Title

Working Memory Components as Predictors of Word Problem Solving: Does Rapid Automatized Naming Speed Mediate the Relationship?

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Publication Date

2015

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA
RIVERSIDE

Working Memory Components as Predictors of Word Problem Solving: Does Rapid Automatized Naming Speed Mediate the Relationship?

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Education

by

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June 2015

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ACKNOWLEDGEMENTS

I would like to express my gratitude to my adviser and mentor Dr. H. Lee Swanson for his continued support during my time at University of California, Riverside and the opportunities he had given me in the past few years. His inspirations, expertise, patience, and enthusiasm made this journey fun and exciting. There were some struggles and bumpy roads in the past few years of this journey, and Dr. Swanson was always there when I need a hand. He is one of the best mentors one could hope for, and I am thankful that I got to be one of his students. He is and will always be an excellent role model and an inspiration for me.

I would also like to thank Dr. Michael Orosco and Dr. Keith Widaman for the helpful comments and suggestions they gave me for this dissertation. I am glad they were part of my academic journey.

There are no words good enough to express the gratitude that I have for my parents, who were there every step of the way through this journey and supported me in all that I do.

I would not have survived this this journey without my friends and colleagues, Cathy Lussier, Mily Kudo, Hapreet Uppal, Jennifer Kong, and Andres Olide. All of you were great inspirations to me, gave me valuable advice, and were always there through the ups and downs. You will always be in my mind and heart. Finally, I want to thank the musicians whose music, Tweets, and Instagram posts never fails to make me smile: Tiffany Alvord, Chrissy Costanza (Against the Current), Jackie Evancho, Kelsie May, Karoline Rhett, and Taylor Swift.

ABSTRACT OF THE DISSERTATION

Working Memory Components as Predictors of Word Problem Solving:
Does Rapid Automatized Naming Speed Mediate the Relationship?

by

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University of California, Riverside, June 2015
Dr. H. Lee Swanson, Chairperson

Numerous studies have established the relationships between working memory (WM), rapid automatized naming speed, reading comprehension, calculation, and word problem solving (WPS). However, there is limited research on the differential effects of the WM components (central executive, phonological loop, visual-spatial sketchpad) on WPS, and how speed, reading, calculation, and knowledge of word problem components mediate that relationship. The sample consisted of 413 between ages 6-10 (*M* = 8.38, *SD* = .51). The results yielded three important findings: 1) without any mediators, the phonological loop was the best predictor of WPS; 2) when other variables were entered into the model, reading and calculation were the strongest direct predictors of WPS; 3) results of the mediation model showed that speed mediated the relationship between the central executive component of WM and WPS, while reading mediated the relationship between the central executive and phonological loop and WPS. The educational implications of these findings are discussed.

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CHAPTER 1

INTRODUCTION

The Importance of Mathematics

Mathematics skills are important in everyday life, and are used in activities such as shopping, cooking, and banking. More importantly, mathematics is the foundation of numerous career fields, particularly science, technology, and engineering. The developments and innovations in these fields, in turn, are essential to the nation's economic prosperity and the ability to compete on a global scale (National Mathematics Advisory Panel, 2008). History is full of examples of nations with citizens that have good mathematics skills, and who lead the country in advances in technology, medicine, defense, finance, and numerous other fields (National Mathematics Advisory Panel, 2008). During most of the 20th century, U.S. has been a leader in engineering and technology, partly due to mathematical skills of its citizens. Examples include the development of nuclear weapons and other uses of nuclear energy beginning in the 1940s (Atomic Central, n.d.), manned moon landings in the late 1960s and early 1970s, personal computers in early 1980s (Computer History Museum, 2006), and the Microsoft Windows operating system in mid 1980s (Microsoft, 2013). These are only a few of the most important engineering and technology innovations that propelled the U.S. into a world leader. More importantly, mathematics is one of the foundations in the fields of engineering, physics, and computer science that brought about these innovations, and illustrate the importance of mathematics in sustaining economic and technological competitiveness.

Unfortunately, in the past few decades the U.S. has been lagging behind in its mathematical prowess compared to other nations, particularly Asian countries. In first grade, both Chinese and Japanese children scored higher than U.S. children, and the differences persisted through fifth grade (Stevenson, Lee, & Stigler, 1986). A follow up study a decade later revealed that the situation did not improve, and had even gotten worse (Stevenson, Chen, & Lee, 1993). While the mathematics scores of U.S. children in 1980 and 1990 were similar, the scores of Chinese children improved, and thus, widening the mathematics achievement gap. Furthermore, when the top 10% of children were compared, the scores of the top performing U.S. children were similar to that of average Chinese children. More recent data suggests that the mathematics achievement gap still exist today (OECD, 2012a; National Center for Education Statistics, 2009). The most recent data from the Trends in International Mathematics & Science Study (TIMSS; National Center for Education Statistics, 2011) indicated that 4^{th} and 8^{th} grade children in 11 countries scored higher than U.S. children (57 countries participated in the assessment). The top five countries were Singapore, Korea, Hong Kong, China, and Japan. Although the average mathematics score of the U.S. (541) was above the TIMSS average (500), the average scores of the top five countries were 606, 605, 602, 591, and 585. The most recent data from the Programme for International Student Assessment (PISA; OECD, 2012a) indicated that the U.S. was below average in mathematics, and was ranked 27^{th} among the 34 countries that participated in the assessment. Furthermore, the scores of China (the top performing country) is equivalent to over two years ahead of Massachusetts (the top performing state in the U.S.). A deeper analysis of the PISA data revealed that the weaknesses of mathematics skills of U.S. students were not in basic calculation, but in mathematics skills that require higher cognitive thinking, such as real-world problems, which require comprehension of the problems, translating them into mathematical terms, and finally doing the basic calculations. The National Mathematics Advisory Panel (2008) noted in a report: "During most of the 20th century, the United States possessed peerless mathematical prowess… But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century" (p. xi). In sum, mathematics skills are an important educational foundation, and are important for the U.S. to continue to be a leader in the fields of science, technology, and engineering, and to remain competitive globally.

The importance of mathematics is also revealed on a more individual level. Those who have low mathematics skills as children have higher risk of lower mathematics skills when they complete their education as adults, beyond the influence of family socioeconomic status, reading skills, and intelligence (Raghubar, Barnes, & Hecht, 2010). Children who do not have good mathematics skills are also less likely to continue into higher education (OECD, 2012b). For children who do pursue high education, having a good mathematics foundation gives them more college and career options (National Mathematics Advisory Panel, 2008), and influence what they chose for college major (Paglin & Rufolo, 1990). In other words, when children did not have a good mathematics foundation, and thus, poorer mathematics performance, it is less likely that they will choose a field that requires good mathematics skills such as science, technology, and engineering. Upon graduating from college, young adults are more likely to have full-time employment when they have good basic mathematics (e.g., algebra; National Mathematics Advisory Panel, 2008), and is beyond the influence of intelligence, reading skills, and ethnicity (Rivera-Batiz, 1992). Furthermore, mathematics skills also influences earnings (Ashenfelter & Krueger, 1994; National Mathematics Advisory Panel, 2008) and productivity at the workplace (Rivera-Batiz, 1992)

Word Problem Solving

Because of the importance of mathematics skills for individuals and society, it is essential for children to develop good mathematical skills at an early age. While basic calculation are an important part of mathematics competence, another aspect of mathematics that may be as important and warrants further investigation is word problem solving. Word problem solving are mathematics exercises where background information is presented as texts rather than mathematics notations and equations, and is one of the most important methods through which students can learn to select and apply the appropriate strategies for solving real-world problems. Because of its text format, word problems incorporate reading fluency, reading comprehension, and calculation. Furthermore, word problem solving involves multiple domains (i.e., reading and mathematics) and is considered a form of higher cognitive processing that involves multiple cognitive systems (e.g., Andersson, 2007; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Swanson, 2006a; Swanson & Beebe-Frankenberger, 2004; Zheng, Swanson, & Marcoulides, 2011).

The PISA assessment indicated that U.S. children show weaknesses when asked to solve word problems (OECD, 2012a), which suggests a need to further understand the cognitive mechanisms and processes that underlie word problem solving, so that educators can develop better teacher education and instructional practices, and provide more effective strategies help these children (National Mathematics Advisory Panel, 2008). Several lines of research have already begun in attempts to understanding cognitive predictors of word problem solving. As expected, because word problems require children to first understand the text, select the correct strategies for finding the solution, and finally do the actual calculations, basic reading and calculation skills are strongly associated with word problem solving (e.g., Andersson, 2008; Kyttälä, & Björn, 2014; Swanson, 2004, 2006a, 2011; Swanson & Beebe-Frankenberger, 2004; Swanson, Jerman, & Zheng, 2008; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008; Zheng et al., 2011). However, reading and calculation skills do not entirely explain word problem solving performance. Domain general cognitive factors also come into play.

Another line of research that attempts to explain word problem solving performance is in the field of working memory. By far the most utilized framework of working memory is Baddeley's multiple-components model (Baddeley, 1986, 2006, 2007, 2012; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Gathercole, 1998). In this model, working memory consists of three components: visual-spatial sketchpad, phonological loop, and central executive. The visual-spatial sketchpad is for the temporary storage of visual and spatial information, and it is important for manipulation of mental images, such as mathematical symbols and shapes. The phonological loop is for the temporary storage of verbal information, and it is important for temporary storage of text and verbal information, such as the story in a word problem. It is also used for encoding and maintaining calculation operands (Furst & Hitch, 2000; Noël, Désert, Aubrun, & Seron, 2001). The central executive coordinates activities between the two subsystems (i.e., visual-spatial sketchpad and phonological loop), and increases the amount of information that can be stored in the two subsystems. As expected, the capacity and efficiency of working memory is associated with word problem solving performance (e.g., Lee, Ng, Ng, & Lim, 2004; Swanson, 2004, 2006a, 2011, 2014; Swanson et al., 2008; Zheng et al., 2011).

Related to working memory, a third line of research on rapid automatized naming (also referred to as naming speed) suggests that it may be related to word problems solving. Rapid automatized naming is the ability to name as quickly and accurately as possible a series of objects, colors, or numbers (Norton & Wolf, 2012). Researchers have suggested that development of working memory is not the increase of *capacity*, but is the increase in *efficiency* (Case, Kurland, & Goldberg, 1982; Roodenrys & Hulme, 1993). Since working memory capacity is limited, the faster information is processed through each of the three components, the more capacity it frees up and becomes available for the next set of information. Rapid automatized naming has been found to be strongly associated with reading fluency and comprehension (e.g., Blachman, 1984; Compton, 2003; Georgiou, Aro, Liao, & Parrila, 2014; Lepola, Poskiparta, Laakkonen, & Niemi, 2005; Moll, Gobel, Gooch, Landerl, & Snowling, 2014; Savage & Frederickson, 2005; Schatschneider, Fletcher, Francis, Carlson, & Foorman, 2004) and calculation (e.g., Cirino, 2011; Geary, 2011; Hecht, Rogesen, Wagner, & Rashotte; 2001; Moll et al., 2014; Koponen, Aunola, Ahonen, & Nurmi, 2007; Swanson & Kim, 2007), which as discussed earlier, are strongly associated with word problem solving. Since naming speed measures operational efficiency of working memory and has a linear relationship with working memory span (Case et al., 1982; Roodenrys & Hulme, 1993), and is associated reading and calculation, which in turn is associated with word problem solving, it is possible that naming speed mediates the relationships between working memory and word problem solving.

Another line of research that is relatively new in the field of word problem solving is children's knowledge of the components that underlie word problem solving. These word problem solving components include identifying the question, numerical information, goals, operation, and algorithm to use for each word problem (Swanson, 2004). The question, number, and goals form the *representation* construct, while the operation and algorithm form the *planning* construct. Swanson (2004) found in sample of 8- and 11-year-old children, that the operation and algorithm components were moderately correlated with word problem solving, but the question, number, and goal components were not. The operation and algorithm components also uniquely predicted word problem solving in hierarchical regression models that included reading, calculation, and three components of working memory. Zheng et al. (2011) used structural equation modeling techniques (path modeling) and found that a latent variable composed of all of the word problem solving components did not have direct effects on word problem solving, and it did not mediate the relationship between working memory and word problem solving. However, note that Zheng et al. (2011) did not separately examine the effects of representation (i.e., the question, number, and goals components) and planning (i.e., the operation and algorithm components).

In summary, mathematics is one of the most important skills to have, not only for everyday life but also for individuals who work in the science, technology, and engineering fields. The developments and innovations in these fields, in turn, help the U.S. remain competitive globally. However, children in U.S. have been lagging behind other countries in mathematics performance, particularly in the area of word problem solving. It is essential to understand the factors that affect mathematics performance so that researchers and educators can better help children who have poor mathematics performance. Several lines of research have attempted to identify some of these factors, including reading and calculation, working memory, rapid automatized naming (naming speed), and word problem components. All of these proximal (reading, calculation, and word problem components) and cognitive (working memory and naming speed) factors have been found to directly or indirectly predict word problem solving. However, despite several decades of research, there still remain areas that needed to be clarified or explored.

