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Pulsed Radiation by a Phased Semi-Infinite Periodic Planar Array of Dipoles

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I. Introduction

To gain an understanding of the sparsely explored time domain (TD) behavior of periodic arrays of radiating or scattering elements (phased array antennas, frequency selective surfaces and related applications), we have initiated a systematic investigation of relevant canonical TD dipole-excited Green's functions (GF), which so far include those for infinite and truncated line periodic arrays [1], [2], as well as for infinite planar periodic arrays [3]. Such Green's functions have been parameterized in terms of TD-Floquet waves (FW) of cylindrical [1] and planar type [3], and truncation-induced TD-FW-modulated diffractions [2].

Such waves on semi-infinite and finite square arrays of dipoles have been investigated in the frequency domain (FD) [4], and shown to be useful in practical array applications [5]. The present contribution extends our TD studies to an infinite periodic sequentially pulsed *semi-infinite planar* array. The phenomenology associated with truncated TD-FW and truncation-induced diffraction is explained in terms of instantaneous frequencies aided by asymptotic parameterization. Preliminary numerical results demonstrate the efficiency of the TD-FW algorithms.

II. Statement of the Problem

The geometry of the semi-infinite planar array of dipoles oriented along the $\mathbf{J}_0$ direction and excited by transient currents in free space is shown in Fig.1a. The period of the array is d_x and d_z in the x and z directions, respectively. The $\mathbf{E}$ field component is simply related to the $\mathbf{J}_0$-directed magnetic scalar potential A which shall be used throughout. A caret ^ tags time-dependent quantities; bold face symbols define vector quantities; $\boldsymbol{i}_x$, $\boldsymbol{i}_y$ and $\boldsymbol{i}_z$ denote unit vectors along x, y, and z, respectively. FD and TD quantities are related by the Fourier transform pair $A(\omega)=\int_{-\infty}^{\infty}\hat{A}(t)e^{-j\omega t}dt$, $\hat{A}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)e^{j\omega t}d\omega$. The phased array FD and TD dipole currents $J(\omega)$ and $\hat{J}(t)$, respectively, are given by

$$\left.\begin{array}{c} J(\omega) \\ \hat{J}(t) \end{array}\right\} = \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\delta(x'-nd_x)\delta(z'-md_z)\left\{\begin{array}{c} e^{-j\omega(\eta_x x'+\eta_z z')/c} \\ \delta(t-(\eta_x x'+\eta_z z')/c) \end{array}\right\} \qquad (1)$$

In the m,n-dependent element current amplitudes multiplying the delta function in (1) the FD portions $\omega\eta_x x'/c$ and $\omega\eta_z z'/c$ account for an assumed (linear) phase difference between adjacent elements in the x and z directions, respectively, and η_x/c and η_z/c denote interelement phase gradients normalized with respect to ω. The TD portion identifies sequentially pulsed dipole elements, with the element at $(x',z')=(nd_x,md_z)$ turned on at time $t_{mn}=(\eta_x nd_x+\eta_z md_z)/c$.

III. FD and TD Floquet Waves for the Infinite Array

A):**Frequency Domain FW.** Applying the infinite Poisson summation formula to the doubly-infinite sum over the radiation by each dipole we obtain the total field expressed as $A^{tot}(\mathbf{r},\omega)=\sum_{p,q=-\infty}^{\infty}A_{pq}^{FW}$ where the FW $A_{pq}^{FW}(\mathbf{r},\omega)=e^{-j\mathbf{k}_{pq}^{FW}\cdot\mathbf{r}}/(2jd_1d_2k_{ypq})$. Here, $\mathbf{k}_{pq}^{FW}=k_{xp}\boldsymbol{i}_x+k_{ypq}\boldsymbol{i}_y+k_{zq}\boldsymbol{i}_z$ denotes the total FW_{pq} propagation vector, and $\mathbf{r}=x\boldsymbol{i}_x+y\boldsymbol{i}_y+z\boldsymbol{i}_z$ the distance to the observer. The spectral wavenumbers

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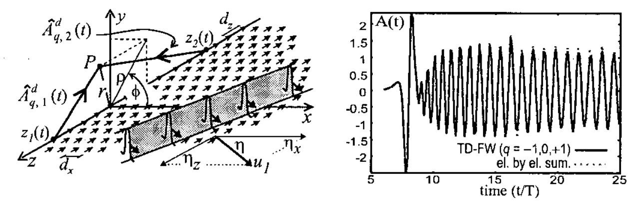

