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Essays on Trade Dynamics

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Essays on Trade Dynamics

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by

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2024

The Dissertation of Carlos André Bezerra de Góes is approved, and it is acceptable in quality and form for publication on microfilm and electronically.

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2024

DEDICATION

To my wife, Nicole —for collecting the pieces whenever all seemed broken, holding my hand, and pushing me forward.

To my parents, Dulce and Jos’e de Ribamar —for your unconditional, unquestioning, and wholehearted love.

To my *vovós*, Dulcinéa and Alda —for blazing a long trail that now extends from *Maranhão* to California.

EPIGRAPH

If one assumes that the essence of economic development is just to do more of the things that the economy already does, the costs of realistic trade distortions cannot be too large.

Paul Romer, 1994

Innovative countries are the most productive, but their innovations also drive growth elsewhere.

Jonathan Eaton & Samuel Kortum, 1999

Les tocó en suerte una época extraña. El planeta había sido parcelado en distintos países, cada uno provisto de lealtades, de queridas memorias, de un pasado sin duda heroico, de derechos, de agravios, de una mitología peculiar, de próceres de bronce, de aniversarios, de demagogos y de símbolos. Esa división, cara a los catógrafos, auspiciaba las guerras.

Jorge Luis Borges, 1977

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ABSTRACT OF THE DISSERTATION

Essays on Trade Dynamics

by

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Doctor of Philosophy in Economics

University of California San Diego, 2024

Professor Marc Muendler, Chair

This dissertation lies on the intersection between international trade and macroeconomics. It brings together time and space. Its chapters embed growth or adjustment over time, emphasized in macroeconomics, into trade models, which focus on the distribution of economic activity over space.

Chapter 1 shows that a plausibly exogenous increase in market access increases the probability of product innovation in the context of the enlargement of the European Union. Then, it rationalizes these findings through a new quantitative framework that integrates the forces of specialization and market size and nests the Eaton-Kortum trade model and the Romer growth model as special cases. In this framework, the product innovation growth rate increases with higher market access. The key result is an analytical expression to decompose gains from trade into dynamic and static components. A quantitative version of the model suggests that the EU enlargement increased its long-run yearly growth rate by 0.10pp; and dynamic gains account for as much as 90% of total gains from trade.

Chapter 2 focuses on the potential effects of global and persistent geopolitical conflicts on trade, technological innovation, and economic growth. In conventional trade models, the welfare costs of such conflicts are modest. Using a dynamic trade model, it shows that welfare losses of a decoupling of the global economy can be drastic, as large as 12% in 20 years for some regions, with more significant losses in lower-income countries. Two mechanisms are essential to capture these effects: technological diffusion and input-output linkages, both of which magnify welfare losses.

Chapter 3 proposes a new theory explaining why trade flows adjust slowly after a shock. The model features staggered sourcing decisions, nests the Eaton-Kortum model as the limiting long-run case, and provides a quantitative framework that accounts for

the time-varying trade elasticity. In doing so, it microfounds the gap between empirical estimates of the trade elasticity in the short and long run. Simulations in the context of the US-China trade war suggest that the short-run welfare impact can be smaller than the long-run level for the United States but larger for China despite the same low short-run trade elasticity, while third countries such as Mexico and Vietnam may experience welfare losses in the short run but welfare gains in the long term.

Chapter 1

Trade, Growth, and Product Innovation

1.1 Introduction

Over the last decades, the trade literature converged to a broad consensus regarding how to summarize the static gains from trade. But there is no similar consensus on how to measure dynamic gains from trade[1]. In this paper, I address this topic by examining the mechanisms through which trade integration can induce product innovation. Economic theory presents conflicting viewpoints regarding this question. *Canonical trade theory* typically suggests that increased economic integration should cause countries to produce a *smaller range* of produced goods[2]. Models that emphasize growth and innovation, such as those common in *macroeconomics*, often emphasize the role of market size for having an incentive to innovate and produce a *large range* of goods[3].

[1]For a comprehensive review of the literature and the different mechanisms that link trade, growth, and innovation, see the paper by Melitz and S. Redding (2021)

[2]In the class of Ricardian models, this follows naturally: as a country opens up to trade, it specializes in a smaller set of goods. But this also happens in the class of Melitz models. As a country opens up to trade, due to the selection effect, the least productive firms of each country exit the market, which results in a smaller range of firms (or, equivalently, goods) in either market. This result holds with asymmetric populations and symmetric productivity distributions or even with asymmetric productivity distributions, as long as the countries are not too dissimilar —for the latter see Demidova (2008).

[3]This is true of a very large class of endogenous growth models in macroeconomics, both with and without scale effects. See, for instance, Chapter 13 of Acemoglu (2008).

This paper integrates these two traditions by conceiving a global marketplace in which the economic forces of *specialization* and *market access* are jointly operating and developing tractable and intuitive ways of modeling them in a dynamic framework fit for policy evaluation. First, I show that after large events of trade integration —the expansion waves of the European Union (EU) —the countries that joined the EU started producing more product varieties, investing more in research and development (R&D), and trading more compared to candidate countries that did not join at a given horizon. Additionally, I show that a plausibly exogenous increase in market access increases the probability of a given country starting production of and exporting a given product. These facts are all suggestive of a *dynamic market access effect*. Second, to rationalize this reduced-form evidence, I propose a new dynamic general equilibrium model of frictional trade and endogenous growth with arbitrarily many asymmetric countries that nests the Eaton and Kortum (2002) model of trade and the P. M. Romer (1990) growth model as special cases. Third, I provide analytical expressions decomposing gains from trade into dynamic and static components; growth and welfare into "Romer" and "Eaton-Kortum" parts; and show analytically that the product innovation and R&D growth rates increase with higher market access, which is consistent the facts I used as motivation for the model. Lastly, I use a numerical version of the model to estimate the welfare effects of 2004 enlargement of the EU, in this framework: (a) the enlargement increased its long-run yearly growth rate by about 0.10pp; and (b) dynamic gains can account for between 65-90% of total welfare gains from trade.

My focus on *product innovation* stems from two key reasons, one theoretical and one empirical. From a theoretical standpoint, the new product margin can have large welfare implications. Empirically, around trade liberalization episodes, the bulk of trade creation comes from the extensive margin[4].

[4]For the former, P. Romer (1994) has shown that in a simple trade model, adding extensive margin can make welfare costs of a 10% tariff increase from 1% to 20%. For the latter, Kehoe and Ruhl (2013)

The paper starts by documenting a set of facts related to the Eastwards enlargement of the EU. Compared to countries that selected into being candidates of the EU but were not yet members, New Member States (NMS) started: (a) producing more product varieties; (b) spending more on private R&D per capita; and (c) having larger trading values.

Later, in order to go beyond correlational analysis, I exploit the fact that, once NMS join the EU, they not only have preferential access to the European market, but they also have to adhere to the Common Commercial Policy of the European Union. NMS have immediate preferential access to third-party markets via pre-existing trade agreements between the EU and these third-party markets.

Importantly, the NMS did not get to negotiate the tariff variation that they face —these were only a byproduct of the EU accession process. In this context, through an event-study design, a plausibly exogenous increase in market access leads to a higher probability of initiating production and exporting a given product —i.e, leads to product innovation in the extensive margin.

I develop a dynamic general equilibrium model that is consistent with both the stylized facts and the market access mechanism to rationalize this reduced-form evidence. Like much of the trade and growth literature, the model presented in this paper incorporates forward-looking dynamics. However, unlike much of the literature, it shies away from stylized simplifications, such as symmetric countries or two-country cases. It encompasses an arbitrary number of asymmetric countries, costly trade, and is fit for counterfactual quantitative exercises. Therefore, it fits neatly into the tradition of quantitative trade models in international trade or policy counterfactuals using dynamic stochastic general equilibrium models in macroeconomics.[5]

provide an extensive documentation of the empirical facts.

[5]In the trade literature, this is the modern world of "trade theory with numbers" (Costinot and

In the model, in each source country, there are producers of final goods varieties that combine labor and intermediate goods using a constant returns to scale technology. They source differentiated intermediate varieties from foreign countries. These countries differ in their product spaces: some countries have a measure of intermediate goods that are larger than others.

Intermediate goods are *non-rival* in the same spirit as in the endogenous growth literature. As new varieties are invented, they can be simultaneously and immediately sourced by final goods producers everywhere, inducing increasing returns.

In this framework, international trade induces substitutability across non-rival goods. Trade also implies that the measure of intermediate varieties that effectively diffuses to each country will be a price-weighted average of the measure of varieties imported from all trade partners. If there are no trade costs, all countries will share the same effective measure of varieties. Conversely, in autarky, each country will only take advantage of its own varieties.

At each destination, final good varieties from every source are aggregated into a final composite good with some probability. Those actually sourced for aggregation will be only the lowest-cost varieties at each destination. Prices depend on productivities, taken to be the realization of a random variable.

The final composite good is used for household consumption and as an input for the production of intermediate varieties and research & development (R&D). Once a new blueprint is invented, each intermediate goods producer has perpetual rights over the production of its variety. They produce under monopolistic competition and set prices optimal prices accordingly through market-specific price discrimination.

Rodríguez-Clare, 2014). In the macroeconomics literature, this is the use of macro models as "the leading tool" for assessing the effect of policy changes in "an open and transparent manner." (Christiano, Eichenbaum, and Trabandt, 2018).

Since this model embeds an input-output structure, the optimal monopolist prices will depend on the price of the final composite good at the intermediate source countries.

Forward-looking households use equity markets to invest their savings in the R&D of new goods. For each unit of the final good invested in a new R&D project, there is a risky return on investment with probability determined by a *Poisson* process. At an aggregate level: domestic households hold a balanced portfolio of infinitely many small firms, such that they face no idiosyncratic risk; savings equal investment; investment flows determine the growth of varieties; and a non-arbitrage condition connects the real interest rate (the asset market) to real returns on R&D (the equity market).

Over the balanced growth path (BGP), I prove that the equilibrium will be characterized by a stable distribution of income and measures of varieties, the real interest rates will equalize across countries, and all countries will grow at the same rate. Even though there are no international capital markets, trade acts as a vehicle that will integrate R&D stocks and returns. Countries with larger labor forces will have larger equilibrium measures of varieties, which is a fact also observed in the cross-section of countries in the data.

Exploiting the linearity of income in the measure of varieties, I derive an analytical decomposition for the BGP growth rate across labor and capital income shares of GDP, whose elements can be further interpreted as "Romer" and "Eaton-Kortum" components, giving intuitive meaning to the results. The Eaton-Kortum component of growth is very much Ricardian, i.e., related to technology, while the Romerian is related to market access, both domestically and internationally.

Another contribution of the paper is to provide a formula for welfare gains from trade that decomposes welfare into static and dynamic components. The welfare formula subsumes the static results of Arkolakis, Costinot, and Rodríguez-Clare (2012) into a dynamic framework. Like the growth formula, the static component of welfare also has Ricardian and Romerian margins, with the Romerian margin augmenting the Ricardian one through an extensive margin. One of the technical contributions of the model is a tractable way of integrating a new product margin into the Eaton-Kortum framework, which is one of the workhorse models in the international trade literature and lacks such a margin.

By comparing the static and dynamic components of welfare, the model clarifies that they work through different mechanisms, rationalizing the two forces of market access and specialization. The reason is that the former operates on households as consumers and the latter on households as producers and investors

An additional theoretical insight lies in accounting for market access as an avenue for growth and product innovation. Increased market access is related to a higher steady-state equilibrium product innovation growth rate. This finding highlights the positive impact of trade integration on fostering product innovation and is consistent with the reduced-form evidence presented in the beginning of the paper.

The final contribution is to set up and calibrate a quantitative version of the model that solves for the endogenous balanced growth path of the model with an experiment of asymmetric country groups and costly trade. I then use this framework and apply trade cost shocks to replicate the policy scenario of the 2004 Eastwards enlargement of the European Union.

The outcome of the numerical exercise is a set of results and decompositions of both static and dynamic welfare as a result of the EU enlargement. This toolkit suggests that: (a) the EU enlargement increased its long-run yearly growth rate by about 0.10pp; (b) the share of "Eaton-Kortum" share in static gains from trade can vary widely across countries, being as large as 90% for some countries and as small as 10% for some other countries; (c) dynamic gains can account for a large share of total welfare gains from trade; and (d) the share of dynamic gains also varies across countries, ranging from 65-90% of total welfare gains from trade.

Related Literature

This paper adds to the theoretical literature on trade and growth —and in particular to trade and product innovation. The literature can be traced back to the seminal paper by P. M. Romer (1990). While Romer does not develop a full model, he mentions in the paper that a natural extension of his model "pertain to its implications for growth, trade, and research."[6] Extensions of the Romer model of endogenous growth of product innovation to a two-country framework were later done by Rivera-Batiz and P. M. Romer (1991a) and Rivera-Batiz and P. M. Romer (1991b) as well as Grossman and Helpman (1990), in a very similar framework. I extend the Romer growth model to a multiple asymmetric country framework and combine it with a modern quantitative Ricardian trade model of Eaton and Kortum (2002).

The model is also related to the work by Acemoglu and Ventura (2002), who proposed a model with Armington trade that features an AK-model of trade and growth with a stable distribution of income over the balanced growth path. While groundbreaking, they restrict their analysis to the costless trade case, while in this paper trade costs can be positive with much more heterogeneity across countries.

Since modeling the complete state space of dynamics and countries is nontrivial, most of the trade and growth literature has to make compromises. Part of the literature simplifies by assuming a world of symmetric countries (Perla, Tonetti, and Waugh, 2015; Sampson, 2016) or a two-country world (Eaton and Kortum, 2006; Hsu, Riezman,

[6]This is in section VII of P. M. Romer (1990).

and Wang, 2019; Helpman, 2023). Another part, while adding the heterogeneity to the cross-section, rules out forward-looking dynamics and models growth as some external diffusion process (Buera and Oberfield, 2020, Cai, Li, and Santacreu, 2022). My model departs from most of the literature by having both asymmetric countries and forward-looking dynamics in a theoretical and quantitative framework.

As will be clear in the next section, it is a "true macro model" combined with a "true trade model." In this sense, it is more similar to the very recent models of Sampson (2023) and Kleinman et al. (2023). However, unlike mine, the latter is a model of convergence rather than a model of long-run growth and the former is a model of firm-productivity growth rather than product innovation.

My paper makes two sets of contributions to the empirical literature. First, it documents a collection of facts using production-and-trade data around the enlargement episodes of the European Union. This first part of the analysis is more akin to papers like Hummels and Klenow (2005), Andrew B Bernard et al. (2009), Kehoe and Ruhl (2013), and Arkolakis, Ganapati, and Muendler (2020), which provide noncausal documentation of novel stylized facts regarding the extensive margin. But the paper also goes beyond that, using plausibly exogenous variation in an event-study design using a very detailed source-destination-product-year dataset. In doing so, it relates more papers like Goldberg et al. (2010), Bas (2012), Argente et al. (2020), and Rachapalli (2021), which estimate well-identified empirical effects regarding product innovation.

The paper is organized as follows. Section 1.2 introduces the empirical evidence that motivates the work, first summarizing some stylizing facts and then providing some causal evidence on the relationship between market access and product innovation. Later, Section 1.3 lays down the theory, introduces the model, defines the equilibrium, and states the main results regarding the existence of the balanced growth path and the equilibrium growth rate being decreasing in trade costs. Afterward, Section 1.4 uses a numerical version of the model to estimate the dynamic welfare effects of the 2004 enlargement of the European Union and decomposes them using the key results of the previous section. Finally, I conclude by trying to relate the main takeaways to the general literature and where the main advances were.

1.2 Empirical Evidence

This section describes the evidence related to international trade and product innovation in the context of the different enlargement waves of the European Union (EU). First, it describes the data. Then it presents some stylized facts comparing new member states (NMS) of the EU relative to candidate countries. Finally, it uses an event-study approach to isolate some plausibly exogenous variation of trade costs on the probability of initiating production of a new product.

Data sources

Production data comes from Eurostat's Prodcom (*Production Communautaire*), which is an annual full coverage survey of the European mining, quarry and manufacturing sectors, reporting the value of production of 4,000+ different product-lines of EU members and candidate countries. Data are really high-quality and coverage error is estimated to be below 10%. These data allows one to create a time-series of product counts for products actually produced in each member state and candidate countries of the European Union.

Bilateral tariff data come from WITS (World Integrated Trade Solution Trade Stats). It consolidates tariff data from the UNCTAD's Trade Analysis Information System (TRAINS) as well as from the WTO. Bilateral trade flow data comes from UNCOMTRADE.

I matched all of these to the production data using Eurostat's concordance between Prodcom product-codes, which are finer than Harmonized System (HS) 6-digit level, to a HS-6 digit level. WITS and UNCOMTRADE data come natively at an HS-6 digit level. Combined, these constitute a novel production-and-trade matched database.

I also collected data on (a) the dates of accession of new member states to the European Union; (b) trade agreements existent and entered into force between the European Union and third parties before 2004; and (c) expenditure in private research & development expenditures per capita. The first two come from hand collecting documents and tables from the European Commission's official websites while the latter comes from Eurostat.

Further details on the data used, data matching, and data construction are on Appendix 1.7.4.

Institutional Context

Throughout this paper, the institutional setting will be expansion of the European Union —or its *enlargement*, how it is typically called in EU language. The enlargement happened in different waves as the EU included more members from the six original members that created the group in 1957, as shown in Figure 1.1.

I use this setting in three ways. First, I exploit the staggered nature of the EU expansion to summarize some stylized facts regarding some key statistics of New Member States (NMS) of the EU relative to candidate countries who are not yet members of the EU at a given horizon.

Then, focusing on the largest expansion wave in 2004, I show that that the adoption of the EU's Common Commercial Policy induced a plausibly exogenous variation in market access between 9 NMS that joined the EU in 2004[7] and 12 third-party countries

[7]While ten countries joined the EU in 2004, I do not have product-level PRODCOM data for Malta.

with previously existing trade agreements with the EU. Finally, in the numerical quantification exercise, I will also use the enlargement waves as natural country groups to estimate the welfare effects of the 2004 enlargement.

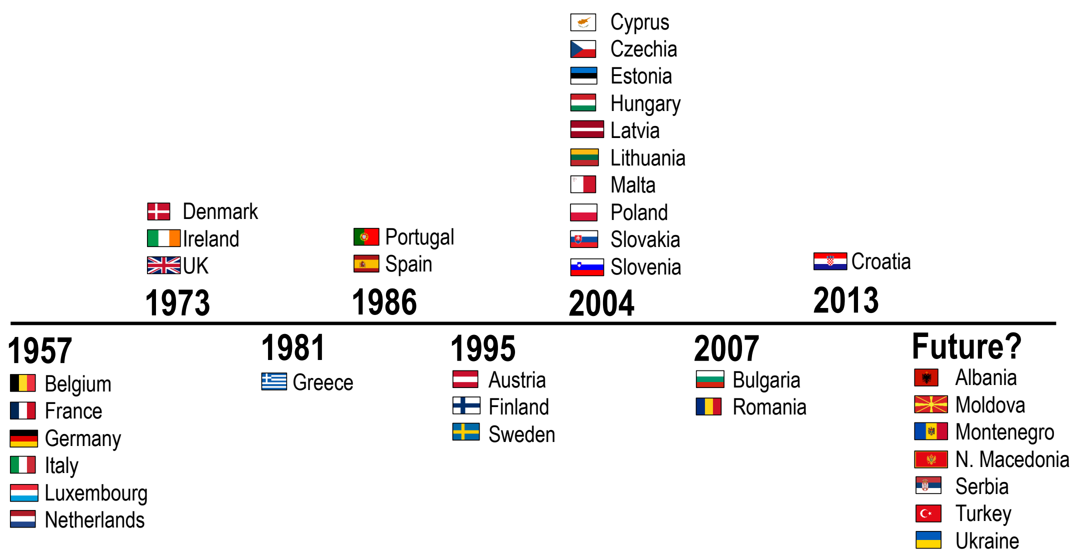


Figure 1.1. Institutional Context: Timeline of European Union Enlargement. The EU enlargement comes in waves. The future cohorts serve as comparison groups, for some time, to previous cohorts. The largest expansion wave is 2004.

Stylized Facts

My stylized facts compare two groups: countries that became new members of the European Union *relative to countries that self-selected into becoming candidates for EU membership but were not yet members at a given time horizon*, exploiting the staggered nature of the enlargement. Here, one can think of countries that became EU members as individual members of a "treatment group" and candidate countries that applied for EU membership but had not yet become members by that time as individual members of a "control group." Of course, since treatment assignment, in this case, is not random, this is not actually a true experiment.

In this paper, to avoid the potential biases of the Two-Way Fixed Effects (TWFE) estimator in summarizing the data, I adopt the Callaway-Sant'Anna (CS) estimator[8]. In a nutshell, CS calculates group-specific treatment effects by: (a) comparing the treated group with either the not-yet-treated groups or the never-treated groups; and then (b) aggregating them into an average treatment on the treated given a specific set of weights. This estimator is consistent even if true treatment effects are heterogeneous.

Therefore, even if the objective is to simply summarize the data rather than to make causal claims, one would still want to avoid making "forbidden comparisons." The CS estimator, in this case, will simply recover the average difference in outcomes for NMS relative to countries that are candidate countries but are not yet members, at different horizons around EU enlargement events. In Appendix 1.7.4, I formal description of the CS estimator used here.

Given the weighting scheme of the CS estimator, in the estimates reported below, the event that will have the largest weight will be the 2004 EU expansion, which enlarged membership by ten countries, but only nine are observed in PRODCOM data. The other episodes of expansion – Bulgaria and Romania, in 2007; and Croatia, in 2013 – influence the estimates with proportional weights for the horizons in which data is available. It is important to highlight that throughout the sample, there is readily available data for candidate countries that never became EU members, which serve as a natural comparison group.

Here, I run the staggered difference-in-differences event study regressions for a set of variables, using similar models. First, using the *measure of produced varieties* as the dependent variable. Then with *log of real private research and development expenditures*

[8] Goodman-Bacon (2021) shows that the TWFE estimator is a weighted average of all possible two period-two group comparisons and that, as emphasized by Borusyak, Jaravel, and Spiess (2022), it is biased if treatment effects are heterogeneous. Sun and Abraham (2021) proposed a new estimator that accommodates treatment effect heterogeneity, which was later generalized by Callaway and Sant'Anna (2021).

and the *log of real value of yearly trade* as dependent variables.

The frame of reference is to take these variables as aggregate *macro moments*. Relative to a candidate country that did not fully integrate its economy with the European Union and did not have preferential access to the trade partners of the EU, what happens, *on average*, to these variables in New Member States?

As shown in Figure 1.2, fifteen years after membership, the expected differential increase in varieties is 306, or about 17% relative to the year of membership[9]. The differential effect seems to cumulatively increase after the year of membership.

The effects regarding private R&D, shown in Figure 1.3 show a clear break in trend in the differential averages around the year of membership. Fifteen years after the expected differential growth in private R&D expenditures is about 60%.

Finally, the results relative to trade also show a differential growth, as illustrated by Figure 1.4. There are no signs of pre-treatment trends and, seven years after membership, the expected differential growth in the value of yearly trade is about 50%.

[9]The average treatment on the treated is 306.23 and the conditional average number of produced varieties in treatment year zero is 1804.6.

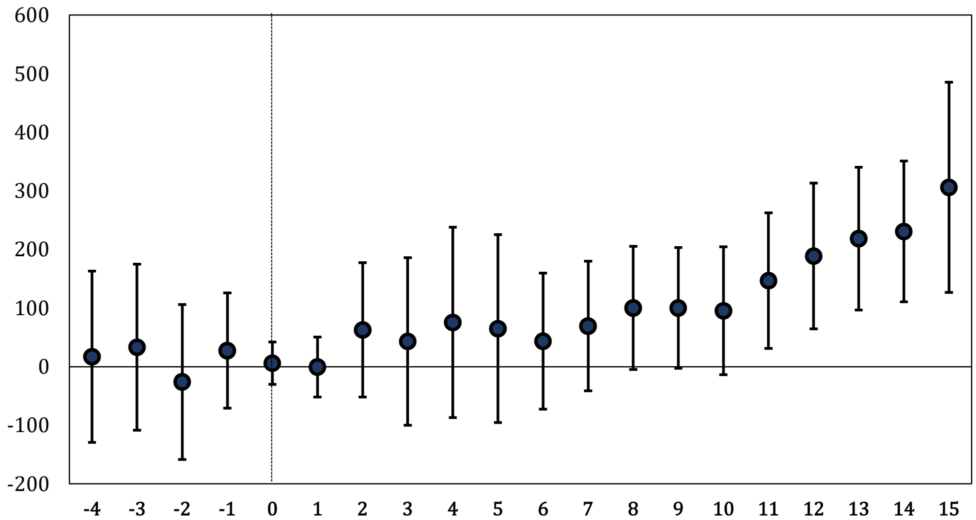


Figure 1.2. Staggered difference-in-differences: Measure of Varieties. ***X-axis: years around EU enlargement event. Y-axis: in number of produced varieties.*** This plot shows the estimated coefficients $\theta(t)$ time-specific average treatment on the treated coefficient described by equation (1.35) at the and aggregate level. The bars around the red line denote 95% bootstrapped standard errors.

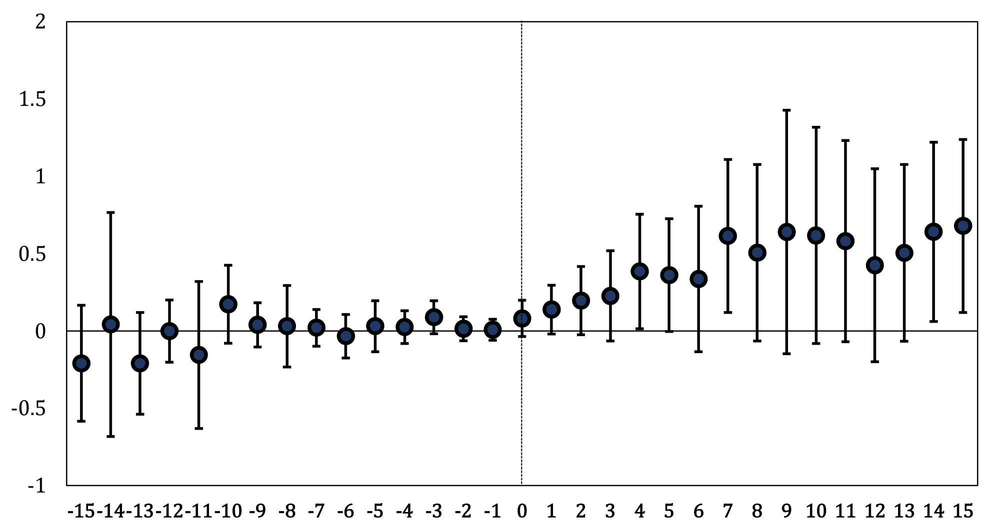


Figure 1.3. Staggered difference-in-differences: Log of Private Research and Development Expenditures Per Capita. ***X-axis: years around EU enlargement event. Y-axis: in log value private yearly R&D expenditures per capita (thousand euro).*** This plot shows the estimated coefficients $\theta(t)$ time-specific average treatment on the treated coefficient described by equation (1.35) at the aggregate level. The bars around the red line denote 95% bootstrapped standard errors.

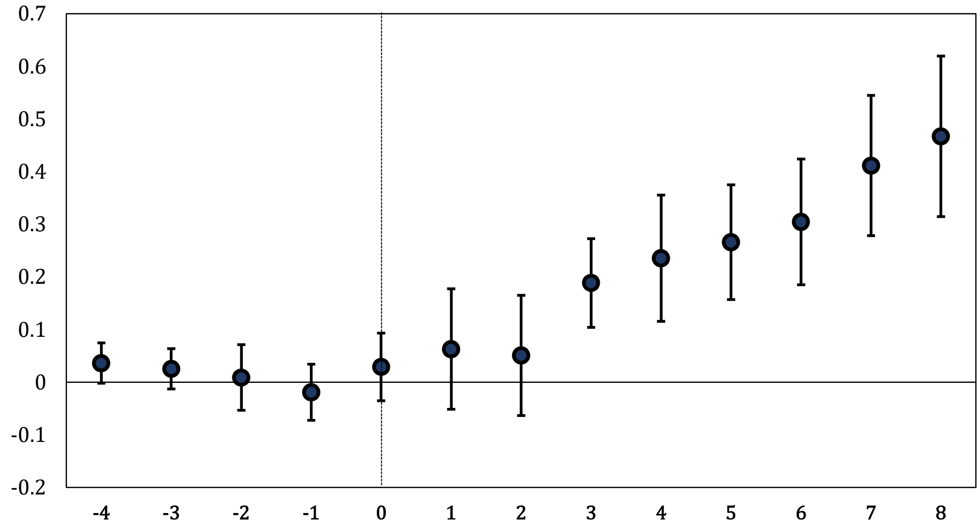


Figure 1.4. Staggered difference-in-differences: Log of Real Value of Yearly Trade. ***X-axis: years around EU enlargement event. Y-axis: in log real value of yearly trade (thousand US Dollars).*** This plot shows the estimated coefficients $\theta(t)$ time-specific average treatment on the treated coefficient described by equation (1.35) at the aggregate level. The bars around the red line denote 95% bootstrapped standard errors.

Causal Evidence

Once NMS joins the EU, they not only have preferential access to the European market, but they also have to adhere to the Common Commercial Policy of the European Union. This means that these countries have immediate preferential access to third-party markets via previously existing trade agreements between the EU and these third parties. Furthermore, since these trade agreements previously existed, while the NMS had immediate access to them, they did not get to negotiate the tariff variation that they faced —these were only a byproduct of the EU accession process.

Figure 1.5 illustrates how this happened in a specific example: the Free Trade Agreement between the EU and Mexico. The EU joined a FTA with Mexico in 2000, but the NMS only joined the EU in 2004, so in 2004 the latter immediately adhered to these previously negotiated tariff schedule. The product-level bilateral tariff variation $\Delta\tau_{sdip,2004}$ which was a by-product of the EU accession process is my measure of the market accession shock.

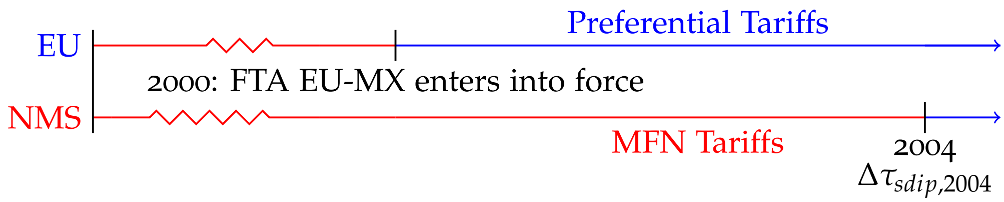


Figure 1.5. Event Study Design, Constructing the Trade Shock: I use the fact that when the NMS joined the EU in 2004, they had immediate preferential access to third-party markets via previously signed EU trade agreements which the NMS did not get to negotiate. The product-level bilateral tariff variation $\Delta\tau_{sdip,2004}$ which was a by-product of the EU accession process is my measure of the market accession shock. In the example above, the EU joined a FTA with Mexico in 2000, but the NMS only joined the EU in 2004, so in 2004 they immediately adhered to these previously negotiated tariff schedule.

I focus on the largest wave of enlargement was in 2004. The source of variation is at the source-country $\times$ destination-country $\times$ HS-6-code product level. The metric of the tariff shock change is simply $\Delta\tau_{sdip,2004} \equiv -(\tau_{sdip,2004} - \tau_{sdip,2003})$, which is the change in the level of effectively applied bilateral tariffs at the product level between 2003 and 2004. In each year, there are about 300 thousand observations.[10]

I estimate a sequence of cross-sectional local-projection linear probability models, which estimate what is the marginal effect of a *decrease* in the tariffs on exports of a given product p, *conditional on that country s not producing that particular product before joining the EU in 2003*. The fact the data is highly granular permits me to exploit within *industry* $\times$ *source* $\times$ *destination* $\times$ *horizon* (across product) variation.

Formally, I estimate the following equation:

$$P\left(X_{sdip,h} > 0 \middle| Y_{s\cdot ip,2003} = 0\right) = \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h} \quad \text{for } h \in \{2000, \cdots, 2010\} \tag{1.1}$$

[10]Leading up to 2004 there were no large changes in bilateral tariffs between NMS and third parties but between 2003 and 2004 there was a median drop of about 2.5 percentage points. In the years after the enlargement, there was also not a large change in the distribution of bilateral tariff rates, which is shown in Figure 1.15 in Appendix 1.7.4. Figure 1.16 in Appendix 1.7.4 plots the distribution of $\Delta\tau_{sdip,2004}$, excluding the zero-valued observations.

where $X_{sdip,h}$ is the market value of exports between country s and country d of product p of industry i at horizon h; $Y_{s \cdot ip,2003}$ is the market value of production in country s of product p of industry i in 2003; α_h are horizon (time) fixed-effects; $\gamma_{sdi,h}$ are *source* $\times$ *destination* $\times$ *industry* interactions fixed-effects for each h. These types of cross-sectional event studies with local projections can be interpreted as differences in differences with continuous treatments[11].

This strategy takes seriously the assertion in Baier and Bergstrand (2007) that countries engage endogenously in free trade agreements (FTAs). The identification assumption is that conditional on the very saturated fixed effects that this model includes, the unobserved components $\nu_{sdip,h}$ are uncorrelated with the change in tariffs $\Delta\tau_{sdip,2004}$. Intuitively, the identification is robust to a NMS (say, Poland's) policymakers endogenously targeting EU accession to have preferential access to a third-party's (say, Mexico's) car industry (relative to other industries and countries), but not if they want to have preferential access to compact cars relative to SUVs in Mexico. For further details on the methodology, see Appendix 1.7.4.

As shown in Figure 1.6, an increase in market access by 1 percentage point increases the probability of starting to produce and export a given product by about 1 percent by 2010. To benchmark this result, it is about one-third of the conditional mean $\mathbb{E}[X_{sdip,h} > 0 | X_{s \cdot ip,h} > 0, h > 2003] = 2.9\%$. There are no signs of a pre-existing trend before 2004: both the magnitude of the coefficients and the standard errors are very small before the treatment date.

Summary of Empirical Evidence

I have documented the following novel facts: as *New Member States* go through a large trade integration event with the European Union, they *start producing new product*

[11] See Chodorow-Reich (2019) and Dube et al. (2023).

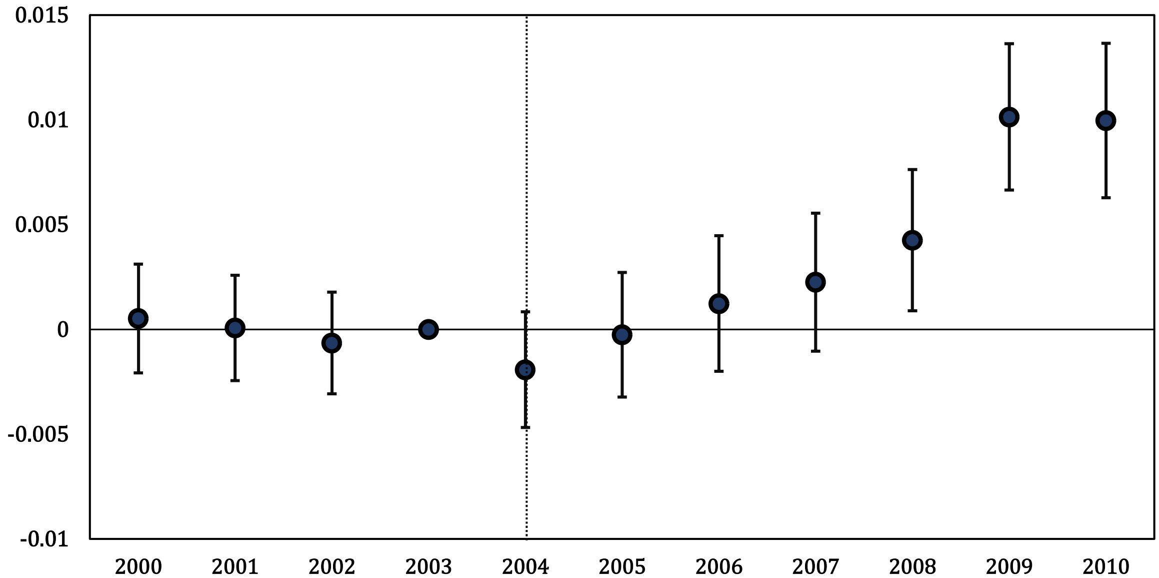


Figure 1.6. Entry Regressions. This plot shows the coefficients β_h of the local projection linear probability models specified in equation (1.1). Each year is a different cross-sectional regression with approximately 300 thousand observations. The whiskers show 95% confidence intervals with robust standard errors clustered at the source-destination-industry level.

varieties; investing more in research and development; and trading more in real values relative to candidate countries that are not yet members at a given horizon. Furthermore, I have shown that as they are exposed to a *plausibly exogenous new market accession shock*, as it happened due to the idiosyncrasies of the Common Commercial Policy of the EU, *they increase their probability of starting production of a new product variety*, which suggests that market access increases the rate of product innovation.

To rationalize this reduced-form evidence, I develop a dynamic general equilibrium model that is consistent with both the stylized facts and the market access mechanism. This is what I do in the following section.

1.3 Theory

Here I present a dynamic multi-country model of the world economy with intertemporal optimization, investment in research and development, and trade in final and intermediate goods. In this economy, time is continuous with $t \in \mathcal{T} \equiv [0, \infty)$ and countries indexed by $s \in \boldsymbol{K} \equiv \{1, \cdots, N\}$.

Every country has the ability to produce final goods $\omega \in [0,1]$. However, they differ in their ability to produce non-rival intermediate inputs $\nu \in [0, M_s(t)]$, where the upper bound of the interval $M_s(t)$ defines the product space of a particular country. Intermediate goods are *non-rival* in the same spirit as in the endogenous growth literature: new blueprints can be simultaneously used by multiple producers at the same time, inducing increasing returns to scale[12].

As intermediate goods are invented, trade acts as a mechanism that diffuses new blueprints: producers expand their production function by sourcing newly minted inputs from around the world. Exporters are monopolists in their intermediate varieties and therefore have the incentive to invest in the development of new varieties, thereby propelling growth. Therefore, international trade will work as a vehicle that integrates global research and development stocks and induces growth-rate convergence over the balanced growth path.

My goal is to make this model easily accessible and recognizable for someone who is familiar with either modern trade theory or modern growth theory. This model will recover, as special cases, the Eaton and Kortum (2002) model of trade and the P. M. Romer (1990) model of growth. Some functional form assumptions will be such that this nesting is clear.

[12]See Jones (2005) and Jones (2019) for extensive reviews.

1.3.1 Demand

In each country $s \in \boldsymbol{K}$, there is a representative household the maximizes its lifetime utility according to:

$$\begin{aligned} \max_{C_s(t)} & \quad \int_0^\infty \exp\{-\rho t\} \log\left(C_s(t)\right) dt \\ s.t. \quad P_s(t) I_s(t) + P_s(t) C_s(t) & = r_s(t) A_s(t) + w_s(t) L_s \end{aligned}$$

where $P_s(t) C_s(t)$ are aggregate consumption good prices and quantities in country s; $I_s(t)$ are instantaneous investment flows; and $w_s(t)$,$r_s(t)$ are wages and interest rates. At any instant, the state of asset holdings is simply the cumulative investment flows: $A_s(t) \equiv \int_0^t I(s) ds$[13].

Households choose a sequence of consumption quantities for the aggregate good, satisfying the Euler Equation:

$$\frac{\dot{C}_s(t)}{C_s(t)} = \frac{r_s(t)}{P_s(t)} - \rho \tag{1.2}$$

1.3.2 Production and Trade in Varieties

There are three kinds of producers in each country: *those who produce varieties of the final good*, *those who produce varieties of intermediate goods*, and *those who invest in research and development*. This section will focus on the two first ones.

Final Goods Producers.

In each country, a local assembler for the final composite good $Y_d(t)$ who operates under perfect competition uses the least expensive variety $\omega \in [0,1]$ available at $d \in \boldsymbol{K}$

[13]This, of course, implies that one can write investments as $I_s(t) = \dot{A}_s(t)$, which clarifies the optimal control problem at hand.

with the following technology:

$$Y_d(t) = \left[\int_0^1 y_d(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is a constant elasticity of substitution across sourced varieties ω. Under these assumptions, the ideal price index of the final good satisfies $P_s(t)$ satisfies:

$$P_s(t) = \left[\int_0^1 p_s(t,\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

A producer of each variety $\omega \in [0,1]$ of the final good is endowed with a constant returns to scale technology that combines labor and intermediate inputs $\nu \in [0, M_s(t)]$ coming from multiple countries $k \in \boldsymbol{K}$:

$$y_s(t,\omega) = z_s(t,\omega)[\ell_s(t,\omega)]^{1-\alpha} \left(\frac{1}{\alpha} \sum_{k \in \boldsymbol{K}} \int_0^{M_k(t)} [x_{ks}(t,\omega,\nu)]^{\alpha} d\nu \right) \tag{1.3}$$

where $z_s(t,\omega)$ is total factor productivity; $\ell_s(t,\omega)$ is factor demand for labor for variety $\omega \in [0,1]$ located in country s; and $x_{ks}(t,\omega,\nu)$ is the demand for a intermediate good of variety $\nu \in [0, M_k(t)]$ sourced from country k for production as an input of a final good in country s.

Non-rival intermediate goods varieties are differentiated across countries: an input $\nu \in [0, M_k(t)]$ is different from $\nu \in [0, M_n(t)]$, even if it is indexed by the same symbol. For instance, the first one may be a twelve-core computer chip from Estonia while the second one may be a large language model from Malta. Additionally, note that countries differ in their ability to produce intermediate goods, which is denoted by the upper bound of the integral $M_k(t)$. Optimal demand for an intermediate good satisfies:

$$x_{ks}(t,\omega,\nu) = \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot z_s(t,\omega)^{\frac{1}{1-\alpha}} \tag{1.4}$$

Intermediate Goods Producers.

Each intermediate goods producer in country s has perpetual rights over the production of each variety $\nu \in [0, M_s(t)]$. They are endowed with a linear technology that transforms one unit of the final good into one unit of the intermediate good.

Assumption 1 (Trade Costs)**.** *Trade is subject to iceberg trade costs, which implies that shipping a final or intermediate good variety from source region s to a consumer in region d requires producing* $\tau_{sd} \geq 1$*, where* $\tau_{dd} = 1$ *and* $\tau_{sd} = \tau_{ds}$ *for all* $s, d \in \boldsymbol{K}$.

Given assumption (1), intermediate goods producers face heterogeneous marginal costs and set optimal prices accordingly through market-specific price discrimination. They take marginal costs and demand curves as given and choose optimal prices to maximize profits, with the optimal price being a mark-up over marginal costs for every variety ν and ω:

$$p^M_{ks}(t) = \frac{\tau_{ks} P_k(t)}{\alpha} \qquad \forall \omega \in [0,1], \quad \nu \in [0, M_k(t)]$$

Note that this is the standard result of profit maximization under monopolistic competition with two variations. First, as in most trade models, prices are differentiated by destination and are inclusive of trade costs τ_{ks}. Second, since intermediate goods use one unit of the final good at the origin country k to produce one unit of the intermediate good, its marginal cost is $P_k(t)$. Optimal monopolist prices being independent of variety ν imply that demand for is symmetric:

$$\bar{x}_{ks}(t,\omega) \equiv \left[z_s(\omega) \cdot \frac{p_{ss}(t,\omega)}{p^M_{ks}(t)} \right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \qquad \forall \nu \in [0, M_k(t)]$$

Given the result above, rewrite the final goods firm maximization problem in the following way:

$$\max_{\ell_s(t,\omega)} \frac{1}{\alpha} [p_{ss}(t,\omega) \cdot z_s(t,\omega)]^{\frac{1}{1-\alpha}} \cdot \tilde{M}_s(t) \cdot \ell_s(t,\omega) - \ell_s(t,\omega) w_s(t) \tag{1.5}$$

which comes from substituting for $\bar{x}_{ks}(t)$ and defining:

$$\underbrace{\tilde{M}_s(t)}_{\substack{\text{effective}\\ \text{measure of}\\ \text{input varieties}}} \equiv \sum_{k \in \boldsymbol{K}} \underbrace{M_k(t)}_{\substack{\text{measure of}\\ \text{varieties}\\ \text{in each } k}} \cdot \left(\underbrace{p^M_{ks}(t)}_{\substack{\text{optimal monopolist}\\ \text{price from } k \text{ to } s}} \right)^{1-\eta} \qquad \text{where } \eta = \frac{1}{1-\alpha} \tag{1.6}$$

The effective measure of input varieties is a key object in this model that captures the diffusion of non-rival intermediate goods to country s. It measures input varieties sourced from each country weighted by marginal cost. The first term on the right-hand side captures heterogeneity in the source-country measure of varieties since final goods producers are sourcing intermediate varieties internationally. The second term captures the substitutability across intermediate goods, controlled by the elasticity of substitution η. The exponent $1 - \eta < 0$ down-weights the relative importance of intermediate goods coming from source countries k with relatively more expensive intermediate inputs.

This object also makes explicit how the model nests both the Eaton-Kortum model of trade and the Romer growth model. If $\alpha \to 0$, then there is no intermediate sector. The technology (1.5) collapses into a linear production function as in Eaton and Kortum (2002). If the world is in autarky —i.e., if $\tau_{sd} \to \infty$ for all $d \neq s$ —then, after setting $P_s(t) = 1$ as the numéraire of the home economy, $\tilde{M}_s(t) = \alpha^{\frac{\alpha}{1-\alpha}} M_s$ and the final goods technology becomes linear in labor with an extensive margin M_s, as in P. M. Romer (1990)[14].

Even outside the limiting cases expressed above, one should observe that the final goods producer's technology in this model is related to those in both the Eaton-Kortum and the Romer models. It is equivalent to a simple Eaton-Kortum model that uses the final good as an intermediate input with an added extensive margin shifter $\tilde{M}_s(t)$. It also is related to the technology of the Romer model, which is linear in labor (or human capital), except that in this model the measure of varieties component is a weighted average of inputs coming from domestic and international suppliers.

Note that with a slight redefinition, one can also interpret a transformation of $\tilde{M}_s(t)$ as the price of a composite basket of intermediate goods, as it is standard in many models that resort to a constant elasticity of substitution:

$$P_s^M(t) \equiv \tilde{M}_s(t)^{\frac{1}{1-\eta}} = \left(\sum_{k \in \boldsymbol{K}} M_k(t) \cdot p_{ks}^M(t)^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{1.7}$$

which is how, due to notational convenience, this object will appear throughout the rest of the paper. Define the value in final goods in country $d \in \boldsymbol{K}$ to be $P_d(t)Y_d(t)$. Then, using the definitions above and the properties of C.E.S. and Cobb-Douglas, intermediate goods sales by country s in country d equal:

[14]For a more detailed description of the nesting, see Appendix 1.7.2.

$$M_s(t)p_{sd}^M(t)x_{sd}(t) = \alpha \cdot M_s \left(\frac{p_{sd}^M(t)}{P_d^M(t)}\right)^{1-\eta} \cdot P_d(t)Y_d(t) = \alpha \cdot \lambda_{sd}^M(t) \cdot P_d(t)Y_d(t)$$

The last equation follows from defining intermediate trade shares $\lambda_{sd}^M(t) \equiv \frac{M_s p_{sd}^M(t)^{1-\eta}}{\sum_{k \in \boldsymbol{K}} M_k p_{kd}^M(t)^{1-\eta}}$.

In the standard P. M. Romer (1990) model, assemblers source intermediate goods exclusively from domestic suppliers. One important implication of that assumption is symmetry: the price of all intermediate goods will be the same. Conversely, in this framework, when sourcing intermediate goods from multiple countries $k \in \boldsymbol{K}$, the prices of these goods will no longer be necessarily the same.

Economically, it is this lack of symmetry in prices that will induce substitutability across varieties sourced from different countries, which is reflected in the composite price of intermediate goods above. The more dissimilar countries are in terms of their relative unit costs, the more substitution across intermediate goods will occur. Hence, accommodating asymmetric countries in a dynamic framework will be an important feature of this model.

Trade in final goods.

The factory gate price $p_{ss}(t,\omega)$ for a variety has three components: the unit production cost $w_s(t)$, the price of intermediate goods $P_s^M(t)$, and a producer-specific productivity $z_s(t,\omega)$. Destination prices also include iceberg trade costs. Under perfect competition, consumers in country d choose the lowest price variety ω available at the domestic market:

$$p_d(t,\omega) = \min_{s\in \boldsymbol{K}} \left\{ p_{sd}(t,\omega) \right\} = \min_{s\in \boldsymbol{K}} \left\{ \tau_{sd} p_{ss}(t,\omega) \right\} = \min_{s\in \boldsymbol{K}} \left\{ \frac{P_s^M(t)^{\alpha} w_s(t)^{1-\alpha} \tau_{sd}}{z_s(t,\omega)} \right\} \tag{1.8}$$

Assumption 2 (Productivity draws)**.** *Following Eaton and Kortum (2002), assume that $z_s(t,\omega)$ is an iid random variable drawn from a market-specific Fréchet distribution*

$$F_s(t)(z) = \exp\left\{ -T_s z^{-\theta} \right\}.$$

where T_s is the the scale parameter and θ is the shape parameter.

Given assumption (2), both prices and demanded quantities (which are functions of productivity draws) are also random variables. By the law of large numbers, the share of varieties sourced from s to d equals[15]:

$$\lambda_{sd}^F(t) \equiv \frac{E_{sd}^F(t)}{E_d^F(t)} = \frac{T_s (P_s^M(t)^{\alpha} w_s(t)^{1-\alpha} \tau_{sd})^{-\theta}}{\sum_{n\in \boldsymbol{K}} T_n (P_n^M(t)^{\alpha} w_n(t)^{1-\alpha} \tau_{nd})^{-\theta}} \tag{1.9}$$

where $E_{sd}^F(t)$ denotes the expenditure on final goods going from country s to country d; $E_d^F(t)$ denotes total expenditure on final goods in country d.

1.3.3 Research and Development

The *research sector* creates new varieties of the intermediate good. One can think of this sector as investing in the invention of new machines, which result in new blueprints. These firms use ψ units of the final good as inputs to research and development (R&D), but success is not guaranteed.

[15]Since there are infinitely many varieties ω and productivities are iid random variables, by the law of large numbers, the share of varieties sourced from s to d converges almost surely to the probability of sourcing a specific variety from s to d.

Assumption 3 (Research and Development Process)**.** *The success rate of R&D follows a Poisson process with flow arrival rate equal to* $\psi I_s(t)dt$*, where* $I_s(t)$ *is the research input, measured in units of the final good per time unit.*

Once researching firms invent a new machine, they hold perpetual monopoly rights over the new variety ν. They can either set up their own shop to produce and enjoy the profits of producing such variety at the market or, alternatively, they can sell the rights to this patent to an intermediate variety producer. In either case, domestic households, that finance the invention of new varieties through capital markets, will collect the profits.

The economic value of a new variety is the present value of producing the new varieties and selling them as intermediate inputs to final goods producers, which is, at period t:

$$V_s(t,\nu) = \int_t^{\infty} \exp\left\{ -\int_t^{\tau} \frac{r_s(k)}{P_s(k)} dk \right\} \pi_s(\tau,\nu) d\tau \tag{1.10}$$

where $\pi_s(\tau,\nu)$ is the flow profit per variety per unit of time. Research firms will only invest if the expected return of their investment is positive, that is $\psi V_s(t,\nu) I_s(t,\nu) - P_s(t) I_s(t,\nu) \geq 0$. With free entry, this condition holds with equality and in equilibrium it pins down the value of each variety: $V_s(t,\nu) = P_s(t)/\psi$.

Since the only asset market in this economy is the domestic equity market, domestic households save by funding investments in new varieties through a balanced portfolio of infinitely many small firms, such that they face no idiosyncratic risk. At the aggregate level, then, $\dot{M}_s(t) = \psi I_s(t)$, where $I_k(t)$ is the level of aggregate investment in the domestic economy measured in units of the final good. The value of aggregate assets is simply the value of all invented varieties $P_s(t) A_s(t) = M_s(t) V_s(t)$ and, since the arrival rate of ideas is constant, the total stock of assets is at any instant a function of the total measure of varieties $A_s(t) = M_s(t)/\psi$.

Taking the derivative of both sides of (1.10) with respect to time and noting that both $V_s(t,\nu)$ and $\pi_s(\tau,\nu)$ are independent of ν pins down the real interest rate in this economy. The result is a non-arbitrage condition relating returns on assets to returns on R&D:

$$\frac{r_s(t)}{P_s(t)} = \underbrace{\frac{\psi \cdot \pi_s(t,\nu)}{P_s(t)}}_{\text{flow dividend rate}} + \underbrace{\frac{\dot{P}_s(t)}{P_s(t)}}_{\text{capital gains}} \tag{1.11}$$

1.3.4 Market Clearing and Equilibrium

Factor Market Clearing

Let $Y_d(t)$ denote the total output of the final good and $X_d(t), I_d(t)$ denote the use of the final good as inputs for the production of intermediate inputs and R&D, respectively. Then total output in the final good for a given country must satisfy:

$$Y_d(t) = C_d(t) + I_d(t) + X_d(t) \tag{1.12}$$

where $I_d(t)$ and $C_d(t)$ are pinned down by the dynamic problem, described below, and $X_d(t)$ can be expressed as a function of aggregate demand in all destinations.