Extending the Research

One of the areas that needed further research is the differential contribution of the three working memory components (visual-spatial sketchpad, phonological loop, and central executive) on word problem solving. While previous research has established that working memory is a strong predictor of word problem solving, only recently have research focused on examining the direct and mediated influences of all three working memory components on word problem solving (Zheng et al., 2011; Meyer et al., 2010). Meyer et al. examined the relationships between working memory components and mathematics among a sample of 2nd to 3rd graders, and found that for 2nd graders, the phonological loop and central executive predicted mathematics reasoning skills (counting, identifying geometric shapes, solve word problems); for 3rd graders, the visual-spatial sketchpad (but not phonological loop nor central executive) predicted mathematical reasoning and numerical operations skills (identifying and writing numbers, counting, number production, written calculation problems). Zheng et al. (2011) conducted similar study with a sample of 2nd to 4th graders. They found direct relationships between all three components of working memory and word problem solving, but the strength of these relationships differed. The beta path coefficients for central executive, visual-spatial sketch pad, and phonological loop were .64, .39, and .26, respectively. This indicates central executive is the most influential among the three working memory components. Zheng et al. also tested a mediation model, with calculation, reading, and word problem components as mediators. The significant indirect paths from working memory to word problem solving were through calculation and reading. In the presence of these mediators, the direct paths from working memory to word problem solving were nonsignificant, suggesting that mediators are an important part of understanding the mechanisms underlying word problem solving.

A limitation of the Meyer et al. (2010) study is that no mediation analysis was conducted. Although direct relationships exist and different components of working memory have different strength of relationship with word problem solving, once mediators were introduced the direct relationship became nonsignificant (Zheng et al, 2011). This suggests the need to examine mediators, as Zheng et al. (2011) have done. Another limitation of previous research is that rapid automatized naming was not included as a mediator. Several studies conducted by Swanson and colleagues have included rapid automatized naming, but not as mediators between working memory and word problem solving (e.g., Swanson, 2004, 2006a, 2011; Swanson & Kim, 2007; Swanson & Beebe-Frankenberger, 2004).

In summary, this proposed study will attempt to address the above limitations in the following ways. First, this study will examine differential contribution of working memory components. Second, this study will include multiple mediators. While working memory components are good predictors of word problem solving, it only explains approximately 30% of the variance in mathematics performance (e.g., Holmes & Adams, 2006; Swanson, 2006a, 2011), and approximately 50% of the variance even when calculation, reading, and fluid intelligence were included in the models (Lee et al., 2004; Swanson, 2004). Because of the close relationships between the working memory components and calculation and reading, and the close relationships between working memory and rapid automatized naming speed, the inclusion of these variables into a model should play a significant role to identifying the underlying processes affecting word problem solving performance.

CHAPTER 2

REVIEW OF LITERATURE

Word Problem Solving and Its Relationship with Calculation and Reading

Word problem solving consists of background information that is presented as texts rather than mathematical equations and notations. Example of a simple one-step word problem is: “Nick bought 7 ice cream cones. He ate 5 ice cream cones. How many does Nick have now?” Example of a multi-step word problem is: “Lana picked 36 tulips and 37 roses to make flower bouquets. If she only used 70 of the flowers though, how many extra flowers did Lana pick?” Mathematics word problem solving is essentially composed of two components, reading and calculation. When solving a word problem, children have to first read and understand the problems, which is where decoding and comprehending the texts (i.e., reading proficiency) become important. Second, children have to select the correct strategy do the actual calculations, which is where calculation skills become important. As expected, because word problems consist of basic reading and calculation, these basic skills and particularly reading, are strongly associated with word problem solving.

Vilenius-Tuohimaa et al. (2008) examined reading comprehension and its relationship with word problem solving in a sample of 4th grade children. Their measures included reading fluency (i.e., technical reading skills; speed and accuracy in separating words in texts) and reading comprehension (i.e., understanding what was read). Children were tested on 20 word problems that included questions such as, “Suvi has 9 liquorice candies and she eats 5 of them. How many liquorice candies does Suvi have now?” and

“There are 7 girls in the class. What is the number of the boys, when there are 16 pupils in the class altogether?” Examination of poor and good readers (based on reading fluency scores) indicated that poor readers had significantly lower scores on word problem solving than good readers (poor readers M = 9.25, SD = 2.69; good readers M = 11.97, SD = 3.25; F = 37.95, $p < .001$). Results of a path analysis showed that reading fluency significantly predicted both word problems solving and reading comprehension. Furthermore, the standardized path coefficients from reading fluency to word problem solving and reading comprehension were the same (.36), indicating that reading fluency is equally important in predicting these two constructs. Reading comprehension was also found to be significantly correlated with word problem solving (r = .38). Mothers education and gender also predicted reading fluency, reading comprehension, and word problem solving, but the standardized path coefficients indicated the strength of these relationships was not as strong as the relationships of the other constructs (i.e., between reading fluency and word problem solving and reading comprehension, or between reading comprehension and word problem solving. Similar results were found by Kyttälä and Björn (2014) in a sample of 8^{th} grade children. In the reading fluency measure, in addition to asking children to separate words, as in the Vilenius-Tuohimaa et al. (2008) study, children in the Kyttälä, & Björn study was also asked to find spelling errors. Gender and visual-spatial ability (a component of working memory) was also measured and controlled in the analysis. The results showed that reading fluency and comprehension predicted word problem solving. The coefficients indicated that reading fluency is a stronger predictor (15.78) than reading comprehension (9.27). Furthermore, while both reading fluency and comprehension had moderate correlation with word problem solving, reading fluency (r = .54) was stronger than that of reading comprehension (r = .50). Calculation was also measured (i.e., basic calculation and equations), but was not examined as a predictor of word problem solving. It was examined as criterion variable, where they found that reading fluency predicted calculation skills. However, calculation skills was significantly correlated with word problem solving (r = .38).

Reading comprehension was also examined as a predictor of word problem solving in the presence of other cognitive factors. In a sample of 1st through 3rd graders, Swanson et al. (2008) found that Year 3 word problem had moderate correlation with Year 1 reading comprehension (r = .59) and Year 3 calculation skill (r = .66). Hierarchical regression analysis indicated that Year 1 reading comprehension predicted Year 3 word problem solving, even in the presence of other factors, including working memory and rapid automatized naming. As with the Kyttälä & Björn (2014) study, the calculation was not a predictor in the Swanson et al. (2008). Unfortunately, the Vilenius-Tuohimaa et al. (2008) study did not examine the contribution of calculation, and the Swanson et al. (2008) and Kyttälä, & Björn (2014) studies did not enter both reading and calculation into the same models as predictors. The inclusion of calculation (and other variables) may lessen the strength of the relationship between reading comprehension and word problem solving. In more comprehensive studies that included domain general cognitive factors, similar relationships between reading and word problem solving were found. Swanson (2006a) examined a sample of 1st to 3rd graders' growth of several cognitive factors and their relationship with word problem solving. Year 1 reading comprehension was a significant predictor of Year 2 word problem solving, even in the presence of other cognitive factors, including working memory. However, calculation was not significant in the model. In a two-year longitudinal study of work problem solving in 1st to 3rd graders, Swanson (2011) found that Grade 1 reading comprehension and Grade 3 calculation skill was correlated with Grade 3 word problem solving (reading comprehension $r = .76$; calculation skill $r = .75$). However, when cognitive factors (e.g., working memory) were entered into the hierarchical regression models, Grade 1 reading comprehension and Grade 3 calculation did not predict Grade 3 word problem solving. Finally, Zheng et al. (2011) examined a sample of 2nd to 4th graders, using path modeling techniques. Predictors include working memory, age, reading, calculation, and word problem components. They found that while both reading and calculation directly predicted word problem solving, reading was a stronger predictor ($\beta = .54$) than calculation skill ($\beta = .34$).

In summary, research suggests that reading skill is more important than calculation skill when it comes to word problem solving. However, there are three main limitations with previous studies. First, some studies did not include both reading and calculation as predictors of word problem solving. Because word problem solving utilizes both reading and calculation skills, it is important to include both of these basic skills when examining word problem solving. Second, there is limited research on how working memory influences reading and calculation, which in turn, affect word problem solving. Studies showed that both reading and calculation were significantly correlated with word problem solving, but when working memory and other cognitive factors were entered into the models (hierarchical regression models or path models), some studies showed that reading comprehension was still a significant predicator of word problem solving but calculation was not. It is essential to further explore these relationships, and examine reading and calculation as possible mediators between working memory and word problem solving.

Working Memory

Working memory are domain-general factors that is important in understanding the mechanisms underlying children's word problem solving proficiency. There are several working memory frameworks, including Cowan's Embedded Processes Theory, Engel's theory on inhibitory processes, Jonidess' theory on the anatomical locations of different memory processes (Baddeley, 2012). The framework that is most utilized by researchers in the field of education is Baddeley's multiple-components model (Baddeley, 1986, 2006, 2007, 2012; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Gathercole, 1998). According to Baddeley and colleagues' model, working memory consists of three major components that are essential for the temporary storage and manipulation of information. Because word problem solving, and its underlying basic reading and calculation skills, require the manipulation of information such as texts, selecting the correct strategies to use, and mathematics symbols and equations, etc., understanding the relationships between working memory, word problem solving, reading, and calculation are essential to understanding why some children are better at word problems than others.

Working Memory, Short-term Memory, Long-term Memory, and Intelligence

Working memory is a distinct system that is separate from short-term and long-term memory (Alloway & Copello, 2013). As the name suggests, working memory is our ability to work with (manipulate) current information. Although the capacity of working memory is limited (Baddeley, 1986), like that of short-term memory, it is a different system. Short-term memory is for the brief storage of information. For example, when children are solving a word problem, the short-term memory is used to store facts of the problem (e.g., "Billy went to the store and bought 5 apples. The next day, he went to the store again and brought 3 more apples. How many apples does Billy have now?"). When they attempt to solve the problem, they will select the algorithm to use (e.g., addition) and manipulate the equations needed to solve the problem (e.g., 5+3). At this point, they are using working memory. Long-term memory is also distinct from working memory. Long-term memory is for the storage of accumulated information (e.g., "more" is equal to the mathematical operation "addition"). The importance of working memory cannot be underestimated. It has been found to be closely related to basic reading, calculation, and word problem solving (Alloway, 2006; Alloway & Copello, 2013; Gathercole, 1999; Gathercole, 2008; Swanson, 2004, 2006a, 2011; Swanson et al., 2008; Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001).

Working memory is also a distinct concept from fluid intelligence and a better predictor of academic achievement in children (Alloway, 2009; Alloway & Alloway, 2010; Alloway & Copello, 2013). While fluid intelligence measures what have already been learned, working memory provides an indicator of the potential to learn. This may be why working memory is a better predictor of word problem solving than fluid intelligence (Swanson, 2004; Swanson & Beebe-Frankenberger, 2004).

Working memory can also affect classroom behaviors (Alloway, 2006; Alloway, Doherty-Sneddon, & Forbes, 2012; Gathercole, 2008; Gathercole, Durling, Evans, Jeffcock, & Stone, 2008), and because working memory is a measure of the potential to learn, deficits in working memory can impair children's learning opportunities (Gathercole, 1998). Children with poor working memory tend to be reserved in group activities, difficulties in following lengthy instructions, difficulties in keeping places in lengthy procedures, appears to have short attention span, and appear to be inattentive (Gathercole, 2008). A study of classroom teachers found that teachers' ratings of students' classroom behavior were associated with working memory deficits (Alloway et al., 2012). That is, students that teachers consider to have problematic classroom behaviors also had lower working memory scores (measured by the Working Memory Rating Scale developed by the authors). These students were also report to have disruptive and moody. Gathercole et al., (2008) examined a sample of 2, 5, and 6 year-old children in a laboratory setting, and found that working memory is associated with the ability to follow instructions involving sequences of actions.

Components of Working Memory and Their Functions

Visual-spatial Sketch Pad. The visual-spatial sketchpad (VSSP) component of working memory is responsible for the temporary storage of visual and spatial information, and it is important for manipulation of mental images (Baddeley, 1986, 2006, 2007, 2012; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Gathercole, 1998). This includes mathematical symbols and equations when attempting to solve word problems. The manipulation of physical shapes, color, and movement is also processed by the VSSP. Younger children tend to be more dependent on the VSSP than older children, and their VSSP capacity increases from age 5 to 11 (Gathercole, 1998). After age 11, the VSSP capacity is similar to that of adults. Gathercole (1998) suggested that the developmental increases in VSSP capacity is due to an increase in actual capacity rather than processing efficiency, and also due to the increasing use of nonvisual processing, including the phonological loop and central executive components of working memory.

