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When Bonds Matter: Home Bias in Goods and Assets[*]

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Abstract

This paper presents a model of international portfolios with real exchange rate and non financial risks that accounts for observed levels of equity home bias. A key feature is that investors can trade domestic and foreign bonds in addition to equities. Bonds matter: in equilibrium, investors structure their bond portfolio to hedge real exchange rate risks and equity home bias arises when non-financial income risk is negatively correlated with equity returns, *after controlling for bond returns*. Our framework allows us to derive equilibrium bond and equity portfolios in terms of sufficient statistics— directly measurable hedge ratios. We estimate equity and bond portfolios implied by the model for G-7 countries an find strong empirical support for the theory. We are able to account for a significant share of the equity home bias and obtain a currency exposure of bond portfolios comparable to the data.

Keywords : International risk sharing, International portfolios, Equity home bias
JEL codes: F30, F41, G11

1   Introduction

The current international financial landscape exhibits two critical features. First, the last thirty years witnessed an unprecedented increase in cross-border financial transactions (see Lane and Milesi-Ferretti (2007)). Second, despite this massive wave of financial globalization, international portfolios remain heavily tilted toward domestic assets. This is the well-known equity home bias (See French and Poterba (1991) and Coeurdacier and Rey (2013) for a recent survey). As of 2008, the share of US stocks in US investors' equity portfolios was 77.2%, despite the fact that US equity markets account for only 32% of world market capitalization.[1] The importance of these two features has spurred renewed interest for theories of optimal international portfolio allocation.

Two important strands of literature aim to account for the observed bias. In both approaches, investors depart from the perfectly diversified portfolio of frictionless general equilibrium models à la Lucas (1982), in order to insulate their consumption stream from additional sources of risk. Differences in equilibrium portfolio holdings across countries thus reflect the equilibrium hedging properties of domestic equity returns, relative to foreign ones. Generically, consider a risk-factor X that impacts *negatively* domestic wealth relatively more than foreign wealth. In equilibrium, the difference between domestic and foreign own-equity holdings (the degree of equity home bias) will be proportional to the following hedge ratio:

$$\frac{\text{cov}(X, R)}{\text{var}(R)}, \tag{1}$$

where R denotes the difference between domestic and foreign equity returns. Domestic equity bias arises when this relative equity return is *positively correlated* with X, that is, when domestic equities constitute a better hedge for domestic investors against risk factor X.

The two strands of literature differ in which risk factor they consider. One approach, following Obstfeld and Rogoff (2000), explores the link between consumption expenditures and international portfolios in stochastic general equilibrium models where investors have different consumption baskets.[2] In this class of models, investors face real exchange rate risk: $X = (1 - 1/\sigma)\Delta \ln Q$ where $\Delta \ln Q$ is the rate of change of the real exchange rate and an increase denotes an appreciation, and σ is the coefficient of relative risk aversion. The hedging demand for equities is then proportional to $(1 - 1/\sigma)\text{cov}(\Delta \ln Q, R)/\text{var}(R)$.[3] With a coefficient of relative risk aversion σ above unity, domestic equity bias arises when

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[1]The equity home bias is a general phenomenon. The share of home equities in other G7 countries portfolios in 2008 are as follows: 80.2% in Canada, 73.5% in Japan, 66% in France, 53% in Germany and 52% in Italy. All these countries account for less than 10% of world market capitalization.

[2]A non-exhaustive list of contributions –some of which precedes Obstfeld and Rogoff (2000)– includes Dellas and Stockman (1989), Uppal (1993), Baxter, Jermann and King (1998), Serrat (2001), Kollmann (2006), Obstfeld (2007), Heathcote and Perri (2007), Coeurdacier, Kollmann and Martin (2009), Collard, Dellas, Diba and Stockman (2007), Coeurdacier (2009) and Benigno and Nistico (2011).

[3]See Kouri and Macedo (1978), Krugman (1981) and the references in Adler and Dumas (1983) for an early derivation of this result under partial equilibrium.

relative equity returns are *positively* correlated with an appreciation of the domestic real exchange rate. The reason is as follows: with $\sigma > 1$, efficient risk sharing requires that domestic consumption expenditures increase when the real exchange rate appreciates, i.e. when their relative price increases.[4] If domestic equity returns are high precisely at that time, they provide the appropriate hedge against real exchange rate risk, and domestic investors will optimally tilt their portfolio towards domestic equity. As shown by van Wincoop and Warnock (2010), this line of research faces a serious challenge: for many countries, the empirical correlation between excess equity returns and the real exchange rate is close to zero.

The second strand of literature focuses on the hedging properties of domestic stocks against fluctuations in non-financial incomes (e.g. labor income).[5] In that case, the risk factor is $X = -R^n$, where R^n denotes the return to domestic non-financial income relative to the rest of the world. The hedge ratio takes the form $-\text{cov}(R^n, R)/\text{var}(R)$: if returns on domestic equities are high precisely when returns on non-financial wealth are low, then investors will favor domestic stocks. This line of research also faces an important empirical challenge as initially shown by Baxter and Jermann (1997). These authors find that financial and non-financial returns appear to be positively correlated, suggesting that optimal portfolios should be biased towards *foreign* equity.[6]

The first contribution of this paper is to merge and improve upon these two strands of literature by showing that many of the earlier results are not robust to the introduction of domestic and foreign bonds, whether nominal or real. We establish this point in a generic setting, characterizing jointly the optimal equity and bond portfolios in environments with multiple sources of risk and different degrees of completeness of financial markets. Our approach allows us to characterize the optimal equity and bond portfolios in terms of *sufficient statistics* that can easily be estimated, in the spirit of Chetty (2009). These sufficient statistics take precisely the form of the hedge ratios of equation (1), extended to the case of multiple asset classes. While these hedge ratios are equilibrium objects and thus depend on the specific details of the model (the structure of shocks, preferences etc...), it is sufficient to measure them to back out efficient portfolios.

The key economic insight of our paper is that in most models of interest, as well as in the data, nominal or real relative bond returns are strongly positively correlated with real exchange rate fluctuations. As a result, it is optimal for investors to use bond holdings to hedge real exchange rate risks. In that sense, bonds matter. All that is left for equities is to hedge the impact of any *additional* source of risk on investors' wealth, more specifically its non-financial income component. Again, the precise structure of these additional risk factors matters for optimal portfolio holdings, but the general portfolio structure can be estimated

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[4]Under efficient risk sharing, the Backus and Smith (1993) condition applies and relative consumption expenditures satisfy $(PC/P^*C^*) = (P/P^*)^{1-1/\sigma}$.

[5]A non-exhaustive list of contributions includes Bottazzi, Pesenti and van Wincoop (1996), Baxter and Jermann (1997), Julliard (2003), Heathcote and Perri (2007), Engel and Matsumoto (2009), Berriel and Bhattarai (2013), Mukherjee (2013) and Arespa (2015).

[6]Other empirical papers found more mixed results. See Bottazzi et al. (1996) and Julliard (2003).

independently of the specificities of the model. Generically, equity home bias arises if non-financial income risk is negatively correlated with equity returns, *after controlling for bond returns*. This conditioning is important: to the extent that *unconditional* and *conditional* hedge ratios for non-financial income risk are different in the data, bonds also matter for the insurance properties of equities against fluctuations in non-financial wealth.

A fully specified model provides the mapping from these hedge ratios to the structural parameters. Different models will imply different mappings. We show how this can be done in a two-country two-good model with stochastic endowments and redistributive shocks between capital and labor. This particular example also serves to illustrate starkly how failure to allow for trade in bonds can lead to incorrect inference on the structure of optimal equity portfolios. The same model without bond trading (equity-only) predicts that investors should short domestic equities, as in Baxter and Jermann (1997). By contrast, the model with equity and bond predicts full home equity bias.

The second important contribution of this paper is to confront the theory to the empirical evidence. We show how to estimate the sufficient statistics—and hence efficient portfolios—from observable data on bond returns, real exchange rates and the estimated returns to financial and non-financial wealth. Simple regressions of real exchange rate fluctuations and the return on non-financial wealth on bond and financial returns are sufficient to back out empirical estimates of the hedge ratios and thus equilibrium portfolios from the data. This provides an important link between recent theoretical work on international portfolios and data on asset returns. We use quarterly data on market returns, non-financial and financial income for the G-7 countries since 1970 to ask wether asset returns are theoretically consistent with observed portfolios. Since returns on non-financial and financial wealth are not directly observed, we consider a number of different approaches, such as Campbell (1996) (our benchmark estimation) or Lustig and Nieuwerburgh (2008) to construct alternate measures of these returns.

For all G-7 countries, and across all specifications, we find that the presence of bonds is key to obtaining more reasonable equity positions. Without bond trading, 'the international diversification puzzle is worse than you think' as Baxter and Jermann (1997) argued. However, once we allow for bond holdings, we predict, in line with the data, significant levels of equity home bias for all G-7 countries. Put differently, financial and non-financial returns are significantly *negatively* correlated, once investors are able to control their real exchange rate exposure with domestic and foreign bonds. Finally, our empirical estimates also predict short but fairly small domestic currency positions for a reasonable degree of relative risk aversion.

Section 2 presents our basic framework and characterizes optimal equity and bond portfolios in terms of hedge ratios. Section 3 presents a fully-specified version of the model with endowment and redistributive shocks and characterizes the equity-only and the full portfolios. Section 4 presents our empirical results. Section 5 concludes.

3

2 A Benchmark Model

2.1 Set-up

Preferences. We consider a two-period model ($t = 0, 1$) with two symmetric countries, Home (H) and Foreign (F), each with a representative household. Country i's representative household has standard Constant Relative Risk Aversion (CRRA) preferences, with a coefficient of relative risk aversion $\sigma \geq 1$ defined over a consumption index C_i, and a discount factor $0 < \xi \leq 1$:

$$U_i = \frac{C_{i,0}^{1-\sigma}}{1-\sigma} + \xi E_0 \left[\frac{C_{i,1}^{1-\sigma}}{1-\sigma} \right], \tag{2}$$

where E_0 denotes expectations conditional on date $t = 0$ information. The ideal consumer price index in country $i = H, F$ is denoted $P_{i,t}$, in terms of an arbitrary numeraire.

Financial markets and budget constraints. Trade in financial assets occurs in period 0. In each country there is a 'Lucas tree' whose supply is normalized to unity. In both periods, a cash-flow $d_{i,t}^f$ is distributed to owners of this financial asset (stockholders) as dividend. Another cash-flow $d_{i,t}^n$ is distributed to households of country i as non-financial income. At the simplest level, one can think of $d_{i,t}^n$ as representing 'labor income.' More generally, it describes all of country i's income sources that cannot be capitalized into financial claims.[7]

Agents can also trade Home and Foreign one-period bonds. Both bonds are in zero net supply. Buying one unit of the bond of country i in period $t - 1$ yields a cash-flow $d_{i,t}^b$ at date t. These bonds are riskless but pay in different units. If the bonds are risk-free in real terms, a unit of country i's bond purchased at date $t - 1$ yields $d_{i,t}^b = P_{i,t}$ at date t, i.e. enough resources to purchase one unit of country i's consumption index.

The representative household from country i enters period $t = 0$ with an initial portfolio of stocks $\{S_{ij,0}\}$ and bonds $\{B_{ij,0}\}$ from country $j \in \{H, F\}$ and faces the following budget constraint:

$$P_{i,0} C_{i,0} + \sum_j \left(p_S^j S_{ij,1} + p_B^j B_{ij,1} \right) = d_{i,0}^n + \sum_j \left(S_{ij,0} \left(p_S^j + d_{j,0}^f \right) + B_{ij,0} d_{j,0}^b \right) \tag{3}$$

where p_S^j (resp. p_B^j) denotes the price of a stock (resp. of the bond) from country j at date 0. The right hand side of equation (3) measures sources of funds, non-financial and financial. The left hand side captures uses of funds: consumption and portfolio investment.

At date $t = 1$, all income is spent. The period budget constraint takes the following form:

$$P_{i,1} C_{i,1} = \sum_j \left(S_{ij,1} d_{j,1}^f + B_{ij,1} d_{j,1}^b \right) + d_{i,1}^n \tag{4}$$

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[7]This could be due, *inter alia*, to domestic financial frictions, capital income taxation or poor enforcement of property rights.

Lastly, market clearing in markets for stocks and bonds of country $i \in \{H, F\}$ requires at both dates:

$$\sum_j S_{ji,t} = 1; \quad \sum_j B_{ji,t} = 0. \tag{5}$$

2.2 Equilibrium portfolios

Portfolios decisions. The optimal portfolio allocation results from maximizing (2) subject to (3) and (4). The corresponding optimality conditions for stocks and bonds holdings in country i are given by the usual Euler equations:

$$E_0 \left(\mathcal{M}_i R_j^f \right) = E_0 \left(\mathcal{M}_i R_j^b \right) = 1 \quad ; \quad i, j \in \{H, F\}, \tag{6}$$

where $R_j^f = d_{j,1}^f / p_S^j$ and $R_j^b = d_{j,1}^b / p_B^j$ denote the gross return on stocks and bonds respectively in country j and $\mathcal{M}_i = \xi \left(P_{i,0} / P_{i,1} \right) \left(C_{i,1} / C_{i,0} \right)^{-\sigma}$ is the stochastic discount factor in country i.

Log-linearization of the budget constraint. While it is not generically possible to derive exact solutions for portfolios, we can characterize approximate optimal consumption and portfolio decisions around the symmetric equilibrium where both countries have the same distribution of financial, non-financial and bond cash flows, households hold similar initial portfolios and have no initial net foreign asset positions, using standard log-linearization techniques as in Devereux and Sutherland (2011) and Tille and van Wincoop (2010).[8] Before we do so, we need to introduce a bit of notation. First, we use variables without country indices to denote *differences* across countries: $x = x_H - x_F$. Second, we use Jonesian hats for the log-deviation of a variable x from its (symmetric) steady state value $\bar{x}$: $\hat{x} \equiv log(x/\bar{x})$. Finally, we use the operator Δ to denote first differences: $\Delta x = x_1 - x_0$.

Define the Home country real exchange rate as the foreign price of the domestic good, $Q \equiv P_H / P_F$, so that an increase in the real exchange rate represents a real appreciation. Log-linearizing yields:

$$\hat{Q}_t = \hat{P}_{H,t} - \hat{P}_{F,t}. \tag{7}$$

Define aggregate nominal expenditures $X_{i,t} = P_{i,t} C_{i,t}$, and denote $1 - \delta = \bar{d}_t^n / \bar{X}_t$ the steady state share of non-financial income in total expenditures, assumed common in both periods. We then take the difference between Home and Foreign budget constraints in both periods from (3) and (4), log-linearize around the symmetric equilibrium, and use the market clearing conditions (5), to obtain:

$$\hat{X}_0 = (1 - \delta) \hat{d}_0^n + (2S - 1) \delta \hat{d}_0^f + 2b \hat{d}_0^b$$
$$\hat{X}_1 = (1 - \delta) \hat{d}_1^n + (2S - 1) \delta \hat{d}_1^f + 2b \hat{d}_1^b,$$

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[8] Formally, we log-linearize around $d_{i,0}^l = \bar{d}_0^l$ and $E_0 d_{i,1}^l = \bar{d}_1^l$, and assume that $S_{ii,0} = S$ and $B_{ii,0} = B$ for $i \in \{H, F\}$ and $l \in \{n, f, b\}$. Appendix A.1 derives the more general case where countries are asymmetric ex-ante. In that case, the optimal portfolio contains an additional intertemporal component.

where $S = S_{ii,0} = S_{ii,1}$ and $B = B_{ii,0} = B_{ii,1}$ denote the (symmetric) optimal holdings of a country's own equities and real bonds and $b = B/\bar{X}_0$ denotes the steady state ratio of bond holdings to aggregate expenditures.

Taking the difference between the two previous equations, using the definition of the stochastic discount factor $\mathcal{M}_i = \xi \left(P_{i,0}/P_{i,1} \right) \left(C_{i,1}/C_{i,0} \right)^{-\sigma}$ and the fact that $\Delta \hat{X} = \Delta \hat{Q} + \Delta \hat{C}$, we obtain:

$$\Delta \hat{X} = \left(1 - \frac{1}{\sigma} \right) \Delta \hat{Q} - \frac{1}{\sigma} \hat{\mathcal{M}} = (1 - \delta) \Delta \hat{d}^n + (2S - 1) \delta \Delta \hat{d}^f + 2b \Delta \hat{d}^b \tag{8}$$

The left hand side of this equation determines relative consumption expenditure growth $(\Delta \hat{X})$ as a function of the rate of change of the real exchange rate $(\Delta \hat{Q})$ and the relative stochastic discount factor $(\hat{\mathcal{M}})$. The right hand side expresses relative income growth for a given portfolio choice (S, b), as a function of non-financial income growth $(1 - \delta)\Delta \hat{d}^n$, the relative return on equities $(2S - 1)\delta \Delta \hat{d}^f$ and the relative return on bonds $2b \Delta \hat{d}^b$. More formally, we can write the (log-linearized) relative return on equities R^f, non-financial wealth R^n and bonds R^b as:

$$\hat{R}^f = \Delta \hat{d}^f - E_0 \Delta \hat{d}^f \quad ; \quad \hat{R}^n = \Delta \hat{d}^n - E_0 \Delta \hat{d}^n \quad ; \quad \hat{R}^b = \Delta \hat{d}^f - E_0 \Delta \hat{d}^f.$$

Substituting into (8), we obtain:

$$\Delta \hat{X} - E_0 \Delta \hat{X} = \left(1 - \frac{1}{\sigma} \right) \left(\Delta \hat{Q} - E_0 \Delta \hat{Q} \right) - \frac{1}{\sigma} \left(\hat{\mathcal{M}} - E_0 \hat{\mathcal{M}} \right) = (1 - \delta)\hat{R}^n + (2S - 1) \delta \hat{R}^f + 2b \hat{R}^b \tag{9}$$

Equation (9) is a key equation for our analysis. It tells us how relative aggregate consumption expenditures must vary with the portfolio structure, for any realization of portfolio returns.

Hedge Ratios. If relative bond and equity returns are not perfectly correlated, it is always possible to 'project' the rate of change of the real exchange rate and the return on non-financial income on stock and bond returns.[9] This projection takes the form:

$$\begin{cases} \Delta \hat{Q} - E_0 \Delta \hat{Q} & \equiv \quad \beta_{Q,b} \hat{R}^b + \beta_{Q,f} \hat{R}^f + u_Q \\ \hat{R}^n & \equiv \quad \beta_{n,b} \hat{R}^b + \beta_{n,f} \hat{R}^f + u_n \end{cases} , \tag{10}$$

where the residual terms u_i are orthogonal to asset returns $\hat{R}^j$, i.e. $E_0[u_i \hat{R}^j] = 0$ for $i \in \{Q, n\}$ and $j \in \{f, b\}$. The coefficients $\beta_{i,j}$ capture the loading of asset return j on risk factor $i = \{Q, n\}$. These loading factors, also called *hedge ratios*, have the usual interpretation in terms of covariance-variance ratios:

$$\beta_{n,j} = \frac{\text{cov}_{\hat{R}^l} \left(\hat{R}^n, \hat{R}^j \right)}{\text{var}_{\hat{R}^l} \left(\hat{R}^j \right)} \; ; \; \beta_{Q,j} = \frac{\text{cov}_{\hat{R}^l} \left(\Delta \hat{Q} - E_0 \Delta \hat{Q}, \hat{R}^j \right)}{\text{var}_{\hat{R}^l} \left(\hat{R}^j \right)},$$

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[9] Appendix A.1 shows formally that a *rank condition* needs to be satisfied. This will generically be the case if the dimension of the underlying shocks is larger or equal to 2.

where $j \neq l \in \{f, b\}$ and $\mathrm{cov}_z(x, y)$ (resp. $\mathrm{var}_z(x)$) denotes the covariance between x and y (resp. the variance of x), *conditional on z*. While these factor loadings are equilibrium objects and model-dependent, their empirical counterpart can be obtained simply from the reduced form multivariate regressions of equation (10), independently of the specifics of the model and the source of shocks driving asset returns.

