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Production of Mesons by Photons and Nucleons

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The Production of Mesons by Photons

The production of mesons by photons has recently been accomplished experimentally,[1] and at present extensive experimental work is being done on the problem.[2] It has been found that the present experimental techniques permit the study of the production of mesons by photons with more accuracy and in more detail than is possible for production by nucleon-nucleon collision. The theoretical analysis of such information is also considerably simplified by the nature of the interaction. It will be shown that the results depend very markedly on the nature of the coupling of the mesons to the electromagnetic field. Since processes involving such coupling can be handled very well for non-relativistic energies by treating the interactions as weak, the usual methods of perturbation theory can be applied with some confidence to this aspect of the production process. The uncertainties of the nature of the coupling of mesons to nucleons, which, as is well known, lead to incorrect prediction of scattering phenomena, do not strongly affect this process. In fact, the very characteristic differences between the behaviour of the photon-ejected spin zero and spin one mesons will be shown to be due almost entirely to the nature of the meson coupling to the electromagnetic field.

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[1] McMillan, Peterson, and White; Science 110, 579 (1949)

[2] Cook, Steinberger, McMillan, Peterson, White; private communications

The theory of the production of photo-mesons has been studied by a number of people,[3] with the most complete work being the recent contribution of Feshbach and Lax. The methods used have been those of perturbation theory in the weak coupling approximation. Effects of the recoil of the nucleons have been neglected. The results obtained differ markedly for the various theories. It is not immediately apparent how the characteristic differences are related to the detailed features of the theories. It is therefore of interest to attempt to understand by classical argument, without reference to perturbation theory, some details of the process. It has further been thought worthwhile to carry out the calculations using the new covariant formalism, eliminating unnecessary approximations and exhibiting the simplicity of the methods. Finally, the higher order corrections in the meson-nucleon interaction have been calculated for the pseudoscalar theory, using the new subtraction techniques, and found to give finite and unambiguous results.

I General Description

A. Ratio of Cross Section for Production of Negative and Positive Mesons

One of the most striking of the early experimental observations on the production of mesons by photons was the excess of negative over positive mesons.[1] At present this is still the best established experimental fact. It was pointed out by Brueckner and Goldberger[4] that very simple classical

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[3] W. Heitler, Proc. Roy. Soc. 166, 529 (1938)
M. Kobayashi and T. Okayama, Proc. Phys. Math. Soc. Japan 21, 1 (1939)
H. S. W. Massey and H. C. Corben, Proc. Camb. Soc. 35, 84 (1939) and 35, 463 (1939)
L. Nordheim and G. Nordheim, Phys. Rev. 54, 254 (1938)
H. Feshbach and M. Lax, Phys. Rev. 76, 134 (1949)
L. Foldy, Phys. Rev. 76, 372 (1949)

[4] K. Brueckner and M. Goldberger, Phys. Rev. 76, 1725 (1949)

arguments could give an explanation of this result. These arguments simply pointed out that there is an essential asymmetry in the production of negative and positive mesons. When the former are produced from neutrons, the charge-carrying nucleon is the final proton with large recoil velocity. When positive mesons are produced, the proton is initially at rest and does not interact through its charge. Using the interaction

$$e \, \frac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta} \tag{1}$$

which differs from the non-relativistic $e\,\vec{v} \cdot \vec{A}$ because of retardation effects in the interaction of charge with the electromagnetic field, one obtains for the ratio of the interactions leading to the production of positive and negative mesons

$$\frac{I\ (\text{positives})}{I\ (\text{negatives}} = \frac{e \, \frac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta}\ (\text{meson})}{-e \, \frac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta}\ (\text{meson}) + e \, \frac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta}\ (\text{recoil proton})} \tag{2}$$

The ratio of the cross sections then is the square of the ratio of the interactions. Using over-all energy and momentum conservation, this ratio can be written

$$\frac{\sigma\ (\text{positives})}{\sigma\ (\text{negatives})} = \left[1 - \frac{q_o}{Mc^2} (1 - v/c \cos\Theta) \right]^2 \tag{3}$$

where

q_o = meson energy including rest energy

M = nucleon mass

v = meson velocity

θ = angle between meson and photon

The dependence on meson energy and angle of this function is given explicitly in Figure 1.

In this very simple argument, the effects of the magnetic moments of the particles have been ignored. It is of interest, therefore, to look in more detail at the nature of the interaction, including such effects. This will indicate what information can be given by a careful measurement of the negative-to-positive ratio. One can formulate the argument in the following manner: the interaction with the electromagnetic field leading to the ejection of mesons is of the form

$$I = A_\mu \int j_\mu (\vec{r}\,',t)\, e^{i(\vec{K}\cdot\vec{r}\,' - K_o t)}\, d\vec{r}\,' dt \tag{4}$$

where j_μ is the total current carried by the interacting particles. The current can be separated into a curl-free and a divergence-free part, the former corresponding to a linear motion of charge and the latter to circulating currents.

$$j_\mu = qv_\mu + \frac{\partial M_{\mu\nu}}{\partial x_\nu} \tag{5}$$

where v_μ is the relativistic velocity with spacial components $\vec{v}/(1-\beta^2)^{1/2}$, q the charge, and $M_{\mu\nu}$ is an anti-symmetric tensor. The interaction then is

$$\begin{aligned} I &= A_\mu \int \left(qv_\mu + \frac{\partial M_{\mu\nu}}{\partial X_\nu}\right) e^{i(\vec{K}\cdot\vec{r}\,' - K_o t)}\, d\vec{r}\,' dt \\ &= A_\mu \int (qv_\mu - iM_{\mu\nu}K_\nu)\, e^{i(\vec{K}\cdot\vec{r}\,' - K_o t)}\, d\vec{r}\,' dt \end{aligned} \tag{6}$$

We can easily evaluate this integral, considering each interacting particle separately. If the wave-length of the radiation is large compared with the region over which the charges and currents are distributed, or if one assumes a delta-function of position for the spacial distribution, the dependence on $\vec{r}\,'$ can be given by

$$\begin{aligned} v_\mu(\vec{r}\,',t) &= \vec{v}(t)\,\delta\left[\vec{r}\,' - \vec{r}(t)\right] \\ &= v_\mu(t)\,\delta\left[\vec{r}\,' - \vec{r}(t)\right](1-\beta^2)^{1/2} \\ M_{\mu\nu}(\vec{r}\,',t) &= M_{\mu\nu}(t)\,\delta\left[\vec{r}\,' - \vec{r}(t)\right](1-\beta^2)^{1/2} \end{aligned} \tag{7}$$

The integral for each particle then is

$$I = A_\mu \int \left[qv_\mu(t) - iM_{\mu\nu}(t)K_\nu\right](1-\beta^2)^{1/2}\, e^{i\left[\vec{K}\cdot\vec{r}(t) - K_o t\right]}\, dt \tag{8}$$

Changing variable and partially integrating gives

$$I = A_\mu \int \frac{d}{ds} \left[\frac{v_\mu(s) - iM_{\mu\nu}(s)K_\nu}{K_o(1-\beta \cos\theta)} \right] e^{is} (1-\beta^2)^{1/2} \tag{9}$$

If the interaction takes place over a time much less than the period of the radiation, the variation of the exponential term can be ignored. This gives

$$I = -\Delta\left[\frac{A_\mu P_\mu + \frac{m}{2} M_{\mu\nu} F_{\mu\nu}}{\underset{\sim}{K \cdot P}}\right] \tag{10}$$

where the four-vector product is

$$\underset{\sim}{K \cdot P} = -K_o P_o + \vec{K} \cdot \vec{P}$$

and $\Delta[\quad]$ denotes the change during the interaction.

In this expression for the interaction of the electromagnetic field with the charges and currents of the meson-nucleon system, we have not included the dependence of the meson emission on such factors as the strength of the coupling of the mesons to the nucleons and the spins of the nucleons. We shall assume that a simple multiplicative factor U gives the spin dependence of the forces and the strength of the coupling. When we consider the application of field theory to this process, we shall see that this actually is so for spin-zero mesons. The quantum nature of the process also has not been considered. We actually need the matrix element between the initial and final nucleon states of the operator which we have derived. The interaction therefore can be written for each particle

$$\begin{aligned} I &= \psi_F^\dagger \left\{ U\Delta\left[\frac{q\vec{A}\cdot\vec{P} + \frac{m}{2} F_{\mu\nu} M_{\mu\nu}}{\underset{\sim}{K \cdot P}}\right]\right\} \psi_I \\ &\equiv \overline{U}\Delta\left(q\frac{\vec{A}\cdot\vec{P}}{\underset{\sim}{K \cdot P}}\right) + \frac{m}{2} F_{\mu\nu}\Delta(\overline{U M_{\mu\nu}}) \end{aligned} \tag{11}$$

We can use this expression to evaluate the ratio of the cross sections for negative and positive mesons. For simplicity we will take the mesons to have spin zero, i.e., no magnetic moment, and first assume that the nucleons interact only in the proton state. When a positive meson is produced, the proton is the initial nucleon at rest and so interacts only through its magnetic moment with the transverse photon.

$$I(+) = -e \frac{\vec{A} \cdot \vec{q}\, \overline{U}}{\underset{\sim}{K \cdot q}} + \frac{m}{2} \frac{\overline{U M_{\mu\nu}}}{\underset{\sim}{K \cdot I}} F_{\mu\nu} \tag{12}$$

q = meson 4-momentum

I = initial nucleon 4-momentum

When a negative meson is produced, the proton is the final recoil nucleon carrying current $e\,\vec{F}/m$

$$I(-) = e\left(\frac{\vec{A} \cdot \vec{q}}{\underset{\sim}{K \cdot q}} - \frac{\vec{A} \cdot \vec{F}}{\underset{\sim}{K \cdot F}}\right)\overline{U} - \frac{m}{2} \frac{\overline{UM_{\mu\nu}}}{\underset{\sim}{K \cdot F}} F_{\mu\nu} \tag{13}$$

F = final nucleon 4-momentum

If we now use over-all 4-momentum conservation, we find that

$$\vec{A} \cdot \left(\frac{\vec{q}}{\underset{\sim}{K \cdot q}} - \frac{\vec{F}}{\underset{\sim}{K \cdot F}}\right) = \frac{\vec{A} \cdot \vec{q}}{\underset{\sim}{K \cdot q}} \frac{\underset{\sim}{K \cdot I}}{\underset{\sim}{K \cdot F}}$$

and

$$I(-) = -\frac{\underset{\sim}{K \cdot I}}{\underset{\sim}{K \cdot F}} \left[e \frac{\vec{A} \cdot \vec{q}}{\underset{\sim}{K \cdot q}} \overline{U} + \frac{m}{2} \frac{\overline{UM_{\mu\nu}}}{\underset{\sim}{K \cdot I}} F_{\mu\nu}\right] \tag{14}$$

$$= -\frac{\underset{\sim}{K \cdot I}}{\underset{\sim}{K \cdot F}} I(+)$$

Therefore, under these assumptions, the plus-to-minus ratio would be

$$\frac{\sigma(+)}{\sigma(-)} = \left(\frac{\underset{\sim}{K \cdot F}}{\underset{\sim}{K \cdot I}}\right)^2$$

$$= \left[1 - q_o/Mc^2(1 - v/c \cos\theta)\right]^2 \tag{15}$$

This is the same result obtained when magnetic moment terms were ignored (Equation 3). It is interesting to observe that the ratio of the linear current interactions is the same as the ratio of the magnetic-moment interactions, so that the plus-to-minus ratio is independent of the values of

$$\overline{U} = \psi_F^{\dagger} U \psi_I$$

$$\overline{UM_{\mu\nu}} = \psi_F^{\dagger} UM_{\mu\nu} \psi_I$$

If the nucleons do not interact only in the proton state, but instead with anomalous moments, we can write

$$M_{\mu\nu}\,(\text{proton}) = \gamma_P\, M_{\mu\nu}$$
$$M_{\mu\nu}\,(\text{neutron}) = -\gamma_N\, M_{\mu\nu} \tag{16}$$

where γ = magnitude of magnetic moment of nucleon in Bohr magnetons

$$\gamma_P = 2.87$$
$$\gamma_N = 1.91 \tag{17}$$

This gives for the ratio of the interactions

$$\frac{I(+)}{I(-)} = \frac{-e\,\frac{\vec{A}\cdot\vec{q}}{K\cdot q}\,\overline{U} - \frac{m}{2}F_{\mu\nu}\left(\frac{\gamma_P}{K\cdot I} + \frac{\gamma_N}{K\cdot F}\right)\overline{M_{\mu\nu}\,U}}{e\,\frac{\vec{A}\cdot\vec{q}}{K\cdot q}\,\overline{U}\,\frac{K\cdot I}{K\cdot F} + \frac{m}{2}F_{\mu\nu}\left(\frac{\gamma_P}{K\cdot F} + \frac{\gamma_N}{K\cdot I}\right)\overline{M_{\mu\nu}U}} \tag{18}$$

If $\overline{M_{\mu\nu}\,U} = \overline{M_{\mu\nu}}\;\overline{U}$

then the factor $\overline{U}$ cancels. The ratio of the cross section averaged over photon polarization and nucleon-moment orientation gives

$$\frac{\sigma\,(+)}{\sigma\,(-)} = \left(\frac{K\cdot F}{K\cdot I}\right)^2\left[1 + 0\left(\frac{\mu^2}{M^2}\right)\right]$$

Therefore, if the interaction is of this type, the ratio of the cross sections is nearly unchanged by the anomalous moments, since $\frac{\mu^2}{M^2} \sim 2$ percent. This is a result of the predominance of the coupling of the electromagnetic field to the linear motion of charge, i.e., to the electric dipole formed by the meson-nucleon charges.

If $\overline{M_{\mu\nu}U} >> \overline{M_{\mu\nu}}\,\overline{U}$, or what is equivalent, the magnetic moment terms are predominant in the interaction,

$$\frac{\sigma\,(+)}{\sigma\,(-)} = \left[1 - \frac{\gamma_P - \gamma_N}{\gamma_P + \gamma_N}\,\frac{q_o}{Mc^2}\left(1 - \frac{v}{c}\cos\theta\right)\right]^2$$
$$= \left[1 - .20\,\frac{q_o}{Mc^2}\left(1 - \frac{v}{c}\cos\theta\right)\right]^2 \tag{19}$$

This function is given graphically in Figure 1. It is apparent that, if nucleon moment interactions are important, the introduction of a magnetic moment for the neutron removes much of the asymmetry in the process leading to the production of positive and negative mesons, leading to a plus-to-minus ratio which is nearly one.

We remark here that we have assumed that, with photons at energies of about the meson rest energy, the nucleons interact with the anomalous moments observed in a static field. This will be true only if the circulating currents which give rise to the anomalous moments are confined to a region small compared with the wave-length of the radiation. If this is not true, the anomalous moments will show energy dependence and the values of the static moments cannot be used.

We have ignored the interaction of the meson magnetic moments. These could contribute only if the meson were a vector particle, i.e., with spin one. It is apparent that such interactions are symmetrical for the production of either negative or positive mesons. Therefore strong meson-moment interactions would give a negative-to-positive ratio close to one.

One can conclude that a verification of the results (3) would indicate that the meson does not interact strongly through a magnetic moment and that the neutron anomalous magnetic moment does not play an important part in the process. We have seen that these conclusions do not depend on the nature of the coupling of the mesons to the nucleons.

B. Angular Distribution

If the photon is absorbed by the ejected meson at photon energies for example of 200-300 Mev, $\beta = v/c$ for the meson will not be small compared with one. The angular distribution will show large asymmetry about 90 degrees due to the presence in the differential cross section of the denominator

$$\frac{1}{(1 - \beta \cos\theta)^2} \qquad (20)$$

which appears because of the retardation effects in the interaction of charge with the electromagnetic field. However, if the absorption of the photon is through its coupling with the nucleon, $v/c << 1$ for the nucleon and the angular distribution will be nearly symmetric about 90 degrees. One would therefore expect that the degree of symmetry would indicate which particle interacts most strongly with the photon.

If the interactions are principally of the form $e \vec{v} \cdot \vec{A}$ then, since the photon field is transverse, the angular spectra must fall off to zero at 0 or 180 degrees. Spectra which do not exhibit this behaviour must be due to magnetic moment-like interactions.

II Principal Features of Various Theoretical Results

The calculations, unless otherwise specified, are for the first non-vanishing order in which the process can take place. The nucleons are treated as Dirac particles and the effects of their recoil are fully taken into account. The scalar, pseudoscalar, vector, and pseudovector theories are considered, using the couplings with the nucleon field which do not involve derivatives of the meson field.

The spectra shown in Figures 2, 3, 4 and 5 are for mesons produced by a dK/K photon spectrum. This approximates the bremsstrahlung energy distribution at high energies which is used in the laboratory to produce mesons. The relation between the meson and the photon energy, as a function of angle, is given in Figure 9.

A. Scalar Meson

The scalar meson characteristically shows a dipole angular distribution (Figure 2) at low energies which is strongly distorted forward at energies above a few Mev by the denominator (Equation 20). This is due to the predominance of the photon-meson interaction and the absence of a magnetic moment for the meson. This simple result is quite analogous to the low-energy photo-electric effect in an atom, since the mesons are spherically symmetric in distribution about the nucleon, and the photon interacts relatively weakly with the nucleon. This is in accordance with the fact that the meson cloud about the nucleon extends to distances of the order $\hbar/\mu c$, while the circulating currents associated with the magnetic moment of the nucleon are distributed over a region of order $\hbar/Mc$ and therefore give contributions which are smaller in the ratio $\left(\frac{\mu}{M}\right)^2$. The plus-to-minus ratio from the lowest order calculations is the same as that obtained by the classical argument, as would be expected since the nucleons are treated as Dirac particles and the meson has no magnetic moment interaction.

B. Pseudoscalar Meson

The pseudoscalar theory shows a roughly isotropic angular distribution (Figure 3) indicating the predominance of the coupling of the photon to the magnetic moment of the nucleon. The unimportance of the electric-dipole terms in the interaction, i.e., the coupling to the linear motion of charge, is due to the close binding of the meson cloud to the nucleon. The probability of finding a meson at a distance $\hbar/\mu c$, which is the wave-length of the photon, is relatively small. However, the direct coupling of the photon to the magnetic moment of the nucleon, preceded or followed by the emission of the final meson, is relatively probable. Examination of the matrix elements involved in these transitions indicates that this behaviour is due to the existence of intermediate states in which the nucleon undergoes transitions to negative energy states. For such processes the matrix elements of the couplings to the electromagnetic and meson fields are of the order one instead of v/c for the nucleons. The ratio of negative to positive mesons given by the lowest order calculation is the same as that for the scalar theory.

