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ABSTRACT

Some modes of propagation of Alfvén waves in a cylindrical hydromagnetic waveguide have been investigated experimentally. The radial distribution of the wave magnetic field can be described by the theoretically predicted Bessel functions. The rapid attenuation of a higher mode as the wave propagates has been observed. The reflection of waves from the end of the hydromagnetic waveguide is also reported.

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INTRODUCTION

Propagation of torsional hydromagnetic (Alfvén) waves in a cylindrical plasma has been reported.[1] These experiments have been extended to include measurements of the radial distribution of the oscillating magnetic field which is associated with the wave, and of reflections of the wave from the end of the cylindrical plasma.[2] These observations make possible a quantitative comparison between predictions of hydromagnetic theory and experimental results.

We first sketch the derivation of the pertinent wave quantities from Maxwell's equations, Ohm's law, and Newton's second law of motion. We then describe a new method of plasma preparation that results in a well-defined ionization front which proceeds down the tube and produces a plasma that is relatively free of impurities. The experimental measurements of the various wave quantities are reported and are found to be in good agreement with the theoretical predictions. A discussion of a rather surprising boundary condition is given. Finally, the observation of the wave reflections is reported and discussed.

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*This work was done under the auspices of the U.S. Atomic Energy Commission.

THEORY

We consider a torsional hydromagnetic wave propagating in a cylindrical plasma with finite conductivity. Azimuthal symmetry is assumed (see EXPERIMENTAL discussion) and we will look for solutions to the equations in which the various field quantities have the form $F(r)\,e^{i(pz\, -\, \omega t)}$, i.e., are plane waves with sinusoidal time variation, propagating only in the z direction. Displacement current and plasma pressure are neglected. The wave frequency ($\sim 3 \times 10^{6}$ radians/sec for this experiment) is assumed low with respect to the ion cyclotron frequency (typically 1.5×10^{8} radians/sec for this experiment). We use rationalized MKS units in this discussion. Following Newcomb,[3] we begin with the linearized form of Maxwell's equations, Ohm's law, and Newton's second law of motion:

$$\nabla \times \underset{\sim}{b} = \mu_0 \underset{\sim}{j} \tag{1}$$

$$\nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{b}}{\partial t} \tag{2}$$

$$\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}_0 = \frac{\underset{\sim}{j}}{\sigma} \tag{3}$$

$$\rho_0 \frac{\partial \underset{\sim}{v}}{\partial t} = \underset{\sim}{j} \times \underset{\sim}{B}_0 \tag{4}$$

where $\underset{\sim}{b}$ is the oscillating magnetic field associated with the wave, $\underset{\sim}{j}$ is the current density associated with the wave, $\underset{\sim}{E}$ is the wave electric field, $\underset{\sim}{v}$ is the plasma velocity, $\underset{\sim}{B}_0$ is the static axial magnetic field supplied by external coils, ρ_0 is the plasma mass density, σ is the plasma conductivity, and μ_0 is the permeability of free space. Introducing the $e^{-i\omega t}$ dependence and solving Eqs. (1) to (4) for the wave magnetic $\underset{\sim}{b}$ yields:

$$\mathbf{b} + \frac{1}{\mu_0 \omega^2 \rho_0} \nabla \times \left\{ \mathbf{B}_0 \times \left[\mathbf{B}_0 \times (\nabla \times \mathbf{b}) \right] \right\} - \frac{1}{i \mu_0 \omega \sigma} \nabla \times \nabla \times \mathbf{b} = 0 \tag{5}$$

This can be simplified to give

$$\nabla^2 b_z \hat{\mathbf{z}} + \frac{\partial^2 \mathbf{b}}{\partial z^2} - i \alpha \nabla^2 \mathbf{b} - \nabla \frac{\partial b_z}{\partial z} + \frac{\omega^2}{V^2} \mathbf{b} = 0 , \tag{6}$$

where $V = B_0 / (\mu_0 \rho)^{1/2}$ is the Alfvén velocity, $\alpha = \omega / \mu_0 \sigma V^2$, $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$, and $\hat{\mathbf{z}}$ is a unit vector in the axial direction.

Extracting the equation for the perpendicular component $b_\perp$, we have

$$\frac{\partial^2 b_\perp}{\partial z^2} - i \alpha \nabla^2 b_\perp - \nabla_\perp \frac{\partial b_z}{\partial z} + \frac{\omega^2}{V^2} b_\perp = 0 \tag{7}$$

where $\nabla_\perp$ is the perpendicular component of the gradient operator. Inserting the e^{ipz} dependence and rearranging, we have

$$\nabla_\perp^2 b_\perp + k_c^2 b_\perp + \frac{p}{\alpha} \nabla_\perp b_z = 0 \tag{8}$$

where

$$k_c^2 = -p^2 \left(1 + \frac{i}{\alpha}\right) + \frac{i}{\alpha} \frac{\omega^2}{V^2} . \tag{9}$$