Fig. 1. a) Semi-infinite periodic planar array of electric dipoles. u_l points in the direction of the phasing η/c with $\eta = (\eta_x^2 + \eta_z^2)^{1/2}$. b) Radiated fields. Parameters in Sec.V.

$$k_{xp} = \omega\eta_x/c + \alpha_p, \quad \alpha_p = 2\pi p/d_x, \qquad k_{zq} = \omega\eta_z/c + \alpha_q, \quad \alpha_q = 2\pi q/d_z \tag{2}$$

characterize FW propagation along x and z, respectively, and $k_{ypq}(\omega) = (k^2 - k_{xp}^2 - k_{zq}^2)^{-1/2}$, where $k = \omega/c$, with k the ambient wavenumber, c the ambient wave speed, and $\Im m\, k_{ypq} \leq 0$ on the top Riemann sheet. Floquet waves with transverse propagation constants $k_{t,pq} < k$ or $k_{t,pq} > k$, $k_{t,pq} = (k_{xp}^2 + k_{zq}^2)^{-1/2}$, are propagating or evanescent, respectively, in the y-direction.

B):**Time Domain FW.** The inversion of the FD-FW is written as $\hat{A}_{pq}^{FW}(\mathbf{r},t) = \int_{-\infty}^{\infty} F(\omega)\exp(-j\hat{\psi}(\omega))d\omega$ in which $F(\omega)$ accounts for the slowly varying amplitude terms and $\hat{\psi}(\omega) = k_{xp}x + k_{ypq}y + k_{zq}z - \omega t$, with k_{xp}, k_{ypq} and k_{zq} functions of ω, accounts for all the ω phase terms in the exponent. For $p = q = 0$, the phase is linearly dependent on ω and the inverse Fourier transform is evaluated as $\hat{A}_{00}^{FW}(\mathbf{r},t) = (2d_xd_z\sqrt{1-\eta^2})^{-1}cU(t-t_0)$, in which $\eta = \sqrt{\eta_x^2+\eta_z^2}$; $U(\tau) = 0$ or 1 for $\tau < 0$ or $\tau > 0$, respectively, is the Heaviside step function; and $t_0 = (\eta_x x + \eta_z z + \sqrt{1-\eta^2}y)/c$ is the turn on time [3]. For $p \neq 0$ or $q \neq 0$, the phase $\hat{\psi}(\omega)$ contributes to the inverse Fourier integral through the asymptotic local frequencies $\omega_{pq}(\mathbf{r},t)$ which satisfy the saddle point condition $(d\hat{\psi}/d\omega)_{\omega_{pq}} = 0$, and parameterize the TD-FW wave dynamics. The solutions [3]

$$\omega_{pq,i}(\mathbf{r},t) = \bar{\omega}_{pq} + \frac{(-1)^i\tilde{\omega}_{pq}\tau}{\sqrt{\tau^2-\tau_0^2}}, \quad i = 1,2, \quad \tau = t - (\eta_x x + \eta_z z)/c, \quad \tau_0 = \sqrt{1-\eta^2}y/c \tag{3}$$

with $\bar{\omega}_{pq} = c(\eta_x\alpha_p + \eta_z\alpha_q)/(1-\eta^2)$, $\tilde{\omega}_{pq} = [\bar{\omega}_{pq}^2 + \alpha_{pq}^2c^2/(1-\eta^2)]^{1/2}$, are real in the causal domain $t > t_0 = (\eta_x x + \eta_z z)/c + \tau_0$ $(\tau > \tau_0)$. Standard asymptotic evaluation of the pqth TD-FW ω-integral leads to [3] as

$$\hat{A}_{pq,i}^{FW}(\mathbf{r},t) \sim \frac{ce^{-j(\alpha_p x+\alpha_q z)}e^{-j(-i)^i\pi/4}}{d_1d_2\sqrt{2\pi(1-\eta^2)}} \frac{e^{j(\bar{\omega}_{pq}\tau+(-1)^i\tilde{\omega}_{pq}\sqrt{\tau^2-\tau_0^2})}}{\sqrt{\tilde{\omega}_{pq}}(\tau^2-\tau_0^2)^{1/4}}U(\tau-\tau_0) \tag{4}$$

The unit step function $U(\tau-\tau_0) = U(t-t_0)$ arises because real saddle point frequencies $\omega_{pq,i}$ are restricted to $\tau > \tau_0$ $(t > t_0)$.

