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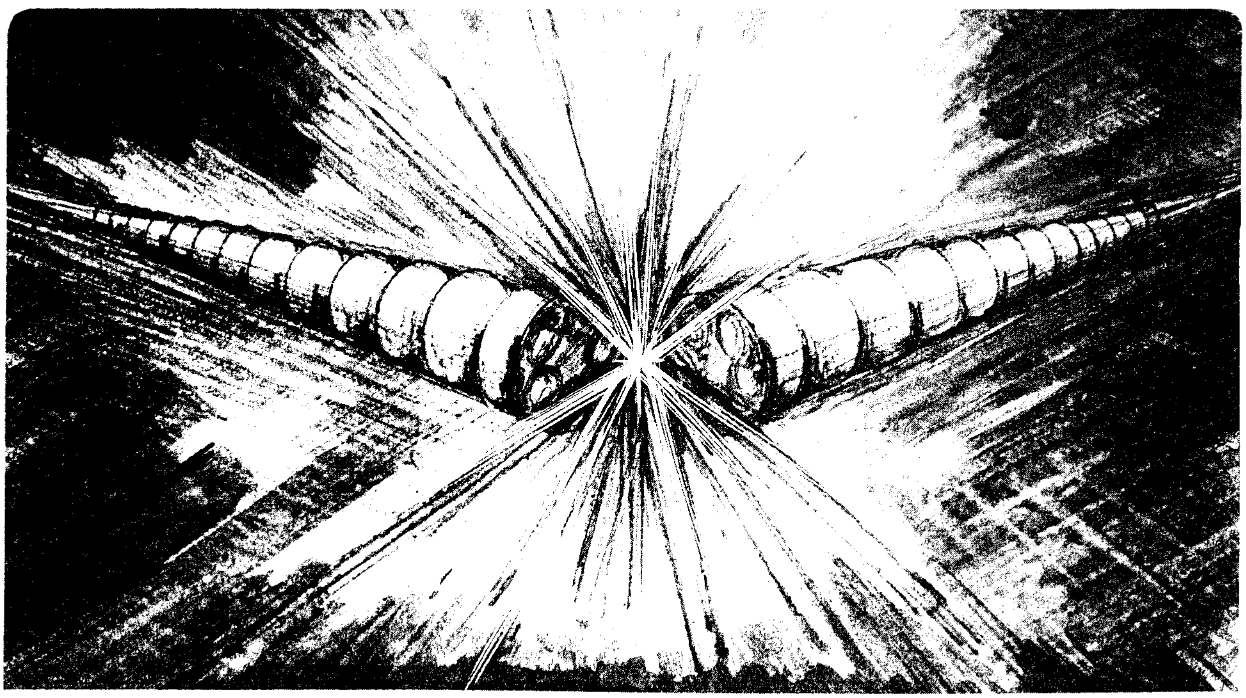

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Theoretical Examination of Transfer Cavities in a Standing-wave Free-electron Laser Two-beam Accelerator

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Theoretical examination of transfer cavities in a standing-wave free-electron laser two-beam accelerator

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ABSTRACT

Recent analysis of the Two-Beam Accelerator (TBA) by Wurtele, Whittum and Sessler[1] has shown that the transfer cavities, both in the relativistic klystron version (RK/TBA) and the standing-wave free-electron laser version (SWFEL/TBA), can be characterized by a simple coupling impedance. In the two cases the radiation process is very similar: only the modes that couple to the electron beam are different. As a result, computer programs that are able to handle realistic cavities (with beam ports and coupling ports, etc.) can be employed to evaluate the performance of either version of the TBA. We have employed the code URMEL[2] to study the proper coupling impedance for a number of realistic cavities for a SWFEL.

1. INTRODUCTION

A Two-Beam Accelerator[3,4,5] is a high-gradient device for accelerating particles to very high energies. The power for the high-gradient structure is extracted from a high-current low-energy drive beam, through interaction with a relativistic klystron (RK) or a standing-wave free-electron laser (SWFEL). The energy of the drive beam is replenished by means of re-acceleration units placed periodically. In the SWFEL version, power is extracted through a series of uncoupled cavities operating at the desired frequency. Uncoupled cavities are used to alleviate the problem of rf power extraction and to avoid slippage of the electron phase with respect to the rf phase, a problem which is encountered when using long

FELs. Since the rf power does not propagate between the cavities, the only link between the cavities is provided by the energy and phase of the electrons.