Equilibrium portfolios. From the Euler equations (6) of the investor problem, observe that the relative stochastic discount factor $\hat{\mathcal{M}}$ satisfies:

$$E\left[\hat{\mathcal{M}} \ \hat{R}^i\right] = 0 \text{ for } i \in \{f, b\} \tag{11}$$

Using equation (11) to project the budget constraint on relative asset returns $\hat{R}^f$ and $\hat{R}^b$, we obtain the following key property:[10]

Property 1 (Optimal Portfolios in terms of Hedge Ratios) *Under the rank condition of Appendix A.1, the optimal portfolio is unique and can be expressed in terms of the loading factors $\beta_{i,j}$ as follows:*

$$b^* = \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\beta_{Q,b} - \frac{1}{2}\left(1 - \delta\right)\beta_{n,b} \tag{12a}$$

$$S^* = \frac{1}{2}\left[1 - \frac{1-\delta}{\delta}\beta_{n,f} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f}\right]. \tag{12b}$$

Property 1 has two key implications that are central to our analysis. First, it establishes that the loading factors (or hedge ratios) $\beta_{i,j}$ provide *sufficient statistics* for the optimal portfolios, in the sense of Chetty (2009). The structural details of a general equilibrium model will generically provide a mapping of the loading factors into the primitive characteristics of the model. Yet the portfolio predictions remain identical across models, conditional on a set of loading factors $\beta_{i,j}$. Second, these loading factors can easily be measured empirically.

Let's now discuss the structure of equilibrium portfolios implied by Property 1. Consider first the bond portfolio b^* in equation (12a). It contains two terms. The first term $(1 - 1/\sigma)\beta_{Q,b}/2$ reflects the role of bonds in hedging real exchange rate risk. When $\sigma > 1$, the household's relative consumption expenditures $\hat{X}$ increase when the real exchange rate appreciates. If, after controlling for equity returns, domestic bonds deliver a high relative return precisely when the currency appreciates (i.e. $\beta_{Q,b} > 0$), then domestic bonds constitute a good hedge against real exchange rate risk. The second term $-(1 - \delta)\beta_{n,b}/2$ captures the role of bonds in hedging non-financial income risk. When domestic bonds and the return to nonfinancial wealth are positively conditionally correlated ($\beta_{n,b} > 0$), investors want to short the domestic bond to hedge the implicit exposure from their non-financial wealth. Equation

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[10]The proof is relegated to Appendix A.1. It relies on observing that the relative stochastic discount factor is orthogonal to asset returns, using Eq.(11).

([12a](#)) indicates that investors will go long or short in their domestic bond holdings depending on the relative strength of these two effects.

Consider now the equilibrium equity position S^* in equation ([12b](#)). The first term inside the brackets represents the symmetric risk-sharing equilibrium of Lucas (1982): $S^* = 1/2$. This is the optimal equity portfolio if equities are not useful to hedge real exchange rate or non-financial risk ($\beta_{Q,f} = \beta_{n,f} = 0$).

The second term, $(1 - 1/\sigma)\beta_{Q,f}/\delta$, is similar to the term that has been emphasized in Coeurdacier (2009), Obstfeld (2007) and others, with one important difference. It represents the demand for domestic equity that arises from hedging the real exchange rate risk, corresponding to the hedge portfolio in equation ([1](#)). This demand is driven by the conditional correlation between equity returns and the real exchange rate, $\beta_{Q,f}$. If this loading is positive, domestic stock returns are relatively high when the currency appreciates. The important difference is that this hedge ratio is *conditional on bond returns*. As we will establish shortly, conditional and unconditional hedge ratios can differ greatly, with important implications for optimal portfolios.

The last term, $-(1 - \delta)\beta_{n,f}/\delta$, determines how equity portfolios are structured to hedge non-financial risk. Investors optimally want to undo the endowed equity exposure implicit in their non-financial wealth and measured by $\beta_{n,f}$. To fix ideas, consider the case where bonds are risk-free in real terms so that $d_{i,t}^b = P_{i,t}$. In that case, it is immediate, using equation ([10](#)), that $\beta_{Q,b} = 1$ and $\beta_{Q,f} = 0$ since $\hat{R}^b = \hat{Q} - E_0\hat{Q}$. In the absence of non-financial income (i.e. when $\delta \to 1$), the optimal portfolios are the same as in Adler and Dumas (1983). Since bonds hedge perfectly real exchange risk, risky asset holdings are fully diversified: $S^* = 1/2$. Equation ([12b](#)) extends Adler and Dumas (1983) to the case with non-financial income ($\delta < 1$):

$$S^* = \frac{1}{2}\left(1 - \frac{1 - \delta}{\delta}\beta_{n,f}\right). \tag{13}$$

This result is reminiscent of Baxter and Jermann (1997) who find that financial and non-financial returns are (*unconditionally*) positively correlated and conclude that the optimal portfolio should therefore be tilted towards foreign equities ($S^* < 0.5$). However, unlike Baxter and Jermann (1997), equation ([13](#)) indicates that the relevant hedge ratio is *conditional on bond returns*. Our model predicts that home equity bias arises if $\beta_{n,f} < 0$. To our knowledge, this condition has not been empirically investigated in the literature.[11]

Finally, observe that our approach is valid as long as equities and bonds are not redundant assets (the rank condition is satisfied), regardless of the degree of completeness of financial markets. Technically, we show in Appendix [A.1](#) that if an additional spanning condition is

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[11]Engel and Matsumoto (2009) also note that this is the relevant condition in presence of bond holdings, or forward exchange contracts. See also Coeurdacier et al. (2009) and Coeurdacier, Kollmann and Martin (2010).

satisfied, markets are *locally* complete, in the sense that the efficient risk sharing condition of Backus and Smith (1993) holds locally:

$$\hat{\mathcal{M}} = -\sigma \Delta \hat{C} - \Delta \hat{Q} = 0. \tag{14}$$

When this condition holds bonds and equities are sufficient to span the relevant sources of risk in the economy. This implies that the decomposition in Eq. (10) is exact and $u_i = 0$. The fact that our approach works both in complete and incomplete market environments gives it a great deal of generality.

3   Closing the Model: The Case of Redistributive Shocks

We have established that the partial equilibrium hedge ratios $\beta_{i,j}$ provide sufficient statistics for a full characterization of optimal portfolios. Yet, by fleshing out the remaining details of the model, we can link these hedge ratios to the structural parameters of the model. While providing a full fledged general theory of the factor loadings $\beta_{i,j}$ is beyond the scope of this paper, this section presents a simple model with endowment and redistributive shocks.[12] We use the model to contrast conditional and unconditional equilibrium factor loadings—and therefore optimal equity portfolios in environments with and without bond trading. These results serves to illustrate the potential pitfalls of using an equity-only model as commonly done in the literature. Readers only interested in the empirical implications of Section 2 can go directly to Section 4.

3.1   A model with endowment and redistributive shocks.

The structure of the model borrows all the elements introduced in Section 2. In addition, one needs to specify further the production and the demand side as well as the nature of the shocks.

Endowments and shocks. Each country receives an endowment of a country-specific tradable good each period. The endowment in country i at date t is denoted $y_{i,t}$. $y_{i,0}$ is known while $y_{i,1}$ is stochastic and symmetrically distributed with mean $\bar{y}_1$ common to both countries.

We denote $p_{i,t}$ the price at date t of country i's good in terms of the numeraire. At each date, the financial cash-flow represents a share $\delta_{i,t}$ of output at market value $p_{i,t}y_{i,t}$: $d_{i,t}^f = \delta_{i,t}p_{i,t}y_{i,t}$. $\delta_{i,0} = \delta$ is known while $\delta_{i,1}$ is stochastic and symmetrically distributed with mean δ. Shocks to $\delta_{i,t}$ represent redistributive shocks, i.e shocks to the share of total output distributed as financial income in country i. While we take these fluctuations as exogenous, they can occur endogenously in a world where capital and labor enter into the production function with a non-unit elasticity of substitution, in presence of capital and

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[12]The working paper version Coeurdacier and Gourinchas (2011) provides additional examples with, e.g., fiscal shocks, nominal shocks or non-traded goods. This example with redistributive shocks is similar to Coeurdacier et al. (2009) and Engel and Matsumoto (2009).

labor augmenting productivity shocks or in presence of biased technical change in the sense of Young (2004).[13]

In each country the representative consumer enters period $t = 0$ with a given financial portfolio of financial assets, receive financial and non-financial income as described in Section 2, consume and trade financial claims. In period $t = 1$, stochastic endowments and stochastic shocks to δ are realized, households consume using the revenues from their financial portfolio and their non-financial endowment.

Preferences. Each representative household consumes both goods with a preference towards the domestic good. For $i, j \in \{H, F\}$ and $t = 0, 1$ the consumption index $C_{i,t}$ is a constant-elasticity aggregator:

$$C_{i,t} = \left[a^{1/\phi} c_{ii,t}^{(\phi-1)/\phi} + (1-a)^{1/\phi} c_{ij,t}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)},$$

where $c_{ij,t}$ denotes country i's consumption of the good from country j at date t. ϕ is the elasticity of substitution between the two goods and $1 \geq a \geq 1/2$ captures preference for the home good (mirror-symmetric preferences). With these preferences, the Fisher-ideal price index for consumption is:

$$P_{i,t} = \left[a p_{i,t}^{1-\phi} + (1-a) p_{j,t}^{1-\phi} \right]^{1/(1-\phi)}. \tag{15}$$

Financial and non-financial cash-flows. We assume that each country's bonds are risk-free in terms of that country's consumption index, that is $d_{i,t}^b = P_{i,t}$. With the notations of Section 2, financial and non financial cash-flows at date t are given by:

$$d_{i,t}^f = \delta_{i,t} p_{i,t} y_{i,t} \quad ; \quad d_{i,t}^n = (1-\delta_{i,t}) p_{i,t} y_{i,t} \quad ; \quad d_{i,t}^b = P_{i,t} \quad \text{for} \quad i \in \{H, F\}.$$

With these definitions of cash-flows, budget constraints can be written as in Eqs. (3) and (4) and portfolio equations as in Eq. (6).

Goods markets equilibrium. In each period, optimal intratemporal allocation of consumption requires:

$$c_{ii,t} = a \left(\frac{p_{i,t}}{P_{i,t}} \right)^{-\phi} C_{i,t} \quad ; \quad c_{ij,t} = (1-a) \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\phi} C_{i,t} \quad \text{for} \quad i \neq j. \tag{16}$$

Resource constraints are given by:

$$c_{ii,t} + c_{ji,t} = y_{i,t} \quad \text{for} \quad i \in \{H, F\}, j \neq i. \tag{17}$$

Define q_t as Home's terms of trade, i.e. the relative price of the Home tradable good in terms of the Foreign tradable good: $q_t \equiv p_{H,t}/p_{F,t}$. An increase in q represents an improvement in

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[13]See also Ríos-Rull and Santaeulàlia-Llopis (2010).

Home's terms of trade.

Using (16) together with the resource constraints (17) yields the following expression for relative output:

$$\frac{y_{H,t}}{y_{F,t}} = q_t^{-\phi} \Omega_a \left[(\frac{P_{F,t}}{P_{H,t}})^\phi \frac{C_{F,t}}{C_{H,t}} \right] \tag{18}$$

where $\Omega_a(x) \equiv \left[1 + x(\frac{1-a}{a}) \right] / \left[x + (\frac{1-a}{a}) \right]$. Without home bias in preferences ($a = 1/2$), $\Omega_{1/2}(x) = 1$ identically and Eq. (18) simplifies to $y_{H,t}/y_{F,t} = q_t^{-\phi}$: the price elasticity of relative output is ϕ, independently of the distribution of relative expenditures. As emphasized by Obstfeld (2007), the term $\Omega_a(.)$ captures the Keynesian *transfer effects* due to consumption home-bias: with $a > 0.5$, a reallocation of wealth towards the home country requires an improvement in the domestic terms of trade since it shifts relative demand towards the domestic good.

Log-linearization of returns. We show in Appendix A.1 that the model with independent endowment and redistributive shocks satisfies the rank and spanning conditions so that markets are locally complete. This simplifies greatly the characterization of the equilibrium. Formally, it implies that the Backus and Smith (1993) efficient risk sharing condition, Eq. (14), holds locally and $\Delta \hat{C} = -1/\sigma \Delta \hat{Q}$.

Log-linearizing the goods' market equilibrium condition Eq. (18) substituting the above expression, and using the fact that $\Delta \hat{Q} = (2a-1)\Delta \hat{q}$ from Eq. (15), we obtain a relationship between relative output and the terms of trade:

$$\Delta \hat{y} = -\lambda \Delta \hat{q} = -\lambda (2a-1)^{-1} \Delta \hat{Q} \tag{19}$$

In this expression, $\lambda \equiv \phi \left(1 - (2a-1)^2 \right) + (2a-1)^2 / \sigma > 0$ represents the equilibrium terms of trade elasticity of relative output. Without home bias in preferences ($a = 1/2$), $\lambda = \phi$, the elasticity of substitution between Home and Foreign goods. When $a > 1/2$, the additional term $(2a-1)^2(1/\sigma - \phi)$ captures the required change in the terms of trade needed to accommodate transfer effects. The last equality uses the fact that terms of trade and real exchange rates are perfectly correlated in this model, with $\hat{q} = (2a-1)\hat{Q}$.

Define aggregate nominal income $x_{i,t} = p_{i,t} y_{i,t}$. We can write the (log-linearized) relative return on equities $\hat{R}^f$, bonds $\hat{R}^b$ and non-financial income $\hat{R}^n$ as:

$$\hat{R}^f = \Delta \hat{\delta} - E_0 \Delta \hat{\delta} + \Delta \hat{x} - E_0 \Delta \hat{x} \tag{20a}$$

$$\hat{R}^n = -\frac{\delta}{1-\delta} \left(\Delta \hat{\delta} - E_0 \Delta \hat{\delta} \right) + \Delta \hat{x} - E_0 \Delta \hat{x} \tag{20b}$$

$$\hat{R}^b = \Delta \hat{Q} - E_0 \Delta \hat{Q} \tag{20c}$$

Financial and non-financial returns co-move positively with innovations to nominal income growth $\Delta \hat{x} - E_0 \Delta \hat{x}$ and negatively with redistributive shocks $\Delta \hat{\delta} - E_0 \Delta \hat{\delta}$.

Projection of risk factors on asset returns. Using (19), we get immediately that $\Delta \hat{x} = (1 - \lambda)(2a - 1)^{-1}\Delta \hat{Q}$. Substituting into asset returns, we obtain the following equilibrium projection of the real exchange rate and the return to non-financial wealth on equity and bond returns—the model-specific counterpart of equation (10):[14]

$$\Delta \hat{Q} - E_0 \Delta \hat{Q} = \hat{R}^b \tag{21a}$$

$$\hat{R}^n = \frac{1 - \lambda}{(1 - \delta)(2a - 1)}\hat{R}^b - \frac{\delta}{1 - \delta}\hat{R}^f \tag{21b}$$

Equation (21) characterizes the equilibrium hedge ratios $\beta_{i,j}$ in terms of the structural parameters of the model:

$$\beta_{Q,b} = 1 \quad ; \quad \beta_{Q,f} = 0 \quad ; \quad \beta_{n,b} = \frac{1 - \lambda}{(1 - \delta)(2a - 1)} \quad ; \quad \beta_{n,f} = -\frac{\delta}{1 - \delta} \tag{22}$$

In Eq. (22), two elements are essential: first, since investors can trade real risk-free bonds, relative bond returns and the real exchange rate are perfectly correlated ($\beta_{Q,b} = 1$ and $\beta_{Q,f} = 0$). Second and more importantly, despite positive co-movements between financial and non-financial returns driven by innovations to nominal income growth ($\Delta \hat{x} - E_0 \Delta \hat{x}$), the loading of non-financial income risk on financial asset returns $\beta_{n,f}$ is always strictly negative. The reason is that nominal income growth is tightly linked to terms-of trade and the real exchange rate in this model through Eq. (19), and thus can be hedged using real bonds.

Equilibrium portfolios. Substituting the equilibrium loadings (22) into equation (12), the optimal portfolio satisfies:

$$S^* = 1 \quad ; \quad b^* = \frac{1}{2}\left[(1 - \frac{1}{\sigma}) + \frac{\lambda - 1}{2a - 1}\right]. \tag{23}$$

Since purely redistributive shocks only affect the distribution of total output, but not its size, the optimal hedge is for the representative domestic household to hold all the domestic equity. This perfectly offsets the impact of the redistributive shocks on total income. Consequently, the model implies full equity portfolio home bias.

Observe that this result does not depend upon the size of the redistributive shock. If we denote the relative variance of redistributive and endowment shocks by $v^2 = \sigma_\delta^2/\sigma_y^2$, then the model predicts that $S^*(v) = 1$ as long as $v > 0$.[15]

The optimal bond position is the outcome of two forces: first, investors hedge real exchange risk when $\sigma \neq 1$. This is the term $(1 - 1/\sigma)/2$. Second, investors are fully exposed to domestic endowment shocks given their equity holdings. The bond portfolio makes sure

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[14]Since markets are locally complete, residual (non-insurable) noise u_Q and u_n in Eq. (10) are zero in all states.

[15]When $v = 0$, portfolios are indeterminate. Equity and bond returns are perfectly correlated when there is endowment risk only and the model fails the rank condition of App. A.1.

that endowment risk is equally shared between home and foreign investors. This is the term $(2a - 1)^{-1}(\lambda - 1)/2$. The bond position can be long or short depending on whether $\lambda < 1 - (1 - 1/\sigma)(2a - 1)$ or not, i.e depending on whether relative income growth co-move positively or negatively with relative bonds returns, or equivalently the real exchange rate.

3.2 The pitfalls of equity-only models

To illustrate the pitfalls of using equity-only models, consider what happens in the previous model if households can only trade equities. Following similar steps as in Section 2, one can derive the equilibrium equity-only optimal portfolio:

$$S^u = \frac{1}{2} \left[1 - \frac{1 - \delta}{\delta} \beta^u_{n,f} + \frac{1 - \frac{1}{\sigma}}{\delta} \beta^u_{Q,f} \right].$$

This equation is similar to Eq. (12b) except that the loadings $\beta^u_{i,f} = \text{cov}(\hat{R}^i, \hat{R}^f)/\text{var}(\hat{R}^f)$ are *unconditional* loadings. The formula has the same interpretation: the optimal equity portfolio hedges both non-financial and real exchange rate risk.

With two sources of uncertainty (endowment and redistributive shocks) and only equities, markets are incomplete, even locally. Solving for the optimal hedge ratios $\beta^u_{i,f}$ requires to follow the approach of Devereux and Sutherland (2011). This is done in Appendix A.2 for the general case. We consider here the limit of $S^u(\upsilon)$ as $\upsilon \to 0$, i.e. as redistributive shocks become vanishingly small. Intuitively, in that case financial and nonfinancial returns become positively correlated (see Eqs. (20)) so that $\beta^u_{n,f} > 0$. In the limit of $\upsilon = 0$, markets are locally complete again, and following the same steps as before, one can establish that:

$$\beta^u_{n,f} = 1 \quad ; \quad \beta^u_{Q,f} = (2a - 1)/(1 - \lambda). \tag{24}$$

It follows that the optimal equity portfolio of the equities-only model satisfies:

$$S^u(0) = \lim_{\upsilon \to 0} S^u(\upsilon) = \frac{1}{2} \left[1 - \frac{1 - \delta}{\delta} + \frac{1 - \frac{1}{\sigma}}{\delta} \frac{(2a - 1)}{1 - \lambda} \right] \tag{25}$$

As before, this portfolio is the sum of three terms: a Lucas pooled portfolio $(1/2)$, a term due to hedging of nonfinancial income risk $(-(1 - \delta)/(2\delta))$ and a final term hedging real exchange rate risk $(1 - 1/\sigma)(a - 1/2)/(\delta(1 - \lambda)))$. In the absence of bond trading, however, the hedging term for nonfinancial risk always imparts a large foreign equity bias, as in Baxter and Jermann (1997): financial and nonfinancial income are perfectly correlated when υ goes to zero and $\beta^u_{n,f} = 1$. While in principle the last term can be positive or negative, depending on whether the equilibrium terms of trade elasticity of relative output λ is smaller or greater than 1, we know from van Wincoop and Warnock (2010) that the unconditional loading factor $\beta^u_{Q,f}$ is positive but small in the data, so that the portfolio $S^u(0)$ should typically

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exhibit foreign bias.[16]

Thus, as in Baxter and Jermann (1997), the equity-only model cannot account for the home-equity bias for υ sufficiently small. Once bond trading is allowed, the same model imparts full equity home bias, independently on model parameters. This striking example shows the potential crucial role of bond trading for the composition of equity portfolios. From an empirical perspective, our portfolio results are driven by the stark difference between *unconditional* and *conditional* hedge ratios: in the above example, $\beta_{n,f}^u$ is typically positive—at the limit, when υ goes to zero, $\beta_{n,f}^u = 1$. To the opposite, $\beta_{n,f}$ is unambiguously negative, thus for any value of υ. If data were generated by such a model, measuring the unconditional hedge ratio would lead to the conclusion that the international diversification puzzle in worse than we think as in Baxter and Jermann (1997), while measuring the conditional hedge ratio would lead to the opposite conclusion.

