Title
A Handbook For Inter-vehicle Spacing In Vehicle Following (includes Disk)

Authors
Sun, Y.
Ioannou, P.

Publication Date
1995

CALIFORNIA PATH PROGRAM
INSTITUTE OF TRANSPORTATION STUDIES
UNIVERSITY OF CALIFORMA, BERKELEY

A Handbook for Inter-Vehicle Spacing in Vehicle Following

Y. Sun
P. Ioannou

University of Southern California

California PATH Research Report
UCB-ITS-PRR-95-1

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration.

January 1995

ISSN 1055-1425

A Handbook for Inter-Vehicle Spacing in Vehicle Following

Y. Sun and P. Ioannou

Center for Advanced Transportation Technologies
University of Southern California

January 1995

A Handbook for Inter-Vehicle Spacing in Vehicle Following

by

Y. Sun and P. Ioannou

Abstract

The inter-vehicle separation in vehicle following is an important parameter that affects both safety and highway capacity. For a collision-free vehicle following, this separation should be large enough so that under a worst case stopping scenario no collision will take place. For a high capacity highway system, the inter-vehicle separation setting should be as small as possible. Since safety cannot be easily traded-off, the choice of the minimum safety inter-vehicle distance for a collision-free environment is important both from safety and capacity points of view. In this paper we consider a general worst case stopping scenario and use it to develop algorithms for generating the minimum safety spacing (MSS) for collision-free vehicle following. We use these algorithms to study the effects of vehicle characteristics and other parameters on the value of MSS. Furthermore, we consider the case where the choice of a smaller value of inter-vehicle separation leads to a rear-end collision. We study the effects of the various parameters on the severity of collision by using a proposed algorithm. The results and algorithms developed in this paper can be used to study the effect of new technologies and automated vehicle functions on safety of vehicle following and on highway capacity.

Keywords: Automatic Incident Detection, Car Following, Collision Avoidance Systems, Collision Dynamics, Highway Capacity, Human Factors, Intelligent Vehicle

Highway Systems, Safety, Traffic Accidents, Vehicle Dynamics, Vehicle Follower Control.

Executive Summary

In this paper, a general worst case stopping scenario, which includes previously studied stopping scenarios as special cases, is defined for vehicle following operations. Mathematical equations are developed to represent the minimum inter-vehicle separation for such stopping scenario to achieve collision-free vehicle following operation. Also, two algorithms are developed to calculate the numerical value of such minimum separation. By computer simulation studies using these algorithms, we find out that some vehicle operating parameters (e.g., initial velocity difference between the two consecutive vehicles) have greater effect on the minimum inter-vehicle separation setting for collision-free vehicle operation than some other parameters (e.g., initial velocity of the leading or trailing vehicle).

If the inter-vehicle separation is set to be less than the minimum value for collision-free vehicle following operation, then a rear-end collision will take place when the worst case stopping scenario occurs. We develop an algorithm to calculate the relationship between the vehicle separation setting and the severity of the collision. Computer simulations show that collision severity can be reduced if the trailing vehicle improves its reaction time or braking capability.

Contents

1 Introduction 1

2 Vehicle Following 3

3 General Stopping Scenario for Vehicle Following 6

3.1 Minimum inter-vehicle seperation for collision-free vehicle following 12

3.2 Examples 18

4 Accident Severity of Rear-End Collisions 24

4.1 Relationship between accident severity and inter-vehicle separation setting . 24

4.2 Examples 26

5 Conclusion 31

List of Figures

1 Vehicle following situation 3

2 Acceleration profile for the leading vehicle in the worst case stopping scenario 6

3 Longitudinal forces on the leading vehicle 7

4 Acceleration profile of the trailing vehicle in the worst case stopping scenario 9

5 Algorithm 1 15

6 Algorithm 2 17

7 The effect of trailing vehicle reaction time 19

8 The effect of deceleration difference between the leading and trailing vehicles 20

9 The effect of road-tire friction coefficient under constant AA, ... 21

10 The effect of initial vehicle velocity 22

11 The effect of initial velocity difference 23

12 Computational procedure for $(h, \Delta V^2)$ curve 27

13 (h, ΔV^2) curves for different values of t_{fc} 29

14 (h, ΔV^2) curves for A_{lm} = $0.8g$ 29

15 (h, ΔV^2) curves for AA, $=0.1g$ 30

List of Tables

1 Parameters of Figure 2 7

2 Parameters of the trailing vehicle 10

3 Parameter values for simulation study 19

4 Parameter values for simulation study 28

1 Introduction

Traffic accidents involve various types of vehicle crashes, such as rear-end collisions, backing collisions, single vehicle road departure accidents, etc [Knipling, R. 1993]. During vehicle following operations, rear-end collision is the most common type of accident. In 1990, 23 % of all police-reported crashes were rear-end collisions that caused 4.7 % of all fatalities [Knipling, R. 1993]. Current statistics portray these rear-end crashes as resulting largely from driver delayed recognition and relatively long reaction time when driving under high speed and close inter-vehicle separation [Knipling, R. 1993][Treat, J. R. *et al.* 1979].

In principle, the possibility of a rear-end collision can be reduced by reducing vehicle speed and increasing inter-vehicle spacing. Since roadway capacity is proportional to vehicle speed and inversely proportional to inter-vehicle spacing [Chien, 1994], large reduction in speed or large increase in spacing lead to a low capacity vehicle/highway system. This is the so called safety/capacity trade-off that is well known in the area of transportation.

The choice of the operating vehicle speed (V) and inter-vehicle spacing (S) for maximum capacity under the constraint of collision-free vehicle following environment is a big challenge in the design of roadway/vehicle systems. The design of vehicles imposes an upper bound on the maximum velocity a vehicle can attain. State and federal regulations also impose upper bounds on the maximum allowable velocity. The spacing S can be reduced as much as possible under the imposed safety constraints. One way to characterize these safety constraints is to consider a worst case stopping scenario in a vehicle following operation.

Such a scenario may be used to calculate the minimum value of S for collision-free vehicle following. This minimum value of S is referred to as the minimum *safety spacing* (MSS). Lenard (1969) and Sklar *et al.* (1979) derived equations for MSS under the assumption that the leading and trailing vehicles have the same deceleration characteristics. MacKinnon (1975) and Shladover (1979) calculated the MSS for particular emergency braking scenarios without the constraint of similar braking properties for consecutive vehicles. By taking the characteristics of future automated vehicles into account, Chien (1993) developed an expression for MSS in the case of successive vehicles with the same acceleration and deceleration capabilities.

The values of vehicle speed *(V)* and spacing (S) in the case of rear-end collision have a significant effect on the severity of the collision or the level of vehicle damage. Their effect manifests itself in the amount of kinetic energy dissipated at impact. Several studies have been performed to quantify and understand the severity of rear-end collisions. Rahimi *et* al. (1971) defined a *safety index* as a function of vehicle operating characteristics that are related to the kinetic energy dissipated during the rear-end collision. Calson (1977), Lenard (1969) and Glimm and Fenton (1980) defined another measure of the dissipated kinetic energy called *accident severity* index. They formulated this accident severity index in different ways. Various functions of collision speeds are used to calculate the accident severity index.

In this paper, a general worst case stopping scenario for vehicle following is used to develop general formulae for calculating the MSS and accident severity index. The formulae include all the scenarios considered in the previous studies as well as new ones. Simulation results are presented to demonstrate some of the main factors of safe vehicle following operation.

2 Vehicle Following

Consider the case of vehicles following each other in a single lane with no passing or other external interruptions. The only interference that a single vehicle may experience is due to other vehicles in the same lane. Figure 1 shows two such vehicles traveling in the same lane with different speeds and accelerations.

