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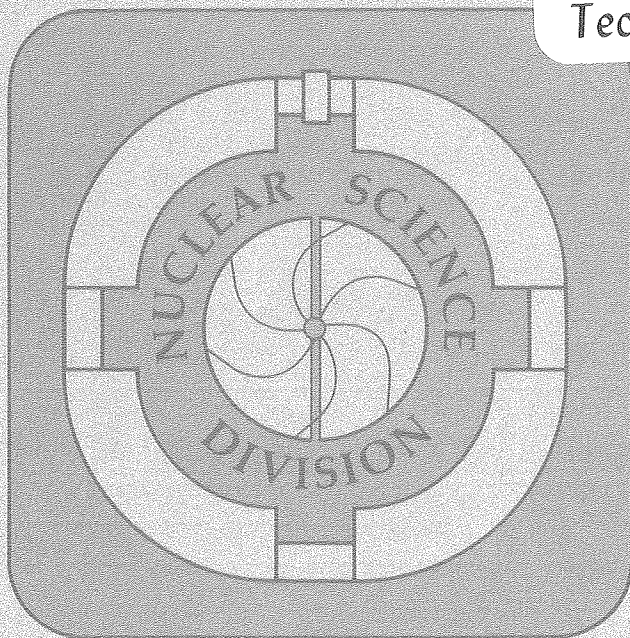
TEST OF SQUARE LAW FOR DEUTERON FORMATION  
IN RELATIVISTIC NUCLEAR COLLISIONS

C.C. Noack, M. Gyulassy, and S.K. Kauffmann

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TEST OF SQUARE LAW FOR DEUTERON  
FORMATION IN RELATIVISTIC NUCLEAR COLLISIONS

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ABSTRACT

We analyze a large body of data on nuclear collisions between 200 MeV and 2 GeV per nucleon in order to test the relation between the deuteron and proton invariant inclusive cross sections. The empirical "square law" stating that the deuteron yield is proportional to the square of the proton yield is found to hold remarkably well although small systematic deviations are also found.

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## INTRODUCTION

One of the striking features of nuclear collisions in the 0.2-2.0 GeV/nucleon range is the copious production of light composite fragments. In Ne+U at 0.4 GeV/nucleon,<sup>1</sup> for example, roughly two-thirds of the protons with energy greater than 20 MeV are bound up in light fragments such as d, t, <sup>3</sup>He, <sup>4</sup>He. Furthermore, the ratios of various composite inclusive cross sections, e.g., d/p, vary strongly with energy and angle as seen in Figs. 12 and 13 of Ref. 1.

The first attempt at understanding light composite production was in terms of the coalescence model of Ref. 2. In this model it is assumed that whenever A nucleons emerge from the reaction with momentum differences less than some coalescence radius  $p_0 \sim 100-200$  MeV/c, then they coalesce into one composite fragment. Since the probability that a nucleon is found in a coalescence volume is proportional to the invariant proton inclusive cross section:  $\sigma_1(p) = \omega d\sigma_1^3/dp_1^3$ , a simple power law is predicted for the inclusive cross section of a fragment with A nucleons,

$$\sigma_A(p) \equiv \omega_A \frac{d^3 \sigma_A}{d^3 p_A} \propto (\sigma_1(p))^A \quad (1)$$

where  $\omega_A$  and  $p_A$  are the energy-momentum per nucleon of fragment A.

For deuterons, Eq. (1) just states that the deuteron yield should be proportional to the square of the proton yield.

Subsequently, it was pointed out<sup>3</sup> that Eq. (1) also follows from the law of mass action if the light composites were produced in chemical and thermal equilibrium in the nuclear fireball.

Recently, Lemaire et al<sup>4</sup> have tested the validity of the square law

for high energy deuterons and protons produced in nuclear collisions. They found that the square law for deuterons was accurate over many orders of magnitude of the deuteron cross section.

