off. The mathematical degrees of freedom decrease across these levels, as there are fewer ways to define higher-order polygons (e.g., pentagons and hexagons) vs. lower-order polygons (e.g., triangles) on the sensor array. However, players' coordination challenge increases. With more active vertices, players must develop a stronger sense of their location on the mat relative to the digital display—that is, a stronger figural mapping—in order to adjust their actions. As such, players must coordinate their actions more precisely to manage this increasing number of vertices within the time limit, making play more difficult.

Geometris was designed so that the target mathematics is intrinsic to gameplay, that is, the disciplinary content is instantiated in the game's tactics as well as its strategy (Habgood & Ainsworth, 2011; Holbert & Wilensky, 2014; Kafai, 1996). As such, the moment-to-moment goals of mapping between the game's two displays both mobilize authentic geometrical reasoning in the game's granular tactics and exercise spatial reasoning in the game's broad strategy. In this sense, *Geometris* is more similar to games such as *The Logical Journey of the Zoombinis* (Broderbund, 1996*)* or *Rolly's Adventure* (Williams-Pierce, 2016), in that mathematics is intrinsic to play; and less similar to edutainment games, such as *Math Blaster*, that present educational content between rounds of content-irrelevant play.

3.2 Design Rationale: Implementing Constructivist and Semiotic–Sociocultural Theory

Geometris was designed so that participants' attempts to perform the task exercise a set of disciplinarily favorable perceptual skills that most young children have not yet developed (Piaget et al., 1960). Given appropriate mediation from accompanying adults, young *Geometris* players could thus become enculturated into forms of spatial reasoning believed to serve their mathematics learning. From a cultural–semiotic perspective, *Geometris*'s figural-mapping challenge demands perceiving two sensory displays as mutually referential (Duval, 2006) or otherwise equivalent (Sfard & Lavie, 2005). By what conventions might a child come to perceive a collection of four distributed points as a geometrically significant form, that is, as a rectangle? These displays are superficially different, yet they could become affiliated as "the same" by endorsing mathematical perspectives (Abrahamson, 2002; Bartolini Bussi & Mariotti, 2008; Newman et al., 1989; Sfard, 2002)[2]. As such, teaching new mathematical concepts could be viewed as fostering learners' perceptual signification of the discipline's iconic displays in terms of selected features of sensory-rich concrete situations (Abrahamson, 2009b, 2012a, 2012b, 2014; Abrahamson & Wilensky, 2007; Fyfe et al., 2014). Rather than uni-directional fading from concrete to abstract, as Fyfe et al. (2014) propose, we interpret this activity as encouraging repeated back-and-forth referencing and linking between concrete, enactive resources on the game mat and iconic shapes on the display screen. In a similar vein, *Geometris* seeks to foster geometrical semiosis through occasioning opportunities for the mathematical practice of figural correspondences between polygons (vertices connected by edges) and their schematic rendition (vertices only). These pedagogically targeted norms are designed to emerge authentically through the collaborating players' efforts to communicate about figural elements relevant to the coordinated enactment of their joint actions (Abrahamson & Sánchez-García, 2016; Barnes et al., 1996; Flood, 2018; Shvarts & Abrahamson, 2019; Wittgenstein, 1953). Finally, some researchers argue that distributing tasks across physical and digital displays interferes with learning, by increasing cognitive load beyond productive levels (Mayer, 2005; Rau & Schmidt, 2019; Sweller et al., 1998). In *Geometris*, however, the use of two displays intentionally introduces a figural mapping challenge whose perceptuomotor solution, in the form of new geometrical structures, is the activity's pedagogical objective.

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[2] Whereas perceptual affiliation of sensory stimuli is a Gestalt perception, highlighting a Gestalt in the context of a mathematics activity marks it for learners as a culturally significant referent.

The *Geometris* environment is designed to support players in recognizing figural similarities across its two displays. As shapes appear on-screen, players move on the physical interface[3]. Once players realize that their actions on the mat are reflected on-screen, they learn to attend to the screen for feedback, even as they move on the physical interface. Thus, the game is designed to foster bi-lateral significations of its displays: the digital display mediates perception of the mat in terms of imagined lines and shapes, and the physical display mediates perception of the digital shapes by highlighting their vertices. Additional geometry notions, such as side length, angle measure, translation, dilation, rotation, and symmetry, are embedded into the activity design as pre-symbolic embodied experiences. Such context-bound skills can later be reconceived as instantiations of disciplinary mathematics content (see DeLiema et al., 2019 for examples in STEM-based play).

Having outlined *Geometris*'s design and rationale, we next review a trio of theoretical frameworks that collectively offer both task-specific and socio-cultural perspectives on informal learning. We then describe the setting, methods, and analytic practices for a subsequent discussion of empirical results. Our analysis considers the varied, multimodal resources that parents, in three different study groups, used in scaffolding their children's *Geometris* play. Certain forms of that scaffolding activity, we explain, were productively constrained by particular design decisions, which we elaborate. To close, we reflect on those design decisions we interpret as most pedagogically influential and suggest their potentially broader value within mathematics learning environments.

3.3 Theoretical Frameworks

In evaluating *Geometris* gameplay, we draw on three theoretical frameworks. At the most granular level, we draw on ecological dynamics to consider players' actions as constrained by elements and interactions within the collaborative task environment. We then draw on cognitive anthropology to implicate the central challenge in performing those actions as entraining the perception–action routine of figural mapping. Finally, we take a socio-cultural perspective on means by which this perception–action routine are entrained through collaborative work.

3.3.1 The Dynamics of Joint Action

To make sense of players' collaborative, goal-oriented work in *Geometris*, we follow Abrahamson and Sánchez–García (2016) in applying constructs from ecological dynamics to mathematics-education research. Researchers of *ecological dynamics* model skill acquisition in terms of the evolving dynamics between actors and the task environment (Vilar et al., 2012). Three categories of constraints in the actor–environment system fundamentally shape players' activity: *task*, *environmental*, and *organismic* constraints (Newell, 1986).

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[3] Very young children occasionally try to interact with the projection screen, perhaps based on experiences of touchscreens.

The *Geometris* task is to reconstruct shapes. Task constraints are: (1) players must simultaneously activate precisely those pads corresponding to a shape's vertices; and they must do so (2) within a limited time duration; and (3) in collaboration with a partner. Players must also develop shared, mutually intelligible vocabulary and frames of reference to convey their confusion, planning, instruction, and feedback.

Environmental constraints include: (a) the game's deliberately disjoint physical layout, which requires that players develop fluent perceptual routines for figural mapping between two spatially disparate displays; and (b) the range of deliberate features of the activity resources, such as the mat's large size, as well as incidental features, such as color selections for the sensor pads.

Finally, players are constrained by their own organismic qualities, such as their capacity to access information vital for task completion, physical size, number of limbs, and dexterity. The game's set of features—simultaneous sensor activation (task constraint) and a mat interface (environmental constraint) that is larger than the players' body size (organismic constraint)— limit an individual's ability to enact a solution alone[4]. In turn, the multi-element quality of solution inputs enables a flexible distribution of labor among collaborating players. For example, a dyad could share a target set of 5 vertices at ratios of 1–4, 2–3, 3–2, or 4–1.

Players can also alter task and environmental constraints for one another through feedback and instruction that contain more information than the recipient might otherwise access (Newell & Ranganathan, 2010). For example, one player might confirm their partner's hesitant movements toward a particular vertex, offering otherwise unavailable intermediate task feedback. Note that players who are new to the game but slightly more expert than their partner(s) likely continue to learn even as they teach their partner(s). As such, it could be expected that their learning and teaching goals may, at times, compete, such as in offering incorrect instructions.

3.3.2 Entrained Perception as *the* Problem

By design, the *Geometris* environment requires players to develop a cross-display figural mapping. Such entrained or skill-mediated perceptual routines are well-documented within the research literature, whether as professional vision (Goodwin, 1994), disciplined perception (Stevens & Hall, 1998), or educated perception (Goldstone et al., 2009). We agree with Goodwin that "all vision is perspectival" (1994, p. 606), and we believe that within *Geometris*, entrained perception is not just *a* problem but rather *the* problem. We draw on the tripartite role that Goodwin outlines for relative experts in entraining novices' perception: (1) *highlighting* elements of the environment as task-relevant; (2) *coding* those elements into disciplinary categories; and (3) *creating and*

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[4] We observed one teenaged player make a hexagon by himself using his head, knees, feet, and elbows. Such contortion is atypical within *Geometris* play.

interpreting graphical representations that collapse information across space and time. We furthermore draw on Abrahamson, Gutiérrez, Charoenying, Negrete, & Bumbacher's (2012) expansion of Goodwin's framework from perception per se to perception-for-action. That is, to develop a successful strategy, a novice must entrain their "perceptuo*motor*—not just perceptual—orientation toward the activity" (Abrahamson et al., 2012, p.77). Novices must learn how to orient and adjust their physical position, for example, maintaining their gaze on the screen, in ways that support sense-making of environmental information that changes in response to their movements. We see *Geometris* players guiding novices to achieve these entrainments to varying extents, as we aim to illustrate.

3.3.3 Scaffolding Perception Within Cooperative Work

Finally, we draw on socio–cultural theories of learning to describe how cooperative activity entrains new action–perception routines. In studying childhood development, Vygotsky (2001) differentiated between *real forms*—the intuitive ways that children perceive and act upon the world—and *ideal forms*—culturally specific and sanctioned ways of perceiving and acting. Importantly, the gradual transition between the two occurs through co-enacting ideal forms. Vygotsky's (1978) famed zone of proximal development captures the difference in operational outcomes when children enact real forms vs. when they co-enact ideal forms with adults or more capable peers. Taking a systemic reading of Vygotsky (Shvarts & Abrahamson, 2019), we apply these concepts to *Geometris* gameplay, interpreting figural mapping as an ideal perceptual form that players can learn through co-enactment. We also attend to the means by which relative experts scaffold novice partners toward this ideal.

Inspired by several Soviet researchers, such as Nikolai Bernstein (for a review, see Shvarts & Bakker, 2019), the construct of scaffolding has come to be understood as "'controlling' those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon […] those elements that are within his range of competence" (Wood et al., 1976, p. 90). Wood et al. (1976) delineate categories of these "controlling" actions: reducing degrees of freedom, highlighting salient environmental features, modeling desired actions, and offering feedback and encouragement. As we aim to illustrate below, parents perform many of these functions in *Geometris* play with their children. In so doing, we invoke Cazden (1981) to differentiate between scaffolds oriented toward *performance*, that is, completing the task at hand, and those oriented toward *competence*, that is, gaining "understanding from which answers to similar questions can be generated alone" (p. 7). Importantly, Cazden does not valorize one form of assistance over the other. We take similar care to consider both as pedagogically useful within *Geometris* play.

3.4 Research Questions

1. What are common participatory-facilitation techniques in informal, body-scale, collaborative mathematical play?

2. How is effective participatory facilitation of informal, body-scale, collaborative mathematical play enabled and constrained by design features?

Answering these questions, we maintain, could contribute both towards developing theories of learning and optimizing those environments for engagement and learning.

3.5 Methods

Details of *Geometris* design and data collection are presented above in Section 2. In first reviewing video of *Geometris* play, we noted the following features of each visitor group: how many players were present and their approximate ages; how many levels they played and of what difficulty; characteristic play behaviors; and was an adult present and, if so, did they observe, play, or coach. This initial review revealed that visitor groups exhibited a wide range of play behaviors that seemed to vary with group dynamics and game level.

Of particular interest for this study are cases in which one player developed a figural-mapping strategy before their partner(s) and began to facilitate the group's play toward apparently shifting goals. At times, these relative experts[5] leveraged their figural mapping skill to create scenarios to help their partners develop *competence* in figural mapping. At other times, especially when time was running low, these relative experts seemed to prioritize *performance*, focusing the group's energies on completing each shape. Relatively expert players seemed to transition between these two goals, apparently driven by ad hoc design features. We term such play, in which one player supports another player's work within collaborative play, *participatory facilitation*. As we conceive it, *participatory facilitation* incorporates informal facilitation and scaffolding toward goals of both competence (learning an ideal perceptual form) and performance (winning the game[6]). We are interested in the conditions that seem to support pursuit of one goal or the other.

In the course of the data analysis, the research team came to consider participatory facilitation as a potentially valuable pedagogical technique in informal mathematics learning environments. We therefore decided to narrow our study focus to groups who displayed this interaction style, and we investigated the pedagogical methodologies

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[5] We describe these players as *relative* experts, because they, along with their partners, are equally new to the game. Nevertheless, their perceived expertise *relative to their partners* seems to sanction their informal teaching behaviors through facilitation of play, which is our phenomenon of interest.

[6] Because *Geometris*'s design includes figural mapping as part of the game strategy, performance-oriented facilitation could still expose relative notives to game states through which they could learn this ideal form, if incidentally.

employed by the relatively expert players to scaffold novices' play. Three focal groups were selected for analysis based on their frequency of verbal utterances. In these groups, the expert players seemed to shift their facilitation goals. By examining why they shifted their goals, we observed consistent relations between design factors and facilitation characteristics. Incidentally, these same groups exhibited a moderate to high frequency of utterances, which was conducive for our qualitative analysis of observed behaviors. We analyzed these relative experts' multimodal utterances, including speech, gestures, body position, and gaze, and categorized them using constructs from the reviewed theoretical frameworks of constructivism, ecological dynamics, and sociocultural theory (see Table 1).

Table 1. Categories of Scaffolding in *Geometris* Play

Code	Description	Example
Reduce complexity	Reduce degrees of freedom in the task	Expert activates 2 of 3 vertices of a target triangle
Direction maintenance	Direct novice's attention to the task	"Where do you have to go?"
Highlight	Point out features of the environment as task-relevant	
Mat		"That one (*points to pad*)."
Screen		"We're trying to make that triangle (*points to screen*)."
Across displays		"The yellow dot here (*points to mat*) is the yellow circle there (*points to screen*)."
Feedback	Evaluate completed action	"That's not right, is it?"
Instruction	Coach the novice on future action using…	
Direct	… specific commands.	"Get the blue one (*points to pad*)."
Indirect	… general guidance.	"Keep going..."

The video data were divided into segments by utterance and coded independently by two researchers. The researchers first trained their coding on three practice groups and then coded the three focal groups. Inter-rater reliability was above 80%.

3.6 Results and Analysis

We first describe the play of each focal group and then highlight patterns in participatory facilitation across all three groups.

3.6.1 Case 1: Jared & Audrey

Jared[7] (~5 years old) and his mother Audrey began in the Practice level, which is not time constrained. Audrey activated pads with her feet and hands, while Jared remained in one spot, looking between the mat, his mother, and the screen. When Audrey indicated they were ready, the volunteer facilitator helped them select Level 1.

In Level 1, Audrey quickly and consistently activated the central yellow pad and, after the first shape, one other vertex (Figure 6). Jared worked to complete each shape. Audrey typically described this task, pointing to the screen and saying, “Ok, now we have to make that one” (4 times) and asking her son, “Where do you need to go?” (6 times). Over a period of 2 minutes, Audrey often highlighted features of the mat (9 times) and screen (8 times), though she did not overtly link them. She also gave frequent feedback (11 times), both redirecting her son’s efforts (“That’s not right, is it?”) and affirming his work (“You got it!”). Audrey’s instructions were predominantly indirect (9 indirect vs. 4 direct), encouraging Jared to “try it” and “keep going.” On the last shape, Audrey posed an extra challenge for her son. Previously, she had activated vertices such that Jared was relatively close to the last necessary vertex (1-2 pads away, Figure 6, A-G). On the last shape (Figure 6, H), Audrey moved her right foot forward from one necessary vertex to a different one, forcing Jared to move all the way around the circle. After quite a few steps (and mis-steps), Jared completed the shape in time.

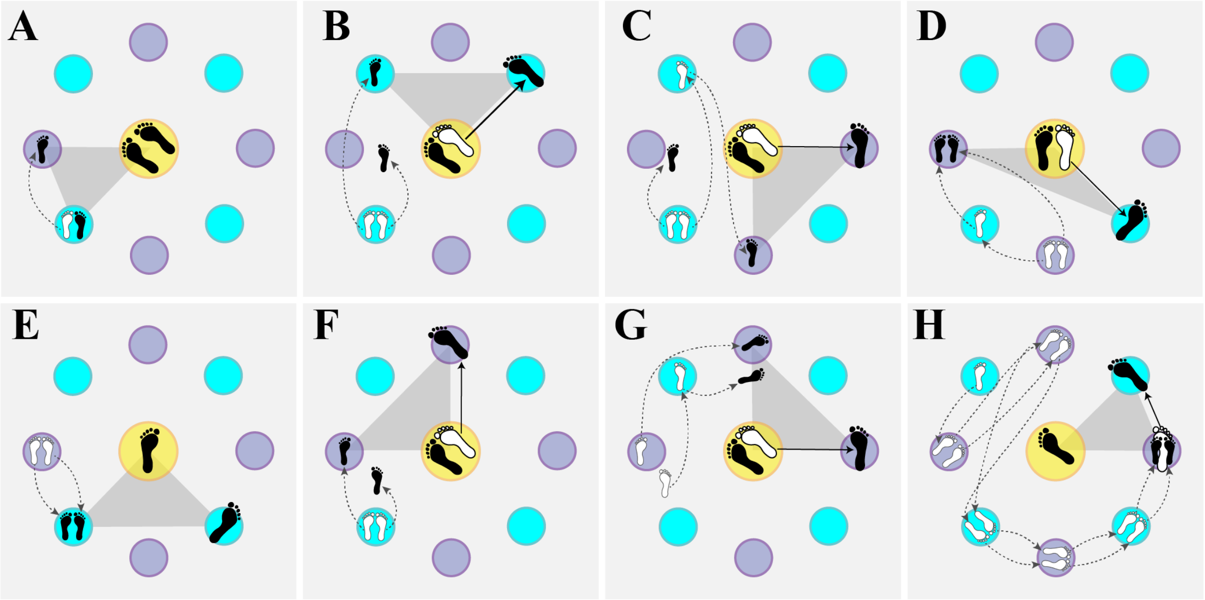

Figure 6. Audrey (big feet) and Jared’s (little feet) positions during Level 1. Each square represents work on one target shape (in gray). White footprints represent temporary positions. (Icon by James Keuning of Noun Project.)

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[7] All names are pseudonyms.

As the pair played Level 2, they communicated much less (~15 utterances vs. ~40 in Level 1). Audrey again occupied 2 target vertices, leaving Jared to find either 1 or 2 vertices of each shape. She took longer to establish her position and repositioned herself on half the shapes, moving simultaneous to her son. As a result, Audrey had less time to focus on instructing her son, and Jared had less independent movement time, that is, time when he was the only player moving. Audrey reiterated the shape-making task objective only once, and she tended to give direct instruction (5 direct vs. 4 indirect), often highlighting a place on the mat by pointing ("Go there"; "Get these ones"). She offered about half as much feedback as in Level 1. For their last shape, Audrey physically positioned Jared, nudging his hips forward so he stepped off an unnecessary pad.

3.6.2 Case 2: Joy & Mike

Joy (~8 years old) and her father Mike approached the *Geometris* exhibit while a previous pair was playing. When this earlier dyad left half-way through Level 1, Joy and Mike stepped in.

