UC Santa Barbara
Core Curriculum-Geographic Information Systems (1990)

Title
Unit 47 - Fractals

Permalink
https://escholarship.org/uc/item/3329925c

Authors
Unit 47, CC in GIS
Klinkenberg, Brian

Publication Date
1990

Peer reviewed

eScholarship.org
Powered by the California Digital Library
University of California

UNIT 47 - FRACTALS

UNIT 47 - FRACTALS

Compiled with assistance from Brian Klinkenberg, University of British Columbia

UNIT 47 - FRACTALS

Compiled with assistance from Brian Klinkenberg, University of British Columbia

A. INTRODUCTION

Why learn about fractals?

- fractals are not so much a rigorous set of models as a set of concepts
- these concepts express ideas which have been around in cartography for a long time
- they provide a framework for understanding the way cartographic objects change with generalization, or changes in scale

- they allow questions of scale and resolution to be dealt with in a systematic way

Length of a cartographic line

- if a line is measured at two different scales, the second larger than the first, its length should increase by the ratio of the two scales
  - areas should change by the square of the ratio
  - volumes should change by the cube of the ratio

- yet because of cartographic generalization, the length of a geographical line will in almost all cases increase by more than the ratio of the two scales
  - new detail will be apparent at the larger scale
  - "the closer you look, the more you see" is true of almost all geographical data
  - in effect the line will behave as if it had the properties of something between a line and an area

- a fractal is defined, nontechnically, as a geometric set - whether of points, lines, areas or volumes - whose measure behaves in this anomalous manner
  - this concept of the scale-dependent nature of cartographic data will be discussed in more detail later

Where did the ideas originate?

- term was introduced by Benoit Mandelbrot to the general public in his 1977 text Fractals: Form, Chance and Dimension
  - a second edition in 1982 is titled The Fractal Geometry of Nature

  - some of Mandelbrot's earliest ideas on fractals came from his work on the lengths of geographic lines in the mid 1960s

- fractals may well represent one of the most profound changes in the way scientists look at natural phenomena
  - fractal-based papers represent over 50% of the submissions for some physics journals
  - many of the studies of the fractal geometry of nature are still at the early stages (especially those in geomorphology and cartography)
  - the results presented in some fields are very exciting (e.g., see Lovejoy's (1982) early work on the fractal dimensions of rain and cloud areas)

B. SOME INTRODUCTORY CONCEPTS

Euclidean geometry

- in traditional Euclidean geometry we work with points, lines, areas and volumes
  - Euclidean dimensions (E) are all positive whole numbers

- the Euclidean dimension represents the number of coordinates necessary to define a point
  - to specify any point on a profile requires two coordinates, thus a profile has a

  Euclidean dimension of two
  - to define a point on a surface requires three dimensions, therefore a surface has a Euclidean dimension of three

- closely allied with Euclidean dimensions are the topological dimensions (DT) of phenomena
  - on a flat piece of paper (which has a Euclidean dimension of 2) you can draw a two-dimensional figure (DT= 2), a one-dimensional line (DT= 1), and a zero-dimensional point (DT= 0) (compare 0-cell, 1- cell and 2-cell notation)

- in fractal geometry we work with points, lines, areas and volumes, but instead of restricting ourselves to integer dimensions, we allow the fractal dimension (D) to be any real number
  - the limits on this real number are that it must be at least equal to the topological dimension of the phenomenon, and at most equal to the Euclidean dimension (i.e., 0&LT=DT&LT=D&LT=E)
  - a line drawn on a piece of paper can have a fractal dimension anywhere from one to two

- the term fractals is derived from the same Latin root [fractus] as fractions; therefore: fractional dimensions

- the fractal dimension summarizes the degree of complexity of the phenomenon, the degree of its 'space-filling capability'

  overhead - Lines of different fractal dimensions

  - straight line will have equivalent topological and fractal dimensions of 1
  - slightly curved line will still have a topological dimension of 1, but a fractal dimension slightly greater than 1
  - highly curved line (DT= 1) will have a much higher fractal dimension
  - line which completely 'fills in' the page will have a fractal dimension of 2
  - many natural cartographic lines have fractal dimensions between 1.15 and 1.30
  - a surface can have a fractal dimension anywhere from 2 (perfectly flat) to 3 (completely space-filing)

- fractal dimension indicates how measures of the object change with generalization
  - e.g. a line with a low fractal dimension (straight line) keeps the same length as scale changes
  - a line with fractal dimension 1.5 loses length rapidly if it is generalized

- topological dimension tells us little about how shapes differ
  - e.g. all coastlines have the same topological dimension
  - however, sections of many coastlines have been found to have very different fractal dimensions

- fractal dimension quantifies the metric information in lines and surfaces in a new and unique manner

C. SCALE DEPENDENCE

- the scale dependent nature of measurements (especially those made on maps) has been observed by many people
  - e.g. as you measure the length of a natural boundary on maps of larger scales, or make your measurements with more precise instruments, the length appears to increase
  - this is known as the "Steinhaus Paradox"