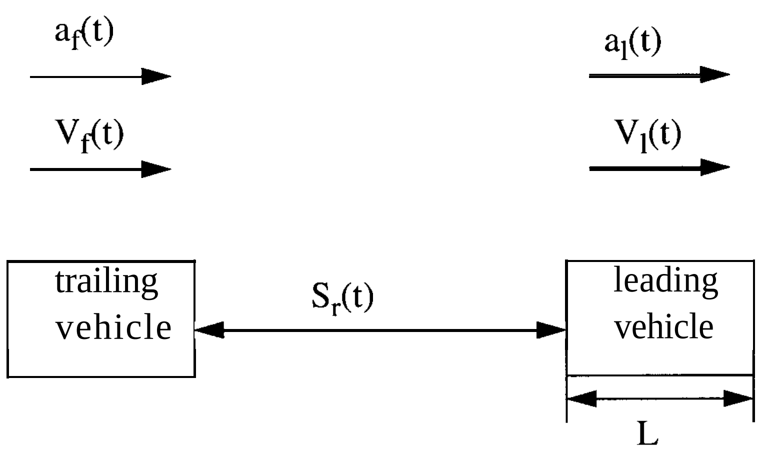

Figure 1: Vehicle following situation

The velocities of the leading and trailing vehicles are denoted by T/;(t) and $V_f(t)$ and the accelerations by $a_l(t), a_f(t)$ respectively. The leading vehicle is assumed to have a total length of L meters. If we use *S,(t), S,(t)* to denote the absolute positions of the leading and trailing vehicles measured from a common reference point, the relative distance S_r between the two vehicles measured from the front of the trailing vehicle to the rear of the leading vehicle is then given by

$$S_r(t) = S_l(t) - L - S_f(t)$$

The inter-vehicle distance S,. could be characterized using different units according to the following definitions [2][3]:

Definition 2.1 Space seperation (S,)

The inter-vehicle distance is represented by the inter-vehicle distance in units of length.

Definition 2.2 Time seperation (h)

The inter-vehicle distance is represented by a time index, which is the time required for the trailing vehicle to travel through the inter-vehicle distance, i.e.,

$$S_r = hV_f$$

The motion of the two vehicles in figure 1 can be described by the following equations:

Leading vehicle:

$$V_l(t) = V_l(0) + \int_0^t a_l(\tau)d\tau \tag{1}$$

$$S_l(t) = S_l(0) + \int_0^t V_l(\tau)d\tau \tag{2}$$

Trailing vehicle:

$$V_f(t) = V_f(0) + \int_0^t a_f(\tau)d\tau \tag{3}$$

$$S_f(t) = S_f(0) + \int_0^t V_f(\tau)d\tau \tag{4}$$

Using equations (1) through (4) the relative distance $S_r = S_l - L - S_f$ can be shown to satisfy

$$S_r(t) = S_r(0) + [V_l(0) - V_f(0)]t + \int_0^t \int_0^\tau [a_l(s) - a_f(s)]dsd\tau \tag{5}$$

It is clear that if the relative distance $S_r(t) > 0$, $\forall t \in [0, T]$ and some $T > 0$, where $[0, T]$ is the time interval of vehicle following operation, then no collision takes place.

If $S_r(t)$ becomes equal to zero at a particular time $t \in [0, T]$, then two cases are possible:

Case I : $S_r(t) \geq 0$ for all $t \in [0, T]$ and there exists at least one time t_{mc} for which $S_r(t_{mc}) = 0$.

In this case, the leading and trailing vehicles touch each other at least once during the whole operation, but no collision force exists between the two vehicles. This situation is defined as Marginal Collision (MC).

Case II : There exists at least one time $t = t_c$ such that $S_r(t_c) = 0$ and $S_r(t_c^+) < 0$.

In this case, a rear-end collision is initiated at *t = t,*. The negative value of *S,(t)* is a fictitious one since the motion of the vehicles after collision could change considerably. The after collision motion of vehicles is outside the scope of this paper. The negative value of *S,(t)* indicates that there is an additional force that could cause the trailing vehicle to travel beyond the position of the rear-end of the leading vehicle if the leading vehicle was not an obstacle.

In the following sections, we study the above two cases for a general stopping scenario where the leading vehicle executes an emergency stop and the trailing one follows with certain time delays and a different deceleration response.

3 General Stopping Scenario for Vehicle Following

We consider the following worst case stopping scenario in a single lane vehicle following situation: The leading vehicle commences a stopping maneuver at the time the trailing vehicle is accelerating. Other emergency stopping scenarios, such as "brick-wall" scenario, can be considered as special cases of this one. The behavior of the vehicles is described as follows:

Leading vehicle : At time instant $t = 0$, the leading vehicle brakes with maximum jerk (J_{lmax}) until it reaches its maximum deceleration $(-a_{lm})$, and then keeps this deceleration until a full stop is achieved. The acceleration profile of the leading vehicle is shown in figure 2. The parameters in figure 2 are described in table 1 and explained as follows:

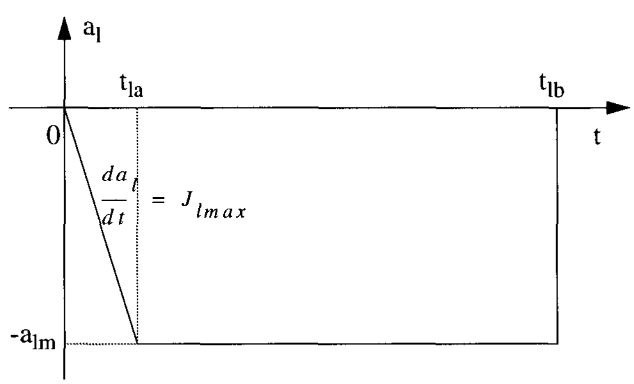

Figure 2: Acceleration profile for the leading vehicle in the worst case stopping scenario

The jerk value J_{lmax} is the maximum changing rate of deceleration and is constrained by the mechanical limits of the vehicle braking system. The maximum deceleration

a_{lm}	the maximum deceleration of the leading vehicle
J_{lmax}	the maximum jerk of the leading vehicle
t_{la}	the time required for the leading vehicle to reach $-a_{lm}$
t_{lb}	the time for the leading vehicle to come to a full stop

Table 1: Parameters of Figure 2

a_{lm} is determined by the condition and properties of the vehicle brake system as well as the condition of the road. The impact of the road geometry and condition on the braking ability of the leading vehicle is illustrated in figure 3.

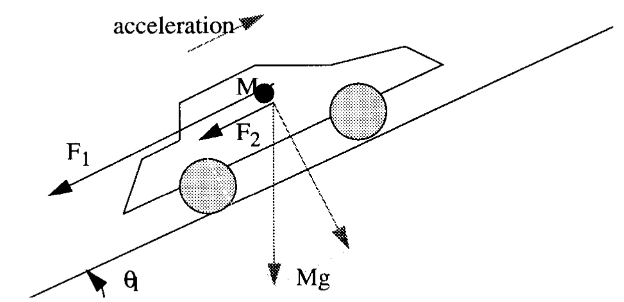

Figure 3: Longitudinal forces on the leading vehicle

In figure 3, M is the mass of the leading vehicle; θ_l denotes the road slope angle; F_1 denotes the maximum braking force; F_2 is the force due to the gravity along the road slope. According to general formula of friction force, the maximum braking force (F_{max}) of the leading vehicle on normal dry road is proportional to its pressure force (N_{press}) perpendicular to the road surface, i.e., under zero road slope,

$$F_{max} = \beta N_{press}$$

$$= \beta Mg$$

$$= MA_{lm}$$

where $\beta > 0$ is a constant. A_{lm} denotes the maximum deceleration rate of the leading vehicle under normal dry road with zero slope angle. With θ_l road slope angle, we have $N_{press} = Mgcos\theta$ and thus

$$F_{max} = MA_{lm}cos\theta$$

Under various road surfaces, the actual maximum braking force of the leading vehicle F_1 usually does not have the same value as F_{max}. We can express F_1 as

$$F_1 = \mu_{lmax}MA_{lm}cos\theta_l$$

where $\mu_{lmax} \leq 1$ denotes the so called "maximum road-tire friction coefficient". The other force F_2 in figure 3 can be calculated using

$$F_2 = Mgsin\theta_l$$

Using Newton's Second Law, the maximum deceleration of the leading vehicle can be written as

$$a_{lm} = gsin\theta_l + \mu_{lmax}A_{lm}cos\theta_l \tag{6}$$

The time values t_{la} and t_{lb} in table 1 can be calculated from other parameters shown in table 1 and figure 2, i.e.,

$$t_{la} = \frac{a_{lm}}{J_{lmax}} \tag{7}$$

$$V_l(0) + \int_0^{t_{lb}} a_l(t)dt = 0 \Rightarrow t_{lb} = \frac{V_l(0) - \frac{1}{2} J_{lmax} t_{la}^2}{a_{lm}} + t_{la} \tag{8}$$

Trailing vehicle : At time $t = 0$, the trailing vehicle is accelerating with a constant acceleration (a_{fac}). After a certain time delay (T_1), the driver or the automated vehicle control system of the trailing vehicle detects the braking maneuver of the leading vehicle. Then after some reaction delay (τ), the trailing vehicle starts to brake with certain jerk (J_{fc}) and deceleration rate (a_{fauto}) at $t = t_{fa}$. Since the trailing vehicle does not readily know in the absence of vehicle-vehicle communication that the leading vehicle is executing an emergency stop, its initial braking is done in order to control the speed and its spacing relative to that of the leading vehicle. After the trailing vehicle detects that the leading vehicle is in the emergency stopping mode, it brakes with its maximum jerk (J_{fmax}) to achieve the maximum deceleration $(-a_{fm})$ at $t = t_{fc}$. It keeps this deceleration until it reaches a full stop. The acceleration profile of the trailing vehicle is shown in figure 4. The parameters in figure 4 are explained in table 2.