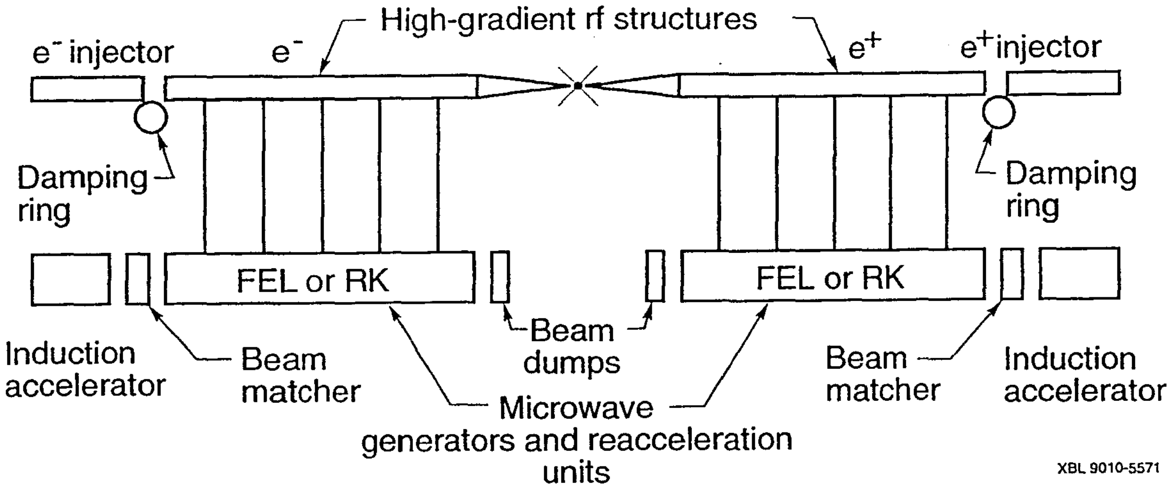

Fig. 1. A schematic of the structure of a TBA

Wurtele, Whittum and Sessler[1] have developed a formalism which enables the study of both the RK/TBA and the SWFEL/TBA in a parallel way by introducing a "coupling impedance" for both the RK and the SWFEL. The coupling impedance can include realistic cavity features, such as beam ports, in a simple manner. Section 2 provides a brief summary of their results. Section 3 gives the coupling impedance for several realistic cavities evaluated using the code URMEL.

2. FORMALISM

In an FEL, the electron beam couples to a transverse electric (TE) mode through its transverse velocity caused by the wiggler magnets. In a relativistic klystron, the beam couples to the axial electric field of a transverse magnetic (TM) mode through the axial current. The coupling in each case is determined by the phase $\psi = \varphi + \theta$ where θ is the particle phase, φ the wave phase and ψ is the relative phase between electrons and the fields in the cavity. If the wave is traveling in the z direction, then for an FEL, $\theta = (k_w + k_z)z - \omega t$, where k_w is the wiggler wave number and k_z the axial wave number of the cavity mode. For a steady state klystron, $\theta = k_z z - \omega t - \theta_r$, where θ_r is a reference particle's phase.

For either an RK or a SWFEL, we may proceed by observing that in Lorentz gauge, the vector potential in a cavity can be decomposed into,

$$\vec{A} = \frac{mc^2}{e} \sum_{\alpha} q_{\alpha}(t) \vec{a}_{\alpha}(\vec{r}), \tag{1}$$

where m is the electron mass, c is the speed of light, $-e$ is the electron charge, and α is the mode index. The quantity q_α is the dimensionless mode amplitude and $\vec{a}_\alpha$ describes the spatial dependence of the mode with the normalization

$$\int_V d^3r' \, \vec{a}_\alpha(\vec{r}\,) \bullet \vec{a}^*_\alpha(\vec{r}\,) = V \quad , \tag{2}$$

where V is the cavity volume.