We are interested in the azimuthal component of Eq. (8). In cylindrical coordinates this is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b_\theta}{\partial r}\right) - \frac{b_\theta}{r^2} + k_c{}^2 b_\theta = 0. \tag{10}$$

Then the general solution is

$$b_\theta = \sum_{n=1}^{\infty} b_{\theta n} J_1 (k_{cn} r) \exp\left[i(p_n z - \omega t)\right] , \tag{11}$$

where $J_1 (k_{cn} r)$ is the first-order Bessel function, and n designates the mode of propagation. The $b_{\theta n}$ are constants determined by the form of the initial perturbation which induces the wave, and the k_{cn} are constants determined by a radial boundary condition. The form of the boundary condition will be discussed in the EXPERIMENTAL section. Experimentally, we shall find that one or two of the principal modes are sufficient to describe the observed distributions. The other wave quantities can be obtained from substitution in Eqs. (1) to (4):

$$v_\theta = -\sum_{n=1}^{\infty} \frac{p_n V^2}{\omega B_0} b_{\theta n} J_1 (k_{cn} r) \exp\left[i(p_n z - \omega t)\right] \tag{12}$$

$$j_r = -\sum_{n=1}^{\infty} \frac{ip_n}{\mu_0} b_{\theta n} J_1 (k_{cn} r) \exp [i(p_n z - \omega t)] \tag{13}$$

$$j_z = \sum_{n=1}^{\infty} \frac{k_{cn}}{\mu_0} b_{\theta n} J_0 (k_{cn} r) \exp [i(p_n z - \omega t)] \tag{14}$$

$$E_r = \sum_{n=1}^{\infty} p_n \left(\frac{V^2}{\omega} + \frac{i}{\mu_0 \sigma}\right) b_{\theta n} J_1 (k_{cn} r) \exp [i(p_n z - \omega t)] \tag{15}$$

$$E_z = \sum_{n=1}^{\infty} \frac{k_{cn}}{\mu_0 \sigma} b_{\theta n} J_0 (k_{cn} r) \exp [i(p_n z - \omega t)] . \tag{16}$$

The attenuation of the various principal modes that is caused by ohmic losses will be needed for the analysis of the experimental results. If we set $p_n = k_n + i/L_n$, where k_n is the propagation constant, and L_n is the attenuation length for the nth principal mode (i.e. the distance in which a wave amplitude decreases by $1/e$), then separating real and imaginary parts of Eq. (9) yields

$$L_n = \frac{2\mu_0 \sigma k_n V^2}{\omega (k_{cn}^2 + k_n^2)} \tag{17}$$

and

$$k_n^2 = \frac{1}{(1+a^2)} \left[\frac{\omega^2}{V^2} - a^2 k_{cn}^2 \right] . \tag{18}$$

For the experimental conditions reported here, a good approximation is

$$k_n^2 = \frac{\omega^2}{V^2} = k^2 . \qquad (19)$$

Thus under these conditions the waves have no dispersion and no cutoff.

PLASMA PREPARATION

The geometry of the hydromagnetic waveguide is shown in Fig. 1. A copper cylinder 86.4 cm long and 14.6 cm in diameter is placed in a uniform axial magnetic field of about 16 kgauss and filled with hydrogen gas to a pressure of 100 microns. The ionizing current is supplied by a lumped-constant pulse line, so that the current and voltage are nearly constant during the period of ionization, as shown in Fig. 2. The pulse line is connected between one electrode and the copper cylinder through a 1-ohm series resistance and ignitron switching tubes. When this voltage is switched on, a local breakdown of the gas occurs at the end of the tube, and then a well-defined ionization front moves axially down the tube at a typical velocity of 5 cm/μ sec. The voltage at the coaxial electrode on the receiving end (not shown in Fig. 1) remains at zero during the time required for the ionization front to move down the tube. When the front reaches the receiving electrode, the voltage there rises abruptly to maximum value, as shown in the lower trace of Fig. 3. The progress of the front down the tube can be observed by noting its time of arrival at each of five radial-current probes (Fig. 4) mounted in the cylinder wall. The radial-current probe consists of a 1/4-in.-diam. circular section of the cylinder wall which is isolated electrically and connected to the adjacent cylinder through a small resistance. The position of the front as a function of time is shown in Fig. 5. Spectroscopic observations indicate that the plasma is relatively free of impurities while the front is progressing down the tube, but after it has reached the far end of the tube, impurity lines from the insulator material become prominent. If the current from the ionizing condenser bank is abruptly stopped (crowbarred) within a few microseconds