Preliminary experimental results indicate that the spectrum of mesons observed is roughly isotropic in angular distribution as predicted by the pseudoscalar theory. Therefore this theory was selected for examination of the corrections of higher order in the nucleon-meson coupling constant g. This involves another emission and reabsorption of a virtual meson, as indicated in Figure 7. It was hoped that the investigation of the corrections would answer questions concerning the contribution of the anomalous magnetic moments of the nucleons and the possible large effects of corrections to processes in which the photon is coupled directly to the meson or its associated Dirac vacuum field. Examination of the effects of higher order processes would also indicate the probability that the expansion in powers of $g^2/4\pi$ would actually lead to a convergent series.

The calculation of the next higher order terms shows that they give contributions about as large as the first order terms, therefore casting grave doubts on the convergence of the expansion and the validity of the application of perturbation methods of this kind.

The largest contribution to the corrections comes from the anomalous magnetic moments of the nucleons, Figures 7-B1, 7-B2 and 7-B3. The anomalous moments differ considerably from those which the nucleons exhibit in a static field, as calculated by Case.[5] Non-static effects occur in the interaction which decreases the magnitude of the moments and also change the sign of the proton moment. This result indicates that it may not be correct to assume that the nucleons interact, with their static

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[5]Phys. Rev. 76, 1725 (1949)

moments, in high energy processes. The actual magnitude of these corrections, however, probably can be believed only in a very qualitative way.

Corrections in which the meson creates virtual nucleon pairs (Figure 7-A_1), one of which may interact directly with the photon (Figure 7-A_2), give negligible contributions and so do not affect the predominance of the coupling of the photon to the nucleon through the Dirac and anomalous moments.

C. Vector and Pseudovector

The vector theory shows a strongly asymmetric angular distribution (Figure 4), indicating the predominance of the photon-meson interaction. This distribution is also peaked markedly forward showing that the interaction is due at least in part to the magnetic moment of the meson. The pseudovector theory shows a nearly isotropic angular spectrum (Figure 5). This, however, is not due to the largeness of the nucleon photon coupling but to the large magnetic moment interaction of the meson. The negative to positive ratio (Figure 6) also reflects the large effects of the magnetic moment of the spin-one mesons, differing considerably from the result obtained by ignoring the meson moment. Also very characteristic of vector and pseudovector meson is the rapid increase of the cross section with energy (Figures 4 and 5). This is due both to the strong energy dependence of the electromagnetic field coupling with longitudinally polarized mesons and to the magnetic moment interactions.

Higher order corrections probably would not particularly affect these results, since processes involving production of virtual nucleon pairs by the mesons seem to give negligible contributions. Higher order processes involving the coupling of the photon to the nucleon and its associated meson field would give corrections to terms which are already relatively unimportant in the process.

III Method of Calculation, Lowest Order

The calculation of matrix elements can be greatly simplified by use of the Feynman-Dyson methods.[6] The necessary operators can be derived by a technique due to Feynman, the correctness of which can also be demonstrated by the Dyson methods. The calculations can also be carried out by the older methods of perturbation theory to give exactly the results derived here. The meson couplings to the nucleon field that are used are those which do not involve derivatives of the meson field, since these introduce non-renormalizable singularities in higher order processes.

In the following, the notation used has $\hbar = c = M = 1$, where M is the nucleon mass. Therefore, all energies and momenta will be measured in units of the nucleon mass. All products of the form $\underset{\sim}{A}\cdot B$ will be understood to be 4-vector products, with $\underset{\sim}{A}\cdot B = -A_0 B_0 + \vec{A}\cdot\vec{B}$. We also use the notation $\underset{\approx}{U} = U_\mu \gamma_\mu$ with the Dirac matrixes $\gamma_i = i\,\alpha_i\,\beta$ $(i = 1,2,3)$, $\gamma_4 = \beta$. The adjoint operator $\psi^\dagger$ is related to the complex conjugate by $\psi^\dagger = i\psi^* \gamma_4$.

The equation of motion for the fields are

A. Free Particles

$$(\underset{\approx}{P} - i)\psi = 0 \qquad \text{Dirac} \qquad (21)$$

$$(\Box - \kappa^2)\phi = 0 \qquad \text{spin zero} \qquad (22)$$

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \phi_\nu}{\partial x_\mu} - \frac{\partial \phi_\mu}{\partial x_\nu}\right) - \kappa^2 \phi_\nu = 0 \qquad \text{spin one} \qquad (23)$$

The last equation can also be written

$$(\Box - \kappa^2)\phi_\nu = 0 \qquad \text{spin one}$$

with the divergence condition

$$\frac{\partial \phi_\mu}{\partial x_\mu} = 0 \qquad (24)$$

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[6] R. P. Feynman, Phys. Rev. 76, 769 (1949)
F. J. Dyson, Phys. Rev. 75, 1736 (1949)

B. Interaction Of Mesons With Dirac Field

$$(\Box - \kappa^2)\phi = g\psi^\dagger U \psi \qquad U = (-1)^{1/2} \quad \text{scalar} \qquad (25)$$

$$= \gamma_5 \quad \text{pseudoscalar}$$

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \phi_\nu}{\partial x_\mu} - \frac{\partial \phi_\mu}{\partial x_\nu}\right) - \kappa^2 \phi_\nu = g\psi^\dagger U_\nu \psi \qquad (26)$$

$$U_\nu = \gamma_\nu \quad \text{vector}$$

$$= \gamma_5 \gamma_\nu \quad \text{pseudovector}$$

The last equation can be written

$$(\Box - \kappa^2)\phi_\nu = \left(\delta_{\mu\nu} - \frac{\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu}}{\kappa^2}\right) g\psi^\dagger U_\mu \psi \qquad (26')$$

$$(\underset{\approx}{P} - i)\psi = g\phi U \psi \qquad \text{spin zero} \qquad (27)$$

$$= g\phi_\nu U_\nu \psi \qquad \text{spin one}$$

C. Interaction With Electromagnetic Field

$$(\underset{\approx}{P} + e\underset{\approx}{A} - i)\psi = 0 \qquad \text{Dirac} \qquad (28)$$

$$\left[\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right)\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right) - \kappa^2\right]\phi = 0 \qquad \text{spin zero} \qquad (29)$$

$$\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right)\left[\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right)\phi_\nu - \left(\frac{\partial}{\partial x_\nu} + ieA_\nu\right)\phi_\mu\right] - \kappa^2\phi_\nu = 0 \qquad \text{spin one} \qquad (30)$$

To order e, these can be written

$$(\underset{\approx}{P} - i)\psi = -e\underset{\approx}{A}\psi \qquad \text{Dirac} \qquad (28')$$

$$(\Box - \kappa^2)\phi = -2ceA_\mu \frac{\partial \phi}{\partial x_\mu} \qquad \text{spin zero} \qquad (29')$$

$$(\Box - \kappa^2)\phi_\nu = -ie\left(2A_\lambda \frac{\partial}{\partial x_\lambda}\delta_{\mu\nu} - A_\nu \frac{\partial}{\partial x_\mu} - \frac{\partial A_\nu}{\partial x_\mu} - A_\mu \frac{\partial}{\partial x_\nu}\right)\phi_\mu \qquad \text{spin one} \qquad (30')$$

If we consider the direct solution of these equations of motion, following Feynman's general arguments, we find the following expressions to be inserted into the Feynman-Dyson diagram:

B'. Emission Of a Meson Of Momentum P_μ — gU — spin zero (31)

$$g\left(\delta_{\mu\nu} + \frac{P_\mu P_\nu}{\kappa^2}\right)U_\mu$$ spin one, polarization ν

C'. Absorption Of a Photon

$-e\underset{\approx}{A}$	Dirac	
$2e\underset{\sim}{A}\cdot\underset{\sim}{P}'$	spin zero, momentum P'_μ	
$e\left[2\underset{\sim}{A}\cdot\underset{\sim}{P}'\delta_{\mu\nu} - A_\nu P'_\mu - K_\mu A_\nu - A_\mu q_\nu\right]$	spin one, momentum P'_μ, initial polarization μ, final polarization ν	

D. Propagation of an Intermediate Particle With Momentum P'_μ

$$\frac{1}{\underset{\approx}{P}' - i} \qquad \text{Dirac} \tag{33}$$

$$\frac{-1}{P'^2 + \kappa^2} \qquad \text{boson, mass } \kappa$$

Exponential factors of the form $e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}}$ have been omitted since after spacial integrations have been carried out, they simply give 4-momentum conservation at each point of the diagram and for the over-all process.

Using these expressions we can now write down matrix elements directly. A virtual meson emitted by a nucleon can absorb a photon and go into a free meson. This is represented by the diagram A, Figure 7.

$$ge\,\psi_F^\dagger\left[2\underset{\sim}{A}\cdot q \;\frac{-1}{(q-K)^2 + \kappa^2}\,U\right]\psi_I\,\phi \qquad \text{spin zero} \tag{34}$$

$$ge\,\psi_F^\dagger\left(\delta_{\mu\nu} + \frac{q'_\mu q'_\nu}{\kappa^2}\right)U_\mu\left(\frac{-2\underset{\sim}{A}\cdot q\,\delta_{\lambda\nu} + A_\lambda q'_\nu + K_\nu A_\lambda + A_\nu q_\lambda'}{(q-K)^2 + \kappa^2}\right)\psi_I\,\phi_\lambda \qquad \text{spin one} \tag{35}$$

The nucleon can emit a real meson, going into a virtual intermediate state, and then absorb the photon. The photon absorption can also come first, followed by the meson emission. This process is represented by the diagram B, Figure 7.

$$-ge\,\psi_F^\dagger\left(\underset{\approx}{A}\,\frac{1}{\underset{\approx}{F}-\underset{\approx}{K}-i}\,U\right)\psi_I\,\phi \qquad \text{spin zero} \qquad (36)$$

$$-ge\,\psi_F^\dagger\left[\underset{\approx}{A}\,\frac{1}{\underset{\approx}{F}-\underset{\approx}{K}-i}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{\kappa^2}\right)U_\mu\right]\psi_I\,\phi_\nu \qquad \text{spin one} \qquad (37)$$

If the nucleons are treated as Dirac particles, the photon can be absorbed only by a proton. Therefore, this diagram represents the production of a negative meson. For production of a positive meson, simply replace I by F and invert the order of the operators.

Combining these two contributions from diagrams A and B, we obtain for the lowest order matrix element for the transition

$$M_2 \;=\; ge\,\psi_F^\dagger U\left(\frac{\underset{\sim}{A}\circ q}{\underset{\sim}{K\circ q}}-\frac{\underset{\sim}{A}\circ \underset{\sim}{I}}{\underset{\sim}{K\circ I}}-\frac{\underset{\approx}{KA}}{2\underset{\sim}{I\circ K}}\right)\psi_I\,\phi \qquad \text{spin zero} \qquad (38)$$

For negative mesons replace I by F in the bracket. This is of the form (Equation 11) derived above, with the moment tensor for the Dirac field

$$M_{\mu\nu} = \frac{e}{2}\,\frac{\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu}{2i} = \frac{e}{2}\sigma_{\mu\nu}$$

$$M_2 \;=\; ge\,\psi_F U_\nu\left[\frac{2\underset{\sim}{A}\circ q\,\phi_\nu + A_\nu\,\underset{\sim}{\phi}\circ K - K_\nu\,\underset{\sim}{\phi\circ A}}{2\underset{\sim}{q\circ K}} - \phi_\nu\,\frac{\underset{\sim}{A\circ I}}{\underset{\sim}{K\circ I}}\right. \qquad (39)$$

$$\left.-\phi_\nu\,\frac{\underset{\approx}{KA}}{2\underset{\sim}{I\circ K}}-\frac{F_\nu-I_\nu}{\kappa^2}\left(\frac{\underset{\sim}{q\circ K}\,\underset{\sim}{\phi\circ A}-\underset{\sim}{A\circ q}\,\underset{\sim}{\phi\circ K}}{2\underset{\sim}{q\circ K}}\right)\right]\psi_I \qquad \text{spin one}$$

For negative mesons replace I by F in bracket and

$$U_\nu\underset{\approx}{KA} \quad \text{by} \quad -\underset{\approx}{AK}\,U_\nu \qquad (40)$$

In these expressions we can demonstrate gauge invariance by substituting

$$A_\mu \;=\; A_\mu' + \frac{\partial\Lambda}{\partial x_\mu}$$

$$= \; A_\mu' + iK_\mu\Lambda \qquad (41)$$

which should leave the matrix element for the transition M_2 unchanged. This is equivalent to showing that replacing A_μ by K_μ reduces M_2 to zero. That this is so can be seen by inspection, using

$$\underset{\approx}{K}\,\underset{\approx}{K} \;=\; K_\mu K_\mu \;=\; 0$$

If we now specialize to the transverse vector potential, A•I is zero since the momentum I_μ is along the direction of the photon momentum. We then have for the spin-zero mesons

$$M_2 \text{ (positive)} = ge\,\psi_F^\dagger \phi\, U \; \frac{\vec{A}\cdot\vec{q}}{K\cdot q} - \frac{KA}{2 I\cdot K} \tag{42}$$

$$M_2 \text{ (negative)} = -ge\,\psi_F^\dagger \phi\, U \left[\bar{A} \cdot \left(\frac{\vec{q}}{K\cdot q} - \frac{\vec{F}}{F\cdot K} \right) - \frac{KA}{2F\cdot K} \right] \psi_I$$

$$= \frac{K\cdot I}{K\cdot F} M_2 \text{ (positives)}$$

This is the same result as that obtained above (Equation 7).

The differential cross section in the laboratory system then can be obtained from

$$d\sigma = 2\pi \left| M_2 \right|^2 P_F \tag{43}$$

where M_2 is to be summed over meson and final nucleon spins and averaged over initial nucleon spin and photon polarization. The sum over meson spin can be made easily. If ϵ_μ is the 4-vector representing the direction of polarization of the meson which satisfies

$$\epsilon_\mu q_\mu = 0 \qquad \text{divergence condition (44)}$$

$$\epsilon_i^2 + \epsilon_4^2 = \frac{1}{2 q_o} \qquad \text{normalization condition (45)}$$

then

$$\sum_{\text{spin}} \epsilon\cdot A\, \epsilon\cdot B = \frac{1}{2 q_o} \left[A\cdot B - \frac{A\cdot q \; B\cdot q}{q\cdot q} \right] \tag{46}$$

Carrying out the indicated sums and averages gives, after simplification of resulting expressions in the laboratory system, for positive mesons

$$\left[\frac{g^2}{4\pi} \frac{e^2}{4\pi} \frac{\pi}{2} \frac{1}{K_o^2} \right]^{-1} \frac{d\sigma}{dq_o} = \frac{q^2 \sin^2\theta \,(2 - \kappa^2/2)}{(q\cdot K)^2} + 1 - \frac{q\cdot K}{I\cdot K} \qquad \text{scalar} \tag{47}$$

$$\frac{q^2 \sin^2\theta}{(q\cdot K)^2} (-\kappa^2/2) + 1 - \frac{q\cdot K}{I\cdot K} \qquad \text{pseudoscalar} \tag{48}$$

$$\frac{1}{(\underset{\sim}{q\cdot K})^2}\left[(\underset{\sim}{I\cdot K})^2 - 3\ \underset{\sim}{I\cdot K}\ \underset{\sim}{q\cdot K} + 15/4\ (\underset{\sim}{q\cdot K})^2 + 1/2\ (\underset{\sim}{q\cdot K})^3 \left(\frac{-4}{\underset{\sim}{K\cdot I}} - \frac{1}{\kappa^2}\right)\right. \tag{49}$$

$$\left. + \frac{q^2\sin^2\theta}{2\kappa^2}\left[(\underset{\sim}{I\cdot K})^2 - 2\kappa^4 - 4\kappa^2\right]\right] \qquad \text{Vector}$$

For pseudovector, add to vector

$$\frac{1}{(\underset{\sim}{q\cdot K})^2}\left[-\frac{2\ (\underset{\sim}{q\cdot K})^2}{\kappa^2} - \frac{2\ (\underset{\sim}{q\cdot K})^3}{\kappa^4} + 6\ q^2\ \sin^2\theta\right] \tag{50}$$

Finally, if we wish to apply these expressions to the calculation of the spectrum of mesons produced by a photon beam which is the result of the bremsstrahlung of energy electrons, we can represent the distribution of energies in the photon beam by

$$\phi\ (K)\quad \frac{dK}{K} \tag{51}$$

where ϕ (K) is a factor, nearly one over the part of the spectrum of interest, which indicates the degree of departure from the simple $\frac{dK}{K}$ distribution.[7] At a given meson energy, the distribution in photon energies leads to a distribution in meson angles, with the energies and angle related by conservation of energy and momentum

$$K = \frac{q_o - \mu^2/2}{1 - q_o + q\cos\Theta} \tag{52}$$

If

$$\frac{d\sigma}{dq_o} = f\ (\Theta,\ q_o) \tag{53}$$

then

$$\frac{d\sigma}{dq_o d\Omega} = f\ (\Theta,\ q_o)\frac{1}{K}\ \frac{dK}{d\Omega}\ \phi(K)$$

$$= \frac{Kq}{q_o - \kappa^2/2}\ \frac{\phi(K)}{2\pi}\ f\ (\Theta,\ q_o)$$

This spectrum is that shown in Figures 4, 5, 6 and 7, with ϕ (K) set equal to one.

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[7] W. Heitler, The Quantum Theory of Radiation, page 170

IV Higher Order Corrections

The perturbation calculations carried out in II are for the lowest non-vanishing order in the coupling constants g and e. The corrections of order e^3 and higher are considered negligible, which is probably justified for non-relativistic energies because of the smallness of the expansion parameter $\frac{e^2}{4\pi} = \frac{1}{137}$. Therefore, only terms of order eg^3 are calculated. The pseudoscalar theory is considered because it gives predictions in qualitative agreement with the experimental results of McMillan, et al.,[1] and because of the simplicity of the theory. Pseudoscalar coupling is used, which is equivalent to pseudovector coupling in lowest order but not in the higher order processes. The calculations made are for energies near threshold where the momenta of the particles can be ignored relative to the rest energies.

The possible diagrams for the corrections to the lowest order result for the production of positive mesons are given in Figure 7. The diagrams B_3, C, C_1 are forbidden since the nucleons indicated as interacting with the photon are in the neutron state. Diagrams involving neutral mesons are also omitted. Following Dyson,[6] we will first evaluate the corrections to the basic operators which are inserted into the matrix element representing an irreducible diagram, and then specialize to the diagrams listed in Figure 7. Corrections of the following types appear (Figure 8):

I. A nucleon, propagating as a virtual particle, can emit and reabsorb a virtual meson. This corresponds to Dyson's vacuum polarization of the second kind. The diagram includes nucleon mass and mesonic charge renormalization effects.

II. A meson, propagating as a virtual particle, can produce a nucleon pair, which then annihilate to give the meson again. The diagram includes meson mass and mesonic charge renormalization effects.