Expenditure Determination

Flow aggregate profits $\Pi_s(t) \equiv \int_0^{M_s(t)} \pi(t,\nu)d\nu$ are a constant fraction of revenue:

$$\Pi_s(t) = \frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} \lambda_{sd}^M(t) \cdot P_d(t) Y_d(t) \tag{1.13}$$

On the expenditure side, GDP of each destination country $s \in \boldsymbol{K}$ country will be exhausted as the combination of the total expenditures of labor and capital income:

$$P_s(t)Y_s(t) = w_s(t)L_s + \Pi_s(t) \tag{1.14}$$

From the income side, nominal GDP must equal the sum of total flow payments received domestically and from the rest of the world:

$$P_s(t)Y_s(t) = \sum_{d \in \boldsymbol{K}} \left[(1-\alpha)\lambda_{sd}^{F}(t) + \frac{\alpha}{\eta}\lambda_{sd}^{M}(t) \right] P_d(t)Y_d(t) \tag{1.15}$$

Trade Balance

Since savings equals investment and there is no access to international capital markets in this economy, GDP accounting requires that trade balances in each country and net exports are equal to zero at any instant:

$$\begin{aligned} &\sum_{d \neq s \in \boldsymbol{K}} \lambda_{sd}^{F}(t)P_d(t)Y_d(t) + \alpha \sum_{d \neq s \in \boldsymbol{K}} \lambda_{sd}^{M}(t) \left[\sum_{k' \in \boldsymbol{K}} \lambda_{dk'}^{F}(t)P_{k'}(t)Y_{k'}(t) \right] = \\ &[1-\lambda_{ss}^{F}(t)]P_s(t)Y_s(t) + \alpha[1-\lambda_{ss}^{M}(t)] \left[\sum_{k' \in \boldsymbol{K}} \lambda_{dk'}^{F}(t)P_{k'}(t)Y_{k'}(t) \right] \end{aligned} \tag{1.16}$$

Dynamic Equilibrium

The dynamics in each of the countries of this world economy are governed by the following system of differential equations:

$$\begin{aligned}\dot{C}_s(t) &= \left[\frac{r_s(t)}{P_s(t)} - \rho\right] C_s(t) \\ \dot{M}_s(t) &= \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(t)\end{aligned} \tag{1.17}$$

As it is clear from the system above, the dynamics of the model are essentially neoclassical. However, since openness to trade impacts the cross-sectional distribution of wages and prices, it will also impact the path of consumption product measures over time.

The first equation —the Euler Equation —states that the household in a country $s \in \boldsymbol{K}$ will choose an upward-sloping consumption path if the real interest rate is greater than the rate of time preference. The higher this gap, the more a household will be willing to defer current consumption and take advantage of higher returns in the asset and R&D markets.

The second equation is less obvious to interpret in its current form, but it states that the growth in the product measure in each country is proportional to the net investment rate. Since expected profits of new varieties are always positive, the net investment rate is also always positive, which means that new varieties are always created, inducing growth in this model.

A more explicit way to observe the net investment rate is by writing the second equation in its equivalent asset representation. Since $M_s(t) = \psi A_s(t)$, then:

$$\dot{A}_s(t) = I_s(t) = \underbrace{\frac{r_s(t)}{P_s(t)} A_s(t)}_{\text{real capital income}} + \underbrace{\frac{w_s(t)}{P_s(t)} L_s}_{\text{real labor income}} - \underbrace{C_s(t)}_{\text{real consumption}}$$

which, along with the discussion regarding the non-arbitrage condition in the previous section, helps clarify that asset markets and varieties markets are two sides of the same coin.

The assumption of log preferences substantially simplifies the dynamic problem. In Appendix 1.7.2, I show that instantaneous consumption is always well-defined as a constant fraction of lifetime wealth:

$$C_s(t) = \rho \left[\underbrace{A_s(t)}_{\text{wealth at } t} + \underbrace{\int_t^\infty \frac{w_s(\tau)}{P_s(\tau)} L_s \cdot \exp\left\{-\bar{r}_s(\tau) \cdot \tau\right\} d\tau}_{\text{PV of future labor income}} \right]$$

where $\bar{r}_s(\tau) \equiv \frac{1}{\tau} \int_t^\tau \frac{r_s(\nu)}{P_s(\nu)} d\nu$ is the average real interest rate between periods t and τ. Once there is an explicit solution for consumption at every t, the differential equation for $\dot{M}_s(t)$ becomes autonomous, and also has an explicit solution as a function of the path of prices.

It also shows that there is a unique initial choice of consumption that is consistent with the optimal choices described by (1.17) and the transversality condition. Since the other conditions to satisfy the Maximum Principle are satisfied, this is equivalent to showing that the solution to the dynamic problem is unique.

Definition 1 (Dynamic Equilibrium)**.** *The dynamic equilibrium of the world economy is defined by a collection of paths of consumption quantities, assets stocks, and profit flows* $[C_s(t), A_s(t), \Pi_s(t)]$*; paths of final goods varieties output quantities* $[y_s(t,\omega)]$*; paths of intermediate goods varieties output quantities* $[x_{ks}(t,\omega,\nu)]$*; paths of prices* $[w_s(t), r_s(t), P_s(t), p_{ss}(t,\omega), p_{sk}(t,\omega,\nu)]$*; and a vector of fundamentals* $(\theta, \sigma, \boldsymbol{T}, \boldsymbol{\tau})'$ *where* $\boldsymbol{T} \equiv \{T_s\}$ *is a collection of location parameters of the Fréchet distribution and* $\boldsymbol{\tau} \equiv [\tau_{sd}]$ *is a matrix of trade costs, such that: (a) households maximize utility given the path for prices; (b) final goods firms maximize* *profits given the path for prices; (c) intermediate goods firms choose prices to maximize profits given demand functions and final goods prices; (d) trade balances; and (e) factors and goods markets clear.*

Homogeneity of Income in Equilibrium

One of the key properties of this model is that real income, real wages, and real profits are a function of the measure of varieties $M_s(t)$ and of terms that are homogeneous of degree zero in the distribution of the measure of varieties $\{M_k(t)\}_{k\in\mathbf{K}}$. Note that for real aggregate labor and aggregate capital income, respectively, can be expressed as:

$$
\begin{aligned}
\frac{w_s(t)L_s}{P_s(t)} &= M_s(t)\cdot\left(\frac{T_s}{\lambda_{ss}^F(t)}\right)^{\frac{1}{\theta(1-\alpha)}}\cdot\left(\lambda_{ss}^M(t)\right)^{-1}\cdot L_s \equiv M_s(t)\times\mathcal{R}_s^w(t)\\
\frac{\Pi_s(t)}{P_s(t)} &= M_s(t)\cdot\frac{\alpha}{\eta}\sum_{d\in\boldsymbol{K}}\frac{\left(\tau_{sd}P_s(t)\right)^{1-\eta}}{\sum_{k'\in\boldsymbol{K}}M_{k'}\left(\tau_{k'd}P_{k'}(t)\right)^{1-\eta}}\frac{P_d(t)Y_d(t)}{P_s(t)}\equiv M_s(t)\times\mathcal{R}_s^\pi(t)
\end{aligned}
$$

which, of course, means that Real GDP is also a function of $M_s(t)$ times a term that is homogeneous of degree zero in the distribution of the measure of varieties:

$$
Y_s(t)=\frac{w_s(t)L_s}{P_s(t)}+\frac{\Pi_s(t)}{P_s(t)}=M_s(t)\times\left[\mathcal{R}_s^w(t)+\mathcal{R}_s^\pi(t)\right]\equiv M_s(t)\times\mathcal{R}_s(t) \qquad (1.18)
$$

This property is important because it is the mechanism that induces increasing returns to scale in this model. It will also be important to characterize the existence of the Balanced Growth Path (BGP).

Intuitively, even though $\mathcal{R}_s(t)$ is a complicated function, the fact that real income is a function of $M_s(t)$ times a term that is homogeneous of degree zero in $\{M_k(t)\}_{k \in \mathbf{K}}$ means that this model falls within the broader class of AK-models in macroeconomics, a property that is inherited from the Romer side of production. Many of the well-behaved properties of AK-models about long-run growth will carry through to this model.

Furthermore, as it will become clear below, trade will influence both the returns to idea creation ($\mathcal{R}_s$) as well as the stock of ideas (M_s). But it is important to bear in mind that some of the mechanisms behind growth in this economy are increasing returns to scale exemplified by the linearity of the production function.

1.3.5 Balanced Growth Path

Autarky

Under autarky, which is a special case in which trade costs are prohibitively high such that countries are isolated as single-country economies, the BGP exists and is unique for each individual economy.

Proposition 1 (Growth rates under autarky)**.** *If $\tau_{sd} \to \infty$ for all $s \neq d$, then growth rates in real consumption $g_s^{autarky}$ in every country $s \in \boldsymbol{K}$ are proportional to domestic market size:*

$$g_s^{autarky} = \frac{\alpha \cdot \psi}{\eta} \cdot \frac{Y_s(t^*)}{M_s(t^*)} - \rho$$

Proof. Appendix 1.7.2. □

Intuitively, Proposition (1) characterizes the BGP in a collection of closed *AK* economies with expanding varieties each of them as in the original P. M. Romer (1990) model. Growth happens endogenously in each of the countries as households invest in the equity market to fund new intermediate varieties. However, the mass of non-rival goods available for production will be completely different across different countries, since final good producers only have access to domestic intermediate inputs and are therefore less productive than they would be if they were trading internationally. Similarly, in general, BGPs will be characterized by different growth rates. Note that growth rates g_s^{autarky} are indeed constant because $\frac{Y_s(t^*)}{M_s(t^*)} = \frac{M_s(t^*) \times \mathcal{R}_s(t^*)}{M_s(t^*)}$ is homogeneous of degree zero in $M_s(t^*)$ for each $s \in \boldsymbol{K}$.

Zero gravity

Now move on to characterize the equilibrium growth rates under the polar opposite case: zero gravity. This is one in which trade is costless and even geographical barriers are nonexistent. The term comes from Eaton and Kortum (2002).

Proposition 2 (BGP under zero gravity). *If $\tau_{sd} = 1$ for all (s, d), then there is a unique world equilibrium growth rate $g^{zero\ gravity}$ that satisfies:*

$$g^{zero\ gravity} = \frac{\alpha \cdot \psi}{\eta} \cdot \frac{\sum_{d \in \boldsymbol{K}} Y_d(t^*)}{\sum_{d \in \boldsymbol{K}} M_d(t^*)} - \rho \tag{1.19}$$

Proof. Appendix 1.7.2 □

By comparing $g^{\text{zero gravity}}$ and $g_s^{autarky}$, it is immediately clear that while the latter is proportional to *domestic* value added per variety $Y_s(t^*)/M_s(t^*)$, the former is proportional to *global* value added per variety $\sum_{d \in \boldsymbol{K}} Y_d(t^*)/\sum_{d \in \boldsymbol{K}} M_d(t^*)$. Intuitively, under zero gravity, growth happens as if the world were a single integrated Romer economy.

It is clear that the growth rate must be common under zero gravity because the expression on the right-hand side of (1.19) is the same for each country. Since $g^{\text{zero gravity}}$ must be a constant, each element in the sum $\frac{Y_d(t^*)}{\sum_{d \in \boldsymbol{K}} M_d(t^*)} = \frac{M_d(t^*) \times \mathcal{R}_d(t^*)}{\sum_{d \in \boldsymbol{K}} M_d(t^*)}$ must be homogeneous of degree zero in $[M_k(t^*)]_{k \in \boldsymbol{K}}$. This, in turn, implies that returns equalize and $\mathcal{R}_d(t^*) = \mathcal{R}(t^*)$.

In the absence of trade costs, the world economy is fully integrated in terms of final goods varieties suppliers and the law of one price holds in the final good. As the final good serves as an input for intermediate varieties, the price of intermediate varieties equalizes globally. A corollary is that the effective measure of input varieties $\tilde{M}_s(t^*)$ also equalizes globally, indicating that non-rival inputs fully diffuse across the world.

Note, however, that income levels need not be the same in this world economy. In fact, those countries that have a higher relative wage at the start of the BGP will have a higher wage relative forever. Therefore, under zero gravity, this model features a *stable global distribution of income* as in the model of Armington trade and capital accumulation-driven growth of Acemoglu and Ventura (2002).

Using the linearity of income in equilibrium, the growth rate over the BGP be decomposed into the following expression, which relates how growth affects labor and capital income, respectively:

$$g^{\text{zero gravity}} = \psi\rho \left[T_s^{\frac{1}{\theta(1-\alpha)}} \left(\frac{w_s(t^*)^{\theta(1-\alpha)}}{\sum_{k\in\boldsymbol{K}} w_k(t^*)^{\theta(1-\alpha)}} \right)^{-\frac{1}{\theta(1-\alpha)}} L_s \left(\frac{M_s(t^*)}{\sum_{k\in\boldsymbol{K}} M_k(t^*)} \right)^{-1} + \frac{\alpha}{\eta} \frac{\sum_{d\in\boldsymbol{K}} Y_d(t^*)}{\sum_{d\in\boldsymbol{K}} M_d(t^*)} \right]$$

The second term within the square brackets, which is related to capital income, shows that profits per variety equalize under zero gravity, with every country having the same level of market access due to the absence of trade frictions and equalization of global prices. The first term within the brackets relates to labor income and shows how wages $w_s(t^*)$ and the measure of varieties $M_s(t^*)$, which are endogenous objects, interact with parameters such as the labor force size L_s and technology T_s.

Since growth rates must be equal for all countries, those countries with better technology and higher labor forces will have a proportionately higher real wage and a higher share in the global measure of varieties. The relationship between the size of a country's labor force and the measure of varieties can also be observed in the data, as seen in Figure 1.7.

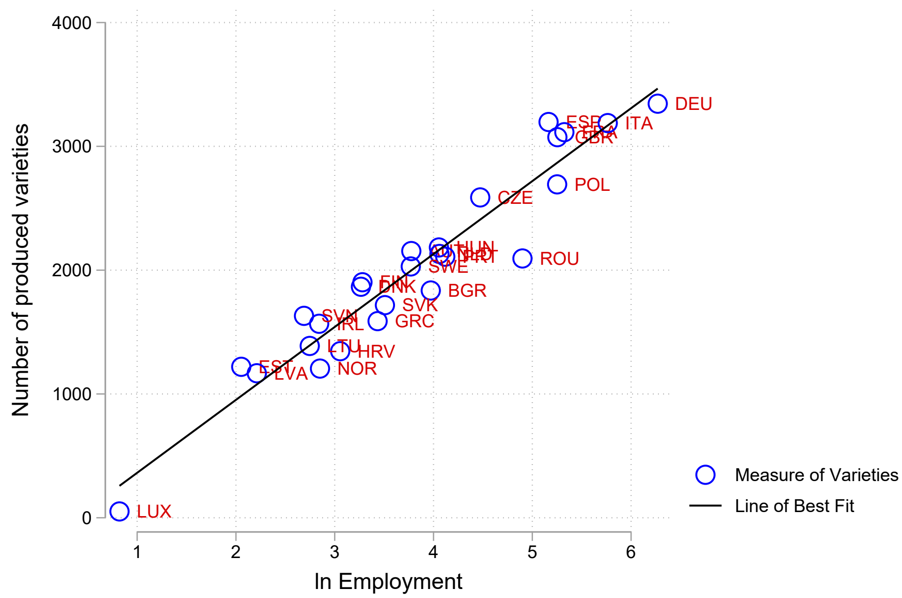


Figure 1.7. Measures of Variety and Labor Force. Long Run Averages between 2000-2020 for the cross-country correlation between the Size of the Labor Force (*ln* of Employment) and the Measure of Produced Varieties. Data come from the Penn World Tables 10.10 and Prodcom, respectively.

Costly but finite trade

I now arrive at the more realistic case of a BGP of positive but finite trade costs.

Proposition 3 (Balanced growth with costly trade). *Given a vector of fundamentals* $(\theta, \sigma, \boldsymbol{T}, \boldsymbol{\tau})$, *if* $\tau_{sd} \in (1, \infty)$ *for all* $s \neq d$, *there exists a balanced growth path world equilibrium growth rate satisfying:*

$$g_s = \psi\rho \left[\underbrace{\left(\frac{T_s}{\lambda^F_{ss}(t^*)} \right)^{\frac{1}{\theta(1-\alpha)}}}_{\text{Eaton-Kortum}} \times \underbrace{\alpha^{1-\eta} \cdot \frac{L_s}{\lambda^M_{ss}(t^*)}}_{\substack{\text{Romer}\\ \text{Domestic}}} + \underbrace{\frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} \lambda^M_{sd}(t^*) \cdot \frac{P_d(t^*) Y_d(t^*)}{P_s(t^*) M_s(t^*)}}_{\substack{\text{Romer}\\ \text{Global}}} \right] \tag{1.20}$$

and where $g_s = g_{s'} (\forall s, s' \in \boldsymbol{K})$. Furthermore, the growth rate can be decomposed into "Eaton-Kortum" and "Romer" components.

Proof. Appendix 1.7.2. □

Like in the previous subsection, equation (1.20) shows how growth affects both components of GDP. One can further interpret these components and relate them to the two canonical models that are the building blocks of this framework.

I termed the capital income part, which is the second term within the square brackets, the *Romer Global* component. The reason is that this component is increasing in each country's market access —i.e., proportional to each country's share of the global intermediate goods market (λ^M_{sd}). Intuitively, profits will be related to a firm's sales and to markets all over the world and, therefore, to its market share in each of those destination markets. This component is also decreasing in the price of the final good (P_d) in the source country since due to the input-output structure embedded in the lab-equipment version of the Romer model the price of the final good is the marginal cost of R&D investment.

Real labor income can be partitioned into two components: an *Eaton-Kortum component* and a *Romer Domestic* component. The *Eaton-Kortum component* is very much Ricardian: real labor income improves with technological improvements (T_s) and as the domestic trade share in final (λ_{ss}^{F}) decreases, consistent with the Ricardian intuition that specialization leads to gains from trade.

The *Romer Domestic* component incorporates domestic market size effects, by integrating the size of the domestic labor force L_s. But it also adjusts for the diffusion of differentiated intermediate goods, which is embedded in the summary statistic of domestic intermediate trade share (λ_{ss}^{M})[16].

The different components of (1.20) make it clear why a BGP requires common growth rates. Both $\lambda_{ss}^{M}(t^*)$ and $\lambda_{ss}^{F}(t^*)$ are homogeneous of degree zero in $[M_k(t^*)]_{k\in\boldsymbol{K}}$ and prices $P_s(t^*)$, $P_d(t^*)$ must be constant along a BGP. For a BGP, g_s must be a constant, and therefore $\frac{Y_d(t^*)}{M_s(t^*)} = \frac{M_d(t^*)\times\mathcal{R}_d(t)}{M_s(t^*)}$ must be homogeneous of degree zero in $[M_k(t^*)]_{k\in\boldsymbol{K}}$. This can only happen if $\mathcal{R}_d(t^*)$ is homogeneous of degree zero in each d, which implies that $\mathcal{R}_d(t^*) = \mathcal{R}(t^*)$.

Intuitively, along the BGP, the real interest rate equalizes globally[17]. Even though there are no international equity markets, the fact that households can invest in new varieties through equity markets and earn expected profits that are linked to exports means that trade acts as a vehicle to integrate international R&D and equity markets. In a balanced growth equilibrium, then, prices and the endogenous distribution of the measure of varieties $[M_s(t^*)]_{s\in\boldsymbol{K}}$ will adjust to make sure that returns and, therefore, growth rates equalize.

How are growth rates related to market access?

After characterizing the existence of the BGP, one can turn to the discussion of what happens to the equilibrium growth rates after there is a change in trade costs.

[16]To see that, note that: $\lambda_{ss}^{M}(t^*) = M_s(t^*)\left(\frac{p_{ss}^{M}(t^*)}{P_s^{M}(t^*)}\right)^{1-\eta} = \frac{M_s(t^*)}{\tilde{M}_s(t^*)}p_{ss}^{M}(t^*)^{1-\eta}$.

[17]One way to see that is through the Euler equation. Since the Euler Equation governs the growth rate in consumption and over the BGP $g_c = g_M$ for all countries, a corollary is that the real interest rate must equalize globally.

Here, these trade costs are directly related to market access, since the mechanism that propels growth is the incentive to have equity claims in the profits of variety exporters. The growth rate is a general equilibrium object that depends on the whole distribution of prices across countries and periods. Therefore, characterizing changes to it is not a trivial task.

Nonetheless, in order to connect the theory to the empirical and quantitative analysis, to gain some intuition, first consider what happens to the *long-run* equilibrium growth rate after a permanent change in trade costs in a world of symmetric countries. While that is an important restriction, it allows for a closed-formed intuitive solution: in that case, g^* can be shown to unambiguously increase in the long run after an episode of trade liberalization.

Proposition 4 (Effects of changes in trade costs over the long run in symmetric economies)**.** *Suppose there exist a collection of symmetric economies that grow over the BGP with costly trade with trade costs* $\tau > 1$*. Then* $\frac{\partial g^*}{\partial \tau} < 0$*.*

Proof. Appendix 1.7.2 □

In this model, the long-run growth rate will change after a permanent change in trade costs if there is a change in the effective market size, represented by how much of the global market exporters can tap into —that is why foreign aggregate demand $\sum_k P_k(t^*)Y_k(t^*)$ is modulated by intermediate trade share $\sum_k \lambda_{sd}^M(t^*)$ in the profit formula.

In a symmetric world, it can be shown in closed form that real profits increase when trade costs go down. The intuition translates to numerical exercises with asymmetric countries. As an example plot in Figure 1.8 shows how these results look like in a numerical exercise with two asymmetric countries.

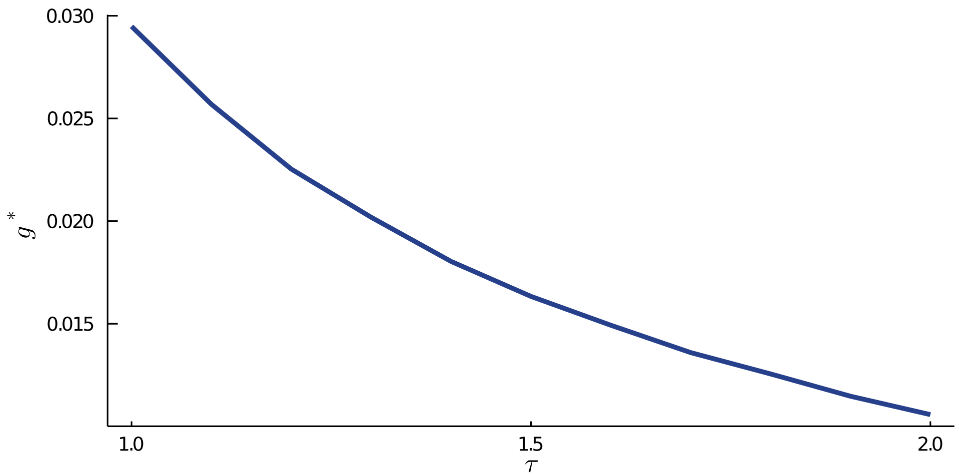


Figure 1.8. Long-run growth in two asymmetric economies as a function of changing trade costs. Results from a numerical simulation of the equilibrium growth rate g^* of two asymmetric economies that differ in their populations but are otherwise equal. Parameters are the following: $L = [1, 1.03]$, $\sigma = 0.77, \theta = 2.12, \alpha = 1/3, \rho = 0.03, T = [1, 1], \psi = 2.46$.

1.3.6 Welfare

With log preferences, at any moment, consumption over the BGP is a fraction of assets plus real labor income. Since such consumption flow grows at a constant rate g^* and the measure of products is simply a linear transformation of assets, as shown in Appendix 1.7.2, welfare along the BGP can be decomposed between a product measure component, a real income component, and a growth component.

$$\begin{aligned} &\int_{t^*}^{\infty} \exp\{-\rho(t-t^*)\} \log\left(\exp\{g^* t\} C_s(t^*)\right) dt \\ =\;& \underbrace{\underbrace{\log\left(\frac{1}{\psi} M_s(t^*)\right)}_{\text{product measure}}}_{\text{transitional}} + \underbrace{\underbrace{\frac{1}{\rho} \log\left(\frac{w_s(t^*) L_s}{P_s(t^*)}\right)}_{\text{real income}}}_{\text{static}} + \underbrace{\underbrace{\frac{g^*}{\rho^2}}_{\text{growth}}}_{\text{dynamic}} \end{aligned} \tag{1.21}$$

Consider what happens to welfare after a change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$, as in Arkolakis, Costinot, and Rodríguez-Clare (2012) —hereinafter ACR. In this dynamic setting, to make a comparison to the static framework, I need to compare what happens across the two BGPs, comparing the preserved value of discounted lifetime utility across the beginning of the two initial equilibria. For that, let me introduce some notation: suppose t^* is the initial period of the original BGP; t^{**} is the first period of the final BGP and let $\hat{x} \equiv x(t^{**})/x(t^*)$.

Then, relative level changes in the first component of welfare across two BGPs can be expressed as $\log\left(\widehat{M}_s\right)$. Changes in the equilibrium product measure will depend on whether the measure of varieties in country s expands or contracts, *relative to the distribution of varieties across countries*, across BGPs. There is no general prediction in the model regarding the direction of this effect.

Countries that have started with a measure of varieties above optimal (relative to other countries) will see a shift in exports (and therefore R&D expenditures) towards other countries and will see their measures of varieties shrink. The opposite is true for countries that started with a measure of varieties below optimal.

Importantly, however, this first component will not compound over time, as highlighted by the fact is not multiplied by the factor ρ^{-1} or ρ^{-2}. This means that it will only change the (relative) income level that a given country arrives with at the BGP and it will have no impact going forward. Therefore, this is a *transitional effect* of welfare. For most reasonable calibrations of ρ, the transitional effect will have a very small weight on total welfare changes.

The second component will be familiar to most trade economists. It looks like the traditional *static welfare formula in ACR*. In the same spirit as ACR, one can also write the static welfare component in changes:

$$\frac{1}{\rho}\log\left(\frac{\widehat{w_s}}{P_s}\right) = \frac{1}{\rho}\log\underbrace{\left(\widehat{\lambda_{dd}^F}^{\,-\frac{1}{(1-\alpha)\theta}}\right)}_{\text{Eaton-Kortum}} + \frac{1}{\rho\eta}\log\underbrace{\left(\sum_{k\in\boldsymbol{K}}\mu_k\cdot\widehat{M}_k\cdot\left(\frac{\widehat{p}_{kd}^M}{\widehat{P}_s}\right)^{1-\eta}\right)}_{\text{Romer}} \tag{1.22}$$

where $\mu_k \equiv \frac{M_k(t^*)\cdot\left(\frac{p_{ks}^M(t^*)}{P_s(t^*)}\right)^{1-\eta}}{\sum_{k\in\boldsymbol{K}} M_k(t^*)\cdot\left(\frac{p_{ks}^M(t^*)}{P_s(t^*)}\right)^{1-\eta}}$.

This component preserves the standard feature that changes in consumer welfare are decreasing in changes in domestic trade share $\widehat{\lambda_{ss}^F}$[18]. This captures the Ricardian intuition of the model: at the margin, there are static gains from specialization in this model.

Like the growth formula, the static component of welfare also has Eaton-Kortum and Romer components. Here, the Romer component impacts welfare by augmenting Ricardian gains through an extensive margin. It is represented by the weighted change in the measure of varieties, accounting for previous weights μ_k, changes in the measures of varieties in each country $k \in \boldsymbol{K}$ across equilibria $\widehat{M}_k$, and changes in the prices of foreign intermediate goods relative to the domestic consumer price index $\left(\frac{p_{ks}^M(t^*)}{P_s(t^*)}\right)$ at the domestic market.

Note that this welfare impact from product innovation resembles how the change in the measure of varieties shows up in the ACR formula in Melitz-type models. This highlights that the nested structure of production featured in this model effectively adds an extensive margin to the Eaton-Kortum framework.

While they do not compound over time, both of these effects have an impact in

[18]The elasticity of this effect is $-\frac{1}{(1-\alpha)\theta}$ rather than the standard $-\frac{1}{\theta}$ due to the input-output structure of the model.

every period over the BGP as it is made clear by it being multiplied by the factor ρ^{-1}. For that reason, these can be understood as a *level effect* or *static effect* of welfare.

The third and last component is the common growth rate g^*. Importantly, since it compounds the BGP level of consumption, it is multiplied by a factor ρ^{-2} rather than ρ^{-1} and it will in general have a larger weight on welfare. This is a metric of *dynamic gains from trade*, which is a *growth effect* of welfare.

Changes in the growth component of welfare will be defined as the change in the growth rage: $\frac{g^{**}-g^*}{\rho^2}$. Since growth rates equalize along the BGP, changes in the growth component of welfare will also be shared across all countries. However, since the other components will differ, the share of the dynamic component of welfare in total gains from trade will therefore be different across countries.

The discount rate ρ will have an important role in attributing weights across the dynamic, static components, and transitional components of welfare. Intuitively, the lower the ρ, the more patient the agent is, and the more relevant the dynamic component of welfare will become.

Welfare as buyers and as sellers

Comparing the dynamic and static components of welfare yields important insights regarding the economic mechanisms behind this model. In fact, the forces of specialization and innovation are reflected in these two components.

To see that, note that, as made clear by (1.22), since $1 - \eta < 1$ country s's *static welfare* is decreasing in the price of foreign intermediate goods. The static welfare formula captures the effect of s as final producers and consumers. As s purchases more foreign intermediate varieties for a cheaper price, it becomes more productive by increasing its effective measure of varieties $\tilde{M}_s(t)$. The other side of the coin is that it decreases the local price index $P_s(t)$, which directly benefits consumers and increases welfare.

By contrast, the growth component of welfare is *increasing* in the price of foreign intermediate goods coming from k, in each destination markets d, relative to the price of intermediate goods from the source country s at those same destination market:

$$g_s \propto \frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} P_d(t) Y_d(t) \left[\sum_{k \in \boldsymbol{K}} M_k(t) \left(\frac{p^M_{kd}(t)}{p^M_{sd}(t)} \right)^{1-\eta} \right]^{-1}$$

The intuition for this contrast is quite straightforward and underscores the different underlying economic mechanisms of the model. The Romerian part of growth captures the effect of d as forward-looking investors in the R&D market and intermediate good producers. Since the intermediate goods are substitutes, all else equal, demand for intermediate goods from s and maximized profits are higher when the price of intermediate goods of foreign competitors from third-party countries k relative to domestic producers from s at each destination market d is higher.

Intuitively, the *growth effect* captures that, from a *seller's perspective*, the domestically produced and exported intermediate variety s is more attractive and competitive when foreign varieties k are more expensive. Conversely, the *static effect* captures that, from a *buyer's perspective*, when foreign varieties k relative to one's domestic purchasing power at d are more expensive, the domestic consumer is worse off. Both channels are economically sensible and the model captures both mechanisms.

Along the BGP, prices and measures of varieties will adjust to make sure that growth rates equalize such that $g_s = g_{s'}$ for every $s, s' \in \boldsymbol{K}$. While the economic mechanisms are still operating under the hood and take over if there is any shock that drives the system off the BGP, these different effects will wash out once differences in prices, measures of varieties and wages endogenously adjust towards a BGP.

1.4 Quantification and Policy Exercise

This section describes a numerical quantification of the model, which solves for three endogenous objects along the Balanced Growth Path: (a) the distribution of wages; (b) the distribution of Measures of Varieties; and (c) the common equilibrium growth rate. I calibrate the model to EU-15 countries and the New Member States (NMS) that joined in the 2004 expansion.

To simplify the exercise, I group these countries into six sets: corresponding to the six waves of the expansion of the European Union of to 2004[19]. The country groups are asymmetric both in terms of labor force and productivity. The groups are:

1. **1957**: Belgium, France, Germany, Italy, Luxembourg, and the Netherlands —the original members;
2. **1973**: Denmark, Ireland, and the United Kingdom;
3. **1981**: Greece;
4. **1986**: Portugal and Spain;
5. **1995**: Austria, Finland, and Sweden;
6. **2004**: Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovia —the New Member States (NMS).

[19]This simplification is just a matter of computational tractability. As described in Appendix 1.7.5, each guess of my solution algorithm solves for a static version of an Eaton-Kortum model with input-output linkages, which itself has multiple steps for solutions. So the problem grows quite fast in complexity in the number of countries. Improving the solution algorithms for this new class of dynamic models is a fruitful avenue of future research.

The solution method is straightforward. I calibrate the model to a baseline scenario and then change iceberg trade costs to induce a trade liberalization shock. By comparing the endogenous equilibria along the Balanced Growth Path of these two scenarios, which include distributions of the measures of varieties $[M_s(t^*)]_{s\in\boldsymbol{K}}$ and wages $[w_s(t^*)]_{s\in\boldsymbol{K}}$ as well as a common equilibrium growth rate g^*, I can infer the welfare consequences of a change in this parameter along the BGP.

Model Calibration

My estimates of the short-term (σ) and long-term (θ) elasticities of trade come from Boehm, Levchenko, and Pandalai-Nayar (2020), which are $\sigma = 0.76$ and $\theta = 2.12$, respectively. The results are not very sensitive to using a $\theta = 4.0$. The vector of labor force $\{L_s\}$ comes from Penn World Tables. The share of intermediate goods $\alpha = 0.36$ is set to equal the average share of intermediate goods in the sample of countries between 2000-2003 from the World Input-Output Database.

I use observed trade flows to infer trade costs. The strategy goes back to Head and Ries (2001). According to the handbook chapter by Head and Mayer (2014), the index is called the Head-Ries Index (HRI) since 2011 (when the working paper version of Eaton, Kortum, et al. (2016) was published). As shown in Appendix 1.7.5, I can write trade costs as:

$$\tau_{sd} = \left(\frac{E^F_{sd}(t^*)}{E^F_{dd}(t^*)} \cdot \frac{E^F_{ds}(t^*)}{E^F_{ss}(t^*)} \right)^{-\frac{1}{2\theta(1-\alpha)}} \tag{1.23}$$

where each flow $E^F_{sd}(t^*)$ defined to be an average between $2000-2003$. The data on bilateral expenditure values $E^F_{sd}(t^*)$ comes from the World Input-Output Database.

Figure 1.9 plots the change in trade costs before and after the 2004 enlargement of the European Union, calculated from an average of for the years 2000-2003 for the immediate "before" period and an average for the years 2004-2007 for the immediate "after" period. Calculations confirm that there were large reductions ($-15-20\%$) in trade costs between NMS and the Western European countries during this period, which is consistent with bilateral tariff data between the NMS and Western Europe from TRAINS and the WTO[20]. Changes in trade costs across the other groups have been comparatively small except for one calculated *increase* in trade costs between Greece (*g1981*) and Austria, Finland, and Sweden (*g1995*).

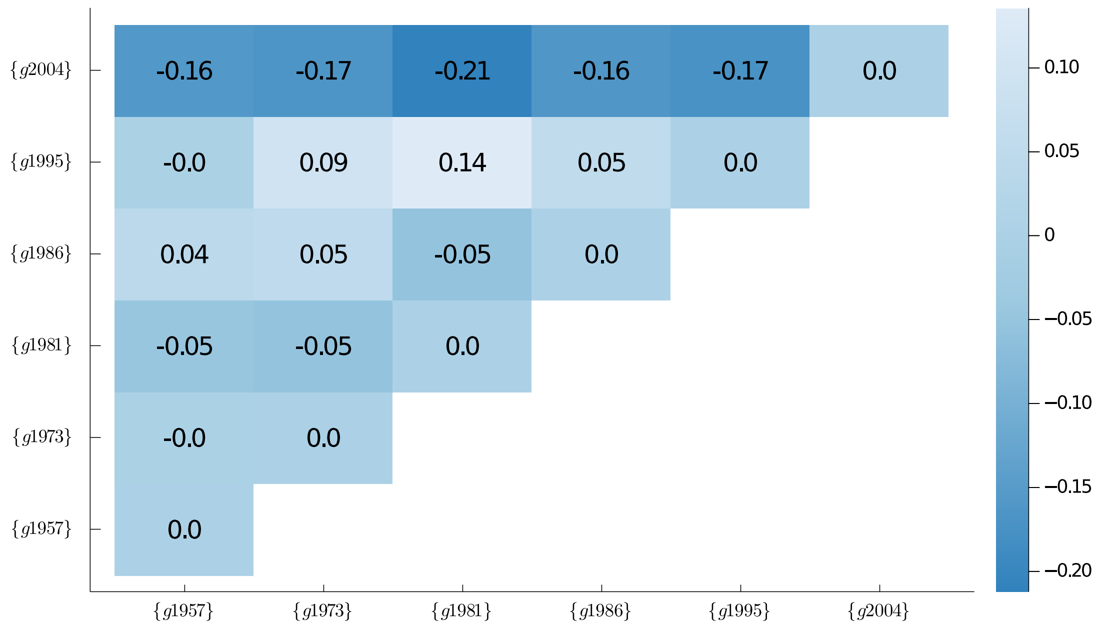


Figure 1.9. Changes in Trade Costs Before and After 2004 EU Enlargement (in percentage terms). This matrix shows the bilateral changes in trade costs, calculated using the method inferred from equation (1.23), before and after the 2004 EU Enlargement. The before period is an average for the years 2000-2003 and the after period is an average for the years 2004-2007. Underlying data comes from the World Input-Output Database.

This is important because these changes in trade costs will act as the main shock across calibrations of BGPs in my numerical exercise. It is relevant that the key driver of changes across equilibria is the enlargement of the EU.

The location parameter of the Fréchet distribution $\{T_s\}$ and ψ are free parameters that I vary to match the distribution of wages and the average growth rate of the EU-15

[20]See the Appendices from Caliendo et al. (2021) for a detailed description of the data.

countries in the $1989-2003$ period —i.e., fifteen years prior to the 2004 expansion of the European Union. The rationale is that I am calibrating this model to BGP growth rate and the EU-15 countries were very likely closer to the BGP than the transition economies of Eastern Europe, so it is reasonable to match the model to their growth rate.

Model Validation

To validate the model, there are some untargeted moments one can look at. First, compare the relative change in real wages across the two BGPs. The predicted changes in the distribution of wages across equilibria in the model can be compared with the relative income growth of each country group around the EU enlargement.

Since the wages distribution is only pinned down up up to the distribution, if real wages of a given group take a larger share of the distribution in the later BGP, an implication is that it must have grown faster than average between those periods. To compare the data with the model, a natural comparison is to use GDP per capita growth rate net of the average of the EU, which yields a income that is normalized for the periods of 1998-2003 and 2005-2010. The way to interpret the data is to see whether or not each group's income per capita grew faster (slower) than average across these periods.

Here, one can see that the model in fact matches the data quite correctly. It predicts relative a catch-up of the New Member States (*g2004*). The model predicts that real wages in NMS would grow about 5.1% faster than the average of the Western European countries, which is very close to observed in the data (5%). As seen in Figure 1.10, most of the other country groups also fall very close to the 45-degree line, suggesting that the model's predictions are reasonable.

One exception is the 1981 wave, for which the model substantially over-predicts

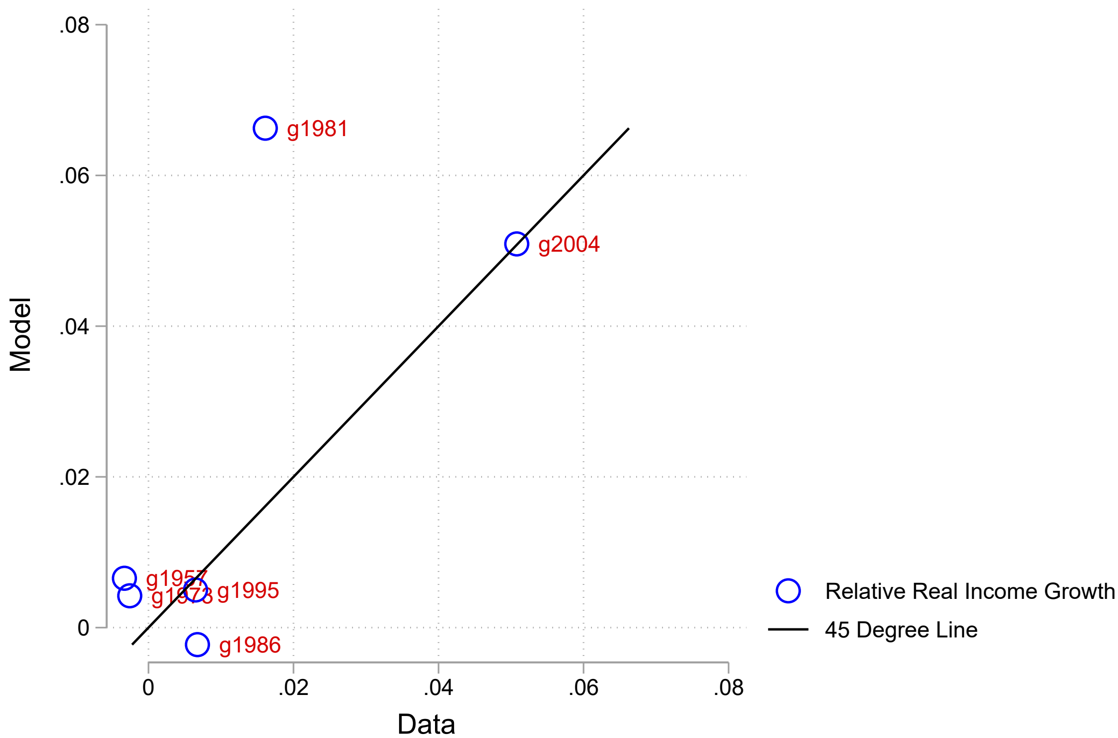


Figure 1.10. Model Validation: Changes in Real Wages, Relative to the Average. In the model, the distribution of wages $\lambda_w \cdot [w_s(t^*)]_{s\in \boldsymbol{K}}$ is normalized with a choice of λ_w such that $\sum_{s\in \boldsymbol{K}} w_s(t^*)L_s = 1$. What is shown in the chart is the percentage change across equilibria $\frac{w_s(t^{**})/P_s(t^{**})}{w_s(t^*)/P_s(t^*)} - 1$, where L_s is assumed to be fixed. In the data, for consistency, I calculated annual GDP per capita then subtracted it from the average of the group for the periods of 1998-2003 and 2005-2010. I then calculated changes and plotted the data. Data comes from the Penn World Tables 10.01.

relative real income growth. The reason being that such a wave consists of a single country: Greece. And the aftermath of the EU enlargement $2005 - 2010$ includes the first years of the Greek debt crisis. Naturally, the model cannot anticipate the negative shocks of the deep recession of the late 2010s in Greece.

Second, compare the (endogenous) distribution of the number of produced varieties in the model to the distribution of the number of produced varieties in the data for the 2000-2003 period. Once again, the observations fall mostly along the 45-degree line, suggesting that the model does a good job in replicating the empirical distribution. As one exception, now the model *underestimates* the actual share of total produced varieties in the NMS (*g2004*).

Finally, compare the changes in trade shares across equilibria. The model captures changes in trade shares really well, as shown in (1.12). Trade expands particularly in exports from NMS to Western European countries, which is captured by the upper quadrant observations that lie close to the 45-degree line. Here, the exceptions are the trade flows from Western Europe towards the NMS. The model predicts a symmetrical response in terms of trade expansion, while in reality, the gains were much more a relative market access from the NMS into the EU market than the other way around.

Results

The main result of this exercise relates to the theoretical welfare decomposition in equation (1.21). One can compare two paths of consumption along the BGP and decompose them into:

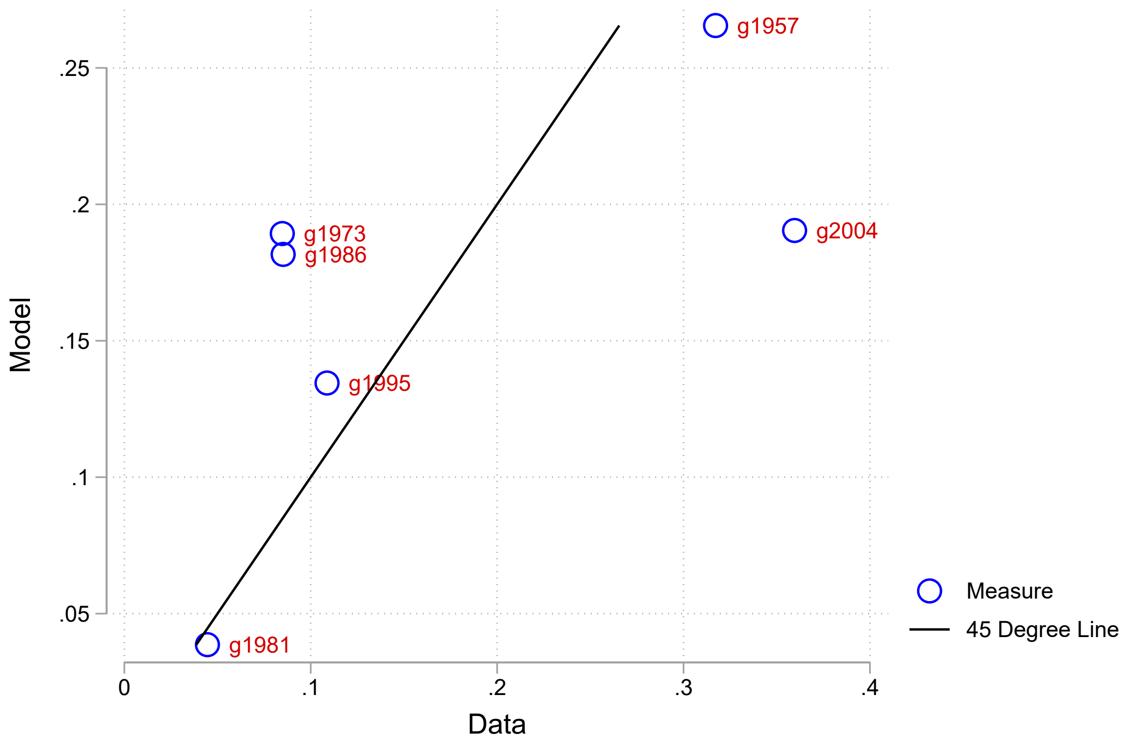


Figure 1.11. Model Validation: Distribution in the Number of Produced Varieties Across Regions. In the model, the distribution of measures of varieties $\lambda_M \cdot [M_s(t^*)]_{s \in \boldsymbol{K}}$ is normalized with a choice of λ_M such that $\sum_{s \in \boldsymbol{K}} M_s(t^*) = 1$. For consistency in the comparison, what I show in the data bars are the relative shares of each country group in the total universe of the product measure, or: $M_s(t) / \sum_{s' \in \boldsymbol{K}} M_{s'}(t)$. This assumes, as in the model, that product varieties in the data are differentiated across countries, so the global product space is $\sum_{s' \in \boldsymbol{K}} M_{s'}(t)$. Data comes from Prodcom (Eurostat) and are averages for the 2000-2003 period.

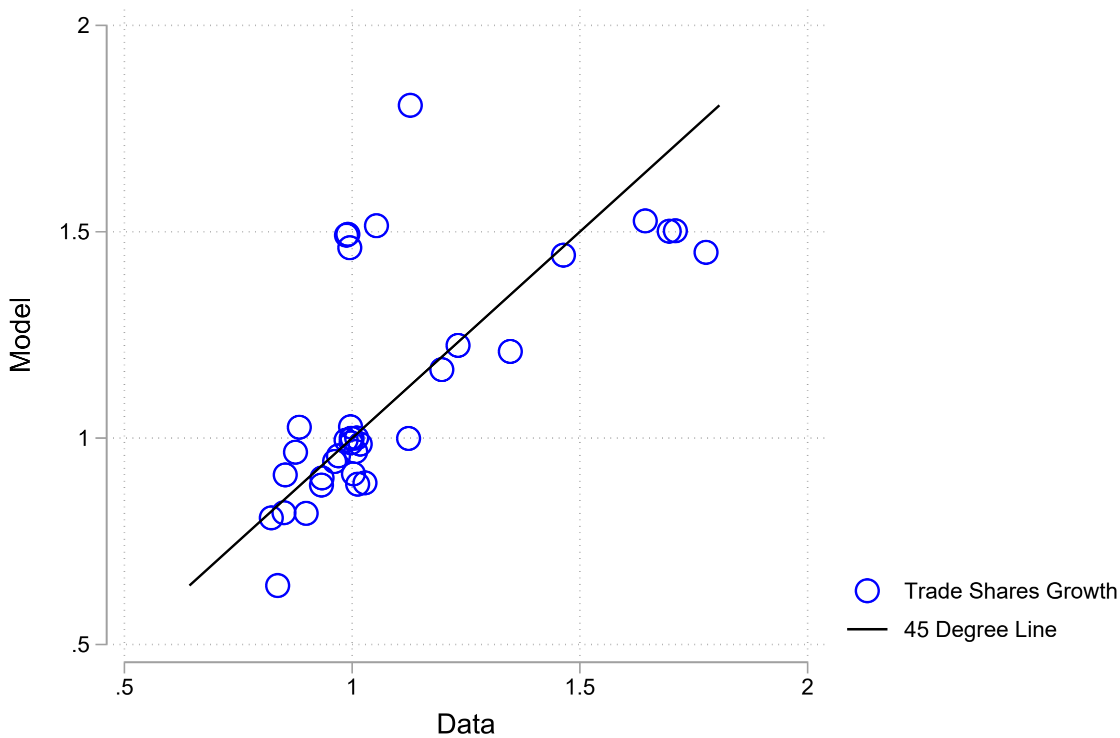


Figure 1.12. Model Validation: Changes in Trade Shares. The model object is plotted is $\hat{\lambda}_{sd}^{F}$: the change in the final sector trade share. In the data, this is total trade shares renormalized to account for the fact that there is no rest-of-the-world in the sample. The before and after periods are 1998-2003 and 2005-2010, respectively.

$$\int_{\tau}^{\infty} \exp\{-\rho(t-\tau)\} \left[\log\left(\exp\{g^{**}t\}C_s(t^{**},\tau)\right) - \log\left(\exp\{g^{*}t\}C_s(t^{*},\tau)\right)\right] dt =$$
$$\underbrace{\log\left(\hat{M}_s\right)}_{\text{transitional}} + \underbrace{\frac{1}{\rho}\log\left(\frac{\widehat{w_s}}{P_s}\right)}_{\text{static}} + \underbrace{\frac{g^{**}-g^{*}}{\rho^2}}_{\text{dynamic}}$$

where $C_s(t^*,\tau), C_s(t^{**},\tau)$ are the paths of consumption along the original and new BGPs, respectively.

For all countries, the transitional component is negligible. They never contribute with more than 0.03% of total absolute value of welfare, in the largest case.

Static gains from trade can be as large as 5-6% of domestic income in the case of NMS (*g2004*) or Greece (*g1981*) or even *negative* or close to zero in the case of the Western European countries such as Portugal and Spain (*g1986*). In the case of Greece and the NMS, they account for 38% and 32% of total welfare gains from trade, respectively.

These changes can be further decomposed into "Eaton-Kortum" and "Romer" parts of static welfare using equation (1.22). Results in Figure 1.13 show a wide variation of the Ricardian component share in total changes in static welfare. While for most country groups that share is about 10% of total changes in static welfare, for Portugal and Spain (*g1986*) it accounts for nearly 25% of static welfare changes while for Greece (*g1981*) the Ricardian share accounts for more than 90% of changes in total welfare.

Finally, the main numerical outcome of the exercise is the differences in growth rates across BGPs $g^{**} - g^{*}$. In the current calibration, *the trade liberalization embedded in the 2004 enlargement of the European Union induced the EU long-run yearly growth rate to increase* $0.10pp$. One implication is that the dynamic part of welfare accounts for the

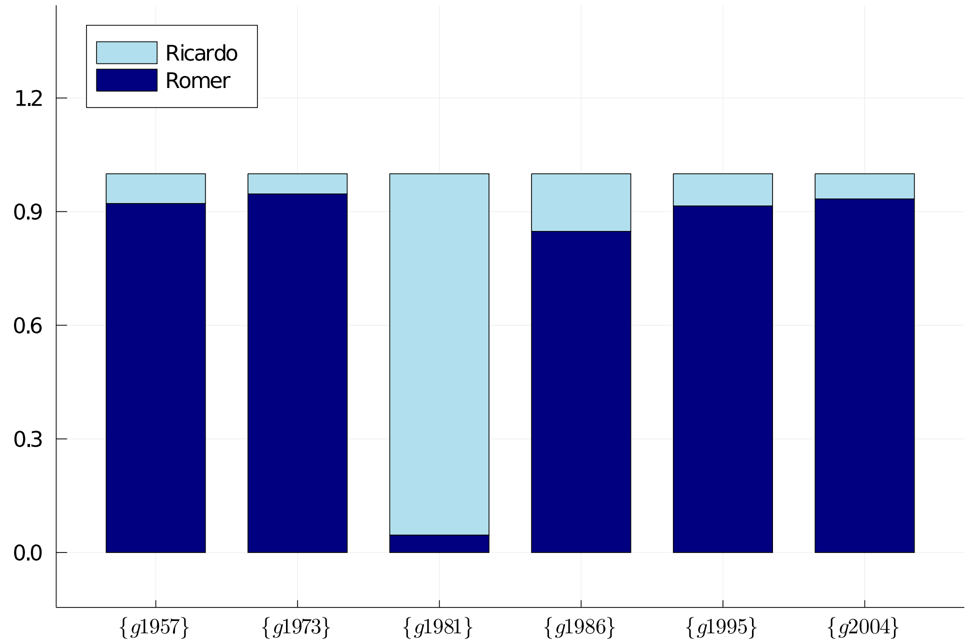


Figure 1.13. Static Welfare Decomposition. Static Welfare Decomposition Across its Eaton-Kortum and Romer Components, according to equation (1.22).

most of gains from trade for all countries. Therefore, not accounting for this channel ignores the majority of gains from trade.

However, the share of total welfare gains it accounts for varies across country groups. According to this model, in the current parametrization, the share of dynamic gains in total welfare gains is between 65% and 90%. This is in line with estimates from Hsu, Riezman, and Wang (2019) (78%) and Perla, Tonetti, and Waugh (2015) (85%).

However, in this model, the change in the equilibrium measure of varieties (and hence the real wage) between one BGP and the other can actually decrease, which implies a negative static welfare share. Therefore, for some countries, such as Portugal and Spain (*g1986*) the share of dynamic welfare in total welfare is *larger than 100%*. These decompositions are in Figure 1.14.