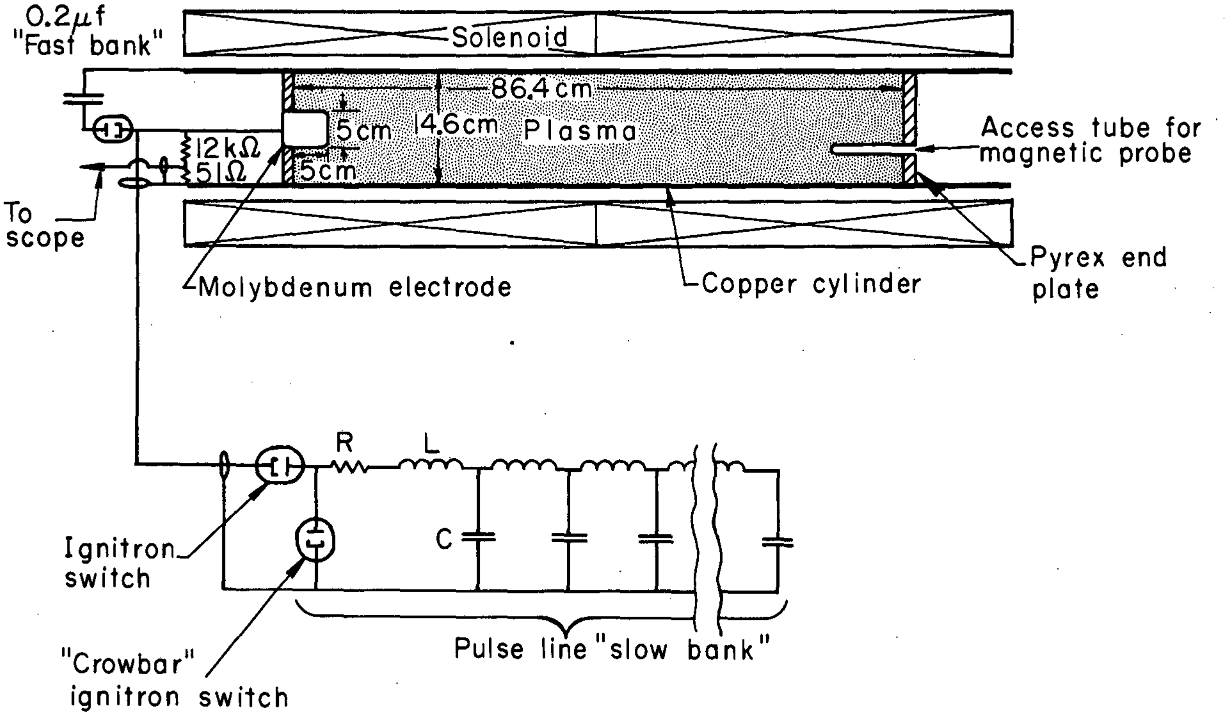

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Fig. 1. Experimental geometry.

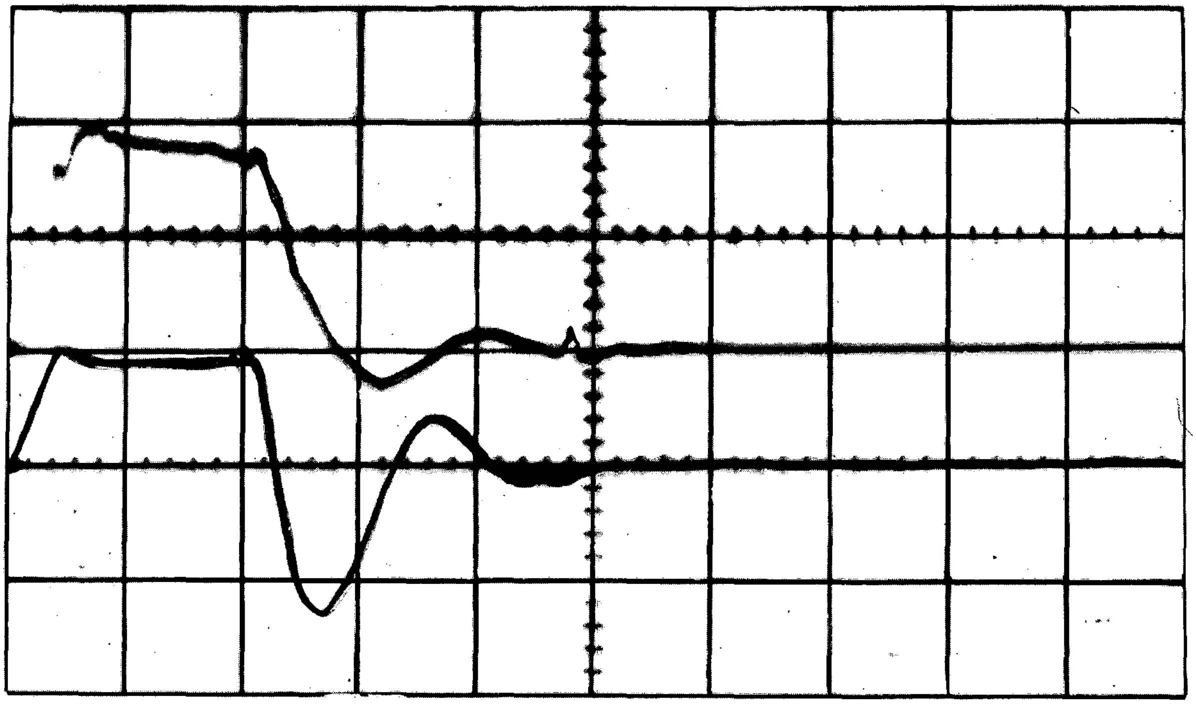

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Fig. 2. Oscilloscope traces showing ionizing conditions. The top trace is voltage on the driving coaxial electrode at 2400 v/large division; the bottom trace is current from a pulse line at 10^4 amp/large division. The horizontal scale is 10 μsec/large division. The tube was crowbarred 22 μsec after the voltage was first applied. A hydromagnetic wave was induced 48 μsec after the voltage was first applied.