More broadly, the overall message is that the optimal equity portfolio will depend on the menu of assets available to investors allowing them to diversify the risks they face. We consider that allowing for bonds is essential since, as we document, they provide a very natural hedge against real exchange rate risk—a point also noted by Adler and Dumas (1983). Other tradable assets may be relevant besides risk-free bonds if they have attractive hedging properties: long term bonds, housing, derivatives... The empirical approach we follow in the next section aims to maintain a parsimonious framework. We will show that we can provide a reasonable account of observed equity portfolios simply by allowing for trade in short term bonds. This does not preclude more sophisticated models from achieving an even better fit with the data.

4 Estimating Optimal Portfolios

A key contribution of the paper is to construct optimal equity and bond portfolios. To do so, we estimate the reduced-form loading factors $\beta_{Q,i}$ and $\beta_{n,i}$ for $i = f, b$ for the G-7 countries. According to Property 1, this is all we need to characterize equilibrium portfolios.

4.1 From theory to data

Two issues arise when mapping the theory into the data, one theoretical, the other empirical. On the theoretical side, one might wonder if our results, derived in a two-period environment survive in a dynamic setting. On the empirical side, our two period model does not allow for time-varying expected returns, an important feature of the data. In Appendix A.5, we show that our results are robust to a dynamic environment with complete markets and i.i.d returns, in a continuous-time model à la Merton (see Merton (1990) and Adler and Dumas

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[16]van Wincoop and Warnock (2010) estimate $\beta_{Q,f}^u = 0.32$. With such a low hedge ratio, the model can deliver equity home bias ($S > 1/2$) only if the share of financial income in total income is very high: $\delta \geq 1 - \beta_{Q,f}^u = 0.68$, a number vastly in excess of any reasonable estimate. In section 4, we estimate $\beta_{Q,f}^u = 0.55$ which would require $\delta > 0.45$ which is still too high.

([1983](#)). Optimal portfolios satisfy Property [1](#). The property holds with factor loadings computed on *total returns*, and thus including any time-varying expected return component — a finding that will matter when computing the empirical counterpart of returns on non-financial wealth.[17] In summary, our results hold in a static context with (locally) complete or incomplete markets, and also in a dynamic context, but under complete markets. Ideally, one would like to derive the equivalent of Property [1](#) for optimal portfolios in a dynamic model with incomplete markets, multiple agents and time-varying expected returns. This is a challenging task that is beyond the scope of this paper.[18]

4.2   The data.

We collect quarterly data for all G-7 countries over the period 1970:1-2008:3, stopping short of the global financial crisis.[19] We consider each member of the G-7 as the Home country in turn, aggregating the remaining countries into a 'Foreign country'.

4.2.1   The easy part: bond returns, real exchange rates, financial and nonfinancial income.

We measure gross real bond returns, R_i^b, as the ex-post gross return on 3-month domestic Treasury-bill converted in constant U.S. dollars.[20] The (log) of the real exchange rate Q_i for country i is defined as the difference between the (log) of the consumer price index in country i, P_i, and the (log) of the consumer price index for the rest of the world, defined as a GDP-weighted average of the price indices of the remaining countries, where all price indices are converted into U.S. dollars:[21]

$$\ln Q_i = \ln P_i - \sum_{j \neq i} \alpha_{ji} \ln P_j,$$

where α_{ji} represents the share of country $j's$ output in the rest of the world outside country i.[22] With this definition, an increase in Q_i represents a real appreciation of the currency of country i. Figure [1](#) reports the real exchange rate for the G-7 countries, normalized to 100 in 2001Q1.

Next, we decompose each country's gross domestic product into a financial and a nonfinancial components using National Income Account data.[23] All variables are converted in US dollars using nominal exchange rates. The decomposition of output Y by income satisfies:

$$Y = COMP + M + \Pi + D + T, \tag{26}$$

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[17]In the dynamic model of Appendix [A.5](#), returns are iid log-normal. Expected returns can be time-varying as long as returns of a given asset are driven by a *unidimensional* Brownian motion to preserve market completeness. Otherwise, the derived portfolio is only valid in the log-case.

[18]See for instance the discussion in Dumas and Lyasoff (2012).

[19]See appendix [B.1](#) for a detailed description of data sources.

[20]Short-term government bond yields and dollar nominal exchange rates are obtained from the *Global Financial Database*.

[21]Consumer Price Indices are from the *OECD Main Economic Indicators*.

[22]Formally, $\alpha_{ji} = Y_j / \sum_{j \neq i} Y_j$.

[23]Data is obtained from the OECD quarterly national income and from U.N. national account statistics.

where $COMP$ refers to the compensation of employees, M to mixed income, Π to the net operating surplus, D to the consumption of fixed capital, and T to taxes minus subsidies on production and imports. According to the 1993 United Nations's System of National Accounts, the net operating surplus Π represents the profits of incorporated entities.[24] By contrast, mixed income M denotes income from self-employment as well as proprietary income.[25] In the model, nonfinancial income denotes the component of aggregate income that cannot be capitalized into financial claims. We follow Gollin (2002) and construct an empirical counterpart W as the sum of the compensation of employees $COMP$, plus a fraction ν of mixed income M:[26] $W = COMP + \nu M$.

Financial income K is then defined as gross operating profits $\Pi + D$ plus the remainder of mixed income $(1 - \nu) M$, net of non-residential gross capital formation I:[27]

$$K = \Pi + D + (1 - \nu) M - I.$$

Using these measures, we construct estimates of the share of financial income δ as $K/(Y - T - I)$. Table 1 summarizes our estimates for the G-7 countries. These estimates range from 13.1 percent for Germany to 25.4 percent for Italy, with an unweighted average of 16.7 percent. For comparison, the table also reports the 'naïve' estimate of δ obtained as one minus the share of compensation of employees in output measured at factor prices, that is $1 - COMP/(Y - T)$. The naïve estimate is much higher, with an average of 41.3 percent.

In what follows, we normalize financial and nonfinancial income by population, and express them in constant U.S. dollars. Figure 1 reports nonfinancial income per capita for each country relative to the nonfinancial income of the remaining G-7 countries. Relative nonfinancial income exhibits marked fluctuations over the period. For instance, for the U.S., it fluctuates between 0.9 and 2.4. It is also strikingly correlated with the real exchange rate, also reported on the same figure.[28]

4.2.2 The harder part: returns to financial and nonfinancial wealth

We now construct empirical counterparts to the return on financial and nonfinancial wealth since neither returns are directly observable.

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[24]It is defined as "the surplus or deficit accruing from production before taking account of any interest, rent or similar charges payable on financial or tangible non-produced assets borrowed or rented by the enterprise, or any interest, rent or similar receipts receivable on financial or tangible non-produced assets *owned by the enterprise.*"

[25]It is defined as "the surplus or deficit accruing from production by *unincorporated enterprises owned by households*; it implicitly contains an element of remuneration for work done by the owner, or other members of the household, that cannot be separately identified from the return to the owner as entrepreneur but it excludes the operating surplus coming from owner-occupied dwellings."

[26]ν is assumed equal to $COMP/(COMP + \Pi)$. The results are very robust to alternative measures of ν, including the polar cases where all mixed incomes are treated as nonfinancial income $(\nu = 1)$ and all mixed incomes are treated as financial income $(\nu = 0)$.

[27]We substract gross capital formation to compute the part of income that flows to owners of financial claims on capital. We adjust gross capital formation for residential investment since the latter does not reflect investment decisions of corporations but of households.

[28]The correlation ranges between 0.68 for Italy and 0.96 for Japan with an average of 0.85

Consider first the return to financial wealth, R^f, where we drop the country subscript i to ease notation. In general, that return is not equal to the return on aggregate equity R^e. In the model, the two are equal because financial wealth is entirely capitalized in the equity market. In practice, firms are financed through equity and corporate debt, among other instruments.[29] What is needed is an estimate of the financial return to the firm. Our benchmark method looks at the liability side of the firms' balance sheet, using observable equity and corporate bond market data. Specifically, we construct the gross return to financial wealth, R^f, as a weighted average of the country's equity (R^e) and corporate debt (R^d) gross constant dollar returns, where the weight μ_t reflects the share of corporate debt in the total value of the firm. These weights are estimated for each country using balance sheet data for non-financial firms from Compustat.[30] Our measure of returns to financial wealth for each country is then:

$$r^f_{t+1} \equiv \log(R^f_{t+1}) = \log\left[(1 - \mu_t)R^e_{t+1} + \mu_t R^d_{t+1}\right]. \tag{27}$$

Section 4.6 presents alternative estimates of R^f as robustness checks.

Consider next the return to nonfinancial wealth, R^n. In a dynamic context, that return differs from the growth rate of real nonfinancial income per capita $\Delta \ln W$: the latter represents only the dividend component and not the total return on the corresponding asset.[31] Measuring the total return on nonfinancial wealth is a difficult issue. We tackle it from a variety of perspectives. First, we follow the standard present-value method of Campbell and Shiller (1988), as detailed in Campbell (1996). Under the assumption that the dividend-price ratio on nonfinancial wealth is stationary, the return on that asset satisfies the following approximation:

$$r^n_{t+1} \equiv \ln\left(W_{t+1} + V^n_{t+1}\right) - \ln V^n_t = k + \phi^n_t - \rho\ \phi^n_{t+1} + \Delta w_{t+1}, \tag{28}$$

where V^n_t denotes nonfinancial wealth at time t, $\phi^n_t = \ln\left(W_t/V^n_t\right)$ is the log dividend-price ratio for nonfinancial wealth, ρ is a number slightly smaller than 1, k is an unimportant constant and we use lower case variables to denote logs.[32]

Using (28) to solve for ϕ_t forward, imposing the equilibrium condition that $\lim_{t\to\infty} \rho^t\left(r^n_t - \Delta w_t\right) = 0$, substituting back into equation (28), and taking conditional expectations, yields the usual present-value relationship:

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[29] One might worry that equilibrium equity positions might differ if firms are able to issue debt as well as equity. We show in Appendix A.3 that in the benchmark model where firms' financing decisions are irrelevant for the value of the firm, this is not the case. In this Modigliani-Miller limit case, the presence of corporate debt has no impact on equity portfolio decisions. In models with some departure from Modigliani-Miller, our results can remain valid as long as changes in the value accrued to debtholders versus shareholders does not interact with the fundamental shocks of the model.

[30] See appendix B.1 for details. The average share of debt in total liabilities is 67.1 percent (Canada), 75.2 percent (France), 75.3 percent (Germany), 76.2 percent (Italy), 70.7 percent (Japan), 59.2 percent (U.K.), 71.8 percent (U.S.). The country equity and corporate debt returns are obtained from the Global Financial Database. For Italy and Japan, corporate bond markets developed only in the late 1980s. We approximate the corporate debt return by using instead the holding return on long-term government debt.

[31] See Baxter and Jermann (1997, p. 175)

[32] One can show that $\rho = 1/\left(1 + \exp\left(\phi\right)\right)$ where ϕ is the steady state value of the log dividend-price ratio. We will use the value of $\rho = 0.98$ in line with standard estimates in the literature. Our results are robust to changes in the value of ρ.

$$r_{t+1}^n - E_t r_{t+1}^n = (E_{t+1} - E_t) \sum_{s=0}^{\infty} \rho^s \Delta w_{t+1+s} - (E_{t+1} - E_t) \sum_{s=1}^{\infty} \rho^s r_{t+1+s}^n. \qquad (29)$$

This expression makes clear that the innovation to the return on nonfinancial wealth on the left hand side of the equation depends positively upon revisions to the path of future expected real nonfinancial income growth—the cash flow component represented by the first summation on the right hand side—and negatively upon revisions to the path of future expected real returns—the discount rate component represented by the second summation on the right hand side.

Our approach consists in constructing the empirical counterpart of equation (29) for each country using a Vector-Auto-Regression (VAR) in first differences of the following form:[33]

$$\mathbf{Z}_{t+1} = \mathbf{A} \ \mathbf{Z}_t + \boldsymbol{\epsilon}_{t+1}$$

where $\mathbf{Z}_t = (\tilde{r}_t, \Delta w_t, \Delta k_t, \Delta \ln Q_t, \mathbf{x}_t')'$. In this expression, $\tilde{r}_{t+s}$ represents a possible proxy for the *expected* return on nonfinancial wealth at time $t + s$, in the sense that $E_t r_{t+s}^n = E_t \tilde{r}_{t+s}$. This proxy is necessary to construct the second summation on the right hand side of (29). In our benchmark approach, we set $\tilde{r} = r^f$, that is, we assume that expected financial and nonfinancial future returns are equal.[34] $\Delta w_t, \Delta k_t$ and $\Delta \ln Q_t$ denote respectively the rate of change of non-financial income, financial income and the real exchange rate. Finally, $\mathbf{x}_t$ denotes a vector of additional controls used to forecast future factor income growth and future returns.

Our VAR specification first-differences financial and non-financial income. We discuss in details in Appendix B.2 why this is the appropriate empirical specification. In short, we find that while we cannot reject the null hypothesis that w and k are integrated processes, we do not find any statistical evidence of a cointegration relationship between the two. This is an important point of departure from the earlier literature. For instance, Baxter and Jermann (1997) estimate a Vector Error Correction Mechanism (VECM) on financial and nonfinancial income, imposing the cointegration relationship that $k - w$ is stationary. This assumption is appealing on theoretical grounds since the share of financial income is bounded between 0 and 1. The null of cointegration is, however, strongly rejected in the data, indicating a very persistent process for income shares, with no apparent error-correction term.[35] Therefore, a stationary VAR in first-differences is appropriate.

Based on our reading of the literature on financial return predictability, we consider a comprehensive list of potential controls for future asset returns: consumption growth; the

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[33]Standard Akaike and Schwarz lag-selection criteria indicate that a VAR of order 1 is the preferred specification for all countries.

[34]Section 4.6 explores various alternatives. One such alternative is to set $\tilde{r} = r^b$, to capture the fact that returns to non financial wealth may have a risk-return profile closer to bonds than equities. Another is to follow Lustig and Nieuwerburgh (2008) and recover returns on nonfinancial wealth from the joint behavior of asset returns, nonfinancial income growth and consumption growth, under the assumption that aggregate consumption satisfies the first-order condition of an optimizing representative household.

[35]Moreover, as discussed in appendix B.2, the assumption that $k - w$ is stationary is also strongly rejected by the data.

dividend-price ratio; the relative T-bill rate (the difference between the yield on 3-month T-bill rate and a 4-quarter moving average); the term premium (the spread between 10 year and 3 months government yields); the yield spread (the spread between the yield on long-term corporate bonds and that on 10-year government bonds); *cay*, the fluctuations in US aggregate consumption-wealth ratio as measured by Lettau and Ludvigson (2001); and *nxa*, the Gourinchas and Rey (2007) measure of US external imbalances. In order to maintain a parsimonious and statistically significant representation, our selection of variables is as follows. First we exclude variables that appear integrated, based on Augmented-ADF tests, since this would violate our stationary VAR assumptions. Second, we select predictive variables based on the Least Angle Regression (LARS) approach of Efron, Hastie, Johnstone and Tibshirani (2004) applied to the financial return equation of the Vector Auto Regression. This selection algorithm efficiently selects a parsimonious subset of predictive variables. In our final specification, only one predictive variable remains: the term premium for the U.S and Italy.

With estimates of $\mathbf{A}$ and $\boldsymbol{\epsilon}_{t+1}$ in hand, the empirical counterpart to $r_{t+1}^n - E_t r_{t+1}^n$ can be obtained from (29) as:

$$r_{t+1}^n - E_t r_{t+1}^n = \left(\mathbf{e}'_{\Delta w} - \rho \mathbf{e}'_{\tilde{r}} \mathbf{A}\right) \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \boldsymbol{\epsilon}_{t+1}, \tag{30}$$

where $\mathbf{e}'_y$ is a row-vector that 'selects' variable y in $\mathbf{Z}$, i.e. such that $\mathbf{e}'_y \mathbf{Z} = y$. The first term, $\mathbf{e}'_{\Delta w} \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \boldsymbol{\epsilon}_{t+1}$, captures the contribution of expected future non financial income growth (the first summation in equation (29)). The second term, $-\rho \mathbf{e}'_{\tilde{r}} \mathbf{A} \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \boldsymbol{\epsilon}_{t+1}$, captures the contribution of expected future returns on nonfinancial wealth (the second summation in equation (29)). Figure 2 reports the return to nonfinancial wealth $r_{t+1}^n - E_t r_{t+1}^n$ for the U.S., together with the growth rate of nonfinancial income Δw. The correlation between the two series is high (0.66), but the striking fact is that the return innovation exhibits much more volatility.[36]

The last step consists in measuring bond, financial and non-financial returns relative to the rest of the world. To this effect, we define the relative returns $\hat{r}_i^l$ of country i as follows:

$$\hat{r}_{i,t+1}^l = \left(r_{i,t+1}^l - E_t r_{i,t+1}^l\right) - \sum_{j \neq i} \alpha_{ji} \left(r_{j,t+1}^l - E_t r_{j,t+1}^l\right),$$

for $l \in \{b, f, n\}$, where α_{ji} is the output weight of country j in the rest of the world outside of country i.

4.3 Estimating the loadings on the real exchange rate

We are now in a position to estimate the key loading parameters in equation (10). We begin with the loadings on the real exchange rate, $\beta_{Q,j}$ for $j = f, b$. These conditional moments can be estimated for each country by the following simple regression for each country i:

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[36]The standard deviation of the return innovations is 3.09% vs. 1.01% for nonfinancial income growth.

$$\Delta \ln Q_{i,t} - E_{t-1}\Delta \ln Q_{i,t} \equiv \mathbf{e}'_{\Delta\mathbf{q}}\boldsymbol{\epsilon}_{\mathbf{i,t}} = \beta^i_{Q,0} + \beta^i_{Q,b}\hat{r}^b_{i,t} + \beta^i_{Q,f}\hat{r}^f_{i,t} + u_{i,t}. \tag{31}$$

where $u_{i,t}$ captures the fluctuations in the real exchange rate that are not spanned by relative bond and financial returns.[37]

Results of regression (31) for each countries are displayed in Table 2. Our empirical results confirm the results of van Wincoop and Warnock (2010) for all the countries considered in the sample: relative bond returns capture most of the variations of the real exchange rate. The coefficient on the relative bond returns in panel A, $\beta_{Q,b}$ is often not statistically different from one, between 0.82 for the U.K and 1.01 for Japan. The R^2 of the regression is also very strong, between 0.86 for UK and 0.95 for France and Japan. Moreover, *conditional on bond returns*, the hedge ratio of financial returns for real exchange rate risk, $\beta_{Q,f}$ is almost never statistically different from zero.[38]

Panel B of the table reports the unconditional loading on the real exchange rate $\beta^u_{Q,f}$ obtained from a regression *only* on the relative financial return $\hat{r}^f$. The coefficients are significantly positive for all countries, between 0.38 (U.K.) and 0.73 (U.S.). This re-emphasizes the importance of properly conditioning on the relative bond returns. Finally, the last column of the table reports the results from a pooled regression with country fixed effects. This can be interpreted as an average loading for all G-7 countries. The estimates, $\beta_{Q,b} = 0.94$ and $\beta_{Q,f} = 0.01$ confirm the strong correlation between relative bond returns and real exchange rates.

4.4   Estimating the loading on the return to non-financial wealth

We now use the returns to non-financial wealth estimated for each country i to estimate the loadings of (relative) bond returns and (relative) returns to financial wealth by estimating the following equation:

$$\hat{r}^n_{i,t} = \beta^i_{n,0} + \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}, \tag{32}$$

where $v_{i,t}$ is attributed both to measurement error in the construction of the return on nonfinancial wealth, and to fluctuations in relative nonfinancial income risk not spanned by relative bond returns and relative returns to financial wealth.

Results of the regression (32) for each countries are shown in Table 3. Panel B reports the estimate of the unconditional loading factor $\beta^{i,u}_{n,f} = cov(\hat{r}^n_i, \hat{r}^f_i)/var(\hat{r}^f_i)$. This coefficient is positive and significant for all countries except Italy, with a pooled estimate of 0.41. This indicates that returns to non financial wealth are positively correlated with returns to

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[37]It is important to note how equation (31) differs from a standard test of uncovered interest rate parity (Fama (1984)). Denote $\check{r}^b_{t-1}$ the *ex-ante* real interest rate differential between $t-1$ and t, expressed in *local units*. Then $\hat{r}^b_t = \check{r}^b_{t-1} + \Delta q_t$ and the coefficient $\beta_{Q,b}$ will be close to 1 if most of the variation in ex-post real interest rate differential $\hat{r}^b_t$ comes from movements in the real exchange rate, regardless of whether uncovered interest rate parity holds. However, under uncovered interest rate parity, $\check{r}^b_{t-1} = -E_{t-1}[\Delta q_t]$ so that $\hat{r}^b_t = \Delta q_t - E_{t-1}\Delta q_t$ measures the innovation to the rate of depreciation and $\beta_{Q,b} = 1$.