As they played the last half of Level 1, Mike consistently positioned himself on 2 of the 3 target vertices. The pair traded off completing shapes, with Mike twice working around Joy's position and twice setting her up to finish the shape. Mike highlighted 3 environmental features and gave feedback 3 times, though once was inaccurate.

Next, the facilitator helped them navigate back to the menu screen, where they selected Level 2. During this level, Mike typically positioned himself on 2 vertices of each shape before posing the problem to his daughter "Where are you going to go?" (6 times). Joy worked to find the remaining 1-2 vertices to complete each shape. Mike scaffolded his daughter's work with frequent feedback (22 times), evenly split between affirming her work ("Yes!"; "Perfect!") and redirecting her efforts ("Nope"; "Not there"). Mike often paired feedback with indirect instruction in the phrase "Yes! And?" (4 times). He highlighted environmental features occasionally (7 times), usually to accompany feedback ("Yup, blue[8]") or instruction ("You do purple").

When the pair played Level 3, their coordination patterns changed. Whereas in Level 2, Mike set his position and then prompted his daughter, in Level 3, he continued to change his position as they worked on 5 of the 8 shapes[9], moving simultaneous to his daughter and leaving less time for dedicated instruction. Mike no longer asked Joy where she should go. Instead of scaffolding his daughter's exploration with feedback, Mike tended to give direct instruction such as "Go there" or "Get that purple one" (17 times). He often paired instructions with gestures that highlighted environmental

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[8] The sensor pads alternated in color between blue and purple (Figures 4, 5, 6). Players often referred to the pads by color.

[9] In repositioning, Mike occasionally activated and held unnecessary vertices.

features, 21 times highlighting parts of the mat and once linking the mat and screen. He gave feedback half as often as in Level 2, skewed more toward redirecting (6 times) than affirming (4 times). As time ran down on one shape, Mike lifted his daughter's feet off the mat to release extra pads.

3.6.3 Case 3: Evan, Max, & Leslie

Evan (~8 years old) and Max (~10 years old) visited the *Geometris* exhibit with Leslie, their mother. As a volunteer facilitator guided Evan and Max through the Practice level, Leslie observed and asked questions from the sidelines.

As the boys played Level 1, Leslie commented from the back of the mat (Figure 7, A) while the volunteer facilitator explained the mapping and gave feedback. On the 2nd shape, Leslie stepped forward to give Evan feedback on his position (Figure 7, B), linking it to the onscreen display ("Do you see how this foot is not where you want it to be? You want it to be... straight shot, right?"). She then explained the figural mapping by linking the mat and screen ("So the yellow dot is the first yellow dot. You want to make your yellow lines go around the shape of the whole shape."). With time running out, Leslie moved to complete the 3rd shape (Figure 7, C), stopping herself as Evan got there. She remained adjacent to the screen just off the mat (Figure 7, D), posing the problem to her children ("How are you going to make this one?") and giving direct instructions ("You're going to get that one."). With time again running low on the 5th shape, Leslie offered a stream of feedback ("Nope nope nope nope nope") and stopped

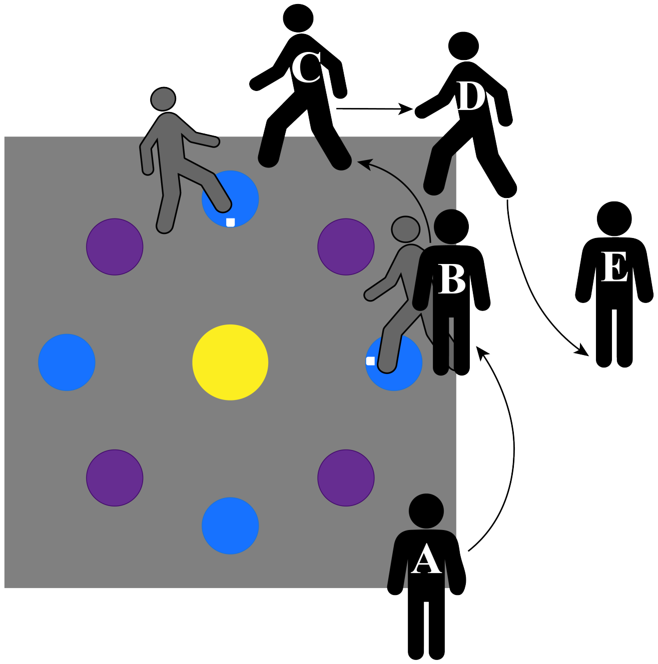

Figure 7. Leslie's positions (black) in Level 1. Starting at A, she moved to B to give feedback ("Do you see how this foot is not where you want it to be?"). She advanced to C ("Top! That one, that one, that one, that one."), stopping short of completing the shape. She continued instructions and feedback from D ("Nope nope nope nope nope"), before stepping back to E for the rest of the level. (Icon by Bradley Avison of Noun Project.)

herself just short of stepping onto the mat. She then brought her hands to her face, grinned sheepishly, and stepped back to the edge of the area (Figure 7, E), where she remained for the rest of the level.

In contrast to the other two groups, Leslie's spontaneous facilitation did not change as dramatically when her boys played Level 2. As in Level 1, the volunteer facilitator continued to give direction and feedback. Leslie remained adjacent to the mat, at times removing her shoe as if to step on the mat but never moving onto it. She directed her sons' attention twice (half as often as in Level 1) but gave feedback over twice as often (17 times in Level 2 vs. 7 times in Level 1). The most notable change occurred in her highlighting patterns. Whereas in Level 1, she highlighted features of the mat and screen and linked them 2-3 times each, in Level 2, Leslie predominantly highlighted features of the mat (5 times), highlighting the screen twice and linking them only once. Consistent with the other groups, she instructed more often in this harder level (9 times in Level 2 vs. 6 times in Level 1), and her instructions were predominantly direct (7 direct vs. 2 indirect).

3.6.4 Patterns of Facilitation—When Time Runs Low, Parents Stop Teaching

Despite their varying group compositions and play styles, we see certain similarities across these parents' participatory facilitation of their children's *Geometris* play. During easier rounds, all three parents repeatedly directed their children's attention to the mapping challenge (Figure 8). They overtly described the task (Leslie: "You want to make your yellow lines go around the shape" and Audrey: "See? We're making that triangle.") or asked their children, "Where do you have to go?" (Audrey and Mike). They made space for their children to explore by setting up a simplified task scenario (Audrey and Mike) or stepping back (Leslie) (Figure 8). They offered frequent feedback on their children's work, and their instruction tended to be indirect, encouraging their children's continued exploration (Audrey's "Keep going" and Mike's "Yes! And?"; Figure 8). We interpret this cluster of facilitation behaviors to suggest an orientation toward the children's developing competence at learning the game's rules and strategy, and—as a by-product—figural mapping skill, in these relatively easy, less time-pressured scenarios.

These facilitation patterns changed as the challenge increased or when time ran low (Figure 8). The parents stopped describing the task or asking their children "Where do you have to go?"[10] Their instructions increased in number and became predominantly direct, typically paired by mat-only highlighting ("Go there [*pointing*]!", "The blue one."; Figure 8). Such direct instruction left little room for the children to explore or get things wrong, thus reducing the frequency of feedback (Figure 8). These changes in facilitation style fundamentally changed the task for the relatively novice players from one of figuring out where to go to one of going where they're told. We interpret this cluster of

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[10] While this decrease could result from fatigue, there is no demonstrable decrease across each group's first round of play.

facilitation behaviors to suggest an orientation toward performance, that is, winning regardless of the child’s understanding, during challenging, time-pressured scenarios. We acknowledge that successful performance of the game task seemed to be the parents’ consistent goal and, through this analysis, highlight parents’ different strategies toward that goal based on task difficulty, with differential impacts on children's opportunities to learn the target figural mapping skills.

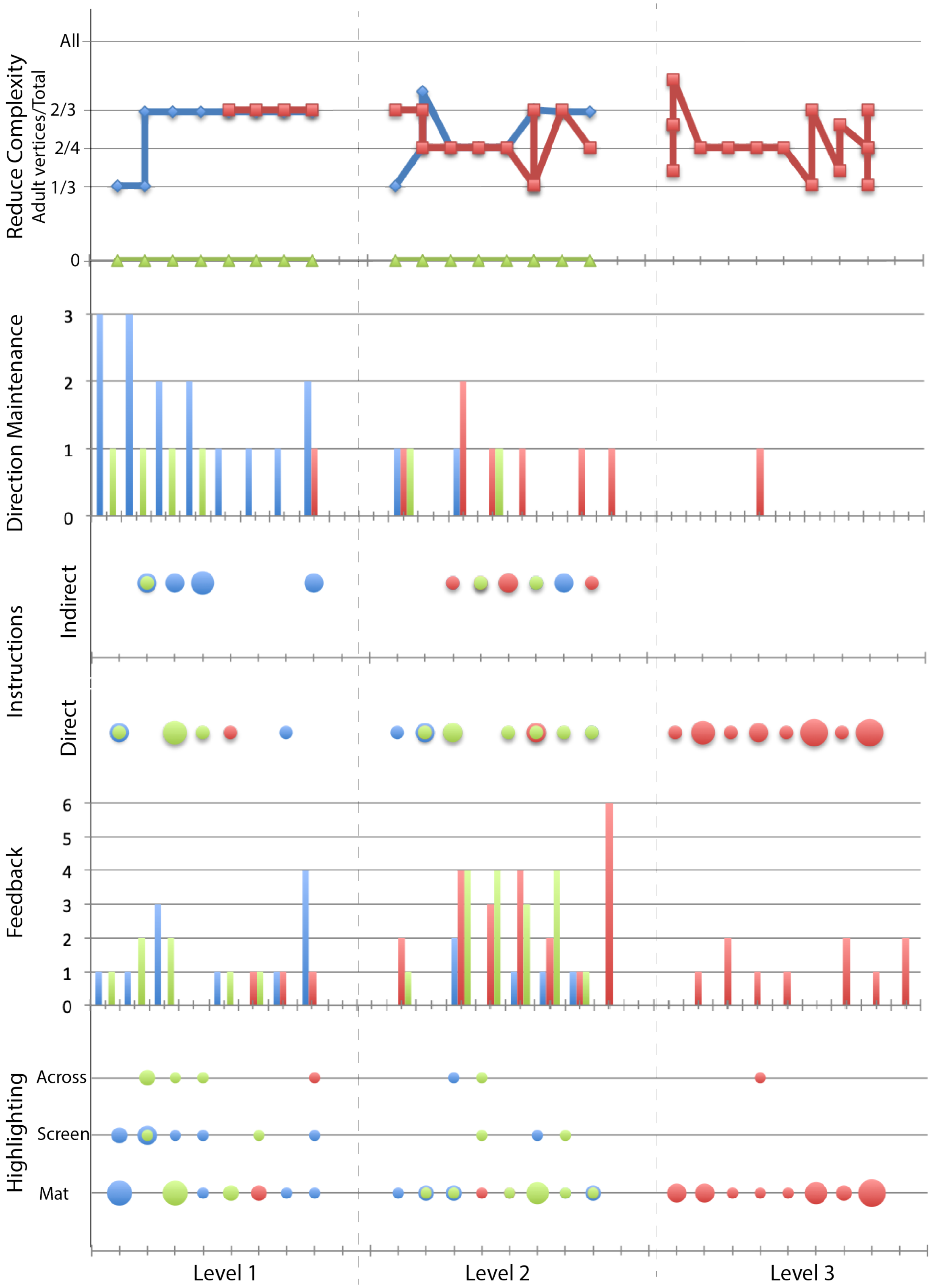

Figure 8. Audrey (blue), Mike (red), and Leslie's (green) participatory facilitation behaviors by level, indicated per shape. From top: the degree to which parents reduced task complexity by activating vertices; frequency of directing children's attention to the task; the type and number (bubble size) of instructions; frequency of feedback; and location and number (bubble size) of highlighted environmental features. Audrey and Jared's Practice round is excluded. Mike and Joy began halfway through Level 1 and were the only group to play Level 3.

We also note a relative infrequency of screen-based and cross-display highlighting (Figure 8). Audrey connected features of the mat and screen only once and Mike only twice. Leslie highlighted cross-display features the most, though this highlighting decreased as the challenge increased (3 times in Level 1 vs. once in Level 2). Considering the literature on entrained perception (Goodwin, 1994; Stevens & Hall, 1998) and entrained perception-for-action (Abrahamson et al., 2012), highlighting the cross-display correspondences more frequently could have better supported children in connecting their on-mat movements with on-screen environmental information, both increasing their efficacy at the game (the parents' goal) and, as a by-product, their figural mapping skill (the designers' goal).

3.6.5 Divisions of Labor on the Large-Scale, Multi-Sensor Interface

We also emphasize the role of the mat interface in parents' participatory facilitation, in particular, their use (or dis-use) of the mat to scaffold their children's play. In easier rounds of play, Audrey and Mike would set their position, activating 1-2 pads, before prompting their children's work ("Where do you have to go?"). This routine established an implicit norm of "I go, you go"—a sequential rather than simultaneous movement coordination that simplified the child's task by: (1) reducing the remaining work; (2) clarifying the task of finding oneself in the display (the moving point is the moving person); and (3) removing the need to negotiate a distribution of labor. That routine broke down during more challenging play. As the increasingly complex shapes increased the mapping challenge, parents took longer to position themselves (see vertical lines in Figure 8, Reduce Complexity), which both decreased time for their children to explore and meant that players moved simultaneously, eliminating the three simplifying benefits describe above. Though she didn't activate pads, Leslie exhibited a similar pattern of behavior. When her sons were exploring or performing well, she stood back from the mat. When they struggled, she stepped forward, stopping herself just short of activating vertices. Ironically, by doing more during challenging moments, parents likely complicated their children's figural-mapping task by introducing more variables into the physical and digital displays, obfuscating cause-and-effect relationships between them.

3.7 Discussion and Implications

By its design, *Geometris*'s distribution across two displays (here, a physical interface and digital screen) poses cross-display figural mapping as the key challenge for players. It also affords resources for scaffolding this learning objective, namely the large-scale, multi-sensor mat interface. As the above cases illustrate, parents' participatory facilitation of their children's play involved flexible use of the game mat among other scaffolding techniques.

The above cases also illustrate that parents' participatory facilitation changed in similar and pedagogically relevant ways. In easier rounds, facilitation supported children in exploring the game's rules and developing successful strategies based on figural mapping. Parents took up consistent positions, posed consistent problems, and guided their children using feedback. When play became more challenging, those facilitation patterns changed, and children's exploration was replaced by instruction-following. Parents moved more, often simultaneous to their children; they stopped overtly framing the task; and they directly instructed their children precisely where to go. From a design perspective, these changes compromised opportunities to notice cross-display similarities in several ways. Simultaneous movement obfuscates cause–effect relationships across displays. At the same time, the decrease in parents' problem-posing and cross-display highlighting means that those relationships were not highlighted verbally or gesturally, either. In these challenging and time-pressured moments, parents made it harder for their children to develop figural mapping skills that would make them more effective players. In noting these pedagogical differences between spontaneous facilitation styles, we aim to avoid the role of critic. Rather, we are encouraged to see example scenarios where each style emerges and to consider design choices that appear to influence facilitation. We also note that these changes may be due to parents' relative, rather than absolute, expertise at the game. With more experience and a stronger sense for the figural mapping across all levels, it could be that these parents would maintain competence-oriented scaffolding techniques throughout play.

We use the remaining space to reflect on *Geometris*'s dual-display design and flexible user interface and to propose directions for future work.

3.7.1 In Dual-Display Designs, as Elsewhere, Mechanics Matter

We claim that *Geometris*'s dual-display design poses for players the cognitive and perceptual challenge of figural mapping. Distributed over physical and digital media, *Geometris* prompts players to establish a mapping between their inputs on the mat and corresponding outputs on the screen. Importantly, this functional mapping alone does not pose the figural mapping challenge that we, as designers, deem relevant to mathematics learning. Rather, figural mapping additionally requires that the means through which players engage with these distributed displays to perform the activity tasks—that is, the sensorimotor actions players enact to solve emergent problems—are designed to constitute the target mathematics skills.

For contrast, consider Brain Dive (Kiili & Perttula, 2012), another body-scale educational game for mathematics. In Brain Dive, a basic arithmetic problem is projected on a large screen. A host of fish also appear onscreen, each labeled with a number. The player, monitored by a motion tracker, jumps to direct their shark up or down in the water column to eat the fish labeled with the answer to the arithmetic problem. Whereas Brain Dive thus requires a particular functional mapping between physical movement and changes to the digital display, the required spatial–dynamical physical act (jumping) does not enact the logico–mathematical process of the content in question (an arithmetic operation). As a result, the fostered sensorimotor perceptions are unrelated to the target concept.

In *Geometris*, the tactic of moving in physical space is precisely reflected in the appearance of points and lines in digital space. Thus, the central figural mapping challenge in *Geometris* arises not from the use of two displays per se but from design choices that engender interactions with those displays that are congruent with the mathematical skills for engaging with the target concepts. In *Geometris*, human bodies can collectively enact, subtend, inscribe, and configure mathematical objects. We propose that sensorimotor congruence between game mechanics and disciplinary practice is a pedagogically useful characteristic of body-scale mathematics learning environments.

3.7.2 Flexible Divisions of Labor Accommodate Flexible Facilitation Goals

We also claim that *Geometris*'s large-scale, multi-sensor mat interface creates opportunities for spontaneous facilitation to scaffold novices' play. Recalling our selection criteria for the data analysis, we chose focal groups in which a relatively expert player both facilitated play and seemed to exhibit a change in their facilitation goals, switching between performance and competence. As we aimed to illustrate above, the physical interface serves both these goals. The game mat is a shared physical interface with sensors agnostic to individual agents. It requires simultaneous physical contact with a discrete set of sensors and, as such, that set is given to multiple valid partitions. Relatively expert players sometimes foster consistent partitions, as in the "I go, you go" routines of Audrey and Mike's first levels, conducive to novices' developing competence. At other times, relative experts leverage this flexible partitioning toward performance, as when each of the three parents, above, moved to complete shapes. Importantly, the transition between these two modes was rapid, reversible, and required little overt coordination. We argue that this ability to switch seamlessly between facilitation goals while maintaining authentic play bears pedagogical value.

For contrast, consider the game of catch: two people must stand at a distance in order to pass a ball back and forth. If a novice demonstrates a need for significant instruction, for example, they keep throwing the ball into the ground or hold their baseball mitt at their hip, the more expert player must leave their position, approach the novice, and guide them, perhaps positioning their hands and limbs, perhaps doing a throw together, etc. By requiring close physical proximity, these moments of competence building are incompatible with normative play[11]. Once the expert deems the novice ready, they return to their separate positions and resume play.