When it comes to mathematics, Gathercole and Pickering (2000) examined a sample of 7 year-old children and their achievement on national curriculum, and found that children with low achievement on mathematics have lower scores on measures of VSSP. Similar results were found with a sample of 8 and 9 year-old children; VSSP predicted mathematics achievement (Holmes & Adams, 2006). When only the three components of working memory were examined, VSSP were good predictor of calculation or word problem solving. For example, Rasmussen and Bisanz (2005) found that VSSP was the best predictor of non-verbal addition problems (presented to the children using blocks) in preschool children (other predictors included phonological loop and central executive). Swanson and Sachse-Lee (2001), Swanson et al. (2008), and Meyer et al. (2010) found that VSSP was a unique predictor of word problem solving, even in the presence of phonological loop and central executive. However, other studies found no direction relationship between VSSP and calculation or reading (Zheng et al., 2011), or that other components of working memory were better predictors than VSSP (Swanson, 2006a, 2011). Although research on the VSSP started in the 1970s, there is relatively less is known about the VSSP compared to the phonological loop and central executive components (Baddeley, 2012).

Phonological Loop. The phonological loop component of working memory is responsible for the temporary storage of verbal information (Baddeley, 1986, 2006, 2007, 2012; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Gathercole, 1998). This includes language, text, and the background story in word problems. One of the functions of the phonological loop is the retaining of serial order, which is important for language/text processing (Baddeley, 2012). Information enters the phonological loop directly through auditory stimulus (e.g., speech), or indirectly through nonauditory internally generated codes (e.g., printed texts or visual objects; Gathercole, 1998). The phonological loop has a limited capacity of approximately 2 seconds; information is lost if there is no attempt to rehearse it (e.g., repeating the information). Developmentally, it has been suggested that rather than actual increase in capacity like the VSSP, older children and adults tend to have better processing speed/efficiency than younger children; the faster information is processed, the more capacity is free to accept incoming information. The research suggests that the phonological loop is responsible for language learning (Baddeley, Gathercole, & Papagno, 1998; Gathercole, 1998).

The phonological loop has also been found to be important for mathematics learning, including encoding and maintaining mathematics operands (Furst & Hitch, 2000; Noël et al., 2001). Furst and Hitch (2000) found that more errors were made in calculation when participants were asked to recite the alphabet or repeat the word "the" while solving calculation problems. Increased calculation errors only occurred when the problem was displayed briefly (4 seconds), but not when they were continuously visible. That is, concurrent articulation interfered with the encoding and rehearsal of operands and mental arithmetic. Noël et al. (2001) went further to manipulate the phonological and visual similarities of addition problems. The results showed that having phonologically similar addends increased calculation errors and lengthened response time. On the other hand, having visually similar addends did not affect calculation errors or response time. Supporting the notion that phonological loop is important not only for language/reading, but also important for mathematics, Gathercole, Pickering, Knight, and Stegmann (2004) found that in both English and mathematics achievement, those who have high achievement scored higher on measures of phonological loop, compared to those who have low or average achievement.

Central Executive. The central executive coordinates activities between the two subsystems (i.e., visual-spatial sketchpad and phonological loop), and increases the amount of information that can be stored in the two subsystems. It is the most complex component of working memory because it is responsible for more than the other components (Baddeley, 2012; Gathercole, 1998), and is thought of as a "virtual homunculus, a little man in the head, capable of doing all the clever things that were outside the competence of the two subsystems" (Baddeley, 2012, p. 14). The central executive is responsible for focusing attention (and blocking irrelevant information), dividing attention between multiple tasks, switching between tasks, and interfacing with long-term memory to retrieve facts accumulated in the past. Given all of its functions, it is no surprise that the central executive is a good predictor of word problem solving. The central executive accounted for 5-22% of the variance in various curriculum-based mathematics skills, including number, algebra, shape, space, measurement, and mental arithmetic (Holmes & Adams, 2006). Other studies have also found the central executive is an important predictor of calculation and word problem solving, even in the presence of other cognitive factors and reading and calculation skills (Ashcraft, 1992; Hecht, 2002; Logie, Gilhooly, & Wynn, 1994; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; Swanson, 2004, 2006a, 2006b, 2011; Swanson et al., 2008; Swanson & Beebe-Frankenberger, 2004).

In a later revision of the three-component working memory model, an episodic buffer was added to this model (Baddeley, 2000). However, the three-component model is still the most widely used framework. Support for the three-component model having been found across various age groups of children starting at age six and upward (Gathercole, Pickering, Ambridge, & Wearing, 2004; Swanson et al., 2008).

Differential Contribution of Working Memory

Since the three components of working memory support different functions, it is no surprise that each component has different strength of influence on word problem solving. Meyer et al. (2010) found that for 2nd graders, the phonological loop and central executive predicted counting, identifying geometric shapes, and word problem solving. For 3rd graders, the VSSP predicted identifying and writing numbers, counting, number production, and calculation. Swanson et al. (2008)'s study of a similar age group (1^{st} to 3^{rd} graders) found that the central executive is the most important when it comes to word problem solving (β = .37), followed by the VSSP (β = .26), and phonological loop (β = .20). Similar results were found by Zheng et al. (2011) in a sample of 2^{nd} to 4^{th} graders; central executive having the most influence (β = .64), followed by the VSSP (β = .39), and phonological loop (β = .26). However, when reading, calculation, and word problem components were entered into the model as mediators, the direct paths from the working memory components and word problem solving became nonsignificant, and instead was significantly mediated by reading and calculation. Similarly, Swanson (2006a) found that in the presence of other predictors (e.g., calculation, reading, word problem components, rapid automatized naming), the central executive and phonological loop were no longer significant predictors of word problem solving. These studies suggest that calculation and reading, and other predictors (e.g., rapid automatized naming and knowledge of word problem components) may mediate the relationship between working memory components and word problem solving.

Rapid Automatized Naming Speed

While basic reading and calculation skills and working memory are important in predicting word problem solving, other predictors have come to the attention of researchers, particularly in areas related to rapid automatized naming (also referred to as naming speed). Rapid automatized naming is the ability to name as quickly and accurately as possible a series of objects, colors, or numbers (Georgiou & Parrila, 2013; Norton & Wolf, 2012). Variations of the task include more or less number of stimuli, mixed stimuli (e.g., mixture of numbers and letters in the same matrix), and naming the stimuli vertically or horizontally. The outcome is measured in total time needed to name all items.

It has been suggested that the developmental increases in working memory is not in the increase of memory *capacity*, but in the increase of *efficiency* (i.e., speed and accuracy), which frees up the limited working memory capacity quicker for processing more information (Case et al., 1982; Roodenrys & Hulme, 1993; Gathercole, 1998). In a series of four experiments involving samples of 3 to 6 year-old children and adults, Case, Harris, and Graham (1992) demonstrated a linear relationship between increases in memory span and increases in articulation rate. In the first experiment, children were asked to complete two tasks. The speed task asked children to listen to a recording of single words and repeat them as quickly as possible. The memory span task asked children to listen to the entire list of words and then repeat them. The results indicated a linear relationship between memory span and naming speed. However, the first experiment did not demonstrate a causal relationship. It could be that naming speed causes the increase in memory span, increase in memory span causes increase in naming speed, or children's more effective use of strategies causing increase in both memory span and naming speed. The second experiment of the Case et al. study addresses this question by controlling for naming speed in children and adults. The results indicated that there were no significant differences in memory span of the younger and older children and adults when naming speed was controlled. This result suggests that the increase in memory span is indeed caused by naming speed. The third and fourth experiments replicated the results of the first two experiments, but with a number counting task (counting number of dots on 8x11.5 inch cards) rather than the word naming task. Roodenrys and Hulme (1993) had similar results with samples of 5 to 6 years-old and 9 to 11 year-old children. In their experiment, children were asked to name as quickly as possible or recall one, two, or three syllable words and nonwords. The results replicated the Case et al. (1982), showing a linear relationship between naming speed and memory span, and that the difference in memory span between the younger and older children are the result of increased naming speed. Furthermore, Roodenrys and Hulme found that naming speed accounted for 62% to 80% of the variance in memory span.

Naming Speed Deficits and Basic Reading/Calculation Skills

Deficit in naming speed has been linked to difficulties and disability in reading and calculation, the basic skills needed for word problem solving. In the area of reading disability research, Wolf and Bowers (1999) have proposed that while phonological deficit is linked to reading difficulties, it does not completely explain it (Norton & Wolf, 2012; Wolf, Bowers, & Biddle, 2000). Wolf and colleagues proposed that naming speed is a second core deficit, separate from phonological processing, in reading difficulties and disability. Furthermore, Wolf and Bowers argue that because naming speed integrates lower level visual processing and higher level cognitive processing, it should be examined as a separate process. Specifically, naming speed requires a) paying attention to the letter or number stimuli, b) visual processes that are responsible for detecting features of letters/numbers, visual discrimination, and identifying patterns; c) integrating visual information with orthographic representations stored in memory; d) integration of visual information with phonological representations stored in memory; e) retrieving phonological labels; f) activating semantic information; and g) activating the motor pathways required for speech. In addition, reviews of several lines of research suggest that naming speed is a separate component from phonological processing (Norton & Wolf, 2012). First, there is only moderate correlation between naming speed and phonological processing, with correlation between .38 and .46. Second, regression and path analyses show that naming speed and phonological processing account for unique variance in reading skills. Third, more recent findings from the fields of genetics neuroimaging indicated different biological bases for naming speed and phonological processing. In summary, naming speed is more than just phonological processing, and Wolf and colleagues argue that it is an essential part of reading. Children with reading difficulties have a double-deficit of phonological processing and naming speed.

Numerous studies have linked naming speed and phonological processing to reading. Cutting, Carlissle, and Denckla (1998; as cited in Wolf & Bowers, 1999), conducted a path analysis that included orthographic processing, phonological processing, memory span, and articulation in predicting reading. They found that naming speed had direct link to reading, whereas phonological processing or articulation did not. Similar results were found by longitudinal studies in the reading literature. A longitudinal study of kindergarten to 2nd grade children, de Jong and van der Leij (1999) found that naming speed and phonological awareness had independent influences on word decoding skills. Lepola et al. (2005) conducted a 2-year longitudinal study of preschool, kindergarten, and 1st grade children, and found in a path analysis that naming speed was a stronger predictor (β = -.44) of word recognition, compared phonological awareness (β = .37). No other predictors of word recognition were in the path model. Furthermore, naming speed contributed unique variance to word recognition at every time point. Compton (2003) examined naming speed and growth in decoding skills (word- and non-word reading) in a sample of 1st graders (measured seven times throughout 1st grade), and found that naming speed of numbers but not colors predicted both word- and non-word decoding, beyond that of phonological awareness. Schatschneider et al., (2004) conducted a study with a sample of two cohorts of 1st to 2nd graders to determine whether four predictors (naming speed, phonological awareness, knowledge of letter names, and letter sound) dominate each other in predicting reading fluency and comprehension. They found that all four were significant predictors of reading fluency and comprehension, but naming speed of letters was more predictive of reading fluency. Similar results were found in an 8-year longitudinal study of German children; the strongest predictor of reading fluency was naming speed (other predictors include phonological awareness, letter knowledge, and IQ; Landerl & Wimmer, 2008).

In summary, the above studies showed the importance of naming speed in reading, and supports Wolf and colleagues' (Norton & Wolf, 2012; Wolf & Bowers, 1999; Wolf et al., 2000) argument that naming speed and phonological processing are separate components in predicting reading skills. Numerous other studies with elementary school children with a wide range of reading abilities in the past few decades have also reached a similar conclusion (e.g., Blachman, 1984; Kibby, Lee, & Dyer, 2014;

Torgesen, Wagner, & Rashotte, 1994; Torgesen, Wagner, Rashotte, Burgess, & Hecht, 1997), and in different language systems in different countries including Germany (e.g., Naslund & Schneider, 1991), Finish (e.g., Korhonen, 1995), Dutch (e.g., Van den Bos, 1998), Chinese (e.g., Pan et al., 2011), English speaking Canadian, and Greek-speaking Cypriot children (e.g., Georgiou et al., 2014).

Since naming speed is more closely related to reading skills, it is no surprise that there is more research related to reading than mathematics. Naming speed has also been found to be associated with mathematics skills throughout elementary school years, but the research in this area is limited. Cirino (2011) examined several mathematics precursors in kindergarteners using path analysis. Quantitative precursors include symbolic and non-symbolic comparisons (which set of dots or numbers are larger), symbolic labeling (identifying and writing missing numbers), rote counting (count starting from 1), and counting knowledge (count objects and knowing the total number of objects). Spatial precursor was visual-spatial working memory. Linguistic precursors were phonological awareness and naming speed (letters, numbers, and objects). Children were also assessed on single-digit addition problems. First, the confirmatory factor analysis showed that phonological awareness and naming speed were moderately correlated ($r = .58$) but separate components. This lends further evidence to the double-deficit hypothesis discussed above (e.g., Norton & Wolf, 2012; Wolf & Bowers, 1999; Wolf et al., 2000), and that naming speed deficit also affects calculation. Second, it was found that naming speed predicted single-digit addition. Its influence on addition ($\beta = .27$) was stronger than that of visual-spatial working memory ($\beta = .20$), and similar to that of phonological awareness (β = .29). Note that all three predictors were present in the model, and all remained significant predictors of addition. Hecht et al.'s (2001) longitudinal study of 2nd to 5th graders found similar results. They assessed the relationships between the phonological loop component of working memory and various calculation skills (multi-digit division and multiplication, fractions, and algebra). The results showed that phonological loop and naming speed had unique contributions to growth in calculation skills.