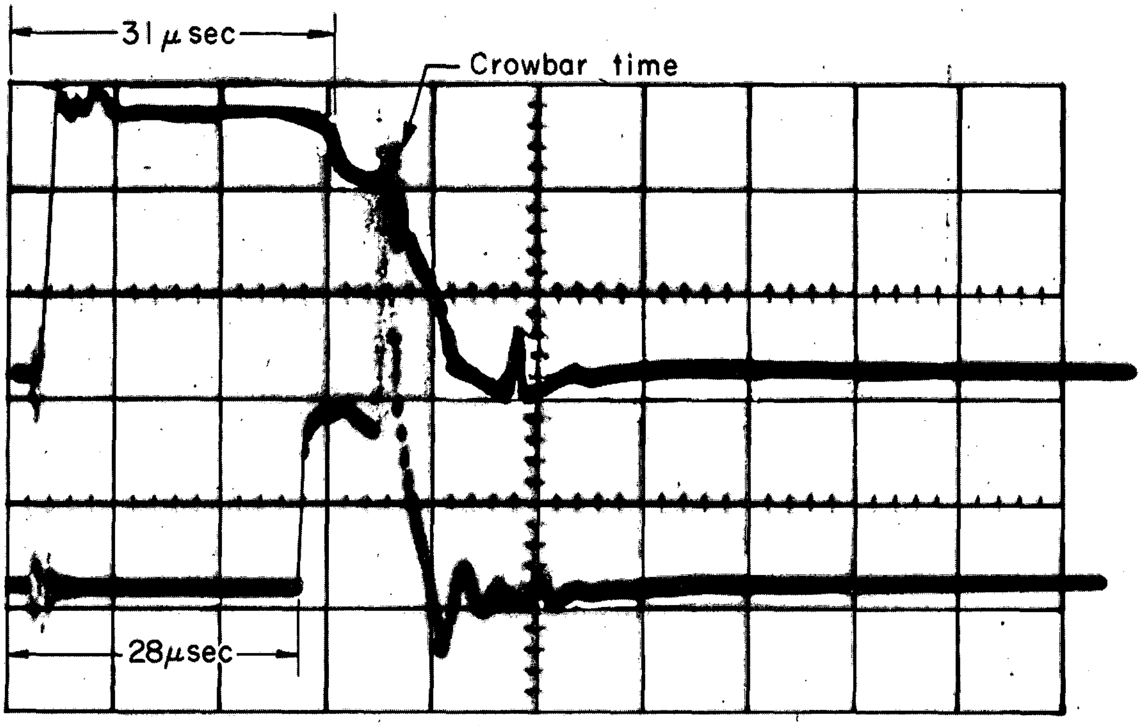

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Fig. 3. Oscilloscope traces showing arrival of ionization front. The top trace is the voltage on the driving coaxial electrode at 1000 v/large division; the bottom trace is the voltage on the receiving coaxial electrode at 1000 v/large division. The horizontal scale is 10 μ sec/large division. The axial magnetic field is 10 kgauss. The abrupt rise of the received voltage is evidence for a well-defined ionization front. Note that the information that the front has reached the load at the receiving end (0.33 ohms) requires one Alfvén transit time (2.8 μsec) to reach the driving end, at which time the driving voltage decreases somewhat.