Since extensive new data for a large variety of projectile and target combinations at various energies have become available,<sup>1</sup> it is now possible to test the square law over a much broader domain of reactions. Furthermore, unlike the high energy deuterons of Ref. 4, which constitute only a small fraction of the deuteron yield, the intermediate energy deuterons (20-100 MeV/nucleon) measured in Ref. 1 can be used to test the square law in regions of momentum space containing the bulk of the deuteron yield. In that region of momentum space most of the nucleons are bound into light composites. Thus the data<sup>1</sup> analyzed here test the square law in the high density regions of momentum space, where in terms of the coalescence model,<sup>2</sup> the probability of finding two or more nucleons in a coalescence volume approaches unity.

The basic idea of the coalescence model<sup>2</sup> is that light composites are formed through final state interactions, after the violent phase of a nuclear collision. That violent phase is thought to be well described via intranuclear cascading.<sup>5</sup> Therefore, before any composites are formed the "primordial" distributions of positive charged and neutral baryons,  $\omega d^3\sigma_{ch}^{pr}/d^3p$  and  $\omega d^3\sigma_{nt}^{pr}/d^3p$ , are assumed to be calculable with any of the available cascade codes. After final state interactions, some of the protons and neutrons are bound in light composites, and we observe the final inclusive cross sections,  $d^3\sigma(Z,N)/d^3p$ , for composites with Z protons and N neutrons. However, baryon number and charge conservation impose the following relations between the "primordial"

distribution and the observed ones,

$$\sigma_{\text{ch}}(\underline{p}) \equiv \omega \frac{d^3 \sigma_{\text{ch}}^{\text{pr}}}{dp^3} = \sum_{Z,N} Z \frac{\omega' d^3 \sigma_{Z,N}}{dp'^3}, \quad (2)$$

$$\sigma_{\text{nt}}(\underline{p}) \equiv \omega \frac{d^3 \sigma_{\text{nt}}^{\text{pr}}}{dp^3} = \sum_{Z,N} N \frac{\omega' d^3 \sigma_{Z,N}}{d^3 p'^3}, \quad (3)$$

where  $\omega'$ ,  $p'$  are the energy-momentum *per nucleon*.

A natural generalization of the coalescence model of Eq. (1), making it more appropriate for the high density region ( $d\sigma_{\text{ch}}^{\text{pr}} \gg d\sigma_{1,0}$ ), is to assume that the  $(Z,N)$ -composite distribution can be written as

$$\sigma_{Z,N}(\underline{p}) = \alpha(Z,N) \cdot \beta(\underline{p}) \cdot (\sigma_{\text{ch}}(\underline{p}))^Z \cdot (\sigma_{\text{nt}}(\underline{p}))^N, \quad (4)$$

where  $\alpha$  is independent of  $\underline{p}$ , and  $\beta$  independent of  $Z$  and  $N$ .  $\alpha$  and  $\beta$  are constrained by Eqs. (2) and (3). For simplicity, we furthermore assume<sup>6</sup> that the "primordial"  $\sigma_{\text{ch}}$  and  $\sigma_{\text{nt}}$  are proportional:

$$\sigma_{\text{nt}}(\underline{p}) = \gamma \cdot \sigma_{\text{ch}}(\underline{p}) \quad (5)$$

with  $\gamma$  independent of  $\underline{p}$ . Equation (4) then simplifies to

$$\sigma_A(\underline{p}) = \alpha'(A) \cdot \beta(\underline{p}) \cdot (\sigma_{\text{ch}}(\underline{p}))^A. \quad (6)$$

Defining

$$R(A) := \frac{\alpha'(A)}{\alpha'(1)} \quad (7)$$

we can rewrite Eq. (6) as

$$\sigma_A(\tilde{p}) = R(A) \cdot \sigma_1(\tilde{p}) \cdot (\sigma_{ch}(\tilde{p}))^{A-1}, \quad (8)$$

where  $\sigma_1$  is the proton distribution, and  $\sigma_{ch}$  the "summed charges" distribution given experimentally in Ref. 1 (cf. Eq. (2)).