In *Geometris*, the physical interface requires no such delineation between teaching–learning and authentic play. Teaching and learning can be seamlessly integrated into play, as when a relative expert completes more or less of the target shape, gives more direct or indirect instruction, or gives more specific or general feedback. Such scaffolds can be taken up, cast off, and taken up again within the norms of gameplay. We do not argue that *Geometris* always results in such teaching behavior—the examples, above, show it does not. Rather, we argue that an interface that supports the ability to

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[11] We can imagine other types of competence-oriented facilitation, such as increasing the distance between players, modeling form, or throwing certain types of passes, as compatible with normative play.

seamlessly switch between performance and competence goals respects novice learners' agency by maintaining their role as contributors to authentic play through varying degrees of scaffolding. Counter to familiar accounts of scaffolds monotonically fading, we document iterative cycles of deploying and fading participation scaffolds. Given that the target figural mapping skill is one of entrained perception, specifically perception of environmental information as mediated by an ideal form of instrument use (Vygotsky, 1978), we argue that learning environments that enable flexibly distributed labor create especially auspicious conditions under which that perception can be encouraged, tested, and developed.

3.7.3 Limitations and Future Work

We identify some limitations to the above analysis and identify areas for continued work. While on display, *Geometris* was monitored by at least one volunteer facilitator at all times. These facilitators adopted a variety of approaches, from encouraging visitors to "figure it out," to overtly explaining the figural mapping and giving frequent feedback, to playing alongside visitors. Such facilitation undoubtedly impacted guests' experiences of the designed environment and, likely, the play behaviors and participatory facilitation we observed from parents. As it was outside the scope of our role as researchers to standardize these volunteers' facilitation behaviors, we simply note for each visitor group the frequency and type of volunteer facilitation.

It could also be that some visitors facilitated their partners' play in ways not captured by our analysis. Visitors may choose to facilitate in these more-subtle ways, without directly observable behaviors, or by choosing not to get involved. While we tracked changes in participation behaviors—for example, Leslie's stepping up and pulling back—our analysis does not include parents who choose non-participation from the start as their means of facilitating their children's playful exploration.

To guide future work, we also identify factors, both from within the designed environment and from the exhibit setting, that seemed to influence parents' participatory facilitation. We associate the major change in facilitation behaviors documented above with the game element of time, specifically lack thereof. As the target shapes became more complex and time ran low, parents tended to shift from posing problems and giving feedback to directly instructing their children. More work is needed to evaluate this apparent relationship. Additionally, the groups varied along multiple dimensions external to the design: child age, number of children, and the volunteer facilitator's engagement level, to name a few. Presumably, all of these factors influence how parents engaged with the game and how their participatory facilitation unfolded. For example, Leslie seemed to attend more to Evan, her 8-year-old son, than to Max, her 10-year-old. Work remains to evaluate the impact of each dimension on facilitation behaviors so that players of all ages and group compositions may access and grapple with the game's figural-mapping challenge in ways conducive to their mathematics learning.

3.8 Conclusions

This paper reported on the empirical evaluation of *Geometris*, a collaborative, body-scale, geometry game. We argued that two qualities of the design support interactions conducive to pursuing the game's learning objective of developing geometrical and spatial reasoning. First, *Geometris*'s spatial distribution across two displays poses figural mapping as the central challenge for players. Second, the flexible divisions of labor enabled by the game's body-scale, multi-sensor interface open possibilities for addressing that challenge. We propose that this dialectical design architecture—engineering activity features that pose domain-relevant problems for task performance even as they create conditions for teaching and learning how to overcome those problems—is pedagogically desirable in educational designs.

We additionally identified, characterized, and exemplified the phenomenon of participatory facilitation, a spontaneous pedagogical practice, in which relative experts alter their own play in consistent ways to facilitate novices' contribution toward shared task outcomes. By delineating the flexible adoption, adaptation, casting-off, and redeployment of scaffolding behaviors that occurs within participatory facilitation, we offer this construct as a contribution to theoretically oriented literature on in/formal instruction. By implicating design decisions that enable and shape particular participatory facilitation behaviors, namely imposing a time constraint on task completion as well as furnishing a large-scale, multi-sensor user interface, we also suggest the value of this construct for pragmatically oriented literature on educational activity design.

More broadly, we see the above cases of *Geometris* play as examples of immersive parent–child co-play in an informal educational setting. Other scholars have identified trade-offs of such immersive educational designs. For example, children may be so immersed in play that they seldom reflect on the domain-relevant relationships that designers intended to highlight (Malinverni et al., 2016), with verbal description implicated as a critical component of reflection on action (Nathan & Walkington, 2017). We have extended these ideas to participatory facilitation of play. Parents who facilitate immersive educational designs for their children may themselves become sufficiently immersed in play so as to change their facilitation behaviors, shifting from a competence orientation to a performance orientation. These findings echo research on informal science education, which suggests that in collaborative tasks, parents may bear the brunt of cognitive work rather than cede control to their children (Gleason & Schauble, 1999). Especially given the informal educational setting of the cases presented in this paper, we aim to avoid elevating certain facilitation goals and behaviors over others. Rather, we suggest that identifying design elements that tip this balance of facilitation goals could be beneficial for the design of immersive activities in contexts that do seek to promote competence-oriented mathematics teaching and learning.

4 Spotlighting spectatorship: Elevating observation-based learning in the design and evaluation of body-scale learning environments

This second paper zooms out from the micro-analysis of a few participant groups to consider the learning opportunities available within a diverse range of participation forms – from spectator to player and some in between – that emerge around *Geometris*. Research on the educational value of play tends to focus on active players, especially when evaluating novel interaction technologies. However, a long history of scholarship underscores observing communal practice as a primary means of enculturation and learning. Considering learning as participation in communal practice, I present frequency analysis of roughly 350 *Geometris* participants followed by thick narrative descriptions of 3 focal groups to characterize the learning opportunities available within diverse forms of participation in this technologically enabled gameplay. I also identify particular design elements – namely the user-agnostic input mechanism and certain crowd control measures – that inadvertently enabled these participation forms. Theoretical implications include the pedagogically relevant range of action–perception possibilities available across participation forms. Additionally, proposed design heuristics could facilitate these diverse forms of participation in other educational designs.

4.1 Introduction

Studies of gaming, especially educational gaming, tend to focus on active players as the unit of analysis. Their actions (e.g. gaze, mouse clicks), multimodal utterances, and demonstrable changes in knowledge (as measured by pre/post assessments) are carefully documented and analyzed. The game itself, much like any learning environment, is evaluated on its ability to induce cognitive change in the learner, with the game and player/learner considered as an exclusive dyad (Duchastel, 1994). Even in cases of collaborative play, the players and game are implicitly taken as a system. This model fits nicely within formal education systems, be they academic or athletic, which tend to measure learning as an individual's quantifiable progress.

But gameplay in real life – in living rooms, schools, public spaces, and sports fields – involves many more forms of participation than just these central players. Relative novices might quietly observe from the sidelines, closely watching players' work. Relative experts might both observe and offer advice, coaching the players. Still others, perhaps disinterested in the game, might socialize in the broader area, offering occasional motivational support by celebrating successes and bemoaning losses. When a round of play ends, people might swap between these roles, with spectators tagging in to play and players stepping back to watch, coach, or socialize. Opportunities for spectatorship of recreational and educational gameplay will only increase as recent and emerging technologies enable large-scale, publicly observable body movements as inputs to gaming systems. How might the design of educational games benefit from expanding its focus to include these spectators? What new learning opportunities and dynamics might emerge? How can game design encourage spectator learning?

This paper attempts to take such a broader view on participation in educational gaming by 1) characterizing diverse ways that people interact with an educational game – from spectator to player, plus a few in between – and 2) identifying particular game design elements that support such varied participation. These dual goals motivate the use of two theoretical frameworks. First, I adopt a perspective on *learning as changing participation in communal practices* (Lave & Wenger, 1991; Rogoff, 2003) rather than as change in a quantifiable measure. This perspective recasts spectators from passive bystanders or players-in-waiting to active learners and contributors to the gameplay scenario. Second, I use constructs of *affordances* (Gibson, 1977) and *constraints* (Newell, 1986) to identify means by which design elements shape participants' behaviors and learning across participation forms. I apply these frameworks to analyze play of *Geometris,* a collaborative, body-scale geometry and spatial reasoning game, in a children's science museum. Throughout, I take an embodied interaction approach (Antle et al., 2009), considering the ways that performance and observation of action serve as resources for learners' sense-making and cognitive development. Through thick narrative description of three cases as well as qualitative coding of roughly 350 visitors, I document diverse forms of participation with this body-scale activity and identify fundamentally influential design elements. I argue that this ecosystem of participation forms supports greater learning opportunities than does the designed-for play alone and that the identified design elements may enable similar spectator learning opportunities in other game play[12]. Implications include the value of designing for and studying spectator forms of interaction so as to more fully capture the available range of learning opportunities within gameplay. Expanding research scope in this way could help to better understand and support peripheral participants' learning experiences, especially in under-resourced settings that may not yet be able to offer all learners extensive time with novel interaction technologies.

4.2 Theoretical Frameworks

To understand learning through spectatorship of collaborative interaction, I draw on socio-cultural perspectives of learning as participation in communal practices, which underscore the central role of observation and gradual apprenticeship into those practices. Additionally, theories from environmental psychology and ecological dynamics suggest psychological bases for these participation behaviors in features of learners' physical and social environments.

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[12] This work is not intended to contradict findings on the value of active participation in learning activities (Cunningham, 2011). Rather, I propose that when educational technologies attract an audience, the forms of learning available to that audience merit study.

4.2.1 Broad foundations of learning through observation

Scholars have long recognized the social, physical, and cultural bases of learning. Within developmental psychology, both Vygotsky and Piaget root development and learning in observation of others and their actions, whether by performing actions socially before internalizing them as psychological constructs (Vygotsky, 1978) or by mimicking “model others” (Piaget, 1954) and adopting rules to coordinate play (Piaget, 1932, 1951). Additionally, scholars of children’s moral development base traits such as aggression and empathy in observation (Bandura et al., 1961). Neuroscience suggests the mirror neuron system – a set of neural pathways that activate both when observing an action and when performing that same action (Gallese et al., 1996) – as a possible mechanism for such learning through observation.

4.2.2 Beyond observation: Learning through pitching in and peripheral participation

Anthropologists have confirmed that observation-based learning is not isolated to the 20th century, predominantly Western contexts of developmental psychology but is widespread, often exhibiting important culture-specific features. Based on decades of research on children’s informal learning in indigenous-heritage communities of the Americas, Rogoff (2014) and her team developed the *learning through observation and pitching in to family and community endeavors* (LOPI) framework. This framework reflects the behaviors of children as young as 2-3 years of age as they carefully observe and productively contribute to collective practices such as meal preparation and household chores. Critically, children’s observation and participation is driven by their own initiative; adult family and community members support that initiative, such as through cooperative action, but do not overtly direct the children’s attention or motivation. Rather, adults are primarily concerned with completing the task at hand. Rogoff contrasts the LOPI framework with the *assembly line* (AL) model, which predominates Western formal education. Within the AL model, adults are primarily concerned with instruction, a task for which they carefully structure, monitor, and (re)direct the learner’s attention and activity. Common techniques include mock excitement, continuous narration, and summative feedback. Any framing task for AL instruction typically has little significance outside of the instructional context. Rogoff emphasizes that LOPI and AL are not mutually exclusive in children’s educational experiences; rather, they constitute distinct patterns of activity between which children and adults may switch frequently and fluidly.

Closely linked to the LOPI model, Lave and Wenger’s (1991) theory of *legitimate peripheral participation* (LPP) outlines diverse forms of participation that constitute collective practices and emphasizes their social and physical bases. Arising from studies of apprenticeship (e.g., tailors, midwives), LPP defines learning in terms of an individual’s progression through a range of participation forms, moving from the periphery to the center of the target activity. For example, a newcomer to a tailor operation might serve as a shop hand (sweeping scraps, fetching orders) before advancing to an apprentice (taking orders, measuring pieces) and eventually to a journeyman (completing pieces, managing others) or even master (running the business operations). All forms of participation are legitimate and necessary. Framing learning as participation creates a range of identities that both drive participation – deferring to more knowledgeable folks – and are negotiated through participation – proving oneself by completing a superior's task. LPP also emphasizes that learning opportunities are spatially produced through the physical arrangement of people and material resources within the task environment. This focus on the social and physical construction of learning extends Lave, Murtaugh, and de la Rosche's (1984) earlier distinction between the socially constructed *setting* and *activity* as experienced by participants and the physical *arena* as a designed space. Consider how different the arena of an elementary school classroom is when used for daytime instruction vs. evening parent teacher conferences; the arena remains the same but the setting and activity differ. This structuring of resources and editing of activity more often occurs in collaboration with (near-) peers than within the master-apprentice relationship, motivating LPP analyses of learning to deemphasize the latter and instead consider holistically how the community engages in collective practice and where learning resources exist within it.

This study both builds on prior applications of these frameworks and departs from some of their commitments. Similar to LOPI- and LPP-informed studies of sport (Ellmer et al., 2019; Larsen, 2016; Ma & Munter, 2014), I apply these frameworks to collaborative activity oriented toward skilled physical performance rather than academic success. This work also departs from the LOPI and LPP frameworks in key ways. Whereas LOPI and LPP study on-going communal efforts, this study addresses shorter interactions with a museum exhibit. Consequently, this study does not focus on the identity development central to the LOPI and LPP perspectives[13]. Additionally, I eschew LOPI and LPP's strict definition of learning as participation to consider how changing levels of participation creates opportunities for individuals to develop their conceptions of the organizing structure within the activity (e.g. strategy in chess, geometry in billiards). Finally, this study does not follow the LOPI and LPP methodological commitment to in-depth ethnography, instead employing observational video analysis methods. Nevertheless, LOPI and LPP offer a valuable perspective to make sense of changing participation forms in the studied collective activity, especially given that many of these forms involve learning of skilled activity through observation.

4.2.3 Context shapes participation and learning by affording and constraining action

In addition to LOPI and LPP's holistic approach, I use constructs from ecological psychology and ecological dynamics for a more fine-grained perspective on the means by which social and physical surroundings shape moment-to-moment activity[14]. These constructs give purchase on how particular design elements shape participation and learning opportunities within collaborative interaction.

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[13] Questions of identity are more likely present as studies of collaborative play grow in timescale, such as for (aspiring) professional athletes and gamers.

[14] I recognize that, ontologically, ecological psychology and ecological dynamics can be interpreted as more deterministic than LOPI and LPP's socio-cultural approach. I reconcile this tension by using the two frameworks at different levels of detail – LOPI and LPP to evaluate the play scenario holistically and ecological psychology and ecological dynamics to consider individual's moment-to-moment behavior within their role in that scenario.

To better understand how designed elements shape participants' behaviors, I draw on Gibson's (1977) theory of affordances. Generally, affordances name the cueing effect whereby physical objects or configurations suggest particular behaviors and counter-indicate other behaviors. For example, a drawer handle affords pulling rather than pushing, turning, or prying. Affordances are task- and culture specific. For example, a pencil affords a different grip when used for writing vs. for popping a balloon, but it affords no particular grip at all for non-writers such as toddlers (Abrahamson, 2019). Additionally, affordances capture the powerful psychological relationship whereby physical objects appear "to be for" certain material or social actions (Heft, 1989) and, just as importantly, not for other actions. Consider two possible seating arrangements. Rows of desks facing a blackboard afford forward attention, suggest the authority of the person at the front, and deter conversation between desk occupants (i.e. it's physically uncomfortable to look at people behind you). Chairs at a round table make it physically easier to attend to others, affording more interactions and decentering any particular authority figure. Importantly, affordances are not features of physical objects alone but of the interaction of actors – with their physical, social, contextual, and interactional interests and possibilities – with those objects.

Whereas Gibson's (1977) theory of affordances helps explain a physical basis for collaborative interaction behaviors, I also attend to social and interactional forces. Newell's (1986) theory of constraints offers such a perspective, specifying how physical and social surroundings shape action and perception. The constraints-based view (Newell, 1986) considers action as shaped by three categories of constraints: *task constraints*, such as rules of interaction; *environmental constraints*, such as the physical setting for the task; and *organismic constraints* related to the participants' physiology. These categories of constraints combine to create possible, probable, and likely action–perception routines that participants will experience. Task constraints might include rules for achieving goals (e.g. how to score points in a game), for allowable moves (e.g. forbidding hands in soccer) and for allowable interactions with others (e.g. forms of communication or contact). Environmental constraints include the interaction space itself (e.g. its scale and material make-up), perceptible indicators within it (e.g. lines, markers, timers), and seemingly incidental features such as ambient noise or weather[15]. Finally, organismic constraints include participants' size, stamina, and task-relevant perceptual routines. Importantly, participants can alter or ameliorate these constraints for one another through collaboration (Newell & Ranganathan, 2010). For example, variable distributions of labor alter both task and organismic constraints, and instructions ameliorate task and environmental constraints (e.g. coaches interpret environmental information for players and suggests next moves). Learning to manage and respond to constraints is central to individuals' increasingly productive participation in play.

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[15] Whereas environmental constraints can be read to encompass affordances, I maintain the utility of both perspectives in studying perceptually rich collaborative activity.

Affordances and constraints are widely adopted constructs within studies of collaboration in play and sport (e.g., Araújo et al., 2009; Immonen et al., 2017; Renshaw et al., 2009). Education researchers, especially those studying novel educational technologies, have also taken them up (e.g., Abrahamson & Sánchez-García, 2016; Kennewell, 2001; Manches & O'Malley, 2016). I see my use of these constructs to study learning within collaborative play as a continuation of this research.

4.2.4 Summary

Considering multiple forms of observation-based and peripheral participation in collaborative interaction offers a holistic perspective on the diverse ways in which people can learn from and contribute to these scenarios. Considering the affordances and constraints on individual forms of participation clarifies the potential actions, perceptions, and behaviors available within that form. After reviewing related works on spectatorship in Section 3, I apply these frameworks to the analysis of a particular play-based task, illustrating the diverse, legitimate forms of participation in that task and identifying design elements that guide participation and learning within those forms.

4.3 Related Works on Spectatorship

An array of fields from art and performance studies (e.g., Helguera, 2011; Rancière, 2014) to the sports sciences (e.g., Wann et al., 2001) have studied the spectator experience. I briefly review research on *collocated* (as opposed to *mass*) spectatorship (Tekin & Reeves, 2017) of intentionally designed, technologically-enabled activity and suggest the need for further study on spectator learning from body-scale collaborative educational technologies.

Design-oriented fields suggest particular practices to engage spectators and encourage learning through observation. As interactive technologies increasingly inhabit public spaces, the field of human-computer interaction has expanded its unit of analysis from a single individual at a computer terminal to include groups and passersby enacting varying depths of interaction, potentially with multiple displays (Brignull & Rogers, 2003; Memarovic et al., 2016; Reeves, 2011). Museum studies also offers insight into how designed features may support varied levels of engagement. A number of studies identify means by which designs can guide visitors from surface-level engagement of how to operate an exhibit (usability) through to deep engagement with the underlying concepts at the heart of the exhibit's design (conceptual learning) (Heath et al., 2005; Horn, 2018; Hornecker et al., 2007). Such work has great practical importance but offers less insight on how spectatorship specifically of movement may create learning opportunities.