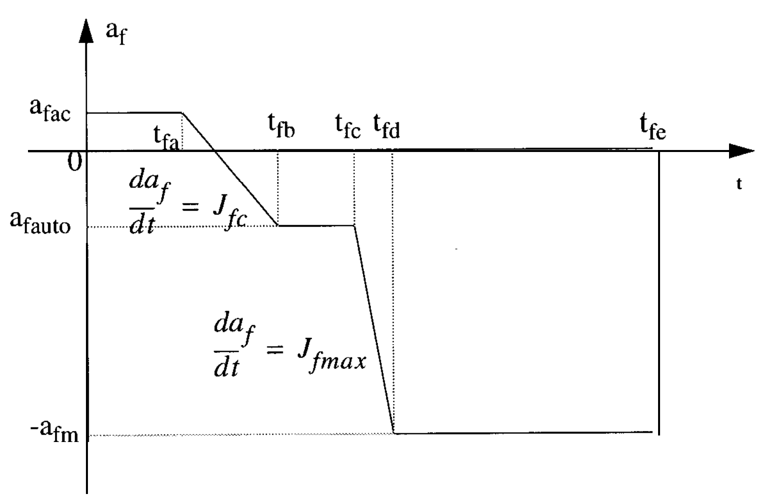

Figure 4: Acceleration profile of the trailing vehicle in the worst case stopping scenario

a_{fac}	the acceleration value under vehicle following situation
a_{fauto}	the acceleration value for soft braking
a_{fm}	the maximum deceleration of the trailing vehicle
J_{fc}	the jerk value for soft braking
J_{fma2}	the maximum jerk of the trailing vehicle
t_{fa}	the time that the trailing vehicle initiates a braking maneuver
t_{fb}	the time that the trailing vehicle reaches a_{fauto}
t_{fc}	the time that the trailing vehicle starts to brake as hard as possible
t_{fd}	the time that the trailing vehicle reaches $-a_{fm}$
t_{fe}	the time that the trailing vehicle comes to a full stop without collision
T_1	the detection delay of the trailing vehicle
τ	the brake actuation delay

Table 2: Parameters of the trailing vehicle

In table 2, the parameter J_{fc} and a_{fauto} indicate a soft braking stage in the stopping maneuver of the trailing vehicle. This braking stage may due to spontaneous response of a driver to the brake lights of the leading vehicle, or due to an automatic soft braking mode of an Intelligent Cruise Control (ICC) system [Chien, C. C. 1994]. The maximum deceleration a_{fm} can be obtained from a similar equation as (6).

The time parameters t_{fa},t_{fb}, ${}_{fd}$ and t_{fe} in figure 4 can be expressed as follows:

$$t_{fa} = T_1 + \tau \quad (9)$$

$$t_{fb} = \frac{a_{fac} - a_{fauto}}{J_{fc}} + t_{fa} \quad (10)$$

$$t_{fd} = \frac{a_{fauto} + a_{fm}}{J_{fmax}} + t_{fc} \quad (11)$$

$$\begin{aligned} t_{fe} &= t_{fd} - \frac{1}{a_{fm}}[\frac{1}{2}J_{fc}(t_{fa}^2 + t_{fb}^2) + \frac{1}{2}J_{fmax}(t_{fc}^2 + t_{fd}^2) + a_{fauto}t_{fb} - V_f(0) - a_{fac}t_{fb} \\ &\quad -J_{fc}t_{fa}t_{fb} - a_{fauto}t_{fd} - J_{fmax}t_{fc}t_{fd}] \quad (12) \end{aligned}$$

where T_1 denotes the detection delay of the driver (or the automated vehicle control system). The actuation delay of soft braking is denoted by $\tau . t_{fc}$ denotes the time when the driver (or the automated vehicle control system) initiates a maximum deceleration maneuver. If the trailing vehicle is driven manually, the value of t_{fc} indicates how fast does the driver perceive and react to an emergency stopping maneuver of the leading vehicle. For automated vehicle driving, this t_{fc} includes the detection, data processing and actuation delays of the automated vehicle control system.

The above stopping scenario includes many cases that have been considered in the past as well as new cases that are relevant to the operation of automated vehicles. For example, by taking $J_{lmax} = \infty$ and $t_{lb} = 0$, we have the so called "brick-wall" scenario. In automated vehicle following operations, advanced hardware and software systems may be implemented to reduce the reaction time of the trailing vehicle. If the automated system of the trailing vehicle can perceive and react to the emergency situation before the soft braking stage (a_{fauto}) is achieved, then we can have a scenario that $t_{fc} < t_{fb}$ [Chien, C. C. 1994]. If vehicle to vehicle communication is available such that the actions of the leading vehicle can be directly communicated to the trailing vehicle, then we may have a deceleration function of the trailing vehicle in which $a_{fauto} = a_{fac}$, $t_{fa} = t_{fb} = t_{fc}$ and t_{fa} is a small time value.

3 . 1 Minimum inter-vehicle seperation for collision-free vehicle following

In this subsection we calculate the minimum inter-vehicle seperation between the leading and trailing vehicles that will guarantee no rear-end collision under the worst case stopping scenario described above. If we let S_0 be the seperation between the leading and trailing vehicles at $t = 0^-$ (just before the stopping scenario is initiated), then the inter-vehicle distance $S_r(t)$ is given by

$$S_r(t) = So + [V_l(0) - V_f(0)]t + \int_0^t \int_0^\tau [a_l(s) - a_f(s)]dsd\tau \tag{13}$$

It is clear that for S_0 large enough, we could have $S_r(t) > 0$, $\forall t \in [0, T]$, where [0, T] is the time interval of vehicle following under consideration. This situation means that no collision takes place. Similarly for S_0 small, we could have case I or II described earlier. We are looking for the minimum value of S_0 which will guarantee that case II cannot happen. This minimum value of S_0 is *the minimum space seperution for collision-free vehicle following* and is denoted by S_{min}. It can be calculated by solving for the marginal collision case as follows:

Consider the following minimization problem

$$\min_{t\in[0,T]}\{S_r(t)\} = \min_{t\in[0,T]}\{S_0 + [V_l(0) - V_f(0)]t + \int_0^t \int_0^\tau [a_l(s) - a_f(s)]dsd\tau\} \tag{14}$$

and let $S_{rmin} \triangleq \min_{t\in[0,T]}\{S_r(t)\}$. As explained in *case I* above, we have an MC situation when $S_{rmin} = 0$, which indicates that the minimum space seperation (S_{min}) for avoiding collision is

$$S_{min} = \max_{t\in[0,T]}\{[V_f(0) - V_l(0)]t - \int_0^t \int_0^\tau [a_l(s) - a_f(s)]dsd\tau \ , \ 0\} \tag{15}$$

Equation (15) indicates that S_{min} is a function of the initial vehicle speeds (T/i(O) , $V_f(0)$) and accelerations (al(t) , $a_f(t)$). The vehicle accelerations are functions of many variables, including time indices, jerks and acceleration limits as shown in figure 2 and 4. The values of these variables can be easily calculated from Equation (6) through (12).