Since developmental increases in working memory is due to the increase in efficiency (i.e., speed and accuracy; Case et al., 1982; Roodenrys & Hulme, 1993; Gathercole, 1998), and working memory has been linked to mathematics (e.g., Meyer et al., 2010; Simmons, Willis, & Adams, 2012; Zheng et al., 2011), recent research have focused on the effects of both naming speed and working memory as predictors of mathematics. Support for the importance of both naming speed and working memory was found in a 5-year longitudinal study of 1st to 5th graders (Geary, 2011). The measures include IQ, three components of working memory (central executive, phonological loop, and visual-spatial sketchpad), naming speed, and mathematics achievement (number, counting, and calculation skills, including multi-digit addition/subtraction, multiplication, and division). The results showed that naming speed, the central executive component of working memory, and IQ predicted mathematics achievement and growth in achievement. More importantly, the effect of naming speed on mathematics and reading achievement was above and beyond that of IQ and working memory. It is important to note that naming speed and working memory are not just proxies of each other. Both contributed unique variance to mathematics skills. Swanson and Kim (2007) examined the relationship between two components of working memory (central executive and phonological loop) and mathematics skills in sample of elementary school children. There were two important findings. First, the path analysis showed that both components of working memory had similar effects on mathematics skills (central executive β = .50, phonological loop β = .50). Second, and more importantly, when naming speed (number and letter naming) was included into the model, the effects of central executive and phonological loop decreased, and the effect of naming speed (β = -.46) was larger than that of both central executive (β = .38) and phonological loop (β = .23). Similarly, Berg (2008) found that the inclusion of naming speed (number naming) in regression analyses did not eliminate the effects of phonological loop and visual-spatial sketchpad on mathematics skills, and each has its unique contribution.

In summary, researchers have suggested that developmental increase in working memory is due to increase in efficiency (naming speed and accuracy; e.g., Case, 1982). Studies have linked deficits in naming speed to reading and calculation difficulties, and naming speed has been suggested as a second core deficit (e.g., Wolf & Bowers, 1999). The research in the reading difficulties/disability literature supports these hypotheses, that naming speed separately and uniquely linked to reading (e.g., Compton, 2003; Lepola et al., 2005; Schatschneider et al., 2004). Furthermore, since developmental increases in working memory is due to efficiency, and working memory is linked to mathematics, studies of these constructs found that both naming speed and working memory uniquely influence mathematics skills (e.g., Berg, 2008; Geary, 2011; Swanson & Kim, 2007). While studies have examined the direct effects of naming speed and working memory on calculation and word problem solving, there is limited research on how naming speed mediates the relationship between the three working memory components and reading and calculation, or between working memory and word problem solving.

Hypotheses

First set of hypotheses is related to reading and calculation skills. The second set is related to working memory. Finally, a third set of hypotheses is related to naming speed.

Reading and Calculation

A1. Based on research that found reading and calculation are associated with word problem solving (e.g., Kyttälä, & Björn, 2014; Vilenius-Tuohimaa et al., 2008; Swanson et al., 2008; Zheng et al., 2011), it is hypothesized that reading and calculation skills have a direct relationship with word problem solving.

A2. Based on the works of Swanson (2006a, 2011), Swanson & Beebe-Frankenberger (2004), and Zheng et al (2011), who found that reading and mathematics skills were significant predictors of word problem solving, even in the presence of cognitive factors, it is hypothesized that reading and calculation will mediate the relationship between working memory and word problem solving.

Working Memory

B1. Based on studies that found differential contributions of working memory components (i.e., central executive, phonological loop, visual-spatial sketch pad) to word to problem solving (e.g., Meyer et al, 2010; Swanson, 2008; Zheng et al., 2011), it is hypothesized that the central executive and the phonological loop will have the strongest direct link to word problem solving.

B2. Based on studies that found some components of working memory have less influence on word problem solving when mediators were included in the model (e.g., Swanson, 2006; Zheng et al., 2011), it is hypothesized that reading, calculation, naming speed, and word problem components will mediate the relationship between the three working memory components and word problem solving.

Rapid Automatized Naming

C1. Based on studies that found links between naming speed and reading (e.g., Landerl & Wimmer, 2008; Lepola et al., 2005; Schatschneider et al., 2004; Wolf & Bowers, 1999) and calculation (e.g., Cirino, 2011; Geary, 2011; Swanson & Kim, 2007), it is hypothesized that naming speed will have a direct relationship to reading and calculation. Furthermore, because reading and calculation are basic skills need for word problem solving, it is hypothesized that naming speed will also have a direct relationship with word problem solving.

C2. Because researchers have suggested that developmental increases in working memory is due to efficiency (i.e., speed and accuracy; Case et al., 1982; Roodenrys & Hulme, 1993; Gathercole, 1998), it is hypothesized that naming speed will mediate the relationship between working memory and word problem solving.

CHAPTER 3

METHOD

Description of Dataset

The dataset to be used in this proposed study was funded from 2009 to 2012 by the Institute of Education Sciences (R324A090002). The principal investigator was H. L. Swanson, Ph.D. The co-principal investigators were Michael Gerber, Ph.D. and Michael J. Orosco, Ph.D. The project director was Cathy Lussier, Ph.D. The purpose of the project was to develop and test a series of interventions for children with mathematics disabilities, particularly mathematic word problems. The interventions were designed to improve working memory in order to improve word problem solving performance. Specifically, the project assessed whether cognitive strategies and direct instruction on the components of word problems (i.e., understanding the underlying concepts and distinguish between different types of word problems) will increase word problem solving performance.

Study 1 of the project assessed whether instruction of word problem solving components (e.g., identifying the numbers in the word problems, and planning the algorithm to use) increased word problem solving performance. An intervention was developed in which the presentation of the components was manipulated (e.g., initially focus on identifying numbers). Study 2 assessed whether two strategies (summarizing word problems and writing out the components of word problems) increased word problem solving performance. Children were taught how to rewrite word problems in order to identify the key components. Study 3 assessed the effects of cognitive strategy training. Children with mathematics disability were trained to use a rehearsal strategy in order to improve word problem solving. Study 4 assessed the effects of the combination of word problem components interventions from Study 1 and 2 with the working memory intervention from Study 3. Children in each study were randomly assigned to treatment or control group. The control group for each study was "business as usual" (e.g., typical mathematics instruction).

This proposed study will only use the first year pretest data, because the effectiveness of the interventions is not the purpose of this study.

Participants

The sample consisted of 413 children (213 females and 200 males) age 6-10, from 35 third to fourth grade classrooms in six Southern California public schools participated in the first two years of the study (the third year was dedicated to data analysis). The sample consisted of 207 were Caucasians, 128 Hispanics, 23 African Americans, 22 Asians, and 32 who identified as Other (e.g., Native American, Vietnamese, Pacific Islander, etc.). The mean socioeconomic status (SES) of the sample was primarily low to middle SES based on free lunch participation, parent education, or parent occupation.

Measures

The following description of the measures is only those that will be used in this proposed study (the original project contains many other measures).

Criterion Measure.

Word problem solving. Three measures were used to assess word problem solving performance. The story problem subtest of the Test of Math Ability (TOMA-2;

Brown, Cronin, & McEntire, 1994), the Story Problem Solving subtest from the Comprehensive Mathematical Abilities Test (CMAT; Hresko, Schlieve, Herron, Swain, & Sherbenou, 2003), and the KeyMath Revised Diagnostic Assessment (KeyMath; Connolly, 1998). In the TOMA, children were asked to silently reading a short story problem that ended with a computational question about the story (i.e. Reading about Jack and his dogs and then ending with, "How many pets does Jack have?") and then working out the answer in the space provided on their own. Reliability coefficient for the subtest is above .80.

The CMAT included word problems that increased in difficulty. The tester read each of the problems to the children, asking them to read along on their own paper. They were then asked to solve the word problem by writing out the answer. Two forms of the measure were created that varied only in names and numbers. The two forms were counterbalanced across presentation order. The manual for the CMAT subtest reported adequate reliabilities (> .86) and moderate correlations (> .50) when compared with other math standardized tests (e.g., the Stanford Diagnostic Math Test).

The KeyMath word problem solving subtest involved the tester reading a series of word problems to the children while showing a picture illustrating the problem and then asking them to verbalize the answer to problem. Both equivalent forms of the KeyMath were used (Form A and Form B). The two forms were counterbalanced across presentation order. The KeyMath manual reported reliability at .90 with split-half reliability in the high .90s. Cross-validation with the Iowa Test of Basic Skills yielded an overall correlation of .76. The KeyMath problem solving subtest involved the tester reading a series of story problems to the student while showing a picture illustrating the problem and then asking the child to verbalize the answer.

Predictor Measures.

Three working memory measures from a normative measure (S-CPT; Swanson, 1995) were administered. Three measures captured executive processing, two that captured visual-spatial sketchpad, and three measures that captured the phonological loop.

Central executive. This component of working memory was measured using three tasks. The Listening Sentence Span task assessed children's ability to remember information embedded in a short sentence (Daneman & Carpenter, 1980; Swanson, 1992). Testers read a series of sentences to each child and then asked a question about a topic in one of the sentences, and then children were asked to remember and repeat the last word of each sentence in order. For example, a set with two sentences: (Listen) "Many animals live on the farm. People have used masks since early times." (Question) "What have been used since early times?"

The Conceptual Span task assessed children's ability to organize sequences of words into abstract categories (Swanson, 1992). Children were presented with a set of words (e.g., "shirt, saw, pants, hammer, shoes, nails") and asked which of the words "go together."

The Auditory Digit Sequence task assessed children's ability to remember numerical information embedded in a short sentence (Swanson, 1992). Children were presented numbers in a sentence context (e.g., "Now suppose somebody wanted to have you take them to the supermarket at 8 6 5 1 Elm Street?") and asked to recall the numbers in the sentence.

Visual-spatial sketchpad. This component of working memory was measured using two tasks. The Mapping and Directions Span task assessed whether the children could recall a visual-spatial sequence of directions on a map with no labels (Swanson, 1992, 1995). Children were presented with a map of a "city" for 10 seconds that contained lines connected to dots and square (buildings were squares, dots were stoplights, lines and arrows were directions to travel). After the removal of the map, children were asked to draw the lines and dots on a blank map. The difficulty ranged from a map with two arrows and two stoplights to a map with two arrows and twelve stoplights. The dependent measure was created by determining the number of correctly answered process questions, recalled dots, recalled lines between the dots, number of correct arrows (to receive credit the arrows had to be both in the correct spot and pointing in the correct direction), and numbers of insertions were also noted (extra dots, lines, and arrows; i.e., errors).

The Visual Matrix task assessed children's ability to remember visual sequences within a matrix (Swanson, 1992, 1995). Children were presented a series of dots in a matrix and were allowed 5 seconds to study the pattern. After removal of the matrix, children were asked to draw the dots they remembered seeing in the corresponding boxes of a blank matrix. The difficulty ranged from a matrix of four squares with two dots to a matrix of 45 squares with 12 dots.

Phonological loop. This component of working memory was measured using three tasks. The Forward Digit Span subtest of the Wechsler Intelligence Scale for Children-Third Edition (WISC-III; Wechsler, 1991) assessed short term memory since it was assumed that forward digit spans presumably involved a subsidiary memory system (the phonological loop). The task involves a series of orally presented numbers which children repeats back verbatim. There are eight number sets with two trials per set, with the number increasing, starting at two digits and going up to nine digits. The WISC-III manual reported a test-retest reliability of .91. Cronbach's alpha was reported as .84.

The Word Span task was previously used by Swanson, Ashbaker, and Lee (1996), and assessed children's ability to recall increasingly large word lists (minimum of two words to maximum of eight words). Testers read to children lists of common but unrelated nouns, and were asked to recall the words. Word lists gradually increased in set size from a minimum of two words to a maximum of eight. Cronbach's alpha was previously reported as .62 (Swanson & Beebe-Frankenberger, 2004).

The Phonetic Memory Span task assessed children's ability to recall increasingly large lists of nonsense words (e.g., des, seeg, seg, geez, deez, dez) ranging from two to seven words per list (Swanson & Berninger, 1995). Cronbach's alpha was previously reported as .82 (Swanson & Beebe-Frankenberger, 2004).

Rapid Automatized Naming Speed. The Comprehensive Test of Phonological Processing's (CTOPP; Wagner, Torgesen, & Rashotte, 2000) Rapid Digit and Rapid Letter Naming subtests were administered to assess speed in recall of numbers and letters. Children received a page that contained four rows and nine columns of randomly arranged numbers (i.e., 4, 7, 8, 5, 2). Children were required to name the numbers as quickly as possible for each of two stimulus arrays containing 36 numbers, for a total of 72 numbers. The dependent measure was the total time to name both arrays of numbers. The Rapid Letter Naming subtest is identical in format and in scoring to the Rapid Digit Naming subtest except that it measures the speed children can name randomly arranged letters (i.e., s, t, n, a, k) rather than numbers. Coefficient alphas for the CTOPP for the Rapid Digit Naming subtest ranged from .75 to .96 with an average of .87, and for the Rapid Letter Naming subtest ranged from.70 to .92 with an average of .82. Coefficient alphas for the Rapid Naming composite score (created from the Rapid Digit and Rapid Letter Naming subtests) ranged from .87 to .96 with an average of .92 indicating a consistently high level of overall reliability. Test-retest reliability for the CTOPP Rapid Digit Naming subtest was .87 and the Rapid Letter Naming subtest was .92 with the Rapid Naming composite test-retest reliability being at an acceptable .90.

