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Fundamental to the Bohr-Mottelson[1-3] representation of nucleonic wave functions for spheroidal nuclei is the separation of a slow rotational motion of the deformed nucleus from a rapid nucleonic motion in the nuclear frame of reference. The treatment introduces two new approximate quantum numbers, Ω and K, the components along the nuclear symmetry axis of the nucleonic angular momentum and of the total angular momentum, respectively. For the lower-lying states, where no "vibrational" excitation is involved, K and Ω are equal. Thus we shall refer hereafter to K-quantum numbers of states with the implicit understanding that Ω remains equal to it. With the availability of calculated nucleon wave functions for spheroidal potentials[4] it is appropriate to check the degree to which the separation of rotational and intrinsic nucleonic motion is justified. That is, we may evaluate and include the usually neglected off-diagonal matrix elements of Bohr's U_1. (Cf. A. Bohr, Ref. 1 pp 29 and 35, also Davidson and Feenberg[5] and Ford.[6]) These matrix elements connect Bohr states which differ in K (and Ω) by one but have like spins and parities. The matrix elements measure the extent to which the rotational motion decouples the intrinsic nucleonic motion from the symmetry axis.

There is considerable experimental evidence from relative gamma and beta transition probabilities to rotational band members that K is quite a good quantum number in even-even nuclei in the regions of high nuclear deformation.[3,7] On the other hand, since no pairing energy need be involved for intrinsic excitation of an odd nucleon, K mixing in odd-mass nuclei could often be important even in the regions of highest deformation. Bohr and Mottelson[8] have recently discussed the role of the matrix elements U_1 in connection with rotational moments of inertia.

In attempts[9] to classify the low-lying nuclear states of Np^{237} the presence of E1 transitions has drawn attention to the even-parity states. Most even-parity states in the 82-126 shell tend in the limit of low deformation toward pure $i_{13/2}$ character, but in the calculations of Nilsson[4] at large deformation ($\delta = +0.3$) become substantially mixed, principally with $g_{9/2}$ and $d_{5/2}$.

As a numerical example illustrative of the special importance of U_1 in certain cases ($j >> \Omega$) some of the lowest nuclear energy levels for a single nucleon in the even-parity states of the 82-126 shell were calculated. The wave functions and eigenvalues of Nilsson at $\delta = +0.3$ (large prolate deformation) were used, with assumption of a rotational quantum energy $\left(\frac{\hbar^2}{2\mathfrak{J}}\right)$ of 7 kev, an average for even-even nuclei in the heavy region. The set of levels at the left in Fig. 1 were calculated neglecting off-diagonal contributions of U_1, and those at the right included them.

There are extensive shifts in the levels and actually a reversal in level order of the lowest two states.

K mixing occurs in the lowest-lying states to an extent that might significantly alter some nuclear properties, such as transition probabilities and magnetic moments. (K mixing cannot, of course, occur in the $I = 1/2$ state.) The spin 5/2 state has 4.4 percent $K = 3/2$ and 0.03 percent $K = 5/2$, and the spin 9/2 state, 9 percent $K = 3/2$ and 0.8 percent $K = 5/2$ character.

For a somewhat larger deformation or larger moment of inertia than used in the numerical case of Fig. 1 the 5/2 state would lie lowest. This configuration would then be possible for the ground state of Np^{237}. A curious feature of Np^{237} is the anomalously large magnetic moment of its ground state, +6.0 ± 2.5 nuclear magnetons,[10] a value outside the Schmidt limits for spin 5/2 nuclei.

Magnetic moment calculations with Nilsson's wave functions of the $I = 5/2$, $K = \Omega = 5/2$ state of predominantly $i_{13/2}$ character give about +3.0 magnetons, not strongly dependent on deformation.[11] Calculations for the $I = 5/2$, $K = \Omega = 1/2$ state without K-mixing give +3.9 nuclear magnetons, virtually independent of the degree of prolate deformation.[12] The partial decoupling of the nucleonic angular momentum from the symmetry axis and the consequent K-mixing have an important influence on the magnetic moment. Indeed for the lowest-lying $I = 5/2$ state of the numerical example of Fig. 1 we calculate a magnetic moment of about +5.2 nuclear magnetons. The magnetic moment would be larger for smaller deformation, approaching a limit of +6.1 nuclear magnetons.