From the value of S_{min} we can calculate the minimum time seperation h_{min} in seconds, i.e.,

$$h_{min} = \frac{S_{min}}{V_f(0)} \tag{16}$$

The values of S_{min} and h_{min} can be calculated by using Equation (14), (15) provided of course the various parameters on which the acceleration responses depend are available. Below we present two algorithms that can be used to generate S_{min}, h_{min} by implementing them on a digital computer.

Algorithm 1: This algorithm is based on Equation (15). Figure 5 illustrates the calculation procedure. This algorithm calculates the total time of the stopping maneuver for the trailing vehicle, i.e., t_{fe}. We divide t_{fe} into small time steps $(0, t(1), t(2), \ldots, t(k), \ldots)$ and

$$t(k+1) - t(k) = \Delta t \;,\; k \in N \text{ and } k < \frac{t_{fe}}{\Delta t}$$

where Δt is a chosen constant time step.

We define a sequence of distance values $S_c(k)$ such that

$$S_c(k) = [V_f(0) - V_l(0)]t(k) - \int_0^{t(k)} \int_0^{\tau} [a_l(s) - a_f(s)] ds d\tau \qquad k \in N \text{ and } k \le \frac{t_{fe}}{\Delta t}$$

The minimum space seperution for collision-free *vehicle* following S_{min} can be obtained by taking the maximum value of the $S_c(k)$

$$S_{min} = \max\{ S_c(k) \mid k \in N \text{ and } k \le \frac{t_{fe}}{\Delta t}\}$$

Accordingly, the minimum time seperation h_{min} can be obtained from Equation (16).

There is one step in this algorithm that needs integral calculation. Such integral function should be carefully programmed to avoid numerical error. The integral algorithm also introduces a calculation loop in the program. Depending on the computer used, such loop may make it time consuming to run this algorithm.

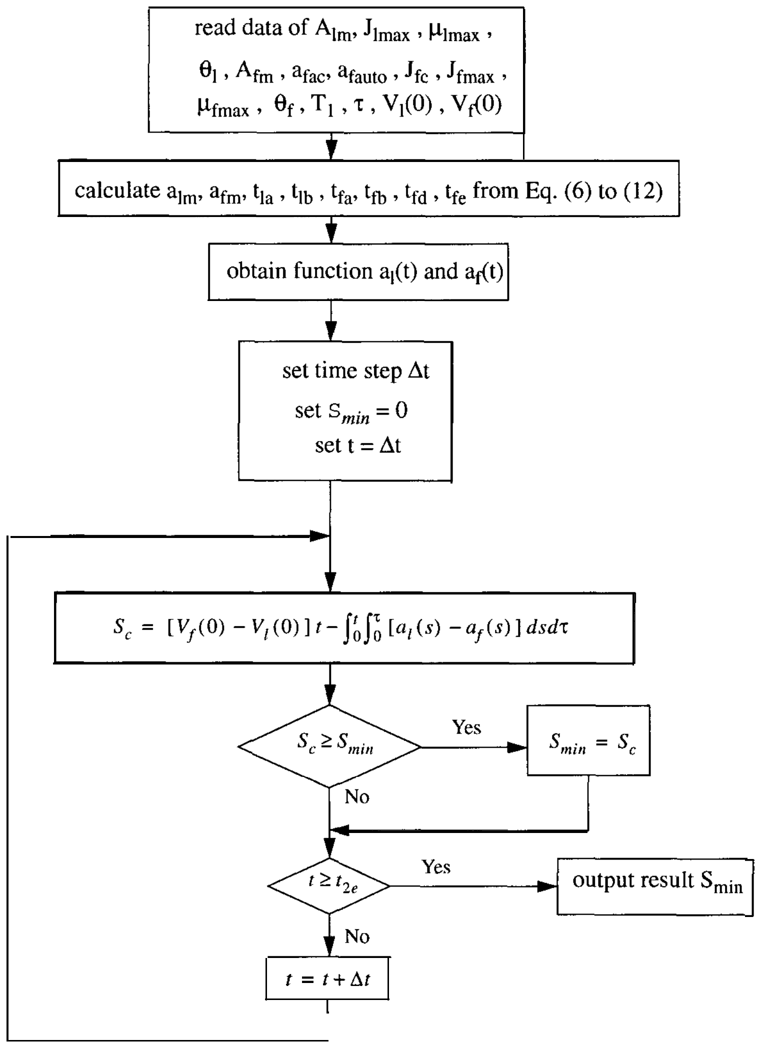

Figure 5: Algorithm 1

Algorithm 2: To avoid integral calculations, algorithm 2 is introduced. This algorithm is based on the physical meaning of marginal collision. That is, MC implies a time t_c exists when the leading and the trailing vehicles touch each other and there is no force at $t = t_c^+$ between the two vehicles. The mathematical description of MC is:

$$V_l(t_c) = V_f(t_c) \tag{17}$$

$$S_l(t_c) = S_f(t_c) + L \tag{18}$$

$$a_l(t_c) \geq a_f(t_c) \tag{19}$$

For the general stopping scenario shown in figure 2 and 4, we divide the time interval [0, t_{fe}] into several subintervals (e.g., $[t_{fc}, t_{fd}] \cap [t_{la}, t_{lb}]$). In each of these time subintervals, $a_l(t)$ and $a_f(t)$ are linear functions so that *T/i(t),* $V_f(t), S_l(t)$ and *S,(t)* can be expressed in simple polynomial forms. Therefore, we can look for the solution of S_{min} in each of these time intervals. The calculation has three steps:

step 1. Solve Equation (17) for t_c in the designated time interval.

step 2. If the solution of step 1 exists, check the acceleration condition in Equation (19).

step 3. If the result of step 2 is true, then solve Equation (2), (4) and (18) to obtain a solution of S,(O) = $S_l(0) - L - S_f(0)$.

The maximum value of the solutions of S,(O) over the time interval [0, t_{fe}] is the result of minimum safety space seperation S_{min}. The value of h_{min} can be calculated from Equation (16). The flow chart of this algorithm is shown in figure 6.