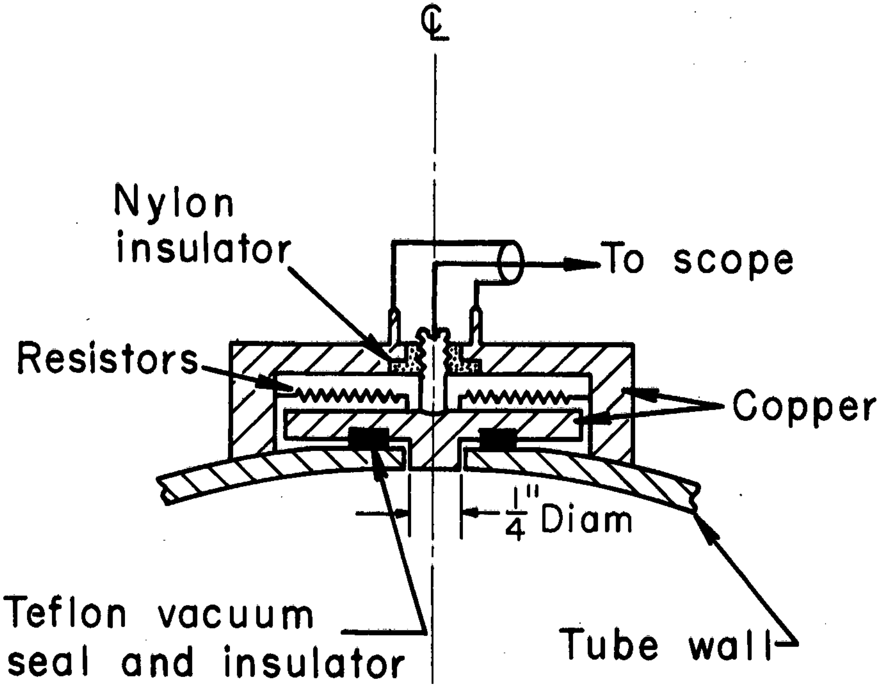

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Fig. 4. Geometry of the radial-current probe. The 0.25-in.-diam. button is connected to the adjacent wall through six parallel 5-ohm resistors. The maximum voltage drop is less than 1 v.

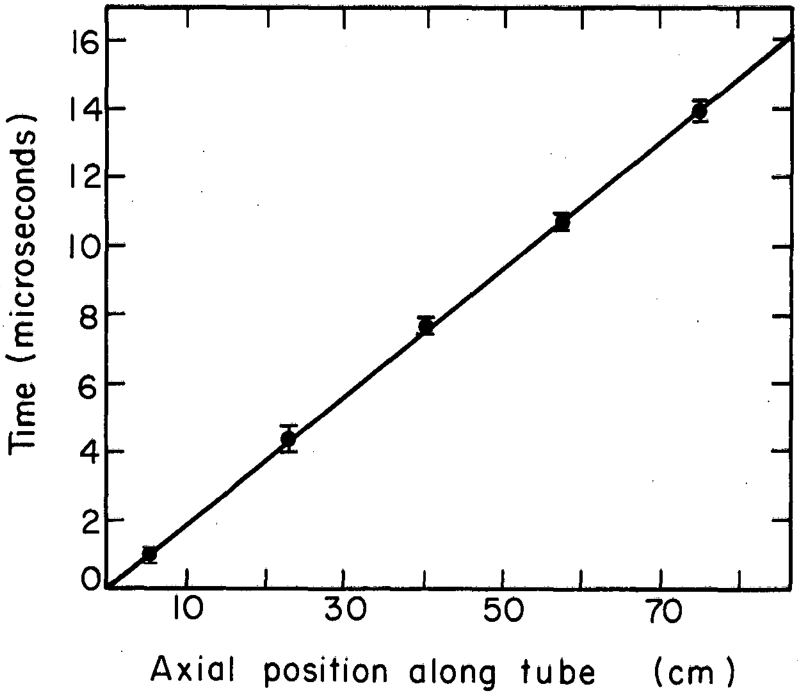

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Fig. 5. Position of the ionization front vs time, as measured with radial current probes. The axial magnetic field was 15.7 kgauss, which resulted in a shorter transit time for the front than in the case shown in Fig. 3.

after the front has reached the insulator at the receiving end, the impurity lines do not arise. Therefore the hydromagnetic wave is induced after the ionizing condenser has been crowbarred, and the wave propagates through a relatively pure, quiescent plasma.

The above discussion has described an ionization front that produces the plasma. Most of the rest of this paper will concern a hydromagnetic wave that is propagated through an already established plasma. The distinction between front and wave must be kept in mind.

EXPERIMENTAL METHOD AND RESULTS

A torsional hydromagnetic wave is induced in the plasma by discharging a 0.2 μf condenser (the "fast bank") through an ignitron switch between the center electrode and the copper cylinder. For the work reported here, this condenser was critically damped so that a single pulse of current flows, as shown in Fig. 6. The resulting wave has been detected with small magnetic probes, which consist of a coil of wire with 75 turns wound on a diameter of 1 mm. The coil is mounted inside a reentrant glass tube that pierces the insulator at the receiving end.

The radial distribution of the oscillating azimuthal magnetic field b_θ which is associated with the hydromagnetic wave has been measured with six magnetic probes of the type described above. They are mounted at six radial positions at a distance of 4-1/2 in. from the receiving end. The magnetic probe signal is displayed on an oscilloscope as shown in Fig. 7. Figure 7 shows the directly transmitted wave signal and also a signal from a wave that has reflected from the receiving end and then from the driving end (i.e. it has made three transits of the tube). The radial distributions of the magnetic field of the directly transmitted wave and of the reflected wave are plotted in Figs. 8 and 9, respectively.