For a mode with frequency ω, the mode amplitude q(t) can be approximated by $\boldsymbol{R}\left\{\mathrm{b}\,\mathrm{e}^{i\varphi}\,\mathrm{e}^{i\omega t}\right\}$. Then the field equation is

$$\frac{\partial}{\partial s} b e^{i\varphi} = ic \frac{1}{\eta} \frac{r}{Q} \frac{I}{I_A} \left\langle e^{-i\theta} \right\rangle, \tag{3}$$

where $s = v_z t - z$, with v_z the beam velocity, I the average beam current, $I_A = mc^3/e \sim$ 17kA, and the brackets indicating an average over a beam slice. The factor η depends on the kind of coupling. For an RK η=2, while for an FEL, $\eta = a_w / 2\gamma$ with γ the Lorentz factor, and a_w the wiggler parameter. The shunt impedance per unit length r is given by [6]

$$\frac{r}{Q} = \frac{4\pi}{VL\omega} \left| \int_{-L/2}^{+L/2} dz \frac{\vec{v}(z)}{v_z} \bullet \vec{a}(z) \exp\left(-\frac{i\omega z}{v_z}\right) \right|^2, \tag{4}$$

where Q is the quality factor and L is the cavity length.

Particle equations linearized about the reference particle are

$$\frac{d\theta}{dz} \approx 2\,\kappa \frac{\delta\gamma}{\gamma_r}\ , \tag{5}$$

$$\frac{d\delta\gamma}{dz} = -\eta \frac{\omega}{c} b \sin(\theta + \varphi) - \frac{eE_z}{mc^2}\ , \tag{6}$$

where $\delta\gamma = \gamma - \gamma_r$, where γ_r is the resonant γ in the case of the FEL, or in the case of an RK , a reference γ. The constant $\kappa = \omega\left(1 + a_w^2 / 2\right) / 2c\gamma^2$ for an FEL, and $\kappa = \omega / 2c\gamma^2$ for an RK. Equations (3), (5), and (6) describe the self-consistent evolution of the beam and the cavity fields in either a SWFEL or an RK. The two cases are distinguished only through the values of η, κ and r/Q.

3. COUPLING IMPEDANCE

Since the radiation field growth is proportional to r/Q (equation 3), we can improve the performance of an FEL by increasing r/Q over an ideal cavity. Consider a SWFEL/TBA with cylindrical rf cavities operating in a TE mode. Although Whittum et. al. have examined rectangular cavities, we choose cylindrical cavity shape for our study because they are simpler to analyze numerically. In an ideal right-cylindrical circular cavity, r/Q for TE_{11p} mode is given by,

$$\frac{r}{Q} = Z_0 \left(\frac{a_w}{\gamma}\right)^2 \frac{(\omega / c)}{32\pi x'^2_{11}[J_1^2(x'_{11}) - J_0^2(x'_{11})]}\ , \tag{7}$$

where J_m represents a Bessel function of m-th order and x'_{mn} is the n-th root of J'_m . For a frequency of 11.424 GHz and a_w / γ=.1013, this evaluates to 11.4 ohms/m.

For cylindrically symmetric geometry, we employ the 2-D code URMEL, which allows for calculation of asymmetric modes, such as dipole modes. We study the coupling impedance of $TE_{1,1,20}$ mode with a frequency of 11.424 GHz. Figure 2 shows a schematic of the structure which is repeated throughout the length of the SWFEL.