Within research on videogame play, spectator participation has been implicated as a key means through which "in-game" activity is contextualized in broader "in-room" and "in-world" activity (Stevens et al., 2008). Spectators engage with both the game and players by offering real-time and retrospective criticism, complements, and scaffolding (Tekin & Reeves, 2017). The spectator experience is influenced by such factors as turn-taking expectations (Downs et al., 2014) and also influences players' gameplay experiences (Kappen et al., 2014; Lin & Sun, 2011). Body-scale gaming (Isbister & Mueller, 2015) that uses the Wii, Kinect, or other motion-tracking interfaces may merit special treatment. The uniquely *expressive* nature of these interfaces (Reeves et al., 2005) publicly reveals both user actions and resulting effects, "enabling the spectator to fully appreciate the performer's interaction" (p. 741). Research suggests that the more physical the game, the more spectators enjoy watching it (Downs et al., 2014), while players themselves report little discomfort at being watched in naturalistic settings (Kappen et al., 2014). Studies of body-scale gaming in home-based, family contexts identify a wide range of spectator roles (Downs et al., 2015) and emphasize the physical humor inherent in operating and observing these systems (Harper & Mentis, 2013). Design efforts to further integrate spectators into body-scale gameplay include using spectator gaze as an input (Maurer et al., 2015).

Despite this breadth of work about the nature and organization of spectatorship including in body-scale environments, less is known about spectator learning in these environments, a topic addressed by a handful of education research studies. The creators of SMALLab (Johnson-Glenberg et al., 2014), an interactive science platform, stipulate that observing students should complete whiteboard activities, though they do not study the difference in learning opportunities or outcomes across participation forms. In classroom studies of their movement-tracking activities, which tessellate captured profile images to create intricate patterns, Mickelson and Ju (2011) document extensive audience participation such as through offering advice, feedback, and critique. King and Smith (2018) offer one experimental study of spectator learning in their evaluation of a movement-based technology for learning about angle measure. Comparing participants who completed a 1-on-1 clinical interview to those who observed their classmates' work, King and Smith found similar learning gains around concepts of angle measure. Noting that performers used more spatial language ("straight out," "90 degrees") while observers used more metaphoric language ("like a person dancing", "like saying hi"), the researchers concluded that the two participation forms offered different perspectives on the central spatial relationships, which other work suggests can be pedagogically valuable when compared and negotiated (Ackermann, 1996; Greeno & Van de Sande, 2007).

This nascent body of work needs further development to better understand the opportunities for learning through observation of embodied learning activities and to identify design practices that support such learning. These needs will only be exacerbated as games and technologies become increasingly immersive and embodied (Antle et al., 2009). This study builds on the available literature through a close analysis of the learning opportunities available within varied participation forms and through identification of design elements that support those learning opportunities.

4.4 Research Questions

1. What forms of spectator participation emerge around collaborative, body-scale educational technologies? What learning opportunities do those participation forms create?
2. In what ways do the material configurations of the designed interaction space enable or constrain players' activity to constitute the varied forms of participation?

4.5 Methods

Details of *Geometris* design and data collection are presented above in Section 2.

4.6 Analysis & Results: The frequency of spectator participation

To characterize visitors' interactions with *Geometris*, I build on Goffman's (1979) terminology for forms of participation in verbal conversation. Goffman distinguishes between people who are *ratified participants* in a scenario (in this case, *ratified players* of the game) and those who eavesdrop or *observe* ratified play[16]. As a middle ground between these roles, Goffman uses *cross-talk* to characterize observers who interact with ratified participants.

I used Goffman's terminology to qualitatively code the interactions of 346 participants[17] in the *Geometris* exhibit. Recall that each level of the game poses 8 target shapes. Participation roles were assigned at the shape grain size. If a person demonstrated multiple participation forms within one shape – for example, observing before offering advice through cross-talk – they were assigned the more active form (e.g. ratified play > cross-talk, cross-talk > observing). Two researchers collaboratively coded 30 minutes of video before independently coding the remaining 5.5 hours. Of the total 6 hours of video, roughly 20% was coded by both researchers, achieving at least 90% agreement.

As detailed in Figure 9, a small majority of people (52%) participated in ratified play. Of all visitors to this exhibit, 35% participated through spectator observation alone, and 1% participated only through cross-talk. A sizeable minority (42%) participated in at least 2 of these ways, including 12% who participated in all three. Such blended participation suggests that people engage with collaborative, body-scale activities in ways that traverse roles from spectator to player. Analyzing ratified play in isolation could overlook a significant portion of an activity's audience. Additionally, such a focus excludes the

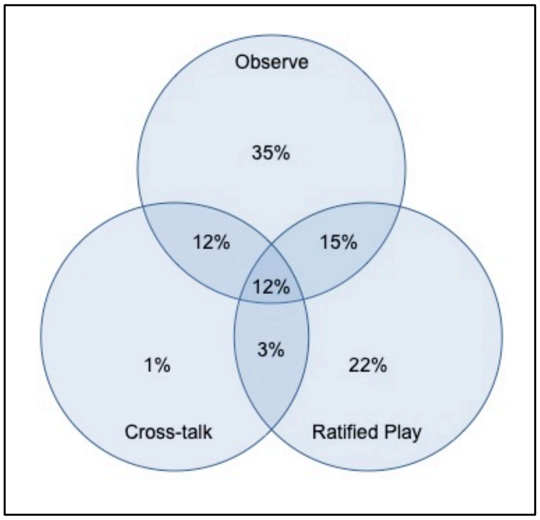

Figure 9. Frequency of observation, ratified play, and cross-talk across all participants (N=346).

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[16] Goffman uses the term *eavesdropping*, which I substitute with *observing* to reflect the relative acceptability of observation in museums compared to other public settings.

[17] Researchers were only counted if they engaged in either cross-talk or ratified play.

observation and cross-talk behaviors that, as demonstrated here, are also part of ratified participants' experiences.

4.7 Analysis & Results: Pathways through spectatorship and play

The frequency of blended participation suggests a closer analysis of how people move between participation forms and how such transitions impact their experience and that of their co-participants. To achieve this more detailed perspective, I offer thick narrative descriptions (Creswell, 2012) of 3 focal cases. Video data of all 58 groups was reviewed and summary descriptions written for each group noting the participation forms enacted. Three cases were selected for thick narrative description based on the following criteria: participants enacted 3 or more forms of participation, and participants transitioned between those forms.

To inform these thick narrative descriptions and enable qualitative visualization, I also apply a 10-level coding scheme to capture how the physical exhibit space and the interaction behaviors introduced above combine to constitute a range of participation forms[18]. I borrow Goffman's (1976) use of the term *bystander* to account for people nearby a ratified interaction, in this case, outside the stanchion posts surrounding the exhibit (Figure 4). I also include a *limbo* region to name the middle ground between the exhibit's stanchions and game mat. Finally, I consider what Goffman calls *side-talk* behaviors, that is, conversations among non-ratified participants. While these interactions are less central to the designed interaction, they are still heard by ratified players and thus might shape their experiences.

Table 2. Forms of Participation within *Geometris*.

Region	Behavior	Example(s)	
Bystander	Observing	Silently watching others play	Increasing centrality to game play ↓
	Side-talk	Chatting to another bystander	
	Cross-talk	Speaking to ratified player(s)	
Limbo	Observing	Silently watching others play	
	Side-talk	Speaking to non-ratified player(s)	
	Cross-talk	Speaking to ratified player(s)	
	Play	Activating pads from limbo, often with hands	
Ratified Player	Play	Activating pads with both feet on the mat	
	Side-talk	Speaking to non-ratified player(s)	
	Cross-talk	Speaking to ratified player(s)	

Taking these geographic regions and behavioral categories together, I consider 10 possible forms of participation with the *Geometris* exhibit (Table 2) and order them by increasing centrality to completion of the gameplay tasks.

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[18] I use this geographically based taxonomy as opposed to a body-based one (e.g., Downs et al., 2015) due to the clearly demarcated physical regions within the game space, as has been done in other museum-based research (e.g., Shapiro et al., 2017).

Participation roles were assigned at the shape grain size. If a person demonstrated multiple participation forms within one shape, they were assigned the more central form according to Table 2. This coding was performed in the same round as that detailed above in which researchers achieved at least 90% agreement on the shared 20% of the total 6 hours of coded video.

4.7.1 Case 1 – A child asserts independent play

Erik[19] (~7 y/o) approached Geometris with his grandparents, Susan and Leon. Molly, the volunteer facilitator, asked Erik if he wanted to play and asked him to remove his shoes. As Molly and Erik collaboratively selected the Practice level, Susan and Leon observed silently from the bystander region with two researchers.

In the Practice level, Molly explained the mat's functionality ("You can go around and try pressing different spots [*activating pads*] and see how they make shapes on the screen [*points to screen*]") and activated pads from limbo, keeping her shoes on. As Erik began to play, Molly coached him (cross-talk) from limbo ("So you made a line. If you step on another one…"). Susan then stepped up to limbo, removed her shoes, and engaged Molly in side-talk ("I didn't know we could use our hands;" Figure 10a). Simultaneously, Leon began coaching Erik (cross-talk) from his bystander position ("Stand on this one back here [*pointing*]"). From limbo, Susan asked broadly "So what are these? What shape are these accomplishing?" before stepping onto the mat (Figure 10b). Susan and Erik completed a triangle together ("Oh, how beautiful!"), after which she directed Erik to another point ("And that one [*pointing*]"). Erik pushed back on her outstretched hand and replied, "Grandma, I know how to do it" (Figure 10c). Susan then backed into limbo, saying "You know how do to it? You can do it on your own? I thought it was two people but if you can play on your own, that's perfectly fine with me." As Erik played alone, Leon and Susan encouraged him and also engaged Molly in side-talk about the game's challenges. When Erik completed an hourglass-like shape, he bid for his grandmother's attention, saying "Look what I made!" (Figure 10d). From limbo, Susan responded, "Cool, I love it!" She then stepped back to the bystander region, put

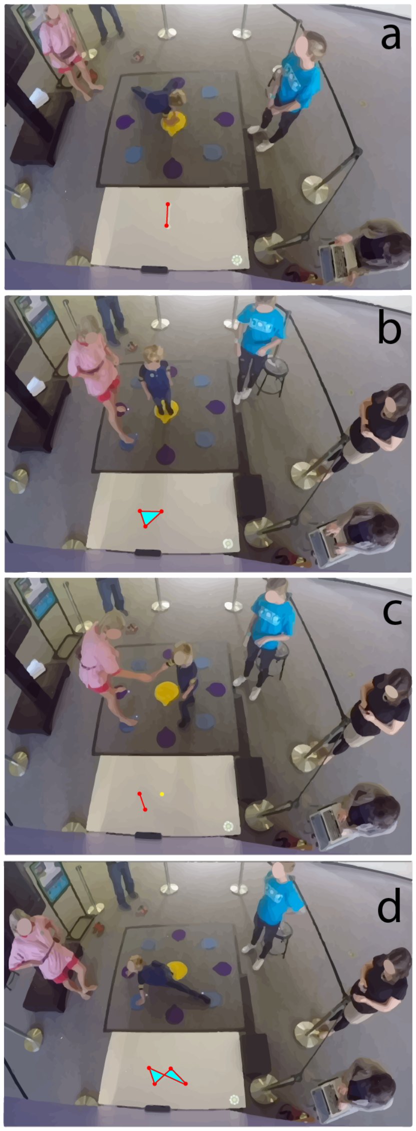

Figure 10. a. Susan (red shorts) and Molly (teal shirt) side-talk in limbo as Erik played. b. Susan joined Erik on the mat. c. When Susan directed Erik, he pushed back. d. Susan stepped back to limbo to observe, where Erik solicits her approval.

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[19] All names are pseudonyms.

her shoes on, and engaged in side-talk with Leon and a researcher. Erik created a few more shapes as Molly observed from limbo, occasionally engaged in side-talk with Susan. Molly then helped Erik back to the main menu.

As Erik played Level 1, Molly coached him (cross-talk) from limbo while his grandparents engaged in bystander side-talk. Early on, Molly knelt in limbo to activate pads with her hands. About halfway through the level, Molly removed her shoes and stepped on to the mat, entering ratified play. She completed one corner of each target triangle and coached Erik to complete each shape. When Erik didn't quite follow Molly's directions, Susan joined in from the bystander region, pointing to the target pad and saying "Over here, over here! Try this!" As Erik completed the shape, Susan resumed side-talk. Molly and Erik played out the rest of Level 1 together. When the level ended, Erik bid for his grandmother's attention, pointing to the screen and saying, "Look at all, look at all the shapes!" before high-fiving Molly with a big smile. Erik then put on his shoes and left the exhibit with his grandparents.

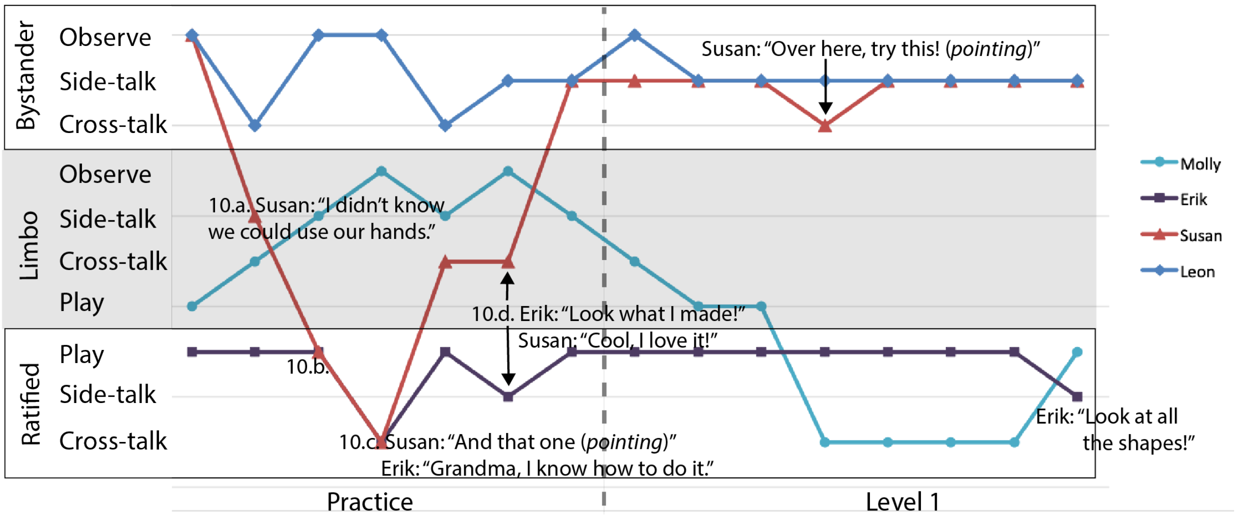

Figure 11. Participants' simultaneous participation forms impact one another, in this case through social pressures (Susan) and task pressures (Molly).

In addition to this thick narrative description, plotting these participation forms over time clarifies patterns and key points in visitors' participation trajectories (Figure 11). Leon (blue diamonds) and Erik (purple squares) had relatively consistent participation throughout. Molly (teal circles) became more involved during timed play in Level 1 than she was during the exploratory, untimed Practice round. Susan (red triangles) traversed all three regions and took on 7 different participation forms. The literal turning point in her participation came when Erik rebuked her direct guidance (point 10.c, corresponding to frame c of Figure 10), reversing her prior trend of increasing involvement to one of visibly pulling back. Interestingly, Erik continued to solicit her input, and Susan continued to pitch in by offering both moral support (10.d) and verbal guidance ("Over here, try this!"). A focus only on ratified play alone (try it by covering up the Limbo and Bystander regions of Figure 11) would miss these forms of continued involvement.

4.7.2 Case 2 – Unfamiliar children negotiate participation forms

Isla (~8 y/o) approached the exhibit with her younger sister, Margo (~5 y/o). Aiden, the volunteer facilitator, asked Isla to remove her shoes and explained that this game is about making shapes. Suggesting she start with Practice, Aiden coached Isla (cross-talk) from limbo on which pads to press (“Step on the yellow pad and another one”) and narrated the results (“See how you created a line?”). Isla alternated her gaze between her feet, the screen, and Aiden, seeming to seek his instruction. Margo remained a bystander. After a minute, Aiden suggested that Isla “play the real game now” and directed her to Level 1.

As Level 1 began, Isla was the only ratified player, and Aiden continued coaching her (cross-talk) from limbo (“So the yellow dot [*points to screen*] means the yellow pad [*points to the mat*]”, “Step on the yellow pad.”). As Isla worked on the first shape, an adult approached and asked Aiden about the allowed number of players and encouraged his toddler to try it (“He’ll tell you what to do.”), but the toddler hid behind his legs. Isla drew Aiden away from this side conversation (side-talk) by repeatedly asking him, “How do I make it?” Aiden returned to coaching Isla from limbo (cross-talk) as the adult and toddler left. Margo, who had been a bystander observer, also left. Shortly thereafter, Maxim (~10 y/o), and his younger sister, Nadia, approached. While Nadia remained a bystander observer, Maxim exclaimed, “I want to do this one! I want to do this one” and observed from limbo for a few seconds before walking onto the mat (Figure 12a). Upon Aiden’s request, Maxim stepped back to remove his shoes before returning to the mat. Aiden began to explain the game to Maxim (“So the yellow dot [*points to screen*] is the yellow pad [*points to the mat*], uh…” but Maxim seemed to ignore him, instead rapidly trying out different pads. Isla seemed put off by Maxim’s actions and proximity, turning to gaze at him while leaning her torso away from him (Figure 12b). Aiden asked Maxim to step back (“You can try after her.”) and began to play from limbo, activating pads with his hands (Figure 12c). Maxim briefly stepped back near his shoes (Figure 12c) before circling around the mat (Figure 12d).

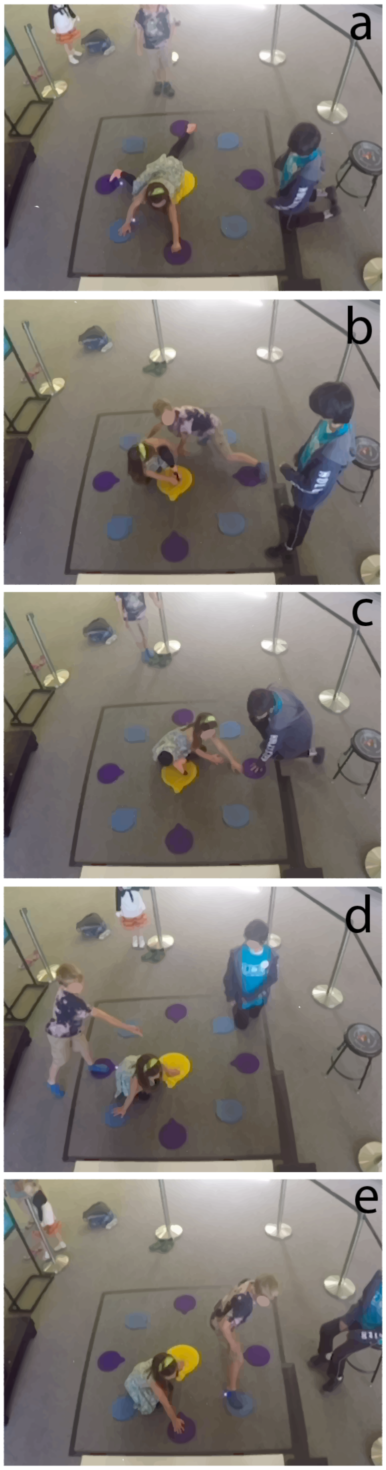

Figure 12. Isla (dress), Aiden (teal shirt), and Maxim (gray shirt). a. Maxim observed from limbo. b. When Maxim joined play, Isla seemed put off, and Aiden asked him to step back. c. Maxim briefly stepped back to his shoes. d. Maxim played and coached from limbo. e. Maxim rejoined ratified play.