Reading. Reading comprehension was assessed by the Passage Comprehension subtest from the Test of Reading Comprehension-Third Edition (TORC; Brown, Hammill, & Weiderholt, 1995). This measure assessed children's text comprehension of a topic's or subject's meaning during reading activities. For each item children were instructed to read silently first the preparatory list of five questions, then read the short story that was presented in a brief paragraph, and finally answer the five comprehension questions (each with four possible multiple choice answers) about the story's content. Coefficient alphas calculated across ages are reported at .90 or above. Test-retest reliability ranged from .79-.88. Inter-rater reliabilities ranged from .87 to .98.

Calculation. The arithmetic computation subtest for the Wide Range Achievement Test-Third Edition (WRAT; Wilkinson, 1993) and the numerical operations subtest for the Wechsler Individual Achievement Test (WIAT; Psychological Corporation, 1992) were administered to measure calculation ability. Both subtests required children to perform written computation on number problems that increased in difficulty, beginning with single digit calculations and continued on up to algebra. The WRAT coefficient alphas were reported as .81 to .92. The WIAT reported similar reliability coefficients .82 to .91.

A version of the Test of Computational Fluency (CBM) adapted from Fuchs, Fuchs, Eaton, Hamlett, and Karns (2000) was also administered. Children were required to write answers, within two minutes, to twenty-five basic math calculation problems that were matched to grade level. The dependent measure was the number of problems solved correctly. Cronbach's alpha has been previously reported as adequate .85 (Swanson & Beebe-Frankenberger, 2004).

Fluid Intelligence. The Colored Progressive Matrices (Raven, 1976) was administered to assess fluid intelligence or IQ. Children were given a booklet with patterns displayed on each page and with each pattern revealing a missing piece. Six possible replacement pattern pieces were presented, and children were required to circle the replacement piece that best completed the pattern. The patterns progressively increased in difficulty. Cronbach's coefficient alpha was adequate .88.

Word Problem Solving Components. This an experimental instrument used to assess the ability to identify components of word problems (Swanson & Beebe-

Frankenberger, 2004; Swanson & Sachse-Lee, 2001). Each booklet contained three problems that included assessing the recall of text from the word problems. To control for reading problems, the examiner orally read (a) each problem and (b) all multiple-choice response options as the students followed along.

After the problem was read, children were asked to turn to the next page on which they see following statement: “Without looking back at the problem, circle the question the story problem was asking on the last page.” The multiple-choice questions for the problem above were: (a) How many pinecones did Darren have in all? (b) How many pinecones did Darren start with? (c) How many pinecones did Darren keep? and (d) How many pinecones did Darren throw back? This page assessed the ability to correctly identify the *question* proposition of each story problem.

On the next page for each problem, instructions were: “Without looking back at the problem, try to identify the numbers in the problem.” The multiple-choice questions for the sample problem above were: (a) 15 and 5, (b) 5 and 10, (c) 15 and 20, and (d) 5 and 20. This page assessed the ability to correctly identify the *numbers* in the two assignment propositions of each story problem.

Instructions on the next page were: “Without looking back at the problem, identify what the question wants you to find.” The multiple choice questions were: (a) The total number of pine cones Darren found all together, (b) What Darren plans to do with the pine cones, (c) The total number of pine cones Darren had thrown away, and (d) The difference between the pine cones Darren kept and the ones he threw back. This page assessed the ability to correctly identify the *goals* in the two assignment propositions of each word problem.

Instructions for the final page were: “Without looking back at the problem, identify whether addition, subtraction, or multiplication was needed to solve the problem.” Children were directed to choose one of the two or three operations: (a) addition, (b) subtraction, and (c) multiplication. After choosing one of the two or three operations, children were then asked to identify the number sentence they would use to solve the problem: (a) 15 x 5 =, (b) 15 + 10 =, (c) 15 – 5 =, or (d) 15 + 5 =. This page of the booklet assessed ability to correctly identify the *operation* and *algorithm*, respectively.

At the end of each booklet, children were read a series of true/false statements. All statements were related to the extraneous propositions for each story problem within the booklet. For example, the statement "Darren used pine cones to make ornaments" would be true, whereas the statement "Darren used pine cones to draw pictures" would be false.

For the purposes of the proposed study, based on Swanson (2004), the word problem solving components were divided into two constructs. The problem representation construct consisted of the question, number assignment, and goals. The solution planning construct consisted of the operations and algorithms components.

Analysis

Data import and cleanup was conducted using SAS 9.3 (SAS Institute, 2012) software on Windows 7 (64-bit) desktop PC. The first step of the cleanup process was the removal of 22 outliers (3.5 standard deviations above or below the mean for each measure), resulting in the final sample size of 413. The path analyses (discussed below) were conducted with and without the outliers (Hendra & Staum, 2010). The inclusion of outliers resulted in higher skewness and kurtosis for several measures, and poorer model fit. In addition, outliers can make models superficially worse (Lee & Xia, 2006; Yuan & Zhong, 2013). Thus, the final analyses were conducted without outliers.

The second step in the cleanup process involved examination of the descriptive statistics to ensure the assumption of normality is met (e.g., skewness below 3 and kurtosis below 4, standard deviations are not larger than the means). The PROC Univariate procedure in SAS was used to examine the distribution of each measure.

Following data cleanup, the dataset was exported as a tab delimited file and used in Mplus 6.11 (Muthén & Muthén, 1998-2011) software for path analysis. Because some measures had missing data, maximum likelihood estimation was used in Mplus. The first two models assessed the influence of working memory components on word problem solving, without mediators. Model 1 was a measurement model of working memory components, fluid intelligence, and word problem solving. Model 2 was a path model with working memory components and fluid intelligence predicting word problem solving, without any mediators. The three working memory components and fluid intelligence latent variables were set to correlate with each other. There are several reasons for these correlations. First, working memory is traditionally conceptualized as one system with multiple components (central executive, phonological loop, visual-spatial sketchpad), with the central executive coordinating activities between the two subsystems (i.e., visual-spatial sketchpad and phonological loop), and also increasing the amount of information that can be stored in the two subsystems (Baddeley, 1986, 2006, 2007, 2012; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Gathercole, 1998). Second, while fluid intelligence is a distinct concept from working memory, studies have shown they have moderate correlations with each other (e.g., Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). Third, setting the correlations between the three working memory components, and between working memory and fluid intelligence, resulted in better model fit.

Models 3 and 4 assessed the direct and mediated effects of naming speed, knowledge of word problem components (representation and planning), reading, and calculation. Model 3 was a measurement model of all latent variables: three working memory components, fluid intelligence, naming speed, representation, planning, reading, calculation, and word problem solving. Finally, Model 4 was a path model testing the direct and mediated effects of naming speed, representation, planning, reading, and calculation on word problem solving.

Various model fit indexes were used to assess the goodness-of-fit for the various models, including chi-square, Bentler Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), Root Mean Square Error of Approximation (RMSEA; along with confidence intervals), and Standardized Root Mean Square Residual (SRMR). These indices were selected because of their widespread use and relative ease of interpretation. For a model to have excellent fit, the following is required: a non-significant χ^2 value; a CFI > .95; a TLI > .90; an RMSEA below .05 with the left endpoint of its 90% confidence interval smaller than .05 (Raykov & Marcoulides, 2008). Because the chi-square test is sensitive to sample size and has a tendency to reject models that are only marginally inconsistent with the data examined, more emphasis is placed on the other reported fit criteria (Raykov & Marcoulides, 2008).

CHAPTER 4

RESULTS

Descriptive Statistics

The means, standard deviations, skewness, and kurtosis are shown on Table 1. The sample size varied from 405 to 413, excluding the 22 outliers. The skewness and kurtosis of all measures were below 3, indicating the data were normally distributed. The estimated correlations are shown on Appendixes A and B.

Structural Equation Modeling

Working Memory Components. Model 1 was a measurement model of working memory components, fluid intelligence, and word problem solving. This model was the baseline model. The central executive component was measured by Conceptual Span, Auditory Digit Sequence, and Listening Sentence Span. The phonological loop was measured by Forward Digit Span, Word Span, and Phonetic Memory Span. The visual-spatial sketchpad was measured by Visual Matrix and Mapping and Directions Span. Fluid intelligence was measured by three subtests of the Colored Progressive Matrices. Word problem solving was measured by WISC, Keymath, TOMA, and CMAT. All factor loadings are above .40, except Phonetic Memory Span (β = .33). The fit indexes indicated excellent model fit: $\chi^2(80) = 98.300$, $p = .08$; *CFI* = .986; *TLI* = .982; *RMSEA* = .024 (.000, .038); Standardized Root Mean Square Residual *(SRMR)* = .034.

Model 2 (Figure 1) was a path model that tested how working memory components and fluid intelligence predict word problem solving. Fluid intelligence was entered into this model because of its close association with working memory (e.g., Conway, Cowan, Bunting, Therriault, & Minkoff, 2002; Engle, Tuholski, Laughlin, & Conway, 1999). Some authors have assumed that the executive component of working memory and fluid intelligence measure the same construct (e.g., Kyllonen & Christal, 1990), while others assume they are distinct concepts (e.g., Alloway, 2009; Alloway & Alloway, 2010; Alloway & Copello, 2013). Thus, fluid intelligence was included in the model to assess whether it has unique variance when in the presence of the three working memory components. The three working memory components and fluid intelligence latent variables were set to correlate with each other. No mediators were present in this model. The fit indexes indicated excellent model fit: $\chi^2(80) = 98.300$, $p = .08$; $CFI = .986$; $TLI = .982$; $RMSEA = .024$ (.000, .038); $SRMR = .034$. As expected, because no paths were removed compared to the baseline Model 1, all fit indexes for Models 1 and 2 were the same. As shown in Figure 1, the central executive, phonological loop, and fluid intelligence were significant predictors of word problem solving. The strongest predictor was phonological loop ($\beta = .373$). Fluid intelligence was also a significant predictor ($\beta = .321$), even larger than that of the central executive ($\beta = .257$). Fluid intelligence was also significantly correlated with all three working memory components. It is important to note that both central executive and fluid intelligence have unique variance, indicating that they are distinct concepts. The three working memory components and fluid intelligence accounted for 57% of the variance in word problem solving.

Since the measures for the visual-spatial sketchpad and fluid intelligence were visually based, the inclusion of fluid intelligence may have made visual-spatial sketchpad a non-significant predictor. Thus, an alternative model was fitted by removing fluid intelligence (Figure 2). The fit indexes indicated excellent model fit: $\chi^2(48) = 67.057$, $p = .04$; *CFI* = .984; *TLI* = .978; *RMSEA* = .031 (.008, .048); *SRMR* = .036. Compared to the previous model that included fluid intelligence, the CFI, TLI, RMSEA, and SRMR for this model indicated a worse fit. However, contrary to the previous model, the visual-spatial sketchpad was a significant predictor of word problem solving ($\beta = .18$), but the standardized coefficients indicated a weak relationship when compared to the effects of the central executive ($\beta = .32$) and phonological loop ($\beta = .45$). The three working memory components alone accounted for 48% of the variance in word problem solving.

Mediation Model. Next, all mediators were added to the model. Model 3 was a measurement model that was used as the baseline model. Speed was measured by Rapid Digit Naming and Rapid Letter Naming. Representation was measured by Question, Number, and Goal components. Planning was measured by Operation and Algorithm components. Reading was measured by TORC and reading subtest of the TORC. Calculation was measured by WIAT, WRAT, and CBM. The fit indexes indicated excellent model fit: $\chi^2(279) = 335.396$, $p = .01$; *CFI* = .984; *TLI* = .980; *RMSEA* = .022 (.011, .030); *SRMR* = .035.

Model 4 (Figure 3) was a path model that assessed the direct and mediating effects of speed, representation, planning, reading, and calculation. The fit indexes indicated good model fit: $\chi^2(289) = 380.964$, $p < .001$; *CFI* = .974; *TLI* = .969; *RMSEA* = .028 (.020, .035); *SRMR* = .041. All factor loadings were above .40, except Phonetic Memory Span ($\beta = .33$) (Table 2). As shown in Figure 3 and Table 3 (for better clarity), the central executive predicted speed, representation, planning, and reading. The phonological loop predicted only reading. The visual-spatial sketchpad did not predict any of the mediators. Speed predicted reading and calculation. As for word problem solving, the direct predictors were speed, representation, planning, reading, and calculation. Reading and calculation being the strongest direct predictors of word problem solving (β = .73 and β = .40, respectively).

Table 4 shows the mediated/indirect paths. Speed, representation, and reading mediated the relationship between the central executive and word problem solving. Reading also mediated the relationship between the phonological loop and word problem solving. Of all six significant mediated relationships, phonological loop and reading was the strongest indirect path predicting word problem solving (β = .23). However, the standardized path coefficients indicated these mediated relationships were weaker than the direct relationships with word problem solving. In Model 4, all of the direct and mediated relationships accounted for 87% of the variance in word problem solving, compared to 57% without the mediators.