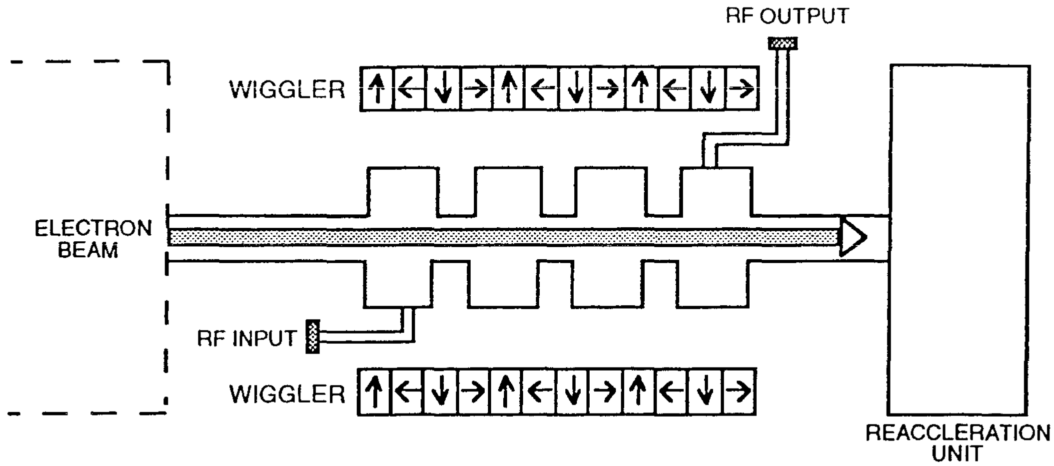

Fig. 2. Diagram of one period a SWFEL/TBA

As a realistic example, we consider a four-cell cavity. The total length of the cavity is a few wiggler wavelengths. To build a standing wave in the structure, the beam port is chosen to have a cutoff frequency higher than 11.424 GHz. To avoid excessive coupling between the cells, the thickness of the discs separating the cells is chosen to be between one and two e-folding lengths. Given these dimensions, the cell length is determined appropriately. FEL interaction leads to significant transverse oscillations, which is not the case in RK interaction. Assuming a 3 mm radius beam, with $a_w / \gamma = 0.067$, we find the transverse oscillations to be about 1.2 mm. Figure 3 shows a schematic of a typical cavity studied.

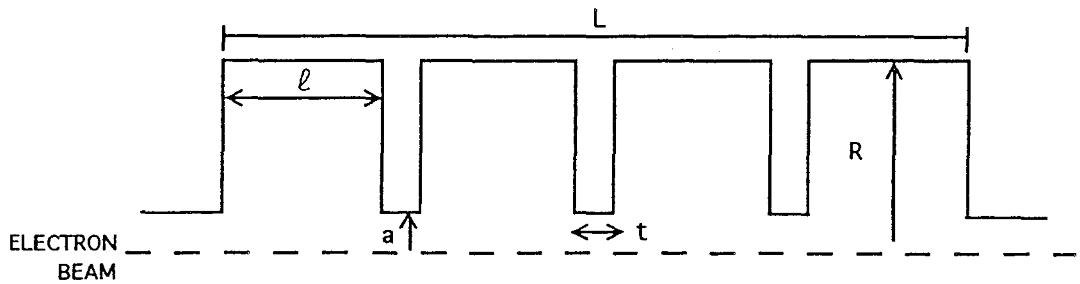

Fig. 3. The shape of a multi-cell cylindrical cavity (not drawn to scale)

Using URMEL's calculation of radial electric fields, we calculate the coupling impedance per unit length. Starting from an ideal pill-box cavity of 31 cm length and 1.5 cm radius, first we put in discs and beam ports in a simple-minded manner without any optimization. Increasing the radius to obtain the same frequency, we find a 25% reduction in r/Q. However, if we fix the cavity radius and increase the length to obtain the right frequency, we find about a 45% increase in r/Q.

As described earlier, r/Q depends on the inner product of the transverse velocity of the electrons and the radial electric field. The transverse velocity of electrons is given by

$$\mathbf{v}_w = c\left(\frac{a_w}{\gamma}\right)\sin(k_w z) \ . \tag{8}$$

In a disc-loaded cavity, the fields are stronger in magnitude near the discs and weaker in the center of the cells. If we use 2 wiggler periods per 4-cell cavity, we find the transverse velocity is high where the fields are low (in the middle of the cells) and low where the fields are high (near the discs). In this case, we are not taking advantage of the geometry of the cavity to optimize the coupling. Moreover, the longitudinal wavelength of the electromagnetic field is shorter near the center of the cells and longer around the discs and ends of the cavity.

For proper lasing, the resonance condition,

$$v_z = \omega/(k_z + k_w) \ , \tag{9}$$

has to be satisfied. In an ideal pill-box cavity, k_z can be easily determined to be $2\pi[L/10]^{-1}$. However, in a realistic case, due to the varying longitudinal wavelength and the fringe fields near the discs and the cavity ends, this wave number is not well defined. So the resonance condition has meaning only in the sense that the proper k_z is the one which maximizes the wave-particle interaction. In addition, the wiggler phase has to be properly arranged to maximize the coupling between electron phase and the rf phase.