III. A meson, interacting with the electromagnetic field, can instead produce a nucleon pair one of which interacts with the field.

IV., V. A nucleon, interacting with the electromagnetic field, can emit a virtual meson. Either particle can then interact with the field. This corresponds to the nucleon anomalous moment terms. Diagrams III, IV, and V include electronic charge renormalization effects.

For diagram I, the correction leads to the replacement of

$$\frac{1}{\underset{\approx}{P} - i}$$

by

$$\frac{1}{\underset{\approx}{P} - i}\left\{\frac{g^2}{(2\pi)^4}\int d^4 \ell \, \gamma_5 \frac{1}{\underset{\approx}{P} - \underset{\approx}{\ell} - i} \gamma_5 \frac{1}{\ell^2 + \kappa^2}\right\}\frac{1}{\underset{\approx}{P} - i} \tag{55}$$

the integral can be evaluated, keeping convergence by integrating to a fixed upper limit which will finally be allowed to become infinite, to give

$$J(P) = \pi^2 \int_0^1 dx \left[-\underset{\approx}{P}(1-x) + i\right]\left[\ln\frac{\lambda^2}{\phi(P^2)} - 1\right] \tag{56}$$

where

$$\phi(P^2) = x\,(P^2 + 1) + (1-x)\kappa^2 - x^2P^2 \tag{57}$$

Following Dyson we now make the expansion

$$J(P) = U(0) + P_\mu\left(\frac{\partial J}{\partial P_\mu}\right)_{P=0} + R_c \tag{58}$$

where R_c is finite as $\lambda^2 \to \infty$. R_c is now separated into

$$R_c = A + B(\underset{\approx}{P} - i) + (\underset{\approx}{P} - i)\, R_c' \tag{59}$$

where A and B are independent of P and $R_c' \to 0$ as $P^2 \to -1$ and $\underset{\approx}{P} \to i$.

The renormalized J(P) then is

$$(\underset{\approx}{P} - i)\pi^2 \int_0^1 dx\left\{\frac{ix\,(\underset{\approx}{P} - i) - 2x - (P^2 + 1)\,(1 - x)}{P^2 + 1}\ln\frac{\phi(-1)}{\phi(P^2)} - \frac{2x^2\,(1-x)}{\phi\,(-1)}\right\} \tag{60}$$

Application to diagram B_1, Figure 7 gives

$$M_4(B_1) = ge\,\psi_F^\dagger\gamma_5\,\frac{g^2}{16\pi^2}\int_0^1 dx\left\{\quad\right\}\frac{1}{\underset{\approx}{P}-1}\underset{\approx}{A}\,\psi_I\,\phi \tag{61}$$

where the curly bracket is the same as above. Now if we use the properties of γ_5 to give

$$\psi_F^\dagger \gamma_5 (\underset{\approx}{P} - i) \equiv \psi_F^\dagger \gamma_5 (\underset{\approx}{F} + \underset{\sim}{q} - i)$$
$$\cong -i(2 + \kappa) \psi_F^\dagger \gamma_5$$

we find

$$M_4(B_1) = M_2(B) \frac{g^2}{16\pi^2} \int_0^1 dx \left\{ (-1 + x/2) \ln \frac{\phi(-1)}{\phi(P^2)} \quad \frac{-2 x^2 (1 - x)}{\phi(-1)} \right\} \tag{62}$$

This integral can be evaluated numerically to give

$$M_4(B_1) = -1.26 \frac{g^2}{16\pi^2} M_2(B) \tag{63}$$

The correction from this process could also be interpreted as a non-static correction to the magnetic moment of the proton.

For diagram II, Figure 8, the correction leads to the replacement of

$$\frac{-1}{P^2 + \kappa^2}$$

by

$$\frac{-1}{(P^2 + \kappa^2)^2} \frac{g^2}{(2\pi)^4} \int sp \left\{ \gamma_5 \frac{1}{-\underset{\approx}{P} + \underset{\approx}{\ell} - i} \gamma_5 \frac{1}{\underset{\approx}{\ell} - i} \right\} d^4 \ell \tag{64}$$

Carrying out the indicated spur sum and integration gives for the integral

$$J(P^2) = -4\pi^2 \int_0^1 dx \left\{ -\lambda^2 + \phi \ln \frac{\lambda^2}{\phi} + \phi \right\} \tag{65}$$

where

$$\phi(P^2) = x(1 - x) P^2 + 1 \tag{66}$$

Expansion about $P^2 = -\kappa^2$ gives

$$J(P^2) = J(-\kappa^2) + J'(-\kappa^2)(P^2 + \kappa^2) + R_c \tag{67}$$

where

$$R_c = 4\pi^2 \int_0^1 dx \left\{ \frac{1/2\, x^2 (1 - x)^2 (P^2 + \kappa^2)^2}{1 - x(1 - x)\kappa^2} \left[1 + 0(P^2 + \kappa^2) \right] \right\} \tag{68}$$

Application to diagram A_1, Figure 7, gives

$$M_4(A_1) \cong ge\, \phi\, \psi_F^\dagger \gamma_5 \psi_I \frac{-2 \underset{\sim}{A} \cdot \underset{\sim}{q}}{(q - K)^2 + \kappa^2} \frac{q^2}{16\pi^2} \left[\frac{(q - K)^2 + \kappa^2}{15} \right] \tag{69}$$

Near threshold $(q - K)^2 + \kappa^2 = 2\kappa^2$

Using this

$$M_4(A_1) = M_2(A) \frac{g^2}{16\pi^2} \frac{2\kappa^2}{15} \tag{70}$$

Since $\kappa^2 \sim 1/40$, the contribution from this higher order process is less than 1 percent.

For diagram III, replace 2 e $\underset{\sim}{A}\cdot\underset{\sim}{P}$

by

$$\frac{g^2 e}{(2\pi)^4} \int d\,4\ell \;\; sp\left\{\gamma_5 \frac{1}{\underset{\approx}{P'} + \underset{\approx}{\ell} - i} \underset{\approx}{A} \frac{1}{\underset{\approx}{P} + \underset{\approx}{\ell} - i} \gamma_5 \frac{1}{\underset{\approx}{\ell} - i}\right\} \tag{71}$$

The integral, after taking indicated spur sum, is

$$J(K) = -8\pi^2 \underset{\sim}{A}\cdot\underset{\sim}{P} \int_0^1 dx \int_0^x dy \tag{72}$$

$$\left\{\frac{(1 - x)\phi(K) + 2x + 1}{2\phi(K)} + (2 - 3x/2)\left[\ln \frac{\phi(K)}{\kappa^2} + 3/2\right]\right\}$$

where

$$\phi(K) = 1 - x(1 - x)\kappa^2 - 2y(1 - x)\,\underset{\sim}{K}\cdot\underset{\sim}{P}$$

and the final momentum $P = K + P'$ has been taken to satisfy the relation for a free particle

$$P^2 + \kappa^2 = 0$$

Subtraction of the charge renormalization term J(0) leaves the finite expression

$$J(K) - J(0) = -8\pi^2 \underset{\sim}{A}\cdot\underset{\sim}{P} \int_0^1 dx \int_0^x dy \left\{\frac{2x + 1}{2}\left(\frac{1}{\phi(K)} - \frac{1}{\phi(0)}\right) + (2 - 3x/2)\ln\frac{\phi(K)}{\phi(0)}\right\} \tag{73}$$

$$\cong 4/3\,\pi^2\, \underset{\sim}{A}\cdot\underset{\sim}{P}\,\kappa^4$$

Application to diagram A_2, Figure 7, gives

$$M_4(A_2) = e'/e\; M_2(A) \tag{74}$$

where

$$e'/e = g^2/16\pi^2 \; 4/3 \; \kappa^4$$

Since $\kappa^2 \sim 1/40$, the contribution from this higher order process is again less than 1 percent.

Diagram IV, Figure 8, also gives the anomalous magnetic moment terms for a static field. The expression to be inserted for such a vertex graph, with an initial proton, is

$$\frac{-g^2 e}{(2\pi)^4} \int d^4 \ell \, \gamma_5 \, \frac{1}{\underset{\approx}{\ell} - i} \, \gamma_5 \, \frac{2A \circ (I - \ell)}{\left[(I - \ell)^2 + \kappa^2\right]\left[(F - \ell)^2 + \kappa^2\right]} \tag{75}$$

For an initial neutron, the interacting meson has a negative charge and this expression changes sign. The integral is

$$\pi^2 \int_0^1 dx \int_0^x \left\{ \underset{\approx}{A} \left[\ln \frac{\lambda^2}{\phi(K)} - 3/2 \right] + 2y \left[\frac{-(x-y)\underset{\approx}{I} - (1-x)\underset{\approx}{F}}{\phi(K)} + i \right] \underset{\sim}{A \cdot I} \right\} \tag{76}$$

where

$$\phi(K) = (1-y)\kappa^2 + y\left[1 + (1-y)I^2 + 2\,\underset{\sim}{I \cdot K}\,(1-x)\right]$$

If one now uses the identity

$$\underset{\sim}{A \circ I} = \frac{\underset{\approx}{AK}}{2} + \underset{\approx}{A}i + \frac{(\underset{\approx}{F} - i)}{2}\underset{\approx}{A} + \underset{\approx}{A}\frac{(\underset{\approx}{I} - i)}{2} \tag{77}$$

and subtracts the term for which K is zero and the nucleons satisfy the Dirac equation for a free particle, i.e.

$$\psi_F^\dagger (\underset{\approx}{F} - i) = (\underset{\approx}{I} - i)\psi_I = 0$$

$$I^2 = F^2 = -1$$

one obtains the finite expression

$$\pi^2 \int_0^1 dx \int_0^x dy \left\{ \underset{\approx}{A} \ln \frac{\phi(0)}{\phi(K)} + 2y \left[\frac{-(x-y)\underset{\approx}{I} - (1-x)\underset{\approx}{F} + i}{\phi(K)} \right] \underset{\sim}{A \cdot I} + \frac{2y^2 \underset{\approx}{A}}{\phi(0)} \right\} \tag{78}$$

Application to diagram B_2, Figure 7, with $\underset{\sim}{A \cdot I} = 0$ for the initial nucleon at rest and a transverse photon, gives

$$M_4(B_2) = e'/e \; M_2(B) \tag{79}$$

If we examine these matrix elements, it is apparent that the corrections from diagrams I, IV, and V are equivalent to an interaction by the nucleons with an anomalous moment in Bohr magnetons

$$\text{proton} \qquad g^2/16\pi^2\ (.88 - 1.26) = -.38$$

$$\text{neutron} \qquad g^2/16\pi^2\ (.88 - .32) = -.56$$

These results are to be contrasted with those of Case,[5] who also calculated the anomalous moments for a static field, using pseudoscalar meson theory. He found that for charge symmetric theory, the anomalous moments were

$$\text{proton} \qquad .65\ \ g^2/16\pi^2$$

$$\text{neutron} \qquad -1.61\ \ g^2/16\pi^2$$

where the notation has been adjusted to agree with that used here.

We find therefore that the correction of order eg^3 is the sum of the contributions from the graphs A_1, A_2, B_1, B_2, C_2, C_3. The first two give corrections which are less than 1 percent and can be neglected relative to the others. The remaining correction is

$$M_4 = g^2/16\pi^2\ \left[.88 - 1.26\right] M_2(B) - g^2/16\pi^2\ \left[-.88 + .32\right]\ M_2(C)$$

$$= 0.18\ g^2/16\pi^2\ M_2(B)$$

Since $g^2/4\pi$ is about 4π, we see that the contribution from the fourth order terms is about one-sixth as large as that from the lowest order. The smallness of the total effect, however, is due to near cancellation of the large contributions from the individual effects. This cancellation is probably fortuitous and cannot be expected to be repeated for the next order processes. We also see that the corrections do not affect the predominance of nucleon moment interactions, characteristic of the pseudoscalar theory, and that this is probably true if effects of even higher order are included.

V Conclusions

We have seen that certain features of the process of production of mesons by photons are nearly independent of the nature of the coupling of where

$$e'/e = g^2/16\pi^2 \int_0^1 dx \int_0^x dy \left[\frac{2y^2}{y^2 + (1-y)\kappa^2} + \ln \frac{\phi_0}{\phi} \right]$$

$$= .88\ g^2/16\pi^2$$

Diagram V, Figure 8, also gives an anomalous magnetic moment term. The expression to be inserted for the vertex graph is

$$\frac{-g^2 e}{(2\pi)^4} \int d\,4\ell\, \gamma_5 \frac{1}{F - \ell - i} A \frac{1}{I - \ell - i} \gamma_5 \frac{1}{\ell^2 + \kappa^2} \tag{80}$$

The integral is

$$2\pi^2 \int_0^1 dx \int_0^x dy \left\{ \frac{\left[-F + i + yF + (x-y) I\right] A \left[I - i - yF - (x-y) I\right]}{2\phi} + A/2\ (\ln \lambda^2/\phi - 3/2) \right\} \tag{81}$$

where

$$\phi\,(K) = x + (1-x)\kappa^2 - 2(1-x)(x-y)\ F\cdot K + x(1-x)\ F^2$$

Subtraction of the renormalization term, using

$$\psi_F\,(F - i) = (I - i)\,\psi_I = 0$$

and setting K = 0 leaves the finite expression

$$\pi^2 \int_0^1 dx \int_0^x dy \left\{ \frac{\left[-F + i + yF + (x-y) I\right] A \left[I - i - yF - (x-y) I\right]}{\phi\,(K)} - \frac{x^2 A}{\phi(0)} + A \ln \frac{\phi(0)}{\phi(K)} \right\} \tag{82}$$

Application to the diagram C_3, Figure 7, taking $A\cdot F = 0$, gives

$$M_4(C_3) = e'/e\ M_2(C) \tag{83}$$

where

$$e'/e = .32\ g^2/16\pi^2$$

and

$$M_2(C) = ge\, \psi_F^\dagger \frac{KA}{2\ I\cdot K}\, \psi_I\, \phi$$

This is equivalent to a magnetic moment interaction for the neutron.

mesons to nucleons. In particular one can expect through the measurement of the distribution in energy and angle of photo-mesons at energies above a few Mev:

1. To determine with some confidence the spin of the meson, through the characteristic behaviour of particles with a magnetic moment in the electromagnetic field. The strong energy dependence of the coupling of spin-one mesons to photons has been used previously by Christy and Kusaka[9] to demonstrate that cosmic ray mesons cannot be of this type.

2. To examine the distribution of mesons about the nucleon, since if the distribution extends to distances of the order $\hbar/\mu c$, one would expect that the electric-dipole terms would be predominant in the coupling to the electromagnetic field. Only if the meson distribution is singular, i.e., closely bound to the nucleons, can one expect the interaction with the circulating currents of the magnetic moments to become relatively important.

3. Through a measurement of the plus-to-minus ratio to determine the effects of a meson magnetic moment and the nature of the anomalous moments for the nucleons. The anomalous moments will be nearly independent of the frequency of the electro-magnetic field only if they are confined to regions small compared with the wave-length of the radiation.

The further details of the calculation made with perturbation theory can probably not be accepted quantitatively since higher order corrections are not negligible. For example, explicit calculation of the higher order effects for the pseudoscalar theory predicts large anomalous magnetic moment interactions for the nucleons including corrections, as large as the lowest order terms, corresponding to polarization of the vacuum by the nucleons. The anomalous moments are considerably

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[9] Phys. Rev. 59, 414 (1941)

smaller than those which the nucleons exhibit in a static field and different in sign for the proton. The importance of the corrections for the pseudoscalar theory appears to be the result of the close binding of the mesons to the nucleons, which in turn leads to the characteristic predominance of the nucleon magnetic moment interactions in the production process. Higher order effects, however, do not change the general features of the lowest order results. There, therefore, seems to be ground for hope that the lowest order calculations for all of the theories may give qualitatively correct results.

The experimental results obtained by the Berkeley workers,[2] although still preliminary, indicate that mesons of 40-100 Mev produced by photons on hydrogen are nearly isotropic in angular distribution from 45-135 degrees in the laboratory system. The cross section for mesons produced in this energy and angular range from hydrogen appears to decrease slowly with increasing meson energy. The magnitude of the total cross section, about 10^{-28} cm^2, can be fitted for the theories giving roughly isotropic angular spectra at low energies (Figures 3 and 5) with a value of the coupling constant $\frac{g^2}{4\pi\hbar c}$ of about 40 for pseudoscalar mesons and 0.4 for pseudovector mesons. The mesons produced from carbon show an excess of negative over positive mesons in the ratio of 1.7 ± 0.2 in the energy range of 30-100 Mev observed at 90 degrees to the photon beam direction. This is in agreement with the ratio given in Figure 1, with the assumption that the neutron does not interact with the electromagnetic field. However, it is not clear that the complications of the binding of the nucleons in the carbon nucleus are unimportant. A more detailed study of the dependence on energy and angle of the negative-to-positive ratio is being carried on at present.

These results all seem to indicate that the general features of the pseudoscalar meson theory are correct, i.e., the zero spin of the meson and the close binding of the meson cloud to the nucleons. It is hoped that the degree of validity of these conclusions will be indicated as the experimental work is completed.

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Titles of Figures

1. The ratio of the cross sections for the production of negative and positive mesons. The solid curves are calculated with the assumption that the neutron does not interact with the electromagnetic field, the dotted curves with the **assumption** that the neutron interacts with the anomalous moment observed in a static field. The energies indicated in this and the following figures are the meson energies, which are related to the photon energy as shown in Figure 9.

2. Angular distribution of scalar mesons from dK/K photon spectrum.

3. Angular distribution of pseudscalar mesons from dK/K photon spectrum.

4. Angular distribution of vector mesons from dK/K photon spectrum.

5. Angular distribution of pseudovector meson from dK/K photon spectrum.

6. Ratio of cross sections for production of positive and negative mesons vector and pseudovector theory.

7. Diagrams for photo-meson production. Diagrams A,B, C are for production in lowest order ge; diagrams with subscripts are for production in order g^3e.