- Richardson (1961) made an extensive study of the cartographic representation of international borders

  suggested overhead Richardson plot, see Mandelbrot 1982, p. 33

  - he observed that there was a predictable relationship between the scale at which the measurement was made, and the length of the line
  - even though the length increased when the borders were measured on maps of larger scale, the increase was predictable
  - plots illustrating the relationship between measurement scale and length have since become known as Richardson plots
  - Mandelbrot subsequently placed Richardson's (and others) work within the framework of fractal geometry, and showed that such behavior is predicted in a fractal world

Determining fractal dimension

- an example of how to determine the fractal dimension of a cartographic line: 1. step a pair of dividers (step size s1) along the line; say it takes n1 steps to span the line 2. the length of the line is equal to s1n1 3. repeat the process, but decrease the step size (to s2); it now takes n2 steps to span the line 4. the length of the line is now s2n2 5. the fractal dimension can be calculated as:

  $D = \log (n2/n1) / \log (s1/s2)$

- worked example:
  - dividers size: 10 m number of steps: 100
  - dividers size: 5 m number of steps: 220

    $D = \log (220/100) / \log (10/5) = \log (2.2) / \log (2.0) = 0.3424 / 0.3010 = 1.14$

  - here used logs to base 10, but any base could be used

- the more irregular the line, the greater the increase in length between the two estimates, and the greater the fractal dimension

- Mandelbrot's texts, the book by Peitgen and Saupe (1988), and the papers by Goodchild and Mark (1987) and Milne (1988) discuss other methods of determining the fractal dimension
  - there are a large number of ways of determining the fractal dimensions of points,

lines, areas, and volumes

Some questions

1. what is the "true" length of a line?

2. how can you compare curves whose lengths are indeterminate?

3. of what value are indices based on length measurements?

    - the perimeter of an area object increases steadily with scale, but the area of an area object deviates up and down by much smaller amounts
    - are analyses based on area less scale-dependent than ones based on perimeter?
    - what does this indicate about measures of shape based on the ratio of perimeter to the square root of area?

- there is no complete solution to these (and similar types of) problems
    - however, use of fractal geometry (especially the fractal dimension) does allow us to make reasonably meaningful comparisons and indices (as illustrated in Woronow, 1981)

- these questions are of special interest to cartographers interested in digital representations of cartographic features (e.g. Buttenfield, 1985)
    - there are implications with respect to: 1. digitizing
        - determination of the appropriate sampling interval 2. generalizing lines

        - the best method for generalizing lines may be that method which best retains the fractal dimension of the line 3. displaying lines at a scale greater than that at which the line was collected
        - introduce additional "information", by adding artificial detail to the line, detail which is a function of the fractal dimension of the original line); 4. incorporating the fractal dimension into traditional cartometry measures
        - see Woronow (1981)

D. SELF-SIMILARITY AND SCALING

Self-similarity

- indicates that some aspect of a process or phenomenon is invariant under scale-changing transformations, such as simple zooming in or out

- can be expressed in two ways:

  overhead - Self-similarity 1. geometric self-similarity, in which there is strict equality between the large and small scales

        - not found in natural phenomena
        - the Morton order, quadtrees use this idea in replicating the same pattern at every level 2. statistical self-similarity, in which the equality is expressed

in terms of probability distributions
- this type of (random) self-similarity is the more common, and is the type found in many natural phenomena, such as coastlines, soil pH profiles, river networks (Burrough, 1981; Peitgen and Saupe, 1988; etc.)

- the simplest test of self-similarity is visual
  - if a phenomenon is self-similar, any part of it, if suitably enlarged, should be indistinguishable from the whole or from any other part
  - if a natural scene is self-similar, it should be impossible to determine its scale
    - e.g. it should be impossible to tell whether a picture of self-similar topography shows a mountain range or a small hill - there are no visual cues as to the picture's scale
  - since many scale cues are cultural, geological or geomorphological, self-similar topographies are most common on lunar or recent volcanic landscapes

Scaling

- not necessarily equivalent to self-similarity, although the two terms are often used interchangably in the literature

- consider a landscape, as represented by a surface and a contour map
  - on the contour map (coordinates in 2 dimensions only) the axes can be switched without fundamentally changing the characteristics of the landscape, i.e. the characteristics of the contour lines
    - contour lines are therefore examples of simple scaling fractals
  - in the case of the surface, with coordinates in 3 dimensions, we cannot interchange the z axes with either of the x or y axes without fundamentally altering the characteristics of the landscape
    - since the z axis has a different scaling parameter than the x or y axes, a three- dimensional representation of the Earth's surface is therefor an example of a non-uniform (or multiple) scaling representation