IV. FD and TD Floquet Waves for the Semi-Infinite Array

A):**Truncation-Induced FD Diffracted Fields.** As shown in [4], truncated FD-FW expressions are obtained by deformation of the relevant spectral integration path, followed by uniform asymptotics,

$$A^{tot}(\mathbf{r},\omega) = \sum_{p,q} A_{pq}^{FW}(\mathbf{r},\omega)U(\phi_{pq}^{SB}(\omega) - \phi) + \sum_q A_q^d(\mathbf{r},\omega) \tag{5}$$

$$A_q^d(\mathbf{r},\omega) \sim \frac{\sqrt{j}e^{-j(k_{\rho q}\rho+k_{zq}z)}}{2d_z\sqrt{2\pi k_{\rho q}\rho}}\left(B(\omega)+\sum_{p=-P}^{P}\frac{F(\delta_{pq}^2)-1}{jd_xk_{\rho q}(\cos\phi_{pq}-\cos\phi)}\right) \tag{6}$$

where $A_{pq}^{FW}(\mathbf{r},\omega)$ are the FWs, and A_q^d are the q-th cylindrical diffracted fields due to the truncation. The radial wavenumber $k_{\rho q}=\sqrt{k^2-k_{zq}^2}$ with $\Im m k_{\rho q}<0$ implies radial exponential decay when $|k|<|k_{zq}|$ so that only a few A_q^d terms in (5) are necessary away from the truncation. $\phi_{pq}^{SB}(\omega)$ is the shadow boundary of the truncated FW_{pq} which, for propagating FWs, coincides with the FW_{pq} propagation angle $\phi_{pq}(\omega)=\cos^{-1}(k_{xp}/k_{\rho q})$; $B(\omega)=\{1-\exp[jd_x(k_{\rho q}(\omega)\cos\phi-\omega\eta_x/c)]\}^{-1}$; $F(x)$ is the transition function of the Uniform Theory of Diffraction (UTD)[4], with argument $\delta_{pq}(\omega)=\sqrt{2k_{\rho q}(\omega)\rho}\sin((\phi-\phi_{pq}(\omega))/2)$ and $\phi=\cos^{-1}(x/\rho)$ is the observation angle measured from the truncation of the array, see Fig.1. Every FW_q in (5) is the same as that for the infinite array, except that its domain of existence is the region $\phi<\phi_{pq}^{SB}$.

B): **Truncation-Induced TD Diffracted Fields.** To parameterize the truncated FW phenomenologies in terms of instantaneous frequencies and wavenumbers, we access the time domain through Fourier inversion of $A^{tot}(\mathbf{r},\omega)$ in (5) which has been obtained by high-frequency asymptotics This restricts the validity of the truncated TD solution to *early observation times* near the wavefronts. Each of the $A_{pq}^{FW}(\mathbf{r},\omega)$ and its corresponding diffracted field $A_q^d(\mathbf{r},\omega)$ has a particular arrival time, near which the FD asymptotics will be most accurate. Again we need to distinguish between the dispersive $q\neq0$ and the nondispersive $q=0$ cases. The asymptotic inversion *procedure* is analogous to that in [2], but differs in detail. We shall only list the results. The "quasi nondispersive" $q=0$ ($\alpha_q=0$) term is decomposed based on the relations $k_{\rho0}(\omega)=\omega/c(1-\eta_z^2)^{1/2}$ and $B(\omega)=1/2+j\sum_{p=-\infty}^{\infty}[\omega/c(1-\eta_z^2)^{1/2}(\cos\phi-\cos\phi_{00})-\alpha_p]^{-1}$. Therefore, Fourier inversion for the terms in (6) corresponding to the $p=0$ contributions can be done in closed form,

$$\hat{A}_0^d(\mathbf{r},t)=\frac{1}{2\pi d_z\sqrt{2\tau_d}}\left[\frac{1}{\sqrt{t-t_d}}+\frac{2c\sqrt{t_d-t_0}\sin^{-1}\left(\frac{\sqrt{t-t_d}}{\sqrt{t-t_0}}\right)}{d_x\sqrt{(1-\eta_z^2)}(\cos\phi-\cos\phi_{00})}\right]U(t-t_d) \tag{7}$$