Keeping one foot in limbo, he activated pads slightly behind Isla. Isla continued her play, overtly asking Aiden (side-talk) for feedback ("Did I do it? Did I do it?"). Maxim also continued playing from limbo, extending and retracting one foot while keeping the other rooted in limbo. At one point, he also coached Isla, "Get that one [*pointing*]!" (Figure 12d) before returning to ratified play for the last shape of Level 1 (Figure 12e). With Level 1 complete, Aiden asked Isla if she'd like to continue playing with Maxim ("Do you want to play with him?"), to which she nodded in reply.

As Maxim selected Level 2, Isla moaned, "No" and dropped her head. Aiden coached extensively from limbo (cross-talk). Maxim seemed to experiment rapidly, switching between pads, while Isla waited for Aiden's instructions. Isla also gave Maxim one instruction ("You don't need the yellow"). When they struggled on long shapes, Aiden activated pads with his hands from limbo. Halfway through, Maxim also coached Isla ("And that one [*pointing*]!"). With the level almost done, Isla left. At this point, Aiden removed his shoes and joined Maxim in ratified play, saying "Umm, ok. I'll play with you then." Aiden continued to coach Maxim (cross-talk) as the pair finished the level together.

Aiden and Maxim then played Level 3 together. Isla returned briefly, commenting from the bystander region that the adjacent video screen "tells you how to do it." Aiden and Maxim finished Level 3, with Aiden coaching Maxim on roughly half the shapes.

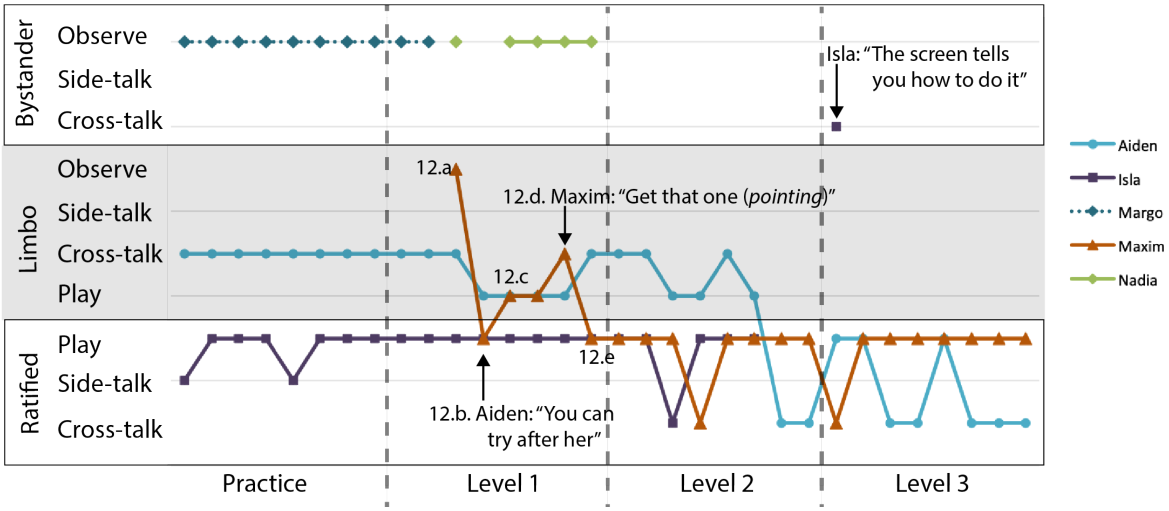

Figure 13. A group's participation is composed of individuals intersecting and interacting participation threads. In this case, arrivals and departures, as well as overt requests, induce visitors to change their participation roles.

Figure 13 offers a visualization of this group's participation forms. Maxim's appearance partway through Level 1 corresponds with a change in Aiden's participation, as the latter begins to play from limbo. Later, Isla's departure midway through Level 2 corresponds with a similar change in Aiden's participation, as he steps up from limbo play to ratified play. Maxim's participation is momentarily rerouted by Aiden's direct request to "try after her" (12.b), though he persisted in playing from limbo and shortly reasserted himself as a ratified player where he remains even after Isla's departure. In this group, the limbo region offered an important space for observing and especially for pitching in, even in ways not entirely welcomed by ratified players.

4.7.3 Case 3 – A summer camper learns from observing

The museum hosting *Geometris* also runs a summer day camp. A group of 8 campers (~7-8 y/o) arrived at the exhibit together and removed their shoes upon request from Kim, the volunteer facilitator. The first two campers on the mat ran and hopped around in Practice. From limbo, Kim explained (cross-talk) how the mat works ("If you look at the screen [*pointing*], and then you press [*stepping on a pad*], you see that the sensors light up and they become the corners of the shape that you're making"). As more children finished removing their shoes, they paused to observe in limbo before finding a spot on the ever more crowded mat. Eventually, all 8 campers played on the mat together where Kim joined them. She prompted a discussion (cross-talk) about the shape they were making ("How many sides does that [*pointing*] have?"). As this discussion ended, Kim stepped back to limbo and selected three campers - Annie, Morgan, and Rhea - to play the game's timed levels. Some of the remaining campers went to watch the large video monitor in the bystander region, as others put on their shoes and left.

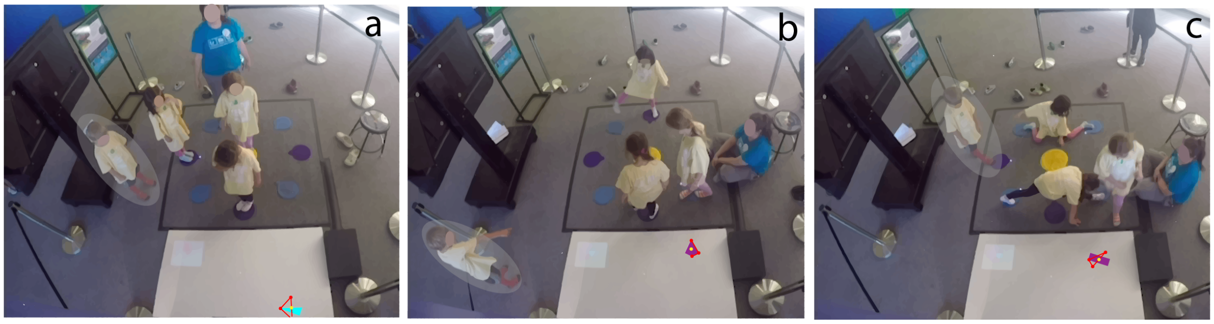

Figure 14. Summer campers play Level 2. a. Ian (gray oval) observed in limbo. b. He stepped back to observe and coach as a bystander. c. Ian returned to limbo, playing covertly.

Kim navigated Annie, Morgan, and Rhea to Level 2, where they remained ratified players throughout (Figure 14). Initially, Kim coached the girls from the mat ("This one [*pointing*]", "Somebody has to stand on this one over here [*pointing*]") and played with them. Around this time, Ian, who had been observing in the bystander region, stepped up to silently observe from limbo (Figure 14a). After a few shapes, he returned to the bystander region where he coached the girls (cross-talk) on one shape ("Right there!"), pointing to the target pad (Figure 14b). Around this time, Kim sat down in limbo, continuing to coach the girls but no longer playing. Ian then stepped up into limbo and covertly played with the girls on the level's last shape (Figure 14c). As the group completed Level 2, Kim helped them interpret their shape score. Kim then selected Ian and 2 other campers to play Level 1, though the campers were called away by their counselors partway through the level.

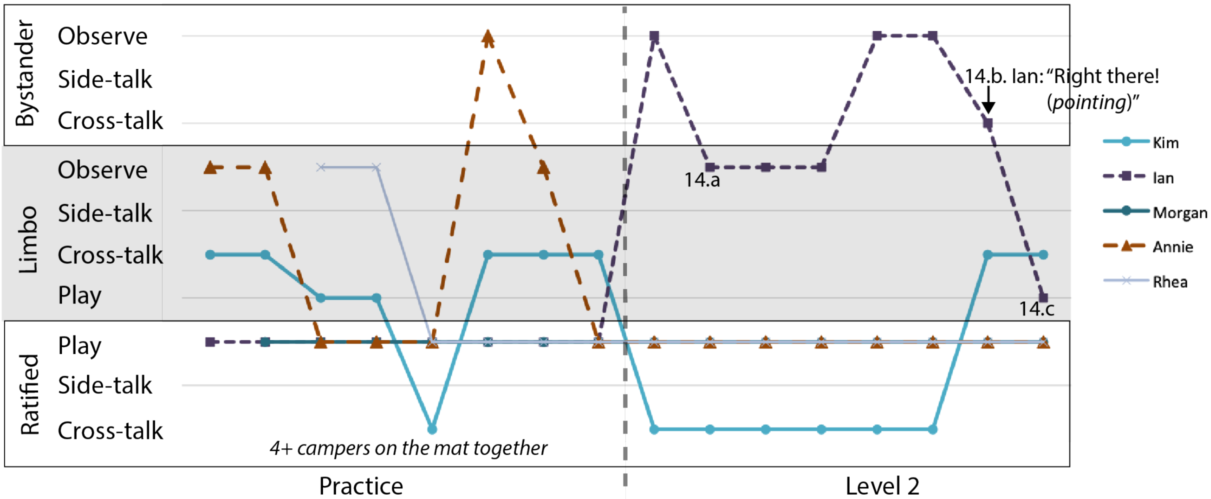

Figure 15. Campers' participation. In Practice, all campers played together. In timed play (Level 2), some campers played throughout while Ian transitioned roles. Additional campers' trajectories were removed for clarity.

Though perhaps less dramatic than the other two groups, these campers' play trajectories (Figure 15) again emphasize the possibilities for LOPI within collaborative, body-scale play. Participation during the Practice round was indeed a bit chaotic (left half of Figure 15), with many campers entering at different times, stepping back to observe and then returning to play, and generally running around the mat. During timed gameplay in Level 1, Morgan, Annie, and Rhea settled into ratified play, as Ian (purple squares) transitioned between roles. After prolonged observation in the limbo and bystander regions, Ian gesturally and verbally coached the ratified players (14.b). He then physically stepped up, too, playing covertly from limbo (14.c). Ian's increasing involvement in completing game tasks seemingly indicated his developing understanding of the game's spatial mapping strategy. This development, as well as the ways in which it was shaped by observing and pitching in, would have been overlooked by considering ratified play alone.

4.8 Discussion

Having described diverse forms of play and spectatorship in *Geometris*, I next propose features of those participation forms that may inform educational design more broadly. Using theories of affordances and constraints, I identify the possibilities for action, perception, and learning within the participation forms detailed above. I then use the LOPI and LPP frameworks to suggest how these diverse forms constitute the collaborative and pedagogically valuable interactions witnessed within *Geometris*. Finally, I propose key design elements as fundamentally enabling these participation forms.

4.8.1 Affordances and Constraints on Action and Perception Shape Learning Opportunities Across Participation Forms

I propose that the exhibit's three participation regions of bystander, limbo, and ratified play uniquely shaped participants' actions and perceptions within those spaces, creating

Table 3. Possibilities for Action and Perception by Region.

Region	Visual Perception		Action	
	Access	Perspective	Possibility	Expectation
Ratified Play	Clear; infrequent interruption	Immersive	Yes	Yes
Limbo	Moderate; occasional interruption	Adjacent	Yes	No
Bystander	Mediocre; frequent interruption, small scale	Distanced	No	No

distinct and synergistic learning opportunities. Of the three regions, ratified play placed the fewest constraints on visual perception and offered the most possibilities for action (Table 3). Players' view of the game environment was subject to only occasional line-of-sight restrictions by their co-players, and their perspective on that information was an immersive one, being literally positioned at shapes' vertices or edges. Ratified players also had the most possibilities for action, activating sensors at will. At the same time, they were subject to all the game's constraints and were expected to navigate these constraints in order to complete the game tasks (i.e. it would be strange for a ratified player to stand on the mat without activating sensors). Ratified players didn't always meet that expectation (e.g. Isla repeatedly asking "How do I do it?"). I suggest that action emerged as both an affordance and a constraint on ratified players' learning; while the possibility to act enabled players to test out spatial mapping strategies, the expectation to act may have at times overwhelmed players, inhibiting their learning of the target spatial mapping skill.

Participants in limbo experienced slightly reduced opportunities for action and visual perception compared to ratified players (Table 3). Their view of the mat and screen was periodically interrupted by ratified players, and limbo participants had an adjacent rather than immersive perspective on the game. This adjacency afforded covert action on the mat through which limbo participants could test their ideas or help players (e.g. Maxim, Ian, and the facilitators playing from limbo). At the same time, limbo participants were not expected to act. They could observe the game and ratified players' work without the time-constrained pressure to act. I suggest that limbo served as a valuable middle ground in more ways than geography; limbo participants enjoyed visual access to task information, the possibility for action, *and* the time to process that information at a self-regulated pace to develop their understanding of the interaction's underlying strategy.

Finally, bystander participants were most limited in visual perception and action possibilities (Table 3). From outside the stanchions, bystanders' view of the game was frequently obscured by players, the scale of the screen was likely smaller than ideal for them, and their perspective on the game was distanced. Bystanders were also most constrained in action; being many feet from the mat, their primary actions were verbal and gestural communication. Even still, Susan and Ian's participation demonstrates that bystanders could intellectually and emotionally engage with ratified play, offering coaching and encouragement especially in time-constrained moments, as has been documented in other studies of body-scale game spectatorship (Downs et al., 2015; Tekin & Reeves, 2017). Additionally, bystanders were afforded almost unlimited observation time with no expectation to act. Some bystander contributions (e.g. Leon and Ian's cross-talk coaching) indicated understanding of the spatial mapping despite having not played themselves, suggesting conceptual learning.

4.8.2 Implications for Study and Design of Educational Technology

In accordance with the LOPI and LPP frameworks, all present participants, no matter their centrality or periphery to the game mat, constituted the gameplay experience documented above. I argue that the non-ratified forms of participation can play a more central role in constructing the collaborative interaction than their geography or names might imply. The cases above illustrate how bystanders can offer strategic and moral support to players through cross-talk (e.g. Leon, Susan, Ian), as has been similarly documented in studies of videogame play (Tekin & Reeves, 2017). And limbo participants can involve themselves in play through verbal and gestural cross-talk with ratified players (e.g. Susan, Maxim, Kim) as well as through covert play (e.g. Maxim, Ian, Aiden, Molly, and Kim). Additionally, participants did not remain in one form throughout but rather transitioned between participation forms. 39% of participants both observed and pitched in (Figure 9), whether through cross-talk (12%), ratified play (15%), or both (12%). The three detailed cases suggest motivations behind these transitions. Some participants chose a new role to test or demonstrate competence (e.g. Ian moving from a bystander role to one in limbo) or to signal disinterest (e.g. Isla moving back from ratified play to bystander observation). Others were overtly signaled by their co-participants to change forms (e.g. Erik pushing Susan out of play; Aiden asking Maxim to step back; Kim selecting campers to play). Related research on videogame play suggests that spectators enjoy their experience more when such transitions are codified and anticipated (Downs et al., 2014).

Given that these relatively brief collaborative interactions with novel educational technology depart from the traditional LOPI and LPP focus on prolonged collective practice, why does it matter that participants take on or transition between multiple participation forms? What's the value of expanding our design and evaluation focus to include spectatorship? At face value, educational technologies and design frameworks conducive to principles of LOPI could more appropriately match the learning models of indigenous heritage students and expand learning models for all students, as Rogoff herself suggests (2014). Moreover, the data presented above suggests that the affordances and constraints on action and perception within each participation form constitute a rich ecosystem of learning opportunities that would be overlooked by designing for or evaluating ratified gameplay alone[20]. Transitioning between participation forms enables participants to take on variable responsibility for action while still remaining a legitimate participant within collaborative play, respecting their agency. Additionally, when participants change geographic region, they change perspective on environmental information. Research suggests that translating between perspectives in this way – as when a participant moves between the distanced bystander perspective,

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[20] I respect Tekin and Reeves' (2017) distinction between "being a spectator" and "doing spectat*ing*" and acknowledge that the existence of this participation ecosystem does not mean that all participants will leverage it.

the adjacent limbo perspective, and the immersive ratified play perspective – helps draw out shared elements of those perspectives and strengthens learning of the target skill (Ackermann, 1996; Greeno & Van de Sande, 2007; King & Smith, 2018; Shusterman & Li, 2016). Thus, I echo calls for design frameworks of body-scale interactions that reflect the complex, inherently social contexts in which learners engage with them (Maurer et al., 2015), as has been more fully elaborated for other technologically-enabled educational spaces (Berman et al., 2016; Blikstein & Krannich, 2013; Dreessen & Schepers, 2019; Rogers & Blikstein, 2013).

4.8.3 Design Elements Shape Participation in Intended and Unintended Ways

Having outlined the possibilities for action and perception within the various participation forms as well as their constitution of the collective play experience, I suggest particular design elements that facilitated those participation forms and learning opportunities.

Among intentional design elements, a) the game's scale and b) its input–output mechanism respectively enabled visual access for all participants and fluid transitions between participation roles. First, the scale of both the mat and screen made the game *expressive* (Reeves et al., 2005), enabling participants other than ratified players to observe the game's input–output relationships and serving as an *entry point* (Heath et al., 2005) for visitors in any region. This scale also demanded a high degree of physicality from players, increasing spectatorship possibilities (Downs et al., 2014; Downs et al., 2015). Secondly, the flexible input capacity of the game mat served as an *access point* (Heath et al., 2005), enabling fluid transitions between participation forms, especially between limbo and ratified regions. The mat's sensors track activations rather than players. Thus, participants can even leave (e.g. Isla, the campers) or join in the middle of a level, perhaps without other players knowing (e.g. Maxim, Ian) or consenting (e.g. Susan). Additional learning implications of this input–output mechanism are further elaborated in Rosenbaum et al. (2020). Adopting similar features in other educational technologies could foster this looseness wherein participants pitch in as they are interested and step back when they are not, arguably fostering participation structures more aligned with the LOPI and LPP perspectives.

Considering incidental design features, the requirement to remove one's shoes and the stanchions respectively delineated ratified play from other participation forms and legitimized spectatorship. To reduce wear and dirt, participants were asked to remove their shoes. Taking off one's shoes became a bid for ratified play, while activating sensors with one's shoes on, often by kneeling or using hands in limbo, emerged as a form of hedging. Shoes became an observable indicator of social positioning between participants and of the ways they negotiated the different expectations for action across participation roles. Intended as a crowd control measure, the stanchions also shaped behavior outside their intended role by legitimizing spectatorship (e.g. Margo, Nadia, and Ian's bystander behaviors). In the absence of a waiting area, intently watching others' play, especially that of strangers, might be awkward. However, the stanchions signaled that this region ~4 feet outside the game mat was intended for waiting and watching. Bystanders arguably edited this space (Ma & Munter, 2014), remaking it for spectatorship and its accompanying behaviors (e.g. side-talk chit chat, cross-talk coaching). Given the pedagogical value of the bystander participation documented above, such a legitimized observation areas could benefit other collaborative play designs.