In monetary terms, a back-of-the-envelope calculation suggests an additional 0.10% yearly growth rate to the aggregate GDP of the Western European plus the New

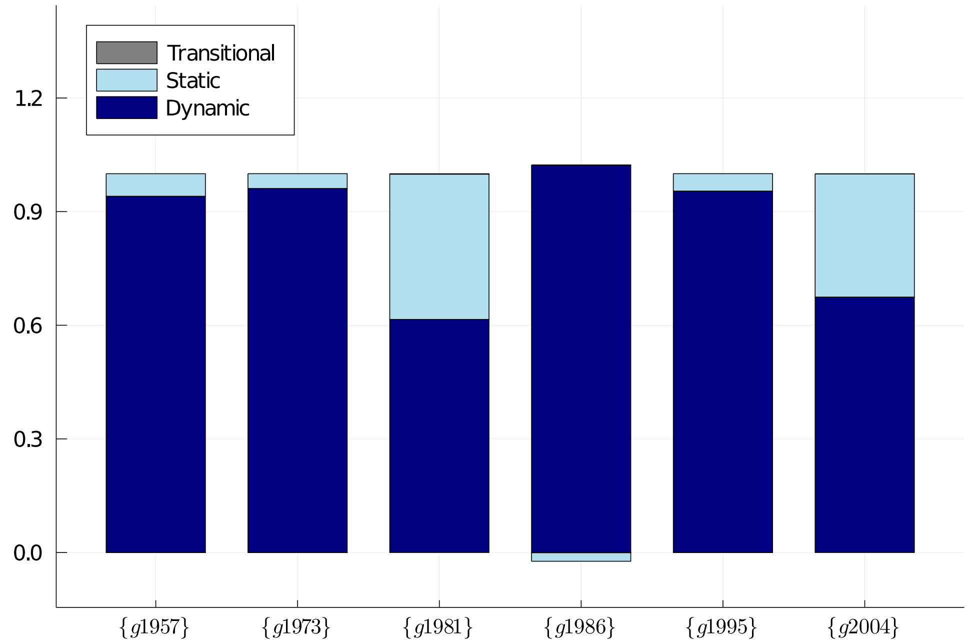


Figure 1.14. Total Welfare Decomposition. Welfare Decomposition Across its Transitional Static and Dynamic Components, according to equation (1.21).

Member States since the year of accession —that is, between 2004 and 2023 —would have induced an additional current production level of approximately \$332 billion in the continent, which accounts for 2.0% of the total level of production of the European Union.

1.5 Conclusion

I focus on the long-lasting question of the relationship between trade and growth and, in particular, trade and product innovation. I make several contributions: theoretical, empirical, and quantitative.

On the theoretical front, my main contribution is a new framework that reconciles the forces of specialization and market size, rationalizes foreign market access as a rationale for growth in a dynamic framework, and provides an analytical formula for dynamic gains from trade. In all of those points, I maintain active dialogues with the literature, such as nesting the Eaton-Kortum model of trade and Romer growth model as special cases of my model and subsuming the ACR static welfare formula in my dynamic welfare formula.

In my empirical work, I rely on the eastward expansion of the European Union and document several new facts that are consistent with the mechanisms of my model. Compared to countries that selected into becoming candidates but had not joined at given horizon, countries started producing more product varieties, investing more in R&D, and trading more.

I go beyond these facts and exploit plausibly exogenous variation to show that a plausibly exogenous increase in market access leads to a higher probability of initiating production and exporting a given product, which is consistent with the main mechanism of the theoretical model.

Finally, I solve for a quantitative model and replicate the 2004 expansion of the European Union in the computer. The results of the simulation imply that: (a) the EU expansion increased its long-run yearly growth rate by about 0.10pp; and (b) dynamic gains from trade account for somewhere between 65-90% of total welfare gains from trade.

This paper points to the fact that dynamic gains from trade are likely too large to be ignored. The big generalizable takeaway is that the previous literature has largely underestimated gains from trade, perhaps by as much as one order of magnitude. Advancing on this agenda, perhaps by understanding the transition dynamics, is a fruitful avenue of future research.

1.6 Acknowledgments

This chapter is in preparation for future publication. It has not yet been submitted to any journal, but it has been presented in the following conferences or institutions: Latin American Meeting of the Econometric Society, 2024; European Trade Study Group Conference, 2024 (Plenary Session); Midwest International Trade Meeting, July 2024 (Plenary Session); Elon University; Texas A&M University; São Paulo School of Economics - Fundação Getúlio Vargas; University of Chile; UC San Diego; 2023 Southern California Graduate Conference in Applied Economics.

1.7 Appendix

1.7.1 Timeline of EU Trade Agreements

Partner	Signed	Provisional application	Full entry into force
Switzerland	1972		1973
Iceland	1992		1994
Norway	1992		1994
Turkey	1995		1995
Tunisia	1995		1998
Israel	1995	1996	2000
Mexico	1997		2000
Morocco	1996		2000
Jordan	1997		2002
Egypt	2001		2004
North Macedonia	2001	2001	2004
South Africa	1999	2000	2004
Chile	2002	2003	2005

1.7.2 Mathematical derivations

Optimal control problem

In the dynamic optimal control problem, the household chooses an optimal path of $C_s(t)$ at every instant, taking as given prices. The problem of choosing varieties $c_s(t,\omega)$ is separable and can be solved conditional on a path for $C_s(t)$, such that only aggregates matter for the dynamic path. Therefore, the current-value Hamiltonian for this problem is:

$$H(t,C,L,\mu) = \log\left(C_s(t)\right) + \mu_s(t)\left[\frac{r_s(t)}{P_s(t)}A_s(t) + \frac{w_s(t)}{P_s(t)}L_s - C_s(t)\right]$$

with optimality conditions satisfying:

$$\begin{aligned}
\frac{1}{C_s(t)} &= \mu_s(t) \\
\frac{\dot{\mu}_s(t)}{\mu_s(t)} &= \rho - \frac{r_s(t)}{P_s(t)}
\end{aligned}$$

and a transversality condition:

$$\lim_{t\to\infty}\left[\exp\{-\int_0^t \frac{r_s(\nu)}{P_s(\nu)}d\nu\}P_s(t)A_s(t)\right] = 0$$

Taking time derivatives of the first optimality condition and then replacing for $\frac{\dot{\mu}(t)}{\mu(t)}$ yields the Euler equation:

$$\frac{\dot{C}_s(t)}{C_s(t)} = \left[\frac{r_s(t)}{P_s(t)} - \rho\right]$$

Solution to the dynamic problem

Growth in each of the $s \in \boldsymbol{K}$ of the national economies evolve according to the following system of differential equations:

$$\begin{aligned}
\dot{C}_s(t) &= \left[\frac{r_s(t)}{P_s(t)} - \rho\right] C_s(t) \\
\dot{M}_s(t) &= \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(t)
\end{aligned}$$

In this subsection, I will first derive this system of equations, then solve it. First, one sees that consumption evolves according to a first-order differential equation. Let $a(t) \equiv \left[\frac{r_s(t)}{P_s(t)} - \rho\right]$ and write the Euler equation as:

$$\dot{C}_s(t) = a(t) C_s(t)$$

Multiplying both sides by the integration factor $\exp\{-\int_0^t a(\tau) d\tau\}$:

$$\dot{C}_s(t) \exp\{-\int_0^t a(\tau) d\tau\} - a(t) C_s(t) \exp\{-\int_0^t a(\tau) d\tau\} = 0$$

Now, using Leibnitz lemma, note that the time derivative of $\exp\{-\int_0^t a(\tau) d\tau\} C_s(t)$ is $\dot{C}_s(t) \exp\{-\int_0^t a(\tau) d\tau\} - a(t) C_s(t) \exp\{-\int_0^t a(\tau) d\tau\}$. Therefore, integrating both sides with respect to time:

$$\exp\{-\int_0^t a(\tau) d\tau\} C_s(t) = C(0)$$

where $C(0)$ is the constant of integration. Dividing both sides by $\exp\{-\int_0^t a(s)ds\}$ and replacing for $a(t)$ yields the solution for the consumption path:

$$C_s(t) = C(0)\exp\left\{\int_0^t \left[\frac{r_s(\tau)}{P_s(\tau)} - \rho\right] d\tau\right\}$$

which can be rewritten as:

$$C_s(t) = C_s(0)\exp\left\{\left[\bar{r}_s(t) - \rho\right] t\right\}$$

where $\bar{r}_s(t) \equiv \frac{1}{t}\int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu$ is the average real interest rate between periods 0 and t. Now recall that the budget constraint is:

$$P_s(t)I_s(t) + P_s(t)C_s(t) = r_s(t)A_s(t) + w_s(t)L_s \tag{1.24}$$

and that $\psi I_s(t) = \dot{M}_s(t)$ and $\psi A_s(t) = M_s(t)$. Replacing above and solving for $\dot{M}_s(t)$ results in:

$$\dot{M}_s(t) = \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(t)$$

which, after replacement, yields the following equation:

$$\dot{M}_s(t) = \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(0)\exp\left\{\left[\bar{r}_s(t) - \rho\right] t\right\}$$

In turn, this equation has a solution satisfying:

$$
\begin{aligned}
M_s(t) &= M_s(0) \cdot \exp\left\{\int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu\right\} \\
&+ \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\left\{-\int_0^\xi \frac{r_s(v)}{P_s(v)} dv\right\} d\xi \cdot \exp\left\{\int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu\right\} \\
&- \int_0^t \psi C_s(0) \exp\left\{\left[\bar{r}_s(\xi) - \rho\right]\xi\right\} \cdot \exp\left\{-\int_0^\xi \frac{r_s(v)}{P_s(v)} dv\right\} d\xi \cdot \exp\left\{\int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu\right\}
\end{aligned}
$$

which, using the definition of $\bar{r}(t)$, becomes:

$$
\begin{aligned}
M_s(t) &= M_s(0) \cdot \exp\left\{\bar{r}(t) \cdot t\right\} \\
&+ \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\left\{-\bar{r}(\xi) \cdot \xi\right\} d\xi \cdot \exp\left\{\bar{r}(t) \cdot t\right\} \\
&- \psi C_s(0) \cdot \int_0^t \exp\left\{\left[\bar{r}_s(\xi) - \rho\right]\xi\right\} \cdot \exp\left\{-\bar{r}(\xi) \cdot \xi\right\} d\xi \cdot \exp\left\{\bar{r}(t) \cdot t\right\}
\end{aligned}
$$

simplifying the last integral:

$$
\begin{aligned}
M_s(t) &= M_s(0) \cdot \exp\left\{\bar{r}(t) \cdot t\right\} \\
&+ \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\left\{-\bar{r}(\xi) \cdot \xi\right\} d\xi \cdot \exp\left\{\bar{r}(t) \cdot t\right\} \\
&- \psi C_s(0) \cdot \int_0^t \exp\left\{-\rho\xi\right\} d\xi \cdot \exp\left\{\bar{r}(t) \cdot t\right\}
\end{aligned}
$$

Finally, note that both $P_s(t)$ and $r_s(t)$ are functions of wages. Therefore, given the initial measure of varieties $M_s(0)$ and the wages for all countries, which are defined at every instance through the trade equilibrium, paths for consumption $C_s(t)$, varieties

$M_s(t)$ and assets $A_s(t) = 1/\psi M_s(t)$ follow the equations above.

As a final step, one needs to pin down the starting values. $M_s(0)$ is given and calibrated to reflect the technological level of country s. Choice of $C_s(0)$, by contrast, is an endogenous object that guarantees that, given lifetime income and the initial level of assets, consumption as governed by the euler equation will be optimal. Start from the equation above, multiply both sides by $\exp\{-\bar{r}(t)\cdot t\}$:

$$\begin{aligned}\exp\{-\bar{r}(t)\cdot t\} M_s(t) &= M_s(0) + \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\{-\bar{r}(\xi)\cdot\xi\} d\xi \\ &- \psi C_s(0) \cdot \int_0^t \exp\{-\rho\xi\} d\xi\end{aligned}$$

Now evaluate this equation taking the limit $t \to \infty$.

$$\begin{aligned}\lim_{t\to\infty}\Big(\exp\{-\bar{r}(t)\cdot t\} M_s(t)\Big) &= M_s(0) + \int_0^\infty \psi \frac{w_s(t)}{P_s(t)} L_s \cdot \exp\{-\bar{r}(t)\cdot t\} dt \\ &- \psi C_s(0) \cdot \int_0^\infty \exp\{-\rho t\} dt\end{aligned}$$

Recall that the transversality condition is:

$$\lim_{t\to\infty}\left[\exp\{-\int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu\} P_s(t) A_s(t)\right] = 0$$

which states that the value of assets cannot grow faster than the interest rate, the standard no-Ponzi scheme condition. Using the fact that $\psi A_s(t) = M_s(t)$, noting that prices $P_s(t)$ are always positive and finite, and dividing both sides by $P_s(t)/\psi$, we can rewrite this as:

$$\lim_{t\to\infty}\left[\exp\{-\bar{r}_s(t)t\}M_s(t)\right]=0$$

Using the fact that $\lim_{t\to\infty}\Big(\exp\left\{-\bar{r}(t)\cdot t\right\}M_s(t)\Big)=0$, we can then solve for $C_s(0)$ as:

$$C_s(0)=\left[\frac{1}{\psi}M_s(0)+\int_0^\infty\frac{w_s(t)}{P_s(t)}L_s\cdot\exp\left\{-\bar{r}_s(t)\cdot t\right\}dt\right]\cdot\left[\int_0^\infty\exp\left\{-\rho t\right\}dt\right]^{-1}$$

Using the fact that $\int_0^\infty\exp\left\{-\rho t\right\}dt=\frac{1}{\rho}$, then:

$$\begin{aligned}C_s(0) &= \rho\left[\frac{1}{\psi}M_s(0)+\int_0^\infty\frac{w_s(t)}{P_s(t)}L_s\cdot\exp\left\{-\bar{r}_s(t)\cdot t\right\}dt\right] \qquad (1.25)\\ &= \rho\left[\underbrace{A_s(0)}_{\text{initial wealth}}+\underbrace{\int_0^\infty\frac{w_s(t)}{P_s(t)}L_s\cdot\exp\left\{-\bar{r}_s(t)\cdot t\right\}dt}_{\text{PV of real labor income}}\right]\end{aligned}$$

Therefore, at any instant t, consumption is proportional to lifetime wealth:

$$\begin{aligned}C_s(t) &= \rho\left[A_s(0)+\int_0^\infty\frac{w_s(\tau)}{P_s(\tau)}L_s\cdot\exp\left\{-\bar{r}_s(\tau)\cdot\tau\right\}d\tau\right]\cdot\exp\left\{\left[\bar{r}_s(t)-\rho\right]t\right\} \qquad (1.26)\\ &= \rho\left[A_s(t)+\int_t^\infty\frac{w_s(\tau)}{P_s(\tau)}L_s\cdot\exp\left\{-\bar{r}_s(\tau)\cdot\tau\right\}d\tau\right]\end{aligned}$$

Final varieties producers problem

Each final goods producer chooses intermediate inputs and labor to maximize profits according to:

$$\begin{aligned}\max_{\ell_s(t,\omega),\{x_{ks}(t,\omega,\nu)\}} \quad & p_{ss}(t,\omega)z_s(t,\omega)[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}\sum_{k\in\boldsymbol{K}}\int_0^{M_k(t)}[x_{ks}(t,\omega,\nu)]^{\alpha}d\nu\right) \\ - \quad & w_s(t)\ell_s(t,\omega)-\sum_{k\in\boldsymbol{K}}\int_0^{M_k(t)}p_{ks}(t,\nu)x_{ks}(t,\omega,\nu)d\nu\end{aligned}$$

There are infinitely many first order conditions for this problem: one for each variety ν and one for labor. These satisfy:

$$\begin{aligned}w_s(t)\ell_s(t,\omega) &= (1-\alpha)\cdot p_{ss}(t,\omega)z_s(t,\omega)[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}\sum_{k\in\boldsymbol{K}}\int_0^{M_k(t)}[x_{ks}(t,\omega,\nu)]^{\alpha}d\nu\right) \\ p_{ks}(t,\nu)x_{ks}(t,\omega,\nu) &= \alpha\cdot p_{ss}(t,\omega)z_s(t,\omega)[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}[x_{ks}(t,\omega,\nu)]^{\alpha}\right)\end{aligned}$$

Solving for $x_{ks}(t,\omega,\nu)$ yields equation (1.4):

$$x_{ks}(t,\omega,\nu)=\left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}}\cdot\ell_s(t,\omega)\cdot z_s(t,\omega)^{\frac{1}{1-\alpha}}$$

Intermediate varieties producers problem

Each intermediate varieties producer holds perpetual rights over variety ν, which they sell to final goods varieties in every country $d\in\boldsymbol{K}$. For each destination, they take demand as given and choose prices to maximize profits at every moment:

$$\max_{p_{ks}(t,\omega,\nu)} \frac{1}{\tau_{ks}} p_{ks}(t,\omega,\nu) x_{ks}(t,\omega,\nu) - P_k(t) x_{ks}(t,\omega,\nu)$$

Replacing for $x_{ks}(t,\omega,\nu)$:

$$\max_{p_{ks}(t,\omega,\nu)} \left[p_{ks}(t,\omega,\nu) - \tau_{ks} P_k(t)\right] \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot z_s(t,\omega)^{\frac{1}{1-\alpha}}$$

which, after taking the FOC and solving for $p_{ks}(t,\omega,\nu)$ yields the optimal price as a mark-up over marginal price, which is independent of ω or ν:

$$p_{ks}(t,\omega,\nu) = \frac{\tau_{ks} P_k(t)}{\alpha} \qquad \forall \omega \in [0,1], \quad \forall \nu \in [0, M_s(t)]$$

Flow aggregate profits $\Pi_s(t) \equiv \int_0^{M_s(t)} \pi(t,\nu) d\nu$ are a constant fraction of revenue:

$$\begin{aligned} \Pi_s(t) &= \frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} \lambda_{sd}^M(t) \cdot P_d(t) Y_d(t) \\ &= \frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} \frac{M_s \left(p_{sd}^M\right)^{1-\eta}}{P_d^M} \cdot P_d(t) Y_d(t) \\ &= \frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} \frac{M_s \left(\tau_{sd} P_{sd}\right)^{1-\eta}}{\sum_{k' \in \boldsymbol{K}} M_{k'} \left(\tau_{k'd} P_{k'}\right)^{1-\eta}} \cdot P_d(t) Y_d(t) \end{aligned}$$

Profits per variety $\pi_s(t,\nu) = \frac{1}{M_s(t)} \Pi_s(t)$ are independent of ν.

Trade in Final Goods

Trade shares

In this model, since there are infinitely many varieties in the unit interval, the expenditure share of destination region $d \in \boldsymbol{K}$ on goods coming from source country $s \in \boldsymbol{K}$ converge to their expected values. Let $\lambda_{sd}(t,\omega)$ denote the probability that consumers in region $d \in \mathcal{D}$ source variety ω from region $s \in \mathcal{D}$. For each each n, let $A_n^{-1}(t,\omega) \equiv \frac{\tilde{x}_{sd}(t)}{\tilde{x}_{nd}(t)}$, with $x_{sd}(t) \equiv (P_s^M(t))^{\alpha} w_s(t)^{1-\alpha}\tau_{sd}$. This probability will satisfy:

$$
\begin{aligned}
\lambda_{sd}(t,\omega) &= Pr\Big(s \text{ is the lowest cost supplier of } \omega \text{ to } d\Big) \\
&= Pr\left(\frac{\tilde{x}_{sd}(t)}{z_s(t,\omega)} < \min_{(n\neq s)}\left\{\frac{\tilde{x}_{nd}(t)}{z_n(t,\omega)}\right\}\right) \\
&= \int_0^{\infty} Pr(z_s(t,\omega)=z)Pr(z_n(t,\omega) < zA_n(t))dz \\
&= \int_0^{\infty} f_s(t)(z)\Pi_{(n\neq s)}F_n(t)(A_n z)dz \\
&= \int_0^{\infty} \theta T_s z^{-(1+\theta)} \exp\left\{-\left(\sum_{n\in \boldsymbol{K}} T_n A_n(t)^{-\theta}\right) z^{-\theta}\right\} dz \\
&= \frac{T_s\left(\tilde{x}_{sd}(t)\right)^{-\theta}}{\sum_{n\in \boldsymbol{K}} T_n\left(\tilde{x}_{nd}(t)\right)^{-\theta}} \\
&= \frac{T_s\left(\tilde{x}_{sd}(t)\right)^{-\theta}}{\sum_{n\in \boldsymbol{K}} T_n\left(\tilde{x}_{nd}(t)\right)^{-\theta}} \\
&= \frac{T_s(w_s(t)^{1-\alpha}P_s^M(t)^{\alpha}\tau_{sd})^{-\theta}}{\sum_{n\in \boldsymbol{K}} T_n(w_n(t)^{1-\alpha}P_n^M(t)^{\alpha}\tau_{nd})^{-\theta}} \qquad (1.27)
\end{aligned}
$$

Now note that $\lambda_{sd}(t,\omega)$ is independent of ω, so the probability of sourcing each variety from s to d is identical. A corollary is that aggregate expenditure trade shares of final goods from s in d will be equal to the probability of sourcing an arbitrary variety from s in d.

Price distributions and ideal price index

Recall that, under the assumption of perfect competition, prices equal their marginal costs, such that the price of a variety ω produced in country s and shipped to d satisfies $p_{sd}(t,\omega) = \frac{\tau_{sd} P_s^M(t)^\alpha w_s(t)^{1-\alpha}}{z_s(t,\omega)}$.

Since $z_s(t,\omega)$ is a random variable, $p_{sd}(t,\omega)$ is also a random variable. We can derive the distribution of prices through the following steps. First, note that $z_s(t,\omega) = \frac{\tau_{sd} P_s^M(t)^\alpha w_s(t)^{1-\alpha}}{p_{sd}(\omega)}$. Then, note that:

$$p_{sd}(t,\omega) < p = \frac{\tau_{sd} P_s^M(t)^\alpha w_s^{1-\alpha}}{z} \iff z_s(t,\omega) > z = \frac{\tau_{sd} P_s^M(t)^\alpha w_s^{1-\alpha}}{p}$$

Therefore:

$$\begin{aligned} G_{sd}(t,\omega)(p) &= Pr(p_{sd}(t,\omega) < p) \\ &= Pr\left(z_s(t,\omega) > \frac{\tau_{sd} P_s^M(t)^\alpha w_s^{1-\alpha}}{p} \right) \\ &= 1 - \exp\{-T_s(\tau_{sd} P_s^M(t)^\alpha w_s(t))^{-\theta} p^\theta\} \qquad \forall \omega \in [0,1] \end{aligned}$$

which is the distribution of prices of any variety ω conditional on s being the lowest cost supplier of such a variety to d. To derive the unconditional distribution of prices at d, realize that:

$$
\begin{aligned}
G_n(t,\omega) &\equiv Pr(p_s(t,\omega) < p) \\
&= Pr((\exists s) \text{ for which } p_s d(t,\omega) < p) \\
&= 1 - Pr((\nexists s) \text{ for which } p_{sd}(t,\omega) < p) \\
&= 1 - \prod_{s \in \boldsymbol{K}} Pr(p_{sd}(t,\omega) > p) \\
&= 1 - \prod_{s \in \boldsymbol{K}} \exp\{-T_s(\tau_{sd} P_s^M(t)^\alpha w_s(t))^{-\theta} p^\theta \\
&= 1 - \exp\{- \sum_{s \in \boldsymbol{K}} T_s(\tau_{sd} P_s^M(t)^\alpha w_s(t))^{-\theta} p^\theta\}
\end{aligned}
$$

Recall that the price index is defined as:

$$
\begin{aligned}
P_d(t) &= \left[\int_0^1 p_d(t,\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^\infty p^{1-\sigma} dG_n(t,p)\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^\infty p^{1-\sigma} \theta p^{\theta-1} \exp\left\{- \sum_{s \in \boldsymbol{K}} T_s(\tau_{sd} P_s^M(t)^\alpha w_s(t)^{1-\alpha})^{-\theta} p^\theta\right\} dp\right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Using a change of variables, let $\nu \equiv \sum_{s \in \boldsymbol{K}} T_s(\tau_{sd} P_s^M(t)^\alpha w_s(t)^{1-\alpha})^{-\theta} p^\theta$ and note that $d\nu = \theta p^{\theta-1} \sum_{s \in \boldsymbol{K}} T_s(\tau_{sd} P_s^M(t)^\alpha w_s(t)^{1-\alpha})^{-\theta} p^\theta dp$. Then:

$$
\begin{aligned}
P_d(t) &= \left[\int_0^\infty \left(\frac{\nu}{\sum_{s\in\boldsymbol{K}} T_s(\tau_{sd}P_s^M(t)^\alpha w_s(t)^{1-\alpha})^{-\theta}}\right)^{\frac{1-\sigma}{\theta}} \exp\{-\nu\}\, d\nu\right]^{\frac{1}{1-\sigma}} \\
&= \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \left(\sum_{s\in\boldsymbol{K}} T_s(\tau_{sd}P_s^M(t)^\alpha w_s(t)^{1-\alpha})^{-\theta}\right)^{-\frac{1}{\theta}} \qquad (1.28)\\
&= \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \times \\
&\quad \left(\sum_{s\in\boldsymbol{K}} T_s(\tau_{sd}w_s(t)^{1-\alpha})^{-\theta}\left(\sum_{n\in\boldsymbol{K}} M_n(t)\left[\frac{\tau_{ns}P_n(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}\right)^{(1-\alpha)\theta}\right)^{-\frac{1}{\theta}}
\end{aligned}
$$

which shows that, given parameters $T_s.\tau_{sd}$ and the vector of state variables $\boldsymbol{M}_s(t) = [M_1(t),\cdots,M_N(t)]'$, the closed form solution for the ideal price index $P_d(t)$ is a function of the vector of wages $\boldsymbol{w}(t) = [w_1(t),\cdots,w_N(t)]'$.

Market Clearing and Trade Balance

Market Clearing

Let $Y_d(t)$ denote the total output of the final good and $X_d(t), I_d(t)$ denote the use of the final good as inputs for the production of intermediate inputs and R&D, respectively. Then total output in the final good for a given country must satisfy:

$$
Y_d(t) = C_d(t) + I_d(t) + X_d(t)
$$

where $I_d(t)$ and $C_d(t)$ are pinned down by the dynamic problem, described below, and $X_d(t)$ can be expressed as a function of aggregate demand in all destinations:

$$X_d(t) \equiv \sum_{k \in \boldsymbol{K}} M_d(t) \cdot \left(\frac{p^M_{dk}(t)}{P^M_k(t)} \right)^{-\eta} \cdot \alpha \cdot \left(\frac{P^M_k(t)}{P_k(t)} \right)^{-1} \cdot Y_k(t)$$

Combining the equations, one can express aggregate output as a function of the state variable $M_d(t)$, parameters, and wages (both $r_d(t)$ and $P_d(t)$ are functions of wages in every country):

$$Y_d(t) = I_d(t) + C_d(t) + \sum_{k \in \boldsymbol{K}} M_d(t) \cdot \left(\frac{p^M_{dk}(t)}{P^M_k(t)} \right)^{-\eta} \cdot \alpha \cdot \left(\frac{P^M_k(t)}{P_k(t)} \right)^{-1} \cdot Y_k(t)$$

Expenditure Determination

Flow aggregate profits $\Pi_s(t) \equiv \int_0^{M_s(t)} \pi(t, \nu) d\nu$ are a constant fraction of revenue:

$$\Pi_s(t) = \frac{\alpha}{\eta} \cdot \sum_{d \in \boldsymbol{K}} \lambda^M_{sd}(t) \cdot P_d(t) Y_d(t)$$

On the expenditure side, GDP of each destination country $s \in \boldsymbol{K}$ country will be exhausted as the combination of the total expenditures of labor and capital income:

$$P_s(t) Y_s(t) = w_s(t) L_s + \Pi_s(t)$$

From the income side, nominal GDP must equal the sum of total flow payments received domestically and from the rest of the world:

$$P_s(t) Y_s(t) = \sum_{d \in \boldsymbol{K}} \left[(1 - \alpha) \lambda^F_{sd}(t) + \frac{\alpha}{\eta} \lambda^M_{sd}(t) \right] P_d(t) Y_d(t)$$

Trade Balance

Total exports are equal to:

$$EX_d(t) = \underbrace{\sum_{d \neq s \in \boldsymbol{K}} \lambda^F_{sd}(t) P_d(t) Y_d(t)}_{\text{exports in final goods}} + \underbrace{\alpha \sum_{d \neq s \in \boldsymbol{K}} \lambda^M_{sd}(t) \left[\sum_{k' \in \boldsymbol{K}} \lambda^F_{dk'}(t) P_{k'}(t) Y_{k'}(t) \right]}_{\text{exports in intermediates}}$$

Total imports are equal to:

$$IM_d(t) = \underbrace{[1 - \lambda^F_{ss}(t)] P_s(t) Y_s(t)}_{\text{imports in final goods}} + \underbrace{\alpha [1 - \lambda^M_{ss}(t)] \left[\sum_{k' \in \boldsymbol{K}} \lambda^F_{dk'}(t) P_{k'}(t) Y_{k'}(t) \right]}_{\text{imports in intermediates}}$$

Since there are no international capital markets in this economy, trade will be balanced at any instant. This means that:

$$\sum_{d \neq s \in \boldsymbol{K}} \lambda^F_{sd}(t) P_d(t) Y_d(t) + \alpha \sum_{d \neq s \in \boldsymbol{K}} \lambda^M_{sd}(t) \left[\sum_{k' \in \boldsymbol{K}} \lambda^F_{dk'}(t) P_{k'}(t) Y_{k'}(t) \right] =$$
$$[1 - \lambda^F_{ss}(t)] P_s(t) Y_s(t) + \alpha [1 - \lambda^M_{ss}(t)] \left[\sum_{k' \in \boldsymbol{K}} \lambda^F_{dk'}(t) P_{k'}(t) Y_{k'}(t) \right]$$

Homogeneity of Income in Equilibrium

The trade share for final goods is:

$$\lambda^F_{sd}(t) \equiv \frac{E^F_{sd}(t)}{E^F_d(t)} = \frac{T_s(P^M_s(t)^\alpha w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{\sum_{n\in\boldsymbol{K}} T_n(P^M_n(t)^\alpha w_n(t)^{1-\alpha}\tau_{nd})^{-\theta}} = \frac{T_s(P^M_s(t)^\alpha w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{P_s(t)^{-\theta}}$$

Evaluating it at $\lambda_{ss}(t)$, noting that $\lambda^M_s(t) = \frac{M_s(t)(p^M_{ss}(t))^{1-\eta}}{(P^M_s(t))^{1-\eta}} = \frac{M_s(t)(\frac{1}{\alpha}P_s(t))^{1-\eta}}{(P^M_s(t))^{1-\eta}}$ and solving it for $P_s(t)$ allows me to write it linear in $M_s(t)$:

$$\begin{aligned}
P_s(t) &= T_s^{-\frac{1}{\theta}}\lambda^F_{ss}(t)^{\frac{1}{\theta}}(P^M_s(t)^\alpha w_s(t)^{1-\alpha}) \\
P_s(t) &= \left(\frac{T_s}{\lambda^F_{ss}(t)}\right)^{-\frac{1}{\theta}}\left(\frac{M_s(t)}{\lambda^M_{ss}(t)}\right)^{\frac{\alpha}{1-\eta}} \cdot \alpha^{-\alpha} \cdot P_s(t)^\alpha (w_s(t)^{1-\alpha}) \\
P_s(t) &= \alpha^{-\frac{\alpha}{1-\alpha}} \cdot \left(\frac{T_s}{\lambda^F_{ss}(t)}\right)^{-\frac{1}{\theta(1-\alpha)}}\left(\frac{M_s(t)}{\lambda^M_{ss}(t)}\right)^{\frac{1}{1-\eta}\frac{\alpha}{1-\alpha}} \cdot w_s(t) \\
P_s(t) &= \alpha^{-\frac{\alpha}{1-\alpha}} \cdot \left(\frac{T_s}{\lambda^F_{ss}(t)}\right)^{-\frac{1}{\theta(1-\alpha)}}\frac{\lambda^M_{ss}(t)}{M_s(t)} \cdot w_s(t) \qquad \left(\because \frac{\alpha}{1-\alpha} = 1-\eta\right)
\end{aligned}$$

which allows me to write real wages as a linear function of $M_s(t)$:

$$\frac{w_s(t)}{P_s(t)} = M_s(t) \times \alpha^{1-\eta} \times \left(\frac{T_s}{\lambda^F_{ss}(t)}\right)^{\frac{1}{\theta(1-\alpha)}} \times \left(\lambda^M_{ss}(t)\right)^{-1} \equiv M_s(t) \times \mathcal{R}^w_s(t) \tag{1.29}$$

Similarly, I can write aggregate profits as a linear function of $M_s(t)$:

$$\begin{aligned}
\Pi_s(t) &= \frac{\alpha}{\eta}\sum_{d\in\boldsymbol{K}} \lambda^M_{sd}(t)P_d(t)Y_d(t) \\
\Pi_s(t) &= \frac{\alpha}{\eta}\sum_{d\in\boldsymbol{K}} \frac{M_s(t)\left(\alpha^{-1}\tau_{sd}P_s(t)\right)^{1-\eta}}{(P^M_d(t))^{1-\eta}}P_d(t)Y_d(t)
\end{aligned}$$

Therefore:

$$\frac{\Pi_s(t)}{P_s(t)} = M_s(t) \times \frac{\alpha}{\eta} \sum_{d \in \boldsymbol{K}} \left(\frac{\alpha^{-1} \tau_{sd} P_s(t)}{P_d^M(t)} \right)^{1-\eta} \frac{P_d(t) Y_d(t)}{P_s(t)} \equiv M_s(t) \times \mathcal{R}_s^{\pi}(t) \tag{1.30}$$

The budget constraint then is:

$$\begin{aligned}
C_s(t) + I_s(t) &= \frac{w_s(t)}{P_s(t)} L_s + \frac{\Pi_s(t)}{P_s(t)} \\
&= M_s(t) \times \\
&\left[\alpha^{1-\eta} \left(\frac{T_s}{\lambda_{ss}^F(t)} \right)^{\frac{1}{\theta(1-\alpha)}} \left(\lambda_{ss}^M(t) \right)^{-1} L_s + \frac{\alpha}{\eta} \sum_{d \in \boldsymbol{K}} \left(\frac{\alpha^{-1} \tau_{sd} P_s(t)}{P_d^M(t)} \right)^{1-\eta} \frac{P_d(t) Y_d(t)}{P_s(t)} \right] \\
&= M_s(t) \times \left[\mathcal{R}_s^w(t) + \mathcal{R}_s^{\pi}(t) \right] = M_s(t) \times \mathcal{R}_s(t)
\end{aligned}$$

Balanced Growth Path

Autarky
Proof of Proposition (1)

Proof. Without loss of generality, choose an arbitrary country $s \in \boldsymbol{K}$. Since this world economy is under autarky, evaluate (1.2) replacing for the real interest rate using equations (1.11) and (1.13) and taking the limit $\tau_{sd} \to \infty (\forall s \neq d)$. By assumption (1), $\tau_{ss} = 1 (\forall s)$. Therefore, (1.2) collapses to:

$$g_s^{\text{autarky}} = \frac{\alpha \cdot \psi}{\eta} \cdot \frac{Y_s(t^*)}{M_s(t^*)} - \rho \tag{1.31}$$

for a BGP inclusive of each period $t \geq t^*$.

The next step in the proof is to show that $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s^{\text{autarky}}$.

Since real wages, real profits, assets, and real output are linear functions of $M_s(t)$ in each period, it follows that $g_{M_s} = g_{Y_s} = g_{w_s} = g_{A_s}$. Since, with log preferences, consumption is a constant fraction of output, $g_{C_s} = g_{Y_s}$. Since the choice of s was arbitrary, this holds for any $s \in \boldsymbol{K}$.

To show uniqueness, one needs to solve for growth rate in terms of parameters. In order to do so, a few intermediate steps are necessary. First, note that one can express the demand for intermediates as:

$$\bar{x}_{ss}(t,\omega) \equiv x_{ss}(t,\omega,\nu) = \left[\alpha z_s(\omega) p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega)$$

which, in turn, implies that the optimal price of intermediate varieties is $p_{ss}(t,\omega,\nu) = \frac{1}{\alpha}$ and I can rewrite the production function of the final goods producer as:

$$\begin{aligned}
y_s(\omega) &= z_s(\omega)\ell_s(t,\omega)^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{M_s(t)}\left[\bar{x}_{ss}(t,\omega)\right]^{\alpha} d\nu\right) \\
&= z_s(\omega)\ell_s(t,\omega)^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{M_s(t)}\left[\left[\alpha z_s(\omega) p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega)\right]^{\alpha} d\nu\right) \\
&= \left[z_s(\omega)\right]^{\frac{1}{1-\alpha}} \cdot \left[\alpha \cdot p_{ss}(t,\omega)\right]^{\frac{\alpha}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot \frac{1}{\alpha} \cdot M_s(t)
\end{aligned}$$

Replacing for $p_{ss}(t,\omega)$ using the assumption of pricing under perfect competition:

$$\begin{aligned}
y_s(\omega) &= \left[z_s(\omega)\right]^{\frac{1}{1-\alpha}} \cdot \left[\alpha \frac{w_s(t)^{1-\alpha}\alpha^{-\alpha(1-\eta)} M_s^{\alpha}}{\alpha \cdot z_s(\omega)}\right]^{\frac{\alpha}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot \frac{1}{\alpha} \cdot M_s(t) \\
&= \alpha^{-(1-\alpha)} \cdot z_s(\omega) \cdot w_s(t)^{\alpha} \cdot M_s(t)^{1-\alpha} \cdot \ell_s(t,\omega)
\end{aligned}$$

By GDP expenditure clearing, total expenditure is equal wages plus profits:

$$Y_s(t) = w_s(t)L_s + \frac{\alpha}{\eta}Y_s(t) \implies \frac{1-\alpha}{\eta}Y_s(t) = w_s(t)L_s \implies Y_s(t) = w_s(t)L_s$$

where the last equation states that, in the last equation, GDP is labor income because labor is the only factor of income in this economy. Hence, value added is equal to labor income.

Integrating the production function over ω and using the fact above gives us:

$$Y_s(t) = \left[\int_0^1 z_s(\omega)\ell_s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \cdot \alpha^{-(1-\alpha)} \cdot w_s(t)^{\alpha} \cdot M_s(t)^{1-\alpha} = L_s w_s(t)$$

solving for $w_s(t)$:

$$w_s(t) = \left(\left[\int_0^{\infty} z\ell_s(t,z)^{\frac{\sigma-1}{\sigma}} dF_s(z)\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{1-\alpha}} \cdot \alpha^{-1} \cdot M_s(t) L_s^{-\frac{1}{1-\alpha}}$$

The term in the integral denotes the joint product of productivity and labor allocation across firms. In aggregate terms, since both the distribution of productivity and the population are fixed for every t; and relative wages are fixed along the BGP, this term will be constant.

Following Alvarez and Lucas (2007), note that all goods enter symmetrically in the definition of the aggregate final good and they differ only by their productivity level. Therefore, one can express the BGP growth rate of the economy fully in terms of exogenous objects:

$$g_s^{\text{autarky}} = \frac{\psi}{\eta} \cdot \left(\left[\int_0^\infty z \ell_s(t,z)^{\frac{\sigma-1}{\sigma}} dF_s(z) \right]^{\frac{\sigma}{\sigma-1}} \right)^{\frac{1}{1-\alpha}} \cdot L_s^{-\frac{\alpha}{1-\alpha}} - \rho \tag{1.32}$$

Since neither the productivity distribution $F_s(z)$ nor the demand functions $\ell_s(t^*,z)$ will change along the BGP and all other terms in the growth rate are parameters, this pins down the uniqueness of the BGP under autarky, which completes the proof. □

Zero gravity
Proof of Proposition (2)

Proof. Without loss of generality, choose an arbitrary country $s \in \boldsymbol{K}$. Since this world economy is under zero gravity, evaluate (1.2) replacing for the real interest rate using equations (1.11) and (1.13) and evaluating $\tau_{sd} = 1(\forall s,d)$.Therefore, (1.2) collapses to:

$$g_s^{\text{zero gravity}} = \left[\frac{\alpha \cdot \psi}{\eta \cdot P_s(t^*)} \cdot \frac{\sum_{k \in \boldsymbol{K}} Y_k(t^*)}{\sum_{k \in \boldsymbol{K}} M_k(t^*)} - \rho \right] \tag{1.33}$$

for a BGP inclusive of each period $t \geq t^*$. Since there are no trade costs, the law of one price holds, and $P_s(t^*) = P_d(t^*) \equiv P(t^*)$ for every $s,d \in \boldsymbol{K}$. Choosing $P(t^*)$ to be numéraire of this economy shows that the growth rate will follow the stated equation.

Since the choice of the s of arbitrary and the expression in the right-hand side of the equation is equal for every $s \in \boldsymbol{K}$, it follows that the $g_s^{\text{zero gravity}} = g^{\text{zero gravity}}$ for all $s \in \boldsymbol{K}$, which shows that the growth rate must be common across all countries. Furthermore, since $Y_k(t^*) = M_k(t^*)\mathcal{R}_k(t^*)$ and the fact that g_s must be constant along a BGP, $\frac{\sum_{k \in \boldsymbol{K}} Y_k(t^*)}{\sum_{k \in \boldsymbol{K}} M_k(t^*)}$ will only be homogeneous of degree zero in $[M_n(t^*)]_{n \in \boldsymbol{K}}$ if $\mathcal{R}_k(t^*) = \mathcal{R}(t^*)$.

With log preferences, households will consume a constant fraction $(1-\rho)$ of their income and invest a fraction ρ. The non-arbitrage condition shows that real interest rate and returns to R&D equalize globally along the BGP:

$$\frac{r_s(t^*)}{P_s(t^*)} = \frac{\psi\pi_s(t^*,\nu)}{P_s(t^*)} = \frac{\psi\Pi_s(t^*)}{M_s(t^*)P_s(t^*)} = \frac{\psi}{M_s(t^*)}M_s(t^*) \times \mathcal{R}_s^{\pi}(t^*) = \psi\rho\mathcal{R}(t^*)$$

The next step in the proof is to show that $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s^{\text{zero gravity}}$. Since real wages, real profits, assets, and real output are linear functions of $M_s(t)$ in each period, it follows that $g_{M_s} = g_{Y_s} = g_{w_s} = g_{A_s}$. Since, with log preferences, consumption is a constant fraction of output, $g_{C_s} = g_{Y_s}$. Since the choice of s was arbitrary, this holds for any $s \in \boldsymbol{K}$.

For uniqueness, one needs to show that the cross-subsectional equilibrium is unique. Start from equation (1.15). Evaluating it under zero gravity and noting that prices of final goods and intermediate goods equalize in that situation results in:

$$P_s(t^*)Y_s(t^*) = \sum_{d \in \boldsymbol{K}} \left[(1-\alpha)\frac{T_s w_s(t^*)^{-(1-\alpha)\theta}}{\sum_{k \in \boldsymbol{K}} T_k w_k(t^*)^{-(1-\alpha)\theta}} + \frac{\alpha}{\eta}\frac{M_s(t^*)}{\sum_{k \in \boldsymbol{K}} M_k(t^*)}\right] P_d(t^*)Y_d(t^*)$$

Recall that $P_s(t^*)Y_s(t^*) = w_s(t^*)L_s + \Pi_s(t^*)$ and note that, under zero gravity, $\Pi_s(t^*) = \frac{\alpha}{\eta}\frac{M_s(t^*)}{\sum_{k \in \boldsymbol{K}} M_k(t^*)} P_d(t^*)Y_d(t^*)$. So, given $M_s(t^*)$ the expenditure determination system becomes a simple system in wages:

$$w_s(t^*)L_s = \sum_{d \in \boldsymbol{K}} \left[(1-\alpha)\frac{T_s w_s(t^*)^{-(1-\alpha)\theta}}{\sum_{k \in \boldsymbol{K}} T_k w_k(t^*)^{-(1-\alpha)\theta}}\right] w_d(t^*)L_d$$

Define the excess demand function:

$$Z_s(\mathbf{w},t) \equiv \frac{1}{w_s(t^*)} \left(\sum_{d\in \boldsymbol{K}} \left[(1-\alpha) \frac{T_s w_s(t^*)^{-(1-\alpha)\theta}}{\sum_{k\in \boldsymbol{K}} T_k w_k(t^*)^{-(1-\alpha)\theta}} \right] w_d(t^*) L_d - w_s(t^*) L_s \right)$$

and note:

$$\frac{\partial Z_s(\mathbf{w},t^*)}{\partial w_d(t^*)} = \frac{1}{w_s(t^*)} (1-\alpha) \lambda^F_{sd}(t^*) \left(L_d + \frac{\lambda^F_{dd}(t^*)}{w_d(t^*)} \right) > 0$$

which shows that it satisfies the gross substitution property and the cross-subsection equilibrium is unique. Therefore, the BGP under zero gravity will be unique.

□

General case
Proof of Proposition (3)

Proof. Without loss of generality, choose an arbitrary country $s \in \boldsymbol{K}$. From (1.18), real GDP is a linear function of $M_s(t)$:

$$C_s(t) + I_s(t) = \frac{w_s(t)}{P_s(t)} L_s + \frac{\Pi_s(t)}{P_s(t)} = M_s(t) \times \mathcal{R}_s(t)$$

Over the BGP, with log preferences, consumption is a constant fraction of GDP: $C_s(t^*) = (1-\rho) M_s(t) \times \mathcal{R}_s(t)$. From the Poisson arrival process, $g_{M_s} = \frac{\dot{M}_s(t^*)}{M_s(t)} = \psi\rho \frac{I_s(t^*)}{M_s(t^*)}$. Since trade is balanced, $I_s(t^*) = \frac{\rho}{1-\rho} C_s(t^*)$ and varieties grow at the following rate:

$$g_{M_s} = \psi\rho\left[\alpha^{1-\eta}\left(\frac{T_s}{\lambda_{ss}^F(t^*)}\right)^{\frac{1}{\theta(1-\alpha)}}\left(\lambda_{ss}^M(t^*)\right)^{-1}L_s + \frac{\alpha}{\eta}\sum_{d\in\boldsymbol{K}}\lambda_{sd}^M(t^*)\frac{P_d(t^*)Y_d(t^*)}{P_s(t^*)M_s(t^*)}\right]$$

The following statements are true:

1. $\lambda_{ss}^F(t^*), \lambda_{ss}^M(t^*)$ are homogeneous of degree zero in $\{M_n(t^*)\}$;
2. $\left(\frac{P_s^M(t^*)}{P_d^M(t^*)}\right), \left(\frac{P_d(t^*)}{P_s(t^*)}\right)$ are homogeneous of degree zero in $\{M_n(t^*)\}$;
3. $\frac{Y_d(t^*)}{M_s(t^*)} = \frac{M_s(t^*)\times\mathcal{R}_s(t^*)}{M_s(t^*)}$ is homogeneous of degree zero in $\{M_n(t^*)\}$ if and only if $\mathcal{R}_s(t^*)$ is homogeneous of degree zero in $\{M_n(t)\}$ for all $s \in \boldsymbol{K}$.

Therefore, for g_{M_s} to be consistent with a BGP it must also be homogeneous of degree zero in $[M_n(t^*)]_{n\in\boldsymbol{K}}$. As a result, if g_{M_s} is consistent with a BGP, $\mathcal{R}_s(t^*)$ must be homogeneous of degree zero in $[M_n(t^*)]_{n\in\boldsymbol{K}}$ for all $s \in \boldsymbol{K}$. As a result, it must be that varieties grow at the same rate across countries, which implies that $\mathcal{R}_s(t^*) = \mathcal{R}(t^*)$.

With log preferences, households will consume a constant fraction $(1-\rho)$ of their income and invest a fraction ρ. The non-arbitrage condition shows that real interest rate and returns to R&D equalize globally along the BGP:

$$\frac{r_s(t^*)}{P_s(t^*)} = \frac{\psi\pi_s(t^*,\nu)}{P_s(t^*)} = \frac{\psi\Pi_s(t^*)}{M_s(t^*)P_s(t^*)} = \frac{\psi}{M_s(t^*)}M_s(t^*)\times\mathcal{R}_s^\pi(t^*) = \psi\rho\mathcal{R}(t^*)$$

The next step in the proof is to show that $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s$. Since real wages, real profits, assets, and real output are linear functions of $M_s(t)$ in each period, it follows that $g_{M_s} = g_{Y_s} = g_{w_s} = g_{A_s}$. Since, with log preferences, consumption is a constant fraction of output, $g_{C_s} = g_{Y_s}$. Since the choice of s was arbitrary, this holds for any $s \in \boldsymbol{K}$.

$\square$

Changes in trade costs
Proof of Proposition 4

Proof. The equilibrium growth rate of varieties:

$$g_{M_s} = \psi\rho\left[\left(\frac{T_s}{\lambda^F_{ss}(t^*)}\right)^{\frac{1}{\theta(1-\alpha)}}\left(\lambda^M_{ss}(t^*)\right)^{-1}L_s + \frac{\alpha}{\eta}\sum_{d\in\boldsymbol{K}}\lambda^M_{sd}(t^*)\frac{P_d(t^*)Y_d(t^*)}{P_s(t^*)M_s(t^*)}\right]$$

Recall that:

$$\sum_{d\in\boldsymbol{K}}\lambda^M_{sd}(t^*) = \sum_{d\in\boldsymbol{K}}\frac{M_s(\tau_{sd}P_s(t^*))^{1-\eta}}{\sum_{k'\in\boldsymbol{K}}M_{k'}(\tau_{k'd}P_{k'}(t^*))^{1-\eta}}\frac{P_d(t^*)Y_d(t^*)}{P_s(t^*)M_d(t^*)}$$

Since these economies are symmetric, then: $P_s(t^*) = P_{s'}(t^*)$, $w_s(t^*) = w_{s'}(t^*)$, $M_s(t^*) = M_{s'}(t^*)$ for every s,s' and $\tau_{sd} = \tau$ for every sd. Evaluated with symmetric economies, the expression above becomes:

$$\sum_{d\in\boldsymbol{K}}\lambda^M_{sd}(t^*) = \frac{(N-1)\tau^{1-\eta}}{[1+(N-1)\tau^{1-\eta}]} + \frac{1}{[1+(N-1)\tau^{1-\eta}]} = 1$$

Therefore, denoting $P_s(t^*) = P(t^*)$, $M_s(t^*) = M(t^*)$ and noting that $Y_d(t^*) = M_d(t^*) \times \mathcal{R}(t^*)$ the growth rate becomes to:

$$g^* = \psi\rho\left[(T_s)^{\frac{1}{\theta(1-\alpha)}}\left(\frac{1}{1+(N-1)\tau^{1-\theta}}\right)^{-\frac{1}{\theta(1-\alpha)}}\left(\frac{1}{1+(N-1)\tau^{1-\eta}}\right)^{-1}L_s + \frac{\alpha}{\eta}\mathcal{R}\right]$$

Then, take the derivative of g^* wrt τ:

$$\begin{aligned}\frac{\partial g^*}{\partial \tau} &= \psi\rho\,(T_s)^{\frac{1}{\theta(1-\alpha)}}\left(\frac{1}{1+(N-1)\tau^{1-\theta}}\right)^{-\frac{1}{\theta(1-\alpha)}}\left(\frac{1}{1+(N-1)\tau^{1-\eta}}\right)^{-1}L_s \times \\ &\quad \left(\frac{(1-\eta)\tau^{-\eta}}{1+(N-1)\tau^{1-\eta}} - \frac{\theta\tau^{-\theta-1}}{1+(N-1)\tau^{-\theta}}\right) < 0\end{aligned}$$

which is negative because $(1-\eta) < 0$ and every other term in the parenthesis is positive.

$\square$

Welfare

Recall that $C_s(t^*)$ can be expressed as a constant fraction of total lifetime wealth:

$$C_s(t^*) = \rho\left[A_s(t^*) + \int_{t^*}^{\infty}\frac{w_s(\tau)}{P_s(\tau)}L_s \cdot \exp\left\{-\bar{r}_s(\tau)\cdot\tau\right\}d\tau\right]$$

where $\bar{r}_s = \frac{1}{\tau}\int_{t^*}^{\tau} r_s(t)dt$ is the average interest rate between t^* and τ. Since this holds along the BGP, $\frac{w_s(\tau)}{P_s(\tau)} = \frac{\exp\{(\tau-t^*)g_{w_s}\}w_s(t^*)}{P_s(t^*)}$. Furthermore, since $\frac{r_s(t^*)}{P_s(t^*)}$ is constant along the BGP, $\bar{r}_s(\tau) = \frac{r_s(t^*)}{P_s(t^*)}$ for all $\tau \geq t^*$. Replacing those above results in:

$$
\begin{aligned}
C_s(t^*) &= \rho\left[A_s(t^*) + \frac{w_s(t^*)}{P_s(t^*)}L_s\int_{t^*}^{\infty}\cdot\exp\left\{-\left(\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s}\right)\cdot(\tau - t^*)\right\}d\tau\right] \\
&= \rho\left[A_s(t^*) + \frac{w_s(t^*)}{P_s(t^*)}\frac{L_s}{\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s}}\right] \\
&= \rho A_s(t^*) + \rho\frac{w_s(t^*)}{P_s(t^*)}\frac{L_s}{\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s}}
\end{aligned}
$$

Since $g_{w_s} = g_{C_s}$ and $g_{C_s} = \frac{r_s(t^*)}{P_s(t^*)} - \rho$, $\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s} = \rho$. Hence, over the BGP, real consumption is a fraction of assets plus real labor income:

$$
C_s(t^*) = \rho A_s(t^*) + \frac{w_s(t^*)L_s}{P_s(t^*)}
$$

Welfare over the BGP is:

$$
\begin{aligned}
\int_{t^*}^{\infty}\exp\{-\rho(t-t^*)\}\log\left(\exp\{g^*t\}C_s(t^*)\right)dt &= \int_{t^*}^{\infty}\exp\{-\rho(t-t^*)\}\log\left(C_s(t^*)\right)dt \\
&+ \int_{t^*}^{\infty}\exp\{-\rho(t-t^*)\}g^*tdt \\
&= \frac{\log\left(C_s(t^*)\right)}{\rho} + \frac{g^*}{\rho^2} \\
&= \log\left(A_s(t^*)\right) + \frac{1}{\rho}\log\left(\frac{w_s(t^*)L_s}{P_s(t^*)}\right) + \frac{g^*}{\rho^2}
\end{aligned}
$$

Finally, using the fact that $\psi A_s(t^*) = M_s(t^*)$, I can write:

$$\int_{t^*}^{\infty} \exp\{-\rho(t-t^*)\} \log\left(\exp\{g^* t\} C_s(t^*)\right) dt =$$
$$\log\left(\frac{1}{\psi} M_s(t^*)\right) + \frac{1}{\rho} \log\left(\frac{w_s(t^*) L_s}{P_s(t^*)}\right) + \frac{g^*}{\rho^2}$$

Static welfare

For real labor income, start from equation (1.9) evaluated at $s = d$ and use the fact that, as shown in equation (1.28) of Appendix 1.7.2,

$$P_s(t) = \gamma \cdot \left[\sum_{n \in \boldsymbol{K}} T_n (w_n(t)^{1-\alpha} P_n^M(t)^\alpha \tau_{nd})^{-\theta}\right]^{-\frac{1}{\theta}}$$

where $\gamma \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$. Then, own trade share in a given country can be represented by:

$$\lambda_{dd}^F(t) = \gamma^\theta \cdot \frac{T_d(w_d(t)^{1-\alpha}(P_d(t)^M)^\alpha)^{-\theta}}{[P_d(t)]^{-\theta}}$$

Solving for $\frac{w_d(t)}{P_d(t)}$ delivers:

$$\frac{w_d(t)}{P_d(t)} \quad = \quad \gamma^{\frac{1}{1-\alpha}} \lambda_{dd}(t)^{-\frac{1}{(1-\alpha)\theta}} T_d^{\frac{1}{(1-\alpha)\theta}} \left(\frac{P_d^M(t)}{P_d(t)}\right)^{-\alpha}$$

Replacing for the definition of $P_d^M(t) = \left[\sum_{k \in \boldsymbol{K}} M_k \left(\frac{\tau_{kd} P_k(t)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-\frac{1-\alpha}{\alpha}}$ results in:

$$\frac{w_d(t)}{P_d(t)} = \gamma^{\frac{1}{1-\alpha}} \lambda_{dd}(t)^{-\frac{1}{(1-\alpha)\theta}} T_d^{\frac{1}{(1-\alpha)\theta}} \left[\sum_{k \in \boldsymbol{K}} M_k \left(\frac{\tau_{kd} P_k(t)}{\alpha P_d(t)} \right)^{-\frac{\alpha}{1-\alpha}} \right]^{1-\alpha}$$

Consider what happens to welfare after a change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$, as in Arkolakis, Costinot, and Rodríguez-Clare (2012). In this dynamic setting, to compare the static component of welfare, I need to compare what happens across the two BGPs, comparing the two initial equilibria. Suppose t^* is the initial period of the original BGP while t^{**} is the first period of the final BGP. To fit this framework to the general trade literature, I will compare the static component of these BGP as if they happened in the same period, and compound the difference over time.