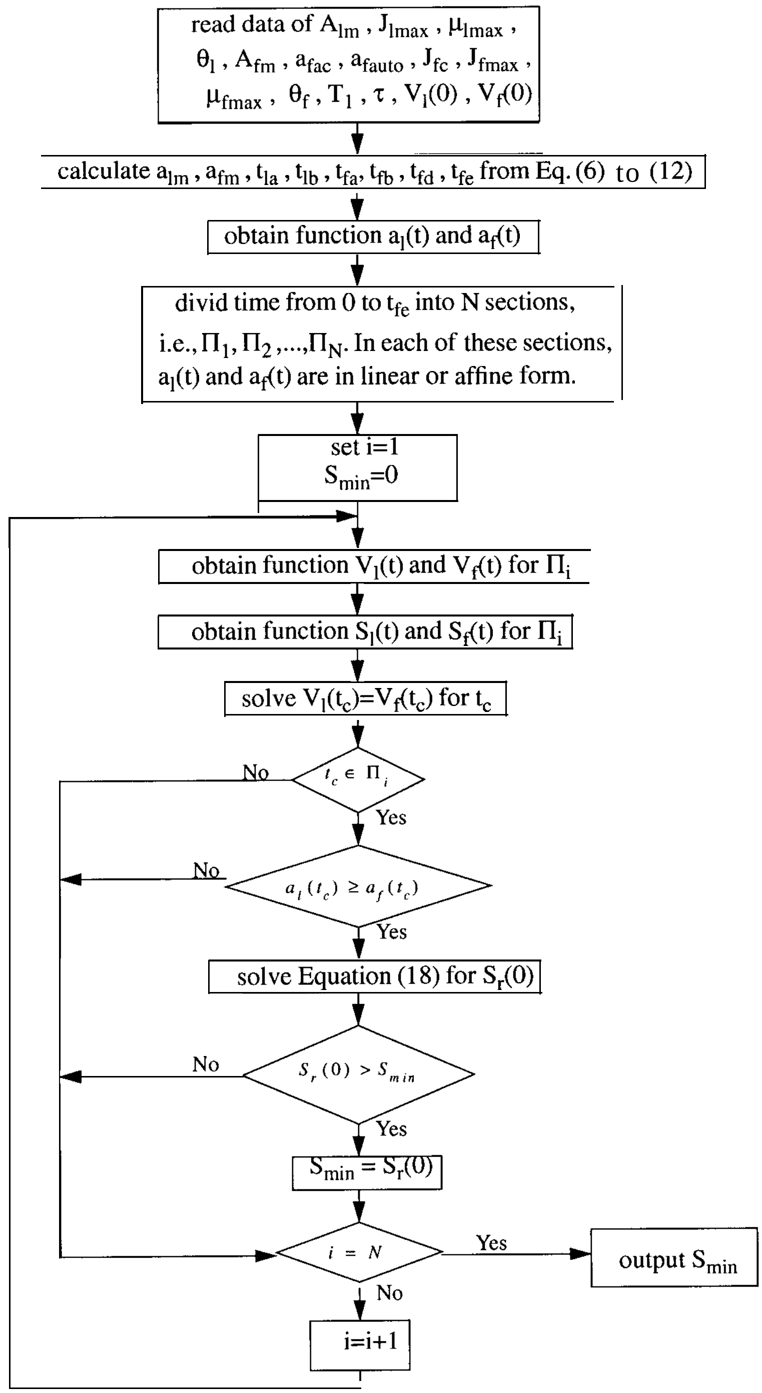

Figure 6: Algorithm 2

Since no integral loop is included in this algorithm, the calculations may be completed much faster than Algorithm 1.

3.2 Examples

Equations (6) through (12) and Equation (15) show that the minimum inter-vehicle separation is related to initial vehicle velocities (T/i(O), *V(O)),* initial trailing vehicle acceleration (a_{fac}), properties of brake systems $(J_{lmax}, A_{lm}, J_{fc}, a_{fauto}, J_{fmax}, A_{fm}, \tau)$, reaction time of driver or automatic vehicle control system (T_1, t_{fc}) and road condition $(\mu_{lmax}, \theta_l, \mu_{fmax}, \theta_f)$. The effects of these factors on the minimum time separation (h_{min}) can be studied by using the results and algorithms of the previous sections to simulate several situations.

If not explicitly stated, the simulation analyses in this subsection are done using the parameter values shown in table 3.

Example 1. *The effect of the driver reaction time or the delay of the automatic vehicle control system* (t_{fc}):

By varying the value of t_{fc}, we calculate a series of h_{min} values using either algorithm 1 or 2. The result is shown in figure 7 as a (t_{fc}, h_{min}) curve.

Figure 7 shows that h_{min} is linear with respect to the reaction delay of the trailing vehicle. If advanced technologies (e.g., vehicle-to-vehicle communication) are implemented to reduce t_{fc}, h_{min} can be reduced accordingly.

T/;(O)	26.667meter/see (60mph)	A_{fm}	$7.85 meter/sec^2$ (0.80g)
$V_f(0)$	26.667meter/see (60mph)	T_1	O.lsecond
J_{lmax}	$72 meter/sec^3$	t_{fc}	$0.35 second$
A_{lm}	$8.34 meter/sec^2$ (0.85g)	τ	O.lsecond
a_{fac}	$0.49 meter/sec^2$ (0.05g)	μ_{lmax}	1
J_{fc}	$20 meter/sec^3$	θ_l	0
a_{fauto}	$-1.96 meter/sec^2$ (−0.20g)	μ_{fmax}	1
J_{fmax}	$72 meter/sec^3$	θ_f	0

Table 3: Parameter values for simulation study

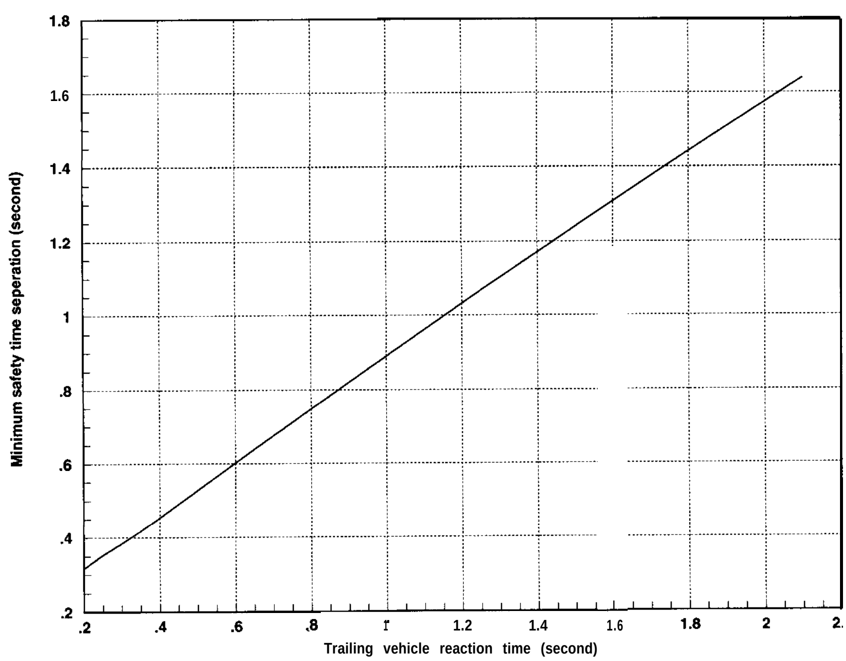

Figure 7: The effect of trailing vehicle reaction time

Example 2. *The effect of the deceleration difference* between *consecutive vehicles* (ΔA_m):

Fix A_{fm} = $0.8g$ and vary the value of A_{lm}, *we* plot h_{min} versus $\Delta A_m \triangleq A_{lm} - A_{fm}$ *as* shown in figure 8.

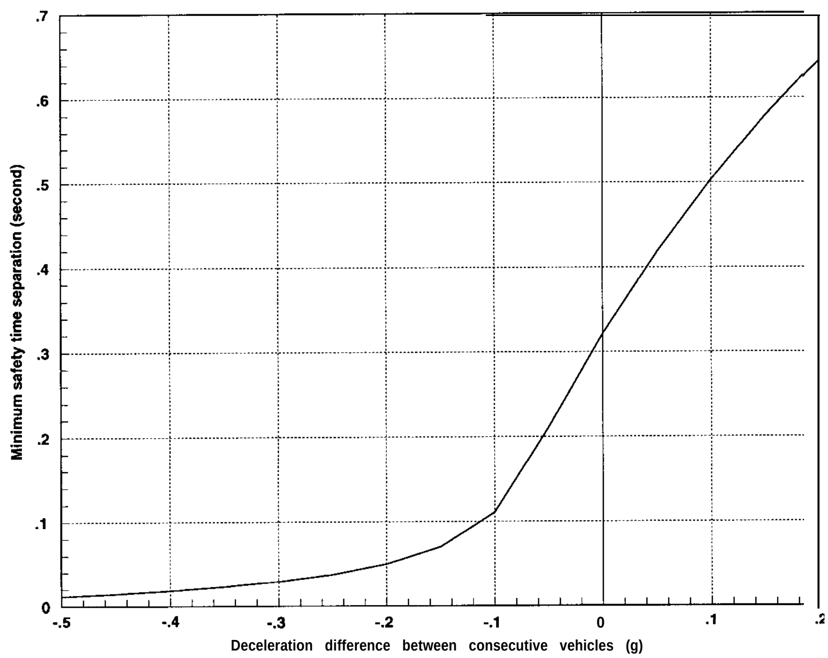

Figure 8: The effect of deceleration difference between the leading and trailing vehicles

Figure 8 shows a non-linear relationship between h_{min} and ΔA_m. If the leading vehicle can brake faster than the trailing vehicle (AA, > 0), then the inter-vehicle separation setting should be increased compared to the case that both vehicles have the same maximum deceleration value (AA, = 0) or the trailing vehicle can decelerate faster than the leading vehicle (AA, < 0).

Example 3. *The effect of road-tire friction coefficients* (μ_{lmax}, μ_{fmax}):

If we ignore the impact of road slope angle and other factors (e.g., loading effect, wind), Equation (6) shows that the maximum deceleration value is proportional to the maximum road-tire friction coefficient. In case that $\mu_{max} \triangleq \mu_{lmax} = \mu_{fmax}$, $A_{fm} = 0.8g$, the effect of road-tire friction coefficients can be illustrated by a plot of h_{min} versus μ_{max} (figure 9).