[38]The exception is the U.K. but even in this case $\beta_{Q,f}$ remains economically very small, less than 7 percent.

financial wealth as in Baxter and Jermann (1997) and the international diversification puzzle is 'worse than you think' when using equities only.

However, the loading factor conditional on bond returns $\beta_{n,f}^i$ reported in panel A is negative and strongly significant for all countries, except Germany. It varies between -0.05 (Germany) and -0.55 (Italy) with a pooled estimate of -0.23. As the previous analysis emphasized, this negative conditional hedge ratio indicates that in all these countries domestic equities constitute a good hedge against shocks to non financial wealth.

Moreover, the positive loadings of (relative) bond returns $\beta_{n,b} > 0$ implies that shorting the local currency bond, and going long in the foreign currency bond, constitutes a good hedge against fluctuations in returns to non-financial wealth (see equation (12)). This is not surprising: in our model, a potentially large part of relative non-financial income comoves with the real exchange rate (see Figure 1), and we know that relative bond returns track almost perfectly the real exchange rate.

4.5   Implied equity and bond portfolios

The previous estimates allow us to back out the implied equity and bond positions using equations (12) when all countries are symmetric. Allowing for different country sizes, equation (12) must be rewritten as follows (see Appendix (A.4)):

$$
\begin{cases}
b^* &= (1 - \omega_i)\left(1 - \frac{1}{\sigma}\right)\beta_{Q,b} - (1 - \omega_i)(1 - \delta)\beta_{n,b} \\[2ex]
S^* &= \omega_i + (1 - \omega_i)\left(\frac{1 - \frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1 - \delta}{\delta}\beta_{n,f}\right)
\end{cases}
\tag{33}
$$

where ω_i is the relative size of country i in world market capitalization.

The implied equity bias and bond portfolios are summarized in table 4 using the loading coefficients from our baseline estimates. As equation (33) indicates, the optimal bond position requires an estimate of the degree of risk aversion σ. We consider the plausible value of $\sigma = 2$ in our benchmark calibration. For the share of financial income δ, we use the average share of financial income across G7 countries in the more recent period (2000-2008): $\delta = 0.191$.

The model is very successful at predicting a significant degree of equity home bias for all countries when bond trading is allowed. Consider first panel B, which excludes bonds, as commonly done in the literature. The baseline refers to the first term in equation (33), that is, a predicted portfolio share equal to the share in world market capitalization ω_i. The second term (Bias due to Q) reflects the contribution of the real exchange rate hedging component: $(1 - \omega_i)(1 - 1/\sigma)\beta_{Q,f}^{i,u}/\delta$. Given the positive unconditional correlation between financial returns and exchange rates ($\beta_{Q,f}^{i,u} > 0$ in table 2), this term is positive, indicating a potential source of home bias. The second term (Bias due to r^n) reflects the contribution of the non-financial income hedging component: $-(1 - \omega_i)(1 - \delta)\beta_{n,f}^{i,u}/\delta$. Since $\beta_{n,f}^{i,u}$ is strongly positive (see table 3), this term contributes negatively to the optimal equity portfolio and dominates the real exchange rate hedge. The result, as in Baxter and Jermann (1997) is a

strong predicted foreign bias, $S^i - \omega_i$ ranging from -8.6 percent for France to -91.5 percent for Germany, in total contrast to the data.[39]

By contrast, Panel A shows that the estimated model accounts for a large share of observed equity home bias once bond trading is allowed. The hedge portfolio is now dominated by the non-financial income component. This term is strongly positive since $\beta_{n,f}^i < 0$ in table 3. The predicted equity portfolio (S) is 29% for Germany, between 59% and 101% for Canada, Japan, U.K. and the U.S. and quite above 100% for France and Italy.[40] Available empirical evidence indicates a home equity position between 55% (Germany) and 85.6% (Canada).[41] Except for Germany, the equity bias predicted by the model is comparable to the amount of bias in the data. Using $\beta_{Q,f}$ and $\beta_{n,f}$ estimated on pooled data for all countries, we get equity portfolios close to 90% for all countries, also fairly close to the data.

The last line (ΔS) reports the change in the predicted equity position between the equity only and the full model. In all cases, the predicted equity position increases substantially, moving the model closer to the data. For instance, in the case of the U.S., in the model with equity only, investors should have a strong foreign bias ($S = 12\%$) while the full model predicts 101% domestic equity holdings, much closer to the empirical estimate (83.2%).

Panel A also reports the model predictions for bond holdings. As for equities, we can decompose the predicted bond position into a real exchange rate hedge component (($1 - \omega_i)(1 - 1/\sigma)\beta_{Q,b}$) and a non financial income component ($-(1 - \omega_i)(1 - \delta)\beta_{n,b}$).[42] We find a strong positive demand for local currency bonds arising from real exchange rate hedging, given the positive loading factor $\beta_{Q,b}$, but an even stronger and negative loading factor for hedging non-financial income risk, given $\beta_{n,b}$.[43] While each of these component can be large relative to output, they offset each other and imply net currency exposure of bond portfolios of reasonable magnitude. Thus, the model predicts that countries should issue bond liabilities in their own currency, between 19 percent (US) and 54 percent (Italy) of their domestic output. Data regarding the net currency exposure of debt positions from Lane and Shambaugh (2010) suggests that G7 countries are on average short in domestic currency (and long in foreign currency) although the positions are smaller than those predicted by the model. The average net currency bond exposure is $b = -6.3\%$ of GDP over 2000-2004.[44] There is some heterogeneity across countries: while US, UK, Japan and Italy are short in

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[39]The exception is Italy, where the unconditional loading $\beta_{n,f}^{\mathrm{u}}$ is insignificant and therefore the model predicts more home bias than observed.

[40]The results for Italy are perhaps to be taken with some caution since we imputed the return on Italian T-bills for the return on corporate bonds.

[41]Data are from Coeurdacier and Rey (2013).

[42]The baseline is zero for the bond position.

[43]This term would only grow stronger relative to the real exchange rate hedge if we decrease the coefficient of risk aversion σ. In the limit where $\sigma = 1$, the real exchange rate hedge component disappears.

[44]In the data, countries often have leveraged external debt position. The counterpart of b in the model is $(b_{HH} - b_{HF})/2$ where b_{HH} denotes the net domestic currency debt exposure, that is, the difference between domestic currency denominated debt assets and domestic currency denominated debt liabilities– and b_{HF} denotes the net foreign currency debt exposure, that is, the difference between foreign currency debt assets and foreign currency debt liabilities. This counterpart generates the same wealth transfer towards a country whose currency depreciates by 1% with respect to all other currencies as in our model.

domestic currency, Germany, France and Canada are long. Our empirical counterpart of b ranges from -16.4% of GDP for the UK to 9.8% for France. Overall, it is fair to say that the fit of the benchmark model in terms of bond portfolios is less impressive. As direct inspection of equation (33) shows, for higher values of σ the hedging of real exchange rate becomes progressively more important, reducing the magnitude of the bond positions.

4.6 Using Different Measures of Returns to Financial and Non-Financial Wealth

A key element of our analysis is the construction of returns to financial and non financial wealth r^f and r^n. If these returns are incorrectly measured, one should be cautious when interpreting the loading factors and predicted portfolios. This section investigates the robustness of our results to various alternative measures of financial and non financial returns.

A first point of departure would be to construct returns to financial wealth using the same approach as for non financial returns, with national income data as in Baxter and Jermann (1997). This approach yields the following expression for the return to financial wealth:

$$r_{t+1}^f - E_t r_{t+1}^f = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta k_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+s}^f \qquad (34)$$

Using the same VAR specification as in section (4.2.2), the empirical estimates of the returns to financial wealth becomes:[45]

$$r_{t+1}^f - E_t r_{t+1}^f = (\mathbf{e}_{\Delta k}' - \rho \mathbf{e}_{\tilde{r}}' \mathbf{A}) (\mathbf{I} - \rho \mathbf{A})^{-1} \boldsymbol{\epsilon}_{t+1}. \qquad (35)$$

The returns on the firm thus obtained may be noisy and imperfectly estimated. Our second approach instruments the returns in equation (35) with the country's equity and corporate debt returns, forcing the weights to sum to one. This is equivalent to choosing different weights $\hat{\mu}$ in equation (27), measuring the leverage implied by national accounts data according to a first stage regression:

$$r_t^f = (1 - \hat{\mu}) r_t^e + \hat{\mu} r_t^d + \nu_t. \qquad (36)$$

The predicted component $\left((1 - \hat{\mu}) r_t^e + \hat{\mu} r_t^d\right)$ of (36) becomes our proxy for returns to financial wealth. This method identifies the variations in financial wealth estimated from national accounts that are reflected in market returns and is therefore potentially more robust to measurement error.

A third approach approach simply sets $\mu = 0$, equating the return to financial wealth with observed equity returns:

$$r_t^f = r_t^e \qquad (37)$$

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[45]The implementation still requires the use of observed market returns to form expectations of future returns. In practice, we use the returns on the firm constructed in the previous section as a proxy.

This approach has the merit of simplicity, but as argued earlier, there are good theoretical reasons why equity returns may differ from the returns to the firm.[46]

Figure 3 reports the innovations to financial returns under these alternative measures for the United States. As can be seen from the figure, the return innovations tend to be positively correlated.[47] The equity return is also the more volatile, with a standard deviation of 8% per quarter against 3.5% for the projected NIPA return.

Lastly, we also consider three different approaches to constructing returns to non financial wealth. The first one assumes that non-financial wealth is discounted using the holding return on long term government bonds, denoted r^{lb}. We follow the exact same methodology as in our benchmark estimates but set $\tilde{r} = r^{lb}$ to construct estimates of returns to non-financial wealth.[48]

The second approach borrows from Lustig and Nieuwerburgh (2008). The basic idea is to recover the unobserved innovation to non financial wealth from the joint behavior of consumption and market returns, under the assumption that aggregate consumption satisfies the first-order condition of an optimizing representative household.[49]

Results from regressions (31) and (32) are displayed in table 5 and table 6 for the different specifications and the different countries. Our empirical results confirm the previous results across all specifications: relative bond returns capture most of the variations of the real exchange rate and claims on financial income are not used to hedge real exchange rate changes (see table 5). Moreover, *conditional on bond returns*, the loadings of non-financial wealth on financial wealth are negative across all specifications and significantly so for most of the countries, implying home bias in our model (see table 6). This confirms the important role of bond holdings as an hedging instrument. Hence, qualitatively, results using these alternative measures of returns are very similar to our benchmark case.

Quantitatively, the magnitude of the loadings $\beta_{n,f}^i$ in table 6 are similar to our benchmark case when using the projection of financial returns estimated from national accounts on market returns (panel B, the pooled estimate of $\beta_{n,f}^i$ is equal to -0.17), when using long term government bond returns to discount non-financial wealth (panel D, pooled estimate of $\beta_{n,f}^i$ equal to -0.24), or when using the method of Lustig and Nieuwerburgh (2008) (panel E, pooled estimate of $\beta_{n,f}^i$ equal to -0.19). As reported in table 7, under all of these specifications, the amount of equity home bias generated by our estimates are in line with the home bias data for most countries.

The results are marginally weaker when using national accounts data (panel A) or equity returns (panel C). More generally, one could also argue that these are noisier measure of

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[46]We also set $\tilde{r} = r^e$ in the VAR and re-run the LARS algorithm to select predictive variables. The following variables are added to the VAR: yield spread (Germany and U.S.) and relative T-bill (U.S.).

[47]The NIPA returns tend to have the lowest correlation with the other variables, between 0.15 and 0.21. All other measures are strongly correlated.

[48]In that specification, the following variables are added to the VAR according to our LARS algorithm: term-premium (Italy and the U.S.), consumption growth (Japan, U.S.), relative T-bill (UK) and yield spread (U.S.).

[49]See appendix B.2 for details.

financial returns causing attenuation bias on our estimates of the loadings. When using equity returns, the pooled estimates of $\beta_{n,f}^i$ is equal to -0.08 (panel C), roughly 40% of the value of our benchmark. Hence in this specification, the model can still explain a significant share of equity home bias (around 40%; see table 7). The estimation using national account data to estimate returns to financial wealth performs qualitatively similarly as our benchmark, except for Italy.[50] When looking at the US more specifically, table 7 indicates that the equity portfolio implied by the model are respectively 82% of domestic equity when using national account data and 70% when using equity returns, only slightly below the measured ones.[51]

As a final check, we consider the relative importance of the cash flow and discount components in equation (29). Unlike our benchmark result, Benigno and Nistico (2011) find that, for the U.S., returns to non-financial wealth are largely uncorrelated with financial returns, even after controlling for bond returns. Their approach ignores the contribution of revisions to the path of future expected real returns to the return on non-financial wealth –the second term in equation (29). Conceptually, it is not clear why one would wish to assume that the expected return to non-financial wealth remains constant given the large body of evidence on time-varying asset returns. Further, as the robustness checks presented above illustrates, our results are robust to many plausible alternative assumptions regarding expected future non-financial returns (equal to expected financial return, expected government bond return, or determined by consumption innovations). Lastly, we do find that our results are qualitatively robust to the restriction that expected non-financial returns are constant. Setting the second summation in (29) equal to zero, we find that the conditional loading of non-financial returns on financial ones remains negative and significant for most countries, although not the US or Germany, accounting perhaps for the findings in Benigno and Nistico (2011).[52]

5   Conclusion

What drives equity home bias? This paper merges and improves upon two strands of literature. The first strand focused on risks to non-financial wealth. It concluded that home equity positions should be even more tilted towards foreign equity since non financial and financial returns appeared positively correlated. The second strand looked at frictions in goods markets and emphasized real exchange rate risks. In this class of models, efficient risk sharing requires holding securities delivering high returns when the domestic currency appreciates. However, the correlation between stock returns and exchange rates is too low to generate significant portfolio biases. This class of models has thus been challenged by its lack of empirical support.

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[50]In panel A, Italy is an obvious outlier with $\beta_{n,f} = 0.51$. However, recall for that country, we do not have a good measure of corporate returns which affects the way non financial wealth is discounted. When dropping Italy from our pooled estimation, $\beta_{n,f}^i$ is equal to -0.1 and highly significant.

[51]Like in our benchmark regression, the unconditional loadings (non-reported) for these two specifications are positive and highly significant ($\beta_{n,f}^{i,u} > 0$) implying a very large foreign bias in the model without bonds.

[52]The estimates vary between 0.004 for Germany and -0.20 for France. Results available upon request from the authors.

This paper shows that both strands of the literature are related, but incomplete. It starts from the observation that relative bond returns (nominal or real) are strongly correlated with real exchange rates. It follows that, in a world where investors can trade both equities and bonds, they will hedge real exchange rate risk with the latter. And once this is achieved, the equilibrium equity position will be a function of the residual risks that investors face, namely the risk to their non-financial wealth, *conditional* on bond returns. Equity home bias will arise if non financial risk is negatively correlated with equity returns, after controlling for bond returns. The paper derives this prediction in a fairly general model and characterizes equilibrium portfolios as a simple function of hedge ratios that can easily be estimated from data on real exchange rates and returns on bonds, financial and non-financial wealth. We implement this empirical strategy for the countries of the G-7 and show that under many reasonable specifications, the conditional correlation between financial and non-financial returns is such that it can empirically account for a significant share of the observed equity home bias. For most countries, the conditional correlation between financial and non financial returns is negative and economically significant. In other words, the international diversification puzzle is not so puzzling anymore! The model also makes predictions about equilibrium bond positions. Here, although we find an implied currency exposure of bond portfolios broadly in line with the empirical evidence, the performance of the model is not as good for international bond portfolios.

It is possible to interpret our results in a broader perspective. Nominal exchange rates present a deep source of puzzles in international finance. They are too volatile and largely uncorrelated with their fundamental determinants — the exchange rate disconnect puzzle. To the extent that nominal exchange rate movements drive real exchange rate fluctuations, real exchange rates too, do not behave as predicted in our models —the Mussa (1986) puzzle. For instance, relative real consumption is not correlated with real exchange rate movements as models of risk sharing predict—the Backus and Smith (1993) puzzle. In the context of international portfolios, this implies that real exchange rates fluctuations are both uncorrelated with relative financial returns, and that relative financial and non-financial returns are positively correlated, since a given change in the nominal exchange rate affects both returns in the same direction. Our paper shows that, once currency fluctuations are controlled for through the use of nominal or real bonds, the structure of international equity portfolios conforms to the predictions of standard portfolio models. This provides a qualified success for the theory, since an empirically successful theory of exchange rate fluctuations remains elusive.

We left open an obvious step for future research. One would want to go back and enrich/discriminate among existing models to fully account for the hedge ratios we obtain from the data. Such a model would be consistent both with observed portfolios (quantities) and with their corresponding loadings, i.e the covariance structure of exchange rates and asset returns (prices). Going from the reduced form estimates to the structural parameters of the model requires taking a stand on the 'correct' model of the economy. A full-fledged structural estimation lies beyond what we attempt in this paper.

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	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Euro	Average
δ	16.4	14.1	13.1	25.4	16.1	18.5	13.3	17.9	16.7
naïve-δ	39.9	39.9	40.5	50.9	43.5	36.7	37.8	42.9	41.3

Table 1: Estimates of the share of financial income in output δ (in percent), defined as the share of financial income $(\Pi + D + (1 - \lambda) M - I)$ in output at product prices net of investment $(Y - T - I)$. The naïve share is estimated as one minus the share of compensation of employees $(COMP)$ in output at factor prices $(Y - T)$. Source: OECD Quarterly National Income and U.N. National Account Statistics. Authors' calculations.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Panel A: Conditional Loadings				
$\beta_{Q,f}$	-0.036	-0.011	0.007	-0.017	-0.030	0.064***	-0.013	0.006
(s.e.)	(0.025)	(0.024)	(0.026)	(0.018)	(0.028)	(0.022)	(0.038)	(0.009)
$\beta_{Q,b}$	1.003***	0.944***	0.946***	0.969***	1.012***	0.821***	0.944***	0.942***
(s.e.)	(0.033)	(0.028)	(0.031)	(0.026)	(0.034)	(0.039)	(0.040)	(0.012)
R^2	0.941	0.947	0.940	0.944	0.947	0.863	0.918	0.929
				Panel B: Unconditional Loadings				
$\beta_{Q,f}^u$	0.579***	0.591***	0.616***	0.447***	0.658***	0.376***	0.733***	0.554***
(s.e.)	(0.040)	(0.043)	(0.043)	(0.042)	(0.040)	(0.034)	(0.048)	(0.016)
R^2	0.579	0.557	0.573	0.424	0.641	0.453	0.611	0.535
Obs.	153	153	153	153	153	153	153	1071

Table 2: Loadings on **real exchange rate changes**: $\Delta \ln Q_{i,t} - E_0 \Delta \ln Q_{i,t} = \beta_{Q,b}^i \hat{r}_{i,t}^b + \beta_{Q,f}^i \hat{r}_{i,t}^f + u_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Unconditional loadings impose $\beta_{Q,b} = 0$. Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Panel A: Conditional Loadings				
$\beta_{n,f}$	-0.186***	-0.327***	-0.053	-0.551***	-0.171***	-0.081**	-0.252***	-0.227***
(s.e.)	(0.072)	(0.057)	(0.062)	(0.098)	(0.053)	(0.036)	(0.099)	(0.026)
$\beta_{n,b}$	1.262***	1.122***	1.073***	1.295***	0.970***	0.967***	1.073***	1.096***
(s.e)	(0.094)	(0.069)	(0.075)	(0.140)	(0.065)	(0.062)	(0.103)	(0.034)
R^2	0.709	0.719	0.759	0.366	0.769	0.706	0.595	0.600
				Panel B: Unconditional Loadings				
$\beta_{n,f}^u$	0.588***	0.389***	0.637***	0.068	0.489***	0.286***	0.595***	0.411***
(s.e.)	(0.064)	(0.060)	(0.060)	(0.089)	(0.046)	(0.043)	(0.074)	(0.024)
R^2	0.362	0.219	0.429	0.004	0.428	0.223	0.300	0.213
Obs.	153	153	153	153	153	153	153	1071