If we interpret the ground state of Np^{237} to be this predominantly $K = 1/2$ state and assume positive intrinsic nuclear deformation, the spectroscopic quadrupole moment, not yet measured experimentally, should be negative. This interpretation also offers the means of explanation for the recent observation by Roberts and Dabbs[13] that aligned Np^{237} nuclei emit alpha particles preferentially in a direction perpendicular to the nuclear spin axis. The opposite behavior would be expected for $K = 5/2$ nuclear state with prolate surface deformation.[14, 15]

Difficulty with the above interpretation is encountered in assignment of the 33-kev excited state, and the experimental limits of error on the Np^{237} magnetic moment are still too large to be as restrictive as desirable for nuclear state assignments.

We may generalize from the above calculation that the even-parity nuclear states with an odd nucleon in the 82-126 shell will usually exhibit badly perturbed rotational bands or, at best, bands with very large apparent moments of inertia. The odd-parity states, having generally lower average intrinsic angular momentum (j content), will have smaller matrix elements U_1 and hence be less subject to K-mixing. They should more generally exhibit normal rotational bands.

I am especially indebted to Dr. S. G. Nilsson for sending his calculated wave functions and to Dr. A. Bohr and B. R. Mottelson for other valuable reports in advance of publication.

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Note added in proof: We have just seen a preprint of a paper by Kerman[16] containing a detailed analysis of effects of U_1, designated RPC in his notation. He finds these matrix elements of importance in the W^{183} spectrum.

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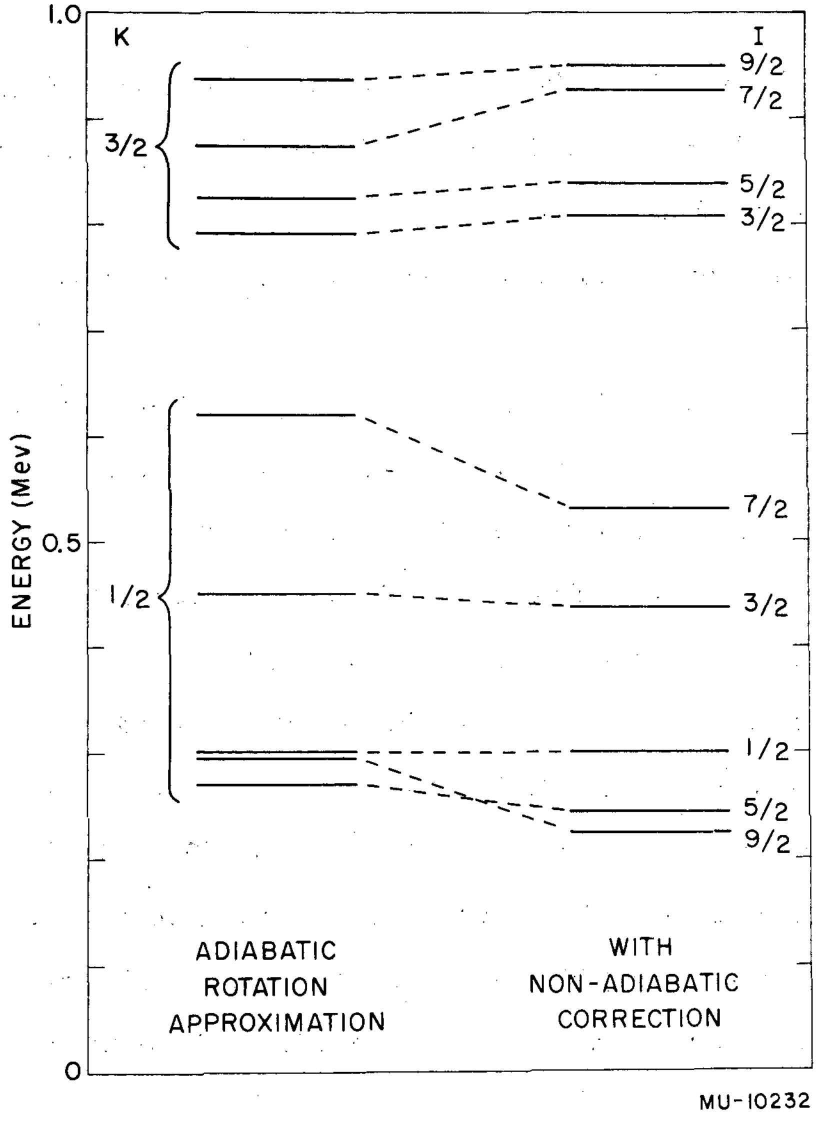

Fig. 1. Theoretical rotational band spectra for an odd nucleon in orbitals of predominantly $i_{13/2}$ character at large prolate deformation ($\delta = 0.3$).