A boundary condition must be found so that the data of Figs. 8 and 9 can be compared with the theory derived above. The radial current probes described above do not show a signal when the hydromagnetic wave is propagated through the plasma. Therefore the radial current density j_r associated with the wave is zero at the cylinder, even though it is a conducting wall. This suggests the presence of a sheath or a layer of neutral particles that insulates the plasma from the copper cylinder. Note that

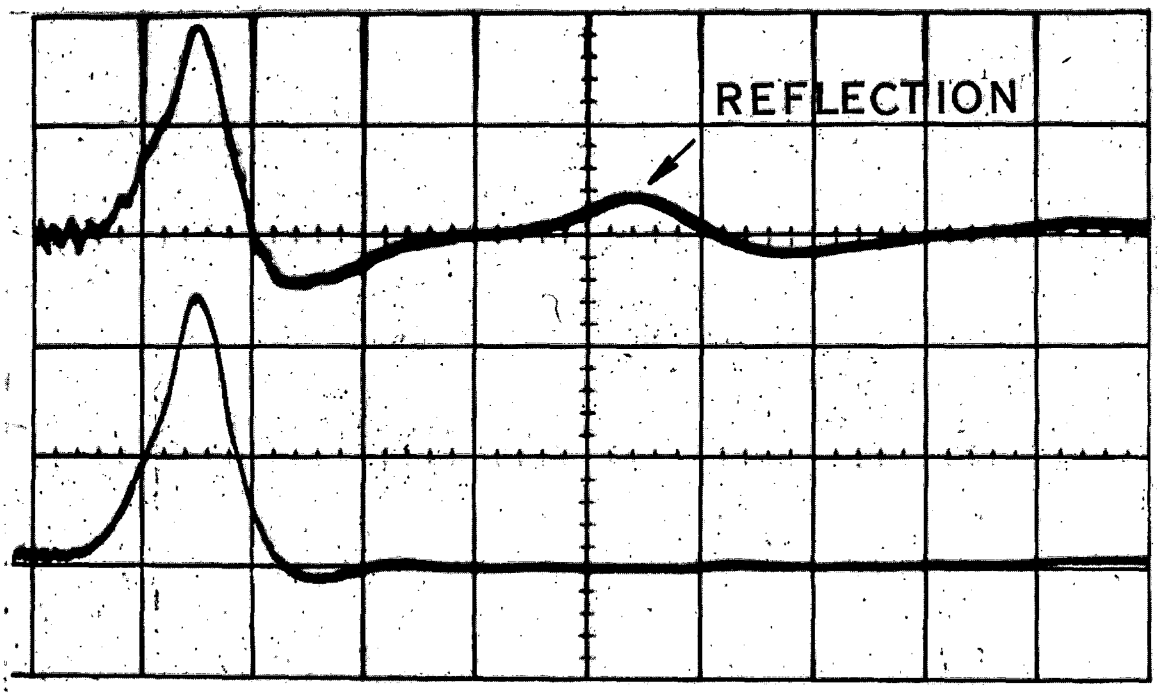

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Fig. 6. Oscilloscope traces showing the driving wave pulse. The top trace is the voltage on the driving coaxial electrode at 500 v/large division; the bottom trace is the current from the capacitor that drives the wave, at 4,000 amp/large division. The horizontal scale is 1 μsec/large division. The voltage and current are essentially in phase, i. e. the plasma load is resistive. A reflection from the receiving end of the waveguide is visible on the voltage trace. This reflection is in phase, corresponding to a reflection from an open-ended waveguide.