8. Details of higher order corrections.

9. Relation between meson and photon energy as a function of the angle of the meson with photon beam direction.

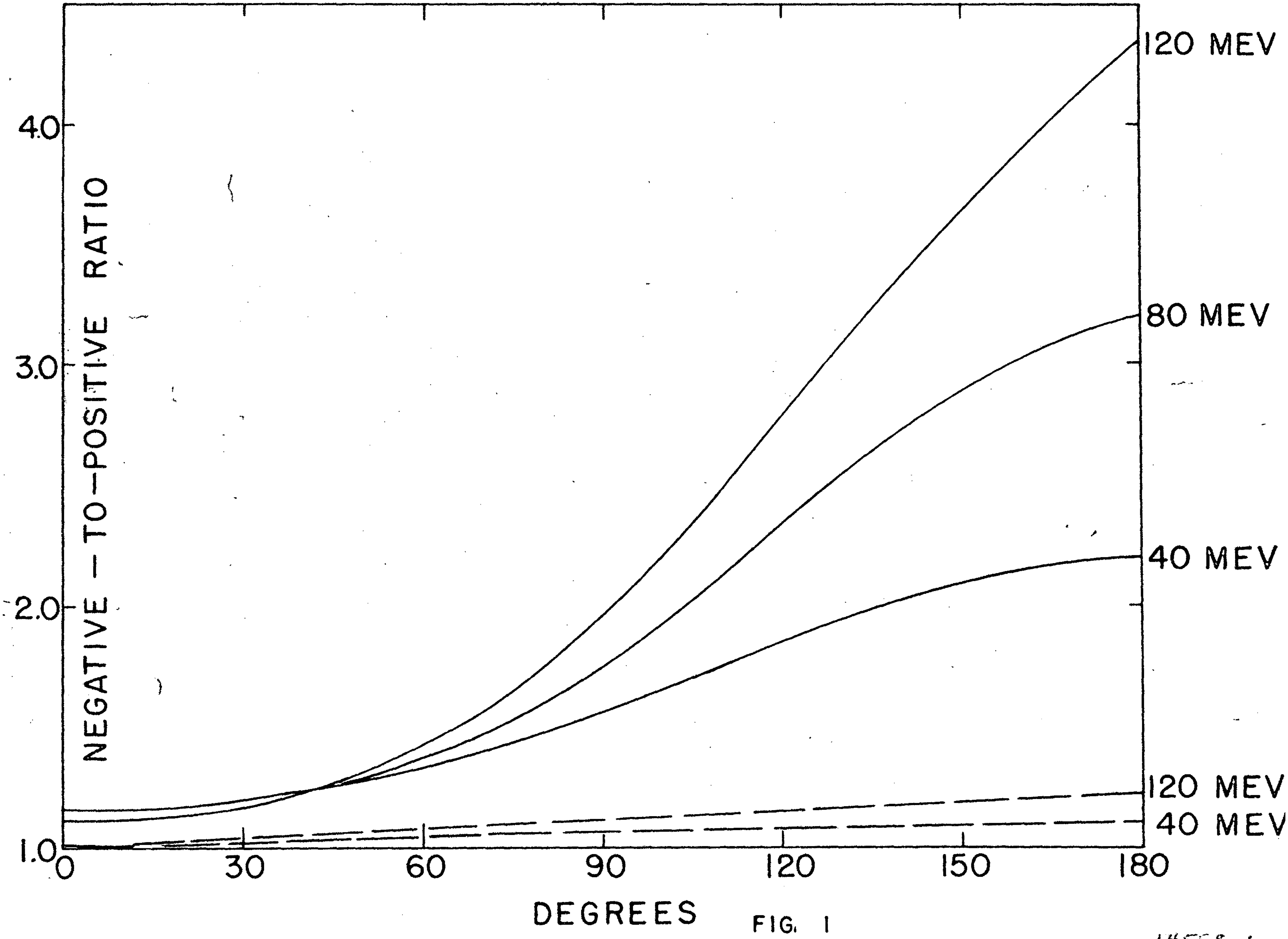

NEGATIVE — TO—POSITIVE RATIO
4.0
3.0
2.0
1.0
0
30
60
90
120
150
180
DEGREES
120 MEV
80 MEV
40 MEV
120 MEV
40 MEV

FIG. 1

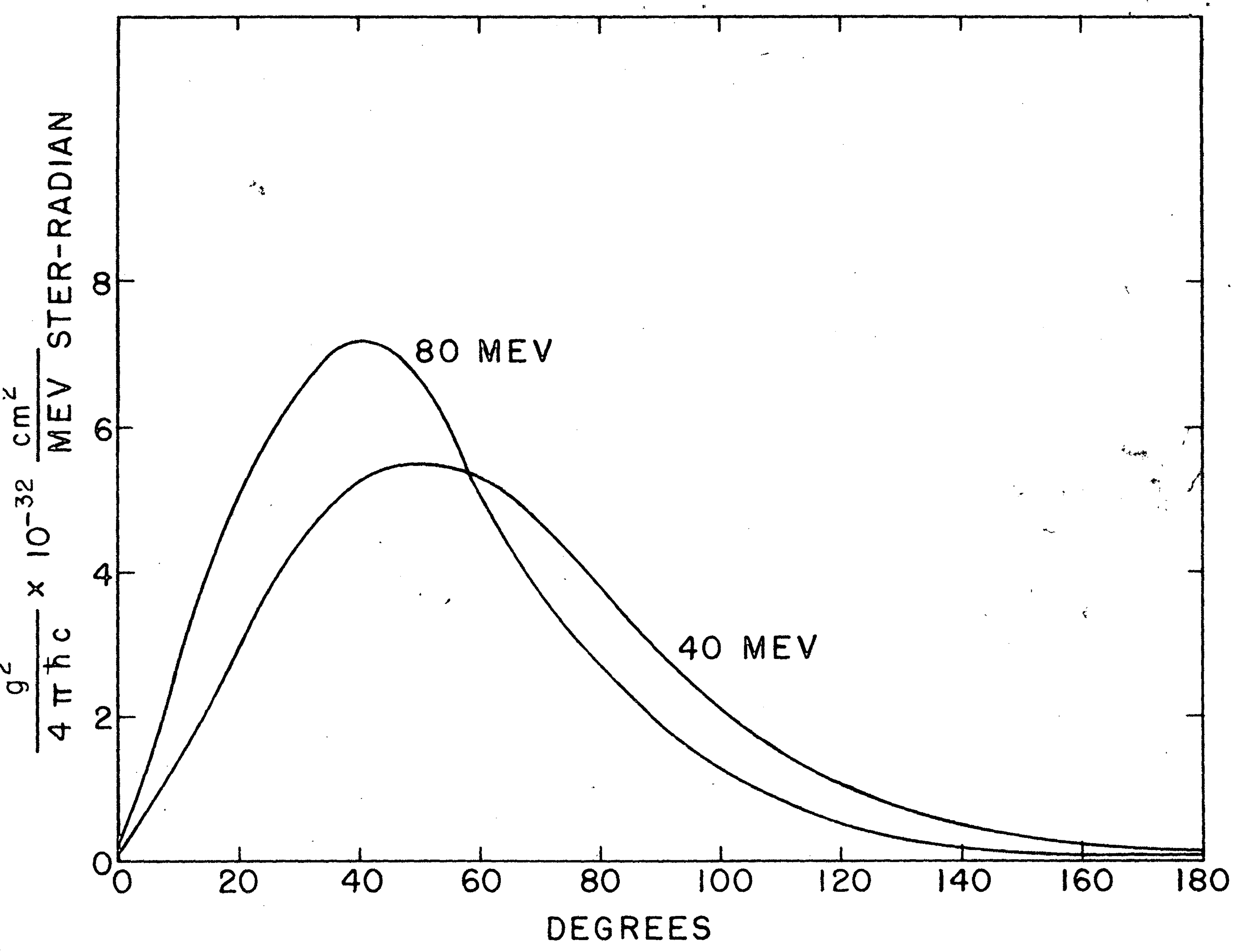
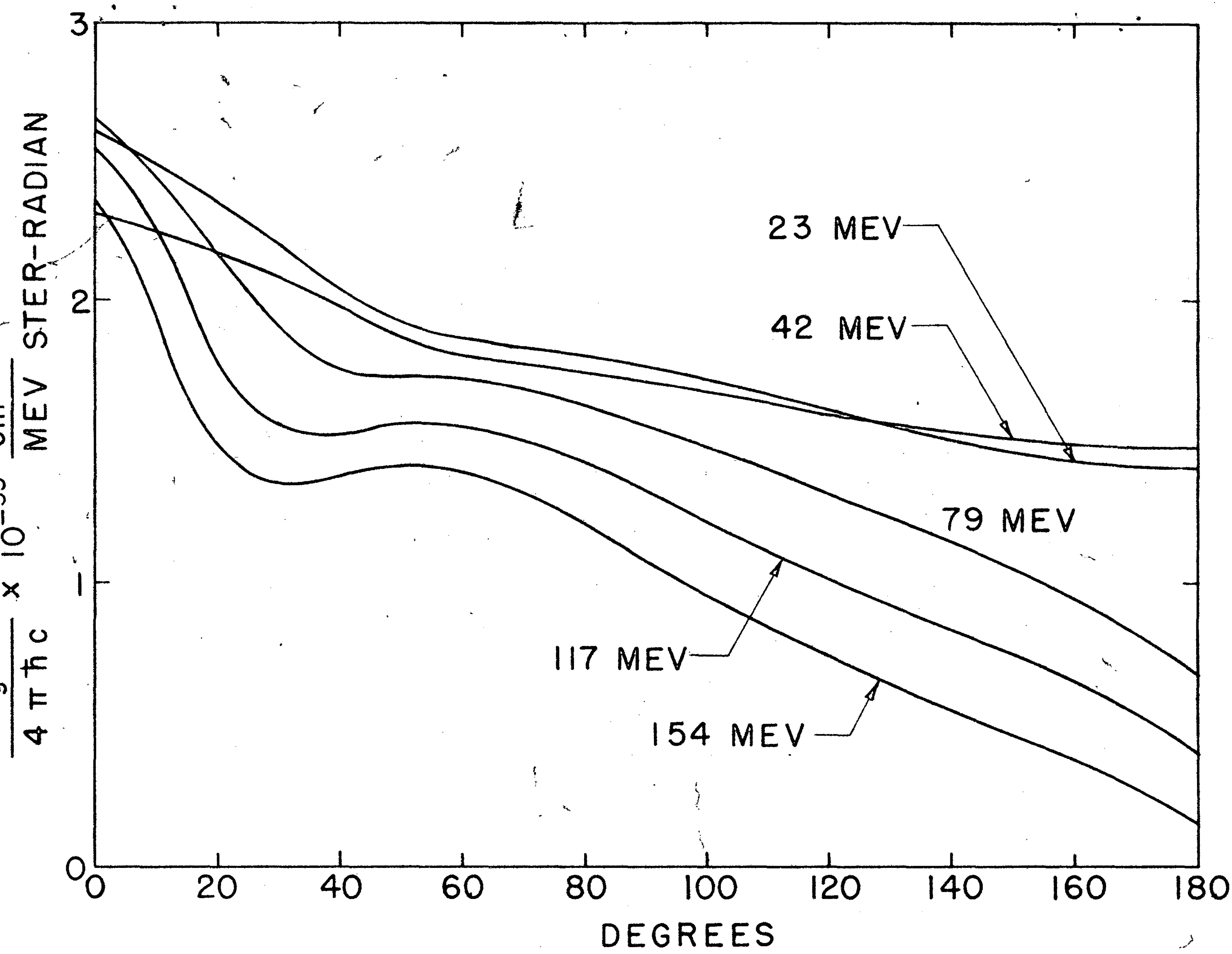

$\frac{g^2}{4\pi\hbar c} \times 10^{-32} \frac{cm^2}{MEV\ STER\text{-}RADIAN}$
8
6
4
2
0
80 MEV
40 MEV
0
20
40
60
80
100
120
140
160
180
DEGREES

FIG. 2

14563-1



3
2
1
0
0
20
40
60
80
100
120
140
160
180
DEGREES
$\frac{g^2}{4\pi\hbar c} \times 10^{-33} \frac{cm^2}{MEV\ STER\text{-}RADIAN}$
23 MEV
42 MEV
79 MEV
117 MEV
154 MEV

FIG. 3

14560-1

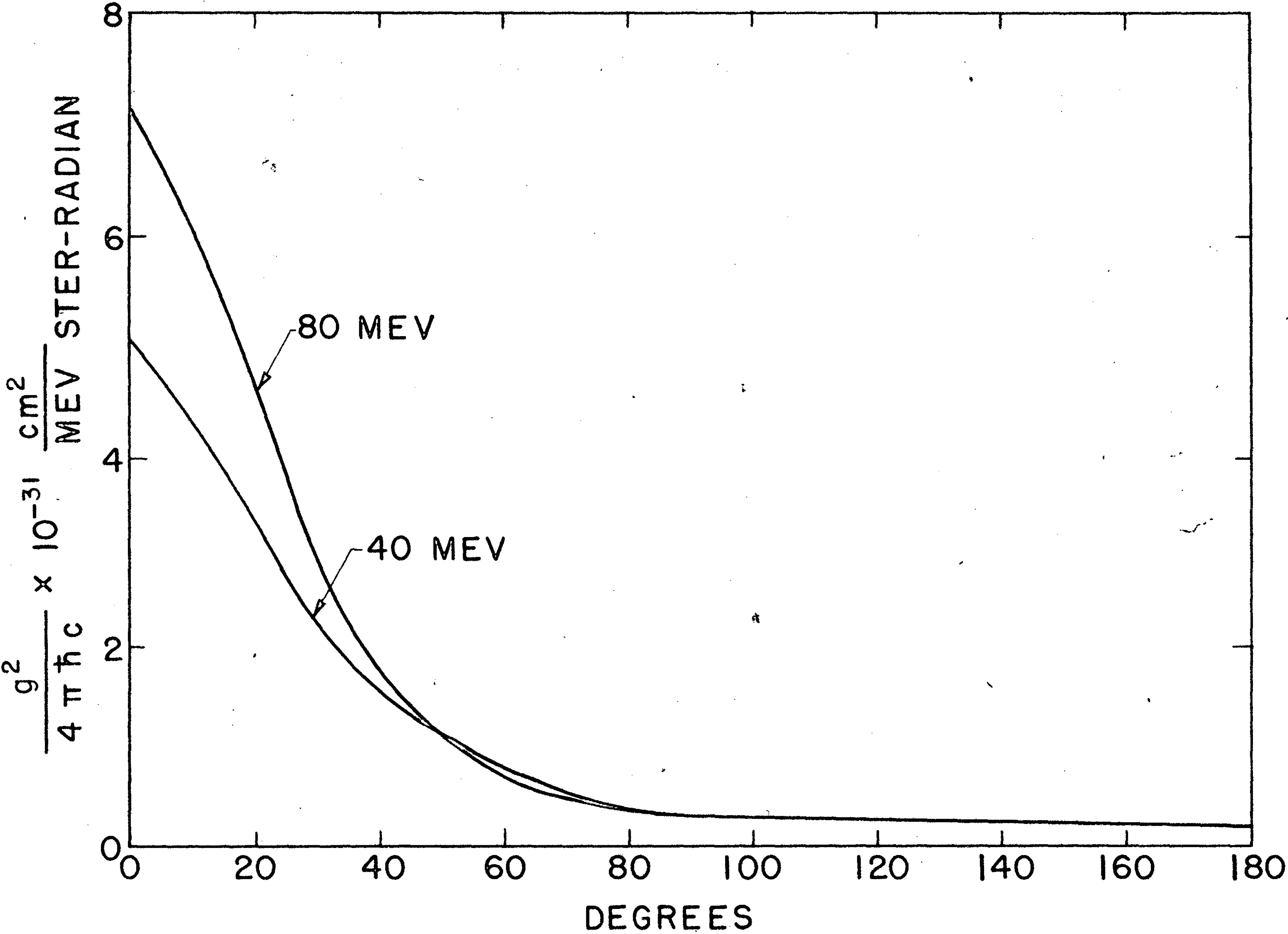
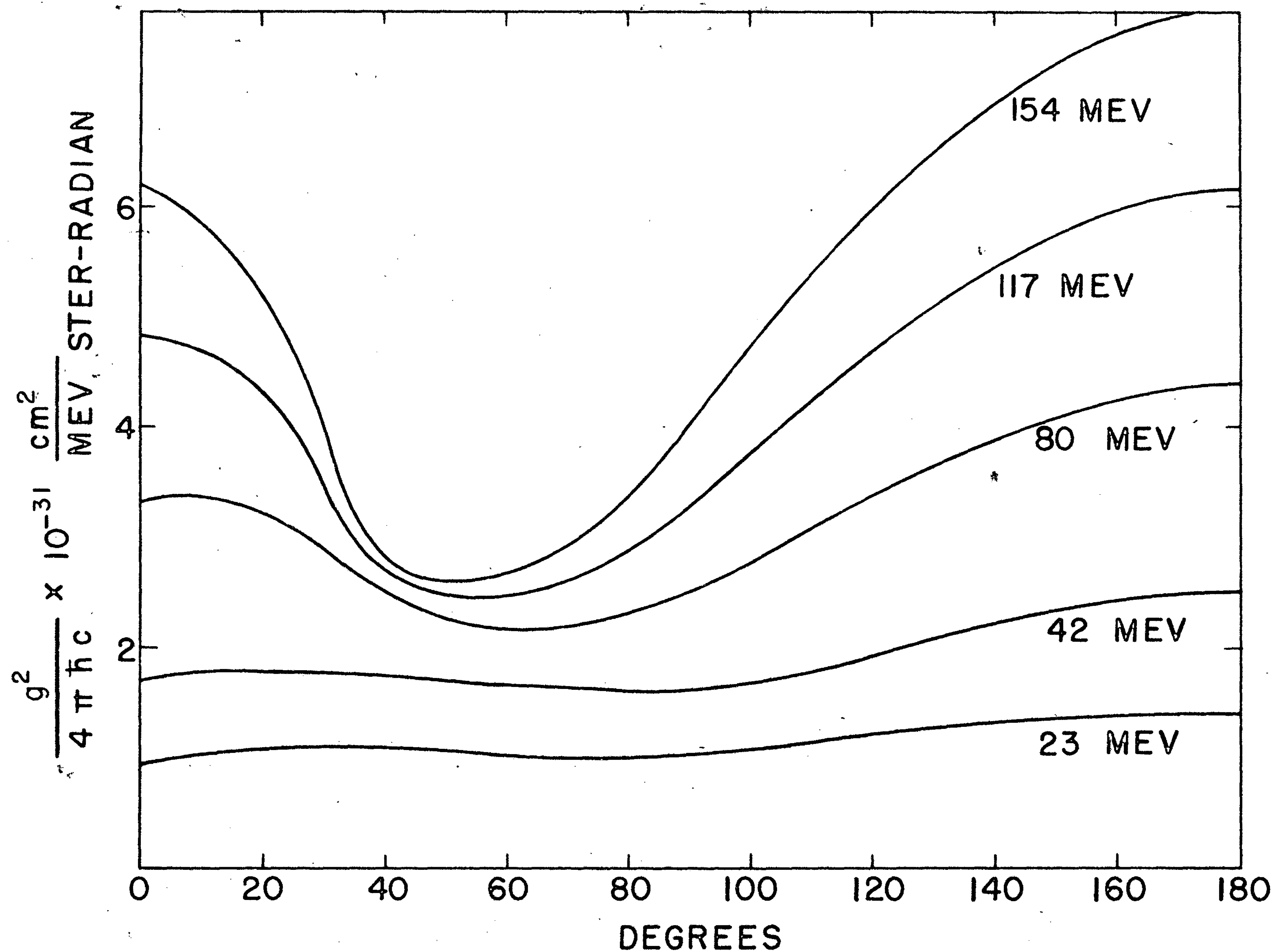