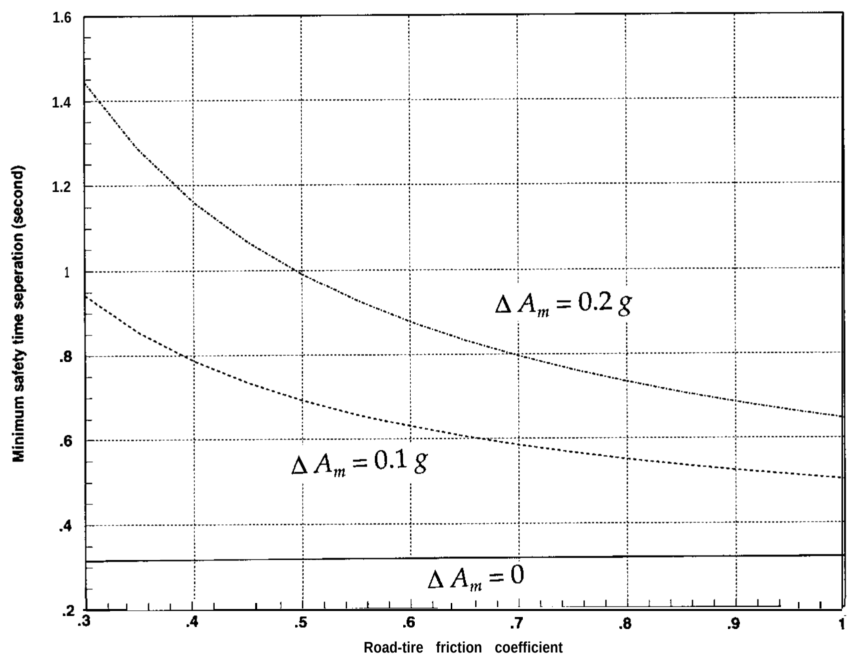

Figure 9: The effect of road-tire friction coefficient under constant AA,

h_{min} has non-linear relationship with the maximum road-tire coefficient. If the leading vehicle can brake faster than the trailing vehicle, and AA, is a constant, then h_{min} increases as the road surface becomes slippery.

Example 4. *The effect of initial vehicle velocities (T/;(O),* $V_f(0)$*):*

Assume that both vehicles are operating under the same velocity, i.e., $V_l(0)=V_f(0)=V_0$. The relationship between the minimum safety time separation and the initial vehicle velocity is shown in figure 10.

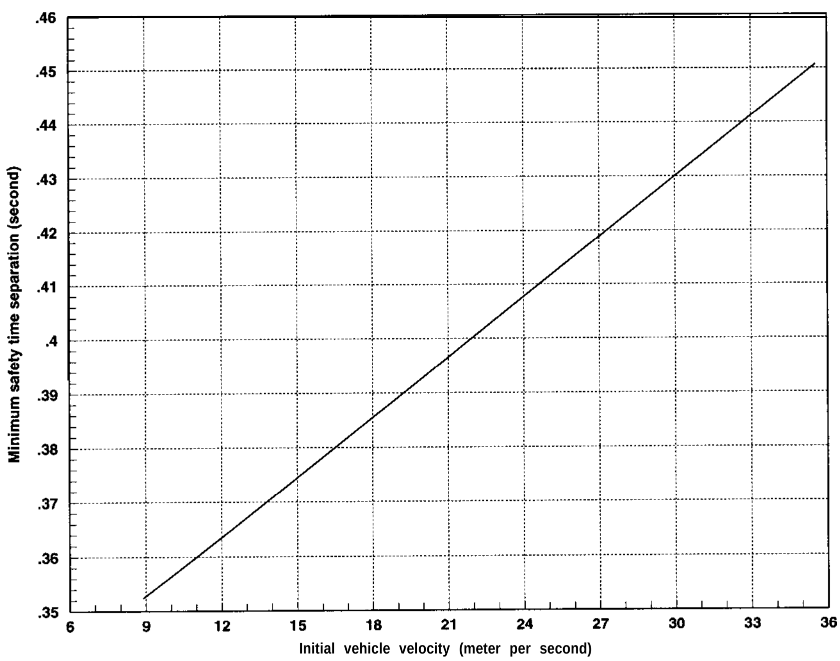

Figure 10: The effect of initial vehicle velocity

Figure 10 shows that the minimum safety inter-vehicle separation increases slowly with increasing speed V_0. The relationship between h_{min} and V_0 is almost linear. The curve indicates that the time separation h_{min} can be taken to be independent of V_0.

Example 5. *The effect* ***of initial*** *velocity difference* $(V_f(0) - V_l(0))$ *:*

When the traffic density is low, the velocity is largely decided by each individual vehicle. The leading and trailing vehicle may have a large velocity difference in this case.

When a vehicle switches from a high speed lane to a low speed lane or vice versa, it may have a leading and/or a trailing vehicle with different velocity. The impact of the velocity difference on the minimum safety time separation is illustrated by a $(V_f(0)-$ ‘/i(O), $h_{min})$ curve (Figure 11).

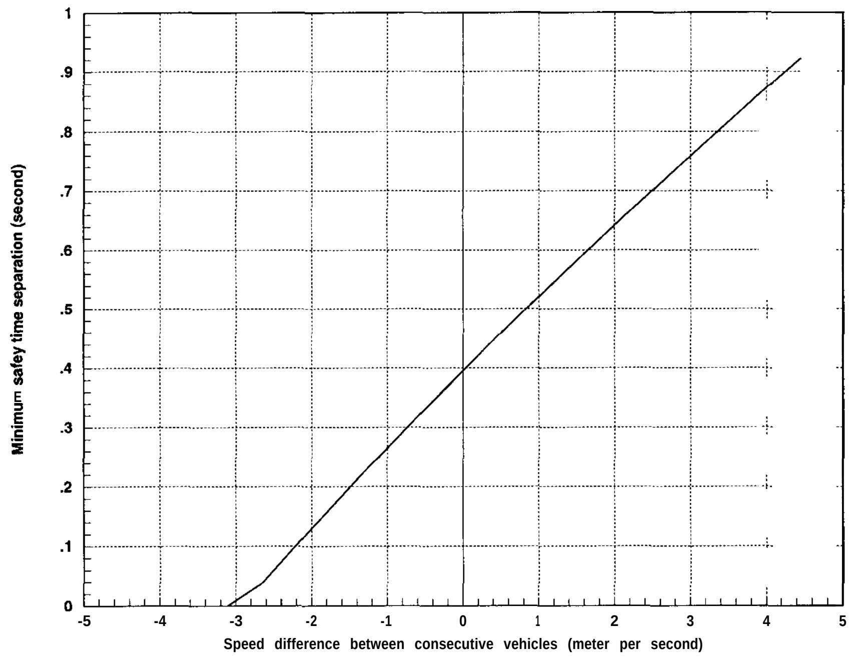

Figure 11: The effect of initial velocity difference

Figure 11 shows that when a vehicle changes from a high speed lane to a low speed lane, a large inter-vehicle spacing is needed to avoid collision with its leading vehicle. Also, when a vehicle changes to a lane that has lower speed, a large separation between this vehicle and its trailing vehicle is needed to avoid possible collisions.

4 Accident Severity of Rear-End Collisions

In this section, we consider the severity of rear-end collisions during vehicle following when the inter-vehicle separation is smaller than the minimum safety spacing developed in Section 3 for the general worst case stopping scenario.

4.1 Relationship between accident severity and inter-vehicle separation setting

The accident severity analysis for rear-end collision has been considered by several investigators. In past studies, the kinetic energy of colliding vehicles at impact is considered as the most important source of collision damage. Accordingly, accident severity index (or safety index) was defined to be proportional to this kinetic energy.

In Equation $(14), S_{rmin} \triangleq \min_{t\in[0,T]}\{S_r(t)\} < 0$ indicates a rear-end collision occurs during the vehicle following operation. If the leading and trailing vehicles have constant jerk and deceleration values during the entire stopping maneuver, then the negative value of S_{rmin} could be taken as an approximate measure of the kinetic energy of the trailing vehicle at impact. Rahimi *et* al. (1971) defined the quantity S_{rmin} as a safety index.