with $\tau_d=\sqrt{1-\eta_z^2}\rho/c$ and $t_d=\eta_z z/c+\tau_d$. All the other p-terms (with $q=0$) yield truncated TD-FW via moving shadow boundaries $\phi_{pq}^{SB}(\omega_{pq,i}(t))$ and the diffracted fields can be approximated in the neighborhood of $\phi_{p0}^{SB}(\omega_{p0,i}(t))$ as in [2] for the truncated line array. For $q\neq0$, TD inversion from the high-frequency result in (6) is based on the stationary (saddle) points ω_q, defined by $(d\widehat{\psi}^d/d\omega)|_{\omega_q^d}=0$, [1] of the composite phase $\widehat{\psi}^d(\omega)=k_{\rho q}\rho+k_{zq}z-\omega t$,

$$\omega_{q,i}^d(\mathbf{r},t)=\frac{c}{1-\eta_z^2}\left(\eta_z\alpha_q+\frac{(-1)^i\sqrt{\alpha_q^2}\tau'}{\sqrt{\tau'^2-\tau_d^2}}\right),\quad i=1,2,\quad \tau'=t-\eta_z z/c,\quad \tau_d=\sqrt{1-\eta_z^2}\rho/c \tag{8}$$

which are real in the diffracted-field causal domain $\tau'>\tau_d$ ($t>t_d$). The two solutions in (8) identify the *local instantaneous* frequencies of oscillation of the q-th diffracted wave at a given point $\mathbf{r}$ and a given instant τ'. The corresponding instantaneous wavenumbers $k_{q,i}(t)=\omega_{q,i}^d(t)/c$, $k_{zq,i}(t)=\eta_z\omega_{q,i}^d(t)/c+\alpha_q$, and $k_{\rho q,i}(t)=(k_{q,i}^2(t)-k_{zq,i}^2(t))^{-1/2}$ are all real for $t>t_d$, and $k_{\rho q,i}(t)\to0$ for $t\to\infty$. Standard asymptotics leads to

$$\widehat{A}_{q,i}^{d,e}(\mathbf{r},t)\sim\frac{e^{-j\alpha_q z_i'(t)}U(\tau'-\tau_d)}{4\pi d_z\sqrt{\tau'^2-\tau_d^2}}\left[B(\omega_{q,i}^d(t))+\sum_p\frac{F[\delta_{pq}^2(\omega_{q,i}^d(t))]-1}{jd_xk_{\rho q,i}(t)[\cos\phi_{pq}(\omega_{q,i}^d(t))-\cos\phi]}\right] \tag{9}$$

$i = 1, 2$. It can be shown that $\omega_q^{d,cutoff} \equiv \omega_q^d(t \to \infty) \leq \omega_{pq,i}^{cutoff} \equiv \omega_{pq,i}(t \to \infty)$, and thus, since $t_d \geq t_0$, at a certain time $t = t_{pq}^{SB} \geq t_d$, the q-th diffracted and pq-th FW local instantaneous frequencies are equal, i.e., $\omega_q^d(t_{pq}^{SB}) = \omega_{pq}(t_{pq}^{SB})$. Furthermore, it can be shown that $\cos\phi_{pq}(\omega_{pq}(t_{pq}^{SB})) = \cos\phi$, which means that at $t = t_{pq}^{SB}$ the pq-th moving SB intercepts the stationary observer at $\mathbf{r}$; there the q-th TD diffracted field has a transitional behavior that compensates for the truncation of the pq-th TD-FW, and restores total field continuity.