Table 3: Loadings on **nonfinancial returns**: $\hat{r}_{i,t}^n = \beta_{n,b}^i \hat{r}_{i,t}^b + \beta_{n,f}^i \hat{r}_{i,t}^f + v_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Unconditional loadings $\beta_{n,f}^u$ impose $\beta_{n,b} = 0$. Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
			Panel A: With Bonds and Equity				
Equity							
Baseline (Market Cap Weights)	5.13	7.30	5.67	3.30	15.71	12.35	50.53
Bias due to:							
Q	-8.96	-2.72	1.77	-4.25	-6.64	14.59	-1.64
r^n	74.56	128.36	21.27	225.88	61.01	30.25	52.76
Total (S)	70.73	132.95	28.71	224.93	70.08	57.18	101.66
Data for (S) (2000-2008)	85.60	71.40	55.40	59.50	84.30	65.20	83.20
Bond							
Bias due to:							
Q	47.58	43.74	44.59	46.85	42.65	35.99	23.35
r^n	-96.85	-84.11	-81.85	-101.33	-66.14	-68.55	-42.92
Total (b)	-49.27	-40.37	-37.26	-54.48	-23.49	-32.56	-19.57
Data for (b) (2000-2004)	9.30	9.90	8.90	-2.70	-12.70	-16.40	-10.90
			Panel B: Equities Only				
Baseline (Market Cap Weights)	5.13	7.30	5.67	3.30	15.71	12.35	50.53
Bias due to:							
Q	145.25	142.53	143.62	108.09	144.20	90.82	104.86
r^n	-238.74	-151.72	-240.46	-26.71	-173.30	-111.90	-137.79
Total (S)	-89.35	-1.29	-85.91	88.97	-12.77	-13.33	12.44
ΔS	160.08	134.24	114.62	135.96	82.84	70.52	89.22

Table 4: Implied Portfolio Equity (S) and bond (b) position for G7 countries. Calculations are done under the assumption that $\delta = 0.19$ and $\sigma = 2$. (S) refers to the percentage of domestic stocks held by domestic residents (data for (S) are averaged over the period 2000-2008). ΔS refers to the difference between the implied S in a model with bonds and equity and the implied S with equities only. (b) refers to the net domestic currency exposure of bond portfolios (as a % of GDP). Data for (b) are computed from Lane and Shambaugh (2010) and refers to the average between net debt assets in domestic currency and net debt liabilities in foreign currency as a % of GDP (averaged over 2000-2004): $b = \frac{b_{HH} - b_{HF}}{2}$.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
			Panel A:	Financial returns estimated using national accounts				
$\beta_{Q,f}$	0.011	0.017	0.037	0.051	0.032	0.047***	0.000	0.027
(s.e.)	(0.010)	(0.010)	(0.014)	(0.012)	(0.013)	(0.019)	(0.019)	(0.005)
$\beta_{Q,b}$	0.954***	0.921***	0.911***	0.923***	0.953***	0.846***	0.934***	0.924***
(s.e.)	(0.022)	(0.019)	(0.025)	(0.019)	(0.022)	(0.035)	(0.027)	(0.009)
R^2	0.941	0.948	0.942	0.950	0.949	0.862	0.918	0.931
			Panel B:	Projection of returns from panel A on market returns				
$\beta_{Q,f}$	-0.052**	-0.021	-0.003	-0.048***	-0.044	0.040	-0.004	-0.019
(s.e.)	(0.028)	(0.029)	(0.026)	(0.019)	(0.033)	(0.033)	(0.043)	(0.011)
$\beta_{Q,b}$	1.020***	0.952***	0.955***	1.006***	1.026***	0.842***	0.938***	0.967***
(s.e.)	(0.035)	(0.032)	(0.033)	(0.028)	(0.039)	(0.051)	(0.046)	(0.014)
R^2	0.941	0.947	0.939	0.946	0.948	0.857	0.918	0.929
			Panel C:	Financial returns based on equity returns				
$\beta_{Q,f}$	-0.003	0.003	0.008	0.011	-0.015	0.040***	-0.007	0.006
(s.e.)	(0.012)	(0.009)	(0.012)	(0.008)	(0.011)	(0.014)	(0.018)	(0.004)
$\beta_{Q,b}$	0.971***	0.935***	0.939***	0.946***	0.996***	0.878***	0.943***	0.946***
(s.e.)	(0.022)	(0.019)	(0.022)	(0.020)	(0.023)	(0.030)	(0.026)	(0.009)
R^2	0.945	0.953	0.935	0.941	0.944	0.889	0.924	0.932
			Panel D: Non-financial returns using bond return discounting: $\tilde{r} = r^b$					
$\beta_{Q,f}$	-0.030	0.001	0.007	-0.018	-0.033	0.053**	-0.030	0.001
(s.e.)	(0.024)	(0.024)	(0.028)	(0.018)	(0.030)	(0.026)	(0.037)	(0.010)
$\beta_{Q,b}$	1.002***	0.929***	0.939***	0.971***	1.006***	0.769***	0.964***	0.937***
(s.e.)	(0.031)	(0.029)	(0.034)	(0.026)	(0.037)	(0.044)	(0.039)	(0.013)
R^2	0.947	0.944	0.931	0.945	0.939	0.807	0.924	0.921
			Panel E: Non-financial returns estimated using Lustig and Nieuwerburgh (2008)					
$\beta_{Q,f}$	-0.027	-0.017	0.017	-0.008	-0.030	0.072***	-0.013	0.011
(s.e.)	(0.028)	(0.024)	(0.028)	(0.018)	(0.030)	(0.023)	(0.038)	(0.010)
$\beta_{Q,b}$	0.980***	0.947***	0.925***	0.958***	1.005***	0.802***	0.945***	0.931***
(s.e.)	(0.037)	(0.029)	(0.034)	(0.026)	(0.036)	(0.040)	(0.040)	(0.013)
R^2	0.927	0.944	0.927	0.945	0.940	0.855	0.918	0.923
Obs.	153	153	153	153	153	153	153	1071

Table 5: Loadings on **real exchange rate changes** for alternative measures of returns: $\Delta \ln Q_{i,t} - E_0 \Delta \ln Q_{i,t} = \beta_{Q,b}^i \hat{r}_{i,t}^b + \beta_{Q,f}^i \hat{r}_{i,t}^f + u_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
			Panel A: Financial returns estimated using national accounts					
$\beta_{n,f}$	-0.093***	-0.041	-0.074**	0.506***	-0.118***	-0.027	-0.149***	-0.004
(s.e.)	(0.027)	(0.026)	(0.034)	(0.062)	(0.025)	(0.031)	(0.047)	(0.015)
$\beta_{n,b}$	1.156***	0.847***	1.104***	0.459***	0.901***	0.900***	0.981***	0.877***
(s.e.)	(0.061)	(0.052)	(0.059)	(0.100)	(0.041)	(0.056)	(0.070)	(0.027)
R^2	0.718	0.662	0.765	0.470	0.786	0.698	0.603	0.572
			Panel B: Projection of financial returns from panel A on market returns					
$\beta_{n,f}$	-0.147**	-0.344***	-0.069	-0.215**	-0.226***	-0.098	-0.196	-0.172***
(s.e.)	(0.081)	(0.071)	(0.062)	(0.111)	(0.063)	(0.052)	(0.113)	(0.031)
$\beta_{n,b}$	1.221***	1.130***	1.093***	0.998***	1.028***	0.998***	1.037***	1.051***
(s.e.)	(0.102)	(0.079)	(0.078)	(0.166)	(0.074)	(0.081)	(0.119)	(0.039)
R^2	0.702	0.703	0.759	0.251	0.773	0.703	0.585	0.584
			Panel C: Financial returns based on equity returns					
$\beta_{n,f}$	-0.109***	-0.053***	0.014	-0.125***	-0.076***	-0.028	-0.099**	-0.080***
(s.e.)	(0.043)	(0.020)	(0.033)	(0.023)	(0.026)	(0.026)	(0.049)	(0.012)
$\beta_{n,b}$	1.287***	1.032***	1.168***	1.240***	0.375***	0.995***	0.926***	0.952***
(s.e.)	(0.079)	(0.044)	(0.058)	(0.058)	(0.053)	(0.055)	(0.071)	(0.025)
R^2	0.678	0.805	0.762	0.751	0.256	0.726	0.571	0.600
			Panel D: Nonfinancial returns using bond returns discounting: $\tilde{r} = r^b$					
$\beta_{n,f}$	-0.148***	-0.289***	-0.100	-0.590***	-0.259***	-0.079	-0.298***	-0.245***
(s.e.)	(0.074)	(0.083)	(0.071)	(0.120)	(0.060)	(0.061)	(0.095)	(0.032)
$\beta_{n,b}$	1.076***	0.951***	0.917***	1.076***	1.073***	0.981***	1.046***	1.012***
(s.e.)	(0.096)	(0.100)	(0.085)	(0.171)	(0.074)	(0.106)	(0.099)	(0.041)
R^2	0.634	0.459	0.612	0.212	0.732	0.457	0.577	0.451
			Panel E: Nonfinancial returns estimated using Lustig and Nieuwerburgh (2008)					
$\beta_{n,f}$	-0.172***	-0.218***	-0.122***	-0.204***	-0.216***	-0.199***	-0.179***	-0.191***
(s.e.)	(0.042)	(0.037)	(0.052)	(0.037)	(0.037)	(0.028)	(0.040)	(0.014)
$\beta_{n,b}$	1.084***	1.141***	0.985***	1.163***	1.165***	1.113***	1.124***	1.116***
(s.e.)	(0.055)	(0.044)	(0.063)	(0.053)	(0.046)	(0.048)	(0.041)	(0.018)
R^2	0.836	0.884	0.765	0.814	0.906	0.815	0.920	0.853
Obs.	153	153	153	153	153	153	153	1071

Table 6: Loadings on **nonfinancial returns** for alternative measure of returns: $\hat{r}^n_{i,t} = \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
	Implied Equity (S) under alternative estimation methods						
Baseline (Market Cap Weights)	5.13	7.30	5.67	3.30	15.71	12.35	50.53
Benchmark estimates	70.73	132.95	28.71	224.93	70.08	57.18	101.66
National Accounts	45.54	27.57	44.34	-191.01	64.94	33.08	81.77
Projection of financial returns	51.19	137.32	32.70	79.24	86.95	58.09	91.05
Equity returns	48.26	28.93	1.99	57.33	39.62	31.96	70.30
Bond returns discounting	57.43	121.10	47.52	240.61	100.76	53.90	109.01
Method of Lustig et al (2008)	67.46	88.67	58.63	84.81	86.32	102.79	86.39
Data for (S) (2000-2008)	85.60	71.40	55.40	59.50	84.30	65.20	83.20
	Implied Bond (b) under alternative estimation methods						
Benchmark estimates	-49.27	-40.37	-37.26	-54.48	-23.49	-32.56	-19.57
National Accounts	-18.43	-29.79	-17.49	-50.01	-26.99	-20.56	-13.11
Projection of financial returns	-45.32	-40.56	-38.34	-29.40	-26.85	-33.84	-18.31
Equity returns	-52.67	-34.02	-44.87	-51.26	16.39	-32.07	-13.71
Bond returns discounting	-35.04	-28.27	-25.68	-37.27	-30.77	-35.86	-17.99
Method of Lustig et al (2008)	-36.71	-41.62	-31.52	-44.69	-37.10	-43.78	-21.61
Data for (b) (2000-2004)	9.30	9.90	8.90	-2.70	-12.70	-16.40	-10.90

Table 7: Implied Portfolio Equity (S) and bond (b) position for G7 countries under alternative methods to compute financial and non-financial returns. Calculations are done under the assumption that $\delta = 0.19$ and $\sigma = 2$. (S) refers to the percentage of domestic stocks held by domestic residents (data for (S) are averaged over the period 2000-2008). (b) refers to the net domestic currency exposure of bond portfolios (as a % of GDP). Data for (b) are computed from Lane and Shambaugh (2010) and refers to the average between net debt assets in domestic currency and net debt liabilities in foreign currency as a % of GDP (averaged over 2000-2004): $b = \frac{b_{HH} - b_{HF}}{2}$.
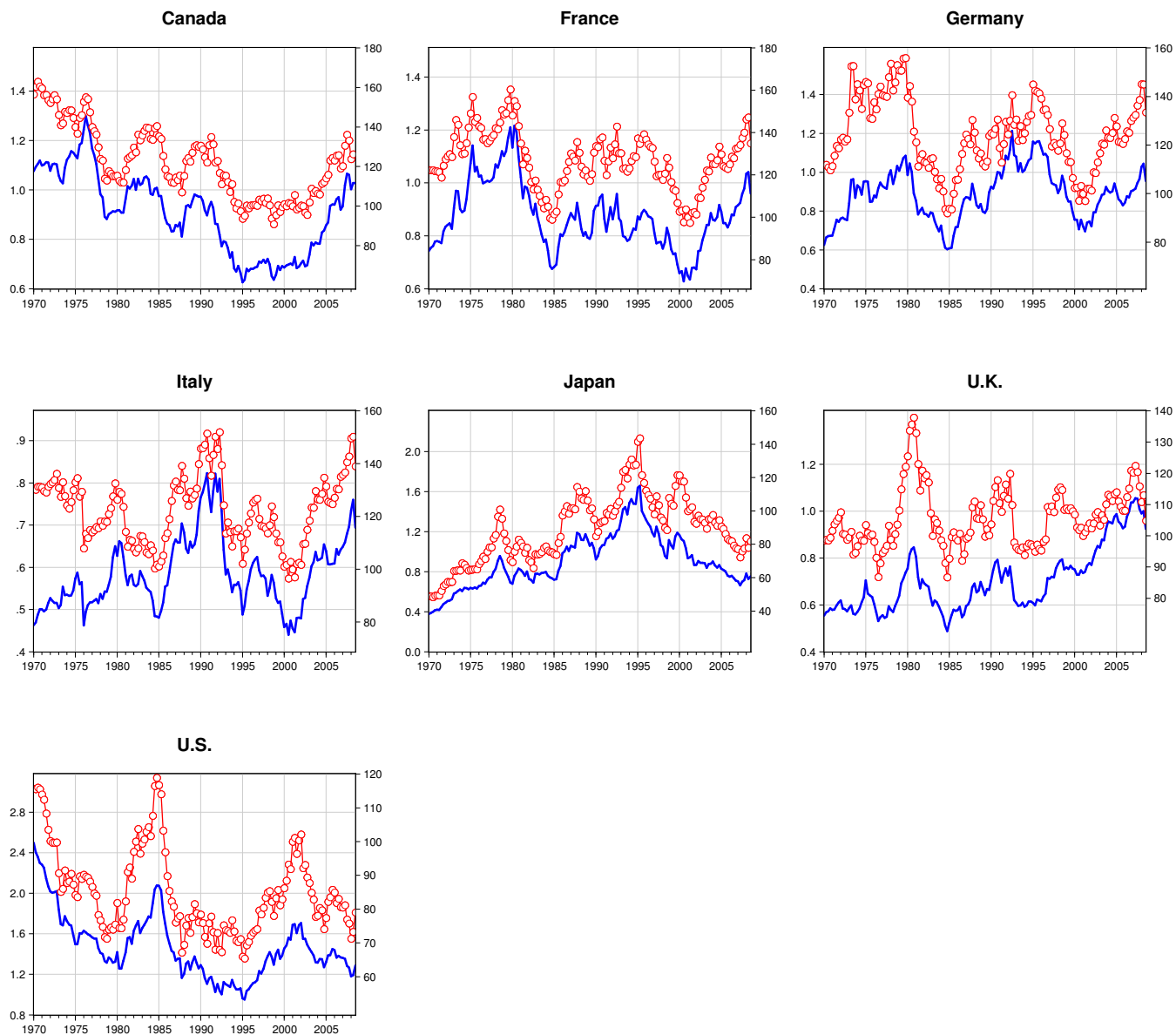
Figure 1: Relative nonfinancial income (left) [–] and real exchange rate [-o-] (100 in 2001Q1, right), G7 countries, 1970:1-2008:3. Data Sources: Global Financial Database, OECD Quarterly National Accounts and UN National Account Statistics. Authors' calculations.
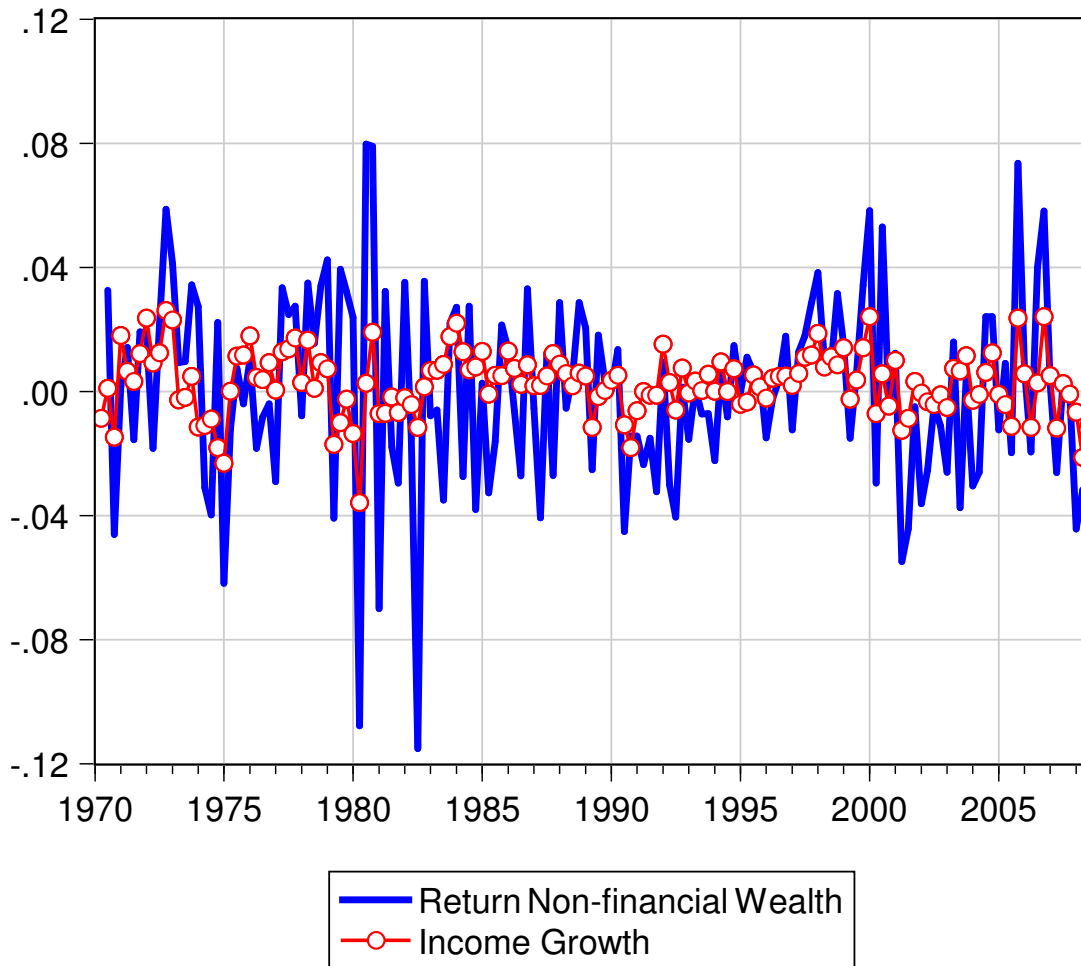
Figure 2: Innovations to returns on nonfinancial wealth $r^n_{t+1} - E_t r^n_{t+1}$, and nonfinancial income growth Δw, United States, 1970:1-2008:3.
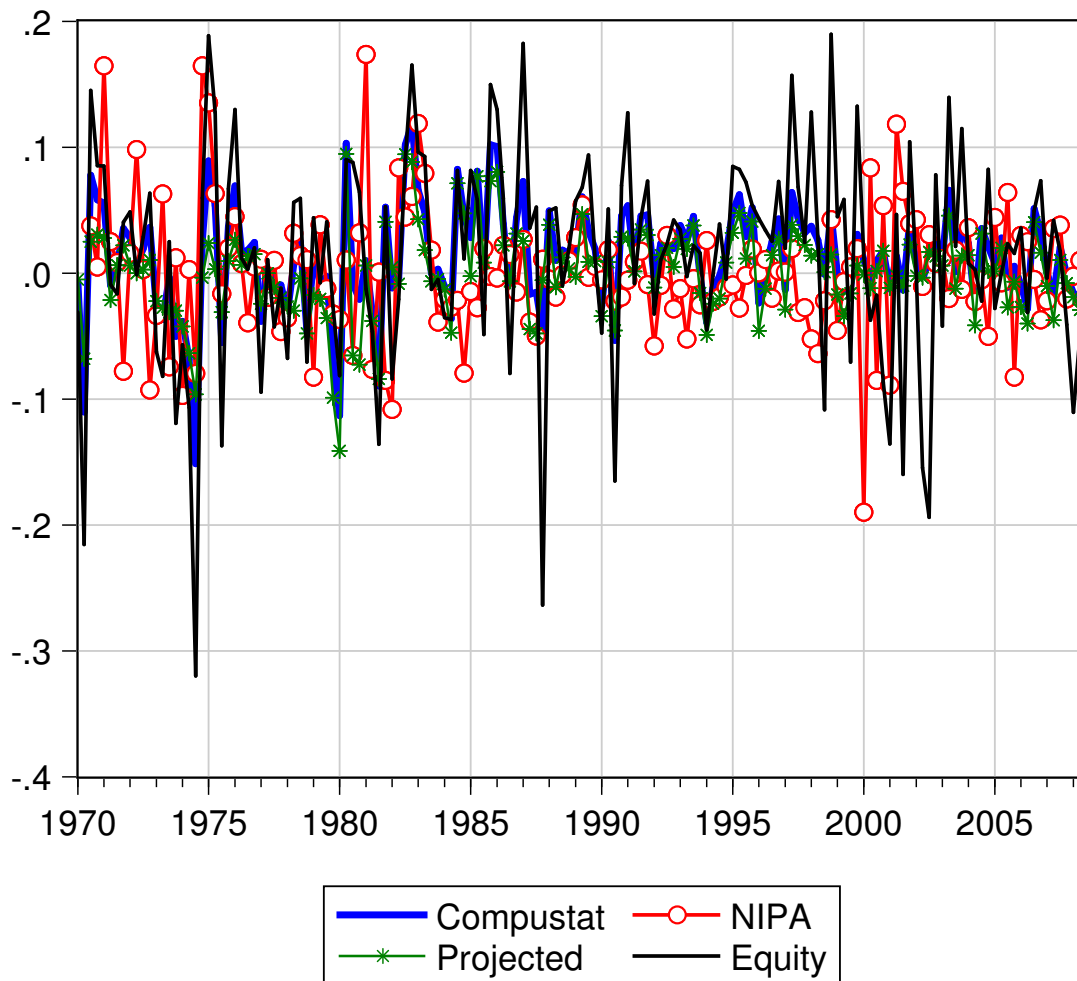
Figure 3: Innovations to returns on financial wealth. Compustat: weighted average of equity and corporate bond returns using share of debt in total assets measured from Compustat; NIPA: innovation to financial return constructed using equation (34); Projected: regresses NIPA returns on equity and corporate bond returns; Equity: S&P-500 Total Return Index. United States, 1970:1-2008:3.