Calson (1977) defined the accident severity index by a function of both the relative speeds of the colliding vehicles and their absolute speeds. Lenard (1969) described the severity of an accident in three ways, all of them are functions of the square of the collision velocity. Glimm and Fenton (1980) simplified Lenard's definition. They expressed the accident severity index (S^2) for a platoon of $(n+1)$ colliding vehicles as

$$S^2 = \sum_{i=1}^{n} \Delta V_{i+1,i}^2(T_{coli})$$

where $\Delta V_{i+1,i}(T_{coli})$ denotes the relative speed at impact between vehicle i and $i+1$.

Here, we only consider two-vehicle collisions. By using Glimm and Fenton's definition, the accident severity index can be simplified as

$$\begin{aligned} S^2 &= \Delta V^2(t_c) \\ &= [V_f(t_c) - V_l(t_c)]^2 \end{aligned} \tag{20}$$

where t_c is the time when a rear-end collision is initiated.

In section 3 of this paper, the minimum safety inter-vehicle separation for vehicle following is calculated. If we set the inter-vehicle separation less than this minimum value, a rear-end collision would take place in the worst case stopping scenario (figure 2 and 4). The relationship between time separation setting (h) and accident severity (S^2) can be illustrated by a *(h,* $\Delta V^2)$ curve according to Equation (20).

In principle, the relationship between *h* and ΔV^2 can be obtained by a two step algorithm:

Step 1 : Solve the equation

$$S_l(t_c) = S_f(t_c) + L$$

$$\Rightarrow \quad S_l(0) + \int_0^{t_c} V_l(t)dt = S_f(0) + \int_0^{t_c} V_f(t)dt + L$$

$$\Rightarrow hV_f(0) = S_r(0) = \int_0^{t_c} [V_f(t) - V_l(t)]dt \qquad (21)$$

and obtain a expression of t_c by a function of *h,* i.e., *t,(h).*

Step 2 : Substitute t_c in Equation (20) to be *t,(h)* and obtain the $\Delta V^2(h)$ function.

However, Equation (21) is hard to solve explicitly since it is a nonlinear integral function. Therefore, computational algorithms are needed. Figure 12 shows a computer algorithm for calculating (h, ΔV^2) curve without solving integral functions.

4.2 Examples

Equation (20) and (21) show that the accident severity index depends on the time separation setting. Also, Equations (1),(3), (6) through (12) indicate that the accident severity depends on parameters such as maximum deceleration values *(Al,,* A_{fm}), trailing vehicle reaction time (T_1, t_{fc}), etc. The effects of these parameters are studied by using the algorithm and results of section 4.1.

If not explicitly stated, the simulation analyses in this subsection are done with parameter values shown in table 4.

Example 1. *The effect of the trailing vehicle reaction time* (t_{fc}):

Varying the value of t_{fc}, we can obtain a series of *(h,* ΔV^2) curves (figure 13).

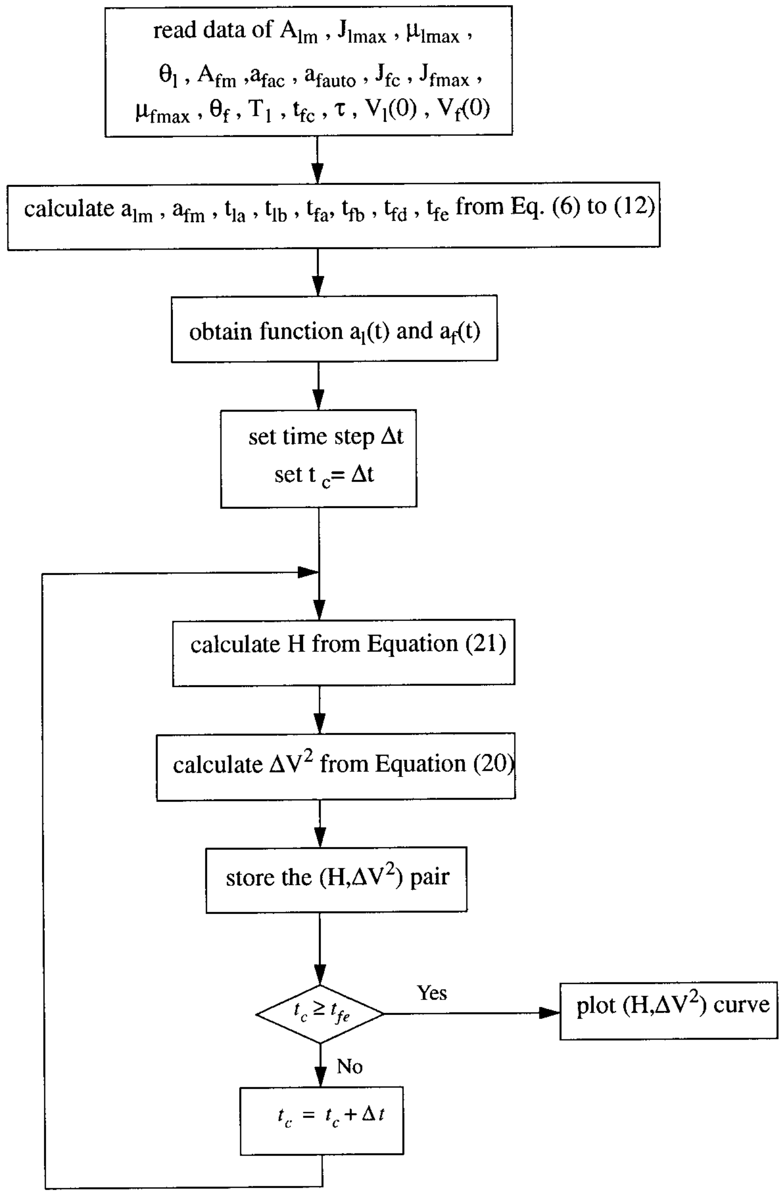

Figure 12: Computational procedure for $(h\ ,\ \Delta V^2)$ curve

I/i(O)	*26.667meter/see (60mph)*	A_{fm}	$6.87meter/sec^2$ $(0.70g)$
$V_f(0)$	*26.667meter/sec (60mph)*	T_1	*O.lsecond*
J_{lmax}	$72meter/sec^3$	t_{fc}	$0.85second$
A_{lm}	$7.85meter/sec^2$ $(0.80g)$	τ	*O.lsecond*
a_{fac}	*0*	μ_{lmax}	1
J_{fc}	*0*	θ_l	*0*
a_{fauto}	*0*	$\mu_{f\ max}$	1
$J_{f\ max}$	$72meter/sec^3$	θ_f	*0*

Table 4: Parameter values for simulation study

Figure 13 shows that when the time separation setting is very small or large enough, the value of accident severity index is small. There is a critical time separation value (h_c) between 0 and the minimum safety time separation that yields the maximum collision damage. As the reaction delay of the trailing vehicle increases, the collision damage increases accordingly.

Example *2. The effect of deceleration difference* between *the leading and trailing vehicles* (ΔA_m):

Fix $A_{lm}=0.8g$ and vary the value of A_{fm}, we obtain a series of *(h,* $\Delta V^2)$ curves for different values of $\Delta A_m \triangleq A_{lm} - A_{fm}$ as shown in figure 14.