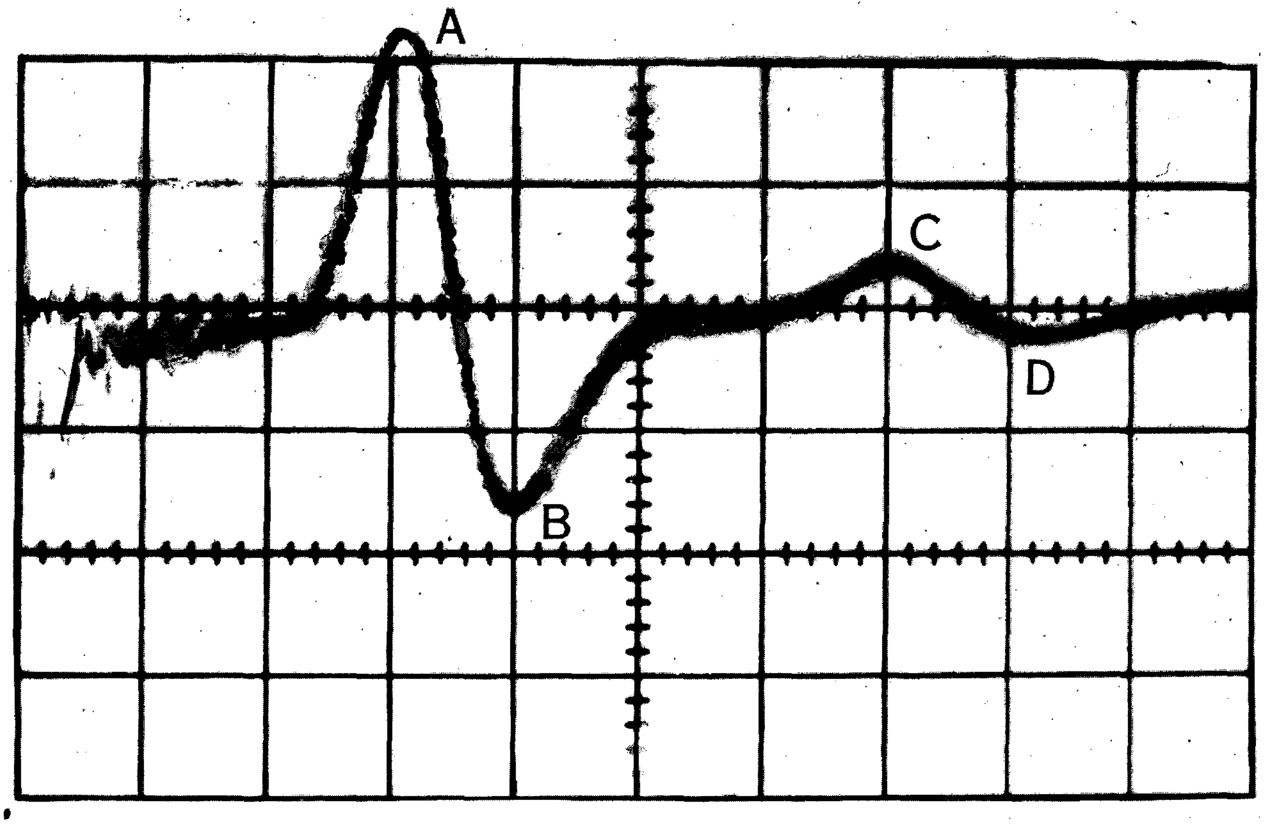

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Fig. 7. Oscilloscope trace showing wave magnetic field b_θ as measured by a probe 12 cm from the receiving end. The vertical scale is 75 gauss/large division and the horizontal scale is 1 μsec/large division. Peak A is the wave pulse arriving from the driving end, and peak B is the out-of-phase reflection from the nearby receiving end. Peak C is a wave which has made three transits of the tube, and peak D is a reflection of this off the receiving end.

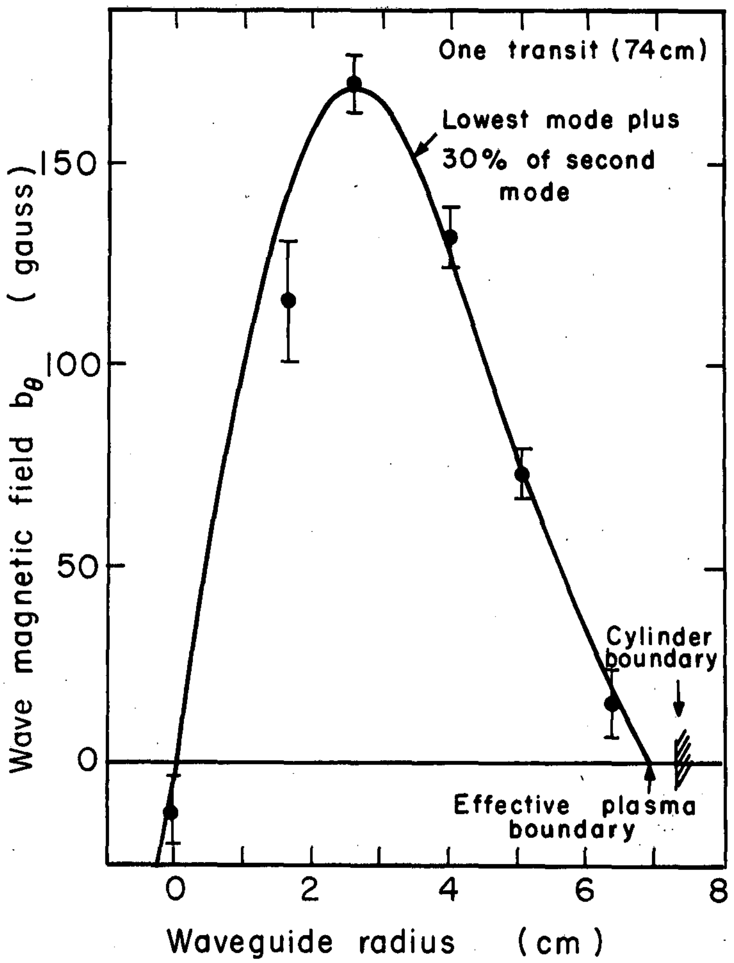

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Fig. 8. Wave magnetic field b_θ vs tube radius, measured at the receiving end of the tube after the wave has made one transit. The solid curve is proportional to $J_1(k_{c1}r)$, the lowest mode, plus 30% of $J_1(k_{c2}r)$, the second mode. The amount of the second mode is determined by a fit to the experimental results. The vertical bars designate the standard deviation of the mean of eight measurements, plus an estimate of the calibration uncertainties.