Since there was a large change in chi-squared compared to the baseline model ($\Delta\chi 2$ = 45.57, Δdf = 10), alternative models were tested to determine which of the removed paths had the largest effect on the model, by adding one path back to the model at a time. None of the individual paths resulted in $\Delta\chi 2 < 10$. Furthermore, the various alternative model tested had only slightly better model fit according to the *CFI*, *TLI*, and *RMSEA*. Thus, Model 4 was kept as the final mediation model.

CHAPTER 5

DISCUSSION

The importance of mathematics, particularly word problem solving, should not be underestimated. It is not only important on an individual level, but also important for the development and competitiveness of the country. Numerous studies have found links between word problem solving and proximal (reading, calculation, and word problem components) and cognitive factors (working memory and naming speed). Research in this area, however, still needs to be advanced. First, there is limited research on the differential contribution of the three working memory components on word problem solving. Second, there is a need to explore how rapid automatized speed may mediate the relationship between working memory and word problem solving. Third, other mediators needed to be considered, including the knowledge of word problem components and the basic reading and calculation skills needed to solve word problems.

The purpose of this study was to test these relationships using a structural equation modeling framework. There are several major findings. First, without any mediators, the phonological loop was the best predictor of word problem solving. Second, when other variables were entered into the model, reading and calculation were the strongest direct predictors of word problem solving. Third, results of the mediation model indicated that speed mediated the relationship between the central executive component of working memory and word problem solving, while reading mediated the relationship between the central executive and phonological loop and word problem solving.

Working Memory Model

Results of the non-mediation model showed that of the three working memory components, the phonological loop was the strongest predictor of word problem solving, followed by fluid intelligence, and the central executive. These results are consistent with previous research that showed phonological loop is important when it comes to mathematics (Furst & Hitch, 2000; Noël et al., 2001). Children's calculation performance suffered when interference of the phonological loop was introduced in experiments (e.g., reciting alphabet while solving problems), which interfered with encoding and rehearsal of operands and mental calculation (Furst & Hitch, 2000). Having phonologically similar addends also increased calculation errors (Noël et al., 2001). The importance of the phonological loop is magnified when it comes to word problem solving, because solving word problems involve another aspect of the phonological loop, language/text processing (Baddeley, 2012; Baddeley et al., 1998; Gathercole, 1998), which is an important first step in understanding the word problem. This may explain why the phonological loop is a stronger predictor of word problem solving than the central executive.

Although results suggest that the phonological loop is the most important when it comes to word problem solving, the central executive is also important (accounting for as much as 22% of the variance in mathematics skills in some studies; Holmes & Adams, 2006), and one of its many responsibilities include focusing attention, dividing attention between multiple tasks, and switching between tasks. A possible reason why the central executive is not as strong a predictor as the phonological loop may be how the measures were administered. All of the measures of word problem solving (WISC, Keymath, TOMA, and CMAT) were administered in the children's classroom in a quiet environment. There was little or no need for the children to focus attention (already focused on the test), divide attention (no interference situations like that of the Furst and Hitch, 2000 or Noël et al., 2001 studies), and switch tasks (the only task were the test in front of them). However, the significant direct path between the central executive and word problem solving suggests that the central executive is still important, perhaps due to its function as an interface with long-term memory (retrieving previously learned language/reading and mathematics facts). Future studies should examine the effects of the central executive and phonological loop on word problem solving when distractions and interference are introduced into a test environment or during timed high-stakes testing.

It should be noted that some studies found that the central executive was a stronger predictor than phonological loop when there were no other variables (e.g., Meyer et al., 2010; Zheng et al., 2011). However, when other variables were present (e.g., reading and calculation), the effects of the central executive and phonological loop were lessened or became non-significant (e.g., Swanson, 2006; Zheng et al., 2011). Similarly, in this study, removing fluid intelligence from the model resulted in increase of the standardized path coefficients for all three working memory components; phonological loop increased from β = .37 to .45, central executive increased from β = .26 to .32, and visual-spatial sketchpad increased from β = .11 (non-significant) to .18 (significant). These results have several implications. First, although working memory components are important when it comes to word problem solving, fluid intelligence is also an important factor. This can be seen more easily when one compares the total variance accounted for in the models with and without fluid intelligence. Without fluid intelligence, the three working memory accounted for 48% of the variance in word problem solving. With fluid intelligence, the model accounted for 57% of the variance. The second implication of these results is that even in the presence of the three working memory components, fluid intelligence had unique variance in predicting word problem solving. These results are in line with researchers who suggested working memory and fluid intelligence are distinct concepts (Alloway, 2009; Alloway & Alloway, 2010; Alloway & Copello, 2013). Working memory (the potential to learn) is important for processing current problems, while fluid intelligence (what have already been learned) is important when one needs to retrieve mathematics facts to solve word problems.

Contrary to some studies (Gathercole & Pickering, 2000; Holmes & Adams, 2006; Rasmussen & Bisanz, 2005), this study found that the visual-spatial sketchpad was not a significant predictor of word problem solving. One would assume that visual-spatial sketchpad is important to mathematics because it is responsible for visual and spatial information, including mathematical symbols, equations, physical shapes, color, and movement (Baddeley, 1986, 2006, 2007, 2012; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Gathercole, 1998). A possible reason why the visual-spatial sketchpad was not a significant predictor in this study may be the inclusion of fluid intelligence, which was also a visually based assessment. In this study, the visual-spatial sketchpad became a significant predictor when fluid intelligence was removed from the model. Since the visual-spatial sketchpad is responsible for manipulation of visual information, and fluid intelligence is a measure of have already been learned, these results suggest that mathematics facts and symbols that were learned is more important than the ability to manipulate them in mental space. An alternative explanation is that the assessments for fluid intelligence (Colored Progressive Matrices) and visual-spatial sketchpad (Visual-matrix and Mapping & Direction) measure essentially the same things. The inclusion of fluid intelligence took away variance from the visual-spatial sketchpad.

Mediation Model

Two major results were found in the mediation model. First, the strongest direct effects on word problem solving were reading and calculation. These results are consistent with previous research that also found reading and calculation to be important factors in word problem solving (Kyttälä & Björn, 2014; Swanson, 2006a; Swanson et al., 2008; Vilenius-Tuohimaa et al., 2008). This is intuitive because reading and calculation are the basic skills necessary to successfully solve word problems. The results also indicated that reading comprehension is more important than calculation (β = .73 vs. β = .40) in predicting word problem solving, lending more evidence that understanding the story is the first step in solving word problems, followed by the actual calculations. If one does not understand the story, it is more difficult to extract the mathematical information from word problems to put into equations and solve them.

Another finding is that knowledge of word problem representation (question, number, and goal) but not planning (operation and algorithm) had a direct relationship with word problem solving. This extended the research of Zheng et al. (2011) by separating the representation and planning components. The results suggest that in addition to understanding the word problem (i.e., reading comprehension), an additional step may be required before the actual calculations. Simply understanding the story is not enough; children need to be able to identify and extract the mathematical information from the story.

Finally, another significant direct relationship worth noting is between speed and word problem solving, which is consistent with studies that found speed to be an important predictor (Berg, 2008; Geary, 2011; Swanson & Kim, 2007). Furthermore, in line with other research, speed was a better predictor of word problem solving than fluid intelligence (e.g., Geary, 2011). In addition to predicting word problem solving, and similar to previous studies (Cirino, 2011; de Jong & van der Leij, 1999; Hecht et al., 2001; Lepola et al., 2005; Schatschneider et al., 2004), this study also found that speed was a significant predictor of the basic reading and calculation that is required for solving word problems. Overall, these direct relationships (speed / reading / calculation → problem solving, and speed → reading / calculation) suggest that speed plays an important role in both basic skills (reading comprehension and calculation) as well as higher skills that integrates both.

The more important findings are the indirect/mediated relationships. First, rapid automatized naming speed mediated the relationships between the central executive and problem solving (central executive → speed → problem solving), and between central executive and reading/calculation (central executive → speed → reading/calculation → problem solving). This lends evidence to the theory that developmental increases in working memory is not in the increase of memory capacity, but in the increase of efficiency (i.e., speed and accuracy), freeing up working memory capacity quicker for processing more information (Case et al., 1982; Roodenrys & Hulme, 1993; Gathercole, 1998). However, the mediated relationship is weak (β = -.08). The weak mediation may suggest that developmental increase is not just efficiency or capacity, but both. That is, as children grow, their working memory capacity and efficiency increases. Future studies should examine developmental growth of both capacity and efficiency using a longitudinal framework. The literature on aging shows that young children and adolescents already have moderate to high processing speed, increasing from age 6 and reaching the maximum at approximately age 20 (e.g., Kail & Salthouse, 1994). It is possible that at such a young age of the sample in this study, working memory capacity matters more than speed. Studies conducted with adults suggest that speed is an important factor in terms of age differences among adults (e.g., Kail & Salthouse, 1994; Salthouse, 1992), but there is limited research on speed among younger populations. Futures studies should examine how speed mediates the relationship between working memory and problem solving among the adolescent and adult populations, and compare them to that of young children.

A second mediation result worth noting is that reading was the strongest among all mediators. Reading mediated the relationship between the phonological loop and word problem solving (β = .23), and stronger than that of central executive and speed (β = -.08). This result, in combination with the strong direct relationship between reading and word problem solving (β = .73), suggest that language processing (the responsibility of the phonological loop) and understanding the story (reading comprehension) is the most important in solving word problems. Furthermore, the results also indicated that calculation also had a direct relationship with word problem solving, but at a much weaker level (β = .40) compared to reading, and that calculation did not mediate the relationship between working memory and word problem solving. These results lend further evidence that reading comprehension is more important than calculation when it comes to solving word problems. This is intuitive because one has to first understand the problem before doing any calculations. Future studies should extend this study by examining different facets of reading, such as fluency and word identification.

Limitations

This study has several limitations. First, because the majority of the children in the sample were age 8, it was not possible to include age as a covariate in the models. It is possible that age can influence the effects of the three working memory components and speed, because developmentally, young children's working memory capacity and speed is still increasing at a rapid rate as they age. Second, the dataset does not have reading fluency and word identification. These variables may have important influence on word problem solving, and thus, future studies should compare the effects with reading comprehension. Third, although this study found that reading comprehension and phonological loop have the strongest relationship with word problem solving, it is possible these relationships will differ for older children (e.g., high school). At upper grades, the mathematics required to solve word problems is more difficult, and thus, calculation and planning (i.e., knowledge of operation and algorithm) may have a strong relationship with word problem solving. Future studies should examine this in a longitudinal framework; examining the changes in relationship between working memory, basic reading and calculation skills, and word problem solving. Fourth, this study did not separately examine children with and without learning disability/difficulty. The relationships between working memory, speed, and word problem solving may differ among average achievers and those with mathematics and/or reading disability.

Conclusions

The U.S. is behind many countries in mathematics (National Center for Education Statistics, 2011; OCED, 2012a), which in turn, puts the U.S. at a developmental and economic disadvantage (National Mathematics Advisory Panel, 2008). When one further examines the PISA assessments, we can see that U.S. children shows weakness in word problem solving. This suggest further understanding the mechanism that underlie word problem solving among young children. In the past few decades, several lines of research have begun to address these issues, including the study of working memory, speed, and basic reading and calculation skills. This study further addressed some of the issues by examining the differential effects of the three working memory components, and the mediated effects of speed, reading, calculation, and knowledge of the components of word problems.

The results suggest that language/text processing is one of the most important factors. This includes the phonological loop component of working memory and reading comprehension. These results are intuitive because one must first understand the language/text in the word problem before even begin to extract the relevant mathematical information and calculation the solution. An important implication for education is that some focus should be on basic reading skills. Teachers may spend more time on developing their students' reading skills, and interventions can focus on reading and the phonological loop component of working memory.

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Table 1

Descriptive Statistics for All Measures

Measure	N	Mean	SD	Skew	Kurt
Age	413	8.3820	0.5114	0.2613	1.3978
Central Executive					
Conceptual Span	406	10.0064	1.7310	0.4176	0.8832
Auditory Digit Seq	406	10.0122	1.8114	0.4075	0.0190
Listening Sentence	406	9.9994	1.5811	0.4988	1.0744
Phonological Loop					
Digit Forward	408	6.7549	1.6644	0.1912	-0.0349
Word Span	411	8.2263	3.3720	0.2862	-0.6533
Phonetic Memory	412	3.1359	2.1435	0.9173	1.5364
Visual-spatial					
Visual Matrix	405	10.0073	1.6080	0.4099	0.1729
Mapping/Direction	411	9.9928	1.7474	0.9514	1.5944
Reading					
TORC (raw)	408	13.9338	5.8160	0.1784	-0.2133
TORC (std)	408	9.3529	2.2427	-0.1198	0.0885
WRAT Read (raw)	410	31.1463	4.2430	0.2837	1.1132
WRAT Read (std)	410	104.3756	11.8282	0.2047	0.8637
Calculation					
WIAT (raw)	412	16.1699	3.1685	-0.0937	0.3021
WIAT (std)	412	98.9272	12.4384	0.0856	-0.2616
WRAT (raw)	412	23.9296	2.7290	0.3211	0.4158
WRAT (std)	412	99.1650	10.2072	0.0531	0.6320
CBM	407	4.7961	3.5030	0.8515	0.5575

Note. TORC = Test of Reading Comprehension, WRAT Read = Reading subtest of the Wide Range Achievement Test, WIAT = numerical operations subtest of the Wechsler Individual Achievement Test, CBM = Test of Computational Fluency.
*p < .05, **p < .01, ***p < .001.