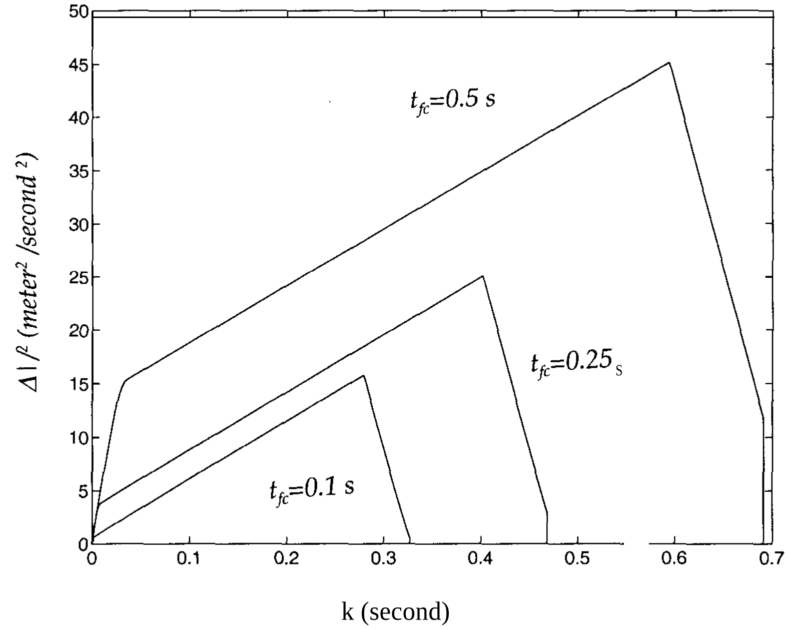

Figure 13: $(h, \Delta V^2)$ curves for different values of t_{fc}

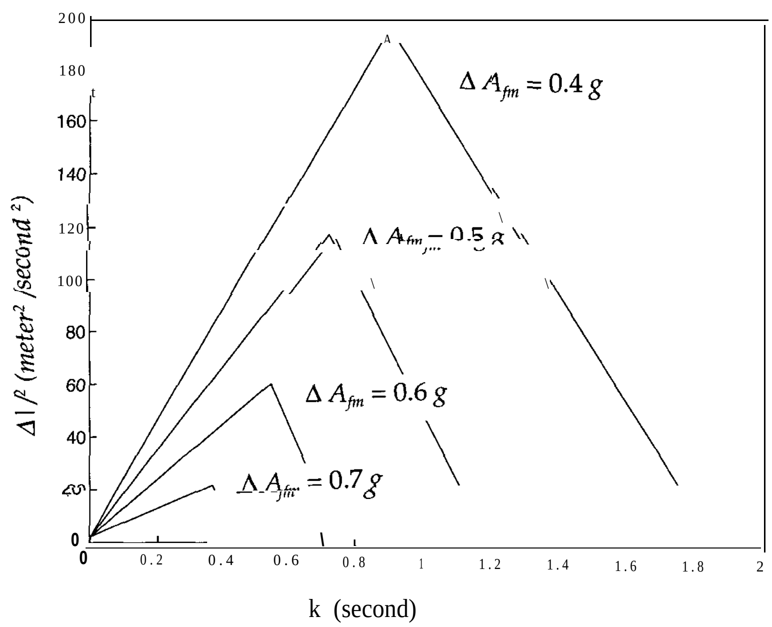

Figure 14: $(h, \Delta V^2)$ curves for A_{lm} = $0.8g$

As the value of AA, increases, the collision damage increases accordingly. When the leading and trailing vehicle have almost the same deceleration capability, the value of accident severity index is bounded by a small value.

Example 3. *The effect of the maximum deceleration values (Al,, A_{fm}):*

Fix the deceleration difference AA, to be O.lg. A series of accident severity index curves can be obtained by varying the value of A_{lm} or A_{fm} as shown figure 15.

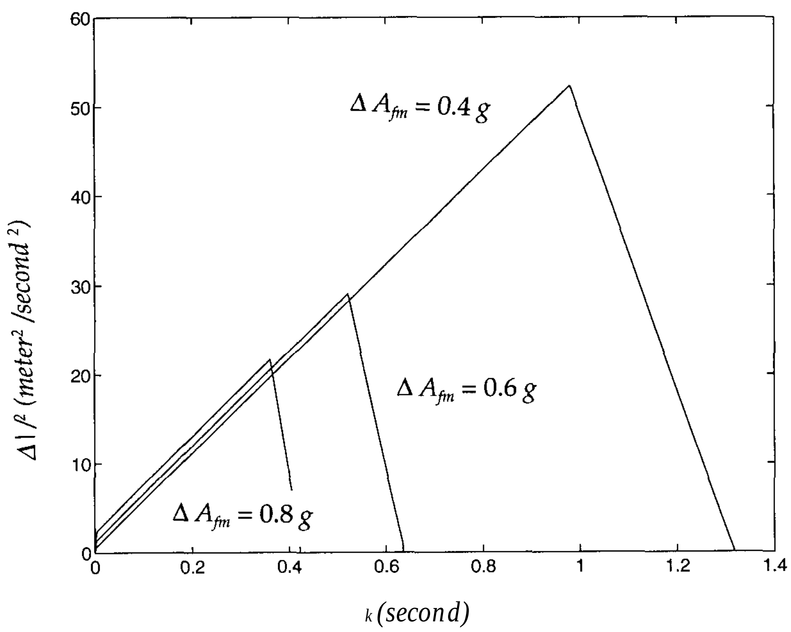

Figure 15: *(h,* ΔV^2) curves for AA, = O.lg

The simulation results show that the maximum collision damage increases when the deceleration capability is reduced. If both vehicles have approximately the same high value of maximum deceleration value, then the probability and severity of rear-end collisions can be reduced.

5 Conclusion

In this paper, a general worst case stopping scenario, which includes previously studied stopping scenarios as special cases, is defined for vehicle following operations. Mathematical equations are developed to represent the minimum inter-vehicle separation for such stopping scenario to achieve collision-free vehicle following operation. Also, two algorithms are developed to calculate the numerical value of such minimum separation. By computer simulation studies using these algorithms, we find out that some vehicle operating parameters (e.g., initial velocity difference between the two consecutive vehicles) have greater effect on the minimum inter-vehicle separation setting for collision-free vehicle operation than some other parameters (e.g., initial velocity of the leading or trailing vehicle).

If the inter-vehicle separation is set to be less than the minimum value for collision-free vehicle following operation, then a rear-end collision will take place when the worst case stopping scenario occurs. We develop an algorithm to calculate the relationship between the vehicle separation setting and the severity of the collision. Computer simulations show that collision severity can be reduced if the trailing vehicle improves its reaction time or braking capability.

References

[1] Calson, W. L. 1977. Crash Injury Loss: The Effect of Speed, Weight, and Crash Configuration. *Accident Analysis and Prevention, vol. 9.*

[2] Chien, C. C. 1994. Intelligent Vehicle Highway System (IVHS): Advanced Vehicle Control System. *Doctoral Dissertation, University of Southern California.*

[3] Garrard W. L. and Caudill R. J. 1977. Dynamic Behavior of Strings of Automated Transit Vehicles. *SAE Paper Series, No. 770288.*

[4] Glimm, J. and Fenton, R. 1980. An Accident-Severity Analysis for a Uniform-Spacing Headway Policy. *IEEE Transactions on Vehicular Technology, vol. ut-29, No. 1.*

[5] Knipling, R. 1993. IVHS Technologies Applied to Collision Avoidance Perspective on Six Target Crash Types and Countermeasures. *Presented at Safety & Human Factors Session of IVHS America Annual Meeting, April 14-17.*

[6] Knipling, R. *et al.* 1993. Assessment of IVHS Countermeasures for Collision Avoidance: Rear-end Crashes. *U.S. Department of Transportation, National Highway Traffic Safety Administration, DOT HS-807 995.*

[7] Knipling, R. *et al.* 1993. Rear-end Crashes: Problem Size Assessment and statistical Description. *U.S. Department of Transportation, National Highway Traffic Safety Administration, DOT HS-807 994.*

[8] Lenard, M. 1969. Safety Consideration for a High Density Automated Vehicle System. *General Electric* Company, *Erie, Pennsylvania.*

[9] MacKinnon, D. 1975. High Capacity Personal Rapid Transit System Developments. *IEEE Transactions of Vehicular Technology, vt-24:8-14.*

[10] Rahimi, A. *et al.* 1971. Safety Analysis for Automated Transportation Systems. *Transportation Science, page l-l 7.*

[11] Sklar, S. J. *et al.* 1979. "Safe-Approach" Vehicle-Follower Control. *IEEE Transactions on Vehicular Technology, vol. vt-28, No. 1.*

[12] Shladover, S. 1979. Operation of Automated Guideway Transit Vehicle in Dynamically Reconfigured Trains and Platoons: Systems Operations Analysis (Part III). U.S. *Department of Transportation, UMTA-MA-0085-79-3.*

[13] Treat, J. R. et *al.* 1979a. Tri-Level Study of the Causes of Traffic Accident: Final Report Volume I: Causal Factor Tabulation and Assessments. *Institute for Research in Public Safety, Indiana University, DOT HS-805 085.*

[14] Treat, J. R. *et al.* 1979b. Tri-Level Study of the Causes of Traffic Accident: Final Report Volume II: Special Analyses. *Institute for Research in Public Safety, Indiana University, DOT HS-805 085.*