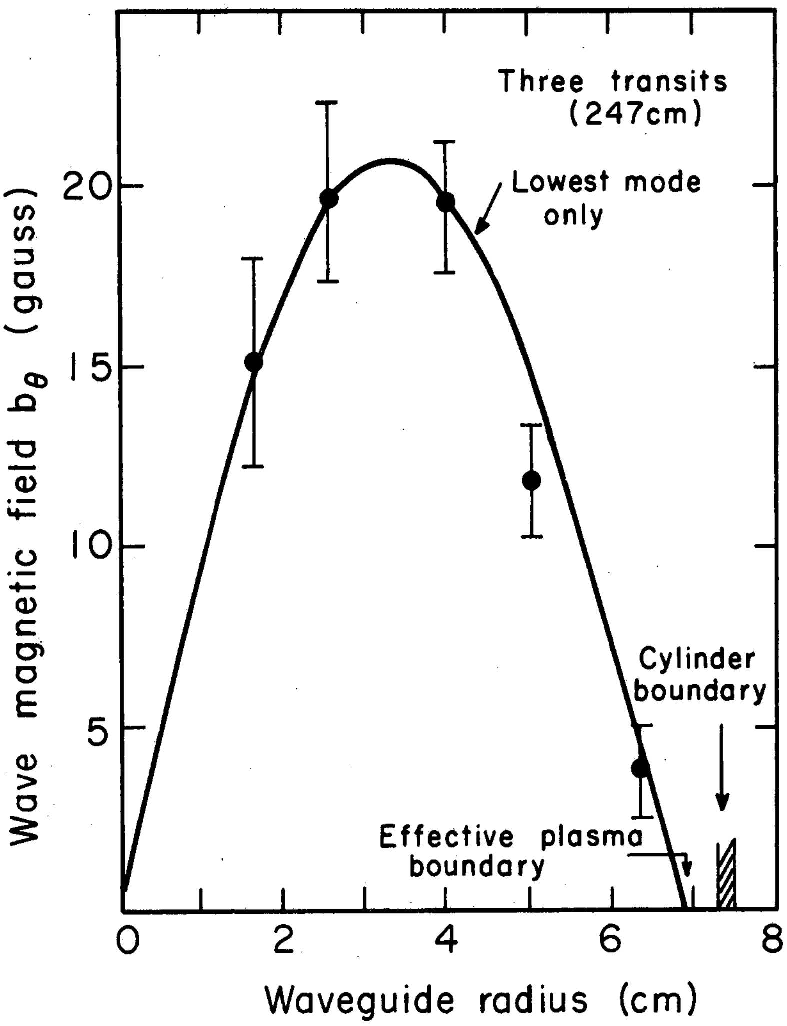

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Fig. 9. Wave magnetic field b_θ vs tube radius, measured at the receiving end of the tube after the wave has made three transits. Theory predicts that the amplitude of higher modes remaining after the wave has traveled this distance is negligible. The solid line is proportional to only $J_1(k_{c1} r)$, the lowest mode.

the direction under discussion is perpendicular to the magnetic field. Since the wave is propagated about 25 μsec after the ionizing condenser has been crowbarred, one might expect that a layer of poorly ionized gas would exist next to the walls. An effective thickness of 4 mm for this layer has been established by a best fit to the data of Figs. 8 and 9.

From Eq. (13) we see that the radial current density j_r is proportional to $J_1(k_{cn} r)$, so that the k_{cn} are now determined by the condition

$$J_1(k_{cn} a) = 0, \tag{17}$$

where a is the effective plasma radius.

The solid line in Fig. 8 is computed from Eq. (11), where $b_{\theta 2}$ is 30% of $b_{\theta 1}$, and all other $b_{\theta n}$ are zero (i.e. one can fit the experimental data with the first mode plus 30% of the second mode). Using the measured attenuation length, L_1 of 96 cm for the first mode one can then compute from Eq. (17) that after the wave has made three transits of the waveguide, the second mode is only 0.4% of the first mode. Therefore for Fig. 9, which gives the reflected wave data, the solid line represents the first mode only. Thus we find experimentally that the radial distribution of the wave magnetic field agrees with theoretical predictions and also that higher modes are damped out as the wave propagates.

Our theoretical analysis postulates an azimuthally symmetrical wave propagation. The coaxial driving-electrode system has this property, and the copper cylinder was carefully aligned with the axial magnetic field by using a method which has been previously reported.[4] The azimuthal symmetry was measured experimentally with four magnetic probes disposed

90 deg apart on the same base circle. A shot-to-shot variation of 10 to 20% was observed in individual probe signals, but the average of several shots gave azimuthal symmetry.

REFLECTIONS

An Alfvén wave propagating in the hydromagnetic waveguide should reflect when it encounters an abrupt discontinuity such as the end of the tube. These reflections have been observed as a voltage appearing on the coaxial electrode at the driving end of the tube (Fig. 6) and also with magnetic probes (Fig. 7). In each case the time interval observed corresponds to the wave making an appropriate number of transits of the tube traveling at the Alfvén velocity. The phase of the reflected voltage signal is in phase with the original signal (Fig. 6) as would be expected for reflection from a high-impedance (open-end) termination. The azimuthal magnetic field changes phase on such a reflection (Fig. 7), as is expected from theoretical considerations.

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