4.8.4 Limitations

This study's claims are tempered by some procedural shortcomings. First, the camera focused on the ratified play and limbo regions (Figure 4), capturing only a few feet of the bystander region and possibly under-representing the frequency of bystander participants. Second, despite precedent for conducting interviews in museum settings (e.g., Gleason & Schauble, 1999; Palmquist & Crowley, 2007; Shine & Acosta, 2000), it was decided in consultation with site experts that interviews might disrupt visitors' experiences. Thus the above data are based on observation alone, missing participants' motivations, frustrations, and perspectives that could have been revealed through interview. Finally, without a comparable installation minus the design features highlighted above, claims about the influence of those features are correlational rather than causal. Such work is part of larger research efforts on body-scale, collaborative interaction technologies.

4.9 Conclusions

Blending scholarship on educational technology with scholarship on learning through observation, this paper reported on varied forms of participation – from spectator to player and some in between – that emerged around *Geometris*, a collaborative, body-scale geometry game. Framing learning as participation in communal practice (Lave & Wenger, 1991; Rogoff, 2003), I presented frequency analysis of roughly 350 visitors as well as thick narrative descriptions of 3 visitor groups to characterize these diverse participation forms and the learning opportunities available within them. Varied perspectives on the game task and differential opportunities for action emerged as salient dimensions across participation forms, shaping participants' learning opportunities as they transitioned between forms. I suggest that such varied forms of participation should be both designed for and studied in other educational technologies, particularly given the often-limited availability of such designs within educational settings. I also identify how design elements of scale and distributed input mechanism enabled engagement across participation forms. Simultaneously, incidental crowd control measures (i.e. stanchion posts, removing shoes) served to delineate participation forms, both for participants and for researchers. Design frameworks sensitive to observation-based learning could facilitate similarly diverse forms of participation and learning in other educational designs.

5 Move with whom? The impact of social relationships on collaborative embodied learning

In this third paper, I again step back to consider how the inherently social context of collaborative play might shape learners' behaviors in embodied learning environments. I develop a framework to characterize participants' physical proximity – a marker of familiarity – as they work on *Geometris* tasks and apply it to dyads of varying social composition. Video recordings and analysis of 41 visitor dyads of families, friends, and strangers suggest that all dyads established successful collaborative movement patterns, though strangers shared space and established physical touch markedly less than family members and friends. Learning implications include the possibly distracting effect of maintaining socially acceptable distance among stranger players. Considering these patterns in the design of other collaborative embodied learning activities could create a more comfortable and supportive environment for people to move and learn together.

5.1 Introduction

Imagine you're preparing a meal with someone. You both gather ingredients, access tools and dishware, prepare the food, set the table. In accomplishing this work, you and your culinary collaborator negotiate not only access to the material stuff of cooking but also how to move around one another. You stand, bend, and reach around each other, sometimes with hot or sharp objects. How you move and work when cooking with your partner, child, or close friend is likely different than with an in-law, a newish roommate, or your cousin's new partner whom you're meeting for the first time. While you might be casual, relaxed, even loving with some cooking partners – leaning in to moments of proximity for a playful nuzzle or hug – you may be stiff, formal, even uncomfortable around others. Maintaining appropriate personal space might require overt communication or planning, even distracting from the cooking tasks at hand, or it may be a non-issue.

What is ostensibly the same collaborative task can feel very different depending on one's collaborator. While the ramifications for daily life may be minimal, these distinctions matter for designers and researchers of collaborative embodied learning environments. Embodied learning activities are predicated on the idea that cognition is fundamentally grounded in perceptuomotor experience. Prompting learners to discover and enact certain perceptuomotor routines as useful within a contextualized task forms the cognitive foundation for their later reflection, expression, and abstraction of that experience in disciplinary terms. Collaborative tasks offer additional learning opportunities as people overtly coordinate and negotiate their work, communicating and expressing their thinking through verbal or gestural means. An increasing number of designs aim to leverage these resources for disciplinary learning in mathematics (e.g., Chorney & Sinclair, 2018; Ma, 2016; Mickelson & Ju, 2011) and science (e.g., Adachi et al., 2013; Carreras & Parés, 2009; Enyedy et al., 2015) as well as language (e.g., Lee et al., 2012), and communication (e.g., Chen et al., 2020).

But as the above cooking example illustrates, when it comes to collaborative action, not all partners or partnerships are the same. The ways in which people move and touch can feel very different based on the people with whom we move and touch (Cekaite & Goodwin, 2021). To offer a few more examples, a parent brushing a young child’s teeth elicits a different feeling than a dental hygienist performing ostensibly the same procedure. Co-workers at a team-building event, say a ropes course or laser tag, move around one another very differently than would a group of friends or family members in the same space. More distanced still, strangers navigating a grocery aisle work to maintain further physical distance than might co-workers in that store. Understanding how various social relationships impact people's willingness to move in proximity can provide insight into the ways they might interact within collaborative embodied learning environments, in turn shaping their potential learning from these activities. Additionally, such insight could inform design heuristics to circumvent potentially restrictive social norms and encourage people to move together in intended ways.

This paper reports on the installation of *Geometris*, a collaborative, embodied learning environment, within a children’s science museum. Visitors to the exhibit had a range of social relationships, from siblings and family members to familiar acquaintances to total strangers. This observational study documents trends in collaborative movement patterns across these social groups. I use a framework of interaction patterns to capture differences in how 41 participant dyads move within the physical space (territoriality) and relative to one another (touch), with proximity as a primary measure of social comfort between collaborators (Hall et al., 1968). I then document how these interaction patterns seem to trend with social familiarity across family, friend, and stranger dyads. Finally, I evaluate the implications of these differences for participants’ learning opportunities and how these implications can inform the design and implementation of collaborative embodied learning environments. As such, this work aims not to evaluate the learning outcomes of *Geometris*, the particular design used in this case, but to evaluate comfort moving together as a precursor for such learning that applies to collaborative embodied designs more broadly.

5.2 Theoretical Frameworks

In this section, I briefly review the theoretical commitments and design principles of embodied design for learning. To understand collaborative movement within these environments, I then draw on theoretical tools that identify the forces that shape collaborative movement and characterize solutions to the challenges of collaborative movement. Specifically, I draw on the work of cultural anthropologists Charles Goodwin and Marjorie Goodwin as well as work from the field of joint action to interrogate the social dynamics and implications of coordinating interaction in multi-party activities. I also review constructs from gesture studies and professional education to characterize particular spatial arrangements of participants in multi-party interaction.

5.2.1 Embodied Design for Learning

While theories of embodied cognition differ along some dimensions – for example, conceptions of the mind (e.g., Gallagher, 2015; Glenberg, 2010; Goldman, 2012; Kelton & Ma, 2018; Wilson, 2002) and the would-be existence and roles of representations (e.g., Anderson, 2003; Hutto & Myin, 2013; Shapiro & Stolz, 2019; Wilson & Golonka, 2013) – they all maintain a central role for perceptuomotor activity in cognitive work. Perceptuomotor processes have been implicated in a wide range of mathematical activities, from teaching (Alibali et al., 2013) and learning (Abrahamson, 2004; Goldin-Meadow et al., 2009; Lemke, 2003; Radford, 2003) to problem-solving (Goldin-Meadow et al., 2001) and argumentation (Ochs et al., 1996; Schwarz & Prusak, 2016). Similarly, the arrangement and malleability of task-related resources – be they representational or semiotic – alter cognitive processes. For example, manipulating physical resources can increase learners' understandings of fractions (Martin & Schwartz, 2005), and perceptual features of symbolic-notation arrays, such as visual spacing or grouping, can interrupt normative arithmetic calculations (Landy et al., 2014).

Adherents to embodied cognition will maintain that all learning is embodied, whether or not the learning activity was designed with these theories in mind. Nonetheless, principles of *embodied design* (Abrahamson, 2009a) aim to create opportunities for students to recognize parity between, on the one hand, their perceptual intuitions and movement inclinations within the environment and, on the other hand, normative mathematical perspectives (Abrahamson, 2014). Within such activities, learners can encounter mathematical concepts as patterns in perceptuomotor activity before generalizing those patterns in semiotic registers (Radford, 2003), helping them recognize those ideas as powerful organizing principles to think with (Papert, 1980) rather than as arbitrary rules to be memorized. Through this embodied learning process, the resulting formal expressions not only represent the target math concept but also carry students' personal, physical experiences (Abrahamson, 2014; Abrahamson et al., 2020; Abrahamson et al., under review).

5.2.2 Coordinated Movement in Multi-party Interaction

This study focuses not only on embodied learning environments but, more specifically, those intended for simultaneous use by multiple co-participants. Such coordinated movement is both fundamental to the human experience and tightly bound by social norms. From infants mimicking their caregivers' actions to pedestrians navigating bustling city sidewalks, we move with and around one another in carefully if subconsciously choreographed daily dances (Sheets-Johnstone, 2017). At the same time, the ways in which we move around each other are heavily bound to and indicative of social relationships. Physical proximity in particular is a strong, if culturally-dependent, indicator of social bonds (Hall et al., 1968). In a form of physical code switching, the kinds of touch, embrace, and other bodily entanglements integral to co-operative action between a child and their parent or between spouses (Goodwin & Cekaite, 2018) might be off-limits (or at least carefully regulated) between non-relatives. This dynamic is further emphasized by the tension evoked by contexts – such as social dance, medical examination, and grappling sports, to name a few – that do involve close touch between strangers. Depending on participants' relationship to one another, interaction within a collaborative embodied learning environment may present another such case of legitimized physical proximity absent its traditional social meaning.

Building on Charles Goodwin's (2017) more specific construct of *co-operative action*, learners in collaborative embodied learning environments iteratively build and negotiate meaning by creating and operating on a pool of shared resources, be they verbal, gestural, physical, or otherwise. Also central to Goodwin's co-operation is the progressive adaptation and reuse of these shared resources. Marjorie Goodwin's (1995) studies of girls disputing a move in hop-scotch – a process that involves carefully constructed and reused verbal utterance, hand and foot gestures, eye gaze, and body position all relative to other bodies and the inscribed hopscotch grid – illustrates the deeply material and social nature of co-operative action.

Cooperative embodied interaction is not only subject to participants' existing social relationships but may also shape those relationships in powerful ways. When co-operating participants share the same goal[21], constructs from the field of *joint action* – which studies how people coordinate their actions when engaged in joint activity (Knoblich et al., 2011) – may be helpful in characterizing their interactions as well as any resulting senses of social intimacy. Coordination patterns between jointly acting individuals may be symmetrical, in which partners mirror or mimic each other's actions, or asymmetrical, in which participants perform complementary roles toward completing the shared task (e.g. carrying heavy furniture (Sebanz et al., 2006)). Such coordination requires carefully trained gaze patterns sensitive both to subtle cues within the partner's body (Vaziri-Pashkam et al., 2017) and to the outcome of their anticipated movements in relation to the environment (Schmitz et al., 2017). These constructs suggest the intersubjective coupling required of participants' action–perception routines within multi-party embodied activities, as has been confirmed by a handful of empirical findings (e.g., Shvarts & Abrahamson, 2019). Additionally, research has documented behavioral and emotional effects of synchronous joint action (Laroche et al., 2014) whereby coordinated activity such as rocking together (Richardson et al., 2007), walking together (Miles et al., 2009), or other simultaneous movement (Bernieri, 1988) engender among co-actors a heightened sense of affect and rapport (Mogan et al., 2017). The stronger the temporal synchrony in joint action, the stronger the induced sense of cohesion and connectedness (Himberg et al., 2018).

5.2.3 Spatial Arrangements of Multi-party Interaction

I next review constructs that characterize the dynamic spatial arrangements of participants in multi-party embodied interaction, capturing and classifying how people move in relation to one another and the environment. Research on *territoriality* (Scott et al., 2004; Tse et al., 2004) describes how participants partition collaborative spaces.

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[21] As Goodwin (2017) notes, co-operation does not always imply a shared goal, such as when children compete against one another or raise a dispute.

This body of literature tends to focus on professional settings and tasks for which movement is secondary to some design, composition, or other task.

Kendon (1976) introduces the idea of a *transactional segment,* the arc of space encompassing the people and objects directly in front of an individual that defines the area in which that individual acts. When multiple people interact, Kendon further defines the overlap of their individual transactional segments as an *o-space*. Within multi-party embodied learning activities, the o-space is fundamental to participants' ability to co-operate and act jointly toward the designed task. I propose that considering the size and location of this o-space relative to the designed embodied learning environment could help characterize the degree to which co-operating participants are comfortable moving together within the designed space.

Finally, constructs from studies of entrained movements – as in professional education (Guo et al., 2020; Weddle & Hollan, 2010), midwifery, or musical training (Nishizaka, 2017) – capture how individuals within a multi-party interaction involving proximal physical contact can shape and manipulate each other's bodies. From studies of dental hygienists in training, Weddle and Hollan (2010) highlight the phenomenon of *molding*, in which "a teacher physically manipulates the body of a novice into institutionally established configurations and movement patterns" (Weddle & Hollan, 2010, p. 128). For their dental hygienists in training, molding looked like instructors laying their hands on or around a student's hands, shaping the student's fingers, or moving their hand through the target motion. Other studies of dental practice illustrate how molding, especially of children, may be collaboratively achieved, at times in concert or in conflict with the child subject (Guo et al., 2020). In this sense, molding captures the ways in which co-participants in multi-party interaction can not only coordinate but also directly manipulate each other's actions.

In summary, within multi-party interaction, individuals' movements and activity are inextricably interwoven with that of their co-participants. What and how we move is beholden not only to the task at hand but also to the social relationships at hand. This study aims to evaluate the ways in which the acceptable degrees of physical intimacy carried by different social relationships may either constructively contribute or destructively interfere with how participants collaboratively enact the designed embodied learning tasks.

5.3 Related Works

A wide range of research literature can inform studies of multi-party activity on shared resources, from broad theories of learning through observation (Rogoff, 2014) and legitimate peripheral participation (Lave & Wenger, 1991) to specific studies of recreation (Nasir, 2005), dance (Vogelstein et al., 2019), and classroom group work (Roschelle, 1992; Rosenberg et al., 2006). While learners in these contexts must implicitly negotiate issues of proximity and coordination around their shared resources, these studies rarely consider such work overtly.

For work that foregrounds on proximity within collaborative behavior, I turn to research on human-computer interaction (HCI). Researchers in this field have extensively studied multi-party collaboration and space sharing, often in the context of tabletop and wall-mounted multi-touch displays. While much of this research focuses on adult professionals engaged in design work – a work environment with different goals and expectations than collaborative embodied learning environments – some projects evaluate more diverse participant groups in naturalistic settings such as museums and public spaces (e.g., Hornecker, 2008). These works highlight tensions that can emerge among strangers (Marshall et al., 2011; Peltonen et al., 2008) but also the pro-social effects that such interaction can engender (Jacucci et al., 2010). Even with familiar or familial groups, interactions may be less cohesive than anticipated, with children competing for and blocking access to materials (Marshall et al., 2009) and adults hanging back as children interact with what is perceived as a toy (Hornecker, 2008; Marshall et al., 2011).

A subset of HCI, *exertion games* use full-body movement as input for computer gaming. Much like collaborative embodied learning environments, distributed or social exertion games are designed for multiple co-participants. Researchers note a compounding effect between physical exertion and social play (Mueller et al., 2009). However, even in such environments, collaborative play may not necessarily result, with players sometimes adopting parallel, independent play styles (Mueller et al., 2017).

Though few in number, some education research studies have evaluated the role of proximity and social relationships within collaborative embodied interaction. Gaudreau et al. (2021) suggest that the interactivity of their designed life-sized board game was one feature that prompted more STEM question asking among children and caregivers than a less interactive control exhibit. The Science through Technology and Play (Danish et al., 2015) environment takes physical proximity between co-participants (along with their speed) as a primary input to their motion-tracking software, modeling learners' movements and displaying it back to them as a key parameter in determining states of matter (i.e., solid, liquid, or gas). And Vossoughi et al. (2020) perform valuable analysis of how social relationships shape students' embodied learning experiences. Their study of students in a Makerspace documents how relational histories shape the ways in which learners position and move their bodies over widely ranging time scales, from a few moments to a few years.

These works provide a useful starting point in understanding how social relationships can shape multi-party interaction. However, the works cited above tend to evaluate one social relationship or setting at a time (e.g., laboratory, classroom, or home). Public spaces such as museums offer a learning space in which family members, friends, or strangers might authentically interact. Yet even the studies in museum settings typically do not differentiate between the types of relationships present within visitor groups. This study seeks to add to this body of work by comparing a larger number of groups across different social relationships.

5.4 Research Questions

1. What are the different ways that people move together in multi-party embodied learning environments? What are the resultant opportunities or barriers for learning?
2. What role(s) do social relationships play in shaping the ways people move and thus learn together in these environments?

5.5 Methods & Materials

Details of *Geometris* design and data collection are presented above in Section 2.

5.5.1 Data Selection and Unit of Analysis

To understand multi-party interactions within Geometris, I first partitioned the video data by visitor group. Similar to other semi-public displays, *Geometris* engendered step-wise engagement patterns (Brignull & Rogers, 2003), with participants often attracted by the activities of other visitors and frequently observing from the sidelines before playing themselves. Thus, while visitor groups are delineated according to play activity, visitors typically observed and thus passively participated in other groups' activity. For purposes of this analysis, a new group was marked each time someone entered or left the stanchioned area, which contains the game mat and the floor space immediately surrounding it (Figure 4).

The social relationships among participants were determined in various ways. Some participants self-reported their relationship as prompted by the game, though other participants either skipped this step or seemed unaware of how the input/output worked, making this reporting unreliable. Other observational clues include timing (e.g. family and friends often arriving and departing together; strangers arriving and departing separately) and clothing (e.g. summer campers wore color-coded t-shirts).

Because proximity is used as the primary measure of comfort, analysis was restricted to those visitor groups with the same number of players, in this case, 2. Groups with occasional, brief play by a 3rd individual (often a facilitator or visiting adult who played for fewer than 5 seconds) were included, though this 3rd player's positions were not analyzed. In total, 41 dyads were analyzed, including 16 dyads of strangers (visitor-visitor, visitor-facilitator, researcher-facilitator, research-visitor), 8 dyads of friends, and 17 dyads of family members (similarly aged as well as intergenerational).