Let $\hat{x} \equiv x(t^* *)/x(t^*)$. Then cumulative changes in static welfare are:

$$\frac{1}{\rho} \log \left(\widehat{\frac{w_s(t^{**})}{P_s(t^{**})}} \right) =$$

$$\frac{1}{\rho} \log \left(\widehat{\lambda_{dd}^F}(t^{**})^{-\frac{1}{(1-\alpha)\theta}} \right) + \frac{1}{\rho\eta} \log \left(\sum_{k \in \boldsymbol{K}} \mu_k(t^*) \widehat{M}_k(t^{**}) \cdot \left(\frac{\widehat{\tau}_{kd} \widehat{P}_k(t^{**})}{\widehat{P}_d(t^{**})} \right)^{1-\eta} \right)$$

where $\mu_k(t) \equiv \frac{M_k(t) \cdot \left(\frac{\tau_{kd} P_k(t)}{P_d(t)} \right)^{1-\eta}}{\sum_{k \in \boldsymbol{K}} M_k(t) \cdot \left(\frac{\tau_{kd} P_k(t)}{P_d(t)} \right)^{1-\eta}}$

Nesting of Romer and Eaton-Kortum

In this subsubsection, I will briefly describe how to recover the canonical P. M. Romer (1990) and Eaton and Kortum (2002) models from the framework described above.

Eaton-Kortum

Setting $\alpha = 0$ implies that the value of new varieties is zero since the demand for and profits of varieties is also zero. Therefore, $I_s(t) = 0$ and $A_s(t) = 0$ for all t and s. While the Eaton-Kortum model is a static model, here it can be thought of as an infinite sequence of static models with no intertemporal decision, since there are no longer asset markets that permit households to save:

$$
\begin{aligned}
\max_{C_s(t), c_s(t,\omega)_{\omega \in [0,1]}} \quad & \int_0^\infty \exp\{-\rho t\} \log\left(C_s(t)\right) dt \\
s.t. \quad P_s(t) C_s(t) &= w_s(t) L_s \\
C_s(t) &= \left[\int_0^1 c_s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \\
P_s(t) C_s(t) &= \int_0^1 p_s(t,\omega) c_s(t,\omega) d\omega
\end{aligned}
$$

Furthermore, since $\alpha = 0$, the intermediate and research and development sectors disappear. The problem of the final goods producer becomes:

$$
\max_{\ell_s(t,\omega)} p_{ss}(t,\omega) \cdot z_s(t,\omega) \cdot \ell_s(t,\omega) - \ell_s(t,\omega) w_s(t)
$$

which is identical to the one in the standard Eaton-Kortum model. Equilibrium will take the form of a system of labor market determination equations that solve for N

wages using trade expenditure shares.

Romer

Setting $\tau_{sd} \to \infty$ for $s \neq d$ implies trade costs are prohibitively high internationally, such that varieties of both final goods and intermediate goods become sold only locally. Normalizing the price of the domestic final good to be the numéraire in each country, I write the dynamic household problem as:

$$\begin{aligned}
\max_{C_s(t), c_s(t,\omega)_{\omega\in[0,1]}} \quad & \int_0^\infty \exp\{-\rho t\} \log\left(C_s(t)\right) dt \\
s.t. \quad I_s(t) = \dot{A}(t) \quad &= \quad r_s(t)A_s(t) + w_s(t)L_s - C_s(t) \\
C_s(t) \quad &= \quad \left[\int_0^1 c_s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

Furthermore, redefine assumption (2) in the following terms:

Assumption 4 (Productivity draws to recover Romer)**.** *To recover the Romer model as a special case of the general model, I need to specify productivity terms $z_s(\omega)$ which are homogeneous across firms in each country. In order to do so, redefine the cumulative distribution function $F_s(t)(z)$ of the baseline case to be one of a degenerate random variable with a point mass concentrated at a certain scalar for each country. Formally:*

$$F_s(t)(z) = \begin{cases} 0 \textit{ for } z < T_s \\ \\ 1 \textit{ for } z \geq T_s \end{cases}$$

Using the symmetry assumption above, the numéraire normalization and the unavailability of foreign intermediate goods in the domestic market, the final goods assembler technology becomes:

$$y_s(t,\omega) = T_s[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{M_s(t)}[x_{ss}(t,\omega,\nu)]^{\alpha}d\nu\right)$$

which is identical to the single-country Romer model. Profits and demand per variety $\nu \in [0, M_s(t)]$ will be constant and growth will be driven by the domestic R&D sector. Equilibrium will take the following form: labor markets will clear; total final goods produced being equal to total final goods used for consumption, intermediate production; and R&D production; and optimized household optimal dynamics will be described by an Euler equation and an asset/measure accumulation equation.

1.7.3 Qualitative evidence: life among product innovators

As an initial exploratory part of this research, I conducted a qualitative survey of managers in firms of New Member States. I first collected a list of notable firms from publicly available sources, restricted the sample to those who were active for at least two years before the time their respective countries joined the European Union, and then crawled through their English-language websites to collect the publicly available contact information. I sent the questionnaire below to 221 firms.

My goal was to assess if the description of the world that macro theorists set forth aligns with the practical intuitions of entrepreneurs. And it turns out that, at among the group of managers that responded to my email, they do. I will highlight two illustrative cases in the text.

For instance, the dynamic mechanism that propels growth, as I have described in the theory subsection is that increased access to foreign markets increases expected profits, thereby increasing the incentive to invest in research and development. This is entirely consistent with the description of the facts by one Czech biotech entrepreneur:

> "Once we joined the EU [...] this allowed us to increase our exports and fund our own genetic programmes." *CEO of a Czech Biotech company*

In their comments, they went on to specify the importance of having access of not only to the European market itself, but also third party markets. They mentioned that after the Czech Republic joined the EU, his firm had immediate access to the standards for labeling and certification in existing trade agreements between the EU and third parties, which facilitated their firm's exports. These kinds of non-tariff barriers are typically considered part of trade costs τ in most trade models.

In this firm's particular case, product innovation came through the invention of breeding of new varieties of farm animals, that were then commercialized. But we see a similar story in a very different market: alcoholic beverages:

> "In 2004, we first started producing the ultra-luxury variation of our signature vodka, which became a popular export product [...] and later started production of 18 new products." —Spokesperson of a Latvian liquor manufacturer

In this case, the firm reported having used the European market's exports as a platform for global expansion. For context, 2004 marks the year Latvia accessed the EU —and also the year that this manufacturer decided to expand its product line by introducing the ultra-luxury versions of its signature product, which they claimed was adequate to the Western European market.

Once again, this is qualitatively consistent with the theoretical mechanism proposed in the model, with market access likely inducing product innovation. Of course, these individual experiences are not necessarily representative of a large universe of firms, which is why in the next two subsections I will perform a detailed quantitative exploration of the data, first detailing some stylized facts, then going into causal inference. Nonetheless, the type of qualitative evidence presented here is useful to show to that the big picture is consistent with the individual experiences.

Qualitative Questionnaire

1. After your country joined the European Union, did your company:
    - start producing more products/services or varieties;
    - start producing fewer products/services or varieties; or
    - keep producing about the same number of products/services or varieties?
2. If your company changed the number of products/services or varieties after EU accession, how was the change implemented and what were the results? Please

include any important information or relevant anecdotes.

3. If your company changed the number of products/services or product/service varieties after EU accession, was the decision primarily motivated by access to new technologies/imports, access to new markets/exports, or both? Explain.

4. After your country joined the European Union, did your company:

    - stay in the same industry;
    - expanded to another industry; or
    - move completely to a new industry?

5. If your company expanded to another industry or moved to a new industry. Please explain whether the change was related to your country's EU accession.

1.7.4 Data and Empirical Appendix

Extensive Description of the Data

Production data

Production data comes from Eurostat's Prodcom (*Production Communautaire*), which is an annual full coverage survey of the European mining, quarry and manufacturing sectors, reporting the value of production of 4,000+ different product-lines of EU members and candidate countries. Prodcom reports, for each product line, country, and year, the value (in euros) and volume (in kg, m^2, number of items, etc.) of production. Product lines follow the Statistical Classification of Products by Activity in the EU (CPA).

The target population of the full coverage sample is every enterprise that manufactures some good in the Prodcom List. Data quality is good for member countries since European Law[21] mandates National Statistical Institutes to collect enterprise-level information on the value and volume of production covering at least 90% of national production in each NACE class, defined as the first four digits of each product code. In practice, reporting goes beyond this minimum threshold and, according to Eurostat, the coverage error is estimated to be below 10%.

Let n, i, p, t index countries, sectors, products, and periods, respectively; and denote Y_{inpt} as the market value of production of product p[22]. The set of varieties produced in each sector is $\mathcal{M}_{nit} = \{k : Y_{nikt} > 0\}$. The measure of varieties is simply the cardinality of the set of produced varieties $M_{nit} = |\mathcal{M}_{nit}| = \sum_k \mathbb{1}_{\{k:Y_{nikt}>0\}}$. The overall measure over varieties produced in a country is, then: $M_{nt} = \sum_i M_{nit}$. These measures can be

[21] "PRODCOM statistics are compiled under the legal basis provided by Council Regulation (EEC) NO 3924/1991 of 19 December 1991 and by Commission Regulation (EC) No 0912/2004 of 29 April 2004 implementing the Council Regulation (EEC) No 3924/91 on the establishment of a Community survey of industrial production. Additionally, a Commission Regulation updating the PRODCOM classification is available annually since 2003."

[22] To construct sector codes, I use Eurostat concordances to map Prodcom product codes to Harmonized System (HS) product codes. I then used the respective HS-2 division codes as sector codes.

directly calculated from Prodcom's table.

Oftentimes, the value of production is labeled as confidential information by the National Statistical Institute, particularly in cases in which production is concentrated on a few enterprises. In those cases, while the value and volume are not publicly available, Eurostat reports this number as *confidential*, which still allows one to infer that $Y_{nikt} > 0$ for that particular variety k, implying that the variety is produced.

Typically, production information at the variety level is not available, which pushed researchers to use product-level trade data instead. Some exceptions include Goldberg et al. (2010) and Rachapalli (2021), who use firm-product links from the Indian Survey of Manufacturers; A. B. Bernard, S. J. Redding, and Schott (2011), who use US Manufacturing Censuses firm-product data.

Tariff and trade flow data

Bilateral tariff data come from WITS (World Integrated Trade Solution Trade Stats). It consolidates tariff data from the UNCTAD's Trade Analysis Information System (TRAINS) as well as from the WTO.

To construct effective tariff rates, one starts from baseline tables of most favored nation tariffs at the source-country $\times$ destination-country $\times$ HS6-code $\times$. Then, one superimposes every bilateral product level preferential tariff available in the WITS database on each of these tables. Furthermore, whenever there are gaps between two identical bilateral preferential tariffs, one fills in those gaps. The result is a dataset of effectively applied tariff rates.

Bilateral trade flow data comes from UNCOMTRADE. These data, which are widely used in research, come natively in a source-country $\times$ destination-country $\times$ HS-6 product-code $\times$ year format, which makes it readily compatible with the tariff data mentioned above.

Let s,d,i,p,t index source countries, destination countries, sectors, products, and periods, respectively; and denote X_{sdipt} as the market value of bilateral trade of product p.

The set of traded varieties in each sector is $\mathcal{X}_{nit} = \{k : X_{sdikt} > 0\}$. Analogously as with production, one can observe the total number of traded varieties $\sum_k \mathbb{1}_{\{k:X_{nikt}>0\}}$. To make sure these are comparable to PRODCOM's codes, whenever possible, I used concordances and restricted the set of goods to create a dataset that matched both trade and production.

Other data

I also collected data on (a) the dates of accession of new member states to the European Union; (b) trade agreements existent and entered into force between the European Union and third parties before 2004; and (c) expenditure in private research & development expenditures per capita. The first two come from hand collecting documents and tables from the European Commission's official websites while the latter comes from Eurostat.

Formal Description of the Callaway & Sant'Anna Estimator

Formally, let a "treatment" group g be defined as being treated for all periods $t \geq g$. Note that, since the EU enlargement happened simultaneously for more than one country, there is more than one country n for each $g_n = g$. If some country cluster is in group g, then $G_{nt} = g$ $(\forall t)$. If it is never treated, it is in the control group, and then $G_{nt} = \infty$ $(\forall t)$.

The parameter of interest is the average treatment on the treated for a given treatment group g and horizon t, i.e.:

$$ATT(g,t) = \mathbb{E}[M_{nt}(g) - M_{nt}(0)|G_{nt} = g] \quad (1.34)$$

where $M_{nt}(g)$ is the potential outcome of country n at period t if treated at period g; $M_{nt}(0)$ is the potential outcome country n at period t if untreated; X_{ng-1} are pre-treatment time-invariant covariates; and $G_{nt} = g$ is a group indicator.

Note that the $ATT(g,t)$ is group and period-specific. It can be recovered under assumptions similar to the standard difference-in-differences framework: parallel trends and no-anticipation[23]. The next step is to summarize the ATT across groups by appropriately weighting the results as:

$$\theta(t) = \sum_{g} \mathbb{1}\{g \leq t\} w_{gt} ATT(g,t) \quad (1.35)$$

for some weights w_{gt}. Callaway and Sant'Anna (2021) propose the weights $w_{gt} = P(G_{nt} = g|G_{nt} \leq t)$, which is the share of country clusters from group $g \geq t$ out of all country clusters being treated at time t.

Further Details on Causal Inference

Since the largest wave of enlargement was in 2004, in this analysis I will focus exclusively on that wave. The source of variation is at the source-country $\times$ destination-country $\times$ HS-code product level. In each year, there are about 300 thousand observations. Figure 1.15 shows the interquartile range of bilateral product-level tariff rates between NMS and the set of countries that had concluded trade agreements with the EU prior to 2004.

[23]Formally, parallel trends is the assumption that potential outcomes evolve almost surely equally to the untreated group: $\mathbb{E}[M_{nt}(0) - M_{nt-1}(0)|G_{nt} = g] = \mathbb{E}[M_{nt}(0) - M_{nt-1}(0)|G_{nt} > g]$ for all $t \geq g$. No anticipation means that potential outcomes for a treated group are equal to the untreated group for any date before the treatment —i.e., for all $t < g$, $\mathbb{E}[M_{nt}(g)|G_{nt} = 1] = \mathbb{E}[M_{nt}(0)|G_{nt} = 1]$ almost surely.

It shows that there is not much change in tariffs leading up to membership and then a median drop of about 2.5 percentage points between 2003 and 2004. In the years immediately after membership, there is also not a large change in the distribution of bilateral tariff rates. There are some changes after 2007, possibly because some future provisions in trade agreements kick in.

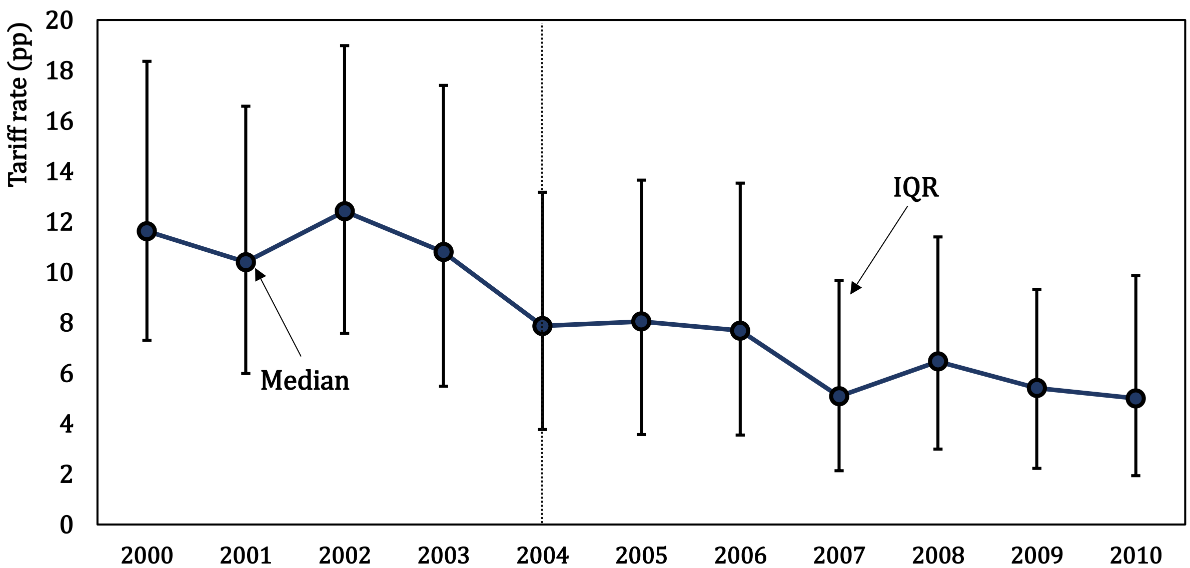


Figure 1.15. Distribution of Tariff Changes Over Time: Interquartile Range Bilateral HS6-Product-Level Tariff Rates Between New Member States (2004 EU Enlargement) and Set of Countries that Concluded Trade Agreements with EU prior to 2004. Data were constructed from WITS Preferential and MFN databases.

The metric of the tariff shock change is simply $\Delta\tau_{sdip,2004} \equiv (\tau_{sdip,2004} - \tau_{sdip,2003})$, which is the change in the level of effectively applied bilateral tariffs at the product level between 2003 and 2004. Figure 1.16 plots the distribution of $\Delta\tau_{sdip,2004}$, excluding the zero-valued observations. The average $\Delta\tau_{sdip,2004}$ is 2.14% and the standard deviation is 12%.

I estimate a sequence of cross-subsectional local-projection linear probability models, which estimate what is the marginal effect of an *increase* in the tariffs on exports of a given product p, *conditional on that country s not producing that particular product before joining the EU in 2003*. The fact the data is highly granular permits me to exploit

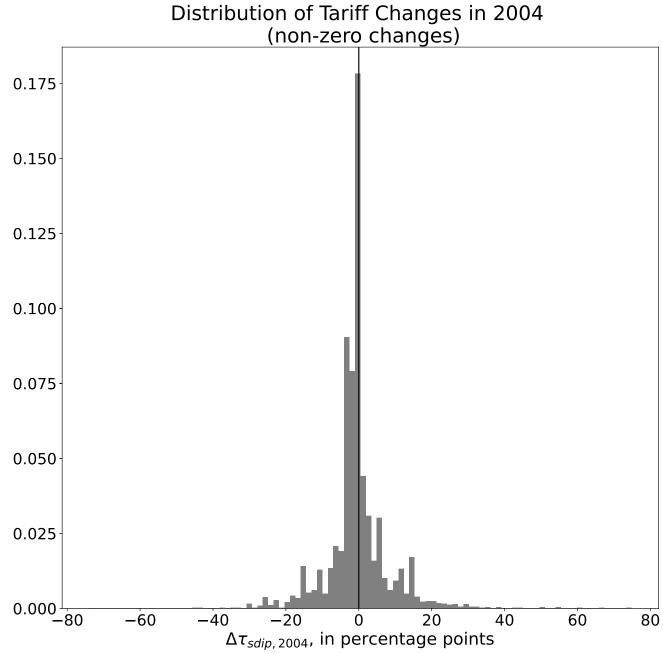


Figure 1.16. Tariff Shock: Distribution of the (non-zero) observations of the changes in Bilateral HS6-Product-Level Tariff Rates Between New Member States (2004 EU Enlargement) and Set of Countries that Concluded Trade Agreements with EU prior to 2004. Data were constructed from WITS Preferential and MFN databases.

within *industry* $\times$ *source* $\times$ *destination* $\times$ *horizon* (across product) variation.

Formally, I estimate the following equation:

$$P\left(X_{sdip,h} > 0 \middle| Y_{s\cdot ip,2003} = 0\right) = \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h} \quad \text{for } h \in \{2000, \cdots, 2010\} \tag{1.36}$$

where $X_{sdip,h}$ is the market value of exports between country s and country d of product p of industry i at horizon h; $Y_{s\cdot ip,2003}$ is the market value of production in country s of product p of industry i in 2003; α_h are horizon (time) fixed-effects; $\gamma_{sdi,h}$ are *source* $\times$ *destination* $\times$ *industry* interactions fixed-effects for each h.

Note that, since these are local projections, the right-hand side coefficients, the regressor $\tau_{sdip,2004}$ is fixed for all horizons, and the coefficients β_h change. As initially argued by Chodorow-Reich (2020) and later formalized by Dube et al. (2023), these types of cross-subsectional event studies with local projections can be interpreted as differences in differences with continuous treatments. If consistently estimated, the estimated coefficients β_h, then, are simply the average treatment on treated compared to the potential outcomes of not being treated, normalized to a treatment of intensity of one unit.

This strategy takes the assertion in Baier and Bergstrand (2007) (henceforth B&B) that countries engage endogenously in free trade agreements (FTAs) and one needs to look for a plausibly exogenous source of variation to check whether or not FTA "actually increase members' international trade" seriously. Here, I rely on their strategy of running dynamic panels with fixed effects to control for unobserved heterogeneity.

Importantly, while they estimate their models at the aggregate country level with *source* $\times$ *destination* $\times$ *period* fixed effects, I have enough variability and data availability to estimate it at the product-level adding *industry* $\times$ *source* $\times$ *destination* $\times$ *period* fixed effects. Hence, this approach adds granularity to B&B's strategy, thereby controlling for more unobserved heterogeneity.

The identification assumption is that conditional on the very saturated fixed effects that this model includes, the unobserved components $\nu_{sdip,h}$ are uncorrelated with the change in tariffs $\Delta\tau_{sdip,2004}$. Intuitively, the identification is robust to a NMS (say, Poland's) policymakers endogenously targeting EU accession to have preferential access to a third-party's (say, Mexico's) car industry (relative to other industries and countries), but not if they want to have preferential access to compact cars relative to SUVs in Mexico.

The identification strategy is plausible. In general, neither lobbyists of industry trade groups nor trade negotiations work in such a disaggregated product-level setting.

Typically, lobbyists consolidate the interests of the producers of many products under the same umbrella and try to influence negotiations. Similarly, even when governments are negotiating tariffs schedule changes —which was not the case in this particular case —these negotiations typically also happen in blocs, with governments exchanging positions in some products for others. Hence, the fact that this is a highly disaggregated dataset at the product level adds a lot of strength to the identification strategy.

As shown in Figure 1.6, an increase in market access by 1 percentage point increases the probability of starting to produce and export a given product by about 1 percent by 2010. To benchmark this result, it is about one-third of the conditional mean $\mathbb{E}[X_{sdip,h} > 0 | X_{s \cdot ip,h} > 0, h > 2003] = 2.9\%$. There are no signs of a pre-existing trend before 2004: both the magnitude of the coefficients and the standard errors are very small before the treatment date.

The related set of continuation regressions, is very similar to the model estimated in equation (1.1), except that now it conditions in initial production being active:

$$\begin{aligned} P\left(X_{sdip,h} > 0 \middle| Y_{s \cdot ip,2003} = 1\right) &= \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h} \\ &\quad \text{for } h \in \{2000, \cdots, 2010\} \end{aligned} \tag{1.37}$$

In this case, there are no effects observed on the extensive margin. When countries already have the ability to produce a given product, additional market access produces very noisy results in the extensive margin. The coefficients are large and bounce between positive and negative and the confidence bands are even larger. One potential explanation is that the countries possibly already had market access before 2004, as illustrated by the positive (albeit insignificant results) for 2000-03, since they already had the production capacity. It is possible that most of the effects concentrate on the intensive margin, something that futures iteration of this paper would need to check.

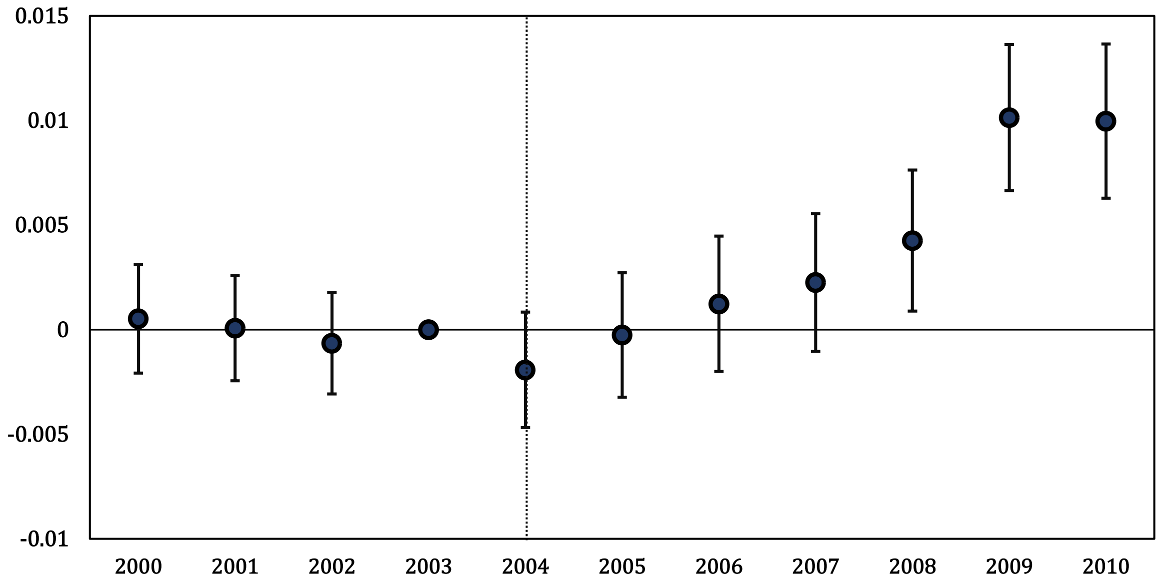


Figure 1.17. Entry Regressions. This plot shows the coefficients β_h of the local projection linear probability models specified in equation (1.1). Each year is a different cross-subsectional regression with approximately 300 thousand observations. The whiskers show 95% confidence intervals with robust standard errors clustered at the source-destination-industry level.

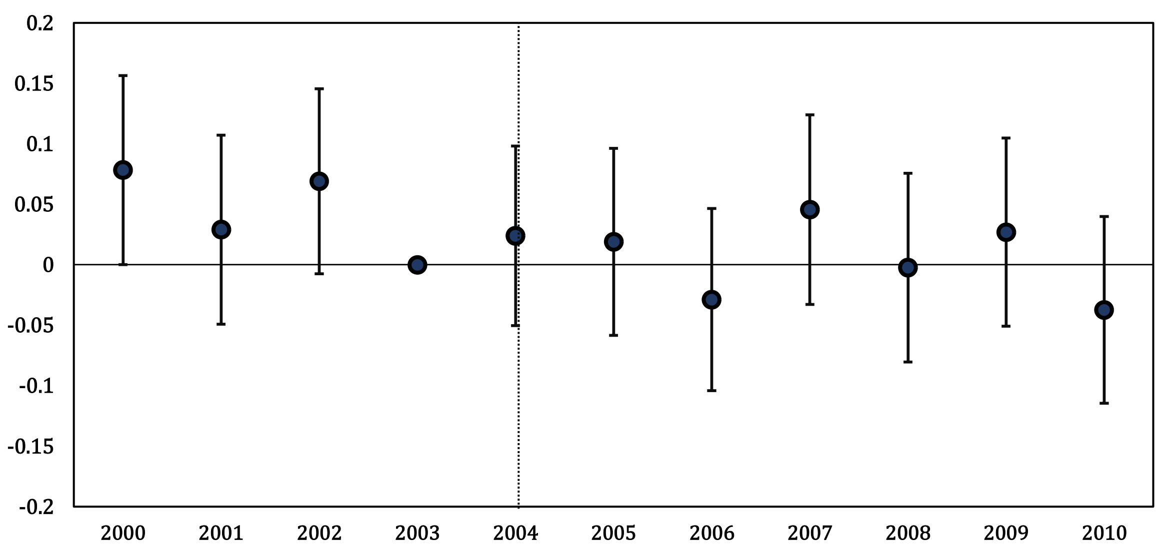


Figure 1.18. Continuation Regressions. This plot shows the coefficients β_h of the local projection linear probability models specified in equation (1.37). Each year is a different cross-subsectional regression with approximately 300 thousand observations. The whiskers show 95% confidence intervals with robust standard errors clustered at the source-destination-industry level.

1.7.5 Computational Appendix

This computation appendix explains how I solve for the BGP growth rate.

1. **Inner loop (Prices of Final Goods)**. Given parameters $\{\theta, \psi, \alpha, \mathbf{L}, \mathbf{T}, \boldsymbol{\tau}\}$ and guesses for wages $\boldsymbol{w}$, measures of varieties $\boldsymbol{M}$ and some common return $\mathcal{R}$, use the input-output structure of the model to solve for the prices of the final goods.

$$
\begin{aligned}
P_s(t) &= \gamma \cdot \left[\sum_{n \in \boldsymbol{K}} T_n \left(P_n^M(t)^{\alpha} w_n(t)^{1-\alpha} \tau_{ns}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \\
P_s(t) &= \gamma \cdot \left[\sum_{n \in \boldsymbol{K}} T_n \left(w_n(t)^{1-\alpha} \tau_{ns}\right)^{-\theta} \left(\sum_{k \in \boldsymbol{K}} M_k(t) \left(\frac{\tau_{kn} P_k(t)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}\right)^{\theta(1-\alpha)}\right]^{-\frac{1}{\theta}}
\end{aligned}
$$

with $\gamma \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$. The last equation makes it explicit that, given parameters, wages, and measures of varieties, this is a system of $|N|$ equations and $|N|$ unknowns in final goods prices. A simple grid search algorithm finds a fixed point for final goods prices.

2. **Intermediate loop**. Given parameters $\{\theta, \psi, \alpha, \mathbf{L}, \mathbf{T}, \boldsymbol{\tau}\}$ and guesses the measures of varieties $\boldsymbol{M}$, some common return $\mathcal{R}$, and the prices from the following step, use the expenditure determination equation to solve for final demand.

$$
P_s(t) Y_s(t) = \sum_{d \in \boldsymbol{K}} \left[(1-\alpha) \lambda_{sd}^F(t) + \frac{\alpha}{\eta} \lambda_{sd}^M(t)\right] P_d(t) Y_d(t)
$$

Update the guess for usages using a constant fraction $(1-\rho)$ of income over the

BGP and taking advantage of returns $\mathcal{R}^g$ and measures M_k^g:

$$w_s(t)L_s(t) = (1-\alpha)\sum_{k\in\boldsymbol{K}} \lambda_{sk}(t)^F (1-\rho)\mathcal{R}^g M_k(t)^g$$

Re-normalize $w_s(t) = \frac{w_s(t)}{L_s \cdot \sum_{k\in\boldsymbol{K}} w_k(t) L_k}$ to ensure it always maps onto a compact space, it is an operator and converges according to the contraction mapping theorem.

3. **Outer loop (Growth rates)**. Given parameters $\{\theta, \psi, \alpha, \mathbf{L}, \mathbf{T}, \boldsymbol{\tau}\}$, prices, wages, and trade shares calculated in the previous steps, update the guesses for M_s^g using:

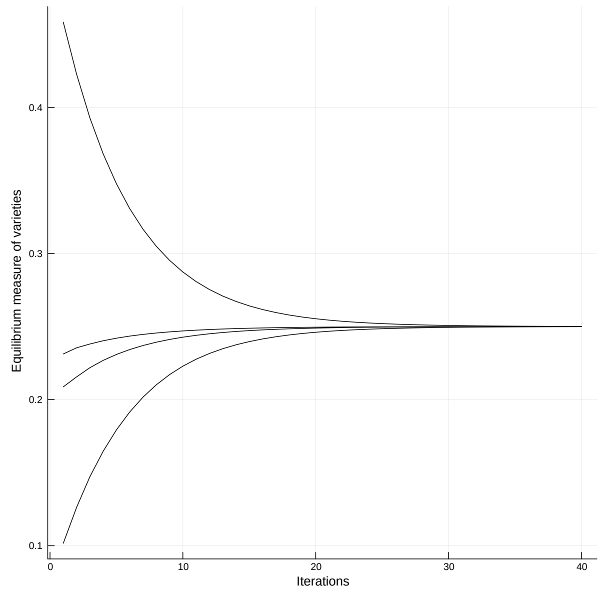
$$M_s^{g'} = \left(\frac{\lambda_{ss}^F(t^*)}{T_s}\right)^{-\frac{1}{\theta(1-\alpha)}} \left(\lambda_{ss}^M(t^*)\right)^{-1} L_s \frac{M_s^g}{\mathcal{R}^g} + \frac{\alpha}{\eta}\sum_{k\in\boldsymbol{K}}\left(\lambda_{sk}^M(t^*)\frac{\sum_{l\in\boldsymbol{K}} \lambda_{kl}^F(t^*)(1-\rho)M_l^g}{P_s(t^*)M_s^g}\right)$$

Again, to make sure it always maps onto a compact space, it is an operator and converges according to the contraction mapping theorem, renormalize the measure of varieties: $M_s^{g'} = \frac{M_s^{g'}}{P_s(t^*) \cdot \sum_{k\in\boldsymbol{K}} M_s^{g'} P_s(t^*)}$

And update the guesses for the global return rates:

$$\mathcal{R}^{g'} = \frac{1}{\sum_{k\in\boldsymbol{K}} M_s^{g'} P_s(t^*)}$$

A test of this algorithm is, starting from a random guess, knowing that a group of symmetric countries will eventually converge towards the same measure of varieties within some tolerance criterion $< \varepsilon$. One numerical illustration of this convergence is the Figure below, for a group of 4 symmetric countries, starting for a random guess, that eventually converge to 0.25 (the sum of the measure of varieties is normalized to sum to 1).



Calibration of Trade Shocks

I use observed trade flows to infer trade costs. The strategy goes back to Head and Ries (2001). According to the handbook chapter by Head and Mayer (2014), the index is called the Head-Ries Index (HRI) since 2011 (when the working paper version of Eaton, Kortum, et al. (2016) was published).

Expenditure in final goods is defined as:

$$E_{sd}^F(t) = \lambda_{sd}^F(t) P_d(t) Y_d(t) = \frac{T_s \left(\tilde{M}_s(t)^{1-\alpha}\right)^\theta (w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{\sum_{n=1}^N T_n \left(\tilde{M}_n(t)^{1-\alpha}\right)^\theta (w_n(t)^{1-\alpha}\tau_{nd})^{-\theta}} \cdot P_d(t) Y_d(t)$$

The ratio between $E_{sd}^F(t)$ and $E_{dd}^F(t)$ is, then:

$$\frac{E_{sd}^F(t)}{E_{dd}^F(t)} = \frac{T_s \left(\tilde{M}_s(t)^{1-\alpha}\right)^\theta (w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{T_d \left(\tilde{M}_d(t)^{1-\alpha}\right)^\theta (w_d(t)^{1-\alpha}\tau_{dd})^{-\theta}}$$

Analogously, the ratio between $E_{ds}^F(t)$ and $E_{ss}^F(t)$ is:

$$\frac{E_{ds}^F(t)}{E_{ss}^F(t)} = \frac{T_s \left(\tilde{M}_d(t)^{1-\alpha}\right)^\theta (w_d(t)^{1-\alpha}\tau_{ds})^{-\theta}}{T_d \left(\tilde{M}_s(t)^{1-\alpha}\right)^\theta (w_s(t)^{1-\alpha}\tau_{ss})^{-\theta}}$$

Therefore:

$$\frac{E_{sd}^F(t)}{E_{dd}^F(t)} \cdot \frac{E_{ds}^F(t)}{E_{ss}^F(t)} = \left(\frac{\tau_{sd}\tau_{ds}}{\tau_{ss}\tau_{dd}}\right)^{-(1-\alpha)\theta}$$

Using Assumption (1), $\tau_{ss} = \tau_{dd} = 1$ and $\tau_{sd} = \tau_{ds}$. Hence, I can express the trade cost τ_{sd} as:

$$\tau_{sd} = \left(\frac{E^F_{sd}(t)}{E^F_{dd}(t)} \cdot \frac{E^F_{ds}(t)}{E^F_{ss}(t)} \right)^{-\frac{1}{2\theta(1-\alpha)}} \tag{1.38}$$

Works cited in Chapter 1

Acemoglu, **Daron** (Dec. 2008). *Introduction to Modern Economic Growth*. New Haven: Princeton University Press.

Acemoglu, **Daron** and **Ventura**, **Jaume** (May 2002). The World Income Distribution*. *The Quarterly Journal of Economics*. 2002. 117, pp. 659–694.

Alvarez, **Fernando** and **Lucas**, **Robert E.** (Sept. 2007). General equilibrium analysis of the Eaton–Kortum model of international trade. *Journal of Monetary Economics*. 2007. 54, pp. 1726–1768.

Argente, **David**, **Baslandze**, **Salome**, **Hanley**, **Douglas**, and **Moreira**, **Sara** (2020). Patents to Products: Product Innovation and Firm Dynamics. *SSRN Electronic Journal*. 2020.

Arkolakis, **Costas**, **Costinot**, **Arnaud**, and **Rodríguez-Clare**, **Andrés** (Feb. 2012). New Trade Models, Same Old Gains? *American Economic Review*. 2012. 102, pp. 94–130.

Arkolakis, **Costas**, **Ganapati**, **Sharat**, and **Muendler**, **Marc-Andreas** (2020). *The Extensive Margin of Exporting Products: A Firm-level Analysis*. en. Tech. rep. w16641. National Bureau of Economic Research. DOI: 10.3386/w16641.

Baier, **Scott L.** and **Bergstrand**, **Jeffrey H.** (Mar. 2007). Do free trade agreements actually increase members' international trade? *Journal of International Economics*. 2007. 71, pp. 72–95.

Bas, **Maria** (Mar. 2012). Input-trade liberalization and firm export decisions: Evidence from Argentina. *Journal of Development Economics*. 2012. 97, pp. 481–493.

Bernard, **A. B.**, **Redding**, **S. J.**, and **Schott**, **P. K.** (Aug. 2011). Multiproduct Firms and Trade Liberalization. *The Quarterly Journal of Economics*. 2011. 126, pp. 1271–1318.

Bernard, **Andrew B**, **Jensen**, **J. Bradford**, **Redding**, **Stephen J**, and **Schott**, **Peter K** (Apr. 2009). The Margins of US Trade. *American Economic Review*. 2009. 99, pp. 487–493.

Boehm, **Christoph E.**, **Levchenko**, **Andrei A.**, and **Pandalai-Nayar**, **Nitya** (Apr. 2020). The Long and Short (Run) of Trade Elasticities. Working Paper. DOI: 10.3386/w27064. URL: https://www.nber.org/papers/w27064 (visited on 02/16/2023).

Borusyak, **Kirill**, **Jaravel**, **Xavier**, and **Spiess**, **Jann** (Apr. 2022). Revisiting Event Study Designs: Robust and Efficient Estimation. arXiv:2108.12419 [econ]. URL: http://arxiv.org/abs/2108.12419 (visited on 10/18/2022).

Buera, **Francisco J.** and **Oberfield**, **Ezra** (2020). The Global Diffusion of Ideas. *Econometrica*. 2020. 88, pp. 83–114.

Cai, **Jie**, **Li**, **Nan**, and **Santacreu**, **Ana Maria** (Jan. 2022). Knowledge Diffusion, Trade, and Innovation across Countries and Sectors. *American Economic Journal: Macroeconomics*. 2022. 14, pp. 104–145.

Caliendo, **Lorenzo**, **Opromolla**, **Luca David**, **Parro**, **Fernando**, and **Sforza**, **Alessandro** (Dec. 2021). Goods and Factor Market Integration: A Quantitative Assessment of the EU Enlargement. *Journal of Political Economy*. 2021. 129, pp. 3491–3545.

Callaway, **Brantly** and **Sant'Anna**, **Pedro H. C.** (Dec. 2021). Difference-in-Differences with multiple time periods. *Journal of Econometrics*. 2021. 225, pp. 200–230.

Chodorow-Reich, **Gabriel** (May 2019). Geographic Cross-Sectional Fiscal Spending Multipliers: What Have We Learned? *American Economic Journal: Economic Policy*. 2019. 11, pp. 1–34.

– (June 2020). Regional data in macroeconomics: Some advice for practitioners. *Journal of Economic Dynamics and Control*. 2020. 115, p. 103875.

Christiano, **Lawrence J.**, **Eichenbaum**, **Martin S.**, and **Trabandt**, **Mathias** (Aug. 2018). On DSGE Models. *Journal of Economic Perspectives*. 2018. 32, pp. 113–140.

Costinot, **Arnaud** and **Rodríguez-Clare**, **Andrés** (2014). Trade Theory with Numbers: Quantifying the Consequences of Globalization. en. In: *Handbook of International Economics*. Vol. 4. Elsevier, pp. 197–261. ISBN: 978-0-444-54314-1. DOI: 10.1016/B978-0-444-54314-1.00004-5. URL: https://linkinghub.elsevier.com/retrieve/pii/B9780444543141000045 (visited on 07/09/2023).

Demidova, **Svetlana** (2008). Productivity Improvements and Falling Trade Costs: Boon or Bane? *International Economic Review*. 2008. 49, pp. 1437–1462.

Dube, **Arindrajit**, **Girardi**, **Daniele**, **Jordà**, **Òscar**, and **Taylor**, **Alan M.** (Apr. 2023). A Local Projections Approach to Difference-in-Differences Event Studies. Working Paper. DOI: 10.3386/w31184. URL: https://www.nber.org/papers/w31184 (visited on 07/02/2023).

Eaton, **Jonathan** and **Kortum**, **Samuel** (2002). Technology, Geography, and Trade. *Econometrica*. 2002. 70, pp. 1741–1779.

Eaton, **Jonathan** and **Kortum**, **Samuel** (July 2006). *Innovation, Diffusion, and Trade*. en. Tech. rep. w12385. Cambridge, MA: National Bureau of Economic Research, w12385. DOI: 10.3386/w12385. URL: http://www.nber.org/papers/w12385.pdf (visited on 01/23/2021).

Eaton, **Jonathan**, **Kortum**, **Samuel**, **Neiman**, **Brent**, and **Romalis**, **John** (Nov. 2016). Trade and the Global Recession. *American Economic Review*. 2016. 106, pp. 3401–3438.

Goldberg, **P. K.**, **Khandelwal**, **A. K.**, **Pavcnik**, **N.**, and **Topalova**, **P.** (Nov. 2010). Imported Intermediate Inputs and Domestic Product Growth: Evidence from India. *The Quarterly Journal of Economics*. 2010. 125, pp. 1727–1767.

Goodman-Bacon, **Andrew** (Dec. 2021). Difference-in-differences with variation in treatment timing. *Journal of Econometrics*. 2021. 225, pp. 254–277.

Grossman, **Gene M.** and **Helpman**, **Elhanan** (1990). Comparative Advantage and Long-Run Growth. *The American Economic Review*. 1990. 80, pp. 796–815.

Head, **Keith** and **Mayer**, **Thierry** (2014). Gravity Equations: Workhorse,Toolkit, and Cookbook. en. In: *Handbook of International Economics*. Vol. 4. Elsevier, pp. 131–195. ISBN: 978-0-444-54314-1. DOI: 10.1016/B978-0-444-54314-1.00003-3. URL: https://linkinghub.elsevier.com/retrieve/pii/B9780444543141000033 (visited on 09/26/2023).

Head, **Keith** and **Ries**, **John** (2001). Increasing Returns versus National Product Differentiation as an Explanation for the Pattern of U.S.-Canada Trade. *The American Economic Review*. 2001. 91, pp. 858–876.

Helpman, **Elhanan** (Nov. 2023). Foreign Competition and Innovation. Working Paper. DOI: 10.3386/w31840. URL: https://www.nber.org/papers/w31840 (visited on 11/08/2023).

Hsu, **Wen-Tai**, **Riezman**, **Raymond**, and **Wang**, **Ping** (Nov. 2019). *Innovation, Growth, and Dynamic Gains from Trade*. en. Tech. rep. w26470. Cambridge, MA: National Bureau of Economic Research, w26470. DOI: 10.3386/w26470. URL: http://www.nber.org/papers/w26470.pdf (visited on 10/18/2023).

Hummels, **David** and **Klenow**, **Peter J** (May 2005). The Variety and Quality of a Nation's Exports. *American Economic Review*. 2005. 95, pp. 704–723.

Jones, **Charles I.** (2005). Growth and Ideas. en. In: *Handbook of Economic Growth*. Vol. 1. Elsevier, pp. 1063–1111. ISBN: 978-0-444-52043-2. DOI: 10.1016/S1574-0684(05)01016-6. URL: https://linkinghub.elsevier.com/retrieve/pii/S1574068405010166 (visited on 12/07/2022).

– (July 2019). Paul Romer: Ideas, Nonrivalry, and Endogenous Growth. *The Scandinavian Journal of Economics*. 2019. 121, pp. 859–883.

Kehoe, **Timothy** and **Ruhl**, **Kim J.** (2013). How Important Is the New Goods Margin in International Trade? en. DOI: 10.1086/670272. URL: https://www.journals.uchicago.edu/doi/epdf/10.1086/670272 (visited on 05/29/2023).

Kleinman, **Benny**, **Liu**, **Ernest**, **Redding**, **Stephen J**, and **Yogo**, **Motohiro** (2023). Neoclassical Growth in an Interdependent World. 2023.

Melitz, **Marc** and **Redding**, **Stephen** (June 2021). *Trade and Innovation*. en. Tech. rep. w28945. Cambridge, MA: National Bureau of Economic Research, w28945. DOI: 10.3386/w28945. URL: http://www.nber.org/papers/w28945.pdf (visited on 06/28/2021).

Perla, **Jesse**, **Tonetti**, **Christopher**, and **Waugh**, **Michael E.** (Jan. 2015). *Equilibrium Technology Diffusion, Trade, and Growth*. en. Tech. rep. w20881. National Bureau of Economic Research. DOI: 10.3386/w20881. URL: https://www.nber.org/papers/w20881 (visited on 01/22/2021).

Rachapalli, **Swapnika** (2021). Learning between Buyers and Sellers Along the Global Value Chain. 2021, p. 81.

Rivera-Batiz, **Luis A.** and **Romer**, **Paul M.** (1991a). Economic Integration and Endogenous Growth. *The Quarterly Journal of Economics*. 1991. 106, pp. 531–555.

– (May 1991b). International trade with endogenous technological change. *European Economic Review*. 1991. 35, pp. 971–1001.

Romer, **Paul** (Feb. 1994). New goods, old theory, and the welfare costs of trade restrictions. *Journal of Development Economics*. 1994. 43, pp. 5–38.

Romer, **Paul M.** (1990). Endogenous Technological Change. *The Journal of Political Economy*. 1990. 98, S71–S102.

Sampson, **Thomas** (Feb. 2016). Dynamic Selection: An Idea Flows Theory of Entry, Trade, and Growth *. *The Quarterly Journal of Economics*. 2016. 131, pp. 315–380.

– (2023). Technology Gaps, Trade and Income. *American Economic Review*. 2023. VOL. 113, p. 66.

Sun, **Liyang** and **Abraham**, **Sarah** (Dec. 2021). Estimating dynamic treatment effects in event studies with heterogeneous treatment effects. *Journal of Econometrics*. 2021. 225, pp. 175–199.

Chapter 2

The Impact of Geopolitical Conflicts on Trade, Growth, and Innovation (joint with Eddy Bekkers)

2.1 Introduction

The last decade has witnessed the beginning of a backlash against global trade integration. Political scientists conjecture that the emergence of China as a new superpower against the incumbent U.S. might lead to strategic competition between these countries —one in which geopolitical forces and the desire to limit interdependence take primacy over win-win international cooperation[1]. Rising support for populist and isolationist parties in many Western countries points towards the same direction[2]. Additionally, the 2022 War in Ukraine and the subsequent strong retaliation of the European Union, the United States, and their allies against Russia suggest that the international economic order based on open markets and expanded globalization could be replaced by a more fragmented international economic system.

[1]See Wei (2019) and Wyne (2020) for a review of the debate among respectively Chinese and American scholars about the shift in foreign policies toward each other.

[2]For evidence of the impact of trade shocks on the rise of populist parties to power, see Colantone and Stanig (2018).

Using these facts as motivation, this paper aims to determine the potential effects of increased and persistent large-scale geopolitical conflicts on trade, economic growth, and technological innovation. Some of the adverse effects are well-known. Increased trade barriers decrease domestic welfare and gains from trade by shifting production away from the most cost-efficient producers and leaving households with a lower level of total consumption.

However, some of the main concerns of policymakers and practitioners regarding potentially detrimental effects of limiting trade are abstracted away in standard models. For instance, these models typically assume a fixed technology distribution for domestic firms, limiting gains from trade to static gains. This assumption renders it impossible to address one of the most important long-term consequences of continued geopolitical conflicts or receding globalization —namely, reduced technology and know-how spillovers that may happen through trade.

In order to realistically assess the impact of trade conflicts on global innovation, we build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion. We model the arrival of new ideas as a learning process for producers in each country-sector cluster. By engaging in international trade, i.e. importing intermediate inputs, domestic innovators have access to new sources of ideas, whose quality depends on the productivity of the countries and sectors from which they source intermediates.

This dynamic mechanism substantially alters the incentives in the face of trade conflicts. In a static setting, countries with large domestic markets are likely to have limited welfare losses if they cut trade ties with a foreign trade bloc. This is usually true even if this foreign trade bloc is of higher productivity and even more so if such a loss can be compensated by decreased trade costs with a third group of countries —as it would be the case with countries aligning in geopolitical blocs. This is no longer true in our model. If those countries lose access to high-productivity suppliers, they also forgo the idea-diffusion aspect of trade. As such, over time, the cumulative dynamic costs of trade conflicts become much larger, especially for countries away from the productivity frontier.

In our model, idea diffusion is mediated by the input-output structure of production, such that both sectoral intermediate input cost shares and import trade shares characterize the source distribution of ideas. Innovation is summarized by describing productivity in different sectors as evolving according to a trade-share weighted average of trade partners' sectoral productivities. This process is controlled by a parameter that determines the speed of diffusion of ideas, which we calibrate using historical data on output growth.[3]

Our approach implies that the strength of ideas diffusion is a function of the strength of input-output linkages in production. Productivity thus evolves endogenously in each sector as a by-product of micro-founded market decisions —i.e., an externality that market agents affect with their behavior but do not take it into account when making decisions. In this framework, the outbreak of large-scale trade conflicts will have spillover effects on the future path of sectoral productivities of all countries. Changes in trade costs induce trade diversion and creation, which, in turn, impact productivity dynamics in a way that is not internalized by agents.

After characterizing the model, we solve it recursively and use it to perform policy experiments in the context of heightened geopolitical conflicts. We explore the potential

[3]Trade costs are calibrated targeting observed trade shares as in new quantitative trade models applying exact hat algebra, whereas initial productivity is calibrated based on actual labor productivity data. With this approach and the chosen calibration of the ideas diffusion parameter, we stay close to observed data. As such, baseline values to which counterfactual experiments are applied are identical to actual values, ensuring that the impact of counterfactual experiments is not distorted.

impact of a "decoupling of the global economy," a scenario under which technology systems would diverge in the global economy. We divide the global economy into a Western bloc and an Eastern bloc based on differential scores in foreign policy similarity. In doing so, we provide the first set of estimates for dynamic losses of economic decoupling.

We provide four sets of estimates. First, we simulate increased trade costs arising from geopolitical circumstances, which increase frictions prohibitively if one country wants to trade with another one outside its bloc. Second, we simulate a scenario of a global increase in tariffs, in which all countries move from a cooperative tariff setting in the context of the WTO to a non-cooperative tariff setting[4]. Third, we explore the potential effect of moving one of the regions from the Western Bloc to the Eastern Bloc. Fourth, we limit decoupling to electronic equipment, the sector displaying so far the most decoupling policies. These four policy experiments are essential to analyze the impact of decoupling, the difference between different ways to decouple (with resource-dissipating iceberg trade costs or rent-generating tariffs), the role of technology spillovers in the model by analyzing bloc membership, and decoupling in the sector most scrutinized. To limit the already large number of policy experiments, we focus on the hypothetical scenario of a complete decoupling into a Western and Eastern Bloc. Hence, we do not explore a scenario with a "neutral" bloc.