V. Band Limited Pulse Excitation

When each dipole in (1) radiates a practically useful band-limited (BL) pulse $\hat{G}[t-(\eta_x x+\eta_z z)/c]$, the corresponding band-limited TD-FW $\hat{A}_{pq}^{FW,BL}$ for p or $q \neq 0$ and TD-diffracted field $\hat{A}_q^{d,BL}$ for $q \neq 0$, can be evaluated by including the pulse spectrum $G(\omega)$ in the impulsive inversion integral. For wideband (short duration) pulses, $G(\omega)$ can be considered slowly varying with respect to the phases $\hat{\psi}(\omega)$ and $\hat{\psi}^d(\omega)$, and can therefore be approximated by its value at the saddle point frequencies $\omega_{pq,i}(t)$, and $\omega_{q,i}^d(t)$ for FWs and diffracted fields, respectively. The asymptotic BL-TD fields $\hat{A}_{pq,i}^{FW,BL}$ and $\hat{A}_{q,i}^{d,BL}$ are found by multiplying the ordinary asymptotic $\hat{A}_{pq,i}^{FW}$ and $\hat{A}_{q,i}^d$ by $G(\omega_{pq,i})$ and $G(\omega_{pq,i}^d)$, respectively. The FD-FW$_{00}$ and $q = 0$ diffracted field are not inverteble by ω-asymptotics and are calculated by convolving $\hat{G}(t)$ with the TD-FW $\hat{A}_{00}^{FW}(t)$ and $\hat{A}_0^d(t)$ in (7), respectively. Preliminary numerical experiments have been carried out to test the accuracy of the asymptotic solutions for $\hat{A}_{pq}^{FW,BL}$, and to compare the results with a reference solution obtained by an element-by-element summation over the pulsed BL radiation from all dipoles, i.e., $\hat{G}[t-(\eta_x x+\eta_z z)-R_{mn})/c]/[4\pi R_{mn})]^{-1}$, with $R_{mn} = ((x-nd_x)^2+(z-md_z)^2+y^2)^{1/2}$, $m, n = 0, \pm1, \pm2, ...$ The mn-series has been truncated at $|m|, |n| < 130$, comprising all non-negligible element radiations. Figure 1b shows plots for a semi-infinite planar array with interelement phasing $\eta_x = \eta_z = 0$ (broadside radiation) and interelement spacing $d_z = 10d_x$ in order to highlight the new phenomena (with respect to the truncation in [2]) due to the z-dispersion relation for diffracted fields (all fields with $p \neq 0$ are negligible). BL excitation: normalized Rayleigh pulse $\hat{G}(t) = \Re e[j/(j + \omega_M t/4)^5]$ (i.e., $\hat{G}(0) = 1$); $G(\omega) = \pi(6\omega_M)^{-1}(j4\omega/\omega_M)^4\exp(-4|\omega|/\omega_M)$; central radian frequency $\omega_M = 2\pi c/d_z$ ($\lambda_M \equiv 2\pi c/\omega_M = d_z$). Observer location: $(x, y, z) = (-d_z/2, 8d_z, 0)$ so that $\phi = 93.6°$, and since $\phi_{0q} = 90°$ diffracted fields are in transitional behavior. Since $U(\phi_{0q} - \phi) = 0$, no FW$_{0q}$ are present. Fields $\hat{A}(t)$ are plotted versus normalized time t/T, with $T = d_z/c$. The almost identical turn-on times $t_0/T = 8$ and $t_d/T = 8.02$ also indicates that diffracted fields are in transitional behavior. The included asymptotic terms $p = 0$ and $|q| \leq 1$ (solid curve), suffice to give good agreement with the reference solution (dotted curve), demonstrating good convergence of the TD-FW representation.

References

[1] L. B. Felsen and F. Capolino, "Time domain Green's function for an infinite sequentially excited periodic line array of dipoles," *IEEE Trans. Ant. Propagat.*, vol. 48, pp. 921–931, June 2000.

[2] F. Capolino, and L. B. Felsen, "Time domain Green's function for a phased semi-infinite periodic line array of dipoles," in *IEEE AP-S/URSI Symp.*, Orlando, FL, July 11-16 1999.

[3] F. Capolino and L. B. Felsen, "Ttime domain green's function for an infinite sequentially excited periodic planar array of dipoles," in *IEEE AP-S Symp.*, Salt Lake City, Utah, July 16-21 2000.

[4] F. Capolino, M. Albani, S. Maci, and L.B. Felsen, "Frequency domain Green's function for a planar periodic semi-infinite dipole array," *IEEE Trans. Ant. Propagat.*, vol. 48, pp. 67–74, Jan. 2000.

[5] A. Neto, S. Maci, G. Vecchi, M. Sabbadini, "Truncated Floquet wave diffraction method for the full wave analysis of large phased arrays," *IEEE Trans. Ant. Propagat.*, vol.48, n.3, March 2000.