Equation (8) is the generalized coalescence law. In the low density region  $\sigma_{ch} \rightarrow \sigma_1$ , and Eq. (8) reduces, of course, to Eq. (1). We shall now confront Eq. (1) and Eq. (8) with the latest data for deuteron formation.

For deuterons the proportionality factor in Eq. (1) is a quantity with the dimension of an inverse invariant cross section. It is therefore not very suitable for comparing the degree of validity of the "square law" for different reactions, with highly varying cross sections. We therefore define a dimensionless ratio  $c$  by setting

$$\sigma_d(\tilde{p}) =: c \cdot R \cdot (\sigma_1(\tilde{p}))^2, \quad (9)$$

where  $R$  is a best fit to the ratio of  $\sigma_d$  to  $(\sigma_1)^2$ :

$$R := \left( \sum_i (\sigma_d(\tilde{p}_i))_{p_i} \Delta E_i \Delta \Omega_i \right) / \left( \sum_i (\sigma_1(\tilde{p}_i))^2_{p_i} \Delta E_i \Delta \Omega_i \right). \quad (10)$$

Here the sums are taken over all (reliable - cf. below) experimental data points  $i$  with bins  $\Delta E_i$  and  $\Delta \Omega_i$ .

Note that there is no unique way of defining a "best fit"  $R$ . The definition used by us gives equal weight to all the experimental data points; it also means that we have normalized  $R$  in such a way that the total number of deuterons *actually observed in any one reaction* (summed over all energies and angles) equals the total number of deuterons



predicted from the proton data points in the same reaction. Because all cross sections decrease rapidly with energy and angle, this amounts essentially to fitting  $c$  to the points with the highest cross sections.

With these definitions, we would have  $c = 1$  for all data points in all reactions if the square law would hold strictly, so that the deviations from  $c = 1$  are a direct measure of the deviations from the square law. The errors given for  $R$  in Table 1 and for  $c$  in the figures are calculated from the experimental errors (essentially count statistics) as given in Ref. 1. The uncertainty ( $\sim 20\%$ ) in the overall normalization has not been included in  $\Delta R$  (it does not enter in  $\Delta c$ , of course). The error bars indicated in the figures are representative examples. At the high-cross-section end of the graphs, the errors are never larger than the print characters.

The whole procedure is completely analogous in the case of the "generalized square law," Eq. (8),

$$\sigma_d(\tilde{p}) = c \cdot R' \cdot \sigma_p(\tilde{p}) \sigma_{ch}(\tilde{p}) . \quad (11)$$

In evaluating  $R'$ , as well as in all the figures, we have excluded some of the data points for reasons of experimental unreliability. These are (cf. Ref. 1 for details):

- i) all extrapolated data points,
- ii) all data points that were interpolated across the dead-layer between counters,
- iii) in some cases (He+U at 1 GeV·A, Ar+Ar, Ar+U at 388 MeV·A), the data points just before the beginning of a dead-layer, which were systematically too high in the deuteron count because of pile-up in the counter.

All fifteen reactions of Ref. 1 have been analyzed in this fashion. For those for which no figures are given, the deviations from the square law are between the worst case (Ne+U at 241 MeV·A) and the best case (Ar+Ca at 388 MeV·A).

The straight lines in Fig. 1a correspond to a proton square law, Eq. (1). The straight lines in Fig. 1b correspond to the generalized square law of Eq. (8). On this log-log plot we see remarkable adherence to both laws within a factor of 2. This form of plotting  $\sigma_p^2$  vs.  $\sigma_d$  emphasizes that the approximate law works within a factor of two over three or four *decades* of cross section!