Our analysis leads to five main findings. First, we show that the projected welfare losses for the global economy of a decoupling scenario can be drastic, as large as 12% in some regions; and are largest in the lower-income regions as they would suffer the most from reduced technology spillovers from richer areas. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas.

[4]Nicita, Olarreaga, and Silva (2018) estimate to increase global tariffs, on average, by 32 percentage points. For simplicity, we use this average number as a reference and we assume that countries in different blocs raise tariffs against countries in the other bloc by this average amount.

In a dynamic setting with diffusion of ideas welfare losses are larger and display more variation. Fourth, if one of the middle income regions, Latin America and the Caribbean (LAC), would switch from the higher-income Western bloc to the lower-income Eastern bloc, its welfare costs of decoupling would be significantly higher. This experiment illustrates that policymakers in low- and middle-income countries would face difficult decisions if decoupling would aggravate. Fourth, the welfare costs of decoupling only in electronic equipment, the sector where decoupling is already taking place, would be much smaller than under full decoupling, albeit sizeable, ranging from $0.4 - 1.9\%$. Finally, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one and due to differences in either trade costs, unit costs and/or productivities between sectors in a country's trading partner; we explore this issue both through theory and simulations[5].

We make five main contributions to the literature. First, we build a multi-sector model of the global economy with Bertrand competition, profits, and technology spillovers which can be solved recursively and permits assessing realistic trade policy experiments. Second, we analyze idea diffusion inefficiencies in a multi-sector framework both analytically and numerically. We show analytically that such inefficiencies come from differences in trade costs and unit costs between sectors and numerically that such inefficiencies tend to be larger in a multi-sector framework. Third, we calibrate the strength of the diffusion of technologies through trade with a tight fit between simulated and historical GDP growth rates, which is appropriate for counterfactual simulation. Fourth, we examine the long-run effects of real-world policy experiments related to the decoupling of the global economy. Last, we draw insights from the Political Science literature to incorporate geopolitical conflicts into a workhorse trade

[5]Before conducting simulations with the multi-sector, multi-region model calibrated to real-world data, we explore the discrepancy between actual and optimal levels of idea diffusion. This comparison shows that to maximize the total diffusion of ideas, trade shares must be at their optimal point *in every sector*.

model.

Our model builds on the work that evaluates the impact of trade on innovation and shows that trade openness can increase the level of domestic innovation, particularly on the single-sector model of Buera and Oberfield (2020). Compared to previous work, we present a recursive model with input-output linkages as in Caliendo and Parro (2015); calibrate the strength of the diffusion of ideas to target historical GDP growth rates across all regions; and explore diffusion inefficiencies in a multi-sector setting.

In our model productivity growth is driven by an autonomous component and a diffusion component which is a function of intermediate trade linkages. Other models such as Cai, Li, and Santacreu (2022) in the spirit of Eaton and S. Kortum (1999) assume that technology diffusion is an autonomous process with the strength of diffusion calibrated to data on patent citations. Since we calibrate the strength of diffusion to target GDP growth controlling for labour and capital growth, in our model trade plays an important role in productivity catch-up. As such our approach constitutes an upper bound for the potential welfare losses associated with trade decoupling through less diffusion of ideas. Furthermore, our results focus on the adjustment costs over the counterfactual transition path after some exogenous shock rather than a long-run balanced growth path.

Related Literature.

Our paper is closely related to the literature that adds dynamics to trade models by incorporating knowledge diffusion channels. The earliest explorations of this topic go back to Eaton and S. Kortum (1999), who developed a multi-country dynamic model in which firms innovate by investing in research and development (R&D) and knowledge diffuses, after some lag, to other markets. In this model, diffusion happened somewhat mechanically, was unrelated to trade, and eventually reached all countries[6].

More recently, Alvarez et al. (2013) combined the Eaton and S. Kortum (2002) Ricardian model of trade with an idea diffusion process first presented by S. S. Kortum (1997). Importantly, the authors conjectured that the diffusion process is proportional to the quality of managers of firms whose products reach a given destination market. Ideas flow from one market to another in proportion to the trade linkages between them. Therefore, impediments to trade have not only static but also dynamic costs —as they decrease knowledge diffusion.

From a theoretical perspective, our work is related to Cai, Li, and Santacreu (2022) and Deng and C. Zhang (2023), who develop multisector dynamic trade models of knowledge diffusion. Cai, Li, and Santacreu (2022) extend Eaton and S. Kortum (1999) to a multi-sector model of trade, innovation, and knowledge diffusion with lag-diffusion dynamics, exploring how the welfare gains from trade are affected by knowledge diffusion through their impact on changes in comparative advantage. There are three main differences between their work and ours. First, our model emphasizes the nexus between trade and idea diffusion, whereas Cai, Li, and Santacreu (2022) model technology spillovers as being independent of the amount of trade. Additionally, while they calibrate knowledge spillovers with data on patent citation, we calibrate the strength of the diffusion of ideas based on the fit between actual and simulated historical GDP growth rates. Third, the papers have a different focus: we focus on policy questions and explore how the effect of potential trade policy changes are affected by the inclusion of ideas diffusion in the model, while they highlight how patterns of comparative advantage change with technology spillovers.

[6]In differentiated varieties of trade models, knowledge diffusion shows up in papers like Romer (1990), Rivera-Batiz and Romer (1991), and Grossman and Helpman (1989). In the text, we focus on papers that incorporate knowledge diffusion to Ricardian models, which is the class of models that this paper falls in.

Deng and C. Zhang (2023) integrates Buera and Oberfield (2020) in a Levchenko and J. Zhang (2016) multi-sector framework, finding strong convergence in comparative advantage, dynamic gains from trade about 1/3 larger than static gains, and identifying central players in technology diffusion. There are two main differences with our work. First, Deng and C. Zhang (2023) employ a different way to calibrate the model, following Levchenko and J. Zhang (2016) to estimate trade costs and productivity parameters. We instead infer trade costs and productivity based on observed data and target GDP growth rates with the diffusion of ideas parameter. Second, they explore issues like convergence in comparative advantage and central players in technology diffusion in a backward-looking non-recursive model, whereas we explore the costs of geopolitical decoupling and the repercussions of bloc membership besides studying the inefficiencies of ideas diffusion in a multi-sector recursive framework fit for policy experiments.

Our work is also related to the literature on the costs of economic decoupling. We share in common with Eppinger et al. (2021) and Felbermayr, Mahlkow, and Sandkamp (2023) that we also use a Ricardian model with input-ouput linkages, as in Caliendo and Parro (2015). The former distinguishes between trade costs for intermediate and final goods (as in Antràs and Chor (2018)) to simulate a decoupling in global value chains. The latter simulates a set of scenarios for East-West decoupling by increasing trade costs in all sectors, and shutting down cross-bloc trade. We differ in that we add dynamics and knowledge diffusion to that framework, extending Buera and Oberfield (2020) to a multisector framework.

Attinasi, Boeckelmann, and Meunier (2023) calibrate a Baqaee and Farhi (2024) model of trade and economic networks to a scenario of economic decoupling. Like us, they divide the world in geopolitical blocs using foreign policy similarity indices and simulate decoupling through an increase of iceberg trade costs. However, they focus on short-term static rigidities and abstract away from knowledge diffusion. Instead, we estimate long-run dynamic losses.

Organization.

This paper is organized as follows. In Section 2.2 we present the model, detailing production, demand, and consumption of the global economy. We also describe the dynamic evolution of productivities in different regions and sectors. In Section 2.3 we describe the discrepancy between the actual and optimal diffusion of ideas in a multi-sector framework. In Section 2.4 we discuss the calibration of the model and underpin the examined policy experiments. In Section 2.5 we present the results of our main policy experiments and some alternative simulations. Finally, we conclude in Section 2.6 summarizing the key takeaways.

2.2 Environment

Time is discrete and indexed by $t \in \mathcal{T}$. There are $d \in \mathcal{D}$ regions in the global economy, which cover every part of the world economy, either as a stand-alone country, or a regional aggregate of countries. In each region, there are multiple industries $i \in \mathcal{I}$.

2.2.1 Demand

In each region d and each period t a representative agent maximizes Cobb-Douglas preferences over consumption of goods in different sectors $i \in \mathcal{I}$, $q_{d,t}^{c,i}$:

$$\begin{aligned} \max_{\{q_{d,t}^{i}\}_{i \in \mathcal{I}}} \sum_{i \in \mathcal{I}} (q_{d,t}^{c,i})^{\kappa_d^i} \quad s.t. \quad & \sum_{i \in \mathcal{I}} \kappa_d^i = 1 \\ & \sum_{i \in \mathcal{I}} p_{d,t}^{i} q_{d,t}^{c,i} \leq (1 - s_{d,t}) Y_{d,t} \\ & Y_{d,t} = w_{d,t} \ell_{d,t} + r_{d,t} k_{d,t} + T_{d,t} + \sum_{i \in \mathcal{I}} \Pi_{d,t}^{i} \end{aligned} \tag{2.1}$$

$Y_{d,t}$ is gross income determined by $w_{d,t}, \ell_{d,t}$, the wage and measure of workers; $r_{d,t}k_{d,t}$, capital income; $T_{d,t}$, transfers; and $\Pi^i_{d,t}$, profits. We set $s_{d,t}$ to be an exogenous savings rate.

Part of the literature assumes capital (sometimes labeled "structures") has a fixed stock[7]. Since we will simulate the model many periods into the future and the labor force is expected to grow, assuming capital stock as fixed structures would induce decreasing returns. As our focus is idea diffusion, an exogenous savings rate is the simplest possible assumption to prevent running into decreasing returns. We abstract from intertemporal optimization and set the exogenous path of savings to keep the capital stock per capita constant.

The preference structure above implies the following demand function for goods in sector $i \in \mathcal{I}$:

$$q^{c,i}_{d,t} = \frac{(1 - s_{d,t})\kappa^i_d Y_{d,t}}{p^i_{d,t}} \tag{2.2}$$

with the aggregate price index satisfies:

$$P^c_{d,t} = K \cdot \Pi_{i \in \mathcal{I}} (p^i_{d,t})^{\kappa^i_d} \tag{2.3}$$

where $K = \Pi_{i \in \mathcal{I}} (\kappa^i_d)^{-\kappa^i_d}$ is a collection of Cobb-Douglas coefficients.

2.2.2 Production

There are many producers of different varieties ω of each commodity i. Firms are endowed with identical technology and combine factors of production $f^i_{d,t}$ and intermediate inputs $m^i_{d,t}$ to produce variety $q^i_{d,t}(\omega)$:

[7]See, for instance, Caliendo, Dvorkin, and Parro (2019).

$$q^i_{d,t}(\omega) = z^i_{d,t}(\omega)\left[(\Psi^{i,f}_{d,t})^{\frac{1}{\rho_i}}(f^i_{d,t})^{\frac{\rho_i-1}{\rho_i}} + (\Psi^{i,m}_{d,t})^{\frac{1}{\rho_i}}(m^i_{d,t})^{\frac{\rho_i-1}{\rho_i}}\right]^{\frac{\rho_i}{\rho_i-1}} \tag{2.4}$$

The cost of the unit input bundle, $c^i_{d,t}$, is a function of the prices of factors of production, $pf^i_{d,t}$, and prices of commodities used as intermediates, $p^i_{d,t}$:

$$c^i_{d,t} = \left[\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\rho_i} + \Psi^{i,m}_{d,t}(p^i_{d,t})^{1-\rho_i}\right]^{\frac{1}{1-\rho_i}} \tag{2.5}$$

Firms combine factors of production ($f^i_{d,t}$) and intermediate commodities ($m^i_{d,t}$) according to the following sub-production functions:

$$f^i_{d,t} = \left[(\Psi^{i,k}_{d,t})^{\frac{1}{\nu_i}} k_{d,t}^{\frac{\nu_i-1}{\nu_i}} + (\Psi^{i,l}_{d,t})^{\frac{1}{\nu_i}} \ell_{d,t}^{\frac{\nu_i-1}{\nu_i}}\right]^{\frac{\nu_i}{\nu_i-1}} \tag{2.6}$$

$$m^i_{d,t} = \left[\sum_{j\in\mathcal{I}} (\Psi^{i,j}_{d,t})^{\frac{1}{\mu_i}} (q^{m,i,j}_{d,t})^{\frac{\mu_i-1}{\mu_i}}\right]^{\frac{\mu_i}{\mu_i-1}} \tag{2.7}$$

The first aggregator combines capital, $k_{d,t}$, and labor, $\ell_{d,t}$, as factors of production, while the second one uses sectoral commodities $q^{m,j}_{d,t}$ as intermediate inputs.

2.2.3 Supply of Factors of Production

The supply of the three factors of production changes over time. They are perfectly mobile and thus have a uniform price across sectors. For each country, an exogenous path of endowments of labor is imposed based on external projections from the United Nations and the International Monetary Fund as described in the data section below.

Aggregate capital, $k_{d,t}$, is a function of capital in the previous period, $k_{d,t-1}$, depreciation, δ, and investment, $in_{d,t}$, evolving according to the following law of motion:

$$k_{d,t} = (1 - \delta_d) k_{d,t-1} + in_{d,t} \quad (2.8)$$

Investment in region d is a Leontief function of sectoral investment, $q^{in,i}_{d,t}$ implying the following expression for sectoral investment demand and the corresponding price index of investment, $p^{in}_{d,t}$:

$$q^{in,i}_{d,t} = \overline{\chi}^i_d in_{d,t} \quad (2.9)$$

$$p^{in}_{d,t} = \sum_{i \in \mathcal{I}} \overline{\chi}^i_d p^{in,i}_{d,t} \quad (2.10)$$

We assume that the ratio of a region's trade balance to its total income is fixed. Abstracting from other components of the current account, the capital account is equal to the trade balance. Assuming a fixed trade balance ratio (relative to income) thus implies that the investment rate is equal to the savings rate minus the trade balance rate, $tb_{d,t}$. Hence, in equilibrium we have:

$$p^{in}_{d,t} in_{d,t} = \left(s_{d,t} - tb_{d,t} \right) Y_{d,t} \quad (2.11)$$

2.2.4 International trade

Trade happens through demand for varieties used as inputs in the production of sectoral goods $q^j_{s,t}$. These goods, in turn, are used in two different ways: as intermediate inputs in the production of varieties and investment goods; and in final consumption. Consumers, investors and firms demand sectoral commodities $q^j_{d,t}$ by amounts $q^{c,j}_{d,t}$, $q^{in,j}_{d,t}$ and $q^{m,j}_{d,t}$, respectively.

A local producer sources the cheapest landed variety $\{q^j_{d,t}(\omega) : \omega \in [0,1]\}$ from all countries $s \in \mathcal{D}$ and produces the sectoral commodity according to the following technology:

$$q^j_{d,t} = \left[\int_{[0,1]} q^j_{d,t}(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} d\omega \right]^{\frac{\sigma_j}{\sigma_j - 1}} \tag{2.12}$$

The price of commodity $j \in \mathcal{I}$ thus satisfies:

$$p^j_{d,t} = \left[\int_{[0,1]} p^j_{d,t}(\omega)^{1-\sigma_j} d\omega \right]^{\frac{1}{1-\sigma_j}} \tag{2.13}$$

Let $x^i_{sd,t}(\omega)$ be the landed unit cost of supplying variety ω of commodity $i \in \mathcal{I}$ produced in source region $s \in \mathcal{D}$ and delivered to region $d \in \mathcal{D}$:

$$x^i_{sd,t}(\omega) \equiv \frac{tm^i_{sd,t} \cdot \tau^i_{sd,t} \cdot c^i_{sd,t}}{z^i_{s,t}(\omega)} = \frac{\tilde{\tau}^i_{sd,t} c^i_{sd,t}}{z^i_{s,t}(\omega)} = \frac{\tilde{x}^i_{sd,t}}{z^i_{s,t}(\omega)} \tag{2.14}$$

where $tm^i_{sd,t}$ are gross import taxes, which can be source and destination specific; $\tau^i_{sd,t} \geq 1$ are bilateral iceberg trade costs; and $z^i_{s,t}(\omega)$ is the firm's productivity. The last equality follows from defining $\tilde{x}^i_{sd,t} \equiv tm^i_{sd,t} \cdot \tau^i_{sd,t} \cdot c^i_{s,t}$ as the landed input bundle costs.

Since varieties can be sourced from every region $s \in \mathcal{D}$, consumers in destination region $d \in \mathcal{D}$ will only buy variety ω from the source with the lowest landed price. Following Bernard et al. (2003), producers engage in Bertrand competition. Hence, the firm with the lowest price of a variety captures the entire market for that variety. The firm will set the price either equal to the marginal cost of the second most efficient firm (domestically or from other regions) or equal to its monopoly price if the marginal cost of the nearest competitor is higher than the monopoly price. More formally, for each country, order firms $k = [1, 2, \cdots]$ such that $z^i_{1s,t}(\omega) > z^i_{2s,t}(\omega), \cdots$. If the lowest-cost provider of the variety ω to country $d \in \mathcal{D}$ is a producer from country $s \in \mathcal{D}$, the price in d satisfies:

$$p^i_{d,t}(\omega) = \min\left\{ \underbrace{\frac{\sigma_i}{\sigma_i - 1}\frac{\tilde{x}^i_{sd,t}}{z^i_{1s,t}(\omega)}}_{\text{optimal monopolist price}}, \underbrace{\frac{\tilde{x}^i_{sd,t}}{z^i_{2s,t}(\omega)}}_{\substack{\text{MC of 2nd most}\\ \text{productive firm from } s}}, \min_{n \neq s} \underbrace{\frac{\tilde{x}^i_{nd,t}}{z^i_{1n,t}(\omega)}}_{\substack{\text{MC of most productive}\\ \text{firm from other countries}}} \right\} \tag{2.15}$$

Assumption 5 (Productivity draws)**.** *We follow the canonical Eaton and S. Kortum (2002) assumption that and take $z^i_{s,t}(\omega)$ to be the realization of an i.i.d. random variable. Productivity is distributed according to a Type II Extreme Value Distribution (Fréchet):*

$$F^i_{s,t}(z) = \exp\{-\lambda^i_{s,t} z^{-\theta_i}\} \tag{2.16}$$

The country-specific Fréchet distribution has a region-commodity-specific location parameter $\lambda^i_{s,t}$, which denotes absolute advantage (better draws for all varieties), and a sector-specific scale parameter θ_i, which governs comparative advantage[8].

We show in the Appendix that prices in the destination region $d \in \mathcal{D}$ will be:

$$p^i_{d,t} = \mathbf{\Gamma}_1 (\Phi^i_{d,t})^{-\frac{1}{\theta_i}} \tag{2.17}$$

where $\mathbf{\Gamma}_1$ is a constant [9]; $\Phi^i_{d,t} \equiv \sum_{s \in \mathcal{D}} \lambda^i_{s,t} (\tilde{x}^i_{sd,t})^{-\theta_i}$.

As there are infinitely many varieties in the unit interval, by the law of large

[8] As in a standard Eaton and S. Kortum (2002) model, the location parameter, $\lambda^i_{s,t}$, describes the productivity of region s in sector i and thus determines its absolute advantages, whereas the dispersion parameter θ_i governs the variation of productivity within sector i between countries and thus determines the strength of comparative advantage. A higher θ_i implies less variability in productivity and thus lower potential for diversification according to comparative advantage.

[9] Specifically, $\mathbf{\Gamma}_1 \equiv \left[1 - \frac{\sigma_i - 1}{\theta_i} + \frac{\sigma_i - 1}{\theta_i}\left(\frac{\sigma_i}{\sigma_i - 1}\right)^{-\theta_i}\right]\Gamma\left(\frac{1-\sigma_i+\theta_i}{\theta_i}\right)$, where $\Gamma(\cdot)$ is the Gamma function

numbers, the expenditure share of destination region $d \in \mathcal{D}$ on goods coming from source country $s \in \mathcal{D}$ converges to its expected value. $\pi^i_{sd,t}$ denotes the expenditure share of demand in region $d \in \mathcal{D}$ on goods coming from region $s \in \mathcal{D}$ as a share of total expenditure on commodity $i \in \mathcal{I}$:

$$\pi^i_{sd,t} \equiv \frac{\lambda^i_{s,t}(\tilde{x}^i_{sd,t})^{-\theta_i}}{\Phi^i_{d,t}} \tag{2.18}$$

Sectoral commodities can be used both for final consumption and as intermediate inputs. Given the assumptions above, $\pi^{i,j}_t$, which is the expenditure on goods coming from s to be used as intermediate inputs in sector i of country d as a share of their total expenditures on goods from sector j is equal to the trade shares for final consumption, i.e: $(\forall i, j)\pi^{i,j}_{sd,t} = \pi^j_{sd,t}$.

In the presence of Bertrand Competition, we show in the Appendix that source firms realize a profit that is proportional to the total expenditure of destination countries. In particular, profits are:

$$\Pi^i_{s,t} = \frac{1}{1+\theta_i}\sum_{d\in\mathcal{D}} \pi^i_{sd,t} e^i_{d,t}; \quad e^i_{d,t} = \sum_{ag\in\{c,in,m\}} e^{ag,i}_{d,t} \tag{2.19}$$

with $e^{ag,i}_{d,t} = p^i_{d,t} q^{ag,i}_{d,t}$.

2.2.5 Equilibrium

Our model is characterized by a sequence of cross-sectional equilibria satisfying equilibrium equations in each of the periods t, consisting of equilibrium in the product market and the factor market, detailed in Appendix 2.8.3. The state variables that link each period are the capital stock $k_{d,t}$ and the country-sector-specific technology parameters $\lambda^i_{d,t}$.

First, given $\{k_{d,t-1}\}_{d\in\mathcal{D}}$, the capital stock in period t is determined by investment in period t and depreciation of previously existing capital, as specified in equation (2.8). Second, given the country-sector-specific technology parameters $\{\lambda^i_{d,t-1}\}^{i\in\mathcal{I}}_{d\in\mathcal{D}}$, technology evolves according to equation (2.21), specified in the following section.

We abstract from intertemporal optimization of consumption, imposing instead a fixed savings rate. This makes the model computationally more tractable and leads to a more straightforward interpretation of the simulation results, as we focus on the counterfactual trajectory of technology given a change in trade costs. However, abstracting from intertemporal optimization implies that potential effects through changes in savings rates on capital accumulation are also abstracted from[10].

2.2.6 Dynamic innovation

Unlike in the standard Eaton and S. Kortum (2002) model or in the Bertrand competition version developed in Bernard et al. (2003), we assume that each region's location parameter evolves over time. Each commodity $i \in \mathcal{I}$ and each country $d \in \mathcal{D}$ has a different period-specific productivity distribution $F^i_{d,t}(z)$.

Our model follows a strand of the literature that models ideas diffusion through random matches between domestic and foreign managers[11]. Seminal examples of this work include Jovanovic and Rob (1989) and S. S. Kortum (1997). More recently, Alvarez et al. (2013) explored how idea diffusion is intertwined with trade linkages. Like Buera

[10]The assumption of a fixed trade balance implies that the capital stock is also not affected by potential changes in the capital balance in response to shocks. However, the international finance literature suggests that standard open economy models with intertemporal optimization have generated counterfactual predictions on the direction of capital flows between developed and emerging countries in the 1990s and 2000s. Capital was flowing on net from emerging to developed economies instead of capital flowing to emerging economies with higher growth rates as predicted by the standard models.

[11]For a detailed review of this literature, see the comprehensive review chapter published by Buera and Lucas (2018).

and Oberfield (2020), we assume that a manager draws new insights as a by-product of sourcing a basket of inputs.

We extend this framework to a model with diffusion of ideas in a multi-sector context and solve it in a recursive fashion that permits the assessment of the long-run effects of policy experiments. The idea diffusion mechanism is mediated by the input-output structure of production, such that both sector cost shares and import trade shares characterize the source distribution of ideas[12].

Assumption 6 (Idea formation)**.** *New ideas are the transformation of two random variables, namely: (i) original insights o, which arrive according to a power law:* $O_t(o) = Pr(O < o) = 1 - \alpha_t o^{-\theta}$*; (ii) derived insights* z'*, drawn from a source distribution* $G^i_{d,t}(z)$*. After the realization of those two random variables, the new idea has productivity* $z = o(z')^{\beta}$*, where o is the original component of the new idea,* z' *is the derived insight, and* $\beta \in [0,1)$ *captures the contribution of the derived insights to new ideas. Local producers only adopt new ideas if their quality dominates the quality of local varieties. Therefore, for any period, the domestic technological frontier evolves according to* [13]*:*

$$F^i_{d,t+\Delta}(z) = \underbrace{F^i_{d,t}(z)}_{Pr\{productivity<z \text{ at } t\}} \times \underbrace{\left(1 - \int_t^{t+\Delta} \int \alpha_\tau z^{-\theta}(z')^{\beta\theta} dG^i_{d,\tau}(z') d\tau\right)}_{Pr\{no\ better\ draws\ in\ (t,t+\Delta)\}}$$

Lemma 1 (Generic Law of Motion, Buera and Oberfield, 2020)**.** *Given Assumption 2, if, for any t,* $F^i_{d,t}(z)$ *is Fréchet with location parameter* $\lambda^i_{d,t} = \int_{-\infty}^{t} \alpha_\tau \int (z')^{\beta\theta_i} dG^i_{d,\tau}(z') d\tau$ *and*

[12]As mentioned earlier, our work is closely related to Cai, Li, and Santacreu (2022), who extend S. S. Kortum (1997) to a multi-sector framework. We differ in that they model diffusion as happening separately from trade, rather than as a trade-externality.

[13]Here we simply use the fact that $o = z(z'^{-\beta}$ and note that, given an insight z', at any moment t the arrival rate of ideas of quality better than z is $Pr(O > o) = Pr(O > z(z'^{-\beta}) = \alpha_t z^{-\theta}(z')^{\beta\theta}$. We then integrate over all possible values of z'.

scale parameter θ_i, the former evolving according to the following law of motion:

$$\Delta\lambda^i_{d,t} = \alpha_t \int (z')^{\beta\theta_i} dG^i_{d,t}(z') \tag{2.20}$$

where α_t is a parameter that controls the arrival rate of ideas and β is the sensitivity of current productivity to derived insights. The integral on the right-hand side of the equation denotes the average productivity of ideas drawn from source distribution $G^i_{d,t}(z')$[14].

To fully characterize (2.20), we need to define the source distribution. We assume that managers learn from their suppliers, such that $G^i_{d,t}(z')$ is proportional to the sourcing decisions in the production of commodity i in country d. Productivity thus evolves endogenously as a by-product of sourcing decisions. Additionally, we assume that insights take time to come to fruition. Rather than drawing insights from interactions with suppliers in the current period, we assume that insights take one period to materialize. Intuitively, we are assuming that entrepreneurs have to study their purchases for one period and only then draw insights. This assumption will be convenient because it will allow us to compute the law of motion for technology without relying on present-period trade shares. Therefore, we will be able to solve the model recursively and use it for counterfactual analysis of the long-run impact of policy experiments.

Assumption 7 (Source Distribution from Intermediates)**.** *The source distribution $G^i_{d,t}(z') \equiv \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} H^{i,j}_{sd,t-1}(z')$, where $\Psi^{i,j}_{d,t}$ is the intermediate cost share of sector j when producing good i in region d; and $H^{i,j}_{sd,t-1}(z')$ is the fraction of commodities for which the lowest cost supplier in period $t-1$ is a firm located in $s \in \mathcal{D}$ with productivity weakly less than z'.*

[14] Equation (2.20) is a discrete-time approximation of the continuous-time law of motion derived in the Appendix.

One final assumption is required to make sure that the law of motion converges: that the rate of arrival of new ideas —and, therefore, the growth rate in productivities —across sectors is not too dissimilar. Provided that $\beta\theta_i < \theta_j$, the law of motion is guaranteed to converge, as shown by Deng and C. Zhang (2023)[15].

Assumption 8 (Divergence in Arrival Rates of Idea)**.** *The arrival rates of ideas across sectors are not too dissimilar. We consider the two cases that yield the same analytical result: either $\theta_i = \theta_j$ or, for each insight that a buyer from sector i draws from supplier sector j, new ideas have productivity $z = o(z')^{\beta\theta_i\theta_{ij}}$, where $\theta_{ij} \equiv \frac{\theta_j}{\theta_i}$.*

Proposition 5 (Law of Motion in a Multi-Sector Framework)**.** *Given Assumptions 1-4, in the multi-sector multi-region economy described in the previous section, the country-sector-specific technology parameter evolves according to the following process:*

$$\Delta\lambda^i_{d,t} = \alpha_t \Gamma(1-\beta) \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \pi^{i,j}_{sd,t-1} \left(\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} \right)^{\beta} \tag{2.21}$$

where $\Gamma(\cdot)$ is the gamma function, $\Psi^{i,j}_{d,t-1}$ are cost shares, and $\pi^{i,j}_{sd,t-1}$ are intermediate input trade shares.

Proof. Appendix 2.8.4. □

This result extends the *learning from sellers* specification of Buera and Oberfield (2020) to a multi-sector framework that can be solved recursively for policy counterfactuals. Diffusion of ideas is proportional to intermediate trade shares from source country-industry pairs, s, j, into destination country d and use industry i. Total diffusion incorporates intermediate cost-shares $\Psi^{i,j}_{d,t-1}$. Embedded in this specification

[15]We thank an anonymous referee for suggesting including the adjustment term of as an alternative specification to keep the baseline law of motion of a previous version of this paper consistent with heterogeneous trade elasticities.

is the fact that whenever trade flows are small only the most high $pi_{sd,t-1}$ is small, producers are drawing insights from the most high quality marginal firms that are able to export, i.e. $\left(\frac{\lambda^{j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}}\right)^{\beta}$ will be higher. We will use equation (2.21) and a calibrated path of $\alpha_t = \alpha_0 \exp\{\gamma t\}$ to solve for an endogenous path for $\lambda^i_{d,t}$.

Another feature of this multi-sector framework is that, for most empirical applications, dynamic losses will be larger with multiple sectors. In the proposition below, we derive a sufficient condition for average diffusion in a multi-sector framework to be smaller than its single-sector counterpart.

Proposition 6 (Diffusion Losses with Multiple Sectors)**.** *Suppose that, for some arbitrary* $t-1$,
$\sum_{i\in\mathcal{I}} \Psi^i_{d,t-1} \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \lambda^j_{s,t-1} \leq \lambda_{s,t-1}$ *for each* s *and* d*. Then:*

$$\left(\sum_{i\in\mathcal{I}} \Psi^i_{d,t-1} \Delta\lambda^i_{d,t}\right)^{\textit{Diffusion Optimum}} < \left(\Delta\lambda_{d,t}\right)^{\textit{Diffusion Optimum}} \tag{2.22}$$

and

$$\left(\sum_{i\in\mathcal{I}} \Psi^i_{d,t-1} \Delta\lambda^i_{d,t}\right)^{\textit{Market Allocation}} \leq \left(\Delta\lambda_{d,t}\right)^{\textit{Market Allocation}} \tag{2.23}$$

where $\Psi^i_{d,t-1} \equiv \frac{e^i_{t-1}}{e_{t-1}}$

Proof. Appendix 2.8.4. □

The proposition above states that if aggregate productivities $\lambda_{s,t-1}$ in each source country s are at least as large as the cost-weighted average of its sectoral productivities in every destination country d, then: (a) the maximum diffusion rate in a single-sector framework will be strictly larger than what happens in a multi-sector framework; and (b) diffusion of ideas under the market allocation in a single-sector framework is guaranteed to be weakly larger than what happens in a multi-sector framework. In the next section, we discuss the intuition behind this result.

2.3 Discussion and Intuition of Ideas Diffusion in a Multi-sector Framework

In this section, we provide some intuition regarding how the idea diffusion mechanism operates in the multi-sector framework. More specifically, we will show how differences in trade costs, unit costs and productivities between sectors lead to deviations between the actual and diffusion maximizing import shares - i.e., diffusion losses. For each sector i, the diffusion maximizing trade shares satisfy the following program:

$$\begin{aligned} &\max_{\{\pi^{i,j}_{sd,t-1}\}_{j,i\in\mathcal{I},s\in\mathcal{D}}} \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} (\pi^{i,j}_{sd,t-1})^{1-\beta} (\lambda^{j}_{s,t-1})^{\beta} \\ &s.t. \quad \forall (i,j) \in \mathcal{I}\times\mathcal{I} \qquad \sum_{s\in\mathcal{D}} \pi^{i,j}_{sd,t-1} = 1 \end{aligned} \tag{2.24}$$

The trade shares that maximize diffusion, which we refer to as *diffusion maximum* and under the market allocation, satisfy, respectively:

$$\left(\pi^{i,j}_{sd,t}\right)^{\text{Diffusion Optimum}} = \frac{\lambda^{j}_{s,t}}{\sum_{k\in\mathcal{D}} \lambda^{j}_{k,t}}, \qquad \left(\pi^{i,j}_{sd,t}\right)^{\text{Market Allocation}} = \frac{\lambda^{j}_{s,t} (\tilde{x}_s d)^{-\theta_j}}{\sum_{k\in\mathcal{D}} \lambda^{j}_{k,t} (\tilde{x}_k d)^{-\theta_j}}$$

The point that maximizes diffusion sets trade shares proportional to the source country productivity in the source industry, $\lambda_{\lambda^{j}_{s,t}}$, as a proportion of the global productivity stock, $\sum_{k\in\mathcal{D}}\lambda^j_{k,t}$. Under the market allocation, price differences induce deviations from the diffusion maximizing point. Trade will be skewed towards source countries with relatively lower input and trade costs.

Consider what happens in some sector i of country h in an economy that is fully symmetric, both across countries and sectors, but where trade costs are present. The strict concavity of the diffusion equation implies that idea diffusion is not uniform as $\pi^{i,i}_{hh}$ varies. The optimal diffusion point is $(\pi^{i,i}_{hh})^{\text{Diffusion Optimum}} = \lambda^i_h/(\lambda^i_h + \lambda^i_f) = 1/2$. Under the market allocation, trade costs induce home bias such that domestic share is $(\pi^{i,i}_{hh})^{\text{Actual Trade}} = 1/(1+\tau^{-\theta}) > 1/2$ and ideas diffusion is below the optimal point. If trade costs increase and $\tau \to \infty$, the home country moves to autarky, and deviations from the optimal idea diffusion reach a maximum. We plot the optimal, actual trade, and autarky points along the ideas diffusion function for sector i on the left-hand side panel of Figure 2.1.

The right-hand side panel illustrates what happens when the home country has a lower productivity in sector i. The curve shifts down at the autarky point and the optimal solution moves to the left (smaller domestic trade share)[16]. When $\lambda^i_h < \lambda^i_f$, diffusion losses from high trade costs are higher. This highlights a key characteristic of this class of models: countries that are *less productive* in a given sector have *higher dynamic gains from trade*[17].

Now, why does a multi-sector framework induces lower diffusion than its single-sector counterpart? The answer hinges on the strict concavity of the diffusion function

[16]Formally, once countries are no longer symmetric, we need to make the following regularity condition to guarantee convergence to the autarky equilibrium: $\lim_{\tau\to\infty}(\tau x^i_f)/x^i_h = +\infty$. Most models make this assumption either explicitly or implicitly.

[17]In fact, for any $\pi^i_h \in (0,1]$, the marginal change in diffusion as π^i_h increases will be increasing in a country's productivity. To see that, take $\frac{\partial\Delta\lambda^i_h}{\partial\pi^i_h} = \alpha\cdot\Gamma(1-\beta)\cdot\Psi^i(1-\beta)[(\pi^{i,i}_h)^{-\beta}(\lambda^i_h)^{\beta} - (1-\pi^{i,i}_h)^{-\beta}(\lambda^i_f)^{\beta}]$, which is increasing in λ^i_h.

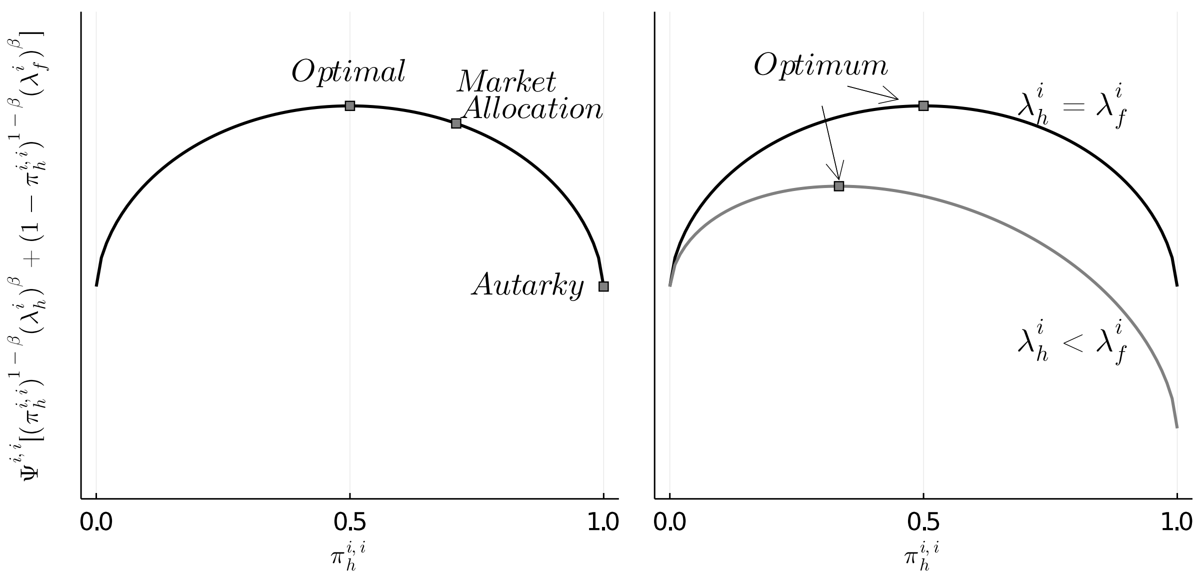


Figure 2.1. Within sector idea diffusion functions in a two-by-two economy. Both panels plot the idea diffusion functions for the home country in a two-by-two model within sector i: $\Psi^i[(\pi_h^{i,i})^{1-\beta}(\lambda_h^i)^\beta + (1-\pi_h^{i,i})^{1-\beta}(\lambda_f^i)^\beta]$. The left-hand side panel shows the optimal, actual trade, and autarky points along the ideas diffusion function when countries are fully symmetric ($\lambda_h^i = \lambda_f^i$). The right hand side panel plots the functions and diffusion optimal points for the cases when countries have identical productivities $\lambda_h^i = \lambda_f^i$ and the home country is less productive $\lambda_h^i < \lambda_f^i$.

(2.21) in trade shares and productivity terms. If one substitutes the solution for diffusion maximizing trade shares, as shown above, into the diffusion function, this will be proportional to:

$$\underbrace{\left(\Delta\lambda_{d,t}\right)^{\text{Diffusion Optimum}}}_{\text{single sector diffusion optimum}} \propto \left(\sum_{s\in\mathcal{D}}\lambda_{s,t-1}\right)^{\beta}$$

$$\underbrace{\sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\left(\Delta\lambda^{i}_{d,t}\right)^{\text{Diffusion Optimum}}}_{\text{average multi-sector diffusion optimum}} \propto \sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\sum_{j\in\mathcal{I}}\Psi^{i,j}_{d,t-1}\left(\sum_{s\in\mathcal{D}}\lambda^{j}_{s,t-1}\right)^{\beta}$$

If aggregate productivities $\lambda_{s,t-1}$ in each source country s are at least as large as the cost-weighted average of its sectoral productivities in every destination country d —i.e., $\lambda_{s,t-1} \geq \sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\sum_{j\in\mathcal{I}}\Psi^{i,j}_{d,t-1}\lambda^{j}_{s,t-1}$, then:

$$\left(\sum_{s\in\mathcal{D}}\lambda_{s,t-1}\right)^{\beta} \geq \left(\sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\sum_{j\in\mathcal{I}}\Psi^{i,j}_{d,t-1}\sum_{s\in\mathcal{D}}\lambda^{j}_{s,t-1}\right)^{\beta} > \sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\sum_{j\in\mathcal{I}}\Psi^{i,j}_{d,t-1}\left(\sum_{s\in\mathcal{D}}\lambda^{j}_{s,t-1}\right)^{\beta}$$

where the last inequality follows from the properties of strict concavity: it is an application of Jensen's Inequality to this particular functional form. A restriction over the parameter space of sectoral productivities is sufficient to guarantee that a single-sector framework will lead to higher maximum diffusion than a multi-sector one. The first inequality follows from the assumption and the fact that the function is increasing.

By a similar reasoning, we can show that the allocation in the single-sector model under the market allocation will be greater than under a multi-sector framework. Note that the diffusion function is strictly concave in both trade shares and productivity states:

$$\underbrace{\sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\left(\Delta\lambda^{i}_{d,t}\right)^{\text{Market Allocation}}}_{\text{average multi-sector diffusion under mkt allocation}} \propto \sum_{i\in\mathcal{I}}\Psi^{i}_{d,t-1}\sum_{j\in\mathcal{I}}\Psi^{i,j}_{d,t-1}\sum_{s\in\mathcal{D}}\left(\pi^{j}_{sd,t}\right)^{1-\beta}\left(\lambda^{j}_{s,t-1}\right)^{\beta}$$

Aggregation guarantees that the aggregate trade share must be a weighted average of sectoral expenditure shares and assures that, for homogeneous productivities across sectors, the market allocation under the multi-sector framework is smaller than in the single-sector framework.

In Figure (2.2), we plot the multi-sector to single-sector diffusion ratio under the market allocation. Extending the two-country example above to two multiple sectors $i, -i$, we assume that each use-sector in h has identical degrees of openness with respect to each supply-sector but they may differ across sectors. We then compare it to its single-sector counterpart.

Intuitively, the more different the degree of openness of sectors are, the lower the level of diffusion in a multi-sector framework is, compared to its single-sector counterpart. In the upper left quadrant of Figure (2.2), sector i sources almost all of its inputs abroad, while sector $-i$ buys most of its inputs domestically. Over the counterdiagonal, the trade shares of the sectors are identical, and differences reach zero.

The explanation again lies in the concavity of the diffusion function. By definition, the aggregate trade shares a convex combination of the sectoral trade shares[18]. The

[18] That is, $\pi_{sh} = \sum_{i\in\{i,-i\}}\Psi^{i}_{h}\sum_{j\in\{i,-i\}}\Psi^{i,j}_{h}\sum_{s\in\{h,f\}}\pi^{j}_{sd}$, where the cost-shares can take any value

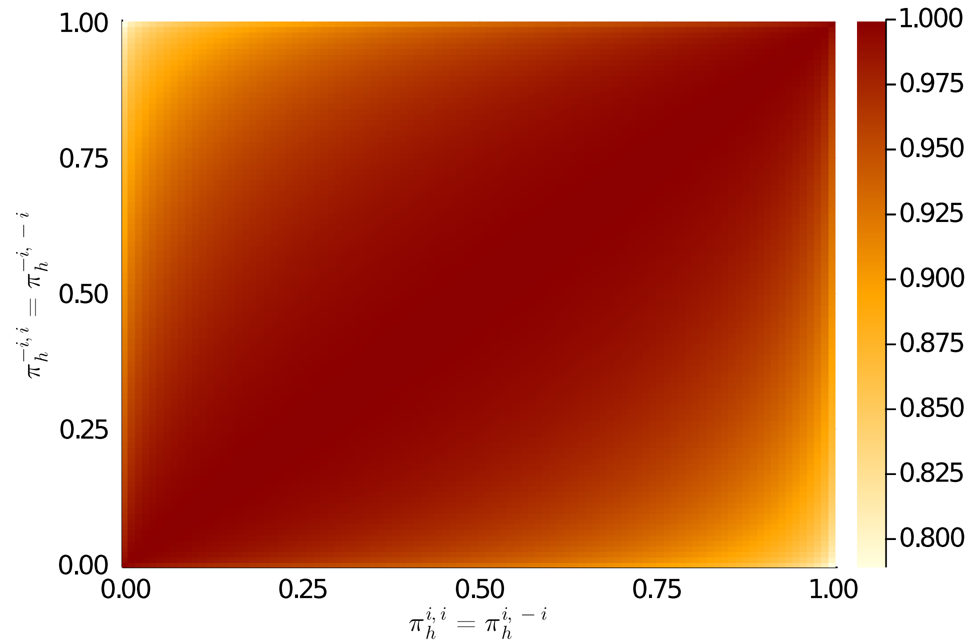


Figure 2.2. Ratio between Market Allocation Diffusion with a multisector and a single-sector framework. Multi-sector to single-sector diffusion ratio under the Market Allocation. Here, we assume each sector use-sector to have identical degrees of openness with respect to each supply-sector ($\pi_h^{i,i} = \pi_h^{i,-i}$, $\pi_h^{-i,i} = \pi_h^{-i,-i}$) but they may differ across sectors. By definition, the aggregate trade shares a convex combination of the sectoral trade share. In the upper left sector i sources almost all of its inputs abroad, while sector $-i$ buys most of its inputs domestically and losses induced by a multi-sector framework reach a maximum. Over the counterdiagonal, the trade shares of the sectors are identical, and differences reach zero.

higher the differences across sectors, the more the concavity of the diffusion function plays a role in inducing diffusion losses in a multi-sector framework.

The assumption stated in Proposition (6) guarantees the same condition on the productivity states. Since productivities are not directly observed and must be calibrated, the assumption stated in the proposition is then a sufficient restriction in the state space that ensures that the comparison satisfies Jensen's Inequality also in its second argument.

It rules out situations in which the calibrated aggregate productivity of a *source country* is not at least as large as the expenditure-weighted average in each *destination country* (because diffusion flows *from source to destination countries*) [19]. If this condition is satisfied, then we achieve the result in Proposition (6).

2.4 Calibration and Setup of Policy Experiments

In this section, we first outline the employed baseline data and behavioral parameters. The parameter determining the strength of diffusion of ideas is calibrated by minimizing the difference between the historical and simulated GDP growth rate across many countries in the world economy. We then motivate and describe the detailed setup of the experiments.

[19] This restriction does not require, however, aggregate productivities to be arbitrarily large. Considering a limiting case (which is not sustainable in equilibrium), helps us to place a natural upper bound. The most restrictive situation would be if some destination country d had the expenditure share $\Psi^{i}_{d,t-1} = 1$ for some arbitrary i and the cost share $\Psi^{i,j}_{d,t-1} = 1$ for some arbitrary j. In that case, a sufficient (but not necessary) condition is that aggregate productivity states in source countries are at least as large as all sectoral productivities $\lambda_{s,t} \geq \lambda^{j}_{s,t}$ for all j.

2.4.1 Data and Behavioral Parameters

Baseline Data

The model is calibrated to trade and production data from the 2014 version of the GTAP Data Base, Version GTAP10A. This means that all spending and cost shares are set equal to the shares in the 2014 database, following the same calibration procedure as in models employing exact hat algebra (Dekle, Eaton, and S. Kortum, 2007). The data are aggregated into 10 regions and 6 sectors as specified in Table 2.1. The model is solved until 2040 in a sequence of recursive dynamic simulations, thus solving the model period per period, using the model solution in the previous period as the starting point for the next period. Population grows based on UN population projections and labor supply grows based on International Monetary Fund projections for employment (until 2025) and United Nations projections regarding working age population (from 2026 until 2040).

Table 2.1. Overview of regions and sectors

Region		Sector	
Code	Description	Code	Description
chn	China	pri	Primary (agri & natres)
e27	European Union 27	lmn	Light manufacturing
jpn	Japan	hmn	Heavy manufacturing
ind	India	elm	Electronic Equipment
lac	Latin America	tas	Business services
ode	Other developed	ots	Other Services
rwc	Rest of the World - Eastern bloc		
rwu	Rest of the World - Western bloc		
rus	Russia		
usa	United States		

The data in the GTAP Data Base do not include profit income as in our model with Bertrand competition. Therefore, we have to modify the baseline data employed, considering that profit income Π_s^i is a share $\frac{1}{1+\theta_i}$ of the total value of sales in sector i in region s. We have done this as follows, proceeding in two steps. First, we reduced the value of payments to the production factor capital (capital income) by 50% and reallocated it to profit income. With this step the share of profit income in the value of sales is not yet equal to $\frac{1}{1+\theta_i}$. Therefore, in a second step we employ our model to modify the base data to target the share of profit income in the value of sales for each country and sector. The reason to proceed in two steps is that capital income in some cases is smaller than profit income required by the model. This is especially the case in sectors with large intermediate linkages and a small trade elasticity, because profit income is a share of gross output in the Bertrand model, whereas capital income is part of net output. The fact that capital income is for some countries smaller than profit income also implies that numerical problems in finding balanced data with profits can appear when raising the number of countries and sectors. Therefore, the number of countries and sectors is limited in the simulations presented here.[20]

Behavioral Parameters

The dispersion parameter of the Fréchet distribution, θ_i, equal to the trade elasticity, is based on the estimates of trade elasticities in Hertel et al. (2007). The substitution elasticity between value-added and intermediates, ρ, between intermediates from different sectors, μ, are equal to zero, implying a Leontief structure. As such we follow the approach employed in most CGE models, which finds empirical support in recent estimates with US data (Atalay, 2017). The substitution elasticities between production factors, ν_i, are based on the values in the GTAP Data Base.

Table 2.2 displays the values of the dispersion parameter of the Fréchet distribution, θ_i and the substitution elasticity between production factors ν_i.

[20] As an alternative approach, we can reduce the value of payments to factors of production by an identical share for all production factors and reallocate this to profit income. The reallocation is set such that profit income $\Pi^i_{s,t}$ becomes a share $\frac{1}{1+\theta_i}$ of the value of sales. However, this approach is not followed because there is a risk that factor income in the data is smaller than profit income implied by the model. As discussed, this is especially a risk in sectors with large intermediate linkages and a small trade elasticity.

Table 2.2. Behavioral parameters

	θ_i	ν_i
Primary (agriculture & natres)	10.09	0.27
Light manufacturing	4.60	1.20
Heavy manufacturing	5.99	1.26
Electronic Equipment	7.80	1.26
Business services	2.80	1.26
Other Services	2.90	1.42
Source	Hertel et al. (2007)	Hertel et al. (2007)

Even though the location parameters of the sector-country specific Fréchet distribution $\lambda_{s,t}^{i}$ evolve endogenously in this model, their starting values need to be calibrated. We calibrate the starting values $\{\lambda_{s,0}^{i}\}_{s\in\mathcal{D},i\in\mathcal{I}}$ using the assumption that this parameter is proportional to PPP-adjusted labor productivity in each sector-country in our baseline year, 2014. This approach is similar to Buera and Oberfield (2020) who infer the location parameters based on total factor productivity calculated as Solow-residuals. An alternative approach would be to estimate both trade costs and the location parameters based on the gravity equation as in Levchenko and J. Zhang (2016). This is the path taken by Deng and C. Zhang (2023), who use historical data for this calibration and decomposition of effects.

We focus on forward-looking policy experiments. With our approach and the chosen calibration of the ideas diffusion parameter, we stay closer to the initially observed data. With an alternative approach, we would need to make assumptions about the autonomous part of growth, not driven by the model mechanism, to set forth a counterfactual. Therefore, we have chosen to identify the location parameters of the Fréchet distribution based on sectoral labor productivity data.

We constructed a database of sectoral productivity by combining two sources: the World Input-Output Database and the World Bank's Global Productivity Database. We provide details of how we aggregated sectors and country groups in the Appendix.

Figure 2.3 shows the distribution of the calibrated $\lambda^i_{s,0}$ parameters across industries i of each region s in our model.

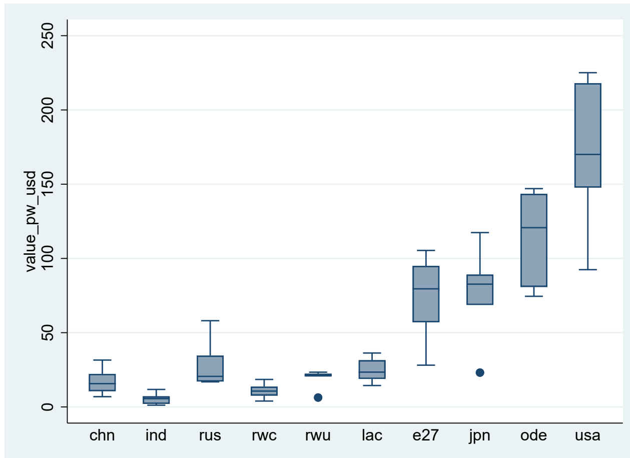


Figure 2.3. Distribution of the calibrated $\lambda^i_{s,0}$ parameters across industries i of each region s in our model. We assume that this parameter is proportional to PPP-adjusted labor productivity in each sector-country. After the initial period, the location parameter of the sector-country-specific Fréchet distribution $\lambda^i_{s,t}$ evolves endogenously according to the model.

Buera and Oberfield (2020) set the growth rate of α_t equal to the population growth rate of the US. We adopt the same heuristics and set the growth rate of the autonomous arrival rate of ideas α_t at 1.18% per year, equal to the projected global population growth rate from 2021 to 2040.

The idea diffusion parameter β is uniform across sectors and determined based on model validation, using simulated methods of moments, as described below. The variance of the growth rates of GDP rise as β increases, because at higher levels of β there is more diffusion of ideas leading to convergence of income levels thus implying larger differences between growth rates of poor and rich countries. As discussed in Section 2.3, countries starting with lower productivity parameters have larger dynamic gains from trade. That effect increases in the value of β[21].

[21] In the limiting point $\beta \to 1$, ideas diffuse instantly and every country not in autarky experiences equal productivity gains in absolute terms. With equal gains in absolute terms, those countries with lower productivity experience larger growth in relative terms as β increases.