Table 2

Standardized Factor Loadings for Mediation Model

Latent Variable	**Indicator**	**Standardized Factor Loading**
Central Executive	Conceptual Span	0.582 (0.046)***
	Auditory Digit Seq	0.653 (0.045)***
	Listening Sentence	0.641 (0.044)***
Phonological Loop	Digit Forward	0.733 (0.048)***
	Word Span	0.617 (0.047)***
	Phonetic Memory	0.331 (0.054)***
Visual-spatial	Visual Matrix	0.666 (0.119)***
	Mapping & Direction	0.646 (0.116)***
Fluid Intelligence	Raven A	0.458 (0.052)***
	Raven AB	0.564 (0.052)***
	Raven B	0.421 (0.053)***
Speed	Rapid Digit Naming	0.890 (0.031)***
	Rapid Letter Naming	0.872 (0.031)***
Representation	Question	0.622 (0.041)***
	Number	0.548 (0.044)***
	Goal	0.706 (0.037)***
Planning	Operation	0.795 (0.037)***
	Algorithm	0.752 (0.038)***
Reading	TORC	0.710 (0.031)***
	WRAT-Read	0.712 (0.031)***
Calculation	WIAT	0.864 (0.019)***
	WRAT	0.866 (0.018)***
	CBM	0.705 (0.029)***
Problem Solving	WISC	0.706 (0.028)***
	Keymath	0.786 (0.023)***
	TOMA	0.646 (0.032)***
	CMAT	0.825 (0.021)***

Note. Standard error in parentheses.
*p < .05, **p < .01, ***p < .001.

Table 3

Standardized Path Coefficients for Mediation Model (Model 4)

Variable	Regressed On	Standardized Coefficient
Central Executive	Speed	-0.274***
	Representation	0.271***
	Planning	0.288***
	Reading	0.197*
	Calculation	0.142^{+}
Phonological Loop	Speed	0.035
	Representation	0.167^{+}
	Planning	0.002
	Reading	0.316***
	Calculation	0.046
Visual-spatial	Speed	-0.115
	Representation	0.050
	Planning	0.044
	Reading	-0.044
	Calculation	0.044
Fluid Intelligence	Speed	0.156
	Representation	0.360***
	Planning	0.357***
	Reading	0.597***
	Calculation	0.503***
	Problem Solving	-0.191
Speed	Reading	-0.400***
	Calculation	-0.333***
Fluid Intelligence	Problem Solving	-0.191
Speed		0.303***
Representation		0.346***
Planning		-0.174*
Reading		0.732***
Calculation		0.403***
	Correlated With	
Phonological Loop	Central Executive	0.386***
Visual-spatial	Central Executive	0.109
	Phonological Loop	0.080
Fluid Intelligence	Central Executive	0.316***
	Phonological Loop	0.332***
	Visual-spatial	0.350***
Representation	Planning	0.526***

*p < .05, **p < .01, ***p < .001, $^{+}$p < .10.

Table 4

Standardized Mediated Path Coefficients for Mediated Model (Model 4)

Mediated Paths	**Standardized Coefficient (SE)**
Central Executive → Speed → Problem Solving	-0.083 (0.036)*
Central Executive → Speed → Reading → Problem Solving	0.080 (0.034)*
Central Executive → Speed → Calculation → Problem Solving	0.037 (0.014)**
Central Executive → Representation → Problem Solving	0.094 (0.038)*
Central Executive → Reading → Problem Solving	0.144 (0.053)**
Phonological Loop → Reading → Problem Solving	0.231 (0.065)***

*$p < .05$, **$p < .01$, ***$p < .001$, $^{+}p < .10$.

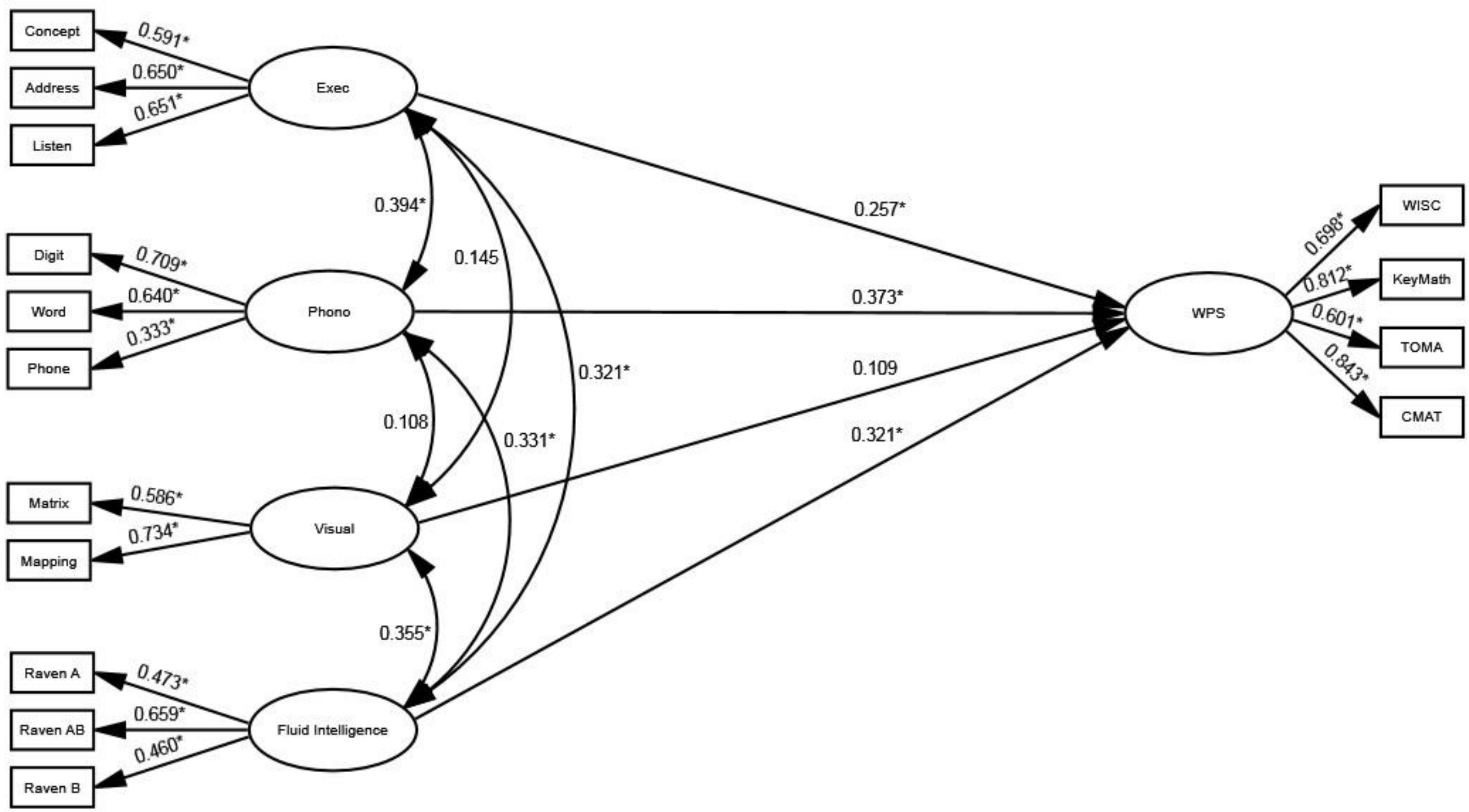

Figure 1. Working memory path model. $\chi^2(80) = 98.300$, $p = .08$; $CFI = .986$; $TLI = .982$; $RMSEA = .024$ (.000, .038); $SRMR = .034$

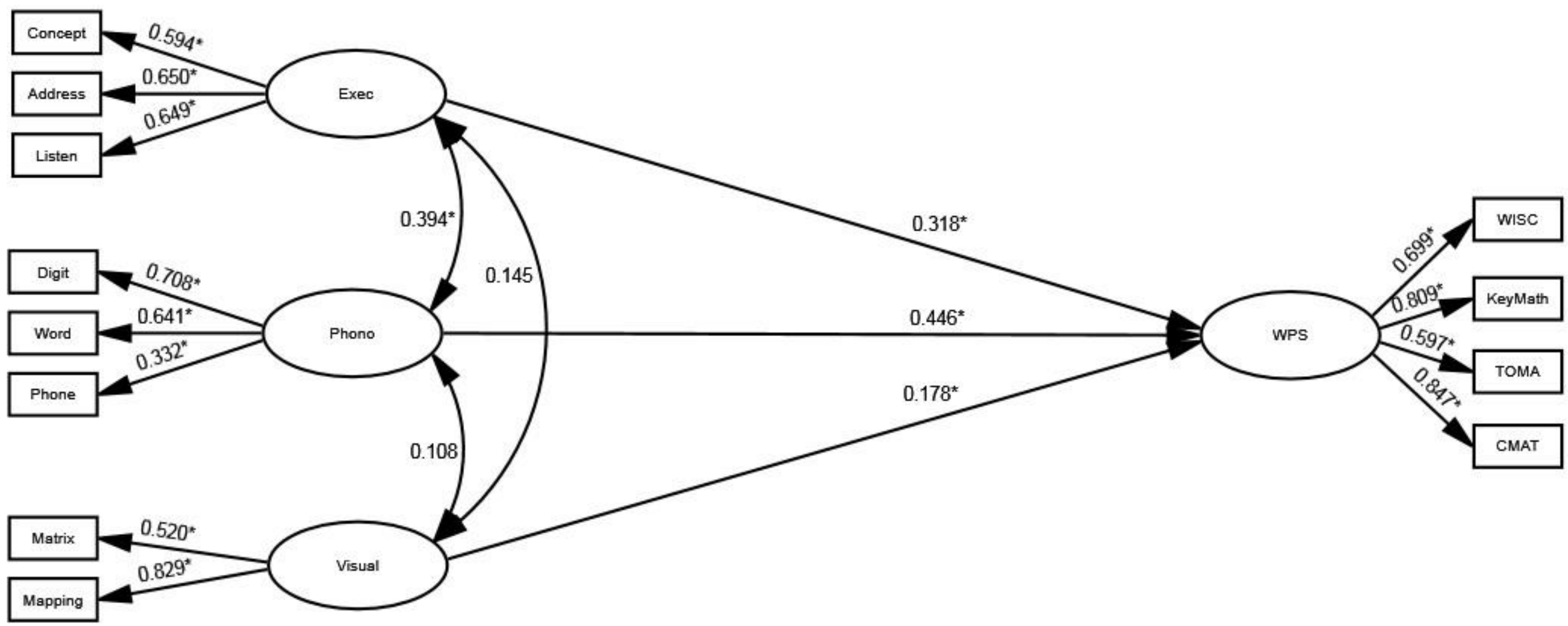

Figure 2. Alternative working memory path model. $\chi^2(48) = 67.057$, $p = .04$; $CFI = .984$; $TLI = .978$; $RMSEA = .031$ (.008, .048); $SRMR = .036$.