On closer examination on a linear plot in Fig. 2, we can see some systematic deviations from the proton square law, Eq. (1), for Reaction 9 (Table 1), Ne+U. In Fig. 2b most of this systematic deviation is removed by the generalized law, Eq. (8). We attribute this to the better representation of the *neutron* data by  $\sigma_{ch}$  at low energies. There are considerably more neutrons than protons near the target rapidity for the U target. Although no neutron data for this particular energy exists, we expect that  $\sigma_d \propto \sigma_p \sigma_n$  would result in just as good a fit as  $\sigma_p \sigma_{ch}$  in Fig. 2b (cf. footnote 6). For the symmetric system in Figs. 2c and 2d, either form of the law works as well.

We again emphasize that in Figs. 1 and 2 the important point to note is that the "square law" is an empirical fact over four decades of deuteron cross section. While some theoretical justification is afforded by the coalescence<sup>2</sup> and thermal<sup>3</sup> models, no rigorous derivation has been put forward. On the other hand, it is probably very hard to prove a law that is only good to 50%, and the existing heuristic proofs may be the best we can do. In the future it will be interesting to test the cube and quartic laws for <sup>3</sup>He and  $\alpha$  production.

### Acknowledgments

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5. For example, see H.Kuhlmann, C.C.Noack, Proc. Symp. on Relativistic Heavy Ion Research, March 1978, GSI-Darmstadt, and Z.Fraenkel and Y.Yariv, Phys. Rev. C20 (1979) 2227.
6. The present experimental data on neutron emission indicate that this is roughly the case.

TABLE 1. Values of  $R$  (mb/sr MeV<sup>2</sup>/c)<sup>-1</sup> for Figs. 1 and 2.

Number	Reaction	$E_{lab}$ (MeV/A)	$R$ ( $\Delta R$ )	$R'$ ( $\Delta R'$ )
1	Ne + Al	393	51.9(0.7)	31.8(0.4)
2	Ne + Ag	393	21.2(0.3)	12.1(0.1)
3	Ne + Au	393	15.7(0.4)	8.2(0.2)
4	Ne + U	393	13.6(0.1)	6.6(0.1)
5	p + U	1041	25.4(0.5)	20.7(0.4)
6	$\alpha$ + U	1049	16.2(0.2)	11.5(0.2)
7	Ne + U	1045	8.9(0.1)	5.7(0.1)
8	Ar + U	1042	6.8(0.1)	4.0(0.1)
9	Ne + U	241	13.9(0.2)	6.8(0.1)
10	Ne + U	393	21.2(0.3)	12.1(0.1)
11	Ne + U	1045	8.9(0.1)	5.7(0.1)
12	Ne + U	2095	8.2(0.1)	4.4(0.1)
13	Ar + Ca	388	18.2(0.4)	12.1(0.3)
14	Ar + Ca	1042	18.5(0.2)	13.6(0.2)
15	$\alpha$ + Al	399	115.0(2.0)	82.0(1.4)

Figure Captions

- Fig. 1. a) Plot of  $R|\underline{p}|\sigma_p^2 = \frac{R}{|\underline{p}|} \left( \frac{d\sigma_p}{dEd\Omega} \right)^2$  vs.  $\frac{d^2\sigma_d}{dEd\Omega} = |\underline{p}|\sigma_d$  for reactions<sup>1</sup> in Table 1. Typical experimental error bars are indicated. The data have been shifted by factors of  $10^n$  to separate the reactions. The straight lines correspond to the "square law". Values of R are given in Table 1.
- b) Plot of  $|\underline{p}|R'\sigma_p\sigma_{ch}$  vs.  $\frac{d^2\sigma_d}{dEd\Omega}$  for the same reactions as in (a). Straight lines correspond to the generalized square law of Eq. (8).
- Fig. 2. Detailed linear plot of ratios  $R\sigma_p^2/\sigma_d$  (2a) and  $R'\sigma_p\sigma_{ch}/\sigma_d$  (2b) for reaction 9 of Table 1; also shown is  $R\sigma_p^2/\sigma_d$  (2c) and  $R'\sigma_p\sigma_{ch}/\sigma_d$  (2d) for reaction 13. Comparison of data to straight lines show approximate validity of square law over three to four decades of deuteron cross section,  $d^2\sigma_d/dEd\Omega$ .