We simulate the model from 2004 to 2019 imposing historical growth rates of population and the labor force (based on IMF data) without further policy changes varying the level of β and evaluate for which values of β the mean and variance of the growth rates of GDP are closest to the mean and variance in historical data.[22] Formally, we are minimizing the following loss function with m the historical moment and $m(\beta)$ the simulated moment for either real GDP per capita growth or real GDP growth in the 2004-19 period [23]

Figure (2.4) plots the loss function for values of $\beta \in [0.4, 0.5]$. Tables A1-A4 in the Appendix display additional summary statistics (mean, standard deviation, maximum and minimum) of average growth rates of GDP and GDP per capita between 2004 and 2019. Regardless of the weight, the loss function takes a minimum when $\beta \in [0.44, 0.45]$, a rather short interval. Taking an agnostic stance and setting the weight $w^{GDPpc} = 1/2$, we find that the loss function is minimized for $\beta = 0.44$, but results are virtually unchanged by setting it to any value in the aforementioned interval.

Figure 2.5 compares projected GDP growth rates for $\beta = 0.44$ in individual regions with historical GDP growth rates. This figure shows that the simulated GDP growth rates are relatively close to the historical growth rates, suggesting that also for individual regions the model does a good job at replicating historical growth rates. Furthermore, these results can also be interpreted as an analysis of the under- and overperformance of different regions compared to the projections of the model with diffusion of ideas through trade.

[22]In this exercise the growth rate of α_t is set at 1.18% per year, the global population growth rate between 2004 and 2019.

[23]We minimize: $\min_\beta \sum_{m \in \{\mu,\sigma\}} w^{GDPpc}(m(\beta)^{GDPpc,model} - m^{GDPpc,hist})^2 + (1 - w^{GDPpc})(m(\beta)^{GDP,model} - m^{GDP,hist})^2$ where w^{GDPpc} is the exogenous weight put on real GDP per capita growth, rather than aggregate real GDP growth. As a first step, we raise β in steps of 0.05 from 0 to 0.6. For values larger than 0.6 the variance in the simulations becomes unrealistically high, so these are disregarded. This exercise indicates that the simulated growth rates are closest to historical growth rates for β between 0.4 and 0.5 (See Appendix Table A2). Therefore, as a second step, we simulate the model raising β in steps of 0.01 from 0.4 to 0.5.

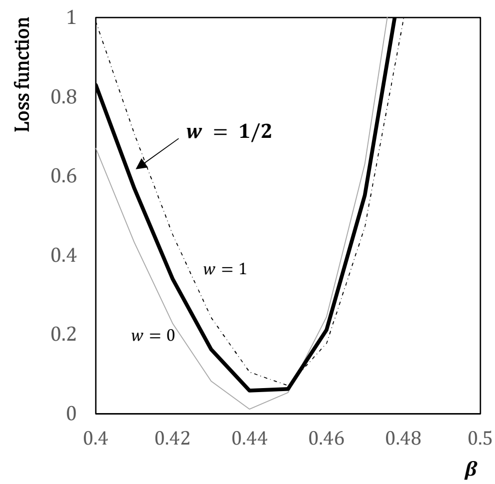


Figure 2.4. Plots the loss function for values of $\beta \in [0.4, 0.5]$. The solid grey line shows the loss function with the parametrization of $w^{GDPpc} = 0$, which is minimized at $\beta = 0.44$. The dotted grey line shows the loss function with the parametrization of $w^{GDPpc} = 1$, which is minimized at $\beta = 0.45$. The thicker black line shows the loss function with the parametrization of $w^{GDPpc} = 1/2$, which is minimized at $\beta = 0.44$.

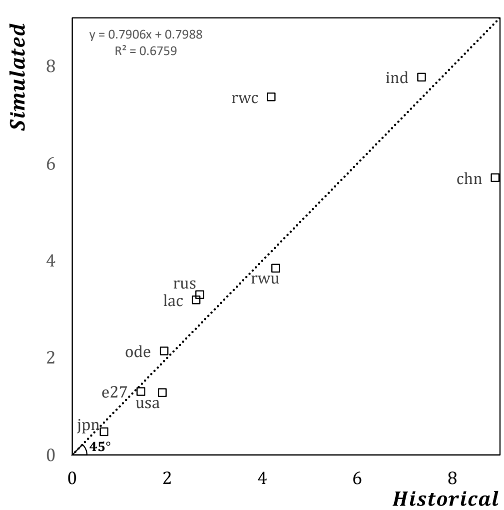


Figure 2.5. Historical and simulated (for $\beta = 0.44$) GDP growth rates (average 2004-2019)

In Appendix Table A3 we display the same summary statistics for β ranging from 0 to 0.6, in steps of 0.05. This table makes clear that calibration to historical data is important. An excessively high value of β will lead to higher than historically observed growth rates in low-income regions. Conversely, a too low value β will fail to show the convergence patterns exhibited in the historical data.

In principle, the described approach could be employed to identify variation in the ideas diffusion parameter between sectors employing data on growth rates of sectoral output. However, consistent data on sectoral growth rates of real output are not available for a broad set of countries employed in this study.

A possible concern of the employed method to calibrate β is that variation in growth between countries is not only driven by ideas diffusion through trade implying technological catch-up. However, the calibration does take variation in labor force growth into account and considers capital accumulation as a source of growth. Furthermore, we follow an approach similar to Buera and Oberfield (2020) who calibrate β based on targeting US growth rates and find a value of β substantially larger than in our calibration, 0.6.

Nonetheless, since we are targeting past GDP growth, it is certainly possible that our calibration over-estimates the impact of this mechanism on growth in the baseline scenario. For that reason, our results should be seen as an upper bound of the impact over the transition path.

Finally, we needed to perform some adjustments in the underlying data to make it consistent with the model. In the calibration we want to make sure that the baseline is consistent with a balanced multiregional input-output dataset, the GTAP Data Base. This has implications for the set-up of the model and the chosen aggregation. First, since a share of household income is saved, savings are incorporated in the model.

Second, since considerable share of factor income constitutes capital income in the data, this production factor is included in the model. Relatedly, investment is a sizeable share of final expenditures and is therefore also incorporated in the model. As explained in the exposition of the model in Section 1 savings and investment are modelled using recursive dynamics. Third, following from the theory the share of income coming from profit income is determined by the dispersion parameter of the productivity distribution which is equal to the trade elasticity in our model. In the calibration we start from an existing balanced database and rebalance this database by assuming that a share of capital income constitutes profit. This limits us in the number of regions and sectors that can be included to avoid that there is not sufficient capital income to be allocated to profit.

2.4.2 Motivation and Set-up of Policy Experiments

Motivation of Policy Experiments

Our main motivation for simulating large-scale trade conflicts is the possibility of receding globalization due to a political backlash. Challenges to the international trade regime (and to globalization at large) might seem like some circumstantial discontinuity in a long-run trend toward increasing openness. However, as we show below, political scientists argue that there is reason to believe that strategic geopolitical rivalries could trump economic gains —at least partially— in their relationships between the U.S. and its allies with China and Russia their allies.

There is ample evidence of substantial gains from trade openness, which can be as large as 50% of national income (Ossa, 2015) even in a static setting. At the same time, recent empirical evidence about frictions in local labor markets (Autor, Dorn, and Hanson, 2013; Dix-Carneiro and Kovak, 2017) highlights the distributional aspects of trade liberalization.

These concerns can translate into political grievances and may have led to an increase in the number of populist and isolationist parties in Western countries calling for less open trade policies (Colantone and Stanig, 2018). An increasing number of political parties use anti-globalization rhetoric to rally the support of constituents that have grievances against the distributional consequences of automation, structural change, offshoring, and trade opening, as shown in the review of the political science literature by Walter (2021).

A clear example of the shift in the consensus during the last decade is the trade conflict between the U.S. and China, which started under the Trump Administration. The economic discourse shifted away from emphasizing the gains from trade to a framing of trade as a zero-sum game and to the use of national-security provisions of the international trade regime to engage in protectionist policy-making[24].

These geopolitical disputes are exemplified not only in the trade conflict between the U.S. and China but also in industry-specific policy changes, such as the U.S. government pressuring allies against allowing the participation of Chinese telecommunications companies in new infrastructure developments or limited cooperation in science and education between the two countries (Tang et al., 2021).

Wei (2019) provides a review of debates among Chinese scholars. Some Chinese analysts see an escalating and continuous conflict between China and the U.S. as a natural and "structural" development of a shifting international system that is moving from a unipolar (the U.S. being the only superpower) to a bipolar (China becoming a superpower on an equal footing to the U.S.) balance of power[25]. They tie a scenario of

[24]For a contemporaneous review of the policies implemented, see Bown and Irwin (2019).

[25]In the context above, the balance of power between functionally equivalent states (the "international structure") provides incentives for strategic behavior by governments that try to maximize their power. We can interpret the unipolar or bipolar configurations as equilibria and disruptions between such equilibria as transition paths. This is known as the "structural realism" theory of international politics, developed by Kenneth Waltz (2010).

a continuous confrontation between the U.S. and China either to strategic geopolitical forces or to domestic political forces in America (Zhao, 2019).

In the West, political scientist Joseph S. Nye Jr. (2020) highlights that, while an abrupt decoupling between the U.S. and China is unlikely, both parties will try to decrease their (inter-)dependence with respect to each other's actions, except where the costs of disengagement are too high to bear[26]. American policymakers and academics also motivate the conflict between China and the U.S. on geopolitical grounds. Although the tone of confrontation is stronger when coming from right-of-center policymakers and scholars[27], both sides of the political spectrum in the U.S. discuss the readjustment of the economic relationship with China due to geopolitical concerns (Wyne, 2020).

The 2022 War in Ukraine and the global-scale retaliation against the Russian Federation that followed is another example of how geopolitical interests can take precedence over gains from economic integration. The escalation began in 2014, after Russia's annexation of Crimea. The U.S. and its allies approved several rounds of sanctions against Russia, culminating in its expulsion from the G8. The confrontation reached another level in the aftermath of Russia's invasion of Ukraine. Western countries imposed sanctions on banking transactions, froze foreign reserves, and closed the airspace for Russian planes. In March 2022, the G7 and the European Union revoked their recognition of Russia's most-favored-nation status, opening the door for large tariff increases, and limited the operation of multinational corporations Russian

[26]Nye Jr. is mostly known for his joint work with Robert Koehane on "complex interdependence" during the post-World War II era (Keohane and Nye Jr, 2011). The authors focus their analysis on the creation of international rules and practices in a world in which the use of military force is very costly due to interdependence between multiple agents that engage both internationally and domestically. For instance, a great degree of trade integration increases the costs (and decreases the probability) of outright military conflict.

[27]See, for instance, the remarks of former White House Trade Council Director to Congress (Navarro, Peter, 2018) or a policy blueprint for decoupling by Scissors (2020), who is a scholar at the America Enterprise Institute, a right-of-center think tank.

subsidiaries[28].

Like in the case of China, the relationship between Russia and the West can also be interpreted through the lens of a strategic game among great powers. Scholars argue about the geopolitical nature of the conflict (Mearsheimer, 2014) and the "reawakening" of geopolitics (Weber and Scheffer, 2022). Therefore, in either case, sanctions and trade conflicts fall within the larger backdrop of a strategic confrontation. These simultaneous conflicts can potentially be interpreted as a larger clash between the U.S. and its allies — a Western bloc; and Russia, China, and their allies — an Eastern bloc. As scholars have argued, we could be observing the emergence of a "China-Russia entente" (Lukin, 2021), which could lead to a "New Cold War" (Abrams, 2022).

We use these facts as motivation to apply our model to conduct hypothetical policy experiments of trade decoupling between East and West: namely, to simulate the effects of large-scale geopolitical conflicts between these blocs, in which players try to limit the level of interdependence between each bloc due to political drivers, even if that leads to economic costs.

The policy experiments focus on the potential effects of decoupling between an Eastern and Western Bloc, since this is most discussed scenario in policy discussions (Aiyar et al., 2023). We do not model the emergence of geopolitical conflict endogenously. The focus of this paper is on the potential consequences of economic decoupling.

Set-up of Policy Experiments

We are agnostic about the degree of future decoupling between East and West. Nonetheless, the fact that international relations scholars envisage disengagement as a real possibility underscores that estimating the potential economic consequences of

[28]For a timeline of sanctions against Russia in the context of the War in Ukraine, see this website maintained by the Peterson Institute for International Economics: https://www.piie.com/blogs/realtime-economic-issues-watch/russias-war-ukraine-sanctions-timeline

decoupling is an important exercise. As our model highlights, changes in trade patterns and sourcing decisions have not only static effects, but also dynamic effects with respect to potential growth and innovation. Our policy experiments try to disentangle the static and dynamic costs of decoupling.

In order to develop the decoupling scenarios, we classify different regions as belonging to either a Western or an Eastern bloc. We do so by resorting to the Foreign Policy Similarity Database, which uses the UN General Assembly voting records for a large set of countries to calculate foreign policy similarity indices for each country pair (Häge, 2011). Intuitively, the index takes countries who vote similarly in the United Nations (compared to the expected level of similarity of a random voting pattern) as being similar in their foreign policy.

We ordered country groups in terms of their foreign policy similarity with China and the United States in order to place the ten regions of our model either in a Western (U.S.) or Eastern (China) bloc. Figure 2.6 shows that Europe, Canada, Australia, Japan, South Korea fall in the Western bloc. Latin America and Sub-Saharan Africa fall somewhere in between, with the former being closer to the U.S. than the latter. India, Russia, and most of North Africa and Southeast Asia fall closer to China.

As a robustness exercise, we repeated the same procedure but replaced China for Russia as the center of gravity of the Eastern bloc. We plot the two Differential Foreign Policy Similarity Indices for each country in our sample in Figure (2.7). Most countries fall very close to the 45-degree line (i.e., the regression coefficient is close to 1) and the correlation between the two series is very high. This suggests that, qualitatively, results will be very similar using either Russia or China as the main country in the Eastern bloc.

After classifying the countries into Eastern and Western geopolitical blocs, we

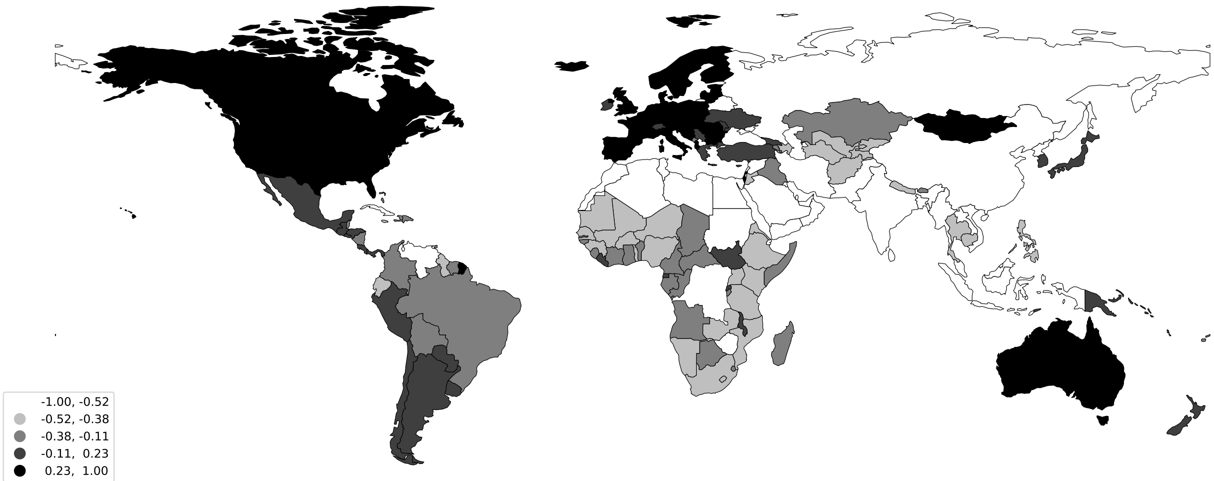


Figure 2.6. Differential Foreign Policy Similarity Index Map. Values are normalized such that 1 represents maximum relative similarity with the U.S. and -1 represents maximum relative similarity with China. The map shows the difference between pairwise similarity indices $\kappa_{i,US} - \kappa_{i,China}$. The parameter $\kappa_{i,j}$ represents the foreign policy similarity of countries i, j, based on vote similarity in the United Nations General Assembly. Given vote possibilities $n, m \in \{1, \cdots, k\}$, one can calculate a matrix $P = [p_{nm}]$, where entry p_{nm} represents the share of votes in which country i took position n and country j took position m. Given matrix P, $\kappa_{i,j} = 1 - \sum_{m \neq n} p_{mn} / \sum_{m \neq n} p_m p_n$, where p_m, p_n are expected marginal propensity of any country to take position m, n at a random vote. For more details, see Häge (2011).

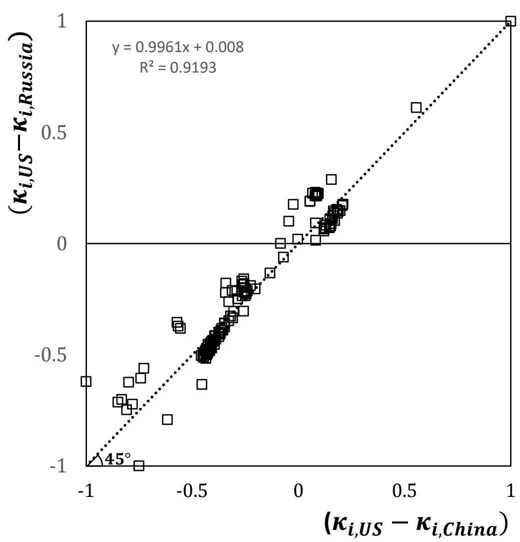


Figure 2.7. Differential Foreign Policy Similarity Index relative to the U.S. and China. Values are normalized such that 1 represents maximum relative similarity with the U.S. and -1 represents maximum relative similarity with China or Russia. See the caption of Figure 2.6 for further details on the indices.

design two different policy experiments. We first increase iceberg trade costs $\tau^{i}_{sd,t}$ to a point where virtually all of the trade happens exclusively within each bloc. In total, we increase bilateral trade costs by ~ 160 percentage points. We assume that the increases in trade costs is permanent. We label this scenario **full decouple**. This provides an important limiting case that can be useful for putting bounds on potential effects.

The second scenario relies on work by Nicita, Olarreaga, and Silva (2018), who estimate that a move from cooperative to non-cooperative tariff setting would increase average tariffs by 32 percentage points globally. We simulate what would happen if countries kept cooperative tariff setting within their trade blocs but moved to non-cooperative tariff setting across trade blocs. For simplicity, we assume that regions in different blocs increase bilateral tariffs $tm^{i}_{sd,t}$ by the globally average increase in tariffs when moving from cooperative to non-cooperative tariffs: 32 percentage point increases in tariff rates against regions outside the bloc. Again, we assume that the increases in trade costs are permanent. These tariff increases seem high. However, the average tariff increase in the China-U.S. trade war was higher than 21pp. We call this scenario **tariff decouple**.

Besides the full and tariff decouple scenarios, we explore two additional policy experiments. First, we evaluate the impact of a switch of the region Latin America and Caribbean (LAC) from the Western bloc to the Eastern bloc. This sheds some light on the question many countries would face if the decoupling would aggravate: which bloc to stay closest to?

Second, we explore a less hypothetical scenario: trade decoupling only in the electronic equipment sector. We perform is a full decoupling between the original blocs (with LAC in the Western bloc) but restrict the increase in iceberg trade costs $\tau^{i}_{sd,t}$ only to the electronic equipment (`elm`) sector. This scenario is motivated by U.S.

and Chinese authorities being increasingly at loggerheads with each other in the technological arena.

One important example of this process has been the conflict involving Chinese telecom giant Huawei Technologies. Since 2019, American corporations have been banned from doing business with Huawei. In a similar move, the New York Exchange delisted China Unicom, China Mobile, and China Telecom. Despite legal challenges and a new administration, as of April 2021, neither decision has been reversed.

Additionally, the U.S. has been using its foreign policy arsenal to pressure allies to join them in limiting Chinese telecom companies' reach. In particular, there is a desire to limit Chinese participation in 5G technology auctions, citing national security and privacy concerns[29]. So far, Australia, the United Kingdom and some European allies have chosen to ban or limit Chinese participation in technological auctions.

This conflict suggests that a large increase in trade costs between the U.S. allies and Chinese allies regarding technological equipment is a positive probability scenario in the future. In this case, decoupling would mean a near-total separation of the electronic equipment sectors of the two blocs.

Huawei and Google break of their business connections after the U.S. government sanctions against the Chinese corporation is a good illustration of what this separation could look like in real life. Huawei used Google's *Android* ecosystem in their smartphones, which gave their users access to Google-approved updates and apps. After the ban issued by the Trump administration, however, Google announced it would comply with the U.S. government directives and Huawei was forced to shift away from Google

[29]North American Treaty Organization (NATO) researchers Kaska, Beckvard, and Minárik (2019) review the arguments put forth from a Western national security perspective. This topic is extremely contentious and some Chinese commentators argue that the U.S. is using national security concerns as an excuse to implement industrial policy.

software and design their own operating system *HarmonyOS*.

Since this separation is driven primarily by regulation rather than tariffs, it is appropriate to think of it as an increase in iceberg trade costs $\tau^i_{sd,t}$ between blocs in the electronic equipment sector and so this is the scenario we will explore.

2.5 Main Results

We have four main scenarios. We simulate full decouple and tariff decouple, defined as explained above. We simulate either scenario both with and without diffusion of ideas, in order to assess the additional impact of the diffusion mechanism. After a discussion of the results of the full and tariff decouple scenarios, we compare the impact of decoupling on productivity in a multi-sector and single-sector setting. We finish this section with a description of the results of the two additional policy experiments that restrict decoupling to the electronics and equipment sector or change LAC from the Western bloc to the Eastern bloc.

In the results below we report the results of a comparison of the simulation results with and without policy experiments. We first simulate the dynamic world economy with no policy change, then do the same with the policy change, and report the long-run cumulative percentage difference between the two: $\hat{x} = \sum_{t=p}^{T}(x'_t - x_t) / \sum_{t=p}^{T} x_t$, where p is the date of the first policy change, x'_t, x_t are the values of variable x with and without the policy change, respectively.

2.5.1 Full and Tariff Decouple

As expected, all scenarios show large negative impacts on cross-bloc trade after the introduction of the policy intervention. In the **full decouple** scenario, trade between the countries in the Western bloc and the Eastern bloc is virtually shut down, with imports and exports dropping by 98%. Those countries also shift a substantial part of their trade to the U.S., with trade flows increasing anywhere between $10 - 42\%$ depending on the region and scenario. The domestic spending share in the U.S. increases by about 7%. The converse happens in the Eastern bloc but with larger dispersion across regions. Trade with the U.S. drops by $65 - 90\%$ while trade with China increases by $9 - 60\%$. The domestic trade share in China increases by 3%. The **tariff decouple** scenario yields qualitatively similar results but with smaller magnitudes. We show the results by region and scenario in Figure 2.8.

One of the reasons behind the asymmetry in trade decreases between blocs is the assumption of a fixed trade-balance-to-income ratio in all regions but one. This implies that regions with large trade surpluses will shift proportionally less of their trade flows away from regions in other trade blocs in order to satisfy the fixed trade balance assumption.

Figure (2.9) shows that both the increases in iceberg trade costs (full decouple) and retaliatory tariff hikes (tariff decouple) induce substantial welfare decreases for all countries. The effects, however, are asymmetric. While welfare losses in the Western bloc range anywhere between -1% and -8% (median: -4%) in the Eastern bloc it falls in the -8% to -12% range (median: -10.5%).

The underlying factor driving the divergence in results between the two blocs is a difference in the evolution of productivity, represented by the scale parameter of the Fréchet distribution of different sectors. Sourcing goods from high productive countries puts domestic managers in contact with better quality designs that inspire better ideas through innovation or imitation.

Importantly, the dynamics governed by equation (2.21) incorporate the input-output structure of production, such that domestic managers in each sector innovate in proportion to the quality and share of their inputs. Losing access to high quality

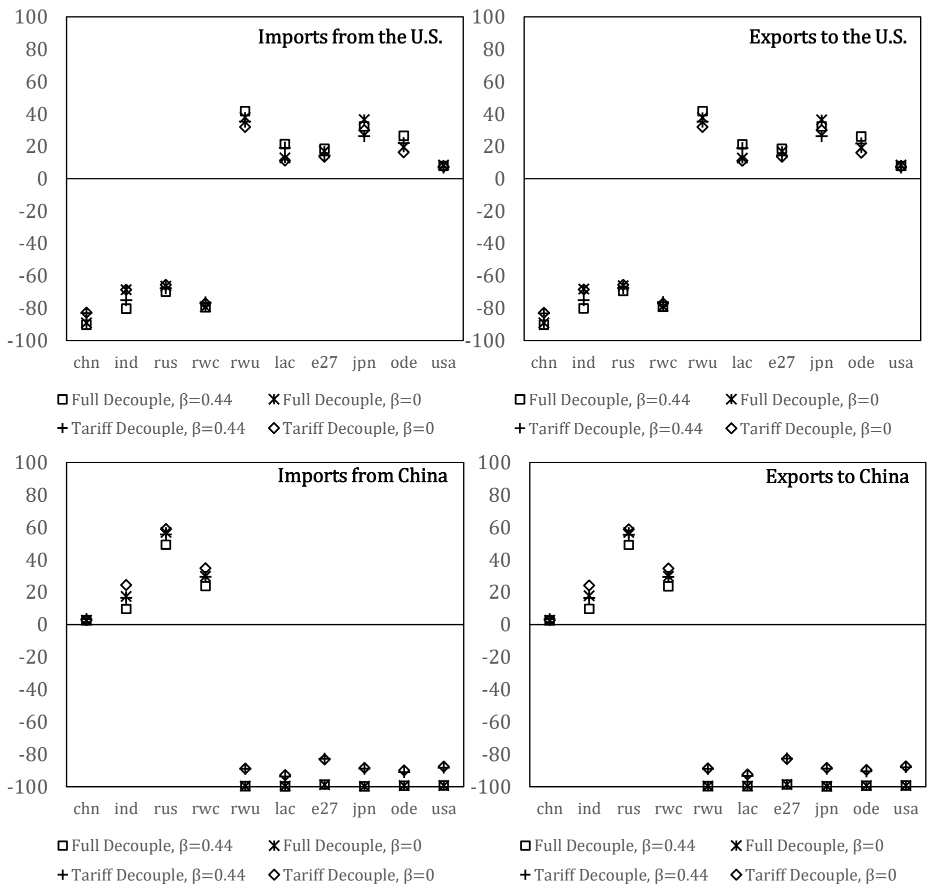


Figure 2.8. Cumulative Percentage Change in Trade Flows with China and the United States, respectively, after the policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^{i}_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^{i}_{sd,t}$, across groups, by 32 percentage points. β is a parameter that controls the diffusion of ideas according to equation 2.21, assumed to be homogeneous across sectors. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

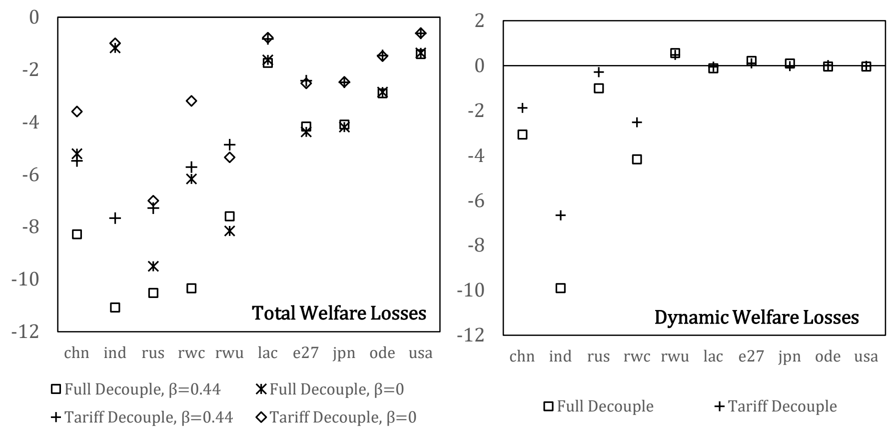


Figure 2.9. Cumulative Percentage Change in Real Income, after the policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. β is a parameter that controls the diffusion of ideas according to equation 2.21, assumed to be homogeneous across sectors. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

designs does not only lead to static losses, but also to a lower level of future innovation, which implies larger dynamic losses. Additionally, the input-output structure of the model implies that cutting ties to innovative regions is particularly costly if the destination country sources many intermediate inputs from such regions prior to the policy change.

For those reasons, in our policy experiments, countries in the Eastern bloc that currently have a lower level of productivity and have larger ties with innovative countries have larger losses. By looking at results in Figure 2.10, one can see the stark contrast between the differential evolution of $\lambda^i_{d,t}$ for those countries in the Western bloc and those in the Eastern bloc. By cutting ties with richer and innovative markets such as OECD countries, destination countries such as China, India, and parts of Asia and Africa shift their supply chains towards lower-quality inputs, which, in turn, induces less innovation. By contrast, while countries in the Western bloc do suffer welfare losses, their innovation paths are virtually unchanged after decoupling,

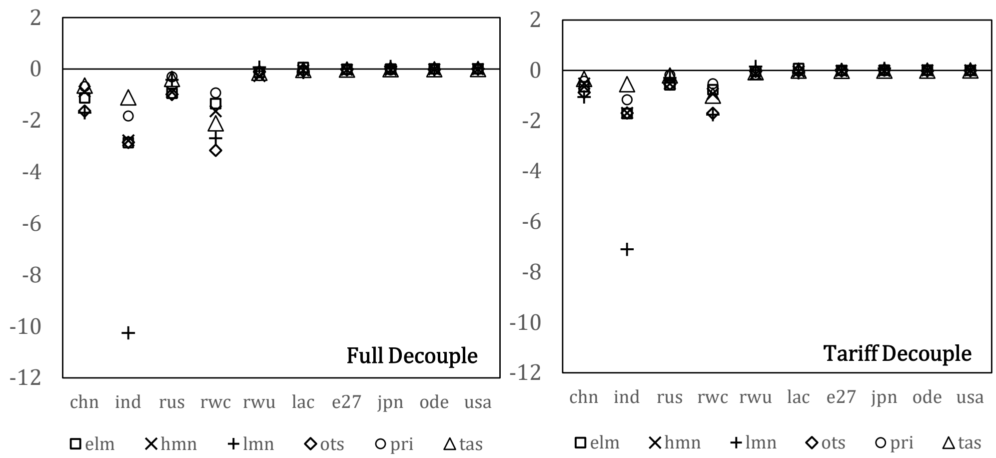


Figure 2.10. Cumulative Percentage Change in the Fréchet Distribution location parameter $\lambda^i_{d,t}$, after policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. β is a parameter that controls the the diffusion of ideas according to equation 2.21, assumed to be homogeneous across sectors. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Sector codes: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. Tables with the values for these charts can be found in the Appendix.

suggesting that nearly all of their losses are static, rather than dynamic.

There is large dispersion across both sectors and countries in differential productivity losses. The two most affected regions are India and the "rest of the Eastern bloc" region. Starting from a lower income level than China and Russia, those regions have a much slower productivity catch-up after severing trade ties with the West. Sectors with larger supply chain linkages to the West prior to the policy change, such as manufacturing in India, experience larger losses.

Among those regions in the Eastern bloc, differential productivity losses are larger in the manufacturing sectors (-1.5% and -3% with full decoupling and tariff decoupling, respectively; this includes `elm`, `lmn`, and `hmn`) than in the services (-0.8% and -1.6%, respectively; `ots tas`) or primary (-0.5% and -1%, respectively; `pri`) sectors.

Finally, we address the contrast between the static effect (when the diffusion of ideas mechanism is shut down) and the dynamic effect. For the two poorer regions of the Eastern bloc, dynamic losses far outsize static losses, which can be explained by the loss of access to higher-quality inputs. In the right panel of Figure (2.9), we show the dynamic losses for each region.

In India, static welfare losses amount to $1 - 2\%$ while dynamic losses range from $7 - 10\%$, depending on the decoupling scenario. Static losses to real income are small because India is a relatively large country and its domestic trade share in the market equilibrium is large, which limits the range of goods affected by changes in terms of trade. However, because it is relatively poor, its losses in the diffusion of ideas version of the model are very large. By severing ties with the Western bloc, it limits the role of trade-induced innovation, which is a by-product of having access to high-quality suppliers.

By contrast, in Russia including dynamics leads only to small additional effects: welfare losses are very similar with or without the ideas diffusion mechanism. As explained above, this stems both from a higher income starting point and relatively limited input-output linkages with the West.

2.5.2 Diffusion Inefficiencies a Multi-sector vs. a Single-sector Framework

In Section 2.3, we stressed that, except in knife-edge cases, within- and between-sector inefficiencies accumulate as the number of countries and sectors increase. The concavity of the diffusion process implies that *total* trade shares being at their optimal points is no longer sufficient for optimal diffusion. Optimal diffusion requires trade shares to be at their optimal points *in every sector*. This suggests that, in most cases, diffusion inefficiencies should increase with the number of sectors.

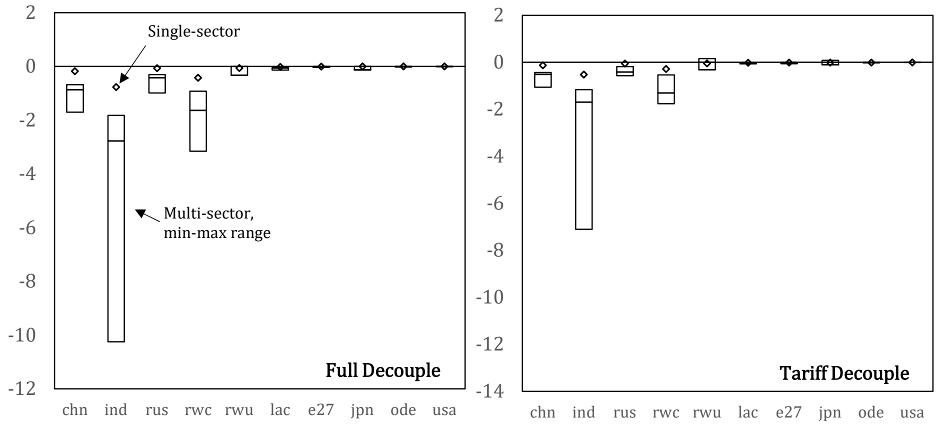


Figure 2.11. Multi-sector vs. Single-sector: Cumulative Percentage Change in the Fréchet Distribution location parameter $\lambda^i_{d,t}$, after policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Sector codes: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. Tables with the values for these charts can be found in the Appendix.

Our numerical results confirm that theoretical intuition. Figure 2.11 contrasts the results of either the **full decouple** or the **tariff decouple** scenarios under the baseline specification presented in the previous section and an alternative simulation in which we collapse the model to a single-sector framework.

In both scenarios, countries in the Easter bloc face higher cumulative diffusion inefficiencies (as measured by the reduction in the Fréchet parameters $\lambda^i_{d,t}$) in a multi-sector framework. In fact, the single-sector dynamic productivity losses are outside the min-max range of the sectoral productivity changes for all countries in the Eastern bloc. These results underscore one important takeaway of this paper: modeling trade diffusion in a simplified single-sector framework can underestimate the level of dynamic losses induced by an increase in trade costs.

2.5.3 Consequences of Bloc Membership

In this section, we consider the consequences of moving one of the regions — Latin America and the Caribbean (LAC) —from the Western to the Eastern bloc. Intuitively, we expect that, by losing access to the highest productivity suppliers, LAC will experience less productivity growth. Nonetheless, the quantitative exercise allows us to have a sense of the magnitude induced by the change in group membership.

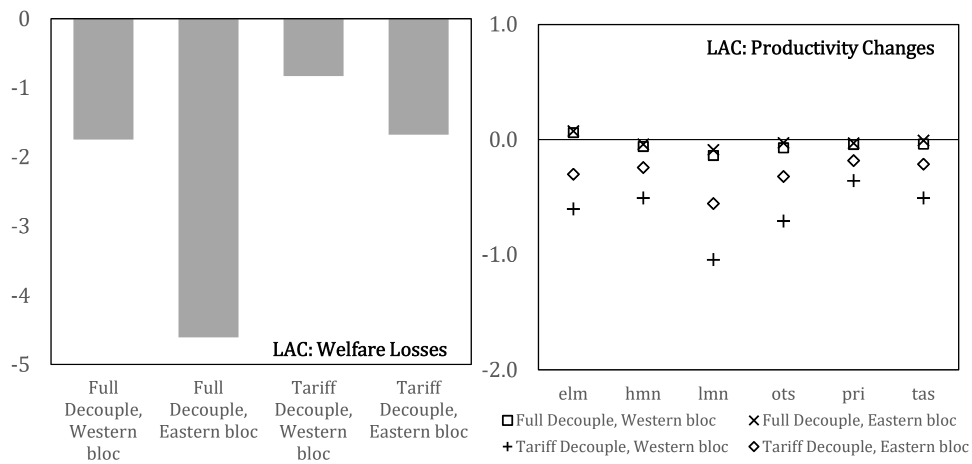


Figure 2.12. Left Panel: Cumulative Percentage Change in Real Income in LAC Region, by scenario. Right Panel: Cumulative Percentage Change of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$ in LAC Region, by scenario. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. In all cases, we set the parameter that controls the diffusion of ideas to $\beta = 0.44$. Sector codes: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. Tables with the values for these charts can be found in the Appendix.

Figure 2.12 compares the results of identical decoupling scenarios, simulating either *full decouple* or *tariff decouple*. The only difference is LAC bloc membership. As expected, most of the changes are concentrated in the LAC region. The left panel of Figure 2.12 shows that welfare losses in LAC are about $100 - 150\%$ larger when it is included in the Eastern bloc, for both scenarios. The domestic trade share in LAC is virtually identical under both settings (with LAC in the Western or in the Eastern bloc), implying similar static welfare losses. This suggests that the increased losses from switching blocs stem almost entirely from dynamic losses.

Moving LAC to the Eastern bloc reduces the welfare losses of decoupling in India and China by about $2 p.p.$ (16%) and $1 p.p.$ (15%), respectively (results not reported). The reason is twofold. First, LAC has a higher income than India and the Rest of the Eastern bloc. All else equal, on average, its inclusion in the bloc raises average productivity and decreases dynamic losses. Second, lower tariff or iceberg trade costs between the Eastern bloc and LAC induce lower static losses for those countries.

The right-hand side panel of Figure 2.12 shows the differential productivity changes in the LAC region for different sectors. When LAC is included in the Western bloc, there are essentially no dynamic productivity losses in any sector: the evolution of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$ in the LAC Region behaves very similarly to a scenario with no policy changes.

In contrast, all sectors have dynamic productivity losses weakly greater than 1% when we simulate decoupling with LAC as part of the Eastern bloc. There is large sectoral heterogeneity. Under full decoupling, productivity losses range from 1% in Electronic Equipment (`elm`) to 0.4% in Business Services (`tas`). These differences are induced by input-output linkages.

This experiment underscores that the costs of decoupling might be unbearably high for low and middle-income countries that are excluded from the Western bloc. Many countries in Latin America and Africa benefit from increasingly large trade ties to China through both having larger market access and access to lower input costs. However, as the dynamic costs of severing ties with the West would be very high, and political leaders in those countries might have the incentive to keep an equidistant relationship between East and West, by preserving both mid-term gains from the relationship with China and longer term dynamic gains from having access to Western supply chains.

2.5.4 Electronic Equipment Decoupling

Finally, we compare our baseline scenario of *full decouple* in **all sectors** with a *full decouple* **restricted to the electronics equipment sector**. In both scenarios, we assume that the ideas diffusion mechanism works as described by equation (2.21) and we set $\beta = 0.44$, according to the calibration described before.

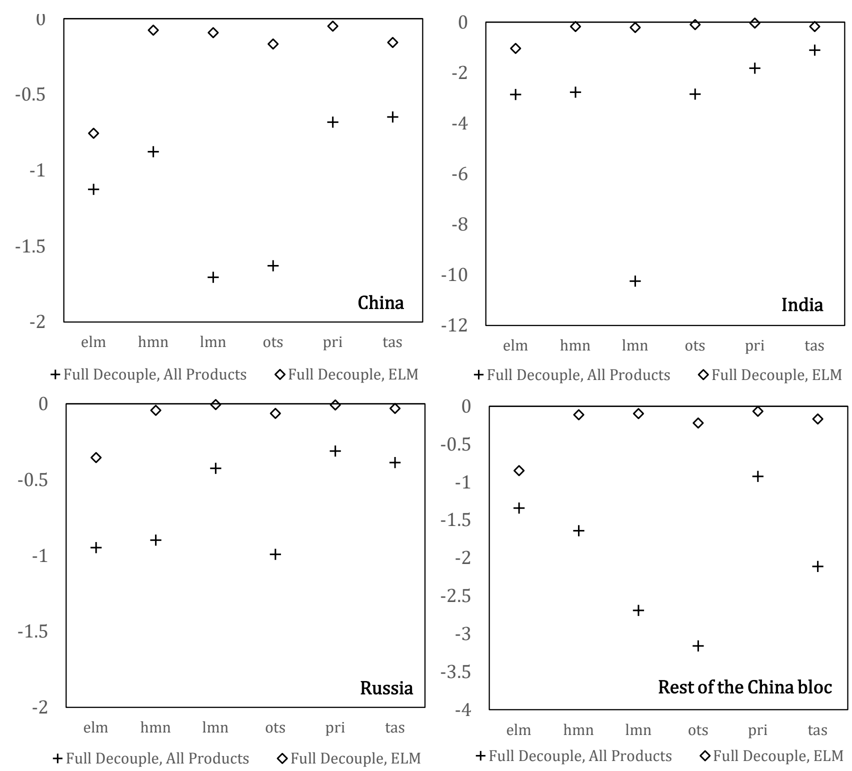


Figure 2.13. Cumulative Percentage Change of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$, by scenario. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points in either all sectors or only in the Electronic Equipment (`elm`) sector. In both cases, we set the parameter that controls the diffusion of ideas to $\beta = 0.44$. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

Note that, due to the multi-sector structure of the model, an increase in iceberg trade costs in one particular sector potentially has an indirect effect in all sectors of the economy. The magnitude of such impact in a given sector can be split between a direct effect (proportional to input use from the `elm` sector as intermediates) and an indirect effect (proportional to the use of the `elm` sector in the production of intermediates inputs).

Results in Figure 2.13 show the productivity losses induced by policy changes represented by the evolution of the Fréchet Distribution scale parameter $\lambda^{i}_{d,t}$ for those regions in the Eastern bloc. Contrasting the full decoupling in all sectors and one restricted to electronic equipment shows that, across all regions, productivity losses are substantially reduced and mostly restricted to the `elm` sector.

While there is some negative spillover effect to other sectors due to input-output linkages, particularly to business services (`tas`), these are very small for most regions. Regions such as Russia, which already had limited exposure to Western intermediate sourcing in the main scenario, see productivity losses go down to nearly zero across all sectors under the scenario that limits decoupling to the `elm` sector. China's losses in the `elm` sector are roughly similar to losses when decoupling happens in all products, but other sectors are not substantially affected.

All other regions have non-negligible losses in the `elm` sector. The largest changes happen for India and the Rest of the Eastern bloc. Those regions have a lower productivity starting point and benefit proportionately more from exposure to higher-quality intermediate inputs. For that reason, full decoupling in all products leads to large differential losses in productivity in those regions. The more restricted full decoupling in `elm` scenario limits losses, since those are proportional to the use of Western electronic equipment as inputs in the production of other sectors.

Changes in productivity map into changes in welfare, pictured in Figure 2.14. While welfare losses are substantial, ranging from $0.4 - 1.9\%$, they are very different in magnitude to the devastating results of a full decoupling in all products, in which

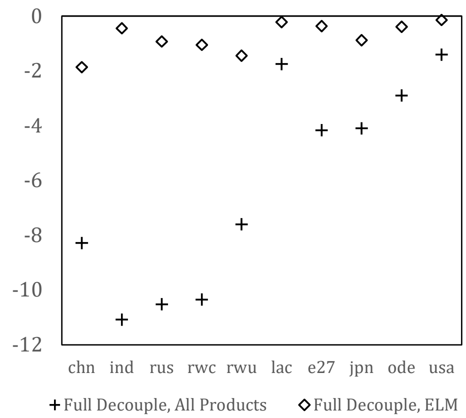


Figure 2.14. Cumulative Percentage Change in Welfare (Real GDP), by scenario. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points in either all sectors or only in the Electronic Equipment (`elm`) sector. In both cases, we set the parameter that controls the diffusion of ideas to $\beta = 0.44$. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

losses range between $8 - 12\%$.

These results underline two important observations. First, the costs of sector-specific decoupling might be limited enough for this scenario to be feasible. Second, input-output structures play an important role in magnifying dynamic losses. Limiting decoupling to one specific sector tapers down indirect magnification effects that happen through the input-output network.

2.6 Conclusion

We build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion in order to realistically investigate the impact of large and persistent geopolitical conflicts on global trade patterns, economic growth, and innovation. Canonical trade models typically start from a fixed technology assumption, which thus misses a crucial source of gains from trade through the diffusion of ideas.

In our theoretical contribution, we show that large trade costs can lead to dynamic inefficiencies in sectoral knowledge diffusion. Furthermore, we show that in a multi-sector framework deviations from optimal knowledge diffusion happen both within and between sectors. Additionally, sectoral deviations accumulate, such that total trade shares being close to their aggregate optimal diffusion points is no longer sufficient to guarantee optimal diffusion. A takeaway of our theoretical discussion is that, as the number of sectors increases, so do the number of deviations from optimality and diffusion losses tend to be higher with multiple sectors.

We then use this toolkit to simulate the trade, innovation, and welfare effects of potential receding globalization characterized by economic decoupling between the East and West, yielding three main insights. First, the projected welfare losses for the global economy of a decoupling scenario can be drastic, being as large as 12% in some regions, and are largest in the lower-income regions as they would benefit less from technology spillovers from richer areas. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. Without diffusion of ideas the size and variation across regions of the welfare losses would be substantially smaller. Third, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one.

This has important implications for the role of the multilateral trading system. First, the current system with global trade rules guaranteeing open and free trade between all major players is especially important for the lowest-income regions. Second, if geopolitical considerations would lead to a split of the big players into two blocs, it would be important that an institutional framework remains in place for smaller countries to keep open trade relations with both blocs, in particular for the lowest income regions.

The toolkit we have built is versatile and can be employed for many other research questions, in particular, focused on the analysis of policy. Future research could be extended in various directions. We mention two. First, there is ample empirical evidence for spillover effects from FDI, so the model could be extended with FDI and sales by multinational affiliates. Second, the framework with both technology spillovers and profits can be employed to analyze the economic effects of subsidy policies in different regions, an important policy topic in the multilateral trading system.

2.7 Acknowledgments

This chapter is in preparation for future publication. It is under review at the Journal of Monetary Economics and has been presented in the following conferences or institutions: OECD/IFC Joint Geoeconomic Fragmentation Workshop, IBRE/Fundação Getúlio Vargas; European Central Bank's Trade Seminar; 8th IMF-WB-WTO Trade Research Conference; 24th Annual Conference on Global Economic Analysis; and WashU at Saint Louis Economics Grad Student Conference. The author of the dissertation was an equal contributor to this paper.

2.8 Appendix

2.8.1 Additional tables calibration exercise

Table 2.3. Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of β

β	Mean	St.Dev.	max	min
GDP				
Historical	3.60	2.66	8.90	0.67
0.40	3.09	2.02	6.26	0.41
0.41	3.21	2.13	6.58	0.43
0.42	3.34	2.26	6.94	0.44
0.43	3.48	2.40	7.34	0.46
0.44	3.64	2.56	7.78	0.48
0.45	3.82	2.73	8.26	0.50
0.46	4.02	2.92	8.79	0.53
0.47	4.24	3.13	9.36	0.56
0.48	4.48	3.36	9.97	0.59
0.49	4.75	3.60	10.62	0.63
0.50	5.04	3.87	11.32	0.68
GDP per capita				
Historical	2.70	2.51	8.36	0.75
0.40	2.20	1.65	4.91	0.47
0.41	2.32	1.76	5.23	0.48
0.42	2.44	1.89	5.59	0.50
0.43	2.59	2.03	5.98	0.51
0.44	2.75	2.19	6.41	0.53
0.45	2.92	2.36	6.89	0.54
0.46	3.12	2.55	7.41	0.57
0.47	3.34	2.76	7.97	0.59
0.48	3.58	2.98	8.57	0.62
0.49	3.84	3.22	9.22	0.65
0.50	4.13	3.48	9.91	0.69

Table 2.4. The squared difference between the sum of the historical and simulated mean and standard deviation of GDP, GDP per capita and their sum

β	GDP	GDP pc	Sum
0.40	0.67	1.00	1.67
0.41	0.43	0.72	1.15
0.42	0.23	0.46	0.69
0.43	0.08	0.25	0.33
0.44	0.01	0.11	0.12
0.45	0.06	0.07	0.13
0.46	0.25	0.17	0.42
0.47	0.64	0.46	1.10
0.48	1.28	0.98	2.26
0.49	2.22	1.79	4.01
0.50	3.54	2.96	6.50

Table 2.5. Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of β between 0 and 0.6

β	Mean	St.Dev.	max	min
GDP				
Historical	3.60	2.66	8.90	0.67
0	2.17	1.10	3.79	0.32
0.5	2.19	1.12	3.84	0.32
0.10	2.20	1.13	3.87	0.32
0.15	2.23	1.16	3.93	0.32
0.20	2.27	1.20	4.02	0.33
0.25	2.34	1.26	4.19	0.33
0.30	2.46	1.39	4.49	0.35
0.35	2.68	1.61	5.08	0.37
0.40	3.09	2.02	6.26	0.41
0.45	3.82	2.73	8.26	0.50
0.50	5.04	3.87	11.32	0.68
0.55	6.89	5.42	15.39	1.01
0.60	9.50	7.32	20.53	1.66
GDP per capita				
Historical	2.70	2.51	8.36	0.75
0	1.29	0.72	2.27	0.40
0.5	1.31	0.75	2.33	0.40
0.10	1.32	0.76	2.36	0.40
0.15	1.35	0.78	2.41	0.40
0.20	1.39	0.82	2.53	0.40
0.25	1.46	0.89	2.74	0.41
0.30	1.58	1.01	3.10	0.42
0.35	1.80	1.24	3.75	0.44
0.40	2.20	1.65	4.91	0.47
0.45	2.92	2.36	6.89	0.54
0.50	4.13	3.48	9.91	0.69
0.55	5.96	5.02	13.92	0.98
0.60	8.54	6.88	18.97	1.55

Table 2.6. Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of β between 0 and 0.6 with an Autonomous Technology Growth Rate of $\alpha = 2.36$

β	Mean	St.Dev.	max	min
GDP				
Historical	3.60	2.66	8.90	0.67
0	2.17	1.10	3.79	0.32
0.5	2.19	1.12	3.84	0.32
0.10	2.21	1.14	3.88	0.32
0.15	2.23	1.16	3.94	0.32
0.20	2.28	1.21	4.04	0.33
0.25	2.35	1.28	4.23	0.33
0.30	2.49	1.41	4.56	0.35
0.35	2.73	1.65	5.22	0.37
0.40	3.17	2.09	6.48	0.42
0.45	3.95	2.85	8.60	0.52
0.50	5.23	4.03	11.76	0.71
0.55	7.16	5.62	15.92	1.08
0.60	9.84	7.52	21.17	1.77
GDP per capita				
Historical	2.70	2.51	8.36	0.75
0	1.29	0.72	2.27	0.40
0.5	1.31	0.75	2.33	0.40
0.10	1.33	0.77	2.36	0.40
0.15	1.35	0.79	2.43	0.40
0.20	1.40	0.83	2.56	0.40
0.25	1.47	0.91	2.79	0.41
0.30	1.61	1.04	3.18	0.42
0.35	1.85	1.28	3.89	0.44
0.40	2.28	1.72	5.13	0.48
0.45	3.05	2.48	7.22	0.56
0.50	4.32	3.64	10.35	0.72
0.55	6.22	5.21	14.45	1.04
0.60	8.87	7.08	19.56	1.66

2.8.2 Construction of Initial Sectoral Productivity Parameters $\lambda^i_{s,0}$

In our model, the location parameter of Fréchet distribution of a given industry-country $\lambda^i_{d,t}$ evolves endogenously according to a law of motion, as described by equation (2.21). To calibrate the model, we need initial values $(\lambda^i_{d,0})_{d\in\mathcal{D},i\in\mathcal{I}}$. We proxy for the initial values using labor productivity in different sectors for each aggregate region and industry in our sample in the base-year 2014.

We do so by combining two different databases: the World Input Output Database's Social Economic Accounts (WIOD-SEA —http://wiod.org/database/seas16) and the World Bank's Global Productivity Database (WB-GPD —https://www.worldbank.org/en/research/publication/global-productivity). WIOD-SEA a reports value added in local currency and employed population for 42 countries and 56 industries. WB-GPD reports value added in local currency and employed population for 103 countries and 9 industries. For countries whose data are available in both databases, we use the data from WIOD-SEA, which is more granular.

The first step is to create a cross-walk between WIOD-SEA industries and the more aggregate sectors in our model, namely: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. We then convert value added in local currency to PPP-USD and market-rate USD using a panel of PPP and market exchange rates from the World Bank's World Development Indicators.

Afterwards, we did a similar cross-walk for the country-sector pairs in the WB-GPD database. The detailed cross-walk can be found at the end of this Appendix. However, WB-GPD only reports one aggregate manufacturing sector, while our model disaggregates manufacturing into three subsectors. In order to make them compatible, we take the following steps: (a) we classify countries as advanced and emerging markets in both the WIOD-SEA and the WB-GPD databases; (b) we calculate the average share of value added and employed workers in total manufacturing for each of the manufacturing subsectors (`elm`, `hmn`, and `lmn`) for emerging markets and advanced economies, respectively, in the WIOD-SEA database; and (c) we use those shares and reported value added and employed workers from the WB-GPD database in order to input, for each country, a disaggregation of total manufacturing into `elm`, `hmn`, and `lmn`. We then convert value added in local currency to PPP-USD and market-rate USD using a panel of PPP and market exchange rates from the World Bank's World Development Indicators.

Finally, we collapse PPP-USD value added, market-rate USD value added, and number of workers for the regions of our model (`chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States), and calculate, for each region-industry pair, labor productivity as:

$$\lambda^i_{d,0} = \frac{PPP\$VA^i_{d,0}}{L^i_{d,0}}$$

using PPP-USD value added per worker.