Appendices

A Theoretical Derivations

A.1 Optimal portfolios in the benchmark model

We use Jonesian hats ($\hat{x} \equiv log(x/\bar{x})$) to denote the log-deviation of a variable x from its mean value $\bar{x}$. Variables x without country indices denotes difference across countries: $\hat{x} = \hat{x}_H - \hat{x}_F$. We denote Δx for first-differences : $\Delta \hat{x} = \hat{x}_1 - \hat{x}_0$.

We assume that countries are symmetric ex-ante, that is $E_{-1}y_{i,t} = \bar{y}$. However, as of time 0, countries can have different expected growth rates. That is, we allow $E_0 y_{i,1}$ to differ across countries. Appendix (A.4) considers the case of ex-ante asymmetries in sizes.

We apply a similar method to Devereux and Sutherland (2011) (see also Tille and van Wincoop (2010)) to characterize equilibrium portfolios. This method relies on deriving:

1. First-order approximation for non-portfolio equations,

2. Second-order approximation of the Euler equations.

Non-portfolio equation. In our generic model of Section 2, there is only one non-portfolio equation, the relative budget constraint. Taking the difference between Home and Foreign budget constraints (4) and using the asset market clearing conditions implies:

$$X_0 = d_0^n + \sum_j (2S_{Hj,0} - 1) d_{j,0}^f + 2 \sum_j (S_{Hj,0} - S_{Hj,1}) p_S^j \tag{A.1a}$$

$$- 2 \sum_j p_B^j B_{Hj,1} + 2 \sum_j d_{j,0}^b B_{Hj,0}$$

$$X_1 = d_1^n + \sum_j (2S_{Hj,1} - 1) d_{j,1}^f + 2 \sum_j B_{Hj,1} d_{j,1}^b \tag{A.1b}$$

Denote (S, B) the optimal holdings of stocks and bonds in the perfectly symmetric equilibrium, i.e. the case considered in the main text where $S = S_{ii,t}$ and $B = B_{ii,t}$. Because we allow for differences in output growth (as of time 0), we can write the optimal stock and bond holdings as $S_{ii,t} = S$ and $B_{ii,0} = B$, $B_{ii,1} = B(1 + \hat{B}_i)$. $\hat{B}_i$ denotes the portfolio rebalancing component of the bond portfolio due to *intertemporal* smoothing between period 0 and 1. Note that with fully symmetric country, as in Section 2, countries have no incentives for reblancing their portfolio, i.e $\hat{B}_i = 0$.[53]

Log-linearizing (A.1a) and (A.1b) and neglecting second-order terms, yields (using the steady-state share of non financial income $1 - \delta = \bar{d}_t^n / \bar{X}_t$):

$$\hat{X}_0 = (1 - \delta) \hat{d}_0^n + (2S - 1) \delta \hat{d}_0^f + 2 \left(\hat{b}_H - \hat{b}_F \right) b p_B + 2b \left(\hat{d}_0^b - p_B \hat{p}_B \right)$$

$$\hat{X}_1 = (1 - \delta) \hat{d}_1^n + (2S - 1) \delta \hat{d}_1^f + 2b \hat{d}_1^b + \underbrace{2 \left(\hat{b}_H - \hat{b}_F \right) \hat{d}_1^b}_{\text{2nd order} \approx 0}$$

---

[53]This *intertemporal* smoothing component could also be implemented through equity holdings. Contrary to the zero-order portfolio, at this order of approximation the rebalancing portfolio $\hat{B}$ is not unique. At this order of approximation, only the change in the net foreign asset position (and not its composition) is pinned down.

where p_S and p_B denotes stocks and bond prices in the symmetric ex-ante equilibrium and $b = B/X$ denotes the ratio of bond holdings over aggregate expenditure.

Taking the difference between period $t = 1$ and $t = 0$ and introducing the relative stochastic discount factor $\mathcal{M} = \mathcal{M}_H/\mathcal{M}_F$, with $\mathcal{M}_i = \xi \left(P_{i,0}/P_{i,1}\right)\left(C_{i,1}/C_{i,0}\right)^{-\sigma}$, we obtain:

$$\left(1 - \frac{1}{\sigma}\right)\Delta\hat{Q} - \frac{1}{\sigma}\hat{\mathcal{M}} = (1 - \delta)\Delta\hat{d}^n + \delta\left(2S - 1\right)\Delta\hat{d}^f + 2b\left(\Delta\hat{d}^b - p_B\hat{p}_B\right) - 2\hat{b}b p_B \qquad (A.3)$$

where $\hat{b} = \left(\hat{b}_H - \hat{b}_F\right)$ is the intertemporal smoothing term between $t = 0$ and $t = 1$.

Portfolio equations. Let us now turn to the second-order approximation of the portfolio equations. The Euler equations for asset returns in country $i \in \{H, F\}$ and asset type $k \in \{f, b\}$ from country $j \in \{H, F\}$ is:

$$E_0\left[\mathcal{M}_i R_j^k\right] = 1$$

Taking differences across country for each asset type $k \in \{f, b\}$, we obtain:

$$E_0\left[(\mathcal{M}_H - \mathcal{M}_F)\left(R_H^f - R_F^f\right)\right] = E_0\left[(\mathcal{M}_H - \mathcal{M}_F)\left(R_H^b - R_F^b\right)\right] = 0$$

The second-order approximation of this expression yields:

$$E\left[\hat{\mathcal{M}}\ \hat{R}^i\right] = 0 \text{ for } i \in \{f, b\} \qquad (A.4)$$

The optimal portfolio is a vector $\left(S, b, \hat{b}\right)$ such that the first order of the non-portfolio condition (A.3) and the second-order portfolio conditions (A.4) are satisfied.

Locally complete markets. Note that, if markets are (locally) complete, we are looking for a portfolio $\left(S, b, \hat{b}\right)$ such that the following risk-sharing condition holds:

$$\hat{\mathcal{M}} = -\sigma\Delta\hat{C} - \Delta\hat{Q} = 0. \qquad (A.5)$$

Such a portfolio trivially satisfies the two (second-order) Euler equation approximations (A.4).

Let us assume that it is possible to find a portfolio such that markets are locally complete. Note that this also implies $\Delta\hat{X} = (1 - 1/\sigma)\Delta\hat{Q}$.

Using Eq. (A.3) and Eq. (A.5), the zero-order portfolio (S, b) must be such that the following equation holds for any (first-order approximation) of the returns innovations $\hat{R}^j$ for $j = \{n, f, b\}$:

$$(1 - 1/\sigma)\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) = (1 - \delta)\hat{R}^n + \delta\left(2S - 1\right)\hat{R}^f + 2b\hat{R}^b \qquad (A.6)$$

The rebalancing portfolio $\hat{b}$ makes sure that Eq. (A.3) holds in expectations together with $E_0\hat{\mathcal{M}} = 0$:

$$\left(1 - \frac{1}{\sigma}\right)E_0\Delta\hat{Q} = (1 - \delta)E_0\Delta\hat{d}^n + \delta\left(2S - 1\right)E_0\Delta\hat{d}^f + 2bE_0\Delta\hat{d}^b - p_B\left(\hat{p_B} + \hat{b}\right) \qquad (A.7)$$

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The rebalancing portfolio $\hat{b}$ only affects the expected component: in other words, deviations form the zero-order bond portfolio will be used for intertemporal smoothing and will be such that:

$$2\left(\hat{p_B} + \hat{b}\right) b p_B = \left[(1-\delta)E_0\Delta\hat{d}^n + \delta\left(2S - 1\right)E_0\Delta\hat{d}^f + 2bE_0\Delta\hat{d}^b - \left(1 - \frac{1}{\sigma}\right)E_0\Delta\hat{Q}\right] \qquad (A.8)$$

The zero-order portfolio (S, b) is the one used for risk-sharing *across states of nature*. The key question is wether one can verify (A.6) in all states of nature. To answer this question, write relative returns $\hat{R}^j$ and the innovation to the real exchange rate $\Delta\hat{Q} - E_0\Delta\hat{Q}$ as a (log-linearized) function of a vector of structural shocks $\hat{\boldsymbol{\epsilon}}$ of dimension $k \times 1$:[54]

$$\hat{R}^j = \mathbf{v}^{\mathbf{j}\prime}\hat{\boldsymbol{\epsilon}}, \quad j = f, b, n; \text{ and } \quad \Delta\hat{Q} - E_0\Delta\hat{Q} = \mathbf{v}^{\mathbf{q}\prime}\hat{\boldsymbol{\epsilon}}, \qquad (A.9)$$

where the vectors $\mathbf{v}$ are also of dimension $k \times 1$.

The portfolio restrictions encoded in (A.6) can be rewritten as:

$$\mathbf{V}\left(\begin{array}{c} \delta\left(2S - 1\right) \\ 2b \end{array}\right) = (1 - 1/\sigma)\mathbf{v}^{\mathbf{q}} - (1 - \delta)\mathbf{v}^{\mathbf{n}}, \qquad (A.10)$$

where $\mathbf{V} = \left(\mathbf{v}^{\mathbf{f}}, \mathbf{v}^{\mathbf{b}}\right)$ is the matrix of loadings for equities and bonds. Because we have two instruments (S and b), markets are locally complete if we have at most two sources of risk: $k \leq 2$. This is our Spanning Condition.

Spanning Condition: The spanning condition holds when $k \leq 2$.

The second condition for the portfolio to be unique and determined is that $k = 2$ and that the matrix $\mathbf{V}$ is invertible.

Rank Condition: The rank condition for the portfolio to be uniquely determined is $k = 2$ and $\det\mathbf{V} \neq 0$.

The Rank Condition is equivalent to assuming that equity and bond excess returns, $\hat{R}^f$ and $\hat{R}^b$, are not perfectly correlated (as well cross-country returns within an asset class). In that case, the unique equilibrium portfolio $(S^*; b^*)$ is determined as follows:

$$\left(\begin{array}{c} \delta\left(2S^* - 1\right) \\ 2b^* \end{array}\right) = \mathbf{V}^{-1}\left[(1 - 1/\sigma)\mathbf{v}^{\mathbf{q}} - (1 - \delta)\mathbf{v}^{\mathbf{n}}\right]$$

Note that, by identifying appropriately the coefficients of vectors $\mathbf{V}^{-1}\mathbf{v}^{\mathbf{q}}$ and $\mathbf{V}^{-1}\mathbf{v}^{\mathbf{n}}$, one can easily show that this last expression is equivalent to:

$$\left(\begin{array}{c} \delta\left(2S^* - 1\right) \\ 2b^* \end{array}\right) = \left(\begin{array}{c} (1 - 1/\sigma)\beta_{Q,f} - (1 - \delta)\beta_{n,f} \\ (1 - 1/\sigma)\beta_{Q,b} - (1 - \delta)\beta_{n,b} \end{array}\right)$$

Where the hedge ratios $\beta_{i,j}$ are uniquely defined when the *rank* and *spanning* conditions are satisfied, and such that:

$$\left\{\begin{array}{rcl} \Delta\hat{Q} - E_0\Delta\hat{Q} & \equiv & \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f \\ \hat{R}^n & \equiv & \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f \end{array}\right.$$

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[54]Without lack of generality, we assume that the structural shocks $\hat{\boldsymbol{\epsilon}}$ are not perfectly correlated.

This ends the proof of Property 1 when markets are locally complete.

Incomplete markets. First note that the rebalancing portfolio $\hat{b}$ is the same as in Eq. (A.8), such that $E_0\hat{\mathcal{M}} = 0$. As a consequence, the zero-order portfolio (S, b) is such that the non-expected component of Eq. (A.3) holds:

$$(1 - 1/\sigma)\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) - \frac{1}{\sigma}\left(\hat{\mathcal{M}} - E_0\hat{\mathcal{M}}\right) = (1 - \delta)\hat{R}^n + \delta\left(2S - 1\right)\hat{R}^f + 2b\hat{R}^b \qquad \text{(A.11)}$$

Markets are not complete (even locally) when $k > 2$ (i.e. there are more shocks than assets). This implies that the risk-sharing condition (A.5) cannot be verified in all states. However, the equilibrium portfolio still has the same expression as in Property 1, as long as a similar Rank Condition is satisfied.

Rank Condition: the equilibrium portfolio under (locally) incomplete markets ($k > 2$) is uniquely determined as long the following rank condition is satisfied: $\text{rank}(\mathbf{V}) = 2$, where $\mathbf{V} = (\mathbf{v^f}, \mathbf{v^b})$ and $\hat{R}^j = \mathbf{v^{j\prime}}\hat{\boldsymbol{\epsilon}}$ for $j \in \{f, b\}$.

This rank condition ensures that $\hat{R}^f$ and $\hat{R}^b$ are not perfectly correlated. In that case, one can always span the vector of structural shocks $\hat{\boldsymbol{\epsilon}}$ on the following basis $(\hat{R}^f, \hat{R}^b, \check{\epsilon}_1..., \check{\epsilon}_{k-2})$ with $E_0(\check{\epsilon}'_s \hat{R}^i) = 0$ for $s = \{1; ...; k - 2\}$ and $i = \{f, b\}$.

Let us rewrite the risk factors in this transformed basis of innovations (with $\check{\boldsymbol{\epsilon}}$ the $(k - 2) \times 1$ vector of innovations $\check{\epsilon}_s$ for $s = \{1; ...; k - 2\}$):

$$\begin{cases} \Delta\hat{Q} - E_0\Delta\hat{Q} &\equiv \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + \check{\mathbf{v}}^{\mathbf{q}\prime}\check{\boldsymbol{\epsilon}} \\ \hat{R}^n &\equiv \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + \check{\mathbf{v}}^{\mathbf{n}\prime}\check{\boldsymbol{\epsilon}} \end{cases}$$

Note that this expression is equivalent to Eq. (10) with $u_Q = \check{\mathbf{v}}^{\mathbf{q}\prime}\check{\boldsymbol{\epsilon}}$ and $u_n = \check{\mathbf{v}}^{\mathbf{n}\prime}\check{\boldsymbol{\epsilon}}$.

The projection of Eq. (A.11) on returns innovations $\hat{R}^i$ for $i = \{f, b\}$, using the portfolio-equation (Eq. (A.4)) gives:

$$\begin{aligned} (1 - 1/\sigma)\beta_{Q,b} &= (1 - \delta)\beta_{n,b} + 2b \\ (1 - 1/\sigma)\beta_{Q,f} &= (1 - \delta)\beta_{n,f} + \delta\left(2S - 1\right) \end{aligned}$$

Such a portfolio implies, by construction, $\frac{1}{\sigma}\left(\hat{\mathcal{M}} - E_0\hat{\mathcal{M}}\right) = (1 - 1/\sigma)\check{\mathbf{v}}^{\mathbf{q}\prime}\check{\boldsymbol{\epsilon}} - (1 - \delta)\check{\mathbf{v}}^{\mathbf{n}\prime}\check{\boldsymbol{\epsilon}}$. This insures that the portfolio-equation (A.4) holds and that (A.11) holds in all states.[55]

Thus, this last expression gives the *unique* equilibrium portfolio as long as the *Rank condition* is verified. This ends the proof of Property 1 when markets are incomplete.

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[55]Note that, by construction, $E_0(\check{\epsilon}'_s \hat{R}^i) = 0$ for $s = \{1; ...; k - 2\}$ and $i = \{f, b\}$—or equivalently, the stochastic discount factor is only correlated with uninsurable risks.

A.2 Closing the model: optimal portfolios with endowment and redistributive shocks

Bonds and equity portfolios. We use the Devereux and Sutherland (2011) approach to characterize the optimal equity and bond positions. To do so, we use the first-order approximations of the non-portfolio equations (see reduced-form of the model below) and the second order approximation of the Euler equations. This pins down a unique equilibrium portfolio. In this appendix, we implement the solution method in a slightly more general model than the one developed in Section 3: asset returns can be driven by structural shocks of dimension higher or equal to two. We do so to show that our derivations do not rely on the locally complete markets assumption of Section 3.

Non portfolio equations. In the general equilibrium of that section, there is an additional non-portfolio equation, which is derived from the optimal intratemporal condition for the allocation of consumption across goods and the market clearing condition in goods markets (equation (18)). The log-linear first-order approximation yields:

$$\hat{y}_t = \left[-\phi + (2a-1)^2(\phi-1) \right] \hat{q}_t + (2a-1)\hat{X}_t \tag{A.12}$$

Taking first differences, we obtain a system of two non portfolio equations:

- Intratemporal allocation across goods:

$$\Delta\hat{y} = -\phi\Delta\hat{q} + (2a-1)[\Delta\hat{X} - (1-\phi)\Delta\hat{Q}] \tag{A.13}$$

- Budget constraint:

$$\Delta\hat{X} - E_0\Delta\hat{X} = (1-\delta)\hat{R}^n + \delta\left(2S-1\right)\hat{R}^f + 2b\hat{R}^b$$

Using a more general expressions of returns than in Section 3, we can express the returns on equities, bonds and nonfinancial wealth as follows:

$$\begin{cases} \hat{R}^f &= \Delta\hat{x} - E_0\Delta\hat{x} &+ \boldsymbol{\gamma}'_f\hat{\boldsymbol{\varepsilon}} \\ \hat{R}^b &= (2a-1)\left(\Delta\hat{q} - E_0\Delta\hat{q}\right) &+ \boldsymbol{\gamma}'_b\hat{\boldsymbol{\varepsilon}} \\ \hat{R}^n &= \Delta\hat{x} - E_0\Delta\hat{x} &+ \boldsymbol{\gamma}'_n\hat{\boldsymbol{\varepsilon}} \end{cases}, \tag{A.14}$$

where $\hat{\varepsilon}$ is a N-dimensional vector of shocks and γ_i for $i = \{b, f, n\}$ is a $N \times 1$ vector that controls the impact of $\hat{\varepsilon}$ on assets returns and non-financial wealth. $\Delta\hat{x} - E_0\Delta\hat{x}$ denotes innovations on (relative) income growth. In Section 3, $\hat{\varepsilon}$ is unidimensional and equal to $\hat{\delta} - E_0\hat{\delta}$ so that the loadings γ_i satisfy: $\left\{\gamma_f; \gamma_b; \gamma_n\right\} = \left\{1; 0; -\frac{\delta}{1-\delta}\right\}$.