5.5.2 Coding Scheme

In evaluating *Geometris* as a collaborative embodied learning environment, I focus on the ways in which participants coordinate and negotiate movement with and around one another in the game space. In keeping with the tradition of diagrammatic coding (e.g. Angelillo et al., 2007; Kendon, 1976, 2010), I

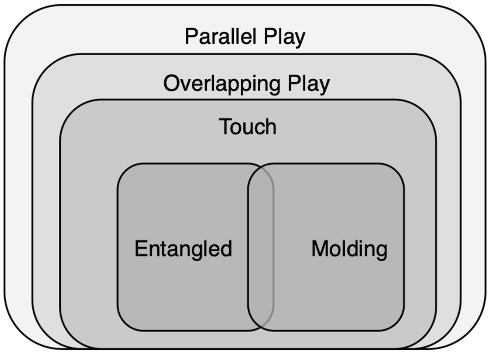
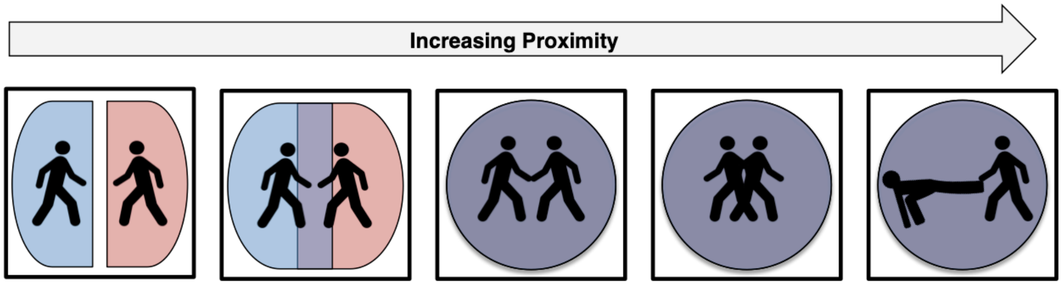


Figure 16. Five categories of interaction in collaborative embodied learning designs.

categorize behavior at the dyad level to capture whole group dynamics. Inspired by theoretical constructs of territoriality, o-space, and molding, I interpolate and consider five modes of coordinated movement (Figure 16) and apply one code per second of the video data. These modes are distinguished based on two dimensions: (1) the arrangement of players' territorial regions within the game space; and (2) the kind of physical contact players enact (Table 4). Please see Appendix B for examples of each coordination mode.

Table 4. Territorial and contact features of the five coordination modes.



Territory:	**Parallel**	**Overlapping**	Shared	Shared	Shared
Contact:	None	None	**Touch**	**Entangled**	**Molding**

At the broadest level, participants engage in *parallel play*, "separately but courteously" (Peltonen et al 2008, p. 1293) co-ordinating their movements so that each participant remains in their own regional zone. Such territorial separation is typical for collaborating strangers or formal acquaintances (Scott et al., 2004; Tse et al., 2004). Territorial demarcation may be overt (e.g., "You take that side, I'll take this side") or de facto, and it maintains physical distance between participants. Parallel play simplifies each player's task – they are responsible only for the sensors in their region – but it also reduces their possibility for action, precluding them from realizing ideas that require sensors in their partner's region.

In *overlapping play*, players establish some shared territory, moving in and out of spaces occupied by their partner only moments before. Inspired by Kendon's (1976) o-space, overlapping play involves relaxed territorial commitments as players maintain distance from one another but also begin to act upon the same physical space and resources. Free to explore more of the game space, players have more agency to test out ideas across the interface but may not yet feel comfortable doing so in their partner's immediate proximity. Additionally, percent overlap is calculated based on the number of mat sensors, out of a total of 9, that have been activated by both players. See Appendix B for more detail.

At the next mode, territorial boundaries dissolve and players establish *touch*, often when simultaneously activating the same sensor. Within a Western cultural context, the occurrence and especially maintenance of physical touch often indicates a close acquaintance or familial bond (Goodwin & Cekaite, 2018). In this coordination mode, players can focus on the geometric and spatial reasoning tasks at hand rather than on maintenance of a socially acceptable personal space.

Closer still, some participants adopt *entangled play*, overlapping their bodies akin to partners in dance or people playing *Twister*®. As the taboo against physical touch dissolves, players may reach or step over one another. In this mode, a partner's location is less of an obstacle to enacting ideas within the game space.

Finally, a few visitor groups even manipulate or *mold* (Weddle & Hollan, 2010) each other's bodies, moving a partner's hand, foot, legs, or entire body to the desired sensors. In this mode, participants explore the space and test ideas not only with their own bodies but with their partner's body as well, effectively using their partner as a tool or puppet for the task at hand and, thus, initiating and managing the distributed solution across two bodies.

Importantly, participants' progression through these coordination modes is not monotonic. A group may momentarily enact a more proximate coordination mode before settling back into a more distanced one. Regardless, each tier of this framework represents a gradual chipping away at the social taboo against physical touch until it becomes no hindrance at all to enacting participants' emergent understanding of the cooperative embodied learning task. For detail on how coordination modes were assigned to the video data, please see Appendix C.

5.6 Results

5.6.1 Illustrative Vignettes

For a sense of what these high-level collaboration patterns mean in terms of dyads' play and learning experiences, consider the following two examples.

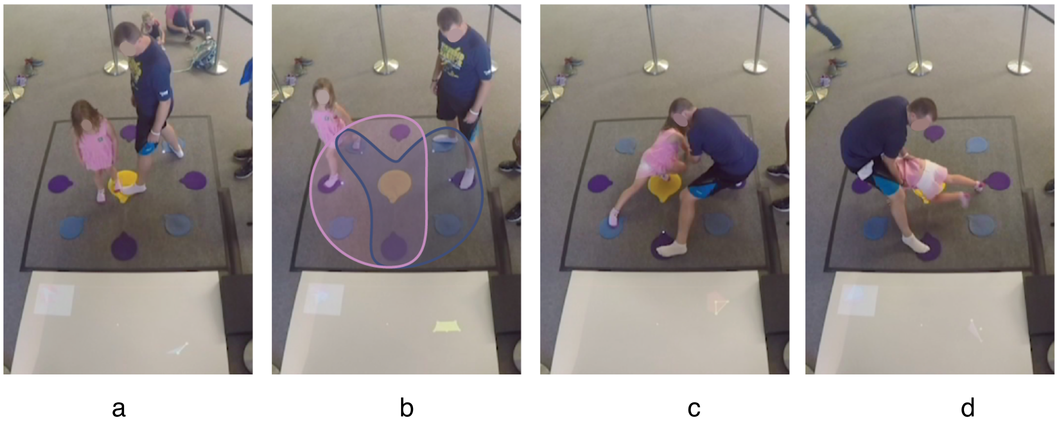


Figure 17. Family: Joy and Mike's Geometris play. They establish and maintain touch during a brief Level 1 (a). During Level 2, they overlap on ~30% of the mat (b, Joy's territory in pink, Mike's in blue). In Level 3, they entangle (c) and Mike molds Joy by lifting her at the waist (d).

Joy (~8 years old) and her father Mike approached the Geometris exhibit while a previous dyad was playing. When that previous dyad left in the middle of Level 1, Joy and Mike stepped in to finish it. They established touch within the first 10 seconds of their play, their feet touching as they both stood on the center pad (Figure 17a), a position they maintained for the short remainder of the level. As they played Level 2, Joy and Mike didn't touch at all. They established overlapping territories, coming to share 3 of the 9 total sensors or 1/3 of the mat (Figure 17b). In Level 3, they demonstrated all five coordination modes. They briefly established touch, then entangled as they both reached on all-fours (Figure 17c). Ultimately, Mike molded Joy by lifting her off the ground from the waist (Figure 17d) so that her feet released unnecessary sensors.

Isla (~8 years old) played with Aaron, a high school-aged volunteer facilitator staffing the exhibit. They began with Level 3, in which they once touch briefly to complete a shape (Figure18a) and gradually grew their overlapping territories until they both activated all the sensors, sharing the whole mat (Figure 18b). They then played Level 2. On their first shape, a rectangle, they established touch by simultaneously activating the same sensor (Figure 18c). Isla, who activated two diagonal vertices of the rectangle, then directed Aaron to get the other two, a move that would require him straddling her (Figure 18c). He shifted back and forth, directing her to change position rather than entangling with her. They did not negotiate their coordination in time and missed the shape. Isla and Aaron completed their play with Level 1, during which they briefly touched twice and overlapped territory on roughly half the mat, sharing 5 of the 9 sensors (Figure 18d).

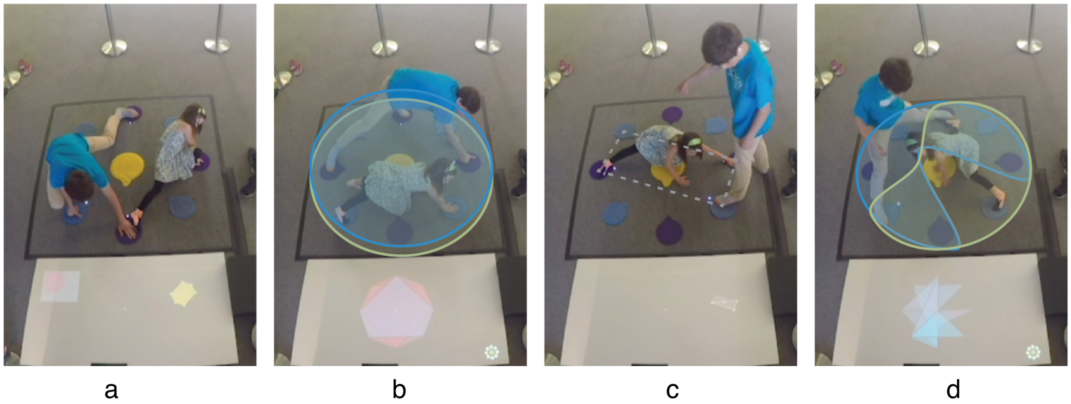


Figure 18. Strangers: Isla and Aaron's Geometris play. They briefly touch to complete a shape in Level 3 (a) and share the whole mat by the level's end (b, Aaron's territory in blue, Isla's in green). They avoid entanglement in Level 2, with Aaron refusing to complete the rectangle (c). They again touch briefly in Level 1 to complete a shape and overlap on roughly half the mat (d, Aaron's territory in blue, Isla's in green).

5.6.2 Patterns in Interaction Behavior by Social Relationship

Applying the above interaction framework to video data of 41 visitor groups suggests that the distinction between strangers and familiars, be they friends or family, matters for coordination patterns, though the degree of familiarity for familiars may matter less.

Table 5. Frequency of coordination modes by social relationship.

Demonstrated Interaction:	**Social Relationship**		
	Strangers (e.g. visitors, facilitators)	**Friends** (e.g. friends, campers, co-workers)	**Family** (e.g. siblings, cousins, parents, children)
	N=16	N=8	N=17
Overlapping Play	100%	100%	100%
Touch	50%	88%	88%
Entangle	6%	25%	29%
Mold	0%	38%	35%

Table 5 summarizes the overall coordination patterns demonstrated by all 41 analyzed dyads. By playing, all dyads automatically engaged in parallel play, so it is not included in the table. All dyads also demonstrated overlapping play, establishing at least some portion of the game mat as shared territory. Thus, willingness to share at least some territory seems unaffected by social relationship (the extent of sharing will be analyzed next).

In coordination modes involving touch, dyads of strangers show marked difference as compared to the more familiar dyads. Half of the stranger dyads established physical touch vs. almost 90% of the family and friend dyads. Only 6% of the strangers (1 dyad) demonstrated entangled play and none exhibited molding. Comparatively, at least a quarter of the friend and family dyads engaged in entangled play and even more exhibited molding.

Friend dyads exhibited coordination modes at a similar rate as family dyads, suggesting that the increase in familiarity from friends to family members may not induce increasingly proximate coordination behaviors. Importantly, the analyzed video included half as many friend dyads as family dyads, so this pattern could be skewed by a few unusually familiar friend dyads. Notably, among friends and family, molding behaviors occurred in more dyads than did entangled play. The increase is small in absolute numbers, with 1 more of each type of dyad engaging in molding than in entangled play.

Territorial Overlap by Social Relationship

Given the ubiquity of overlapping play among dyads of all relationships, I looked more closely at the degree of overlap to analyze how players were sharing space in overlapping play. Once a player activated a sensor, it was considered part of their territory for the remainder of the level. Thus, territories are cumulative within each level and reflect all the sensors a player has touched to that point in the level. Percent overlap was calculated by dividing the number of shared sensors across players' territories by 9 (the total number of sensors). See Appendix B for more detail on determining territories and overlap percentages. Table 6 summarizes the percentage of the game space that was shared by dyad members, stratified by social relationship.

Table 6. Territory overlap by social relationship.

Territory Overlap	**Social Relationship**		
	Strangers (e.g. visitors, facilitators)	**Friends** (e.g. friends, campers, co-workers)	**Family** (e.g. siblings, cousins, parents, children)
	N=16	N=8	N=17
0%	100%	100%	100%
11%	100%	100%	100%
22%	94%	100%	100%
33%	94%	100%	94%
44%	75%	100%	94%
56%	50%	100%	82%
67%	38%	88%	82%
78%	38%	75%	76%
89%	25%	63%	71%
100%	13%	25%	41%

As with the overall coordination modes, dyads of friends behaved similarly to dyads of family members, while dyads of strangers displayed marked difference. Almost all dyads shared up to 33% of the game space, but only half of stranger dyads shared 50% or more. A quarter of friend dyads and over 40% of family dyads shared the whole game space, whereas only 2 strangers dyads (13%) exhibited a similar level of territory overlap.

Coordination Modes over Time

Whereas the above analyses evaluate the presence or absence of coordination behaviors by dyad, I also wanted to get a sense for how quickly these behaviors develop within each relationship type. The following time series analyses depict dyads' territorial overlap over the course of their play. In order to compare across dyads, some of which played many game levels while others played only one, these time series compare only the first level of play for

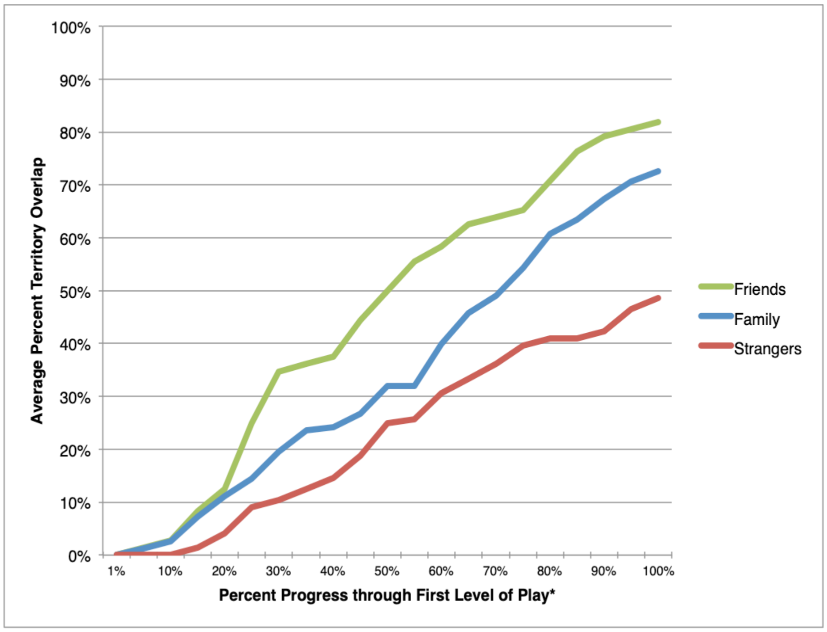


Figure 19. Average overlapping territory by percentage of first level complete.

each dyad. If that first level was a Practice round lasting less than 1 minute, the second level of play was considered.

Within overlapping play, stranger dyads were, on average, slower to begin sharing space, maintaining parallel play (0% overlap) until roughly 15% through their first level[22] (Figure 19). Among stranger dyads, those who achieved the most territory overlap were those who played together the longest (Figure 20). Familiar dyads, on the other hand, began sharing territory almost immediately and increased their shared territory at a faster rate than did strangers (Figure 19). Differences between friend and family dyads should again be viewed in light of the relatively small sample size of friend dyads (N=8).

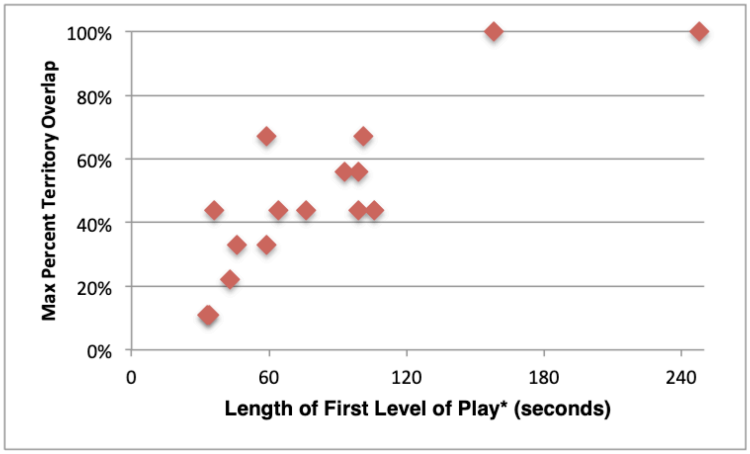


Figure 20. Maximum overlap achieved by stranger dyads during their first level complete.

Summary

All dyads were willing to share the game space to some degree. Analysis of all 41 dyads reflected the differences exemplified by the two illustrative cases. Stranger dyads differentiated from friend and family dyads in that they exhibited coordination modes that require touch (touch, entanglement, and molding) much less frequently than did dyads of friends or family members. Additionally, while all dyads ultimately shared some part of the game space, stranger dyads were both slower to establish shared territory (Figure 19) and shared less territory overall (Table 6), with roughly 50% overlap being the distinguishing threshold between strangers and familiars.

5.7 Discussion

5.7.1 Familiarity Increases Space Sharing and Touch

As predicted by constructs from across theoretical traditions, dyads' movement and coordination behaviors within the collaborative embodied learning environment presented above bore out differences in proximity and touch that aligned with their social relationships. By prompting participants to move together, the exhibit's very design contradicts "the norm of social segregation between the unacquainted" (Peltonen et al., 2008, p.1292), which was reflected in strangers' relatively lower rates of territory sharing and touch-based coordination modes. In line with theory around intimacy

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[22] *So long as the dyad played additional levels, practice levels lasting less than 1 minute were not counted as a first round of play. Six dyads were were affected.

through synchrony, the shared coordination task did increase proximity between strangers. Even periods of a few minutes of play prompted more intimate play (increased territory overlap and proximity), though not to the same level as for friends and family members. Ultimately, the *degree* of territory overlap, the size of the o-space (Kendon, 1976*)*, was more indicative of social relationships than its mere presence or absence. Interestingly, for friend and family dyads, molding occurred more frequently than did entanglement, especially for friend dyads (Table 5). Whereas we might expect that manipulating another's body would be more intimate than inhabiting the same personal space, many instances of molding occurred at maximum distance, such as by moving a foot with a hand (Appendix B, Figure B1e). Instances of entanglement were slightly higher in family dyads than among friends, aligned with Goodwin and Cekaite's (2018) illustration of the close bodily entanglements inherent in some families' daily lives.