Table 2.5. Cross Walk Between WIOD-SEA and Model

WIOD-SEA Sector	Model Sector
A01-03, B	pri
C10-19, C31-32	lmn
C20-25, C28-30	hmn
C26-27	elm
C33, D35, E36, F, G45-47, H50-53, I, L68, N, O84, P85, Q, R, S, T, U	ots
J58, J61, K64-66, M71-73	tas

Table 2.6. Cross Walk Between WB-GPD and Model

WB-GPD Sector	Model Sector
1.Agriculture	pri
2.Mining	pri
3.Manufacturing	(see methodology)
4.Utilities	ots
5.Construction	ots
6.Trade services	tas
7.Transport services	ots
8.Finance amd business services	tas
9.Other services	ots

- **Countries in WIOD-SEA**: Austria, Belgium, Brazil, Bulgaria, Canada, China, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Korea, Rep., Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Norway, Poland, Portugal, Romania, Russian Federation, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

- **Countries in WB-GPD**: Angola, Argentina, Australia, Austria, Azerbaijan, Belgium, Burkina Faso, Bangladesh, Bulgaria, Belize, Bolivia, Brazil, Botswana, Canada, Switzerland, Chile, China, Cameroon, Colombia, Costa Rica, Cyprus, Czech Republic, Germany, Denmark, Dominican Republic, Algeria, Ecuador, Egypt, Spain, Estonia, Ethiopia, Finland, Fiji, France, United Kingdom, Georgia, Ghana, Greece, Guatemala, China, Hong Kong SAR, Honduras, Croatia, Hungary, Indonesia, India, Ireland, Iran, Iceland, Italy, Jamaica, Jordan, Japan, Kenya, Republic of Korea, Lao People's Dem Rep, Saint Lucia, Sri Lanka, Lesotho, Lithuania, Luxembourg, Latvia, Morocco, Mexico, Montenegro, Mongolia, Mozambique, Mauritius, Malawi, Malaysia, Namibia, Nigeria, Netherlands, Norway, Nepal, New Zealand, Pakistan, Philippines, Poland, Portugal, Paraguay, Qatar, Romania, Russian Federation, Rwanda, Senegal, Singapore, Sierra Leone, Serbia, Slovakia, Slovenia, Sweden, Eswatini, Thailand, Turkey, Chinese Taipei, United Republic of Tanzania, Uganda, Uruguay, United States, St. Vincent and the Grenadines, Viet Nam, South Africa, Zambia.

2.8.3 Market Clearing

In every region, factor markets must clear, such that the use of each factor of production summed over all sectors $i \in \mathcal{I}$ in region $s \in \mathcal{D}$ by producers of varieties must equal its supply:

$$\sum_{i \in \mathcal{I}} k^i_{s,t} = k_s, \quad \sum_{i \in \mathcal{I}} \ell^i_{s,t} = \ell_{s,t} \tag{2.25}$$

The main equilibrium conditions are $|\mathcal{D}| \cdot |\mathcal{I}|$ expenditure equations that satisfy:

$$e^j_{s,t} = \sum_{d \in \mathcal{D}} \pi^j_{sd,t} \left(\underbrace{\sum_{i \in \mathcal{I}} \Psi^{i,j}_{d,t} \cdot \Psi^{i,m}_{d,t} \cdot \frac{\theta_i}{1+\theta_i} e^i_{d,t}}_{\text{intermediates}} + \underbrace{\kappa^j_{d,t} \cdot (1 - s_{d,t}) Y_{d,t}}_{\text{final goods}} + \underbrace{\frac{\overline{\chi}^j_d}{\sum_{i \in \mathcal{I}} \overline{\chi}^i_d} (s_{d,t} - tb_{d,t}) Y_{d,t}}_{\text{investment}} \right) \tag{2.26}$$

The part of (2.26) denotes the purchasing by firms of intermediate inputs. $\pi^j_{sd,t}$ is the trade share of s in varieties demanded by the producer of sectoral good j in region d; $\Psi^{i,j}_{d,t}$ is the cost share of sector j in the total intermediate expenditure use in sector i; $\Psi^{i,m}_{d,t}$ is the cost share of intermediates in total cost in sector i; and $\frac{\theta_i}{1+\theta_i} e^i_{d,t}$ is the total cost payments[30].

The second part represents expenditure related to the use of varieties in the production of sectoral goods for final consumption. $\kappa^i_{d,t}$ is the Cobb-Douglas parameter that denotes expenditure share in sector i as a fraction of total final goods expenditures.

The last part represents expenditures on investment goods. $\overline{\chi}^j_d$ are Leontief weights and the paths of savings $s_{d,t}$ and the trade balance $tb_{d,t}$ are given exogenously.

Prices of factors of production are proportional to their use and total cost. Since factors are used in every sector, we aggregate over sectors to calculate aggregate payments to each factor of production:

[30]The cost shares are, respectivelly: $\Psi^{i,j}_{d,t} = \Psi^{i,j}_{d,t}(p^j_{d,t})^{1-\mu_i} / (\sum_{k \in \mathcal{I}} \Psi^{i,k}_{d,t}(p^k_{d,t})^{1-\mu_i})$ and $\Psi^{i,m}_{d,t} = \Psi^{i,m}_{d,t}(pm^i_{d,t})^{1-\rho_i} / (\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\rho_i} + \Psi^{i,m}_{d,t}(pm^i_{d,t})^{1-\rho_i})$.

$$\begin{aligned} w_{s,t}\ell_{s,t} &= \textstyle\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}} \Psi^{i,\ell}_{s,t}\cdot\Psi^{i,f}_{s,t}\cdot\frac{\theta_i}{1+\theta_i}\cdot\pi^i_{sd,t}\cdot e^i_{d,t} && (2.27)\\ r_{s,t}k_s &= \textstyle\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}} \Psi^{i,k}_{s,t}\cdot\Psi^{i,f}_{s,t}\cdot\frac{\theta_i}{1+\theta_i}\cdot\pi^i_{sd,t}\cdot e^i_{d,t} && (2.28)\end{aligned}$$

where $\Psi^{i,f}_{d,t}$ is the cost share of value added in total cost; and, for each factor of production $n\in\{\ell,k\}$, $\Psi^{i,n}_{d,t}$ is the cost share of factor n in total expenditure on factors of production[31].

The government collects tariffs and other taxes and directs them to the representative household as lump-sum transfers:

$$T_{d,t}=\sum_{s\in\mathcal{D}}\sum_{i\in\mathcal{I}}\frac{tm_{sd,t}-1}{tm_{sd,t}}\pi^i_{sd,t}e^i_{d,t} \tag{2.29}$$

Recall that profits are:

$$\Pi^i_{s,t}=\frac{1}{1+\theta_i}\sum_{d\in\mathcal{D}}\pi^i_{sd,t}e^i_{d,t} \tag{2.30}$$

This completes all elements necessary to characterize domestic income:

$$Y_{s,t}=w_{s,t}\ell_{s,t}+r_{s,t}k_{s,t}+T_{s,t}+\sum_{j\in\mathcal{I}}\Pi^j_{s,t} \tag{2.31}$$

Replacing (2.27), (2.28), (2.29), and (2.30) into and the latter into (2.26) provides a system of equations in expenditures $e^i_{d,t}$ that solves for the cross-subsectional equilib-

[31]Once again, the mathematical expressions for the cost shares are: $\Psi^{i,f}_{d,t}=\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\rho_i}/(\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\rho_i}+\Psi^{i,m}_{d,t}(p^i_{d,t})^{1-\rho_i})$; and, for each factor of production $n\in\{\ell,k\}$, $\Psi^{i,n}_{d,t}=\Psi^{i,n}_{d,t}(pn^i_{d,t})^{1-\nu_i}/\sum_{q\in\{\ell,k\}}\Psi^{i,q}_{d,t}(pq^i_{d,t})^{1-\nu_i}$, with $pn^i_{d,t}$ standing for the price of factor n.

rium of this model.

2.8.4 Mathematical Derivations

Evolution of the Productivity Frontier

In this subsection, we largely follow the steps of the mathematical appendix to Buera and Oberfield (2020) to the particularities of our model. In the proofs, for simplicity, we initially abstract away from sectoral specific elasticities, using θ rather than θ_i, but at the end generalize the results to accommodate them. For any period, domestic technological frontier evolves according to:

$$F^i_{d,t+\Delta}(z) = \underbrace{F^i_{d,t}(z)}_{Pr\{\text{productivity}<z \text{ at } t\}} \times \underbrace{\left(1 - \int_t^{t+\Delta} \int \alpha_\tau z^{-\theta}(z')^{\beta\theta} dG^i_{d,\tau}(z')d\tau\right)}_{Pr\{\text{no better draws in } (t,t+\Delta)\}}$$

Rearranging and using the definition of the derivative:

$$\frac{d}{dt}\ln F^i_{s,t}(z) = \lim_{\Delta\to 0}\frac{F^i_{s,t+\Delta}(z) - F^i_{s,t}(z)}{F^i_{s,t}(z)} = -\int \alpha_t z^{-\theta}(z')^{\beta\theta} dG^i_{d,t}(z')$$

Define $\lambda^i_{s,t} = \int_{-\infty}^t \alpha_\tau \int (z')^{\beta\theta} dG^i_{s,\tau}(z')d\tau$ and integrate both sides wrt to time:

$$\begin{aligned}
\int_0^t \frac{d}{d\tau}\ln F^i_{s,\tau}(z)d\tau &= -z^{-\theta}\int_0^t \int \alpha_\tau (z')^{\beta\theta} dG^i_{d,\tau}(z')d\tau \\
\ln\left(\frac{F^i_{s,\tau}(z)}{F^i_{s,0}(z)}\right) &= -z^{-\theta}(\lambda^i_{s,t} - \lambda^i_{s,0}) \\
F^i_{s,t}(z) &= F^i_{s,0}(z)\exp\{-z^{-\theta}(\lambda^i_{s,t} - \lambda^i_{s,0})\}
\end{aligned}$$

Assuming that the initial distribution is Fréchet $F^i_{s,0}(z) = \exp\{-\lambda^i_{s,0} z^{-\theta}\}$ guarantees that the distribution will be Fréchet at any point in time:

$$F^i_{s,t}(z) = \exp\{-\lambda^i_{s,t} z^{-\theta}\} \tag{2.32}$$

Law of Motion of Productivity

As seen above, we have defined:

$$\lambda^i_{s,t} = \int_{-\infty}^{t} \alpha_\tau \int (z')^{\beta\theta^i} dG^i_{s,\tau}(z') d\tau$$

Differentiating this definition with respect to time and applying Leibnitz's Lemma yields:

$$\dot{\lambda}^i_{s,t} = \alpha_t \int (z')^{\beta\theta^i} dG^i_{s,t}(z')$$

We use these results and work with a discrete approximation of the law of motion for productivity:

$$\Delta\lambda^i_{s,t} = \alpha_t \int (z')^{\beta\theta^i} dG^i_{s,t}(z') \tag{2.33}$$

The source distribution $G^i_{d,t}(z') \equiv \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} H^{i,j}_{sd,t-1}(z')$, where $\Psi^{i,j}_{d,t}$ is the expenditure share of sector j in the cost of intermediates when producing good i in region d; and $H^{i,j}_{sd,t-1}(z')$ is the fraction of commodities for which the lowest cost supplier in period $t-1$ is a firm located in $s \in \mathcal{D}$ with productivity weakly less than z'.

We focus our attention on the integral $\int z^{\beta\theta^i} dH^{i,j}_{sd,t}(z)$. Let $F^i_{s,t}(z_2, z_2) = \exp\{-\lambda^i_{s,t} z_2^{-\theta^i}\}$ and $F^i_{s,t}(z_1, z_2) = (1 + \lambda^i_{s,t}[z_2^{-\theta^i} - z_1^{-\theta^i}]) \exp\{-\lambda^i_{s,t} z_2^{-\theta^i}\}$ are, respectively, the probability that a productivity draw is below z_2, and that the maximum productivity is z_1 and the second highest productivity is z_2[32]. Let for each n, $A^i_{n,t} \equiv \tilde{x}^i_{nd,t}\tilde{x}^i_{sd,t}$, such that s will have a lower cost than d iff $A^i_{n,t} z^i_{n,t}(\omega) < z^i_{s,t}(\omega)$. Region s with highest productivity producers z_1, z_2 will supply the commodity $i \in \mathcal{I}$ in region d with the following probability:

$$
\begin{aligned}
\mathcal{F}^{i,j}_{sd,t-1}(z_1, z_2) &= \int_0^{z_2} \Pi_{n \neq s} F^j_{n,t-1}\left(A^i_{n,t} y, A^i_{n,t} y\right) dF^j_{s,t-1}(y, y) \\
&+ \int_{z_2}^{z_1} \Pi_{n \neq s} F^j_{n,t-1}\left(A^i_{n,t} z_2, A^i_{n,t} z_2\right) \frac{d}{dz_1} F^j_{s,t-1}(z_1, z_2)
\end{aligned}
$$

The first term in the right hand side denotes the probability that the lowest cost producer at destination d is from s and has productivity lower than z_2, while the second term accounts for the probability that the lowest cost producer at destination d is from s and has productivity in the range $[z_2, z_1)$. We will evaluate each integral separately. First, take the first term:

[32]To see the latter, note that:

$$
\begin{aligned}
Prob(z_1 \leq Z_1, z_2 \leq Z_2) &= F^i_{s,t}(Z_2) + \int_0^{Z_2} \int_{Z_2}^{Z_1} f^i_{s,t}(y) f^i_{s,t}(y') dy' dy \\
&= F^i_{s,t}(Z_2) + F^i_{s,t}(Z_2)(F^i_{s,t}(Z_1) - F^i_{s,t}(Z_2)) \\
&= (1 + \lambda^i_{s,t}[Z_2^{-\theta} - Z_1^{-\theta}]) \exp\{-\lambda^i_{s,t} Z_2^{-\theta}\}
\end{aligned}
$$

$$
\begin{aligned}
&\int_0^{z_2} \Pi_{n\neq s} F^j_{n,t-1}\left(A^i_{n,t-1}y, A_{n,t-1}y\right) dF^j_{s,t-1}(y,y) \\
&= \int_0^{z_2} \exp\left\{ -\sum_{n\neq s} \lambda^j_{n,t-1}\left(A^i_{n,t-1}y\right)^{-\theta}\right\} \theta\lambda^{m,j}_{s,t} y^{-\theta-1} \exp\{-\lambda^j_{s,t-1}y^{-\theta^j}\}dy \\
&= \lambda^j_{s,t-1}\int_0^{z_2} \theta y^{-\theta^j-1} \exp\left\{ -\sum_{n} \lambda^j_{n,t-1}\left(A^i_{n,t-1}\right)^{-\theta^j} y^{-\theta^j}\right\} dy \\
&= \lambda^j_{s,t-1}\left[\frac{1}{\sum_n \lambda^j_{n,t-1}\left(A^i_{n,t-1}\right)^{-\theta^j}} \exp\left\{ -\sum_{n} \lambda^j_{n,t-1}\left(A^i_{n,t-1}\right)^{-\theta^j} y^{-\theta^j}\right\}\right]_{y=0}^{y=z_2} \\
&= \pi^{i,j}_{sd,t-1} \exp\left\{ -\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\}
\end{aligned}
$$

Now consider the second term.

$$
\begin{aligned}
&\int_{z_2}^{z_1} \Pi_{n\neq s} F^j_{n,t-1}\left(A^i_{n,t}z_2, A^i_{n,t}z_2\right) \frac{d}{dz_1} F^j_{s,t-1}(z_1,z_2) \\
&= \int_{z_2}^{z_1} \exp\left\{ -\sum_{n\neq s} \lambda^i_{n,t-1}\left(A^i_{n,t-1}z_2\right)^{-\theta^j}\right\} \theta^j\lambda^{m,j}_{s,t} z_1^{-\theta^j-1} \exp\{-\lambda^j_{s,t-1}z_2^{-\theta^j}\}dz_1 \\
&= \exp\left\{ -\sum_{n} \lambda^j_{n,t-1}\left(A^i_{n,t-1}z_2\right)^{-\theta^j}\right\} \lambda^j_{s,t-1}\int_{z_2}^{z_1} \theta^j z_1^{-\theta^j-1} dz_1 \\
&= \exp\left\{ -\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\} \lambda^j_{s,t-1}(z_2^{-\theta^j} - z_1^{-\theta^j})
\end{aligned}
$$

Therefore:

$$\mathcal{F}^{i,j}_{sd,t-1}(z_1,z_2) = \exp\left\{-\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\}\left(\pi^{i,j}_{sd,t-1} + \lambda^j_{s,t-1}(z_2^{-\theta^j} - z_1^{-\theta^j})\right) \tag{2.34}$$

Note that:

$$\int z^{\beta\theta^i} dH^{i,j}_{sd,t}(z) = \int_0^\infty \int_{z_2}^\infty z_1^{\beta\theta^i} \frac{\partial^2 \mathcal{F}^{i,j}_{sd,t-1}(z_1,z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \tag{2.35}$$

and that we can calculate the joint density explicitly:

$$\begin{aligned}
\frac{\partial^2 \mathcal{F}^{i}_{sd,t-1}(z_1,z_2)}{\partial z_1 \partial z_2} &= \frac{\partial}{\partial z_2} \exp\left\{-\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\} \theta \lambda^j_{s,t-1} z_1^{-\theta^j - 1} \\
&= \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\left\{-\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\} (\theta^j \lambda^j_{s,t-1} z_1^{-\theta^j - 1})(\theta^j \lambda^j_{n,t-1} z_2^{-\theta^j - 1})
\end{aligned}$$

Plugging this into (2.35):

$$\begin{aligned}
&\int_0^\infty \int_{z_2}^\infty z_1^{\beta\theta^i} \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\left\{-\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\} (\theta^j \lambda^j_{s,t-1} z_1^{-\theta^j - 1})(\theta^j \lambda^j_{n,t-1} z_2^{-\theta^j - 1}) dz_1 dz_2 \\
&= \int_0^\infty \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\left\{-\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta^j}\right\} (\theta^j \lambda^j_{s,t-1} z_2^{-\theta^j - 1}) \lambda^j_{s,t-1} \int_{z_2}^\infty (\theta^j z_1^{\beta\theta^i - \theta^j - 1}) dz_1 dz_2 \\
&= \int_0^\infty \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\left\{-\frac{\lambda^j_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\right\} (\theta^j \lambda^j_{s,t-1} z_2^{-\theta^j - 1}) \lambda^j_{s,t-1} \frac{1}{\beta\theta^i - \theta^j} z_2^{\beta\theta^i - \theta^j} dz_2
\end{aligned}$$

Now, if either $\theta^i = \theta^j$ or Using a change of variables, let $\gamma \equiv \frac{\lambda^i_{s,t-1}}{\pi^i_{sd,t-1}} z_2^{-\theta^j}$, which implies that $d\gamma = -\theta^j \frac{\lambda^i_{s,t-1}}{\pi^i_{sd,t-1}} z_2^{-\theta^j - 1} dz$

Replacing above:

$$
\begin{aligned}
&(\lambda^j_{s,t-1})^{\beta}(\pi^{i,j}_{sd,t-1})^{1-\beta}\frac{1}{1-\beta}\int_0^{\infty}\exp\Big\{-\gamma\Big\}\Psi^{(1-\beta)}d\gamma \\
&= (\lambda^j_{s,t-1})^{\beta}(\pi^{i,j}_{sd,t-1})^{1-\beta}\frac{1}{1-\beta}\Gamma(2-\beta) \\
&= (\lambda^j_{s,t-1})^{\beta}(\pi^{i,j}_{sd,t-1})^{1-\beta}\Gamma(1-\beta) \qquad (\because \Gamma(y+1) = y\Gamma(y))
\end{aligned}
$$

Therefore, replacing into the law of motion for the location parameter of the Fréchet distribution:

$$
\begin{aligned}
\Delta\lambda^i_{d,t} &= \alpha_t \int z^{\beta\theta} dG^i_{d,t}(z) \\
&= \alpha_t \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \int z^{\beta\theta} dH^{i,j}_{sd,t-1}(z) \\
&= \alpha_t \Gamma(1-\beta) \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} (\lambda^j_{s,t-1})^{\beta}(\pi^{i,j}_{sd,t-1})^{1-\beta}
\end{aligned}
$$

which is the same expression as in equation (2.21).

Proof of Proposition 2

Proof. (a) Diffusion Maximum

The trade shares that maximize diffusion solve the program:

$$\max_{\{\pi^{i,j}_{sd,t-1}\}_{j,i\in\mathcal{I},s\in\mathcal{D}}} \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} (\pi^{i,j}_{sd,t-1})^{1-\beta} (\lambda^{j}_{s,t-1})^{\beta} \tag{2.36}$$
$$s.t. \quad \forall (i,j) \in \mathcal{I}\times\mathcal{I} \qquad \sum_{s\in\mathcal{D}} \pi^{i,j}_{sd,t-1} = 1$$

whose solutions satisfy:[33]:

$$(\pi^{i,j}_{sd,t-1})^{\text{Diffusion Optimum}} = \frac{\lambda^{j}_{s,t-1}}{\sum_{k\in\mathcal{D}} \lambda^{j}_{k,t-1}} \tag{2.37}$$

Replacing (2.37) into the objective function results in the following diffusion function:

$$\begin{aligned}
(\Delta\lambda^{i}_{d})^{\text{Diffusion Optimal}} &= \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \left(\frac{\lambda^{j}_{s,t-1}}{\sum_{k\in\mathcal{D}} \lambda^{j}_{k,t-1}} \right)^{1-\beta} (\lambda^{j}_{s,t-1})^{\beta} \\
&= \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \lambda^{j}_{s,t-1} \left(\sum_{k\in\mathcal{D}} \lambda^{j}_{k,t-1} \right)^{-(1-\beta)} \\
&= \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \left(\sum_{s\in\mathcal{D}} \lambda^{j}_{s,t-1} \right)^{\beta}
\end{aligned}$$

[33]Let φ be the Lagrange multiplier. Then, for each (s,i,j) first order conditions satisfy:

$$\begin{aligned}
(1-\beta)\Psi^{i,j}_{d,t-1}(\pi^{i,j}_{sd,t-1})^{\beta}(\lambda^{j}_{s,t-1})^{\beta} &= \varphi \\
(\pi^{i,j}_{sd,t-1})^{\text{Diffusion Optimum}} &= \varphi^{-\frac{1}{\beta}}[(1-\beta)\Psi^{i,j}_{d,t-1}]^{\frac{1}{\beta}}\lambda^{j}_{s,t-1}
\end{aligned}$$

using the constraint:

$$\sum_{s\in\mathcal{D}} (\varphi^{-\frac{1}{\beta}}[(1-\beta)\Psi^{i,j}_{d,t-1}]^{\frac{1}{\beta}}\lambda^{j}_{s,t-1}) = 1 \iff \varphi^{-\frac{1}{\beta}} = [(1-\beta)\Psi^{i,j}_{d,t-1}]^{-\frac{1}{\beta}}(\sum_{s\in\mathcal{D}} \lambda^{j}_{s,t-1})^{-1}$$

whose single-sector analog is: $(\Delta\lambda_d)^{\text{Diffusion Optimal}} = \left(\sum_{s\in\mathcal{D}} \lambda_{s,t-1}\right)^{\beta}$. Let $h(x) = x^{\beta}$ for $\beta \in [0,1)$, a strictly concave function. Then:

$$\begin{aligned}
(\Delta\lambda_d)^{\text{Diffusion Optimal}} &= \alpha_t \Gamma(1-\beta) h\left(\sum_{s\in\mathcal{D}} \lambda_{s,t-1}\right) \\
&\geq \alpha_t \Gamma(1-\beta) h\left(\sum_{i\in\mathcal{I}} \Psi^{i}_{d,t-1} \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \lambda^{j}_{s,t-1}\right) \\
&> \alpha_t \Gamma(1-\beta) \sum_{i\in\mathcal{I}} \Psi^{i}_{d,t-1} \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \times h\left(\sum_{s\in\mathcal{D}} \lambda^{j}_{s,t-1}\right) \\
&= \alpha_t \Gamma(1-\beta) \sum_{i\in\mathcal{I}} \Psi^{i}_{d,t-1} (\Delta\lambda^{i}_{d})^{\text{Diffusion Optimal}}
\end{aligned}$$

where the first inequality comes from the fact that $\sum_{s\in\mathcal{D}} \lambda_{s,t-1} \geq \sum_{i\in\mathcal{I}} \Psi^{i}_{d,t-1} \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \lambda^{j}_{s,t-1}$ and $h(\cdot)$ is an increasing function; and the second inequality comes from applying Jensen's Inequality in the context of strictly concave functions.

(b) Market Allocation

Let $g(\pi, \lambda) \equiv \pi^{1-\beta}\lambda^{\beta}$. Note that $g(\pi, \lambda)$ is strictly concave in both of its arguments. Denote π_{sd}, λ_s the value for aggregate trade shares and aggregate productivity, respectively, in a single sector mode. For ease of exposition, we omit time subscripts below whenever possible. Due to the concavity of $g(\cdot,\cdot)$:

$$g(\pi^{i,j}_{sd}, \lambda^{j}_{s}) \leq g(\pi_{sd}, \lambda_s) + g_{\pi}(\pi_{sd}, \lambda_s)\left(\pi^{i,j}_{sd} - \pi_{sd}\right) + g_{\lambda}(\pi_{sd}, \lambda_s)\left(\lambda^{j}_{s} - \lambda_s\right) \qquad (2.38)$$

Let $\Psi_d^{i,j} \equiv \frac{e_d^{i,j}}{e_d^i}$ and $\Psi_d^i \equiv \frac{e_d^i}{e_d}$ denote the cost shares in the destination country. By definition, $\pi_{sd} = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \Psi_d^i \cdot \Psi_d^{i,j} \cdot \pi_{sd}^{i,j}$[34]. Multiplying both sides of (2.38) by $\Psi_d^i \cdot \Psi_d^{i,j}$ and summing over i and j results in:

$$
\begin{aligned}
\sum_{i\in\mathcal{I}}\Psi_d^i\sum_{j\in\mathcal{I}}\Psi_d^{i,j} g(\pi_{sd}^{i,j},\lambda_s^j) &\leq g(\pi_{sd},\lambda_s) + g_\pi(\pi_{sd},\lambda_s)\left(\sum_{i\in\mathcal{I}}\Psi_d^i\sum_{j\in\mathcal{I}}\Psi_d^{i,j}\pi_{sd}^{i,j} - \pi_{sd}\right) \\
&+ g_\lambda(\pi_{sd},\lambda_s)\left(\sum_{i\in\mathcal{I}}\Psi_d^i\sum_{j\in\mathcal{I}}\Psi_d^{i,j}\lambda_s^j - \lambda_s\right) \\
&= g(\pi_{sd},\lambda_s) + g_\lambda(\pi_{sd},\lambda_s)\left(\sum_{i\in\mathcal{I}}\Psi_d^i\sum_{j\in\mathcal{I}}\Psi_d^{i,j}\lambda_s^j - \lambda_s\right)
\end{aligned}
$$

where the equality follows from the fact that $\pi_{sd} = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \Psi_d^i \cdot \Psi_d^{i,j} \cdot \pi_{sd}^{i,j}$.

Recall the definition of the law of motion for productivity in a multi-sector framework: (2.21):

$$\Delta\lambda_d^i = \alpha\sum_{j\in\mathcal{I}}\Gamma(1-\beta)\Psi_d^{i,j}\sum_{s\in\mathcal{D}}(\pi_{sd}^{i,j})^{1-\beta}(\lambda_s^j)^\beta = \alpha\Gamma(1-\beta)\sum_{j\in\mathcal{I}}\Psi_d^{i,j} g(\pi_{sd,}^{i,j},\lambda_s^j)$$

In a single-sector framework the law of motion of productivity is:
$\Delta\lambda_d = \alpha\Gamma(1-\beta)\sum_{s\in\mathcal{D}} g(\pi_{sd},\lambda_s)$. Using these definitions, multiplying both sides of the inequality above by $\alpha\Gamma(1-\beta)$ and summing over source countries results in:

$$\sum_{i\in\mathcal{I}}\Psi_{d,t-1}^i\Delta\lambda_d^i \leq \Delta\lambda_d + \alpha_t\Gamma(1-\beta)\sum_{s\in\mathcal{D}} g_\lambda(\pi_{sd,t-1},\lambda_{s,t-1})\left(\sum_{i\in\mathcal{I}}\Psi_{d,t-1}^i\sum_{j\in\mathcal{I}}\Psi_{d,t-1}^{i,j}\lambda_{s,t-1}^j - \lambda_{s,t-1}\right) \tag{2.39}$$

[34]To see that, note: $\pi_{sd} = \frac{e_{sd}}{e_d} = \frac{\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} e_{sd}^{i,j}}{e_d} = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \frac{e_d^i}{e_d}\frac{e_d^{i,j}}{e_d^i}\frac{e_{sd}^{i,j}}{e_d^{i,j}} = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \Psi_d^i \cdot \Psi_d^{i,j} \cdot \pi_{sd}^{i,j}$.

Since $g_\lambda(\pi_{sd}, \lambda_s) > 0$, if $\sum_{i \in \mathcal{I}} \Psi^i_{d,t-1} \sum_{j \in \mathcal{I}} \Psi^{i,j}_{d,t-1} \lambda^j_{s,t-1} \leq \lambda_{s,t-1}$ for each s and d, then:

$$\left(\sum_{i \in \mathcal{I}} \Psi^i_{d,t-1} \Delta\lambda^i_{d,t} \right)^{\text{Market Allocation}} \leq \left(\Delta\lambda_{d,t} \right)^{\text{Market Allocation}} \tag{2.40}$$

which completes the proof.

□

Summary Statistics

Note that the aggregate diffusion function is:

$$\Delta\lambda_d = \alpha_t \Gamma(1-\beta) \sum_{d \in \mathcal{D}} \left(\pi_{sd} \right)^{1-\beta} \left(\lambda_s \right)^{\beta}$$

Hence:

$$\frac{\partial \Delta\lambda_d}{\partial \tilde{\tau}_{s'd}} = \alpha_t \Gamma(1-\beta)(1-\beta) \sum_{d \in \mathcal{D}} \left(\pi_{sd} \right)^{1-\beta} \left(\lambda_s \right)^{\beta} \frac{1}{\pi_{sd}} \frac{\partial \pi_{sd}}{\partial \tilde{\tau}_{s'd}}$$

Now calculate $\frac{\partial \pi_{sd}}{\partial \tilde{\tau}_{s'd}}$:

$$\frac{\partial \pi_{sd}}{\partial \tilde{\tau}_{s'd}} = \begin{cases} -\theta \frac{\pi_{s'd}}{\tilde{\tau}_{s'd}} \left[1 - \pi_{s'd} \right], & \text{if } s = s' \\ +\theta \frac{\pi_{sd}}{\tilde{\tau}_{s'd}} \pi_{s'd}, & \text{if } s \neq s' \end{cases}$$

Hence:

$$\frac{\partial \Delta\lambda_d}{\partial \tilde{\tau}_{s'd}} = -\check{\kappa}\left[(1-\pi_{s'd})\left(\pi_{s'd}\right)^{1-\beta}\left(\lambda_{s'}\right)^{\beta} - \left(\pi_{s'd}\right)\sum_{k\in\mathcal{D}\setminus\{s'\}}\left(\pi_{kd}\right)^{1-\beta}\left(\lambda_k\right)^{\beta}\right]$$

where $\check{\kappa} \equiv \theta(1-\beta)\frac{1}{\tilde{\tau}_{s'd}}\alpha_t\Gamma(1-\beta)$. Finally, define $\omega_{sd} \equiv \frac{\alpha_t\Gamma(1-\beta)(\pi_{sd})^{1-\beta}(\lambda_s)^{\beta}}{\Delta\lambda_d}$, the pairwise diffusion term. Then:

$$\begin{aligned}
\frac{\partial \Delta\lambda_d}{\partial \tilde{\tau}_{s'd}}\frac{\tilde{\tau}_{s'd}}{\Delta\lambda_d} &= -\theta(1-\beta)\left[(1-\pi_{s'd})\omega_{s'd} - (\pi_{s'd})\sum_{k\in\mathcal{D}\setminus\{s'\}}\omega_{kd}\right] \\
\frac{\partial \Delta\lambda_d}{\partial \tilde{\tau}_{s'd}}\frac{\tilde{\tau}_{s'd}}{\Delta\lambda_d} &= -\theta(1-\beta)\left[(1-\pi_{s'd})\omega_{s'd} - \pi_{s'd}(1-\omega_{s'd})\right] \\
\frac{\partial \Delta\lambda_d}{\partial \tilde{\tau}_{s'd}}\frac{\tilde{\tau}_{s'd}}{\Delta\lambda_d} &= -\theta(1-\beta)\left[\omega_{s'd} - \pi_{s'd}\right]
\end{aligned}$$

2.8.5 Optimal Diffusion Levels

Two-by-Two Economy

If a Benevolent Planner were to choose domestic trade shares to maximize idea diffusion to a given sector at the home economy, she would solve the following concave programming problem:

$$\begin{aligned} & \max_{\{\pi_h^{i,i}, \pi_h^{i,-i}\}} \Psi^i [(\pi_h^{i,i})^{1-\beta} (\lambda_h^i)^\beta + (1 - \pi_h^{i,i})^{1-\beta} (\lambda_f^i)^\beta] \\ + \quad & (1 - \Psi_{d,t-1}^i) [(\pi_h^{i,-i})^{1-\beta} (\lambda_h^{-i})^\beta + (1 - \pi_h^{i,-i})^{1-\beta} (\lambda_f^{-i})^\beta] \end{aligned}$$

For $\pi_h^{i,i}$, the first-order condition satisfies:

$$\begin{aligned} \Psi^i (1-\beta) [(\pi_h^{i,i})^{-\beta} (\lambda_h^i)^\beta - (1 - \pi_h^{i,i})^{-\beta} (\lambda_f^i)^\beta] &= 0 \\ (\pi_h^{i,i})^{-\beta} (\lambda_h^i)^\beta &= (1 - \pi_h^{i,i})^{-\beta} (\lambda_f^i)^\beta \\ (\pi_h^{i,i})^{\text{Diffusion Optimum}} &= \frac{\lambda_h^i}{\lambda_f^i + \lambda_h^i} \end{aligned}$$

This result is the building block of the ratios that we express in subsection 3. If we want to calculate the within sector ratio of total domestic trade expenditure, we can write:

$$\left(\frac{\Psi^i \pi_h^{i,i}}{\Psi^i (1 - \pi_h^{i,i})} \right)^{\text{Diffusion Optimum}} = \frac{\lambda_h^i}{\lambda_f^i + \lambda_h^i} \times \left(\frac{\lambda_f^i}{\lambda_f^i + \lambda_h^i} \right)^{-1} = \frac{\lambda_h^i}{\lambda_f^i}$$

Similarly, if we want to write a cross-sector ratio of total domestic trade expenditure shares, we can write:

$$\left(\frac{\Psi^i \pi_h^{i,i}}{(1-\Psi^i)\pi_h^{i,-i}}\right)^{\text{Diffusion Optimum}} = \underbrace{\frac{\Psi^i}{1-\Psi^i}}_{\text{cost share}} \times \underbrace{\frac{\lambda_h^i}{\lambda_h^{-i}}}_{\text{own-productivity}} \times \underbrace{\left(\frac{\lambda_h^i + \lambda_f^i}{\lambda_h^{-i} + \lambda_f^{-i}}\right)^{-1}}_{\text{industry-wise productivity}} \quad (2.41)$$

which is the same as equation (22).

Multi-Sector, Multi-Region Economy

For each sector i, the diffusion maximizing trade shares satisfy:

$$\begin{aligned} &\max_{\{\pi_{sd,t-1}^{i,j}\}_{j,i\in\mathcal{I},s\in\mathcal{D}}} \sum_{j\in\mathcal{I}} \Psi_{d,t-1}^{i,j} \sum_{s\in\mathcal{D}} (\pi_{sd,t-1}^{i,j})^{1-\beta} (\lambda_{s,t-1}^{j})^{\beta} \\ &s.t. \quad \forall (i,j) \in \mathcal{I}\times\mathcal{I} \qquad \sum_{s\in\mathcal{D}} \pi_{sd,t-1}^{i,j} = 1 \end{aligned} \quad (2.42)$$

Let φ be the Lagrange multiplier. Then, for each (s,i,j) first order conditions satisfy:

$$\begin{aligned} (1-\beta)\Psi_{d,t-1}^{i,j}(\pi_{sd,t-1}^{i,j})^{\beta}(\lambda_{s,t-1}^{j})^{\beta} &= \varphi \\ (\pi_{sd,t-1}^{i,j})^{\text{Diffusion Optimum}} &= \varphi^{-\frac{1}{\beta}}[(1-\beta)\Psi_{d,t-1}^{i,j}]^{\frac{1}{\beta}}\lambda_{s,t-1}^{j} \end{aligned}$$

using the constraint:

$$\sum_{s\in\mathcal{D}} (\varphi^{-\frac{1}{\beta}}[(1-\beta)\Psi_{d,t-1}^{i,j}]^{\frac{1}{\beta}}\lambda_{s,t-1}^{j}) = 1 \iff \varphi^{-\frac{1}{\beta}} = [(1-\beta)\Psi_{d,t-1}^{i,j}]^{-\frac{1}{\beta}}(\sum_{s\in\mathcal{D}} \lambda_{s,t-1}^{j})^{-1}$$

Therefore:

$$(\pi^{i,j}_{sd,t-1})^{\text{Diffusion Optimum}} = \frac{\lambda^j_{s,t-1}}{\sum_{k\in\mathcal{D}} \lambda^j_{k,t-1}} \tag{2.43}$$

If we want to calculate the within sector ratio of total domestic trade expenditure, we can write:

$$\left(\frac{\Psi^{i,j}_{d,t-1}\pi^{i,j}_{sd,t-1}}{\Psi^{i,j}_{d,t-1}\pi^{i,j}_{nd,t-1}}\right)^{\text{Diffusion Optimum}} = \frac{\lambda^j_{s,t-1}}{\sum_{k\in\mathcal{D}} \lambda^{m,j}_{k,t-1}} \times \left(\frac{\lambda^j_{n,t-1}}{\sum_{k\in\mathcal{D}} \lambda^{m,j}_{k,t-1}}\right)^{-1} = \frac{\lambda^j_{s,t-1}}{\lambda^j_{n,t-1}}$$

Replacing 2.43 into the objective function results in the following diffusion function:

$$\begin{aligned}
(\Delta\lambda^i_d)^{\text{Diffusion Optimal}} &= \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \left(\frac{\lambda^j_{s,t-1}}{\sum_{k\in\mathcal{D}} \lambda^j_{k,t-1}}\right)^{1-\beta} (\lambda^j_{s,t-1})^{\beta} \\
&= \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \lambda^j_{s,t-1} \left(\sum_{k\in\mathcal{D}} \lambda^j_{k,t-1}\right)^{-(1-\beta)} \\
&= \sum_{j\in\mathcal{I}} \Psi^{i,j}_{d,t-1} \left(\sum_{s\in\mathcal{D}} \lambda^j_{s,t-1}\right)^{\beta}
\end{aligned}$$

Recall that under the market allocation:

$$(\pi^{i,j}_{sd,t-1})^{\text{Market Allocation}} = \frac{\lambda^j_{s,t-1}(\tilde{x}_{sd})^{-\theta_j}}{\sum_{k\in\mathcal{D}} \lambda^j_{k,t-1}(\tilde{x}_{kd})^{-\theta_j}} \tag{2.44}$$

Replacing that in the diffusion function results in:

$$
\begin{aligned}
(\Delta\lambda_d^i)^{\text{Market Allocation}} &= \sum_{j\in\mathcal{I}} \Psi_{d,t-1}^{i,j} \sum_{s\in\mathcal{D}} \left(\frac{\lambda_{s,t-1}^{j} (\tilde{x}_{sd})^{-\theta_j}}{\sum_{k\in\mathcal{D}} \lambda_{k,t-1}^{j} (\tilde{x}_{kd})^{-\theta_j}} \right)^{1-\beta} (\lambda_{s,t-1}^{j})^{\beta} \\
&= \sum_{j\in\mathcal{I}} \Psi_{d,t-1}^{i,j} \sum_{s\in\mathcal{D}} \lambda_{s,t-1}^{j} (\tilde{x}_{sd})^{-\theta_j(1-\beta)} \left(\sum_{k\in\mathcal{D}} \lambda_{k,t-1}^{j} (\tilde{x}_{kd})^{-\theta_j} \right)^{-(1-\beta)} \\
&= \sum_{j\in\mathcal{I}} \Psi_{d,t-1}^{i,j} \sum_{s\in\mathcal{D}} \lambda_{s,t-1}^{j} \left(\sum_{k\in\mathcal{D}} \lambda_{k,t-1}^{j} \left(\frac{\tilde{x}_{kd}}{\tilde{x}_{sd}} \right)^{-\theta_j} \right)^{-(1-\beta)}
\end{aligned}
$$

2.8.6 Other Mathematical Derivations

Trade shares

In this model, since there are infinitely many varieties in the unit interval, the expenditure share of destination region $d \in \mathcal{D}$ on goods coming from source country $s \in \mathcal{D}$ converge to their expected values. Let $\pi^i_{sd,t}$ denote the share of expenditures of consumers in region $d \in \mathcal{D}$ on commodity $i \in \mathcal{I}^m$ coming from region $s \in \mathcal{D}$ and, let for each n, $(A^i_{n,t})^{-1} \equiv \tilde{x}^i_{sd,t}\tilde{x}^i_{nd,t}$. This share will satisfy:

$$
\begin{aligned}
\pi^i_{sd,t} &= Pr\left(\frac{\tilde{x}^i_{sd,t}}{z^i_{s,t}(\omega)} < \min_{(n \neq s)}\left\{\frac{\tilde{x}^i_{nd,t}}{z^i_{n,t}(\omega)}\right\}\right) \\
&= \int_0^\infty Pr(z^i_{s,t}(\omega) = z) Pr(z^i_{n,t}(\omega) < zA^i_n) dz \\
&= \int_0^\infty f^i_{s,t}(z)\Pi_{(n \neq s)} F_{n,t}(zA^i_n) dz \\
&= \int_0^\infty \theta\lambda^i_{s,t} z^{-(1+\theta)} e^{-(\sum_{n\in\mathcal{D}} \lambda^i_{n,t}(A^i_n)^{-\theta})z^{-\theta}} dz \\
&= \frac{\lambda^i_{s,t}(\tilde{x}^i_{sd,t})^{-\theta}}{\sum_{n\in\mathcal{D}} \lambda^i_{n,t}(\tilde{x}^i_{md,t})^{-\theta}} \\
&= \frac{\lambda^i_{s,t}(\tilde{x}^i_{sd,t})^{-\theta}}{\Phi^i_{d,t}} \qquad (2.45)
\end{aligned}
$$

Similarly, since countries use the same aggregate final goods as intermediate inputs, cost shares in intermediates for each supplying sector j and region s used in the production of good i in region d satisfies:

$$
\pi^{i,j}_{sd,t} = \frac{\lambda^j_{s,t}(\tilde{x}^j_{sd,t})^{-\theta}}{\Phi^j_{d,t}} \qquad (2.46)
$$

which are the same as expressed in (18).

Price levels

Recall that the prices of commodities and intermediate goods can be expressed, respectively, as:

$$p^i_{d,t} \quad = \quad \left[\int_{[0,1]} p^i_{d,t}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$

Let $\Omega^i_{sd,t}$ and $\Omega^{i,j}_{sd,t}$ denote the subsets of $\Omega = [0,1]$ for which the region $s \in \mathcal{D}$ is a supplier in destination region $d \in \mathcal{D}$. We can then rewrite price levels above as:

$$p^i_{d,t} \quad = \quad \left[\sum_{s\in\mathcal{D}} \int_{\Omega^i_{sd,t}} p^i_{d,t}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$

Similarly, we restate $\mathcal{F}^i_{sd,t}(z_1,z_2)$ and the analogous measure $\mathcal{F}^{i,j}_{sd,t}(z_1,z_2)$:

$$\mathcal{F}^i_{sd,t}(z_1,z_2) \quad = \quad \exp\left\{-\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_2^{-\theta}\right\}\left(\pi^i_{sd,t} + \lambda^i_{s,t}(z_2^{-\theta} - z_1^{-\theta})\right) \qquad (2.47)$$

which denote the fraction of varieties that d purchases from s with productivity up to z_1 and whose second best producer is not more efficient than than z_2. Recall that, from the Bertrand competition assumption, we can write, for each variety ω:

$$p^i_{d,t}(\omega) = \min\left\{\frac{\sigma}{\sigma-1}\frac{\tilde{x}^i_{sd,t}}{z^i_{1s,t}(\omega)}, \frac{\tilde{x}^i_{sd,t}}{z^i_{2s,t}(\omega)}\right\}$$

So we can rewrite the equation $\int_{\Omega^i_{sd,t}} p^i_{d,t}(\omega)^{1-\sigma} d\omega$ in the following fashion:

$$
\begin{aligned}
& \int_{\Omega^i_{sd,t}} p^i_{d,t}(\omega)^{1-\sigma} d\omega \\
= \quad & \int_0^\infty \int_{z_2}^\infty (p^i_{d,t})^{1-\sigma} \frac{\partial^2 \mathcal{F}^i_{sd,t}(z_1, z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \\
= \quad & \int_0^\infty \int_{z_2}^\infty \min\left\{ \frac{\sigma}{\sigma-1}\frac{\tilde{x}^i_{sd,t}}{z_1}, \frac{\tilde{x}^i_{sd,t}}{z_2} \right\}^{1-\sigma} \frac{1}{\pi^i_{sd,t}} \exp\left\{ -\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_2^{-\theta} \right\} (\theta \lambda^i_{s,t} z_1^{-\theta-1})(\theta \lambda^i_{s,t} z_2^{-\theta-1}) dz_1 dz_2
\end{aligned}
$$

With a change of variables, denote $\Psi_1 \equiv \frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_1^{-\theta}$ and $\Psi_2 \equiv \frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_2^{-\theta}$ and $d\Psi_1 = -\frac{\theta \lambda^i_{s,t} z_1^{-\theta-1}}{\pi^i_{sd,t-1}} dz_1$, $d\Psi_2 = -\frac{\theta \lambda^i_{s,t} z_2^{-\theta-1}}{\pi^i_{sd,t-1}} dz_2$, which allows us to rewrite the equation above as:

$$
\begin{aligned}
& \int_{\Omega^i_{sd,t}} p^i_{d,t}(\omega)^{1-\sigma} d\omega \\
= \quad & \pi^i_{sd,t} \int_0^\infty \int_0^{\Psi_2} \min\left\{ \frac{\sigma}{\sigma-1}\frac{\tilde{x}^i_{sd,t}}{z_1}, \frac{\tilde{x}^i_{sd,t}}{z_2} \right\}^{1-\sigma} \exp\left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \\
= \quad & \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \int_0^\infty \int_0^{\Psi_2} \min\left\{ \left(\frac{\sigma}{\sigma-1}\right)^{\theta} \Psi_1, \Psi_2 \right\}^{\frac{1-\sigma}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \\
= \quad & \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \left[\int_0^\infty \int_{\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \Psi_2}^{\Psi_2} \Psi_2^{\frac{1-\sigma}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \right. \\
+ \quad & \left. \int_0^\infty \int_0^{\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \Psi_2} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Psi_1^{\frac{1-\sigma}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \right] \\
= \quad & \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \left[1 - \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} + \frac{\theta}{1-\sigma+\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \cdot \int_0^\infty \Psi_2^{\frac{1-\sigma}{\theta}+1} \exp\left\{ -\Psi_2 \right\} d\Psi_2 \\
= \quad & \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \left[1 - \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} + \frac{\theta}{1-\sigma+\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \Gamma\left(\frac{1-\sigma}{\theta} + 2 \right) \\
= \quad & \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \left[1 - \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} + \frac{\theta}{1-\sigma+\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \frac{1-\sigma+\theta}{\theta} \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \\
= \quad & \left[1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \cdot \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \cdot \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t} (\tilde{x}^i_{sd,t})^{-\theta}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} \\
= \quad & \left[1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \cdot \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \cdot \pi^i_{sd,t} \left(\frac{\lambda^i_{s,t} (\tilde{x}^i_{sd,t})^{-\theta}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} \\
= \quad & \left[1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \cdot \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \cdot \pi^i_{sd,t} \left(\sum_{n \in \mathcal{D}} \lambda^i_{n,t} (\tilde{x}^i_{nd,t})^{-\theta} \right)^{-\frac{1-\sigma}{\theta}}
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
p^i_{d,t} &= \left[\sum_{s\in\mathcal{D}}\int_{\Omega^i_{sd,t}} p^i_{d,t}(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}} \\
p^i_{d,t} &= \left[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\right]^{\frac{1}{1-\sigma}} \\
&\quad \Gamma\left(\frac{1-\sigma+\theta}{\theta}\right)^{\frac{1}{1-\sigma}}\cdot\left(\sum_{n\in\mathcal{D}}\lambda^i_{n,t}(\tilde{x}^i_{nd,t})^{-\theta}\right)^{-\frac{1}{\theta}}\cdot\left[\sum_{s\in\mathcal{D}}\pi^i_{sd,t}\right]^{\frac{1}{1-\sigma}} \\
p^i_{d,t} &= \left[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\right]^{\frac{1}{1-\sigma}}\cdot\Gamma\left(\frac{1-\sigma+\theta}{\theta}\right)^{\frac{1}{1-\sigma}}\cdot\left(\sum_{n\in\mathcal{D}}\lambda^i_{n,t}(\tilde{x}^i_{nd,t})^{-\theta}\right)^{-\frac{1}{\theta}}
\end{aligned}
$$

Which is the same as (17) after allowing the elasticities to be sector-specific.

Marginal costs and profits

From equation (2.12) we can derive standard CES demand functions as:

$$
\begin{aligned}
q^i_{d,t}(\omega) &= \left(\frac{p^i_{d,t}(\omega)}{p^i_{d,t}}\right)^{-\sigma}\frac{e^i_{d,t}}{p^i_{d,t}} && (2.48)\\
c^{i,j}_{d,t}(\omega) &= \left(\frac{p^{m,j}_{d,t}(\omega)}{p^{m,j}_{d,t}}\right)^{-\sigma}\frac{e^{i,j}_{d,t}}{pc^{m,j}_{d,t}} && (2.49)
\end{aligned}
$$

where $p^i_{d,t}$ satisfies equations (17); $e^i_{d,t}$ denotes expenditure on commodity i of macro-sector m in country d; and $e^{i,j}_{d,t}$ denotes expenditure on intermediate input j used in the production of commodity i of macro-sector m in country d.