8
6
4
2
0
$\frac{g^2}{4\pi\hbar c} \times 10^{-31}$ $\frac{cm^2}{MEV}$ STER-RADIAN
80 MEV
40 MEV
0
20
40
60
80
100
120
140
160
180
DEGREES

FIG. 4

14561



154 MEV
117 MEV
80 MEV
42 MEV
23 MEV
$\frac{g^2}{4\pi\hbar c} \times 10^{-31} \frac{cm^2}{MEV\ STER\text{-}RADIAN}$
6
4
2
0
20
40
60
80
100
120
140
160
180
DEGREES

FIG. 5

14562-

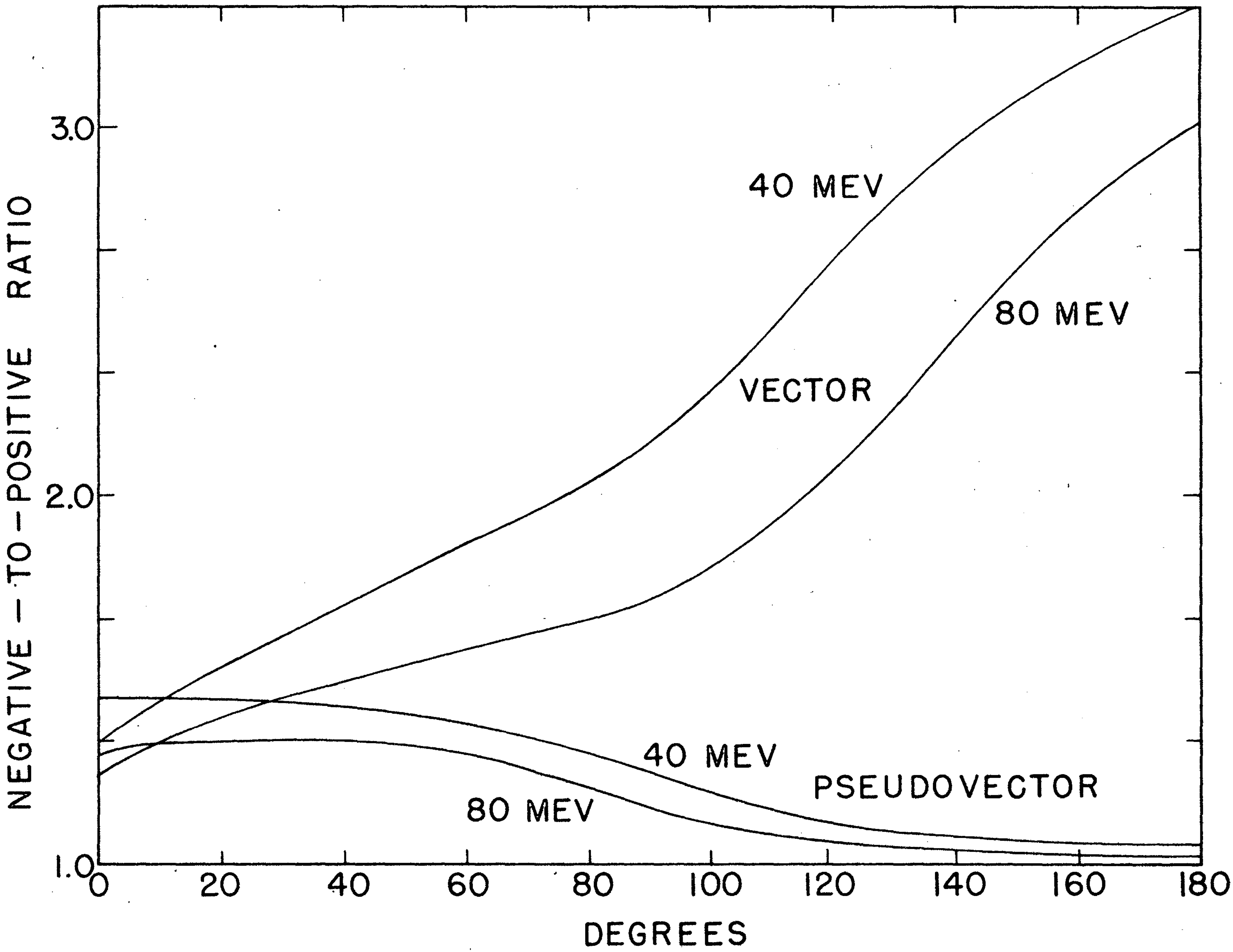

NEGATIVE - TO - POSITIVE RATIO
3.0
2.0
1.0
0
20
40
60
80
100
120
140
160
180
DEGREES
40 MEV
80 MEV
VECTOR
40 MEV
PSEUDOVECTOR
80 MEV

FIG. 6

14559-1

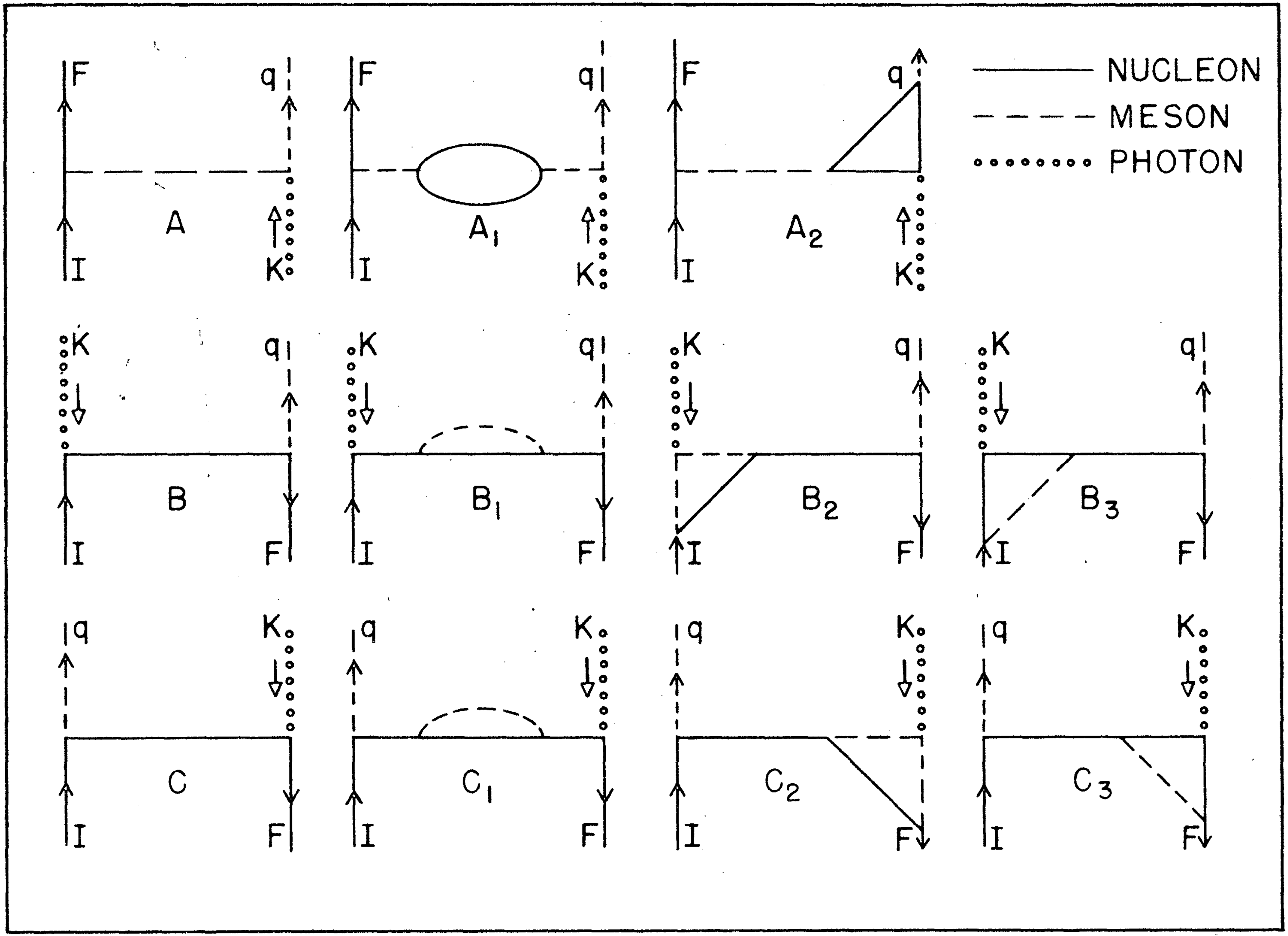
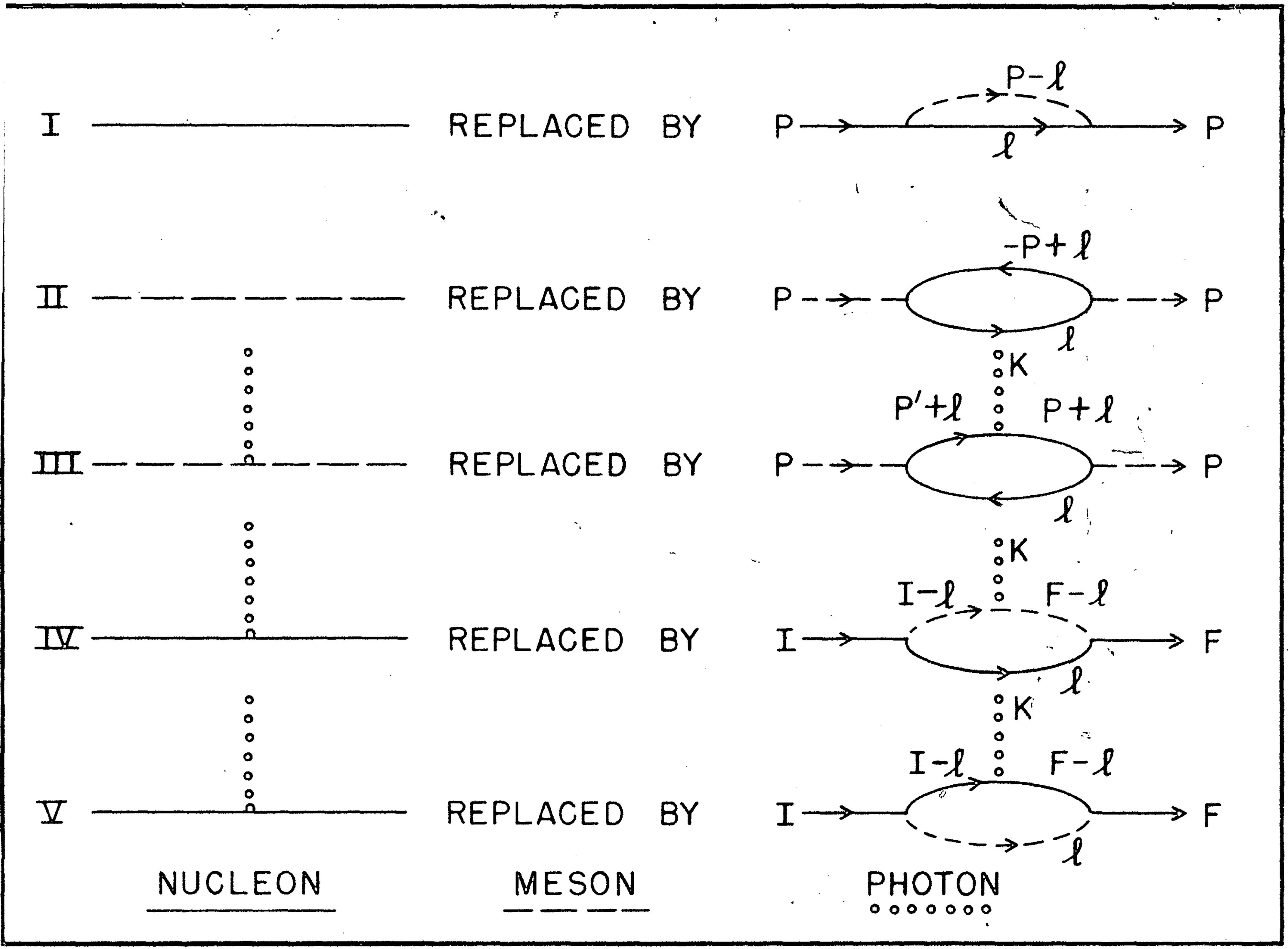


FIG. 7

14565



I
REPLACED BY
P
P−ℓ
ℓ
P
II
REPLACED BY
P
−P+ℓ
ℓ
P
K
III
REPLACED BY
P
P′+ℓ
P+ℓ
ℓ
P
K
IV
REPLACED BY
I
I−ℓ
F−ℓ
ℓ
F
K
V
REPLACED BY
I
I−ℓ
F−ℓ
ℓ
F
NUCLEON
MESON
PHOTON

FIG. 8

14566-1

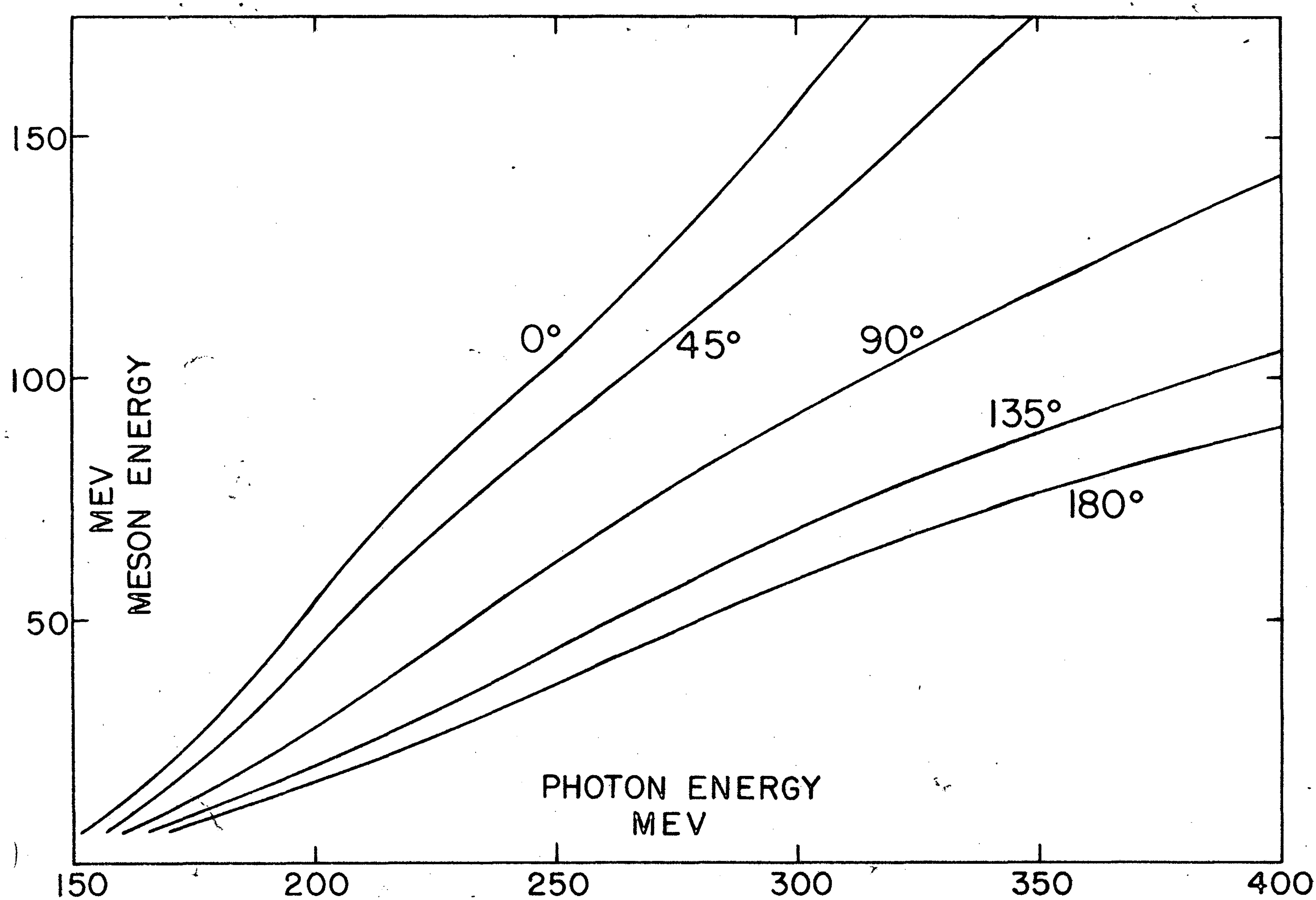

0°
45°
90°
135°
180°
MEV
MESON ENERGY
PHOTON ENERGY
MEV
150
100
50
150
200
250
300
350
400

FIG. 9

14579-

Production of π-mesons in Nucleon-Nucleon Collisions

I Introduction

A problem of considerable interest in meson theory is the production of mesons in nucleon-nucleon collisions. The intimate connection predicted by meson theory between this process and ordinary nucleon-nucleon scattering provided an excellent and basic opportunity for testing the fundamental assumptions of meson theory. The meson theory of nuclear forces assumes that π-mesons, found to interact strongly with nuclei, are responsible for the coupling between nucleons. Since this coupling via the meson field implies the existence of virtual mesons in the mutual field of two nucleons, it should be possible, if sufficient energy is available, for virtual mesons to be materialized as free and observable particles. A comparison of the predictions of the theory with the experimental measurements of nucleon-nucleon scattering and of meson production should indicate whether this basic assumption is quantitatively correct.

Experiments are now being carried out at Berkeley[1] which give information about the production of charged and neutral mesons in neutron-proton and proton-proton collisions. It is of interest to consider in a systematic manner the theory of the production in order to understand what can be learned from these experiments.

The production of mesons in nucleon-nucleon collisions has been studied in considerable detail by a number of theoretical workers.[2] The most thorough theoretical analyses which have been made can be divided into two types: 1) an application of the meson field theory of nuclear forces to the problem considered as a third order process, and 2) an attempt to describe phenomenologically the scattering of the nucleons associated with the meson emission and only the meson emission itself by meson field theory. The first method suffers from the well-known failure of meson theory to describe nuclear forces in more than a qualitative way and from the more general failure of perturbation theory in the weak coupling approximation applied to problems in which the coupling is not weak. This method can, however, be applied rigorously in the lowest order and with this limitation gives a logically complete description of the process. Also, because of the close relation between high energy nucleon-nucleon scattering (virtual meson exhanges) and meson production (virtual meson exchanges together with a real meson emission), a theory which gives qualitatively correct results for the former process might be expected to be correct to a similar approximation for the latter.