Keeping this in mind, we vary the wiggler period and β_z to obtain the maximum interaction. Table 1 gives a summary of the results obtained. The wiggler is assumed to be centered at the middle of the cavity. We optimize r/Q for both "sine" and "cosine" wiggler. A sine (or cosine) wiggler is one that varies as a sine (or cosine) at the center of the cavity. The difference in the maximum r/Q obtained using sine and cosine wigglers is about 10%, with cosine wigglers giving the higher r/Q in most cases. We find that both the wiggler period and the wiggler phase make a significant difference in the strength of the coupling. Although here we optimize the cavities for either a sine or cosine wiggler, further improvement in the coupling impedance can be made by choosing a proper phase (not necessarily 0 or π). In order to improve the electron interaction with rf fields, we try varying the ratio t/ℓ while keeping the cavity length L roughly constant. We find that for L ~ 30 cm the optimum ratio is about 14%.

Cavity Radius R cm	Cavity Length L cm	Cell Length cm	Aperture Radius a cm	Thickness of disc cm	Beta z	aw/gamma	z #	w #	r/Q ohms/m
CLOSED CAVITIES									
1.50	31.0				0.9923	0.1241	9.33	2.45	12.62
1.80	29.0				0.9955	0.0948	9.35	1.75	6.75
DEPENDENCE ON THE RATIO thickness/cell length									
1.80	30.1	7.00	0.65	0.7	0.9918	0.1279	9.22	2.45	8.98
1.80	30.8	7.10	0.65	0.8	0.9917	0.1326	9.22	2.60	9.54
1.80	31.0	7.00	0.65	1.0	0.9899	0.1419	9.20	2.80	9.80
1.80	31.1	6.95	0.65	1.1	0.9892	0.1465	9.19	2.90	9.39
1.80	31.2	6.90	0.65	1.2	0.9933	0.1158	9.82	2.30	7.55
DEPENDENCE ON APERTURE RADIUS a									
1.80	31.0	7.00	0.60	1.0	0.9926	0.1213	9.18	2.85	6.82
1.80	31.0	7.00	0.65	1.0	0.9899	0.1419	9.20	2.80	9.80
1.80	31.0	7.00	0.70	1.0	0.9859	0.1674	9.18	2.80	13.71
1.80	31.0	7.00	0.75	1.0	0.9815	0.1915	9.16	2.80	17.60
1.80	31.0	7.00	*0.80	1.0	0.9765	0.2157	9.14	2.80	21.13
DEPENDENCE OF CAVITY RADIUS R									
1.85	31.0	7.00	0.65	1.0	0.9943	0.1064	9.74	2.10	5.54
1.80	31.0	7.00	0.65	1.0	0.9899	0.1419	9.20	2.80	9.80
1.70	31.0	7.00	0.65	1.0	0.9932	0.1165	9.84	2.30	11.27

Table 1. Dependence of r/Q on various cavity parameters. The columns w # and z # give the number of wiggler periods and longitudinal rf periods in the cavity length. The asterisk implies that a reflecting mesh is required to sustain the mode in the cavity described.

Next we vary the size of the beam port. Of course, the beam port size is restricted by wake field effects, but we do not examine these effects in this study. If we keep the amplitude of transverse oscillations fixed, we find a reduction in r/Q. This is as expected, since the highest fields are near the edges of the discs. However, if we keep the beam dimension fixed and allow bigger transverse oscillation as we open the aperture, r/Q increases significantly. This is so because bigger transverse oscillation corresponds to higher transverse velocity. The size of the beam port is, of course, limited by the desired cutoff frequency. For the TE_{11p} mode with 11.424 GHz frequency, the maximum aperture radius which will not allow propagation of fields is 7.7 mm. However, we may use larger beam ports provided we employ a mesh which reflects the rf fields at the beam ports but lets the beam through. As we make the aperture size larger and larger, we would expect that after some critical size the discs would draw the electric fields farther from the axis, thus reducing the r/Q. Increasing the aperture radius on the discs without changing the beam port size is not advantageous because the electric fields are pulled away from the axis and the transverse oscillations cannot be made larger to compensate.

4. CONCLUSION

As shown above, by using codes such as URMEL, r/Q can be calculated for non-ideal cavities. Simple cylindrical cavities may be loaded with discs and other structures to enhance the coupling impedance. For a given cavity, r/Q contains information about the spacial distribution of electric field in a particular mode. This information can then be used in one-dimensional analyses (theoretical and computer particle simulations) to evaluate, more realistically, the performance of SWFELs or RKs. With additional study we should be able to design structures that further improve the coupling between the rf fields and the electron beam.

5. ACKNOWLEDGMENTS

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