Portfolio equations. Due to symmetry, we can write the Euler equations in relative terms as follows for asset $i = \{f, b\}$:

$$E_0(\mathcal{M} \ R^i) = 0 \text{ for } i = \{f, b\} \tag{A.15}$$

where M is the difference between stochastic discount factor across countries. The second-order approximation of Euler equations is thus:

$$E\left[\hat{\mathcal{M}} \ \hat{R}^i\right] = 0 \text{ for } i = f, b \tag{A.16}$$

Solution method. Using the budget constraint, the intratemporal condition can be rewritten as

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follows, where $\hat{\xi} = \delta\left(2S-1\right)\hat{R}^f + 2b\hat{R}^b = \left(\ \delta\left(2S-1\right),\ \ 2b\ \right)\binom{\hat{R}^f}{\hat{R}^b}$ denotes portfolio excess returns:

$$\Delta\hat{q} - E_0\Delta\hat{q} = q_y\left(\Delta\hat{y} - E_0\Delta\hat{y}\right) + \mathbf{q}'_\varepsilon\hat{\varepsilon} + q_\xi\hat{\xi}$$

where q_y, $\mathbf{q}_\varepsilon$ and q_ξ are derived by substituting portfolio excess returns and the budget constraints into the equilibrium goods market condition.[56] If we rewrite the reduced form model using Devereux and Sutherland (2011)'s notations, we get the following expression for the vector excess returns:

$$\binom{\hat{R}^f}{\hat{R}^b} = \mathbb{R}_1\hat{\xi} + \mathbb{R}_2\binom{\Delta\hat{y} - E_0\Delta\hat{y}}{\hat{\varepsilon}}$$

where $\mathbb{R}_2 = \begin{pmatrix} 1 + q_y & \mathbf{q}'_\varepsilon + \boldsymbol{\gamma}'_f \\ (2a-1)q_y & (2a-1)\mathbf{q}'_\varepsilon + \gamma'_b \end{pmatrix}$ and $\mathbb{R}_1 = \begin{pmatrix} q_\xi \\ (2a-1)q_\xi \end{pmatrix}$.

The first-order approximation of the difference between stochastic discount factor across countries gives:

$$-\frac{\hat{\mathcal{M}}}{\sigma} = \Delta\hat{X} - E_0\Delta\hat{X} + (1/\sigma - 1)(2a-1)\left(\Delta\hat{q} - E_0\Delta\hat{q}\right) = D_1\hat{\xi} + \mathbf{D}_2\binom{\Delta\hat{y} - E_0\Delta\hat{y}}{\hat{\varepsilon}}$$

where $D_1 = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi$ is a scalar and
and $\mathbf{D}_2 = \left(\ (1-\delta)\left(1+q_y\right) + (2a-1)(1/\sigma-1)q_y \quad (2a-1)(1/\sigma-1)\mathbf{q}'_\varepsilon + (1-\delta)\gamma'_n\ \right)$ is a $1 \times N+1$ vector.

Following Devereux and Sutherland (2011), we define $\tilde{\mathbb{R}}_2 = \mathbb{R}_1\tilde{\mathbf{H}} + \mathbb{R}_2$ and $\tilde{\mathbf{D}}_2 = D_1\tilde{\mathbf{H}} + \mathbf{D}_2$ with $\tilde{\mathbf{H}} = (\mathbf{1} - \left(\ \delta\left(2S-1\right)\ \ 2b\ \right)\mathbb{R}_1)^{-1}\left(\ \delta\left(2S-1\right)\ \ 2b\ \right)\mathbb{R}_2$.

Then using the second-order approximation of the Euler equation, we get the following quadratic equation:

$$\tilde{\mathbb{R}}_2\Sigma\tilde{\mathbf{D}}'_2 = 0$$

where Σ is the $(N+1) \times (N+1)$ variance-covariance matrix of the vector of innovations $(\Delta\hat{y} - E_0\Delta\hat{y}, \hat{\varepsilon})'$. Rearranging terms, this equation simplifies into the following expression for portfolios:

$$\begin{pmatrix} \delta\left(2S-1\right) \\ 2b \end{pmatrix} = \left(\mathbb{R}_2\Sigma\mathbf{D}'_2\mathbb{R}'_1 - D_1\mathbb{R}_2\Sigma\mathbb{R}'_2\right)^{-1}\mathbb{R}_2\Sigma\mathbf{D}'_2$$

where we assume that the 2×2 matrix $[\mathbb{R}_2\Sigma\mathbf{D}'_2\mathbb{R}'_1 - D_1\mathbb{R}_2\Sigma\mathbb{R}'_2]$ is invertible (**Rank condition**). When this rank condition is satisfied, the equilibrium portfolio is unique and bond and equity excess returns are not collinear. There also exists a unique decomposition such that:

$$\Delta\hat{Q} - E_0\Delta\hat{Q} \equiv \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + u_Q$$
$$\hat{R}^n \equiv \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + u_n$$

where u_i for $i = \{Q, n\}$ is orthogonal to $\hat{R}^j$ for $j = \{b, f\}$: $E_0\left[u_i\hat{R}^j\right] = 0$. This decomposition allows to rewrite the portfolio as in Section 2, using Property 1.

Equity only portfolios. Using a similar solution technique, one can be the equilibrium portfolio

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[56] $q_y = \frac{[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}[(2a-1)(1-\delta)-1]}{1-[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$, $\mathbf{q}'_\varepsilon = \frac{[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}\gamma'_n$ and $q_\xi = \frac{[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$.

in a model with equities only.

Non portfolio equations.

- Intratemporal allocation across goods:

$$\Delta \hat{y} = -\phi \Delta \hat{q} + (2a-1)[\Delta \hat{X} - (1-\phi)\Delta \hat{Q}]$$

- Budget constraint:

$$\Delta \hat{X} - E_0 \Delta \hat{X} = (1-\delta)\hat{R}^n + \delta (2S-1)\hat{R}^f$$

Portfolio equations. We can write Euler equations in relative terms as follows for asset f:

$$E_0(\mathcal{M}R^f) = 0 \tag{A.17}$$

We use similar expressions as Eq. (A.14) to express returns on financial and non-financial wealth. In the example developed in the core of the paper, $\hat{\varepsilon}$ is unidimensional and equal to $\hat{\delta} - E_0\hat{\delta}$ and $\{\gamma_f; \gamma_n\} = \left\{1; -\frac{\delta}{1-\delta}\right\}$.

Solution method: Using the budget constraint, the intratemporal condition can be rewritten as follows, where we introduce portfolio excess returns $\hat{\xi} = \delta (2S-1)\hat{R}^f$:

$$\Delta \hat{q} - E_0 \Delta \hat{q} = q_y^u (\Delta \hat{y} - E_0 \Delta \hat{y}) + \mathbf{q}_\varepsilon^{u\prime} \hat{\varepsilon} + q_\xi^u \hat{\xi}$$

where q_y^u, $\mathbf{q}_\varepsilon^{u\prime}$ and q_ξ^u are simply derived from the non-portfolio equations where $\delta (2S-1)\hat{R}^f$ has been substituted by $\hat{\xi}$. If we rewrite the reduced form model (Eq. (A.14)) using Devereux and Sutherland (2011)'s notations, we get the following expression for the vector excess returns:

$$\hat{R}^f = \mathbb{R}_1^u \hat{\xi} + \mathbb{R}_2^u \begin{pmatrix} \Delta \hat{y} - E_0 \Delta \hat{y} \\ \hat{\varepsilon} \end{pmatrix}$$

where $\mathbb{R}_2^u = \begin{pmatrix} 1 + q_y^u & \mathbf{q}_\varepsilon^{u\prime} + \boldsymbol{\gamma}_f' \end{pmatrix}$ and $\mathbb{R}_1 = q_\xi^u$.

The first-order approximation of the difference between stochastic discount factor across countries gives:

$$-\frac{\hat{\mathcal{M}}}{\sigma} = D_1^u \hat{\xi} + \mathbf{D}_2^u \begin{pmatrix} \Delta \hat{y} - E_0 \Delta \hat{y} \\ \hat{\varepsilon} \end{pmatrix}$$

where $D_1^u = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi^u$ is a scalar and and $\mathbf{D}_2^u = \begin{pmatrix} (1-\delta)\left(1 + q_y^u\right) + (2a-1)(1/\sigma - 1)q_y^u & (2a-1)(1/\sigma - 1)\mathbf{q}_\varepsilon^{u\prime} + (1-\delta)\boldsymbol{\gamma}_n' \end{pmatrix}$ is a $1 \times N+1$ vector.

Following Devereux and Sutherland (2011), we define $\tilde{\mathbb{R}}_2^u = \mathbb{R}_1^u \tilde{\mathbf{H}}^u + \mathbb{R}_2^u$ and $\tilde{\mathbf{D}}_2^u = D_1^u \tilde{\mathbf{H}}^u + \mathbf{D}_2^u$ with $\tilde{\mathbf{H}}^u = (\mathbf{1} - (\delta (2S^u - 1))\mathbb{R}_1^u)^{-1}(\delta (2S^u - 1))\mathbb{R}_2^u$.

Then using the second-order approximation of the Euler equation, we get the following quadratic equation:

$$\tilde{\mathbb{R}}_2^u \Sigma \tilde{\mathbf{D}}_2^{u\prime} = 0$$

where Σ is the (N+1) x (N+1) variance-covariance matrix of the vector of innovations $\begin{pmatrix} \Delta \hat{y} - E_0 \Delta \hat{y} \\ \hat{\varepsilon} \end{pmatrix}$. Rearranging terms, this equation simplifies into the following expression for portfolios:

$$\delta (2S^u - 1) = \left(\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime} \mathbb{R}_1^{u\prime} - D_1^u \mathbb{R}_2^u \Sigma \mathbb{R}_2^{u\prime}\right)^{-1} \mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime}$$

where we assume that the 2×2 matrix $[\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime} \mathbb{R}_1^{u\prime} - D_1^u \mathbb{R}_2^u \Sigma \mathbb{R}_2^{u\prime}]$ is invertible (**Rank condition** — which states that equity returns across countries are not collinear). When this rank condition is satisfied, the equilibrium equity portfolio is unique. There also exists a unique decomposition such that:

$$\Delta \hat{Q} - E_0 \Delta \hat{Q} \equiv \beta_{Q,f}^u \hat{R}^f + u_Q^u$$
$$\hat{R}^n \equiv \beta_{n,f}^u \hat{R}^f + u_n^u$$

where u_i for $i = \{Q, n\}$ is orthogonal to $\hat{R}^j$ for $j = \{b, f\}$: $E_0 \left[u_i \hat{R}^j \right] = 0$. This decomposition allows to rewrite the portfolio as in Section 2.

In the example of the main text (with $\hat{\varepsilon}$ unidimensional and equal to $\hat{\delta} - E_0 \hat{\delta}$ and $\{\gamma_f; \gamma_n\} = \left\{ 1; -\frac{\delta}{1-\delta} \right\}$), we have:

$$q_y^u = -\frac{1 - (2a-1)(1-\delta)}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$$
$$\mathbf{q}_\varepsilon^u = \frac{(2a-1)\delta}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$$
$$q_\xi^u = \frac{(2a-1)}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$$

together with: $\mathbb{R}_2^u = \begin{pmatrix} 1+q_y^u & 1+\mathbf{q}_\varepsilon^u \end{pmatrix}$ and $\mathbb{R}_1 = q_\xi^u$; $D_1^u = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi^u$ and $\mathbf{D}_2^u = \begin{pmatrix} (1-\delta)\left(1+q_y^u\right) + (2a-1)(1/\sigma - 1)q_y^u & (2a-1)(1/\sigma - 1)\mathbf{q}_\varepsilon^{u\prime} - \delta \end{pmatrix}$.

Equity only portfolio when $v^2 \to 0$. One can verify that when $v^2 \to 0$, only the terms in y remain as $\Sigma \to \begin{pmatrix} \sigma_y^2 & 0 \\ 0 & 0 \end{pmatrix}$. Keeping only the y terms, to compute $(\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime} \mathbb{R}_1^{u\prime} - D_1^u \mathbb{R}_2^u \Sigma \mathbb{R}_2^{u\prime})^{-1}$ and $\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime}$, we make use of:

$$1 + q_y^u = \frac{(\phi - 1)\left(1 - (2a-1)^2\right)}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$$
$$(2a-1)(1/\sigma - 1)q_y^u = -\frac{(2a-1)(1/\sigma - 1)\left(1 - (2a-1)(1-\delta)\right)}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$$
$$(2a-1)(1/\sigma - 1)q_\xi^u = \frac{(2a-1)^2(1/\sigma - 1)}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$$

Thus, $\delta \left(2S - 1 \right) = (\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime} \mathbb{R}_1^{u\prime} - D_1^u \mathbb{R}_2^u \Sigma \mathbb{R}_2^{u\prime})^{-1} \mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime}$ can be rewritten as—noticing that denominator $(\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime} \mathbb{R}_1^{u\prime} - D_1^u \mathbb{R}_2^u \Sigma \mathbb{R}_2^{u\prime})^{-1}$ and numerator $\mathbb{R}_2^u \Sigma \mathbf{D}_2^{u\prime}$ are both multiplied by the term: $\frac{\left(1+q_y^u\right)\sigma_y^2}{\phi + (1-\phi)(2a-1)^2 - (2a-1)(1-\delta)}$:

$$\delta \left(2S^u(0) - 1 \right) = -\frac{(1-\delta)(\phi-1)\left(1 - (2a-1)^2\right) - (2a-1)(1/\sigma - 1)\left(1 - (2a-1)(1-\delta)\right)}{(\phi-1)\left(1 - (2a-1)^2\right) + (2a-1)^2(1/\sigma - 1)}$$
$$= -\frac{(1-\delta)(\lambda - 1) - (2a-1)(1/\sigma - 1)}{\lambda - 1}$$
$$= -(1-\delta) + (1 - 1/\sigma)\frac{2a-1}{1-\lambda}.$$

This is the expression shown in the main text. One can easily verify by identification with the portfolio formula that $\beta^u_{n,f} = 1$ and $\beta^u_{Q,f} = \frac{2a-1}{1-\lambda}$.

A.3 Optimal portfolios with equity and corporate debt [Not for publication]

Consider the benchmark model of Section 2 under locally complete markets.[57]

Assume that firms in country i issue a given amount of corporate debt. We call $D_{i,t}$ the debt payments that have to be paid in period t in country i (we preserve symmetry across countries, i.e $D_{i,0} = D$ and $E_0(D_{i,1}) = D$ but results regarding equity home bias do not depend on this assumption).[58]

We call S_D the (zero-order) share of corporate debt in country i held by country i. Market clearing in the corporate debt market implies that country j holds a share $(1 - S_D)$.

In this environment, Modigliani-Miller theorem holds. This means that equilibrium firms values are independent of the amount of debt issued. In particular, the log-linearized expressions for returns are unchanged and so are the loadings β_{ij} under locally complete markets:

$$\begin{cases} \Delta\hat{Q} - E_0\Delta\hat{Q} & \equiv \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f \\ \hat{R}^n & \equiv \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f \end{cases}$$

Note that $\hat{R}^f$ is *not* relative equity returns anymore (if $D_{i,1}$ non-zero in some states) but relative financial returns, i.e cross country difference in the sum of the returns on equity and returns on corporate debt.

Using similar notations, we introduce $d^f_{i,t}$ the financial income in country i (sum of equity dividends $\left(d^f_{i,t} - D_{i,t}\right)$ and corporate debt payments $D_{i,t}$).

Ignoring portfolio rebalancing terms (assuming perfect symmetry ex-ante and that countries start with the optimal steady state portfolio as in our benchmark case), the budget constraint at date t in country i can be written as:

$$X_{i,t} = d^n_{i,t} + S\left(d^f_{i,t} - D_{i,t}\right) + (1-S)\left(d^f_{j,t} - D_{j,t}\right) + S_D D_{i,t} + (1-S_D)D_{j,t} + B(d^b_{i,t} - d^b_{j,t}) \tag{A.18}$$

$$= d^n_{i,t} + Sd^f_{i,t} + (1-S)d^f_{j,t} + (S_D - S)D_{i,t} + (S - S_D)D_{j,t} + Bd^b_t \tag{A.19}$$

Taking the difference across countries and the first-difference across time gives in log-linearized terms a similar equation to (A.6) once we project on innovations (under locally-complete markets):

$$(1 - \frac{1}{\sigma})\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) = \delta(2S-1)\ \hat{R}^f + (1-\delta)\hat{R}^n + (S_D - S)\Delta\hat{D} + 2b\hat{R}^b$$

If we find a portfolio (S, S_D, b) such that the previous equation holds for arbitrary realizations of the shock innovations (or equivalent asset returns under the Rank condition), markets are locally-complete and such a portfolio is the equilibrium one.

The portfolio $(S, S_D, b) = (S^*, S^*, b^*)$ obviously satisfies this condition and is unique, where (S^*, b^*) are the ones derived in Section (2) (see equations (12a) and (12b)). The intuition for

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[57]Note that our results also hold in the general equilibrium model of Section 3.

[58]We assume that debt issued is bounded above such that at period t equity payments are strictly positive: $\left(d^f_{i,t} - D_{i,t}\right) > 0$ for all states.

the result is quite straightforward: the presence of corporate debt only redistributes income from shareholder to debt holders in some states (without any impact on total financial returns). By holding corporate debt and equity in the same proportion, investors insulate their consumption expenditures from this redistribution.

We can conclude that the presence of corporate leaves the degree of home bias unchanged as well as the expression in terms of factor loadings once we compute the aggregate financial returns $\hat{R}_f$ (equity returns plus corporate debt returns). We also obtain that the (equilibrium) home bias in corporate debt is equal to the one in equity. One assumption is key for this result: returns on financial incomes $hatR_f$ are independent on the capital structure (i.e Modigliani-Miller holds) which makes the projection on the risk factors (and hence the portfolio) unchanged.

A.4   Derivation of portfolios for countries of different sizes

We extend our benchmark model of Section 2 by log-linearizing around the case where countries are of different country sizes. We assume that unconditional means of financial and non-financial incomes, $E(d_{i,t}^j)$, $j = f, n$ and $i = H, F$, are not equal across countries. Calling total income (financial and non financial) in country i, $x_{i,t} = d_{i,t}^n + d_{i,t}^f$, the size of countries is measured by the unconditional mean of $x_{i,t}$, $E(x_{H,t}) = \overline{x_H}$ and $E(x_{F,t}) = \overline{x_F}$. We solve the model when countries start in period zero from their unconditional mean of income $\overline{x_i}$, focusing on the non-expected part of the budget constraints. We denote by ω_i the relative size of country i: $\omega_i = \frac{\overline{x_i}}{\overline{x_i} + \overline{x_j}}$, with $\omega_H + \omega_F = 1$.

Keeping the same notations as in the case of our benchmark model with locally complete markets, projection of the log-linearized budget constraints on shocks innovations in country i gives (using market clearing conditions in the asset market) for $i \neq j$:

$$\Delta \hat{X}_i - E_0 \Delta \hat{X}_i = (1 - \delta)\hat{R}_i^n + \delta S_{ii}\hat{R}_i^f + \frac{\omega_j}{\omega_i}\delta\left(1 - S_{jj}\right)\hat{R}_j^f + \overline{b_{ii}}\hat{R}_i^b - \frac{\omega_j}{\omega_i}\overline{b_{jj}}\hat{R}_j^b$$

where $\hat{R}_i^b$ denotes the return on the bond of country i, $\overline{b_{ii}}$ denotes bonds i held by country i normalized by the unconditional expenditures $\overline{X_i}$ of country i (in the benchmark model $\overline{X_i} = \overline{X}$ and $\overline{b_{ii}} = b_{ii} = b$).[59]

Taking the difference across countries and using $\hat{\mathcal{M}} = 0$, we get:

$$(1 - \frac{1}{\sigma})\left(\Delta \hat{Q} - E_0 \Delta \hat{Q}\right) = (1 - \delta)\hat{R}_i^n + \delta \hat{R}_i^f\left(S_{HH} - \frac{\omega_H}{\omega_F}(1 - S_{HH})\right)$$
$$- \delta \hat{R}_j^f\left(S_{FF} - \frac{\omega_F}{\omega_H}(1 - S_{FF})\right) + \left(1 + \frac{\omega_H}{\omega_F}\right)\overline{b_{HH}}\hat{R}_H^b - \left(1 + \frac{\omega_F}{\omega_H}\right)\overline{b_{FF}}\hat{R}_F^b$$

Rewrite the equilibrium portfolios as:

$$S_{ii} = S = \omega_i + \Omega^f(1 - \omega_i)$$
$$\overline{b_{ii}} = b = \Omega^b(1 - \omega_i)$$

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[59]Note that the steady-state shares of financial and non-financial income are assumed to be constant across countries, only the unconditional mean of total income is different across countries.