The second method based on a more phenomenological approach decribes the meson emission on the basis of field theory but separates the nucleon-nucleon scattering, which gives the momentum transfer necessary for over-all energy and momentum conservation, and attempts to describe this in terms of the experimentally measured potentials. This method is inadequate in as far as processes can occur in the meson production which cannot be described in terms of the scattering process preceded or followed by meson emission. Such processes are for example the interruption of a virtual meson exchange by a real meson emission. These precesses are equivalent to scatterings which take place far off the energy shell, i.e., where energy and momentum are related very differently from the relationship for free particles. Only if such processes give unimportant contributions can the phenomenological approach be approximately correct. However, if this assumption is made, the calculation can be made, the calculation can be made in a manner much more independent of meson field theory than is the third order perturbation calculation.

The rigorous treatment of meson production as a third order process is complicated by mathematical difficulties in integrating the cross-sections over the momenta of the final nucleons to give a result which depends only on the meson momentum. This problem has been partially solved by Cecille Morette[2] for pseudoscalar mesons with pseudoscalar coupling who, however, ignored the effects of the Pauli principle, and so introduced a rather large error (about 50%) in the cross-section near threshold. Her expressions for the cross-sections also have been averaged over meson and nucleon charges and so do not separately give the cross-sections for charged and neutral mesons for neutron-proton and proton-proton collisions. Since these separate quantities are those which can be determined experimentally, it is of interest to calculate them. We therefore have in Section II considered the calculation of the transition matrix elements on the basis of rigorous third order perturbation theory, and obtained the expressions for scalar theory, pseudoscalar theory with pseudoscalar and pseudovector coupling, and vector theory with vector coupling. The calculations are made in the center-of-mass system for energies near threshold where the velocities of the final particles are small. Corrections of the order of v^2/c^2 for the final nucleons and meson are neglected, so the results are only applicable for incident nucleon energies of 350 to 400 Mev corresponding to maximum meson energies in the center-of-mass system of 23 to 44 Mev.

The second method of calculation, treating the nucleon-nucleon interaction phenomenologically, has been carried out by Marshak and Foldy[3] for scalar mesons and for pseudoscalar mesons with pseudovector coupling. They found a zero cross-section for scalar mesons and a very small cross-section (about 10^{-31} at 350 Mev) for pseudoscalar mesons. Both of these results disagree with the experimentally observed[1] large cross section of the order of 10^{-28} cm^2, however, their treatment suffers from an unrealistic choice of the nuclear potentials since they assumed charge independence of the forces at the large momentum transfers necessary for meson production. Experimentally, high energy neutron-proton and proton-proton scattering are qualitatively different. Approximate agreement with the experimental results at high energy is given by the potentials[4]

$$-g_{NP}(1 + P_x)/2 \exp(-\mu r)/4\pi r \quad \text{for N-P scattering,} \quad g^2_{NP}/4\pi = .479$$

$$-g^2_{PP}(\sigma_1 \cdot \nabla/\mu)(\sigma_2 \cdot \nabla/\mu) \exp(\mu r)/4\pi r \quad \text{for P-P scattering,}$$

$$g^2_{PP}/4\pi = .0418$$

where P_x is the space exchange operator. The choice of the P-P potential is not unique; any potential which predicts a very singular and strong interaction in P states would give approximate agreement with the high energy scattering. This potential is chosen since it corresponds to pseudoscalar theory with pseodovector coupling. Using these potentials it is of interest to carry out calculations similar to those done by Marshak and Foldy to see if sufficiently large cross sections can be obtained to explain the experimental results.

This method is applied in Section III to the calculation of the transition matrix elements for scalar theory, vector theory with vector coupling, and pseudoscalar theory with pseudovector coupling. For pseudoscalar coupling, it is possible to generalize the P-P potential to its relativistic form using the equivalence between pseudovector and pseudoscalar coupling pointed out by Nelson[5]. The equivalence theorem gives for the potential the result

$$g^2_{PP}(2M/\mu)^2 \quad (\gamma_5)_1(\gamma_5)_2 \quad \exp(-\mu r)/4\pi r$$

The calculation can then be carried out using this type of interaction. However, it will be shown that for pseudoscalar coupling, processes occur in the production of mesons which cannot be described in terms of a potential interaction and which give the largest contribution to the matrix element. A treatment on the basis of a potential model therefore is not justified. For such a theory the methods of third order perturbation theory should give more reliable results. These phenomenological calculations are made in the center-of mass system and are restricted to energies near threshold, i.e., less than 400 Mev for the incident nucleon in the laboratory system.

An additional important effect which has been ignored in these calculations must be considered before comparison with experiment can be made. In the ordinary approach to the problem of meson production, the approximation is made of replacing the wave functions of the initial and final nucleons by plane waves. This approximation is fairly good for the initial nucleons since it is equivalent to the use of Born approximation in high energy scattering. However, because of the large amount of energy carried off by the meson in its rest mass, the final nucleons are moving slowly, particularly near threshold. It was pointed out by E.W. Hart and Dr. Geoffrey Chew that as a consequence of this it is possible, if the final nucleons are a neutron and a proton, deuteron may be formed. In addition, even if the nucleons do not form a deuteron, use of the plane wave approximation for the final nucleons gives very inaccurate results. In Section IV these effects are considered and shown to be important.

II Third Order Field-Theoretic Calculation of Transition Matrix Elements

The third order matrix element may be written down directly using the methods of Feynman and Dyson.[6] A Feynman-Dyson diagram for meson production is given in Figure 1. Seven additional diagrams may be obtained for emission of the meson by the three other nucleons and for corresponding diagrams in which the two initial or final nucleons are interchanged. The matrix element for the diagram of Fig. 1 is

$$f^{\lambda}(f^{\rho})^2\bar{\Psi}(3)\left[\left(U^2_{\mu}\frac{\tau^{\lambda}\quad 1\quad \tau^{\rho}}{(p_1-q)_{\mu}\gamma_{\mu}-iM}U^1_{\nu}\right)\right]\Psi(1)\;\frac{\bar{\Psi}(4)\;U^1\tau^{\rho}\Psi(2)}{(p_4-p_2)^2+\mu^2}\;\phi^{\lambda}_{\mu}(q)$$

where $U^1_{\mu} = U^2_{\mu} = \delta_{\mu o}$ for S(III)* and $= \gamma_5\,\delta_{\mu o}$ for PsPs(III); $U^1_{\mu} = \delta_{\mu o}\gamma_5/\mu\,(P_4 - P_2)_{\nu}\gamma_{\nu}$ and $U^2_{\mu} = \delta_{\mu o}\gamma_5/\mu\, q_{\nu}\gamma_{\nu}$ for Ps.Pv(III); $U^1_{\mu} = U^2_{\mu} = \gamma_{\mu}$ for V.V(III); $\phi^{\lambda}_{\mu} = \phi^{\lambda}\delta_{\mu o}$ for S(III), Ps.Ps(III), and Ps.Pv(III); and $\phi^{\lambda}_{\mu} = \phi^{\lambda}_{\mu}$ for V.V(III). We use the symmetrical isotopic spin notation[7] $f^{\lambda}\tau^{\lambda} = f_1\tau_1 + f_2\tau_2 + f_3\tau_3 + f_4\tau_4$. Charge symmetry requires that $f_1 = f_2 = \sqrt{2}/2f$; the use of both f_3 and f_4 corresponds to two choices of sign for the coupling of neutral mesons to nucleons, since the expectation value of τ_3 is positive for a neutron and negative for a proton, while the expectation value of τ_4 is positive for both nucleon states. The seven additional matrix elements can be obtained by various permutations of P_1, P_2, P_3, and P_4 and changes of the sign of q_{μ}. These matrix elements are approximately in the center-of-mass system

$$f^{\lambda}(f^{\rho})^2\left[\Psi^*(3)\tau^{\lambda}\tau^{\rho}U_1\Psi(1)\right]\left[\Psi^*(4)\tau^{\rho}U_2\Psi(2)\right]/(2M^2\mu^5)$$

where $U_1 = U_2 = 1$, S(III); $U_1 = 1$, $U_2 = \sigma\cdot p_2\,\mu/4M^2$, Ps.Ps(III); $U_1 = \sigma\cdot q\,\sigma\cdot p_1/\mu M$, $U_2 = \sigma\cdot p_2/M$, Ps-Pv(III); $U_1 = |\vec{q}|/\mu$, $U_2 = 1$, V.V(III).

The total transition matrix elements given by these expressions, squared and averaged over the spins of the nucleons are given by the expressions

$$|H_{if}|^2 = G^6_{III}/(2M^2\mu^5) \qquad \text{S(III), Ps.Ps(III)}$$

$$= G^6_{III}/(2M^2\mu^5)\;(|\vec{q}|^2/\mu^2) \qquad \text{Ps.Pv(III), V.V(III)}$$

where the values of G^6_{III} for the various theories and processes are given in Table I. It is apparent that the results are sensitive to the relative

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*We shall in what follows refer to the 3rd order computations for scalar, pseudoscalar with pseudoscalar or pseudovector coupling, and vector theory with vector coupling as S(III), Ps.Ps(III), Ps.Pv(III), and V.V(III) respectively.

choice of sign for the coupling of neutral mesons to neutrons and protons, since the use of f_3 with f_4 set equal to zero corresponds to the opposite choice of sign for the couplings, while the use of f_4 with $f_3 = 0$ corresponds to the same choice of sign of the couplings. It is interesting to observe that the only theory which predicts production of neutral mesons in P-P collisions with a cross section comparable with that for charged mesons is Ps.Ps(III). For the other theories cancellations occur between matrix elements corresponding to emission of the final meson by the initial or final nucleons which make the cross section the order of $(\mu/M)^2$ smaller than for charged production.

III Phenomenological Calculation of Production

In this type of calculation, we assume that the process of production can be described by meson emission preceded or followed by the nucleon-nucleon scattering. This, however, is not quite true since virtual processes occur which cannot be described in terms of such a separation. An analysis of the process considered as taking place in third order indicates three basic ways in which a typical process can take place. These are

$$P_1 + P_2 \rightarrow \begin{Bmatrix} P_1 + P' + \omega' \rightarrow P' + P_3 & (1) \\ P_2 + P' + \omega' \rightarrow P' + P_3 & (2) \\ P_2 + P' + \omega' \rightarrow P_2 + P_4 + \omega + \omega' & (3) \end{Bmatrix} \rightarrow P_3 + P_4 + \omega$$

It is clear that the virtual meson (ω') exchange in processes (1) and (2) is analogous to that which occurs in scattering; therefore the exchange can be replaced by the effects of the potential which predicts the scattering. However, in process (3) the real meson (ω) is emitted between the emission and reabsorption of the virtual meson. Such a process cannot occur in the scattering of real nucleons; therefore its effect cannot be given in terms of the potential model. However, it can be shown rather easily that such processes will give a rather small contribution, at least for theories in which negative energy states are not important for the virtual nucleons. The energy denominator for these matrix elements is given by

$$1/(E_o - E'_1)(E_o - E'_2)$$

where E_o is the total energy and E'_1 and E'_2 are the energies of the two intermediate states. These denominators are (ignoring negative energy states) for processes (1) and (2)

$$1/(E_o/2 - E_4 - \omega')(E_o - 2E_4) \cong 1/(\omega/2 - \omega')\,\omega$$

and for process (3)

$$1/(E_o/2 - E_4 - \omega')(E_o/2 - \omega - \omega' - E_3) \cong 1/(\omega/2 - \omega')(-\omega/2 - \omega')$$

Now since near threshold the energy ω' of the virtual mesons is much larger than the energy ω of the real meson, these are approximately $-1/\omega\omega'$ for (1) and (2), and $1/\omega'^2$ for (3). Therefore the contribution from process (3) is smaller than the contribution from processes (1) and (2) in the ratio $\omega/2\omega'$. Since the momentum of the virtual meson is equal to the difference of the momentum of the initial and final nucleons which at threshold is about $(\mu M)^{1/2}$, the ratio $\omega/2\omega'$ is about $1/2(\mu/M)^{1/2}$ which is about 20 percent. This contribution is negligible only if the ratio of the masses of the meson and nucleon is small; actually an error of the order of 20 percent in the matrix element can be expected if this term is ignored. A further error arises from the neglect of negative energy states for the virtual nucleons; however, because of the largeness of the energy denominators for such processes, the contribution is negligible for all theories except for pseudoscalar coupling. This case will be discussed in detail below.

We now consider processes leading from the initial state of two nucleons to the final state of two nucleons and a meson. This can take place in two ways, either through a scattering of the two initial nucleons followed by the meson emission, or with the order of these events reversed. We therefore have for the matrix element

$$\int d\vec{r}\, d\vec{r}'\left[\Psi_F^*(\vec{r},\vec{r}')H(\vec{r},\vec{r}')\Psi'(\vec{r},\vec{r}')\right]\int d\vec{r}\, d\vec{r}'\left[\Psi'^*(\vec{r},\vec{r}')\, S_I(\vec{r},\vec{r}')\,\Psi_I(\vec{r},\vec{r}')\right]\Big/E_o-E'$$
$$+\int d\vec{r}\, d\vec{r}'\left[\Psi_F^*(\vec{r},\vec{r}')\, S_{II}(\vec{r},\vec{r}')\Psi''(\vec{r},\vec{r}')\right]\int d\vec{r}\, d\vec{r}'\left[\Psi^{*''}(\vec{r},\vec{r}')\, H(\vec{r},\vec{r}')\,\Psi_I(\vec{r},\vec{r}')\right]\Big/E_o-E''$$

In this expression S_I and S_{II} are the potentials describing the interaction of the initial and final nucleons, H is the operator for meson emission, and E' and E" are the energies of the two intermediate states. The energy denominators are

$$E_o - E' = P_1^2/2M - P_2^2/2M - P'_1{}^2/2M = (P_o^2 - P'^2)/M$$
$$E_o - E'' = P_o^2/M - P''^2/M - \omega$$

A. Scalar meson theory, vector meson theory

We shall now consider the case of two initial protons P_1, P_2 going into a final neutron N_3, proton P_4 and positive meson q. For this case we have the scattering before meson emission in the P-P potential and the scattering after emission in the N-P potential. If we take scalar coupling for the meson, then

$$H = f^\lambda \tau^\lambda \exp(-iq\cdot r)/(2\omega)^{1/2}$$

The case of vector coupling can be considered simultaneously since the coupling is

$$H = f^\lambda \tau^\lambda \gamma_\mu \phi_\mu (\vec{r})$$

where γ_μ is the 4-vector formed from the Dirac matrices. This coupling is approximately for longitudinally polarized mesons

$$f^\lambda \tau^\lambda |\vec{q}|/\mu \exp(-i\vec{q}\cdot\vec{r})/(2\omega)^{1/2}$$

if corrections of the order of v/c for the nucleons are ignored. Therefore we can obtain this result from that for scalar theory by multiplying the matrix element by $|\vec{q}|/\mu$.

We find for the matrix element for S(phen)*

$$-(2)^{1/2} f\, g^2_{PP} M(1 - P_{12}) \frac{(\chi_3^* \vec{\sigma}\cdot(\vec{P}_1 + \vec{P}_4)\chi_1)(\chi_4^* \vec{\sigma}\cdot(\vec{P}_4 + \vec{P}_1)\chi_2)}{(2\omega)^{1/2}(P_1^2 - P_4^2)(\mu^2 + (\vec{P}_1 + \vec{P}_4)^2)}$$

$$+(2)^{1/2} f\, g^2_{NP}(1 - P_{12})\frac{(1 + P_{34})}{2} \frac{(\chi_3^*\chi_1)(\chi_4^*\chi_2)}{(2\omega)^{1/2}\omega(\mu^2 + (\vec{P}_1 + \vec{P}_4)^2)}$$

Near threshold, $P_1 >> P_3$ or P_4. We can also disregard μ^2 relative to P_1^2. We then note that $(1-P_{12})(1+P_{34})(\chi_3^*\chi_2)(\chi_4^*\chi_1) \equiv 0$. Therefore the second term of the matrix element vanishes, and the expression simplifies to

$$- f\, g_{PP}^2 (1-P_{12})(\chi_3^*\sigma\cdot P_1\chi_1)(\chi_4^*\sigma\cdot P_1\chi_2)/M\, \mu^4 (\mu)^{1/2}$$

where we have set $P_1^2/2M = \mu/2$ and $\omega = \mu$, which are their values at threshold.

* We shall refer to the phenomenological treatment of scalar, vector, and pseudoscalar theory with pseudovector coupling as S(phen), V·V(phen), and Ps·Pv(phen), respectively.

A similar expression is obtained for an initial neutron and proton going into two final protons and a meson. For processes involving the scattering of two neutrons, the result depends on the choice of the N-N interaction. However, in the absence of any direct information, we shall assume that this is the same as the P-P interaction. The results for these processes are then identical with those given above.

For the production of neutral mesons, the analysis is similar to that given here. In this case, however, the nucleon charge is unchanged by the meson emission, and the scattering takes place before and after emission in the same potential. We find that cancellation occurs between the terms representing scattering before or after emission so that a zero cross-section is predicted for neutral mesons in either N-P or P-P collisions.

B. Pseudoscalar meson with pseudovector coupling

The analysis for pseudovector coupling can be carried out in an exactly similar way. Here we have

$$H = f^{\lambda}\tau^{\lambda}\sigma\cdot q/\mu \; \exp(-i\vec{q}\cdot\vec{r})/(2\omega)^{1/2}$$

This gives for the production of charged mesons:

$$-f(1-P_{12})(g_{PP}{}^2(\chi_3{}^*\sigma\cdot q\sigma\cdot P_1\chi_1)(\chi_4{}^*\sigma\cdot P_1\chi_2)/\mu^2 - g_{NP}{}^2\frac{(1+P_{34})}{2}(\chi_3{}^*\sigma\cdot q\chi_1)(\chi_4{}^*\chi_2))/M\mu^3(\mu)^{1/2}$$

For neutral mesons from N-P collisions, we have the matrix element

$$g_{NP}{}^2 \; f_3(1-P_{12}P_{34})(\chi_4{}^*\sigma\cdot q\chi_2)(\chi_3{}^*\chi_1)/M\mu^4(2\mu)^{1/2}$$

and for P-P collisions the result is again zero.