A critical qualification to this analysis is the significant cross-cultural variation in practices around physical proximity and touch. Across cultures, behaviors such as kissing, hugging, handholding, or touching of any sort can carry vastly different connotations in the relationships they signify and the spaces and norms under which they are practiced or avoided. Factors such as age, gender, and marital status can also govern the types of touch deemed appropriate. While the dyads considered in this analysis are from the same geographic region, they undoubtedly carry different familial and cultural practices around proximity and touch. Additionally, whereas these data were collected in 2018, any subsequent implementations in the era of COVID-19 may be subject to participants' concerns around health and disease transmission that may compound or usurp typical social norms around proximity.

5.7.2 Implications for Learning

This work aimed not to evaluate learning outcomes of *Geometris*, the particular design used in this case, but to evaluate comfort moving together as a precursor for such learning that applies to collaborative embodied designs more broadly. By design, movement is the primary means by which participants explore, test, and refine their ideas within an embodied learning environment. When that movement is restricted, participants' ability to learn about the environment's governing rules – here, the learning objective – is similarly restricted. As illustrated by the vignettes and overall trends presented above, social pressures seemed to restrict movement, and thus learning opportunities, among some dyads. All analyzed dyads eventually progressed beyond parallel play to overlapping play, where they enjoyed the freedom to test their ideas across more of the game interface. Yet strangers shared less territory than did friends and family members (Table 6), suggesting hesitance to move in their partner's immediate proximity. Stranger dyads also established touch at a much lower rate than did friend and family dyads (Table 5), suggesting that maintenance of socially acceptable personal space may have distracted them from the geometric and spatial reasoning tasks at hand. Only one stranger dyad exhibited entangled play and none exhibited molding, and these coordination modes were relatively infrequent even among friend and family dyads. While lack of necessity may play some role, Isla and Aaron's vignette and the differential frequency across dyad types suggests that strangers are more resistant to entangle or mold one another in ways that could contribute to completion of the goal task.

5.7.3 Implications for Collaborative Embodied Design

There are cultural practices where it is legitimate and anticipated that strangers will consort in close physical contact, and there are other cultural practices where such interaction is not expected and would perhaps be uncomfortable, disconcerting, or taboo. Designers of transformative embodied learning environments bump up against these norms when our designs invite people to physically consort in ways that violate cultural-, context-, or practice-specific norms. Thus, the introduction of a novel design may at times require pushing societal norms around human interaction, in particular around legitimacy of types of interaction in learning contexts. Moreover, the disciplinary trappings of an activity implicitly invoke people's epistemic beliefs regarding the place of the body and movement within that discipline's practices. The STEM domains especially are typically regarded as objective, impartial, and disembodied, independent from individuals' lived and felt experiences. Thus, in addition to the aforementioned norms of social interaction, norms of domain-specific interactions may further discourage collaborative, movement-based learning. As such, transformative design may also require transformation in epistemic beliefs about the role of the body and movement in cognitive practices. This work aims to contribute to efforts begun by other researchers (e.g., Alberto et al., 2021; Danish et al., 2015; Zohar et al., 2018) to challenge implicit beliefs about the role of the body within STEM practice.

I propose that this work suggests means to approach these two transformational challenges. The interactions analyzed in this study suggest that family members and friends were more comfortable moving in close proximity than were strangers. I propose that we leverage this dynamic when creating collaborative embodied designs for learning. When implementing these designs in school classrooms, encourage students to choose their own partners so that proximity is not a hindrance to play. Better yet, it may be that extramural, informal, and community contexts bear richer fruit for these kinds of designs than would traditional school classrooms (Peppler, 2017). Such out-of-school contexts can usefully subvert epistemic beliefs about the (dis)embodied nature of cognitive practices, highlighting their connection to movement and daily life.

5.7.4 Limitations

These findings and implications should be considered in the context of several limitations to this study. First, the relatively smaller sample size for friend dyads precluded the use of statistical tests to potentially distinguish their behaviors from those of family dyads. Additionally, factors such as gender, age, age differentials, and personal history likely also influenced a dyad's comfort moving with and around one another but were not systematically analyzed in this study. Finally, one or both dyad members may have electively adopted a certain facilitation style within their play that was more hands-on, more distant, or a combination thereof (see Section 3). In such cases, proximity could be a pedagogical decision rather than solely an issue of social comfort.

5.8 Conclusions

Innovations both in interaction technology and in theories of cognition have enabled the growth of novel multi-party, embodied learning environments. While research on these environments has contributed to educational design as well as to theories of learning, it is less well understood how participants' relationships with one another shape their behaviors, and thus resultant learning, within these spaces. Using a framework based on territory and touch, this study analyzed dyads composed of strangers, friends, or family members as they played *Geometris*, a collaborative embodied geometry and spatial reasoning game. Rather than evaluating the efficacy of this particular design, this study aimed to evaluate participants' comfort moving together as a precursor for learning that applies to collaborative embodied designs more broadly. Video recordings and analysis of 41 visitor dyads suggest that strangers both share territory and establish physical touch markedly less than family members and friends do. These findings implicate social relationships as a key driver of interaction within embodied learning environments. Consideration of social familiarity in the design, implementation, and evaluation of similar collaborative embodied learning designs could enable the development of educational activities that better support people in moving and learning together in ways that welcome felt experience into disciplinary reasoning.

6 Conclusions & Implications

As a design-based research project, this dissertation aimed to contribute to learning theory on means by which informal interaction can foster mathematics learning. My intention is that such theory could inform the development of other designs to support mathematics learners.

In the first paper, my co-authors and I identified and characterized the phenomenon of *participatory facilitation*, a spontaneous pedagogical practice in which adults altered their play in consistent ways to facilitate children's contribution toward alternating pedagogical and performance goals. This close analysis of interpersonal dynamics within play revealed participants' flexible adoption, adaptation, casting-off, and redeployment of scaffolding behaviors. As such, we offered participatory facilitation as a contribution to theoretically oriented literature on in/formal instruction. By implicating design decisions that enabled and shaped participatory facilitation behaviors – namely a time constraint on task completion and the large-scale, multi-sensor user interface – we also suggested the value of this construct for pragmatically oriented literature on educational activity design. Specifically, interfaces that allow for flexible distribution of labor can accommodate both fading and redeployment of scaffolding within parent-child play.

In the second paper, I broadened my focus beyond the game mat itself to evaluate the varied forms of participation – from player to spectator and some in between – that emerged around the *Geometris* exhibit. Guided by theories of learning as participation in communal practice (Lave & Wenger, 1991; Rogoff, 2003), I presented frequency analysis of all 350 visitors as well as thick narrative descriptions of 3 visitor groups to characterize these diverse participation forms and the learning opportunities available within them. This analysis suggested two dimensions as salient for participants' learning as they transitioned across participation forms: (1) varied perspectives on the game task and (2) differential opportunities for action. I also suggested that design elements of scale and distributed input mechanism enabled engagement across participation forms. Implementing user-agnostic interfaces in other large scale educational designs could allow for similarly flexible and hybrid participation forms.

Finally, in the third paper, I zoomed out again, considering how the social relationships participants brought with them to this embodied activity might impact their collaborative work and resultant learning opportunities. Using a framework based on territoriality and touch, I analyzed 41 dyads of strangers, friends, or family members as they played *Geometris*. Strangers both shared territory and established physical touch markedly less than family members and friends, suggesting that maintenance of social norms around touch may distract from task-oriented activity and associated learning opportunities. This finding suggests that collaborative embodied educational designs may be best suited to environments in which learners are close friends or even family members. Consideration of social familiarity in the design, implementation, and evaluation of similar collaborative embodied learning designs could also enable the development of educational activities that better support people in moving and learning together in ways that welcome felt experience into disciplinary reasoning.

Together, these findings inspire a range of future possibilities for *Geometris* and related designs. One design change, informed by the apparent role of time in shaping participatory facilitation behaviors, would be to allow players to adjust the time constraint, deciding whether to take more time for exploration or to embrace the challenge of racing against the clock. The documented interactions within familial play, both parents and children in the first paper and family members more broadly in the third, also suggest the value of an at-home version of *Geometris* compatible with consumer TV displays. The possibilities for extended use in homes, as compared to the relatively brief use in a museum context, also open up other output options. In addition to the current game levels, players could use the mat as a body-scale sketch pad, forming triangles, quadrilaterals, and other polygons with their bodies to compose larger recognizable patterns or images (similar to extended Tangrams) which could then be displayed. Finally, the findings on spectatorship motivate the exploration of other math and STEM concepts in publically observable ways. Some researchers have taken up this mantle in the design of other body+ scale activities (for example, see Hall et al., 2015; Ma, 2017; Soto, 2019). I am inspired to think about ways these interactions could leverage the physical possibilities of playgrounds, community plazas, and other public spaces where gatherings are both normalized and relatively safe given current pandemic concerns.

Geometris began, as so many design-based research projects do, with a “what if?” question: what if children could make shapes using their whole bodies? The research questions and studies presented above emerged in response to this original question. Considering this dissertation as a whole, I propose the following response: when children can make shapes with their bodies, parents and children learn and play together in complex, exciting, and surprising ways; children carefully observe one another’s work and take up fluid roles, both pitching in and stepping back; people work through the very real relationships and contingencies they bring to this activity. In short, diverse forms of participation in geo-spatial reasoning happen.

This project was guided by a desire to leverage children’s expertise at exploring and operating within their physical world as a resource for their mathematical learning. In light of the studies presented in this dissertation, I suggest that *Geometris* accomplished just that. Contrasted with Bill’s experience of learning his multiplication tables by “making [his] mind a blank” (Papert, 1980), participants in this study made use of their competence within their physical and social worlds as resources for math learning. In so doing, their participation underscored the rich, authentic learning opportunities available within educational activities that don’t dismiss physical bodies and social relationships as distractions but instead embrace them as resources. This work underscores the inextricably situated nature of any activity within learners’ physical and social worlds. We, as designers and researchers, must choose if and how to leverage them.

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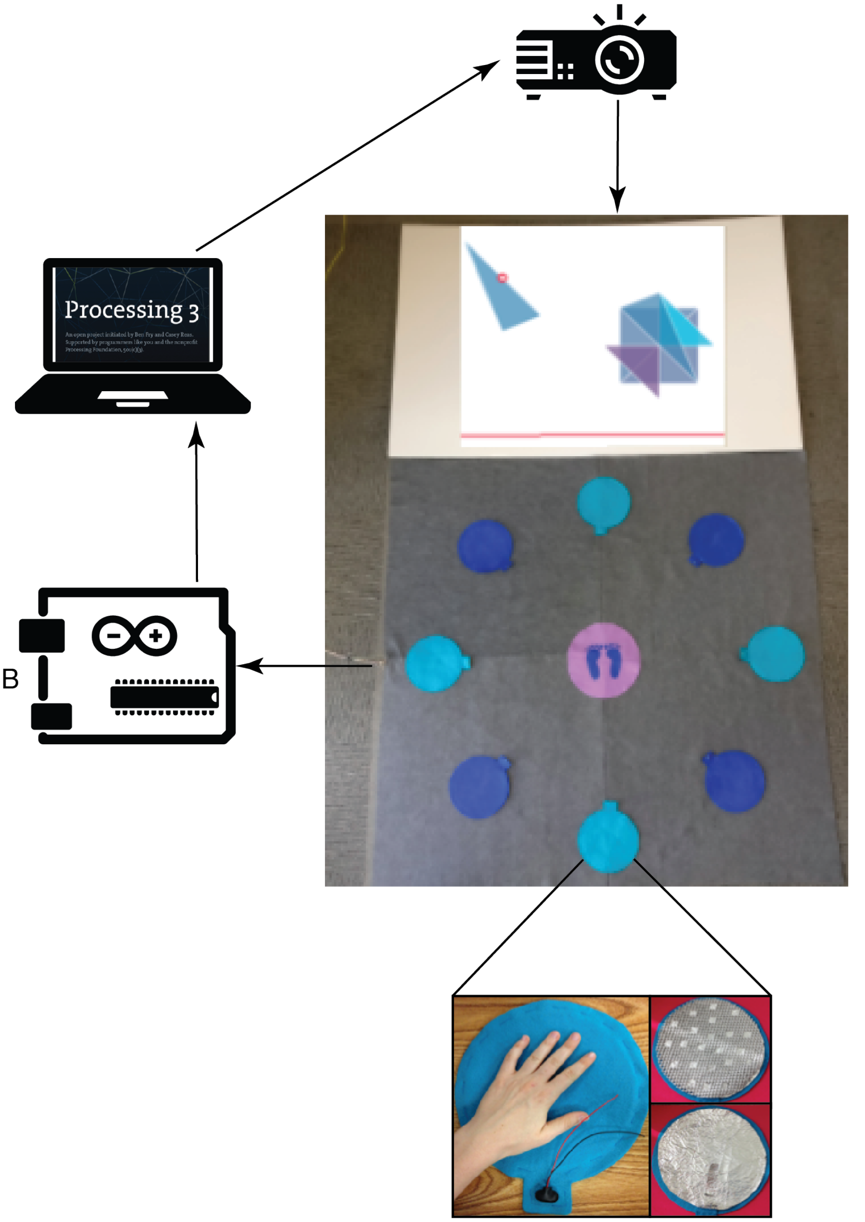
Appendix A – Geometris Schematics



4) A ceiling-mounted projector displays the shapes on the floor screen

3) The Arduino passes touch signals to a computer running Processing 3, which converts the activated sensors into vertices of geometric shapes

2) The 9 sensors (8 radial + 1 center) feed into an Adruino Uno via a custom PCB

1) Custom sensors built from fabric, aluminum foil, and semi-permeable foam detect touch

All hardware and software components were collaboratively designed and implemented by Elena Durán-López, Ganesh V. Iyer, and myself. The above Arduino, laptop, and projector icons were created by uizin, B. Agustín Amenábar Larraín, and Taufik Ramadhan, respectively, of the Noun Project.

Appendix B – Examples of the Five Coordination Modes

In parallel play, players maintain separate territories (Figure B1a). In overlapping play, player’s territories overlap (Figure B1b). Players touch when simultaneously activating the same pad (Figure B1c, top) or when intentionally touching their partner’s body (Figure B1c, bottom). In entangled play, players reach, crouch, step, or stretch within each other’s personal space (Figure B1d). Players mold each other by physically manipulating a part or parts of their partner’s body (Figure B1e).

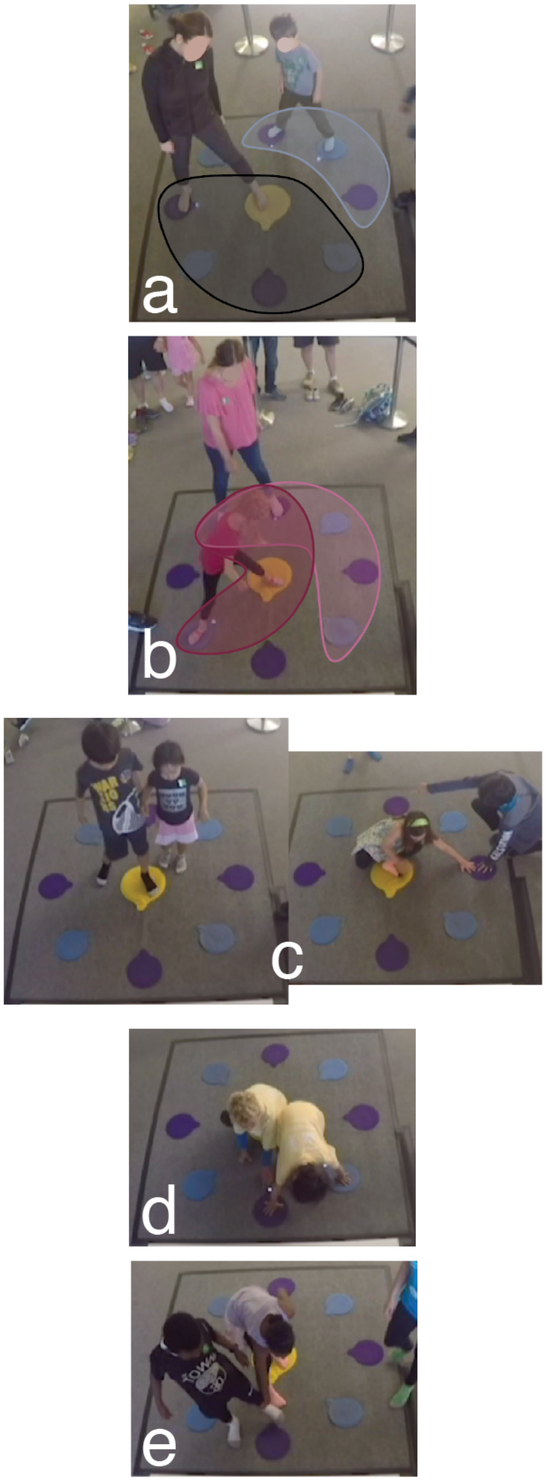


Figure B1. Examples of the five coordination modes. In images a and b, shaded areas indicate each player’s respective territory.

Appendix C – Applying the Coding Scheme to the Video Data

Recall that the coding scheme consists of 5 codes:

1. Parallel play
2. Overlapping play
3. Touch
4. Entangled Play
5. Molding

In order to determine whether players were engaged in parallel or overlapping play, the game mat was divided into 9 regions based on the sensors, 1 region for each of the 8 radial sensors plus 1 region for the center sensor (Figure B1). For each player in the dyad, a 9-element array was constructed to represent their territory. Once a player activated a sensor, that sensor and its corresponding region was considered part of their territory for the remainder of the level. Thus, territories are cumulative and reflect all the sensors a player has touched to that point in the level. Players' territories were reset at the start of each new level of play.

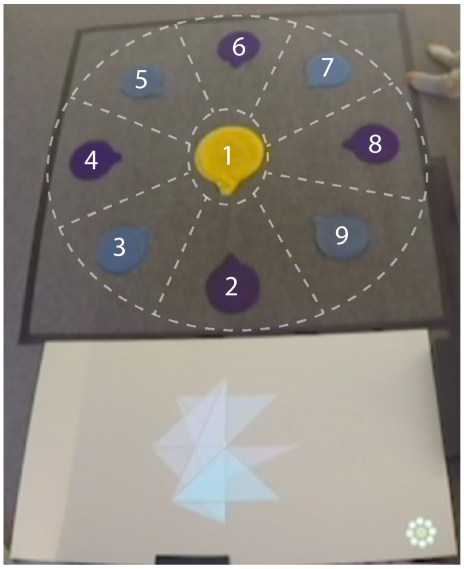


Figure C1. To code participants' locations, the play mat was divided into 9 regions corresponding to the 9 sensor pads.

Codes of “parallel play” and “overlapping play” were determined by comparing the two player's territories for each second of play. If, for a given second, the players' territories were mutually exclusive, that second was coded as “parallel play.” If there was overlap in players' territories, that second was coded as “overlapping play.” Additionally, percent overlap was calculated by dividing the number of shared sensors across players' territories by 9 (the total number of sensors).

The touch, entangled play, and molding codes were applied through inspection of the video data. If players physically touched or simultaneously occupied the same sensor pad on the mat, that second was coded as “touch.” If they overlapped limbs or occupied each other's personal space, that second was coded as “entangled play.” Finally, if a player not only touched but also nudged, pushed, lifted, or otherwise moved part of their partner's body, that second was coded as “molding.”