Ω^f and Ω^b are measures of the size of portfolio biases. Then, keeping the same notations:

$$(1 - \frac{1}{\sigma})\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) = (1 - \delta)\hat{R}^n + \Omega^f\delta\hat{R}^f + \Omega^b\hat{R}^b$$

Assuming the following loadings on $\hat{R}^n$ and $\Delta\hat{Q}$ (see main text):

$$\Delta\hat{Q} - E_0\Delta\hat{Q} = \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f$$
$$\hat{R}^n = \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f$$

Projection on $\hat{R}^n$ and $\Delta\hat{Q}$ gives Ω^f and Ω^b :

$$\Omega^f = -\frac{1 - \delta}{\delta}\beta_{n,f} + \frac{1 - \frac{1}{\sigma}}{\delta}\beta_{Q,f}$$
$$\Omega^b = \left(1 - \frac{1}{\sigma}\right)\beta_{Q,b} - (1 - \delta)\beta_{n,b}$$

Using $S = \omega_i + \Omega^f(1 - \omega_i)$ and $b = \Omega^b(1 - \omega_i)$ gives equilibrium portfolios for countries of different sizes:

$$\begin{cases} b^* &= (1 - \omega_i)\left(1 - \frac{1}{\sigma}\right)\beta_{Q,b} - (1 - \omega_i)(1 - \delta)\beta_{n,b} \\ S^* &= \omega_i + (1 - \omega_i)\left(\frac{1 - \frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1 - \delta}{\delta}\beta_{n,f}\right) \end{cases}$$

A.5 A dynamic portfolio model with complete markets

Set-up. We use a dynamic continuous time portfolio model à la Merton (see Merton (1990) and Adler and Dumas (1983)). Investors have a CRRA utility flow $\frac{c_t^{1-\sigma}}{1-\sigma}$ at date t. We consider two countries $i = \{H, F\}$. Countries are ex-ante *symmetric* except in terms of their initial aggregate wealth W_{it}. Aggregate wealth of country i at date t is the sum of financial wealth W_{it}^f and non-financial wealth W_{it}^n:

$$W_{it} = W_{it}^f + W_{it}^n$$

Each country issues two assets, one bond and one claim on financial wealth. We denote by $k = \{f, b\}$ the asset class. Asset ik denotes the asset of country $i = \{H, F\}$ of class $k = \{f, b\}$. A riskless asset in zero-net supply is traded internationally with riskless rate r, paying in units of the numeraire good. All returns are expressed in the numeraire good. Each asset ik has iid log-normal returns (with common drift across countries):

$$R_{it}^k = \mu^k dt + \sigma_i^k dz_i^k$$

The return on non financial wealth R_{it}^n in country $i = \{H, F\}$ is iid log-normal with common drift across countries:

$$R_{it}^n = \mu^n dt + \sigma_i^n dz_i^n$$

The change in the price index of country $i = \{H, F\}$, denoted π_{it} is iid log-normal with common drift across countries:

$$\pi_{it} = \mu^\pi dt + \sigma_i^\pi dz_i^\pi$$

Complete markets assumption. *The two sources of local risk, returns on human wealth and*

inflation, are *perfectly spanned* by the set of risky securities:

$$R_{it}^n = \lambda^n dt + \sum_{j=\{H,F\}} \sum_{k=\{f,b\}} \beta_{ijk}^n R_{jt}^k \tag{A.20}$$

$$\pi_{it} = \lambda^\pi dt + \sum_{j=\{H,F\}} \sum_{k=\{f,b\}} \beta_{ijk}^\pi R_{jt}^k \tag{A.21}$$

with, for $r = \{n, \pi\}$, $\lambda^r = \left(\mu^r - \sum_{j=\{H,F\}} \sum_{k=\{f,b\}} \beta_{ijk}^r \mu^k \right)$ and $\sigma_i^r dz_i^r = \sum_{j=\{H,F\}} \sum_{k=\{f,b\}} \beta_{ijk}^r \sigma_j^k dz_j^k$.

Note that, following to our notations, the change in the real exchange rate for country H is $\Delta \ln Q_t = \pi_{Ht} - \pi_{Ft}$.

Portfolio choice. The total share of aggregate wealth α_{ijk} invested by country $i = \{H, F\}$ in security j is:

$$\alpha_{ijk,t} = \frac{1}{\sigma} \theta_{jk} + \left(1 - \frac{1}{\sigma} \right) \beta_{ijk}^\pi - \beta_{ijk}^n \left(1 - \frac{W_{it}^f}{W_{it}} \right) \tag{A.22}$$

where θ_{jk} is the market price of risk of security jk defined as the corresponding line of $\Omega^{-1}[\boldsymbol{\mu} - \mathbf{r}]$ with Ω the (4×4) matrix of variance-covariance of returns and $[\boldsymbol{\mu} - \mathbf{r}]$ the vector of excess returns of assets jk.

The proof of this result is immediate using [Adler and Dumas (1983)](#) as the share of aggregate wealth (financial and non financial, $\alpha_{ijk} + \beta_{ijk}^n \left(1 - \frac{W_{it}^f}{W_{it}} \right)$) invested in asset jk has to be equal to $\frac{1}{\sigma} \theta_{jk} + \left(1 - \frac{1}{\sigma} \right) \beta_{ijk}^\pi$. The second term of Eq. ([A.22](#)) is the hedge portfolio of inflation risk and the last term is the hedge portfolio of non-financial wealth risk.

Rewriting Eq. ([A.22](#)) by taking the difference within an asset class k for a country i ($i \neq j$) gives:

$$(\alpha_{iik,t} - \alpha_{ijk,t}) = \left(1 - \frac{1}{\sigma} \right) \left(\beta_{iik}^\pi - \beta_{ijk}^\pi \right) - \left(\beta_{iik}^n - \beta_{ijk}^n \right) \left(1 - \frac{W_{it}^f}{W_{it}} \right) \tag{A.23}$$

assuming that, due to symmetry, assets within an asset class $k = \{f, b\}$ have the same market price of risk ($\theta_{ik} - \theta_{jk} = 0$).

Rewrite the projection of returns on non-financial wealth (Eq. ([A.20](#))) as follows:

$$R_t^n = \sum_{j=\{H,F\}} \sum_{k=\{b,f\}} \left(\beta_{Hjk}^n - \beta_{Fjk}^n \right) R_{jt}^k$$

where $R_t^s = R_{Ht}^s - R_{Ft}^s$ for $s = \{n, f, b\}$ is the cross-country differential in returns.

Symmetry across countries implies: $\beta_{HHk}^n - \beta_{FHk}^n = \beta_{FFk}^n - \beta_{HFk}^n = \beta_{n,k}$. Thus,

$$R_t^n = \sum_{k=\{b,f\}} \beta_{n,k} R_t^k \ .$$

Similarly, using Eq. ([A.21](#)):

$$\Delta \ln Q_t = \sum_{k=\{b,f\}} \beta_{Q,k} R_t^k$$

with $\beta_{HHk}^{\pi} - \beta_{FHk}^{\pi} = \beta_{FFk}^{\pi} - \beta_{HFk}^{\pi} = \beta_{Q,k}$. Eq. (A.23) can be rewritten as:

$$(\alpha_{iik,t} - \alpha_{ijk,t}) = \left(1 - \frac{1}{\sigma}\right)\beta_{Q,k} - \beta_{n,k}\left(1 - \frac{W_{it}^f}{W_{it}}\right) \tag{A.24}$$

Asset market clearing. The market clearing condition in the asset market is for $i \neq j$:

$$\alpha_{iik,t}W_{it} + \alpha_{jik,t}W_{jt} = M_{it}^k$$

with M_{it}^k the total market value of security ik at date t. Equivalently:

$$\alpha_{ijk,t} = \frac{M_{it}^k}{W_{jt}} - \alpha_{iik,t}\frac{W_{it}}{W_{jt}}$$

with $\alpha_{iik,t} = \alpha_{jjk,t}$ and $\alpha_{jik,t} = \alpha_{ijk,t}$ (symmetry).

Equilibrium portfolios. Using market clearing, Eq. (A.24) leads to (abstracting from time indices):

$$\left(\frac{\alpha_{iik}}{1 - \omega_i} - \frac{\delta}{1 - \omega_i}\left(\frac{S_i^k}{W_H^f + W_F^f}\right)\right) = \left(1 - \frac{1}{\sigma}\right)\beta_{Q,k} - \beta_{n,k}(1 - \delta)$$

where $\delta = \left(\frac{W_i^f}{W_i}\right) = \frac{W_j^f}{W_j}$ and $\omega_i = \frac{W_i}{W_H + W_F} = \omega_{i,0}$ (complete markets). Using $M_i^b = 0$ (bonds in net zero supply) and calling b_i the share of wealth invested in domestic bonds and $S_i = \frac{\alpha_{iik}}{\delta}$ the share of financial wealth invested in domestic claims on financial wealth.

$$b_i = (1 - \omega_i)\left[\left(1 - \frac{1}{\sigma}\right)\beta_{Q,b} - \beta_{n,b}(1 - \delta)\right]$$

$$S_i = \left(\frac{S_i^k}{\sum_{i=\{H,F\}} S_i^k}\right) + \left(\frac{1 - \omega_i}{\delta}\right)\left[\left(1 - \frac{1}{\sigma}\right)\beta_{Q,f} - \beta_{n,f}(1 - \delta)\right]$$

These expressions are identical to our static two-period model if the relative stock market size of country i is equal to the relative share of country i in aggregate wealth $\omega_i = \left(S_i^k / \sum_{i=\{H,F\}} S_i^k\right)$.

B   Empirical Appendix

B.1   Data description. [Not for publication]

All data are quarterly, between 1970-Q1 and 2008-Q3.

- Government bond returns: gross return on 3-month domestic Treasury-bill, from Global Financial Database.

- Nominal exchange rates: from Global Financial Database.

- Consumer Price Index (CPI): from OECD Main Economic Indicators

- Gross Domestic Product: OECD Quarterly National Accounts, Seasonally adjusted, except as noted. Notes: Germany: data for West Germany before 1991:Q1. Japan: data before 1979 from 1999 OECD Statistical compendium and seasonally adjusted with X-12 method; Italy: data before 1979 from the 1999 OECD Statistical Compendium (seasonally adjusted).

- Compensation of Employees: OECD Quarterly National Accounts, Seasonally adjusted.except as noted. Notes: Japan: Japan: data before 1998 from 1999 OECD Statistical compendium and seasonally adjusted with X-12 method. Italy: data before 1979 from the 1999 OECD Statistical Compendium (seasonally adjusted). France: data before 1977 from the 1999 OECD Statistical Compendium (seasonally adjusted). Germany: data for West Germany before 1991:Q1.

- Mixed Income:

    - US: fraction of net operating surplus plus mixed income from OECD Quarterly National Accounts. Fraction calculated as the share of mixed income in (mixed income + gross operating surplus - consumption of fixed capital) annual data from UN National Income System of National Accounts. Ratio after 2007 is the average for 2004-2006.

    - UK: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1987-1994. Annual data from 2007 National Accounts Statistics Part III, pp836 for 1995-2005. Ratio after 2005 is the average for 2003-2005. Ratio before 1987 is the average for 1987-1989.

    - Japan: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1980-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1980 is the average for 1980-1982.

    - Italy: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1980-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1980 is the average for 1980-1982.

    - Germany: fraction of net operating surplus + mixed income. Uses data from West Germany before 1991:Q1. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1991-2002. Ratio after 2002 is the average for 2000-2002. Ratio before 1991 is the average for 1991-1993.

    - France: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1978-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1978 is the average for 1978-1980.

    - Canada: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1970-2005. Ratio for 2005-2007 from StatCan. Data after 2007 is the average for 2005-2007.

- Net Operating Surplus and mixed income: from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - France: before 1978 GDP minus compensation of employees, depreciation and indirect taxes.
  - Italy: before 1980 GDP minus compensation of employees, depreciation and indirect taxes.
  - Japan: Before 1998:Q3, net operating surplus + mixed income from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine. After 1998:Q3, defined as GDP minus compensation of employees, depreciation and indirect taxes.
  - United Kingdom: before 1988, GDP minus Compensation of employees, Depreciation and Indirect Taxes.

- Depreciation: OECD Quarterly National Accounts, Consumption of Fixed Capital. Seasonally adjusted, except as noted below.

  - France: Before 1978, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.
  - Germany: Data for West Germany before 1991:Q1.
  - Italy: Before 1980, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.
  - Japan: Before 1998:Q2 from the 1999 OECD Statistical Compendium. After 1998, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from United Nations system of national accounts annual data.
  - United Kingdom: Before 1988, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.

- Indirect Taxes: Taxes less Subsidies on Production and Imports from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 uses data for West Germany.
  - Italy: before 1980, calculated as fraction of GDP, where the fraction is computed annually as the ratio of indirect taxes to GDP from 1999 OECD Annual National Accounts.
  - Japan: before 1998:Q2, from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine. After 1998:Q2 calculated as fraction of GDP, where the fraction is computed annually as the ratio of indirect taxes to GDP from United Nations system of national accounts annual data.

- Gross Fixed Capital Formation: from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - France: before 1978, from 1999 OECD Statistical Compendium.

- Germany: before 1991 uses data for West Germany
- Italy: before 1980, from 1999 OECD Statistical Compendium.
- Japan: before 1994 from ESRI National Accounts Office Total Fixed Investment.

- Residential Investment: OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - Canada: before 1980, assumed to be 25% of total investment.
  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 data for West Germany
  - Italy: before 1980, constructed backwards from the growth rate of total construction, from 1999 OECD Statistical Compendium.
  - Japan: before 1994, from ESRI National Accounts Office Residential Investment.
  - United States: before 1990, from 1999 OECD Statistical Compendium.

- Consumption: OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 data for West Germany.
  - Italy: before 1980, from 1999 OECD Statistical Compendium.
  - Japan: before 1980, from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine.

- Equity Returns: Global Financial Database Total Return index.

- Corporate Bond Returns: quarterly holding return on corporate bond, converted into US dollar, assuming a 10 year maturity. except for Italy and Japan where we use the quarterly holding return on government bonds. Yields on corporate debt from Global Financial Database. Yields on government bonds from IFS (line 61).

- Compustat Weights: For each country and each available year, we construct the share of corporate debt as 1 minus the share of stockholder's equity in total assets for non-financial firms listed in Compustat North America (for the US and Canada) and Compustat Global (for France, Germany, Italy, Japan and the UK). Data start in 1970 for Canada and the US, 1987 for Japan and the UK, 1988 for Germany and France and 1989 for Italy.

B.2   Empirical Issues

B.2.1   VAR diagnostic tests

We specify our Vector Auto Regression in first differences for $\ln w$ and $\ln k$ (see section 4.2.2). This is empirically valid given that:

- w and k are integrated of order 1;

- w and k are *not* co-integrated.

We verify that these conditions are satisfied as follows (detailed results available upon request):

- we conduct Augmented Dickey-Fuller tests of unit roots for both variables. We cannot reject the null of a unit root, except for $\ln w$ in Japan.

- We perform Johansen tests of co-integration for (w, k). We find no cointegration relationship, except for Germany.

- Since theory suggests that the only correct co-integration vector is $w - k$ (see Baxter and Jermann (1997)), we directly test for stationarity for this variable. Using Augmented Dickey-Fuller tests, we cannot reject the null of a unit root, except for Germany (with a p-value of 5.3%).

We conclude from these diagnostic tests that a VAR in first difference is appropriate. Although theory suggests that $w - k$ should be stationary, this variable is extremely persistent even over long periods of time, suggesting that the correcting mechanism does not play an important role at least over the period we consider.

B.2.2 An alternative measure of the returns to non-financial wealth

Lustig and Nieuwerburgh (2008) propose an alternative approach to measuring the returns to human wealth. The key identification assumption consists in assuming that consumption choices are consistent with the choices of a representative agent faced with financial and non-financial wealth. In other words, aggregate consumption satisfies the Euler equation of the representative household when using the total return to the agent's wealth. Since this return is a combination of the return to financial wealth (observable) and non-financial wealth (non-observable), one can then back out the innovation to the return on non-financial wealth.

The Lustig and Nieuwerburgh (2008) method starts with the two equations below:

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{t+1+j}^m - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \tag{B.1a}$$

$$E_t \Delta c_{t+1} = \mu_m + \sigma^{-1} E_t r_{t+1}^m \tag{B.1b}$$

where the first equation is a log-linearization of the intertemporal budget constraint following Campbell (1993), under the assumption that $c_t - v_t^m$ is stationary, where c_t is log-consumption, v_t^m is log-total wealth and $r_t^m = \ln(R_t^m)$ is the return on total wealth: $V_{t+1}^m = R_{t+1}^m (V_t^m - C_t)$. ρ is related to the steady state consumption wealth ratio as $\rho = 1 - \exp(c - v^m)$. Crucially, V_t^m includes non-financial wealth. The second equation is the log-linearized form of the Euler equation that characterizes the slope of the consumption profile. σ is the coefficient of relative risk aversion (inverse of the intertemporal elasticity of substitution) and μ_m captures all variance-covariance terms, assumed constant.

Substituting the Euler equation into the budget constraint, one obtains an expression for the innovation to consumption:

$$c_{t+1} - E_t c_{t+1} = r_{t+1}^m - E_t r_{t+1}^m + \left(1 - \sigma^{-1}\right)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^m \tag{B.2}$$

The next step consists in writing the (log) return on total wealth as :

$$r_{t+1}^m = (1 - \kappa_t) r_{t+1}^f + \kappa_t r_{t+1}^n \tag{B.3}$$

where r^f_{t+1} is the return on financial wealth and r^n_{t+1} the return on non-financial wealth and κ_t is the share of nonfinancial wealth in total wealth (possibly time-varying). Following the usual steps, the innovation to the return on nonfinancial wealth satisfies:

$$r^n_{t+1} - E_t r^n_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta w_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^n_{t+1+j} \tag{B.4}$$

The central idea is to recover $(E_{t+1} - E_t) r^n_{t+1+j}$ from the consumption innovations in (B.2). Substituting (B.3) and (B.4) into (B.2), and assuming constant portfolio shares ($\kappa_t = \kappa$), we obtain:

$$c_{t+1} - E_t c_{t+1} = (1 - \kappa) \left(r^f_{t+1} - E_t r^f_{t+1} \right) + \left(1 - \sigma^{-1} \right) (1 - \kappa) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^f_{t+1+j} \tag{B.5}$$

$$+ \kappa (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta w_{t+1+j} - \kappa \sigma^{-1} (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^n_{t+1+j}.$$

Assuming that financial returns are observed (as in the benchmark case), we can invert this expression to obtain an expression for the innovation to nonfinancial returns that does not involve expected future nonfinancial returns:[60]

$$r^n_{t+1} - E_t r^n_{t+1} = (1 - \sigma) (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta w_{t+1+j} - \sigma \left(\kappa^{-1} - 1 \right) \left(r^f_{t+1} - E_t r^f_{t+1} \right) \tag{B.6}$$

$$- (\sigma - 1) \left(\kappa^{-1} - 1 \right) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^f_{t+1+j} + \sigma \kappa^{-1} (c_{t+1} - E_t c_{t+1}).$$

One can estimate the innovation to non-financial wealth in (B.6) using a Vector Autoregression of the form $\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}$ with $\mathbf{z}'_t = \left(\Delta w_t, \Delta k_t, \Delta c_t, r^f_t, \Delta \ln Q_t, \mathbf{x}'_t \right)$ as:

$$
\begin{aligned}
r^n_{t+1} - E_t r^n_{t+1} = {} & (1 - \sigma) \, \mathbf{e}'_{\Delta w} \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} \boldsymbol{\epsilon}_{t+1} \\
& - \left(\kappa^{-1} - 1 \right) \mathbf{e}'_{r^f} \left[\sigma + (\sigma - 1) \rho \mathbf{A} \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} \right] \boldsymbol{\epsilon}_{t+1} \\
& + \sigma \kappa^{-1} \mathbf{e}'_{\Delta c} \boldsymbol{\epsilon}_{t+1}
\end{aligned}
\tag{B.7}
$$

We implement this VAR estimation in section 4.6 under the assumption that $\sigma = 1$ and $\kappa = 1 - \delta$. In that case, the expression for the innovations simplifies substantially:

$$r^n_{t+1} - E_t r^n_{t+1} = - \left(\kappa^{-1} - 1 \right) \mathbf{e}'_{r^f} \boldsymbol{\epsilon}_{t+1} + \kappa^{-1} \mathbf{e}'_{\Delta c} \boldsymbol{\epsilon}_{t+1}. \tag{B.8}$$

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[60]A similar derivation can be obtained in the case where financial returns are not observed, using equation (34).