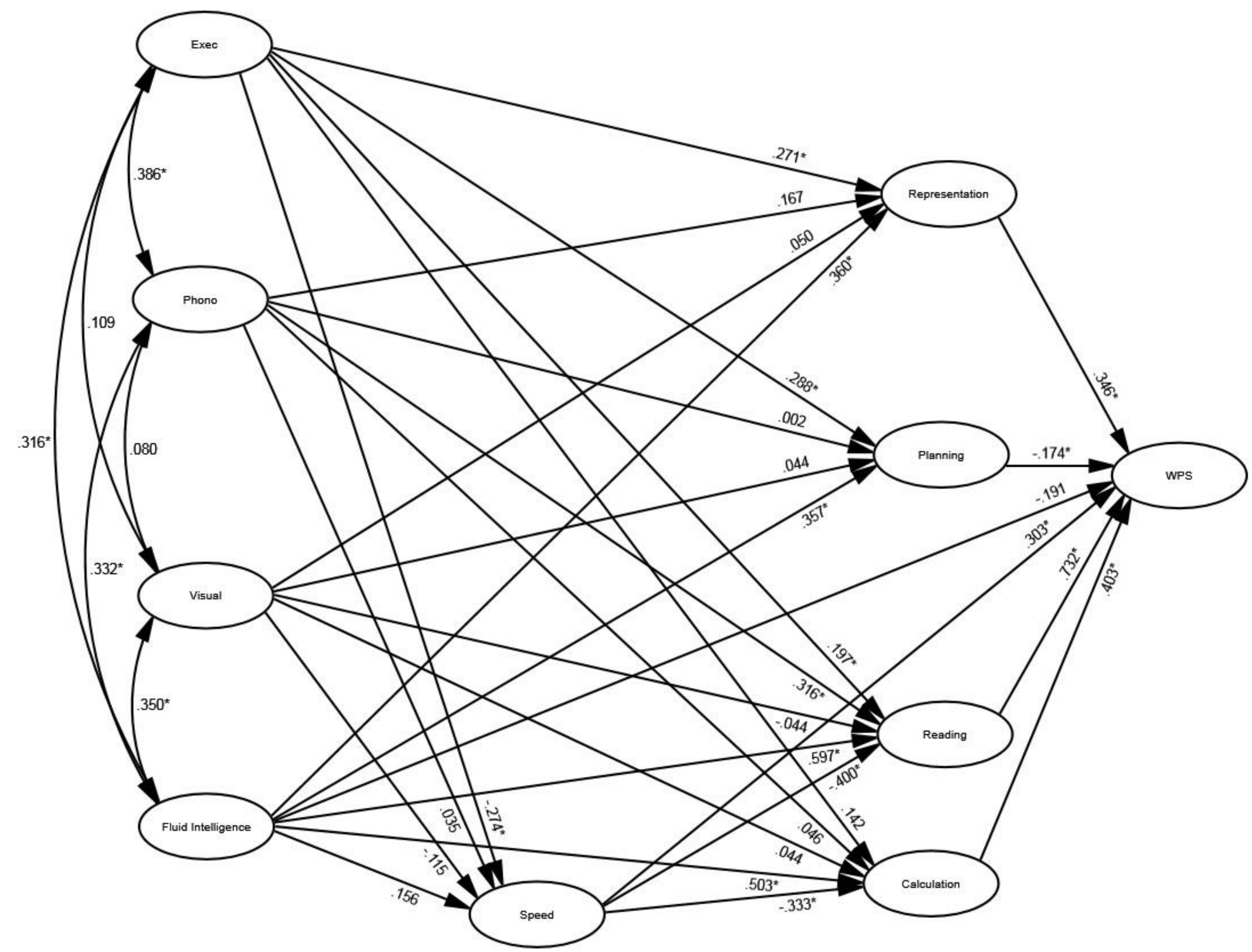

Figure 3. Mediation path model. : $\chi^2(289) = 380.964$, $p < .001$; *CFI* = .974; *TLI* = .969; *RMSEA* = .028 (.020, .035); *SRMR* = .041.

Appendix A
Estimated Correlations for Working Memory Model (Model 2)

	1	2	3	4	5
1. Concept	---				
2. Address	0.418	---			
3. Listen	0.355	0.420	---		
4. Digit	0.159	0.126	0.183	---	
5. Word	0.118	0.151	0.174	0.455	---
6. Phonetic	0.261	0.156	0.174	0.258	0.176
7. Matrix	0.009	-0.008	-0.021	-0.031	0.048
8. Mapping	0.113	0.101	0.093	0.096	0.046
9. RavenA	0.037	0.068	0.105	0.163	0.115
10. RavenAB	0.114	0.136	0.198	0.111	0.149
11. RavenB	0.043	0.110	0.090	0.135	0.054
12. WISC	0.190	0.248	0.300	0.350	0.332
13. KeyMath	0.230	0.214	0.307	0.321	0.303
14. TOMA	0.217	0.254	0.237	0.219	0.200
15. CMAT	0.262	0.225	0.322	0.337	0.342

	6	7	8	9	10
6. Phonetic	---				
7. Matrix	-0.022	---			
8. Mapping	0.063	0.430	---		
9. RavenA	0.081	0.071	0.118	---	
10. RavenAB	0.094	0.159	0.162	0.332	---
11. RavenB	0.026	0.114	0.119	0.167	0.308
12. WISC	0.152	0.109	0.139	0.169	0.247
13. KeyMath	0.117	0.160	0.220	0.269	0.280
14. TOMA	0.124	0.109	0.159	0.215	0.258
15. CMAT	0.139	0.096	0.175	0.205	0.271

	11	12	13	14	15
11. RavenB	---				
12. WISC	0.169	---			
13. KeyMath	0.251	0.558	---		
14. TOMA	0.162	0.409	0.477	---	
15. CMAT	0.273	0.584	0.694	0.512	---

Note. Concept = Conceptual Span, Address = Auditory Digit Sequence, Listen = Listening Sentence, Digit = Forward Digit Span, Word = Word Span, Phonetic = Phonetic Memory Span, Matrix = Visual Matrix, Mapping = Mapping & Directions, Raven A/AB/B = Colored Progressive Matrices Subtests A/AB/B, WISC = Wechsler Intelligence Scale for Children, TOMA = Test of Mathematical Abilities, CMAT = Comprehensive Mathematical Abilities Test.

Appendix B
Estimated Correlations for Mediation Model (Model 4)

	1	2	3	4	5
1. Concept	---				
2. Address	0.417	---			
3. Listen	0.352	0.419	---		
4. Digit	0.160	0.126	0.184	---	
5. Word	0.118	0.152	0.176	0.456	---
6. Phonetic	0.261	0.155	0.174	0.259	0.176
7. Matrix	0.007	-0.006	-0.019	-0.031	0.050
8. Mapping	0.112	0.101	0.094	0.096	0.047
9. RavenA	0.034	0.066	0.104	0.162	0.113
10. RavenAB	0.112	0.134	0.196	0.111	0.147
11. RavenB	0.042	0.107	0.089	0.133	0.054
12. RDN	-0.126	-0.147	-0.061	0.030	-0.030
13. RLN	-0.145	-0.161	-0.068	-0.021	-0.035
14. Quest	0.188	0.131	0.181	0.154	0.156
15. Numb	0.166	0.108	0.131	0.059	0.090
16. Goal	0.178	0.159	0.237	0.199	0.237
17. Oper	0.133	0.181	0.238	0.111	0.150
18. Algor	0.150	0.198	0.226	0.047	0.177
19. TORC	0.206	0.270	0.258	0.281	0.252
20. WRAT-R	0.208	0.282	0.294	0.309	0.224
21. WIAT	0.157	0.190	0.212	0.153	0.157
22. WRAT	0.220	0.228	0.225	0.158	0.160
23. CBM	0.194	0.137	0.191	0.101	0.125
24. WISC	0.190	0.248	0.301	0.351	0.332
25. Keymath	0.228	0.212	0.308	0.322	0.302
26. TOMA	0.213	0.252	0.236	0.222	0.200
27. CMAT	0.261	0.223	0.323	0.338	0.342

Note. Concept = Conceptual Span, Address = Auditory Digit Sequence, Listen = Listening Sentence, Digit = Forward Digit Span, Word = Word Span, Phonetic = Phonetic Memory Span, Matrix = Visual Matrix, Mapping = Mapping & Directions, Raven A/AB/B = Colored Progressive Matrices Subtests A/AB/B, RDN = Rapid Digit Naming, RLN = Rapid Letter Naming, Quest = Word Problem Component Question, Numb = Word Problem Component Number, Goal = Word Problem Goal, Oper = Word Problem Operation, Algor = Word Problem Algorithm, TORC = Test of Reading Comprehension, WRAT-R = Reading subtest of the Wide Range Achievement Test, WIAT = numerical operations subtest of the Wechsler Individual Achievement Test, WRAT = arithmetic computation subtest for the Wide Range Achievement Test, CBM = Test of Computational Fluency, WISC = Wechsler Intelligence Scale for Children, TOMA = Test of Mathematical Abilities, CMAT = Comprehensive Mathematical Abilities Test.

Appendix B (cont.)

	6	7	8	9	10
6. Phonetic	---				
7. Matrix	-0.021	---			
8. Mapping	0.063	0.430	---		
9. RavenA	0.080	0.069	0.117	---	
10. RavenAB	0.094	0.157	0.160	0.332	---
11. RavenB	0.024	0.115	0.120	0.167	0.308
12. RDN	-0.008	-0.080	-0.032	0.073	0.061
13. RLN	-0.021	-0.074	-0.033	0.061	0.002
14. Quest	0.067	0.022	0.135	0.106	0.188
15. Numb	0.080	0.005	0.070	0.038	0.052
16. Goal	0.131	0.083	0.094	0.196	0.177
17. Oper	0.099	0.078	0.139	0.167	0.236
18. Algor	0.098	0.115	0.103	0.143	0.156
19. TORC	0.121	0.101	0.076	0.190	0.257
20. WRAT-R	0.137	0.045	0.097	0.246	0.273
21. WIAT	0.066	0.142	0.127	0.169	0.200
22. WRAT	0.043	0.186	0.129	0.196	0.263
23. CBM	0.047	0.095	0.128	0.136	0.162
24. WISC	0.152	0.110	0.138	0.169	0.246
25. Keymath	0.117	0.160	0.220	0.268	0.280
26. TOMA	0.126	0.110	0.160	0.215	0.259
27. CMAT	0.139	0.097	0.175	0.206	0.272

Note. Phonetic = Phonetic Memory Span, Matrix = Visual Matrix, Mapping = Mapping & Directions, Raven A/AB/B = Colored Progressive Matrices Subtests A/AB/B, RDN = Rapid Digit Naming, RLN = Rapid Letter Naming, Quest = Word Problem Component Question, Numb = Word Problem Component Number, Goal = Word Problem Goal, Oper = Word Problem Operation, Algor = Word Problem Algorithm, TORC = Test of Reading Comprehension, WRAT-R = Reading subtest of the Wide Range Achievement Test, WIAT = numerical operations subtest of the Wechsler Individual Achievement Test, WRAT = arithmetic computation subtest for the Wide Range Achievement Test, CBM = Test of Computational Fluency, WISC = Wechsler Intelligence Scale for Children, TOMA = Test of Mathematical Abilities, CMAT = Comprehensive Mathematical Abilities Test.

Appendix B (cont.)

	11	12	13	14	15
11. RavenB	---				
12. RDN	0.039	---			
13. RLN	-0.021	0.777	---		
14. Quest	0.188	-0.135	-0.124	---	
15. Numb	0.134	-0.049	-0.120	0.380	---
16. Goal	0.176	-0.167	-0.166	0.438	0.368
17. Oper	0.105	-0.038	-0.041	0.323	0.353
18. Algor	0.075	-0.075	-0.068	0.290	0.336
19. TORC	0.211	-0.262	-0.284	0.311	0.212
20. WRAT-R	0.148	-0.252	-0.254	0.297	0.185
21. WIAT	0.231	-0.242	-0.277	0.240	0.152
22. WRAT	0.179	-0.249	-0.257	0.255	0.126
23. CBM	0.226	-0.234	-0.256	0.253	0.156
24. WISC	0.170	-0.169	-0.171	0.276	0.181
25. Keymath	0.251	-0.063	-0.059	0.305	0.210
26. TOMA	0.162	-0.196	-0.178	0.294	0.233
27. CMAT	0.273	-0.122	-0.141	0.359	0.283

	16	17	18	19	20
16. Goal	---				
17. Oper	0.354	---			
18. Algor	0.337	0.599	---		
19. TORC	0.381	0.344	0.271	---	
20. WRAT-R	0.319	0.244	0.249	0.534	---
21. WIAT	0.333	0.180	0.248	0.433	0.409
22. WRAT	0.315	0.188	0.291	0.392	0.460
23. CBM	0.262	0.172	0.238	0.326	0.294
24. WISC	0.362	0.211	0.248	0.455	0.464
25. Keymath	0.412	0.244	0.239	0.426	0.439
26. TOMA	0.362	0.215	0.220	0.462	0.508
27. CMAT	0.440	0.270	0.281	0.455	0.458

Note. Raven A/AB/B = Colored Progressive Matrices Subtests A/AB/B, RDN = Rapid Digit Naming, RLN = Rapid Letter Naming, Quest = Word Problem Component Question, Numb = Word Problem Component Number, Goal = Word Problem Goal, Oper = Word Problem Operation, Algor = Word Problem Algorithm, TORC = Test of Reading Comprehension, WRAT-R = Reading subtest of the Wide Range Achievement Test, WIAT = numerical operations subtest of the Wechsler Individual Achievement Test, WRAT = arithmetic computation subtest for the Wide Range Achievement Test, CBM = Test of Computational Fluency, WISC = Wechsler Intelligence Scale for Children, TOMA = Test of Mathematical Abilities, CMAT = Comprehensive Mathematical Abilities Test.

Appendix B (cont.)

	21	22	23	24	25	26	27
21. WIAT	---						
22. WRAT	0.752	---					
23. CBM	0.607	0.604	---				
24. WISC	0.481	0.447	0.357	---			
25. Keymath	0.468	0.485	0.424	0.558	---		
26. TOMA	0.466	0.487	0.354	0.409	0.477	---	
27. CMAT	0.518	0.494	0.446	0.584	0.694	0.514	---

Note. WIAT = numerical operations subtest of the Wechsler Individual Achievement Test, WRAT = arithmetic computation subtest for the Wide Range Achievement Test, CBM = Test of Computational Fluency, WISC = Wechsler Intelligence Scale for Children, TOMA = Test of Mathematical Abilities, CMAT = Comprehensive Mathematical Abilities Test.