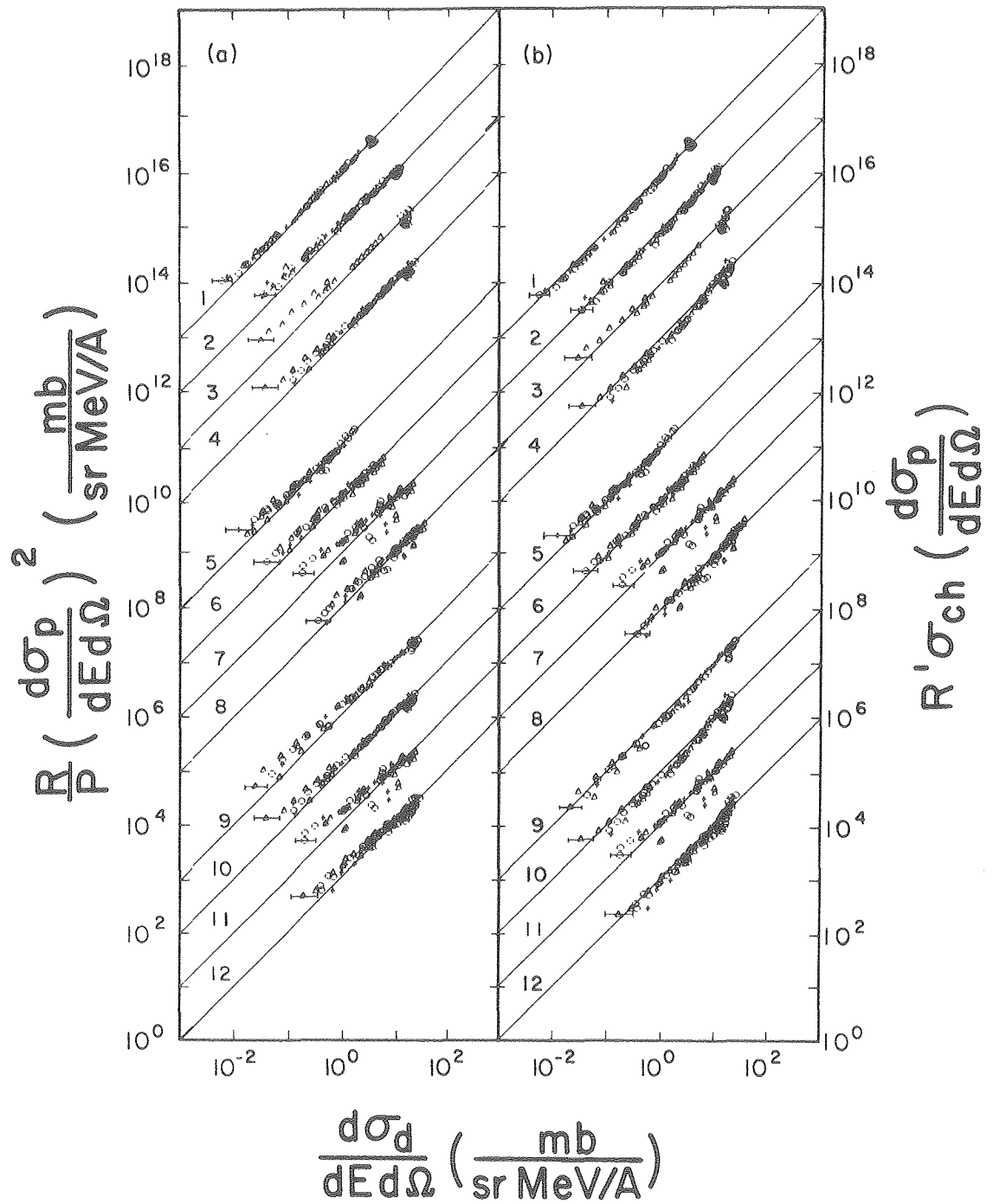


Fig. 1