C. Pseudoscalar meson with pseudoscalar coupling

For the case of pseudoscalar coupling, we have

$$H = f^{\lambda}\tau^{\lambda}\gamma_5 \; \exp(-i\vec{q}\cdot\vec{r})/(2\omega)^{1/2}$$

We shall now use the relativistic generalization of the P-P potential, which is

$$g_{PP}{}^2(2M/\mu)^2 \; (\gamma_5)_1 \; (\gamma_5)_2 \; \exp(-\mu r)/4\pi r$$

Since negative energy states are important with this form of interaction, we must reconsider the energy denominators in their relativistic form. If the intermediate nucleon is in a negative energy state, its energy is approximately equal to the negative of its rest mass, and we have for the energy denominators for processes (1) and (2) approximately $1/2M^2$ and for process (3) $-1/2M\omega'$. We also must consider the behaviour of the matrix elements describing the emission and reabsorption of the mesons. For transitions between positive energy states, $(\gamma_5)\ (\gamma_5) \sim (v/c)^2$ where v is the velocity of the nucleons. For transitions to and from negative energy states, $(\gamma_5)\ (\gamma_5)$ is about 1. Combining these results we find for the approximate magnitude of the matrix elements

Processes (1) and (2) $\quad 2(v/c)^2/\omega\omega' \sim 1/2M\omega'(\mu/M)^{1/2} \cong .40/2M\omega'$ (transitions to positive energy states)

Process (3) $\quad 1/2M\omega'$ (transition to negative energy state)

We see therefore that near threshold we cannot neglect the non-potential like terms corresponding to process (3) with the virtual nucleon undergoing transitions to negative energy states, since they give the largest contribution to the matrix element. We must refer to the third order perturbation theory results for a more adequate description of the process for the anomalous case of pseudoscalar coupling.

D. Summary of results for phenomenological calculation

The squares of the magnitudes of the matrix elements given by the application of the phenomenological method to the meson production problem are given by

$$\left|H_{if}\right|^2 = G^6_{phen}/2M^2\mu^5 \qquad S(phen)$$

$$= (G^6_{phen}/2M^2\mu^5)\ \ q^2/\mu^2 \qquad Ps\cdot Pv(phen),\ V\cdot V\cdot(phen)$$

where the values of G^6_{phen} are listed in Table II. A comparison of these results with those obtained in Section II by the third order calculation for the production of charged mesons is given in Table III. It is apparent that the results obtained by these two methods are approximately equal for not unreasonable choices of the coupling constants. The use of symmetric theory in which $f = f_3$ and f_4 is set equal to zero, however, would predict a zero cross section for S(III) and V·V(III). It is certainly not necessary, however, that such a choice of the coupling constants be made.

IV. Effects of interaction of Final Nucleons

For simplicity we restrict ourselves to the case of two initial protons leading to a final neutron, proton, and positive meson. In the calculations described in Sections II and III, we have made the approximation of representing the wave function of the final nucleons by plane waves. This is equivalent to using the Born approximation to describe the nucleon-nucleon scattering. However, near threshold where the final nucleons have low energies, the scattering into these final states is poorly represented by the Born approximation applied to the potentials. The calculations can be done in a more satisfactory way if the actual wave function of the final nucleons is used. We can then resolve this into plane waves by the relation

$$\Psi_F(r) = \int \alpha_k \exp(ik\cdot r)\, d\vec{k}$$

where

$$\alpha_k = 1/(2\pi)^3 \int \Psi_F(\vec{r}) \exp(-i\vec{k}\cdot\vec{r})\, d\vec{r}$$

The calculations which we have made can then be considered to represent one of the Fourier components of this momentum distribution. We can represent the transition matrix element which leads from the initial state to the final state in which we have plane outgoing waves of relative momentum by $M_{if}(\vec{k})$. The transition matrix element to a state $\Psi_F(\vec{r})$ then will be given by the expression

$$H_{if} = \int d\vec{k}\, \alpha_k\, M_{if}(\vec{k})$$

If we wish further to separate the final state into singlet and triplet spin states, we must consider separately the matrix elements of $M_{if}(\vec{k})$ leading to these spin states.

If we insert the definition of α_k, we have

$$H_{if} = 1/(2\pi)^3 \iint d\vec{r}\,d\vec{k}\ \exp(-i\vec{k}\cdot\vec{r})\,\psi_F(\vec{r})\ M_{if}(\vec{k})$$

Now if we define

$$1/(2\pi)^3 \int d\vec{k}\ \exp(-i\vec{k}\cdot\vec{r})\ M_{if}(k) = M'_{if}(\vec{r})$$

then we have for the matrix element

$$\begin{aligned} H_{if} &= \int \psi_F\ (\vec{r})\ M'_{if}(\vec{r})\ d\vec{r} \\ &= \psi_F(\vec{r}_{average}) \int M'_{if}(\vec{r})\ d\vec{r} \\ &= \psi_F(\vec{r}_{average})\ M_{if}(0) \end{aligned}$$

where $\psi_F(\vec{r}_{av})$ is the value of the final wave function at an average value of $\vec{r}$. This separation can be made more acceptable if we note that $M'_{if}(\vec{r})$ must be large only for $\vec{r}$ considerably less than the range of the forces since the large momentum transfers necessary for meson production lead to a rather singular form for $M'_{if}(\vec{r})$. Then, since for low energy nucleons, $\psi_F(\vec{r})$ is slowly varying over a region of the size of the meson Compton wave length, the result will not be sensitive to the particular value of $\vec{r}_{av}$, as long as $\vec{r}_{av}$ is considerably less than $\hbar/\mu c$. The author wishes to thank Dr. Geoffrey Chew for suggesting the possibility of such a simple separation of the effects of the interaction of the final nucleons.

This simple result can be applied to the problem of interest. If we wish to calculate the probability that a deuteron is formed, we can take the expectation value of the transition matrix element $M_{if}(0)$ between the initial state of arbitrary spin and the final triplet state, and multiply this by the deuteron wave function evaluated at $\vec{r}_{av}$. Similarly, if we wish to include the effects of the interaction of the slowly moving final nucleons in a singlet or triplet state, we multiply the appropriate values of the matrix elements by the singlet or triplet wave functions evaluated at $\vec{r}_{av}$. We shall use the approximate wave functions for a square well[4] of range 1.53×10^{-13}cm, triplet depth V_t of

52.9 Mev, singlet depth V_s of 41.1 Mev. This gives simple analytic expressions for the wave functions of the deuteron and of the unbound system of neutron and proton of low relative momentum. The use of this non-singular potential may underestimate the magnitude of the wave functions for small separations; however, the results will not be qualitatively incorrect. The approximate wave functions are

$$\psi_D\ (r) = \sin((MV_t)^{1/2} r)/r\ \ ((M\mathcal{E}_t)^{1/2}/2\pi)^{1/2}$$
$$\psi_{t,s}\ (r) = \sin((M(V_{t,s} - E_f))^{1/2} r)/r\ \ (M(\mathcal{E}_{t,s} + E_f))^{-1/2}$$

where $\mathcal{E}_s$, $\mathcal{E}_t$ are the singlet or triplet binding energy (in magnitude) and E_f is the energy of the final nucleons. The magnitude of these wave functions is quite insensitive to the choice of $\vec{r}_{av}$ for $\vec{r}_{av}$ less than $\hbar/\mu c$; for simplicity we shall evaluate them at $\vec{r}_{av}$ equal to zero. We then have

$$\psi_D\ (0) = (MV_t)^{1/2}\ ((M\mathcal{E}_t)^{1/2}/2\pi)^{1/2}$$
$$\psi_{t,s}\ (0) = ((V_{t,s} + E_F)/(\mathcal{E}_{t,s} + E_F))^{1/2}$$

It is apparent from these results that the value of the matrix element is considerably increased by the factor $\psi_F(0)$ near threshold, where the final nucleon energy E_f is considerably less than the well depth of about 50 Mev. The factor

$$(V_{s,t}\ E_f)/(\mathcal{E}_{s,t} + E_f)$$

only approaches one for $E_f >> V$; this condition is not satisfied until the incident nucleon energies are the order of a Bev. At 350 Mev where the final nucleon energy varies from 0 to about 25 Mev, this factor varies from about 2 to 25 for the triplet state and from 2 to about 600 for the singlet state. This has the effect of raising the cross section by a factor of about 3 or 4. The effects of the interaction of the final nucleons therefore clearly are large and cannot be ignored.

V Calculation of Differential Cross-Section

We shall consider the specific case of production of positive mesons in P-P collisions. The generalization to other cases can easily be made. To the approximation used in these calculations, the differential cross section in the center-of-mass coordinate system for the production of two nucleons and a meson in a nucleon-nucleon collision is given by the expression

$$d\sigma/d\Omega dT = 2(2)^{1/2} M^2 \mu (T(T_m - T))^{1/2} \left| H_{if} \right|^2 / (4\pi)^3 \pi$$

where T is the meson kinetic energy, T_m is the maximum meson kinetic energy, and H_{if} is the transition matrix element including the effects discussed in Section IV of the interaction of the final nucleons. Using the results of that section, we can write

$$\left| H_{if} \right|^2 = \left| \psi_t(\vec{r}=0) M_{if}(0)(\text{triplet}) \right|^2 + \left| \psi_s(\vec{r}=0) M_{if}(0)(\text{singlet}) \right|^2$$

where $M_{if}(0)$(triplet) is the transition matrix element to a triplet state calculated in Sections II and III, evaluated at zero relative momentum for the final nucleons, similarly for $M_{if}(0)$(singlet). Substituting the values for the wave functions, this becomes (setting $E_f = T_m - T$)

$$d\sigma/d\Omega dt = 2(2)^{1/2} M^2 \mu (T(T_m - T))^{1/2} \Big(\left| M_{if}(0)(\text{triplet}) \right|^2 (V_t + T_m - T)/(\mathcal{E}_t + T_m - T) + \left| M_{if}(0)(\text{singlet}) \right|^2 (V_s + T_m - T)/(\mathcal{E}_s + T_m - T) \Big)$$

The values of $M_{if}(0)$(triplet) and $M_{if}(0)$(singlet) are given in Table IV. The relation between the constants G^6_{III} and G^6_{phen} and the coupling constants f, f_3, and f_4 is given in Tables I and II.

The total cross section then is (dropping small terms in $\mathcal{E}/T_m$ and $\mathcal{E}/V$)

$$= \frac{2(2)^{1/2} M^2 \mu}{(4\pi)^3} \Big(\left| M_{if}(0)(\text{triplet}) \right|^2 (1/4 + V_t/T_m(1 - 2(\mathcal{E}_t/T_m)^{1/2}) \left| M_{if}(0)(\text{singlet}) \right|^2 (1/4 + V_s/T_m(1 - 2(\mathcal{E}_s/T_m)^{1/2})) (2\pi T_m^2)$$

We can now use this cross section ignoring the possibility of deuteron formation to estimate the values of the coupling constants. If we assume a cross section at 350 Mev of 2×10^{-28} cm^2 with the corresponding maximum value of the meson kinetic energy in the center-of-mass system of 24.8 Mev, the corresponding values of the constants $G^6_{III}/(4\pi)^3$ and $G^6_{phen}/(4\pi)^3$ are given in Table V. The values of the coupling constants f, f_3, and f_4 calculated from these values of $G^6/(4\pi)^3$, by the relation given in Tables I and II are given in Table VI. For comparison the values of the coupling constants adjusted to agree with the magnitude of P-P scattering at 350 Mev[4] (about 25 millibarns) are also given. It is apparent that the coupling constants predicted by these two processes, meson production and high energy scattering, are well within the same order of magnitude. Conversely we can conclude that any of the 3rd order or phenomenological results can correctly predict the order of magnitude of the meson production cross section, for not unreasonable choices of the coupling constants.

Finally we must consider the additional process which can contribute to meson production, the formation of a deuteron and a meson in a proton-proton collision. The cross section can be calculated easily; we have

$$d\sigma/d\Omega = \left| M_{if}(0)(\text{triplet})\, \psi_D\ (r=0) \right|^2 Mq\mu/(8\pi^2 P_o)$$
$$= \left| M_{if}(0)(\text{triplet}) \right|^2 4(2)^{1/2} (T_m \mathcal{E}_t)^{1/2}\, V_t\, M^2\mu/(4\pi)^3$$

Using the values of the matrix elements given in Table IV the cross sections are

$$d\sigma/d\Omega = (G^2_{III,phen}/4\pi)^3\ (Tm/\mu)^{3/2}\ 6.66 \times 10^{-26}\ cm^2 \qquad Ps\cdot Pv(III),\ Ps\cdot Pv(phen)$$
$$(G^2{}_{III}/4\pi)^3\ (T_m/\mu)^{1/2}\ 2.22 \times 10^{-26}\ cm^2 \qquad Ps\cdot Ps(III)$$

At 350 Mev, using the values of the constants $G^2/4\pi$ given in Table V, the total cross sections for formation of a deuteron and a meson are

$$\sigma = 5.23 \times 10^{-28}\ cm^2 \qquad Ps\cdot Pv(III),\ Ps\cdot Pv(phen)$$
$$2.57 \times 10^{-28}\ cm^2 \qquad Ps\cdot Ps(III)$$

These cross sections are about the same size as those in which the final nucleons are not bound (assumed to be 2×10^{-28} cm^2).

The differential cross sections at 350 Mev for proton-proton production of positive mesons are given in Figs. 2 and 3 for scalar theory and pseudoscalar theory with pseudovector coupling. For comparison the cross sections obtained when the Born approximation was made of treating the final nucleon wave functions as plane waves are also given (dashed curves). The very striking effects of the interactions of the final nucleons are obvious. The variation of the total cross sections with energy is given in Table VII, including the contribution to the cross section when a deuteron is formed. The normalization is again to a total cross section of 2×10^{-28} cm^2 at 350 Mev for the production in which the two final nucleons are unbound.

VI Conclusions

The experimental results of the Berkeley workers[1] indicate that the cross section per nucleon for protons bombarding carbon is about 2×10^{-28} cm^2 for both charged and neutral mesons. For protons bombarding free protons, the cross section is about the same for production of charged mesons, but appears to be perhaps an order of magnitude smaller for neutral meson production. It is apparent from the results of Sections II, III, and IV that the relative size of these cross sections is predicted successfully only by the third order and the phenomenological results for pseudoscalar mesons with pseudovector coupling. The phenomenological result for scalar and vector theory fails in that a zero cross section is predicted for production of neutral mesons in N-P collisions. The third order result for pseudoscalar theory with pseudoscalar coupling is of the right order of magnitude except for neutral mesons produced in proton-proton collisions, where a cross section is predicted comparable with that for charged mesons, in contradiction with the experimental result. The 3rd order result for scalar and vector theories is peculiar in that the cross section for neutral mesons vanishes for neutron-proton collisions and also vanishes for charged mesons if f^2 is taken equal to $1/2\ f_3^{\ 2}$ (see Table I) corresponding to the use of symmetrical theory. It is interesting to observe that a small cross section is predicted for neutral meson production in proton-proton collisions by all of the theories except pseudoscalar theory with pseudoscalar coupling. It is somewhat questionable, however, that such cancellations as those which appear in these calculations are to be quantitatively believed since it is possible that higher order virtual effects would remove the cancellation which appears in lowest order[8].

The phenomenological calculations for pseudovector coupling are successful in predicting a sufficiently large cross section for agreement with experiment because an unsymmetrical choice of the neutron-proton and proton-proton potentials was made. The calculation made by Marshak and Foldy used a symmetrical interaction and cancellations which occurred reduced the cross section by 2 or 3 orders of magnitude. It is not necessarily true, however, that all symmetrical theories would give a similar result. A theory which while symmetrical could also predict the high energy nucleon-nucleon scattering would presumably give the same general features as the potential models used in these phenomenological calculations. It is interesting to note that the use of symmetrical theory in the third order calculation does predict correctly the general magnitudes of the cross sections for pseudovector coupling but gives zero cross sections for scalar and vector theory.

The experiments of meson production by protons bombarding free protons[1] provide a good opportunity for verifying the detailed predictions of the differential energy spectra. The experimental results of Cartwright, et al, are shown in Fig. 4 in comparison with the predictions of pseudoscalar theory with pseudovector coupling. It is apparent that agreement with experiment can be obtained only if the effects of the interactions of the final nucleons are taken into account. The predicted fine structure of the high energy peak resulting from the deuteron formation cannot be resolved with the present experimental data; presumably an improvement of experimental techniques will make it possible to test this prediction of the theory.

The author wishes to thank Professor Robert Serber and Dr. K. M. Watson for many interesting discussions of the theoretical results derived in this paper. He also wishes to thank the experimental workers at Berkeley for continuous information about the preliminary results of their work, and for aid in interpreting the experiments.

Table I

Values of G^6_{III} for production of charged mesons and neutral mesons of type 3 (coupled through τ_3) or of type 4 (coupled through τ_4).

		S(III), V.V.(III)	Ps.Ps(III) x $\mu^3/16M^3$	Ps.Pv(III)x$(M/\mu)^2$
N-P	π^+, π^-	$f^2(f^2 - 1/2\ f_3^2)^2$	$f^2(f^2 + 1/2\ f_4^2)^2$	$f^2(f^2 + 1/2\ f_3^2)^2$
	$\pi^0(3)$	0	$8f_3^2(f_3^2 - f_4^2)^2$	$8f^4 f_3^2$
	$\pi^0(4)$	0	$8f_4^2((f_3^2 - f_4^2)^2 + f^4)$	0
P-P	π^+	$f^2(f^2 - 1/2\ f_3^2)^2$	$2f^2(3f^4 - f^2 f_4^2 + 3/4\ f_4^4)$	$f^2(f^2 + 1/2\ f_3^2)^2$
	$\pi^0(3)$	0	$16f_3^2(f_3^2 + f_4^2)^2$	0
	$\pi^0(4)$	0	$16f_4^2(f_3^2 + f_4^2)^2$	0

Table II

Values of G^6_{phen} for production of charged mesons and neutral mesons of type 3 and type 4.

		S(phen), V.V.(phen)	Ps.Pv(phen)
Charged		$f^2(M/\mu\ g_{PP}^2)^2$	$f^2(M/\mu\ g_{PP}^2 + g_{NP}^2)^2$
N-P	$\pi^0(3)$	0	$2f_3^2(g_{NP}^2)^2$
	$\pi^0(4)$	0	0
P-P	$\pi^0(3)$	0	0
	$\pi^0(4)$	0	0

Table III

Ratio of 3rd order to phenomenological matrix elements for production of charged mesons.

Scalar, vector	Pseudovector coupling
$(((f^2 - 1/2\ f_3^{\ 2})/4\pi)/.266)^2$	$(((f^2 + 1/2\ f_4^{\ 2})/4\pi)/.114)^2$

Table IV

Values of square of magnitude of transition matrix elements, for P-P production of positive mesons, evaluated at zero relative momentum for the final nucleons, leading to singlet and triplet spin state. All are to be multiplied by $1/2\ M^2\mu^5$.