- shapes that are statistically invariant under transformations that scale different coordinates by different amounts are known as self-affine shapes (Peitgen and Saupe, 1988)
  - the Earth's surface is an example of a self-affine fractal, but it is not an example of a self-similar fractal
  - contour lines, which represent horizontal cross- sections of the land surface, are examples of statistically self-similar scaling phenomenon (because the contour has a constant z value)

- because the land surface is self-affine and not self- similar, those techniques which determine the fractal dimension of the land surface itself produce values which are different than the values produced by those techniques which determine the fractal dimension of the contours derived from that land surface

E. ERROR IN LENGTH AND AREA MEASUREMENTS

scale, through its relationships with generalization and resolution, significantly influences length and area measurements

- problems in estimating line lengths, areas, and point characteristics can be related to the phenomenon's fractal dimension (Goodchild, 1980)
- estimates of area are frequently based on pixel counts, especially in raster-based systems
  - the error in the area estimate is a function of the number of pixels cut by the boundary of the object
  - boundaries with a fractal dimension greater than one will appear more complex as the pixel size decreases (as the resolution increases)
  - the more contorted the boundary, or the higher its dimension, the less rapid the increase in error with cell size

diagram

  - error in a pixel-based area estimate will also be a function of how the phenomenon is distributed about the landscape: the error in area associated with a highly compact phenomenon will be much less than the error in area associated with a widely dispersed, patchy phenomenon
  - Goodchild and Mark (1987, p. 268) show that:
    - the standard error as a percentage of the area estimate is proportional to a(1-D/4) where a is the area of a pixel and D is the fractal dimension of the boundary
    - standard error will thus depend on a1/2 for highly scattered phenomenon and a3/4 for single, circular patches with smooth boundaries

REFERENCES

Only a very small portion of the literature is presented here. For further references you should refer to the Goodchild and Mark (1987) paper; recent issues of Water Resources Research and Science also contain relevant papers

Burrough, P.A., 1981. "Fractal dimensions of landscapes and other environmental data," Nature 294:240-242.

Buttenfield, B., 1985. "Treatment of the cartographic line," Cartographica 22:1-26.

Goodchild, M.F., 1980. "Fractals and the accuracy of geographical measures," Mathematical Geology 12:85-98.

Goodchild, M.F., and Grandfield, A.W., 1983. "Optimizing raster storage: An evaluation of four alternatives," Auto-Carto 6(2):400-407.

Goodchild, M.F., and Mark, D.M., 1987. "The fractal nature of geographic phenomena," Annals AAG 77(2):265-278.

Hakanson, L., 1978. "The length of closed geomorphic lines," Mathematical Geology 10:141-167.

Lovejoy, S., 1982. "Area-perimeter relation for rain and cloud areas," Science 216:185-187.

Mandelbrot, B.M., 1977. Fractals: Form, Chance and Dimension, Freeman, San Francisco.

Mandelbrot, B.M., 1982. The Fractal Geometry of Nature, W.H. Freeman and Co., New York.

Milne, B.T., 1988. "Measuring the fractal geometry of landscapes," Applied Mathematics and Computation 27:67- 79.

Peitgen, H.-O. and D. Saupe (Eds.) 1988. The Science of Fractal Images, Springer-Verlag, New York.

Richardson, L.F., 1961. "The problem of contiguity," General Systems Yearbook 6:139-187.

Unwin, D., editor, 1989. Special issue on fractals. Computers and Geosciences 15(2).

Woronow, A., 1981. "Morphometric consistency with the Hausdorff-Besicovich dimension," Mathematical Geology 13:201-216.

EXAM AND DISCUSSION QUESTIONS

1. Although fractal concepts are important in understanding the error associated with pixel-based area estimates, little has been said about the relationship between fractals and area estimates obtained from vector-based systems. Why? (i.e., would the area of an enclosed figure change significantly? It is expected that the area shouldn't change significantly, as the self-similar detail should increase the area as much as it decreases the area.)

2. Define "fractal". Include in your description terms such as scale dependency, self-similarity and scaling.

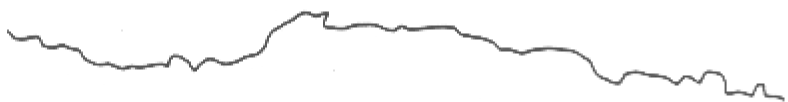
3. Discuss some of the ways in which fractals have changed our way of looking at phenomena. Based on your readings, provide examples from a variety of fields.

4. Theoretically, fractal behavior applies to a phenomenon across all scales. Practically, of course, there are limits to the application of self-similarity to natural phenomena. Where do you think some of these limits occur? (i.e., between what scales do you think portions of coastlines, for example, exhibit self-similar behavior.) What are the implications with respect to the generalization of cartogrpahic lines, if we observe definite limits to the self-similar behavior of cartographic features?

Last Updated: August 30, 1997.

UNIT 47 IMAGES

D=1.1



D=1.2

D=1.4

Geometric Self-Similarity