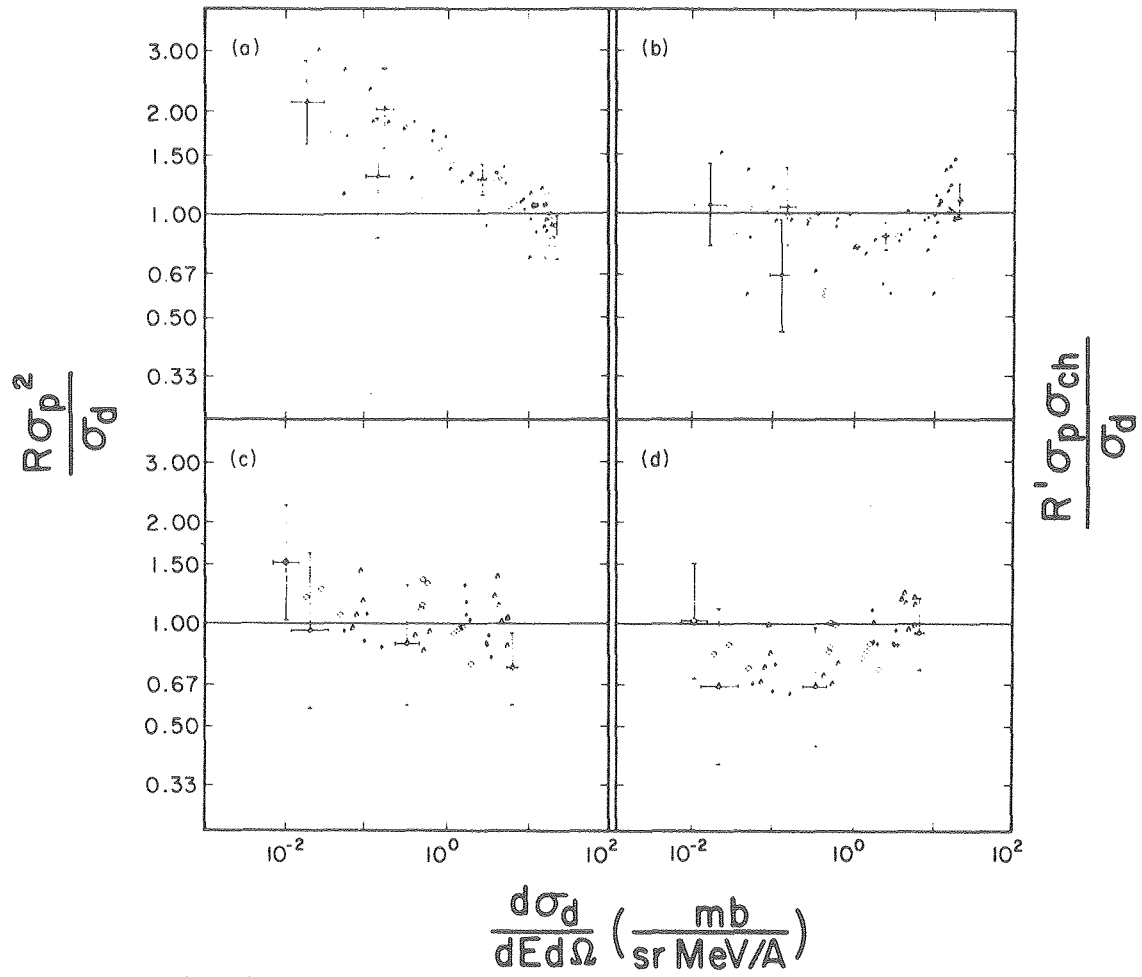


Fig. 2