	S(III)	S(phen)	V.V(III)	V.V(phen)	Ps.Ps(III)	Ps.Pv(III)	Ps.Pv(phen)
$\lvert M_{if}(0)(\text{triplet})\rvert^2$	0	0	0	0	$2/3\ G^6_{III}$	$G^6_{III} 2Tm/\mu$	$G^6_{phen} 2Tm/\mu$
$\lvert M_{if}(o)(\text{singlet})\rvert^2$	G^6_{III}	G^6_{phen}	$G^6_{III} 2Tm/\mu$	$G^6_{phen} 2Tm/\mu$	$1/3\ G^6_{III}$	0	0

Table V

Values of the constants $G^2_{III}/4\pi$ and $G^2_{phen}/4\pi$ to give a total cross section at 350 Mev of 2×10^{-28} cm^2 for production of a positive meson, neutron, and proton in a proton-proton collision.

	S(III), S(phen)	V.V.(III), V.V.(phen)	Ps.Ps(III)	Ps.Pv(III), Ps.Pv(phen)
$G^2/4\pi$	.268	.379	.298	.453

Table VI

Values of the coupling constants f, f_3, and f_4 given by Tables I, II, and V, and also as predicted by P-P scattering at 350 Mev.

	Meson Production	P-P Scattering
S(III)	$(f^2(f^2 - 1/2\, f_3^{\,2})^2)^{1/3}/4\pi = .268$	$(f_3^{\,2} + f_4^{\,2})/4\pi = .283$
S(phen)	$f^2/4\pi = .273$	
V.V.(III)	$(f^2(f^2 - 1/2\, f_3^{\,2})^2)^{1/3}/4\pi = .379$	$(f_3^{\,2} + f_4^{\,2})/4\pi = .283$
V.V.(phen)	$f^2/4\pi = .540$	
Ps.Ps(III)	$(f^2(3f^4 - f^2 f_4^{\,2} + 3/4\, f_4^{\,4})^2)^{1/3}/4\pi = 3.79$	$f^2/4\pi \sim 6$
Ps.Pv(III)	$(f^2(f^2 + 1/2\, f_3^{\,2})^2)^{1/3}/4\pi = .128$	$(f_3^{\,2} + f_4^{\,2})/4\pi = .0418$
Ps.Pv(phen)	$f^2/4\pi = .167$	

Table VII

Variation with energy of total cross section for positive meson production in P-P collisions, in units of 10^{-28} cm^2. The columns headed Unbound are for production leading to a neutron, proton, and meson; those headed Deuteron are for production leading to a deuteron and a meson.

Mev	Scalar	Vector	Ps.Ps.		Ps.Pv.	
	Unbound	Unbound	Unbound	Deuteron	Unbound	Deuteron
290	0	0	0	0	0	0
325	1.10	0.64	0.83	2.05	0.35	2.64
350	2.00	2.00	2.00	2.57	2.00	5.23
375	3.08	4.51	3.48	3.07	5.16	8.89

Fig. 1 Feynman-Dyson diagram for meson production by nucleon-nucleon collisions in lowest order. The solid lines represent the nucleons, the dashed lines represent mesons.

Fig. 2 Differential cross section at 350 Mev for production of positive scalar mesons in proton-proton collisions. The solid curve includes the effects of the interactions of the final particles; the dashed curve is the result of the calculations using Born approximation throughout.

Fig. 3 Differential cross section for production of positive pseudoscalar mesons with pseudovector coupling in proton-proton collisions. The delta function representing deuteron formation is averaged over a 5 Mev energy interval.

Fig. 4 Comparison of the experimental results of Cartwright, et. al., for production of positive mesons by 340 Mev protons bombarding free protons. The curve is in the laboratory system for mesons produced in the direction of the proton beam.

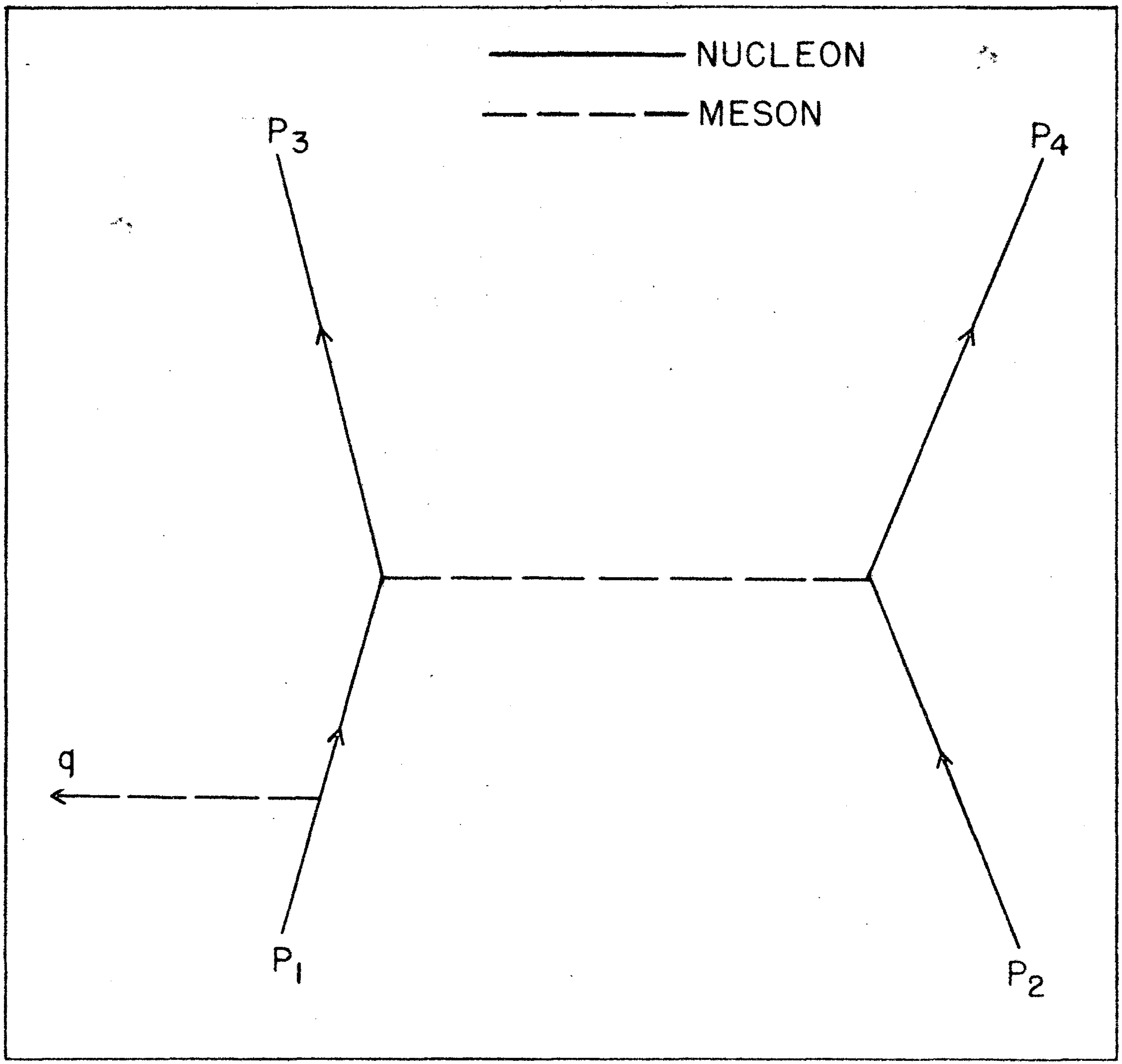


FIG. 1

14626-1

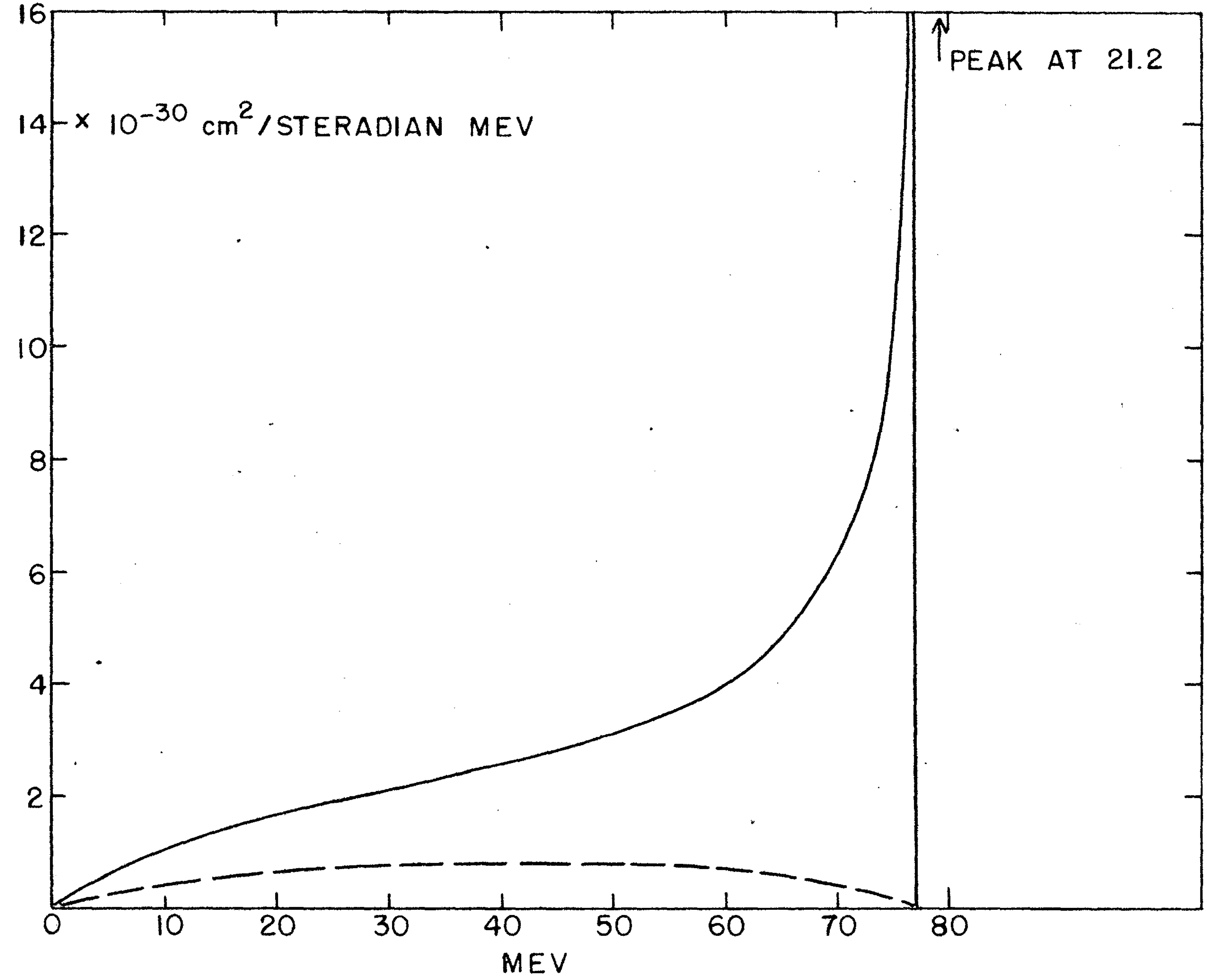

16
14
12
10
8
6
4
2
x 10^-30 cm^2/STERADIAN MEV
PEAK AT 21.2
0
10
20
30
40
50
60
70
80
MEV

FIG. 2

14997-1

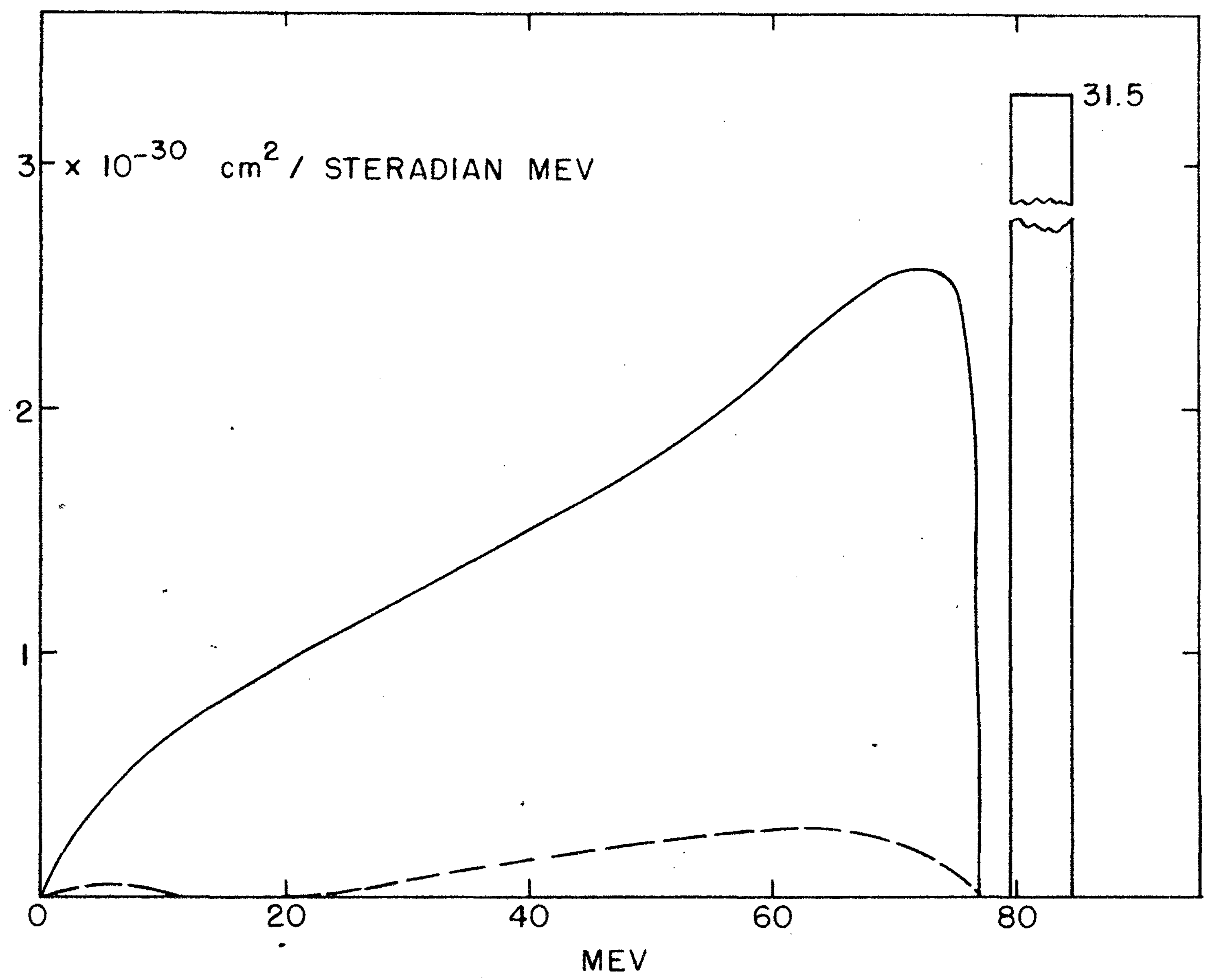
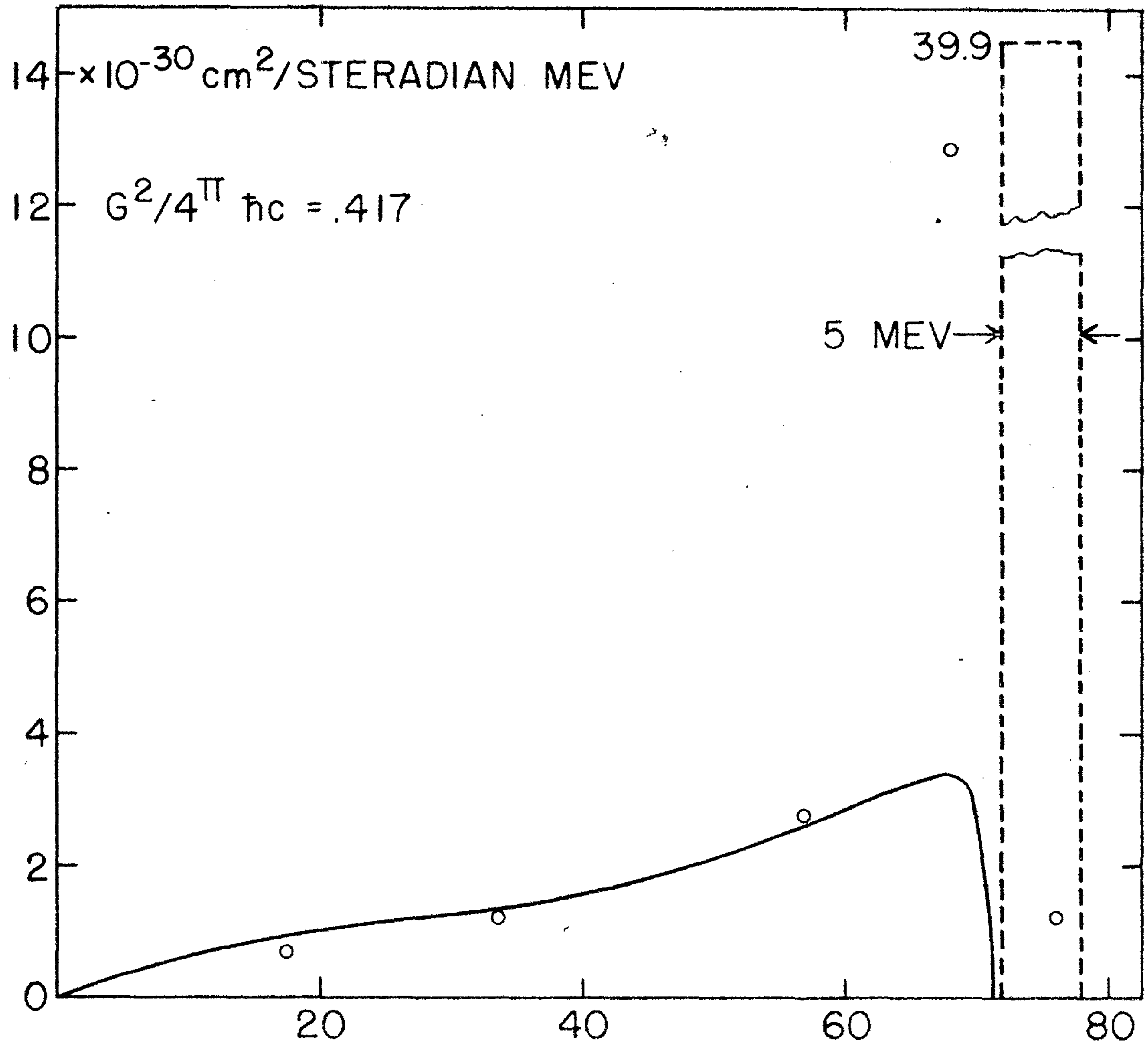


FIG. 3

14998-1



PSEUDOSCALAR THEORY

340 MEV

FIG 4

14900-1

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