As in previous subsubsections of the Appendix, we will derive the expression for the marginal cost and mark-up for the production of variety $q^i_{d,t}(\omega)$ and state a corresponding expression for $c^{i,j}_{d,t}(\omega)$. The marginal cost of producing variety ω sourced in country s and consumed in country s is:

$$\frac{\tilde{x}^i_{d,t}}{z_1(\omega)} q^i_{d,t}(\omega)$$

and total cost of varieties sourced in country s and consumed in country s can be expressed as:

$$\int_{\Omega^i_{sd,t}} \frac{\tilde{x}^i_{d,t}}{z_1(\omega)} q^i_{d,t}(\omega) d\omega = \int_{\Omega^i_{sd,t}} \frac{\tilde{x}^i_{d,t}}{z_1(\omega)} \left(\frac{p^i_{d,t}(\omega)}{p^i_{d,t}} \right)^{-\sigma} \frac{e^i_{d,t}}{p^i_{d,t}} d\omega$$

As in the previous subsection of the Appendix, we let $\Omega^i_{sd,t}$ and $\Omega^{i,j}_{sd,t}$ denote the subsets of $\Omega = [0,1]$ for which the region $s \in \mathcal{D}$ is a supplier in destination region $d \in \mathcal{D}$. We can then rewrite the integral above as:

$$\begin{aligned}
& \int_{\Omega^i_{sd,t}} \frac{\tilde{x}^i_{d,t}}{z_1(\omega)} \left(\frac{p^i_{d,t}(\omega)}{p^i_{d,t}} \right)^{-\sigma} \frac{e^i_{d,t}}{p^i_{d,t}} d\omega \\
= \quad & \tilde{x}^i_{d,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \int_0^\infty \int_{z_2}^\infty (z_1)^{-1} (p^i_{d,t})^{-\sigma} \frac{\partial^2 \mathcal{F}^i_{sd,t}(z_1, z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \\
= \quad & \tilde{x}^i_{d,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \int_0^\infty \int_{z_2}^\infty \frac{1}{z_1} \min \left\{ \frac{\sigma}{\sigma - 1} \frac{\tilde{x}^i_{sd,t}}{z_1}, \frac{\tilde{x}^i_{sd,t}}{z_2} \right\}^{-\sigma} \frac{1}{\pi^i_{sd,t}} \exp\left\{ - \frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_2^{-\theta} \right\} (\theta \lambda^i_{s,t} z_1^{-\theta-1})(\theta \lambda^i_{s,t} z_2^{-\theta-1}) dz_1 dz_2
\end{aligned}$$

Once again, use a change of variables, denote $\Psi_1 \equiv \frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_1^{-\theta}$ and $\Psi_2 \equiv \frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} z_2^{-\theta}$ and $d\Psi_1 = -\frac{\theta \lambda^i_{s,t} z_1^{-\theta-1}}{\pi^i_{sd,t-1}} dz_1$, $d\Psi_2 = -\frac{\theta \lambda^i_{s,t} z_2^{-\theta-1}}{\pi^i_{sd,t-1}} dz_2$, which allows U.S. to rewrite the equation above as:

$$
\begin{aligned}
& \int_{\Omega^i_{sd,t}} \frac{\tilde{x}^i_{d,t}}{z_1(\omega)} \left(\frac{p^i_{d,t}(\omega)}{p^i_{d,t}} \right)^{-\sigma} \frac{e^i_{d,t}}{p^i_{d,t}} d\omega \\
= \quad & \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \int_0^\infty \int_0^{\Psi_2} \Psi_1^{\frac{1}{\theta}} \min \left\{ \left(\frac{\sigma}{\sigma-1} \right)^{\theta} \Psi_1, \Psi_2 \right\}^{-\frac{\sigma}{\theta}} d\Psi_1 d\Psi_2 \\
= \quad & \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \Bigg[\int_0^\infty \int_{\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \Psi_2}^{\Psi_2} \Psi_1^{\frac{1}{\theta}} \Psi_2^{\frac{-\sigma}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \\
+ \quad & \int_0^\infty \int_0^{\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \Psi_2} \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} \Psi_1^{\frac{1-\sigma}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \Bigg] \\
= \quad & \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \Bigg[\int_0^\infty \frac{\theta}{1+\theta} \left[1 - \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Psi_2^{\frac{1-\sigma+\theta}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_2 \\
+ \quad & \int_0^\infty \frac{\theta}{1-\sigma+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \Psi_1^{\frac{1-\sigma+\theta}{\theta}} \exp\left\{ -\Psi_2 \right\} d\Psi_2 \Bigg] \\
= \quad & \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \left[\frac{\theta}{1+\theta} \left[1 - \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] + \frac{\theta}{1-\sigma+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma\left(\frac{1-\sigma}{\theta} + 2 \right) \\
= \quad & \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \Bigg[\frac{\theta}{1+\theta} \left[1 - \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \\
+ \quad & \frac{\theta}{1-\sigma+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \Bigg] \frac{1-\sigma+\theta}{\theta} \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \\
= \quad & \left[1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{\lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}^i_{sd,t})^{1-\sigma} \\
= \quad & \left[1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\frac{(\tilde{x}^i_{sd,t})^{-\theta} \lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{\frac{\sigma}{\theta}} \left(\frac{(\tilde{x}^i_{sd,t})^{-\theta} \lambda^i_{s,t}}{\pi^i_{sd,t}} \right)^{-\frac{1}{\theta}} \\
= \quad & \left[1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \pi^i_{sd,t} \frac{e^i_{d,t}}{(p^i_{d,t})^{1-\sigma}} \left(\sum_{n \in \mathcal{D}} (\tilde{x}^i_{nd,t})^{-\theta} \lambda^i_{n,t} \right)^{-\frac{1-\sigma}{\theta}}
\end{aligned}
$$

Using the expression for $(p^i_{d,t})^{1-\sigma}$:

$$
\begin{aligned}
= \quad & \frac{\left[1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \pi^i_{sd,t} e^i_{d,t} \left(\sum_{n \in \mathcal{D}} (\tilde{x}^i_{nd,t})^{-\theta} \lambda^i_{n,t} \right)^{-\frac{1-\sigma}{\theta}}}{\left[1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \Gamma\left(\frac{1-\sigma+\theta}{\theta} \right) \left(\sum_{n \in \mathcal{D}} \lambda^i_{n,t} (\tilde{x}^i_{nd,t})^{-\theta} \right)^{-\frac{1-\sigma}{\theta}}} \\
= \quad & \frac{\left[1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right]}{\left[1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left(\frac{\sigma}{\sigma-1} \right)^{-\theta} \right]} \pi^i_{sd,t} e^i_{d,t} \\
= \quad & \frac{\theta}{1+\theta} \frac{1+\theta-\sigma+\sigma^{-\theta}(\sigma-1)^{1+\theta}}{1+\theta-\sigma+\sigma^{-\theta}(\sigma-1)^{1+\theta}} \pi^i_{sd,t} e^i_{d,t}
\end{aligned}
$$

Therefore, total cost equals:

$$C_{s,t}^{i}=\sum_{d\in\mathcal{D}}\int_{\Omega_{sd,t}^{i}}\frac{\tilde{x}_{d,t}^{i}}{z_{1}(\omega)}\left(\frac{p_{d,t}^{i}(\omega)}{p_{d,t}^{i}}\right)^{-\sigma}\frac{e_{d,t}^{i}}{p_{d,t}^{i}}d\omega=\frac{\theta}{1+\theta}\sum_{d\in\mathcal{D}}\pi_{sd,t}^{i}e_{d,t}^{i} \tag{2.50}$$

Profits can be expressed compactly as total revenue minus total cost:

$$\Pi_{s,t}^{i}=\sum_{d\in\mathcal{D}}\pi_{sd,t}^{i}e_{d,t}^{i}-\frac{\theta}{1+\theta}\sum_{d\in\mathcal{D}}\pi_{sd,t}^{i}e_{d,t}^{i}=\frac{1}{1+\theta}\sum_{d\in\mathcal{D}}\pi_{sd,t}^{i}e_{d,t}^{i} \tag{2.51}$$

Analogously, total costs and profits of intermediary producers are, respectively:

$$c_{s,t}^{i}=\frac{\theta}{1+\theta}\sum_{d\in\mathcal{D}}\pi_{sd,t}^{i,j}e_{d,t}^{i,j},\qquad \Pi_{s,t}^{i}=\frac{1}{1+\theta}\sum_{d\in\mathcal{D}}\pi_{sd,t}^{i,j}e_{d,t}^{i,j} \tag{2.52}$$

These are allow analogous to the expression in the paper after allowing the elasticities to be sector-specific.

Works cited in Chapter 2

Abrams, **Elliot** (Mar. 2022). The New Cold War. en. URL: https://www.cfr.org/blog/new-cold-war-0 (visited on 03/15/2022).

Aiyar, **Shekhar**, **Chen**, **Jianqian**, **Ebeke**, **Christian**, **Garcia-Saltos**, **Roberto**, **Gudmundsson**, **Tryggvi**, **Ilyina**, **Anna**, **Kangur**, **Alvar**, **Kunaratskul**, **Tansaya**, **Rodriguez**, **Sergio**, **Ruta**, **Michele**, **Schulze**, **Tatjana**, **Soderberg**, **Gabriel**, and **Trevino**, **Juan** (Jan. 2023). Geoeconomic Fragmentation and the Future of Multilateralism. ENG. URL: https://www.imf.org/en/Publications/Staff-Discussion-Notes/Issues/2023/01/11/Geo-Economic-Fragmentation-and-the-Future-of-Multilateralism-527266 (visited on 03/29/2024).

Alvarez, **Fernando E.**, **Buera**, **Francisco J.**, **Lucas**, **Robert E.**, and **Jr** (Nov. 2013). *Idea Flows, Economic Growth, and Trade*. en. Tech. rep. w19667. National Bureau of Economic Research. DOI: 10.3386/w19667. URL: https://www.nber.org/papers/w19667 (visited on 04/08/2021).

Antràs, **Pol** and **Chor**, **Davin** (2018). On the measurement of upstreamness and downstreamness in global value. In: *World Trade Evolution*. URL: https://www.taylorfrancis.com/chapters/oa-edit/10.4324/9781351061544-5/measurement-upstreamness-downstreamness-global-value-chains-pol-antr%C3%A0s-davin-chor?context=ubx&refId=af0896c8-3257-437e-9075-a074a6e36514 (visited on 03/28/2024).

Atalay, **Enghin** (2017). How Important Are Sectoral Shocks? *American Economic Journal: Macroeconomics*. 2017. 9, pp. 254–280.

Attinasi, **Maria Grazia**, **Boeckelmann**, **Lukas**, and **Meunier**, **Baptiste** (2023). The Economic Costs of Supply Chain Decoupling. *SSRN Electronic Journal*. 2023.

Autor, **David H.**, **Dorn**, **David**, and **Hanson**, **Gordon H.** (Oct. 2013). The China Syndrome: Local Labor Market Effects of Import Competition in the United States. *American Economic Review*. 2013. 103, pp. 2121–2168.

Baqaee, **David Rezza** and **Farhi**, **Emmanuel** (2024). Networks, Barriers, and Trade. *Econometrica*. 2024. 92, pp. 505–541.

Bernard, **Andrew B**, **Eaton**, **Jonathan**, **Jensen**, **J. Bradford**, and **Kortum**, **Samuel** (Aug. 2003). Plants and Productivity in International Trade. *American Economic Review*. 2003. 93, pp. 1268–1290.

Bown, **Chad P.** and **Irwin**, **Douglas A.** (2019). Trump's Assault on the Global Trading System: And Why Decoupling from China Will Change Everything Essays. *Foreign Affairs*. 2019. 98, pp. 125–137.

Buera, **Francisco J.** and **Lucas**, **Robert E.** (2018). Idea Flows and Economic Growth. *Annual Review of Economics*. 2018. 10, pp. 315–345.

Buera, **Francisco J.** and **Oberfield**, **Ezra** (2020). The Global Diffusion of Ideas. *Econometrica*. 2020. 88, pp. 83–114.

Cai, **Jie**, **Li**, **Nan**, and **Santacreu**, **Ana Maria** (Jan. 2022). Knowledge Diffusion, Trade, and Innovation across Countries and Sectors. *American Economic Journal: Macroeconomics*. 2022. 14, pp. 104–145.

Caliendo, **Lorenzo**, **Dvorkin**, **Maximiliano**, and **Parro**, **Fernando** (2019). Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock. *Econometrica*. 2019. 87, pp. 741–835.

Caliendo, **Lorenzo** and **Parro**, **Fernando** (Jan. 2015). Estimates of the Trade and Welfare Effects of NAFTA. *The Review of Economic Studies*. 2015. 82, pp. 1–44.

Colantone, **Italo** and **Stanig**, **Piero** (2018). The Trade Origins of Economic Nationalism: Import Competition and Voting Behavior in Western Europe. *American Journal of Political Science*. 2018. 62, pp. 936–953.

Dekle, **Robert**, **Eaton**, **Jonathan**, and **Kortum**, **Samuel** (May 2007). Unbalanced Trade. *American Economic Review*. 2007. 97, pp. 351–355.

Deng, **Liuchun** and **Zhang**, **Chi** (2023). Putting Industries into the Idea-flows Model: A Quantitative Analysis. (Visited on 11/28/2023).

Dix-Carneiro, **Rafael** and **Kovak**, **Brian K.** (2017). Trade Liberalization and Regional Dynamics. *The American Economic Review*. 2017. 107, pp. 2908–2946.

Eaton, **Jonathan** and **Kortum**, **Samuel** (1999). International Technology Diffusion: Theory and Measurement. *International Economic Review*. 1999. 40, pp. 537–570.

– (2002). Technology, Geography, and Trade. *Econometrica*. 2002. 70, pp. 1741–1779.

Eppinger, **Peter**, **Felbermayr**, **Gabriel**, **Krebs**, **Oliver**, and **Kukharskyy**, **Bohdan** (May 2021). Decoupling Global Value Chains — Publications — CESifo. en. URL: https://www.cesifo.org/en/publications/2021/working-paper/decoupling-global-value-chains (visited on 03/28/2024).

Felbermayr, **Gabriel**, **Mahlkow**, **Hendrik**, and **Sandkamp**, **Alexander** (Feb. 2023). Cutting through the value chain: the long-run effects of decoupling the East from the West. *Empirica*. 2023. 50, pp. 75–108.

Grossman, **Gene M.** and **Helpman**, **Elhanan** (1989). Product Development and International Trade. *Journal of Political Economy*. 1989. 97, pp. 1261–1283.

Häge, **Frank M.** (2011). Choice or Circumstance? Adjusting Measures of Foreign Policy Similarity for Chance Agreement. *Political Analysis*. 2011. 19, pp. 287–305.

Hertel, **Thomas**, **Hummels**, **David**, **Ivanic**, **Maros**, and **Keeney**, **Roman** (July 2007). How confident can we be of CGE-based assessments of Free Trade Agreements? *Economic Modelling*. 2007. 24, pp. 611–635.

Jovanovic, **Boyan** and **Rob**, **Rafael** (1989). The Growth and Diffusion of Knowledge. *The Review of Economic Studies*. 1989. 56, pp. 569–582.

Kaska, **Kadri**, **Beckvard**, **Henrik**, and **Minárik**, **Tomáš** (2019). *Huawei, 5G and China as a Security Threat*. en. Tech. rep. Tallinn: NATO Cooperative Cyber Defence Centre of Excellence (CCDCOE). URL: https://www.ccdcoe.org/uploads/2019/03/CCDCOE-Huawei-2019-03-28-FINAL.pdf.

Keohane, **Robert** and **Nye Jr**, **Joseph** (Feb. 2011). *Power & Interdependence*. English. 4th edition. Boston: Pearson. ISBN: 978-0-205-08291-9.

Kortum, **Samuel S.** (1997). Research, Patenting, and Technological Change. *Econometrica*. 1997. 65, pp. 1389–1419.

Levchenko, **Andrei A.** and **Zhang**, **Jing** (Apr. 2016). The evolution of comparative advantage: Measurement and welfare implications. *Journal of Monetary Economics*. 2016. 78, pp. 96–111.

Lukin, **Artyom** (June 2021). The Russia–China entente and its future. *International Politics*. 2021. 58, pp. 363–380.

Mearsheimer, **John J.** (2014). Why the Ukraine Crisis Is the West's Fault: The Liberal Delusions That Provoked Putin. *Foreign Affairs*. 2014. 93, pp. 77–89.

Navarro, Peter (2018). White House National Trade Council Director Peter Navarro on Chinese Economic Aggression June 28th, 2018. 2018, p. 5.

Nicita, **Alessandro**, **Olarreaga**, **Marcelo**, and **Silva**, **Peri** (June 2018). Cooperation in WTO's Tariff Waters? *Journal of Political Economy*. 2018. 126, pp. 1302–1338.

Nye Jr., **Joseph** (Jan. 2020). Power and Interdependence with China. *The Washington Quarterly*. 2020. 43, pp. 7–21.

Ossa, **Ralph** (Nov. 2015). Why trade matters after all. *Journal of International Economics*. 2015. 97, pp. 266–277.

Rivera-Batiz, **Luis A.** and **Romer**, **Paul M.** (May 1991). International trade with endogenous technological change. *European Economic Review*. 1991. 35, pp. 971–1001.

Romer, **Paul M.** (1990). Endogenous Technological Change. *The Journal of Political Economy*. 1990. 98, S71–S102.

Scissors, **Derek** (2020). *Partial decoupling from China: A brief guide*. JSTOR.

Tang, **Li**, **Cao**, **Cong**, **Wang**, **Zheng**, and **Zhou**, **Zhuo** (Oct. 2021). Decoupling in science and education: A collateral damage beyond deteriorating US–China relations. *Science and Public Policy*. 2021. 48, pp. 630–634.

Walter, **Stefanie** (May 2021). The Backlash Against Globalization. *Annual Review of Political Science*. 2021. 24, pp. 421–442.

Waltz, **Kenneth N.** (Feb. 2010). *Theory of International Politics*. English. 1st edition. Long Grove, Ill: Waveland Press. ISBN: 978-1-57766-670-7.

Weber, **Gesine** and **Scheffer**, **Alexandra** (Mar. 2022). Russia's War on Ukraine: the EU's Geopolitical Awakening. en. URL: https://www.gmfus.org/news/russias-war-ukraine-eus-geopolitical-awakening (visited on 03/17/2022).

Wei, **Li** (Dec. 2019). Towards Economic Decoupling? Mapping Chinese Discourse on the China–US Trade War. *The Chinese Journal of International Politics*. 2019. 12, pp. 519–556.

Wyne, **Ali** (Jan. 2020). How to Think about Potentially Decoupling from China. *The Washington Quarterly*. 2020. 43, pp. 41–64.

Zhao, **Minghao** (Sept. 2019). Is a New Cold War Inevitable? Chinese Perspectives on US–China Strategic Competition. *The Chinese Journal of International Politics*. 2019. 12, pp. 371–394.

Chapter 3

Dynamic Adjustment to Trade Shocks (joint with Junyuan Chen, Marc Muendler, and Fabian Trottner)

3.1 Introduction

Innovations and disruptions to global supply chains lead to gradual adjustments in international trade flows. It has long been recognized that the trade elasticity, a key parameter that captures the substitution between imported goods from different countries in response to trade costs, varies by time horizon (e.g. Dekle, Eaton, and S. Kortum, 2008). Boehm, Levchenko, and Pandalai-Nayar (2023) use plausibly exogenous tariff changes to measure the trade elasticity by time horizon and find that the short-run trade elasticity is about half the size of the long-run elasticity. This differential implies substantial frictions in trade adjustment that a static trade model cannot account for. A dynamic framework is needed to provide a rigorous and plausible quantification of the transitory and lasting impacts of shocks to global supply chains.

This paper proposes a dynamic general-equilibrium model of trade with many countries and many industries, where staggered sourcing decisions give rise to horizon-specific trade elasticities. Under the Ricardian trade tenet, products are sourced from the least expensive global supplier. However, the opportunity to switch to a new supplier only arrives randomly following a Poisson process. As a consequence, only some buyers respond to a trade disruption by adjusting to optimal sourcing relations. Other buyers endure a suboptimal sourcing choice until they can adjust. In this framework, disruptions put the world economy through a sustained period of adjustment.

The model preserves the analytical tractability of a class of quantitative Ricardian models based on Eaton and S. Kortum (2002, henceforth EK). We characterize impulse responses in the model using the dynamic hat algebra method. We establish a closed-form expression for the horizon-specific trade elasticity, showing that our model rationalizes empirical estimates of the trade elasticity at different time horizons as a convex combination of short- and long-run elasticity parameters, linked by transitory weights that shift at a constant rate of decay. Furthermore, we derive a novel characterization of the horizon-specific gains from trade that sheds light on the importance of sourcing frictions. Our model shows how the original static welfare formula based on Arkolakis, Costinot, and Rodríguez-Clare (2012) can be augmented to account for dynamic adjustment so it delivers welfare predictions at any time horizon under a time-varying trade elasticity.

Specifically, we assume that intermediate goods are produced using constant returns-to-scale technologies and producers differ by productivity drawn from a country-sector specific Fréchet distribution. Trade is subject to iceberg trade costs. An assembler of an industry's final good at a destination d seeks to buy from the least expensive global supplier, but may not be able to instantaneously switch from one supplier to another. The assembler's sourcing decision is governed by a binary random process: an assembler is either in a position to choose the least expensive global supplier of an intermediate good from any source-industry, or the assembler has to continue purchasing from the same producer as in the preceding period. We can therefore characterize equilibrium as a set of measurable partitions of the space of intermediate goods for each supplier, and then derive the equilibrium distributions. An intermediate good's price at a moment in time equals the initial destination price adjusted for the cumulative changes in marginal costs since the supplier was last selected. We show that a destination country's expenditure shares by source country across intermediate goods take an analytic form as in EK and similar Ricardian frameworks that are consistent with the gravity equation of trade.

The expenditure shares in the augmented gravity equation encode the price that a buyer paid at the time of the last supplier change. Through this unmoved component, while a buyer-supplier relationship lasts, cross price effects of substitution are governed by the short-run trade elasticity, similar to an Armington (1969) model. When all supplier-buyer relationships are reset optimally, the gravity expression simplifies to the common gravity equation in an EK framework, so that the long-run trade elasticity prevails. With the equilibrium relationships at hand, we compute impulse responses recursively, and we analytically derive the trade elasticity ε_i^h for each time horizon h after a shock to the global supply network at time $t = 0$:

$$\varepsilon_i^h \equiv \frac{\partial \log \lambda_{sdi,h}}{\partial \log \tau_{sdi,0}} = -\theta_i \left[1 - (1 - \zeta_i)^{h+1}\right] - (\sigma_i - 1)(1 - \zeta_i)^{h+1},$$

where $\lambda_{sdi,h}$ is destination country d's expenditure share falling on intermediate goods from source country s in industry i in the hth period after the shock, $\tau_{sdi,0}$ is the trade cost component that is shocked at time $t = 0$, θ_i is the long-term trade elasticity as in EK, $\sigma_i - 1$ is the short-term trade elasticity as in Armington, and $\zeta_i \in (0,1)$ is a parameter that describes the frequency at which buyers of intermediate goods from industry i can switch suppliers. The prevailing trade elasticity ε_i^h increases over time in absolute value from the short-run to the long-run level (for the common parametrization $\theta_i > \sigma_i - 1$).

In the long-run, the trade elasticity converges to the familiar Fréchet parameter θ_i as in EK. The rate of convergence depends on the frequency at which buyers can establish a new sourcing relationship ζ_i. The key parameters of our model are therefore identifiable from reduced-form estimates of the trade elasticity at varying time horizons as in Boehm, Levchenko, and Pandalai-Nayar (2023). This characterization of the horizon-specific trade elasticity also implies a horizon-specific welfare formula, which we derive in closed form. The horizon-specific welfare formula features a dynamic adjustment component, which fades over time, and nests the well-known formula from Arkolakis, Costinot, and Rodríguez-Clare (2012) as the limiting case in the long run.

We show how the above results can be used to derive a set of estimation equations for the relevant parameters governing short and long-run trade elasticities, document how existing results from Boehm, Levchenko, and Pandalai-Nayar (2023) can be employed, and quantify our trade model. With the tractability of our model and data on input-output relations, we consider a model world economy consisting of 32 industries across 77 regions. We apply the model to the episode of the US-China trade war started in 2018 and show that rich industry-level dynamics can result, with consequential changes in welfare implications. First, despite the low trade elasticity in the short-run, the United States main suffer a smaller welfare loss over the short run relative to the long-run outcome, when the sourcing frictions are no longer relevant. China, on the other hand, may suffer a short-run welfare loss that exceeds the long-run loss. A lower short-run trade elasticity therefore does not necessarily imply a larger short-run welfare impact in this world economy. Second, a direct application of the static welfare formula from Arkolakis, Costinot, and Rodríguez-Clare (2012), using realized domestic trade shares, can result in qualitatively misleading predictions over finite time horizons. The reason is that sourcing frictions, and the resulting time-varying trade elasticities, can induce substantive and shifting deviations from the long-term welfare outcome. Third, gains from trade can differ between the short and the long run in both sign and magnitude. In the short-run, price disruptions caused by the US-China trade war propagate through the network of existing supply relationships, leading to a global reduction in economic welfare. Those short-run losses, in part, reflect the limited scope for third-party countries to gain from the trade dispute by forming new supply relationships with the United States or China. Gains for third countries may materialize in the medium to long term, however. As a consequence, countries whose previous trade linkages leave them most exposed to the US-China trade war, such as Mexico and Vietnam, experience large initial welfare losses in the short-run, but marked and potentially sizeable increases in welfare in the long-run.

The wide discrepancy between a low (short-run) trade elasticity in international macroeconomics and a high (long-run) trade elasticity in international trade has been documented in, for example, Ruhl (n.d., who calls the discrepancy an "international elasticity puzzle") and Fontagné, Martin, and Orefice (2018). Fontagné, Guimbard, and Orefice (2022), Boehm, Levchenko, and Pandalai-Nayar (2023) and Anderson and Yotov (2022) offer estimation procedures to separately identify short- and long-run trade elasticities. Souza et al. (2024) obtain horizon-specific trade elasticity estimates in a difference-in-differences design for anti-dumping tariff changes. Anderson and Yotov (2022) rationalize their estimation procedure with firm heterogeneity in lag times from recognition to action in the spirit of Lucas and Prescott (1971). In an alternative approach from a macroeconomic perspective, Yilmazkuday (2019) proposes a framework with nested CES models and derives the trade elasticity as the weighted average of macro elasticities. Our general equilibrium model offers a rationalization for the existing estimation methods with a mixture of the Armington and EK elasticities.

The importance of staggered contracts for trade and exchange rate dynamics has been recognized since at least Kollintzas and Zhou (1992) and shares features with staggered pricing Calvo (1983). We generalize deterministic contract ages to supplier relationships that end stochastically. In a related approach, Arkolakis, Eaton, and S. S. Kortum (2011) embed a consumer with no knowledge of the identity of source countries into an EK model. The consumer can switch to the lowest-cost supplier at random intervals but cannot act strategically because the supplier is unknown. We rationalize consumer behavior by introducing assemblers that operate similar to a wholesale or retail firm in that they source bundles of goods at lowest cost while the consumer cannot unbundle the assembled final good. An assembler, in turn, cannot incur losses in imperfect capital markets and thus sources from the current lowest-cost supplier. Our model allows us to derive a stationary equilibrium distribution of supplier prices by age of contract beyond a binary characterization in Arkolakis, Eaton, and S. S. Kortum (2011).[1] Based on the mixture of the stationary equilibrium distributions of prices by contract age, we can fully characterize steady states as well as transition dynamics. As a result, we obtain the original EK model as the limit of the equilibria along the transition path. Our welfare formula therefore endogenously inherits the long-run elasticity as a special case when all supplier contracts are optimally set.

The remainder of the paper is organized as follows. We present the model in Section 3.2, with details on mathematical derivations relegated to the Appendix. In Section 3.3 we turn to the dynamic analysis of the model. Estimation of the key parameters follows in Section 3.4. To illuminate the novel dynamic features of the model for the allocation of economic activities during the adjustment path and the welfare consequences, we present a case study of the US-China trade war in Section 3.5. Section 3.6 concludes.

[1]The underlying stochastic process shares features with the so-called Sisyphos Process Montero and Villarroel (2016).

3.2 Model

3.2.1 Fundamentals

Consider a world economy with N destination countries $d \in \mathcal{D} := \{1, 2, \cdots, N\}$, $s \in \mathcal{D}$ source countries of trade flows, and I industries $i, j \in \mathcal{I} := \{0, 1, 2, \cdots, I\}$. Time t is discrete. Subscripts sdi, t denote a trade flow from source region s to destination d in industry i at time t. Households inelastically supply a single production factor (labor) to domestic firms, and markets are perfectly competitive.

Households.

In each period t, a mass of L_d infinitely-lived households in country d inelastically supplies one unit of the production factor to domestic firms at a competitive wage $w_{d,t}$. Household utility in country d at time t is given by $u(C_{d,t})$, where $C_{d,t}$ is the final good: a Cobb-Douglas aggregate over the composite goods $C_{di,t}$ from each industry with

$$C_{d,t} = \prod_{i \in \mathcal{I}} \left(C_{di,t} \right)^{\eta_{di}} . \tag{3.1}$$

The coefficient η_{di} is the consumption expenditure share of industry $i's$ composite good, with $\sum_{i \in \mathcal{I}} \eta_{di} = 1$. Let $P_{di,t}$ denote the price index of the industry i good in d at time t. Country $d's$ consumer price index is then given by $P_{d,t} = \prod_{i \in \mathcal{I}} \left(P_{di,t} / \eta_{d,i} \right)^{\eta_{di}}$. We assume that households consume their income in every period and discount future utility flows at rate $\beta \in (0, 1)$.

Intermediate Goods.

Every industry i consists of a continuum of producers of intermediate goods $\omega \in [0, 1]$. For each intermediate good, there is a large set of potential producers in each country with different technologies to produce the good. In each industry, producers of an intermediate good ω have an individual productivity z and operate a constant-returns-to-scale technology to produce the good using domestic labor ℓ and composite goods M_{ji} sourced from other industries:

$$y_i(\omega) = z\,(\ell)^{\alpha_{di}} \prod_{j \in \mathcal{I}} (M_{ji})^{\alpha_{dji}}. \tag{3.2}$$

where $y_i(\omega)$ is the output of good ω. The coefficient α_{di} is the value-added share of industry i and the parameters $\alpha_{dij} \geq 0$ are such that $\alpha_{di} = 1 - \sum_{j \in \mathcal{J}} \alpha_{dji}$.

We assume that intermediate goods can be traded across countries subject to an iceberg transportation cost, which implies that shipping one unit of a good in industry i from country s to country d at time t requires producing $d_{sdi,t} \geq 1$ units in s, where $d_{ddi,t} = 1$ for all d. Moreover, goods imported by d from s at t may be subject to an ad-valorem tariff $\bar{\tau}_{sdi,t}$. We combine both trade costs into one parameter $\tau_{sdi,t} \equiv d_{sdi,t} \bar{\tau}_{sdi,t}$.

Given this formulation of trade costs and technologies, there is a *common unit cost component* at destination d for all intermediate goods produced in country s, which we denote with

$$c_{sdi,t} \equiv \Theta_{sj} \tau_{sdi,t}\,(w_{s,t})^{\alpha_{si}} \prod_{j \in \mathcal{J}} (P_{sj,t})^{\alpha_{sji}}, \tag{3.3}$$

where Θ_{sj} is a collection of Cobb-Douglas coefficients. The resulting unit cost of good ω at destination d produced in country s with a productivity $z(\omega)$ is given by $c_{sdi,t}/z(\omega)$.

Production technologies for intermediate goods arrive stochastically and independently at a rate that varies by country and industry. In particular, we follow Eaton and S. Kortum (2012) in assuming that the mass of intermediate goods ω in country s's industry i that can be produced with a productivity higher than z to be distributed Poisson with mean $A_{si} z^{-\theta_i}$.

Assembly of Composite Goods.

In each industry, assemblers bundle intermediate goods into a composite good for consumption or production. An assembler procures intermediate goods at the lowest possible price and costlessly aggregates the sourced intermediates into $Y_{di,t}$ units of industry i's composite good using the technology

$$Y_{di,t} = \left(\int_{[0,1]} y_{di,t}(\omega)^{(\sigma_i - 1)/\sigma_i} \mathrm{d}\omega \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{3.4}$$

where $y_{di,t}(\omega)$ is the quantity purchased of an intermediate good ω by an assembler in country d, and σ_i is the elasticity of substitution between intermediate goods in industry i. We let $p_{di,t}(\omega)$ denote the lowest possible price at which an intermediate good ω can be purchased at destination d. We will explain the exact price at which this intermediate good is available in greater detail below. As we elaborate in Appendix 3.8.1, cost minimization given (3.4) implies that the price of industry i's composite good at destination d satisfies

$$P_{di,t} = \left(\int_{[0,1]} p_{di,t}(d\omega)^{-(\sigma_i - 1)} \mathrm{d}\omega \right)^{-\frac{1}{\sigma_i - 1}}. \tag{3.5}$$

3.2.2 Sourcing Decisions and Trade Flows

Under the Ricardian trade tenet, assemblers seek to source an intermediate good from the least expensive global supplier. However, an assembler may not have the opportunity to adjust its choice of suppliers at any given time due to a sourcing friction, which we describe now. For every intermediate good ω, there is a continuum of producers in every country. Under perfect competition, an assembler optimally sources any given intermediate good ω from only one source country when given the choice.

The assemblers' choice of source country for any given intermediate good ω is governed by an i.i.d. random variable $x_{i,t}(\omega) \in \{0,1\}$ for each industry. If $x_{i,t}(\omega) = 1$, that is if the global draw for an intermediate good ω from industry i gives all assemblers worldwide the green light to switch to their preferred source country, then all assemblers optimally choose to purchase from the least costly source country for variety ω in industry i at time t. Between assemblers in different countries the optimal source country can vary because of different trade costs. Else, if $x_{i,t}(\omega) = 0$, that is if the global draw for intermediate ω turns to red for all assemblers worldwide, then all assemblers must purchase their intermediate goods ω in industry i from the same producer as in the preceding period $t-1$. While the identity of the source country does not change, the quantity procured and the price that the assembler pays can differ from the preceding period if the factory gate price moves (because of changing factor costs) or the currently prevailing trade cost moves.

This formulation of sourcing frictions captures search costs and other types of impediments that prevent the optimal rematch of supply relationships at a moment in time. An implication of the sourcing friction is that price elasticities of demand will differ across intermediate goods according to when their suppliers were last chosen. Let $\Omega^k_{j,t}$ denote the set of industry j goods whose supplier at time t was last chosen k periods ago:

$$\Omega^k_{i,t} = \left\{ \omega : x_{di,t-k}(\omega) = 1, \prod_{\varsigma=t-k+1}^{t} x_{di,\varsigma}(\omega) = 0 \right\}, \tag{3.6}$$

where $\cup_k \Omega^k_{j,t} = [0,1]$. The sets $\Omega^k_{i,t}$ mutually exclusively and exhaustively partition the unit interval of intermediate goods for each industry i.

Demand for Intermediate Goods with Newly Formed Supply Relationships

We now describe the global demand for intermediate goods in each of these sets, beginning with those that are concurrently formed, $\omega \in \Omega^0_{dj,t}$.

If country s is chosen by an assembler in destination d to supply industry i's intermediate good ω at time t, the combination of the producer's productivity ω, factor cost in source country s and the trade cost between s and d in industry i must make the intermediate good the least expensive.

Let $z_{si}(\omega)$ denote the highest realized productivity by any producer in country-industry si. Similar to Eaton and S. Kortum (2002), our distributional assumptions imply that z_{si} has a country-industry specific Fréchet distribution given by[2]

$$\Pr\left[z_{si}(\omega) \leq z | A_{si}, \theta_i\right] = \exp\left\{-A_{si} z^{-\theta_i}\right\}. \tag{3.7}$$

For an assembler in destination d the price of an intermediate good ω from the cheapest available source country at time t is

$$p_{di,t}(\omega) = \min_{s \in \mathcal{D}} \left\{ \frac{c_{sdi,t}}{z_{si}(\omega)} \right\} \tag{3.8}$$

for the common unit cost component $c_{sdi,t}$ given by (3.3) and the producer with the highest realized productivity $z_{si}(\omega)$ in country-industry si.

As in Eaton and S. Kortum (2002), the distribution of paid prices across intermediate goods in the set $\Omega^0_{i,t}$ in destination d at time t satisfies

$$G^0_{di,t}\left[p_{di,t}(\omega) \leq p\right] \equiv \Pr\left[p_{di,t}(\omega) \leq p \middle| x_{i,t}(\omega) = 1\right] = 1 - \exp\left\{-\Phi^0_{di,t} p^{-\theta_i}\right\}, \tag{3.9}$$

where

$$\Phi^0_{di,t} \equiv \sum_{n \in \mathcal{N}} A_{ni} [c_{ndi,t}]^{-\theta_i} \tag{3.10}$$

[2] Our model could also accommodate productivity change over time with a country-industry-time specific Fréchet distribution and resulting $z_{si,t}(\omega)$ realizations that vary over time. To focus most sharply on adjustment to trade shocks, we do not specify productivity shocks.

is a measure of destination d's market access for intermediate goods $\omega \in \Omega^0_{i,t}$, given trade cost and factor prices behind the common unit cost component $c_{ndi,t}$ by (3.3). We relegate the derivation of these results to Appendix 3.8.2. To guarantee that the distribution of paid prices has a finite mean later, we impose the standard parametric restriction that $\theta_i > \sigma_i - 1$ for all $i \in \mathcal{I}$.

The properties of the Fréchet distribution imply that $G^0_{di,t}$ also equals the distribution of prices for intermediate goods $\omega \in \Omega^0_{i,t}$ sourced from any source country s. As a result, country d's expenditure share for each potential source country s across intermediate goods $\omega \in \Omega^0_{i,t}$ must equal the probability that this source country offers the lowest global price:

$$\lambda^0_{sdi,t} = \frac{A_{sj}[c_{sdi,t}]^{-\theta^i}}{\Phi^0_{di,t}}. \tag{3.11}$$

with the common unit cost component $c_{sdi,t}$ given by (3.3).

Within the set of intermediate goods that are sourced through concurrently and optimally formed supply relationships, the partial equilibrium elasticity of trade flows with respect to trade cost is governed by the familiar Fréchet parameter:

$$\left.\frac{\partial \log \lambda^0_{sdi,t}}{\partial \log \tau_{sdi,t}}\right|_{\Phi^0_{di,t}} = -\theta_j.$$

Demand for Intermediate Goods with Continuing Supply Relationships

Intermediate goods $\omega \in \Omega^k_{j,t}$ are purchased from a supplier that was chosen at time $t-k$. To characterize prices and expenditure allocations across these intermediate goods at time t, we denote changes over time for a variable x_t succinctly by $\hat{x}_t \equiv x_t / x_{t-1}$.

Suppose an assembler in d first sourced an intermediate good ω from s at time $t-k$ under the unit input cost $c_{sdi,t-k}/z_{si}(\omega)$, which depends on equilibrium factor prices and parameters by the common unit cost component (3.3). If the intermediate good is still sourced from the same producer at time t, its price will then equal:[3]

$$p^{k}_{sdj,t}(\omega) = \frac{c_{sdi,t}}{z_{si}(\omega)} = \frac{c_{sdi,t-k}\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sid,\varsigma}}{z_{si}(\omega)}, \tag{3.12}$$

which is the initial destination price adjusted for the cumulative changes in iceberg trade costs and factor cost since $t-k$.

We show in Appendix 3.8.3 that country d's expenditure share by source country across intermediate goods $\omega \in \Omega^{k}_{i,t}$ equals

$$\lambda^{k}_{sdi,t} = \frac{\lambda^{0}_{sdi,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sid,\varsigma}\right)^{1-\sigma_i}}{\Phi^{k}_{di,t}}, \tag{3.13}$$

where

$$\Phi^{k}_{di,t} \equiv \sum_{n\in\mathcal{N}}\lambda^{0}_{ndi,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{nid,\varsigma}\right)^{1-\sigma_i} \tag{3.14}$$

reflects the mean price that a buyer pays for the set of intermediate goods $\Omega^{k}_{i,t}$ at time $t-k$ through the trade shares $\left\{\lambda^{0}_{nid,t-k}\right\}_{n\in\mathcal{N}}$.

Comparing equation (3.13) and (3.11) shows how cross-price effects differ across intermediate goods depending on when a supply relationship is formed. If assemblers can source from the least expensive global supplier of an intermediate good at time t, cross-price demand effects are governed the Fréchet parameter θ_i, and trade is governed by comparative advantage.

Conversely, if an assembler is unable to switch suppliers, then the extensive margin is shut down. The only margin of adjustment is the intensive margin, which is captured

[3] Note that $x_t = x_{t-k}\frac{x_{t-k+1}}{x_{t-k}}\cdots\frac{x_t}{x_{t-1}} \equiv x_{t-k}\hat{x}_{t-k+1}\cdots\hat{x}_t$. For a composite variable such as $c_{sdi,t} = \tau_{sdi,t}\,w_{s,t}$, the change over time is $\hat{c}_{sdi,t} = \hat{\tau}_{sdi,t}\,\hat{w}_{s,t}$.

by the terms that collect the product of changes in unit input costs. Effectively, over those partitions, trade happens as if varieties were differentiated across countries with the measure of varieties of each source defined at the last period of adjustment —i.e. at period $t-k$ for partition $\Omega^k_{i,t}$.

In order words, for each partition $\Omega^k_{i,t}$, trade happens under Armington forces. Intuitively, the price elasticity of demand is governed by the elasticity of substitution $\sigma_i - 1$, which captures Armington trade:

$$\left.\frac{\partial \log \lambda^k_{sdi,t}}{\partial \log \tau_{sdi,\varsigma}}\right|_{\Phi^k_{di,t}} = -\left(\sigma_i - 1\right) \quad \text{for } t-k < \varsigma < t.$$

To close the model, we now show how aggregate global demand for industry i's composite good follows from aggregating the trade shares in equations (3.11) and (3.13).

3.2.3 Aggregation

To find aggregate demand, we leverage the homotheticity of assembly. The partial price index for the composite of intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago satisfies $(P^k_{di,t})^{1-\sigma_j} = \int_{\omega\in\Omega^k_{i,t}} p(\omega)^{1-\sigma_j}_{di,t} \mathrm{d}\omega$. The sets $\left\{\Omega^k_{i,t}\right\}^{\infty}_{k=0}$ form a partition of industry i's product space, so we can obtain country d's price index for industry i goods at time t by aggregating these partial price indices over all partitions and find $P^{1-\sigma_j}_{di,t} = \sum^{\infty}_{k=0} \left(P^k_{di,t}\right)^{1-\sigma_j}$.

We establish in Appendix 3.8.2 that the partial price index for the set of intermediate goods whose suppliers are being chosen at time t takes the familiar form

$$P^0_{di,t} = \gamma_i \, \mu_{i,t}(0)^{1/(1-\sigma_j)} \left(\Phi^0_{di,t}\right)^{-\frac{1}{\theta_i}}, \tag{3.15}$$

where $\gamma_i \equiv \Gamma\left([\theta_i - \sigma_i + 1]/\theta_i\right)^{1-\sigma_i}$ is a constant, $\Phi^0_{di,t}$ is given by (3.10), and $\mu_{i,t}(0)$ denotes the measure of the set $\Omega^0_{i,t}$. Following the previous discussion, the endogenous market access term $\Phi^0_{di,t}$ represents the mean price of intermediate goods whose suppliers are chosen at time t. The measure $\mu_{i,t}(0)$ accounts for gains from variety. This measure recursively evolves over time according to the stochastic process that governs sourcing decisions, given by

$$\mu_{i,t}(k) = \begin{cases} \zeta_i, & k = 0 \\ (1-\zeta_i)\mu_{i,t-1}(k-1), & k > 0. \end{cases} \tag{3.16}$$

As we show in Appendix 3.8.3, the partial price index across intermediate goods whose suppliers were last chosen at time $t-k$ is given by

$$P^k_{di,t} = P^0_{di,t-k}\left(\frac{\mu_{i,t}(k)}{\mu_{i,t-k}(0)}\Phi^k_{di,t}\right)^{1/(1-\sigma_i)}, k > 1 \tag{3.17}$$

which is the period $t-k$ price index of the basket of intermediate goods Ω^0_{t-k}, adjusted for the subsequent change in variety composition, captured by $\mu_{i,t}(k)/\mu_{i,t-k}(0)$, and prices, captured by $\Phi^k_{di,t}$.

Given eq:price-index-0 and (3.17), we can solve for the composite price index of industry i goods in country d at time t:

$$P_{di,t} = \gamma_i\left(\Phi^0_{di,t}\right)^{-\frac{1}{\theta_i}}\left[\mu_{i,t}(0) + \sum_{k=1}^{\infty}\mu_{i,t}(k)\left(\frac{\Phi^0_{di,t}}{\Phi^0_{di,t-k}}\right)^{\frac{1-\sigma_i}{\theta_i}}\Phi^k_{di,t}\right]^{\frac{1}{1-\sigma_j}} \tag{3.18}$$

The term $\gamma_i\left(\Phi^0_{di,t}\right)^{-1/\theta_i}$ on the right-hand-side of (3.18) captures the prices paid under flexible supplier choice. The term in brackets quantifies the extent to which current aggregate demand is affected by the stickiness of supply relationships. The term $\Phi^k_{di,t}$ captures differences in demand across intermediate goods driven by differences in the age of their supply relationships and reflect their impact on aggregate demand at time t. The terms $(\Phi^0_{di,t}/\Phi^0_{di,t-k})^{(1-\sigma_i)/\theta_i}$ measure the current demand of a buyer whose supplier relationship from k periods ago differs from that of a buyer who just updated its supplier.

Using the above price indices, we can readily derive country d's expenditure share on industry i goods sourced from country s

$$\lambda_{sdi,t} = \sum_{k=0}^{\infty} \lambda^k_{sdi,t} \left(\frac{P^k_{di,t}}{P_{di,t}} \right)^{1-\sigma_i} . \tag{3.19}$$

where $\lambda^k_{sdi,t}$ is given by (3.11) if $k = 0$ and (3.13) if $k > 0$.

The set of trade shares $\{\lambda_{sdi,t}\}_{s,d \in \mathcal{N}, i \in \mathcal{I}}$ fully characterize demand in the world economy at time t. To close the model, we now describe the conditions for market clearing and define a general equilibrium.

3.2.4 Equilibrium

Denote the total revenue of an industry i in a source country s at time t by $X_{si,t}$. To define equilibrium, we express each industry's revenue in terms of trade shares, given by (3.19), and total expenditures on consumption, $E_{d,t}$, and intermediate inputs in the rest of the world:

$$X_{si,t} = \sum_{d \in \mathcal{N}} \lambda_{sdi,t} \left[\eta_{di} E_{d,t} + \sum_{j \in \mathcal{I}} \alpha_{dij} X_{dj,t} \right] . \tag{3.20}$$

A country's national consumption spending is the sum of its factor income and trade deficit, $E_{d,t} = w_{d,t} L_{d,t} + D_{d,t}$, with $\sum_{d \in \mathcal{N}} D_{d,t} = 0$. We follow the conventional approach in the international trade literature and treat aggregate trade deficits as exogenous. To clear the factor market, wages then adjust to ensure that expenditures equal disposable income,

$$w_{d,t}L_{d,t} = \sum_{i\in\mathcal{I}}(1-\alpha_{di})X_{di,t}, \tag{3.21}$$

and goods market clearing is guaranteed by Walras' law.

We are now ready to define a dynamic general equilibrium and a steady state.

Definition 2. *An economy is described by a set of time-invariant parameters summarizing technologies, preferences and factor endowments,* $\mathbf{\Theta} = \left\{\theta_i,\sigma_i,\{\alpha_{dji}\}_{j\in\mathcal{I}},\varphi_{di},A_{di},\eta_{di},L_d\}_{d\in\mathcal{N}}\right\}_{i\in\mathcal{I}}$, *sourcing frictions* $\boldsymbol{\zeta} = \{\zeta_i\}_{i\in\mathcal{I}}$, *as well as a measure* $\boldsymbol{\mu}_{t_0} = \{\mu_{t_0}(k)\}_{k\in\{0,1,\cdots\}}$ *for some* t_0. *Given histories of trade costs* $\boldsymbol{\tau}_{t-1} \equiv \{\tau_t\}_{\varsigma<t} = \left\{\tau_{sid,\varsigma}\right\}_{s,d\in\mathcal{N},i\in\mathcal{I},\varsigma<t}$ *and their changes* $\hat{\tau}_t \equiv \{\hat{\tau}_{sdi,t}\}_{s,d\in\mathcal{N},i\in\mathcal{I}}$ *as well as nominal wages* $\boldsymbol{w}_{t-1} = \{w_\varsigma\}_{\varsigma<t} = \{w_{d,\varsigma}\}_{d\in\mathcal{N},\varsigma<t}$*:*

1. *A static equilibrium at time t is a vector of wages $w(\hat{\tau}_t \times \tau_{t-1} \cup \boldsymbol{\tau}_{t-1}, \boldsymbol{w}_{t-1}, \zeta, \Theta) = w_t$ that jointly solves* (**??**) *for all $s,d \in \mathcal{N}$ and $i \in \mathcal{I}$.*

2. *A dynamic equilibrium at time t is a history of wages $\boldsymbol{w}_t$ so that, for all $w_\varsigma \in \boldsymbol{w}_t$, $w_\varsigma = w(\hat{\tau}_{\varsigma-1} \times \tau_{\varsigma-1} \cup \boldsymbol{\tau}_{\varsigma-1}, w_{\varsigma-1} \cup \boldsymbol{w}_{\varsigma-2}, \zeta, \Theta)$.*

3. *A dynamic equilibrium at time t is a steady state if $w(\mathbf{1}_{N\times N\times I} \times \tau_t \cup \boldsymbol{\tau}_{t-1}, w_t \cup \boldsymbol{w}_{t-1}, \zeta, \Theta) = w_t$.*

3.2.5 Steady-State Properties

In the following, we show that our model preserves the class of quantitative trade models based on Eaton and Kortum (2002) in the limit when the economy is in steady state, irrespective of the magnitude of the frictions underlying imperfect supplier adjustment, $\zeta_i \in (0,1)$. Intuitively, the transitory effects of trade disruptions that arise in our model reflect how opportunities for finding new suppliers are limited in the short-run but increasing over time. As assemblers get to adjust all supply relationships in the long-run, we then obtain the EK-model as the limit of the equilibria along the transition path.

More formally, let $w^{EK}(\hat{\tau}_t \times \tau_{t-1} \cup \boldsymbol{\tau}_{t-1}, \boldsymbol{w}_{t-1}, 1, \Theta)$ represent the equilibrium allocation in an economy in which suppliers can be flexibly adjusted for all goods, $\zeta_i = 1$ for all i. We can then establish

Proposition 7. *If w_{t^*} is a steady state equilibrium, then*

1. *For any ζ, $w_{t^*} = w(\mathbf{1}_{N \times N \times I} \times \tau_{t^*} \cup \boldsymbol{\tau}_{t^*-1}, w_{t^*} \cup \boldsymbol{w}_{t^*-1}, \zeta, \Theta) = w^{EK}(\mathbf{1}_{N \times N \times I} \times \tau_{t^*} \cup \boldsymbol{\tau}_{t^*-1}, w_{t^*} \cup \boldsymbol{w}_{t^*-1}, 1, \Theta)$.*

2. *For all $k \in \{0, 1, ...\}$, the measure of goods $\omega \in \Omega^k_{i,t}$ equals $\mu_{i,t^*}(k) = (1 - \zeta_i)^k \zeta_i$, and trade flows are given by $\lambda^k_{sdi,t^*} = \lambda_{sdi,t} = \lambda^{EK}_{sid}$ where λ^{EK}_{sid} denotes the trade shares in the frictionless economy.*

Theorem (7) provides numerous useful insights. The first part makes clear that the tools developed by the literature studying the equilibrium properties of static quantitative trade models can be deployed to establish the existence and uniqueness of steady states in our model.

The second part of Theorem (7) highlights properties of the steady states that we later leverage to quantify the model. In particular, it shows that the process governing the evolution of the age distribution of supply relationships over time has a simple geometric stationary distribution. Further, it shows that steady state expenditure allocations are equalized across goods within an industry, irrespective of when their supplier was chosen.

3.3 Dynamic Adjustment to Trade Shocks

In this section, we theoretically characterize the economy's dynamic response to trade disruptions. In particular, we derive a new structural estimating equation for the trade elasticity at different time horizons, and show that transitional dynamics can be characterized using the dynamic hat-algebra. Finally, we provide a new formula for characterizing the horizon-specific gains from trade.

3.3.1 Trade Elasticity by Time Horizon

We begin by showing how the trade elasticity, that is the elasticity of trade flows with respect to transport cost, varies over time. To do so, we let $\varepsilon^h_{sdi,t}$ denote the trade elasticity at horizon h, which we define by:

$$\varepsilon^h_{sdi,t-1} \equiv \left. \frac{\partial \log X_{sdi,t+h}}{\partial \log \tau_{sdi,t}} \right|_{\{\Phi^k_{di,t+\varsigma}\}_{t\leq\varsigma\leq h,k}}, \tag{3.22}$$

which is the elasticity of trade flows in industry i from country s to d at time $t+h$, $X_{sdi,t+h}/X_{sdi,t-1}$ with respect to change in trade costs at t, $\mathrm{d}\log\tau_{sdi,t} = \log\hat{\tau}_{sdi,t}$, holding fixed the general equilibrium terms that summarize changes in market access for industry i goods in destination d. The following derives a closed-form expression for this elasticity.

Proposition 8. *Suppose that the economy is in steady state at $t = -1$. Then, up to a first order, the horizon-h response of trade flows to a shock to trade cost at time $t = 0$ is given by:*

$$\varepsilon^h_i = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] - (\sigma_i - 1)(1-\zeta_i)^{h+1}. \tag{3.23}$$

If $\zeta_i \in (0,1)$, $\lim_{h\to\infty} \varepsilon_i^h = -\theta_i$, where the rate of convergence equals

$$\lim_{h\to\infty} \frac{\varepsilon_j^{h+1} + \theta_j}{\varepsilon_i^h + \theta_i} = \log(1-\zeta_i).$$

Following Proposition (8), the trade elasticity increases over time if $\theta_i > \sigma_i - 1$. In the long-run, it is equal to the Fréchet parameter θ_i, where the rate of convergence, intuitively, depends on the frequency at which buyers can establish a new sourcing relationship ζ_i.

It is worth noting that (3.22) is consistent with reduced-form estimates of the trade elasticity at varying time horizons as in Boehm, Levchenko, and Pandalai-Nayar (2023). Later, we leverage this equivalence to identify the key structural parameters in our model. The horizon-specific formulation of the trade elasticity implied by our model also induces a horizon-specific welfare formula, which we provide next.

3.3.2 The Horizon-Specific Welfare Gains from Trade

When supply relationships are slow to adjust to shocks, trade disruptions can put the economy through a sustained period of readjustment. The following proposition shows that our framework yields a simple formula for welfare analysis, giving changes in real wages associated with an initial set of foreign shocks over varying time horizons.

Proposition 9. *Suppose the economy is in steady state at $t = -1$. Then, the change in real wages in country d at time $h = \{0,1,...\}$, $\hat{W}_d^h = C_{d,h}/C_{d,-1}$, that follows a set of arbitrary shocks to trade cost at time at $t = 0$, is given by*

$$\hat{W}_d^h = \prod_{j\in\mathcal{I}} \left[\left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}} \right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h} \right)^{\frac{1}{\sigma_j - 1}} \right]^{\sum_{i\in\mathcal{I}} \bar{a}_{dji}\eta_i}, \tag{3.24}$$

where

$$\Xi_{dj,h} \equiv \zeta_j \left(\frac{\lambda_{ddj,h}}{\lambda^{k=0}_{ddj,h}}\right)^{\frac{\sigma_j-1-\theta_j}{\theta_j}} + (1-\zeta_j)^{h+1}\left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}}\right)^{\frac{\sigma_j-1-\theta_j}{\theta_j}} + \sum_{\varsigma=1}^{h}\zeta_j(1-\zeta_j)^k\left(\frac{\lambda_{ddj,h}}{\lambda^{k=0}_{ddj,h-\varsigma}}\right)^{\frac{\sigma_j-1-\theta_j}{\theta_j}} \tag{3.25}$$

and $\bar{a}_{dji}$ *is the* (j,i)*-th element of the Leontief inverse* $(\boldsymbol{Id}-\boldsymbol{A}_d)^{-1}$*, with the elements of* $\boldsymbol{A}_d$ *given by* α_{dji}*. If* $\zeta_i \in (0,1)$*, then*
$\lim_{h\to\infty}\hat{W}^h_d = \lim_{h\to\infty}\prod_{j\in\mathcal{I}}\left(\lambda_{ddj,t+h}/\lambda_{ddj,-1}\right)^{-\sum_{i\in\mathcal{I}}\bar{a}_{dji}\eta_i/\theta_j}$.

Although our model features transition dynamics on the supply side, (3.24) shows that welfare analysis can still be conducted using only a few sufficient statistics. These statistics delineate how the impact of trade shocks on real wages varies over time due to staggered sourcing decisions, decomposing the change in real wages associated with foreign shocks into two effects.

The first effect is captured by the terms $(\lambda_{ddj,h}/\lambda_{ddj,-1})^{-1/\theta_j}$ on the right-hand-side of (3.24). Because the Fréchet parameter θ_j gives the price elasticity of trade flows sourced from the currently cheapest global supplier and the share of domestic expenditures the response of trade to prices, each of these terms would give the change in a particular industry j's domestic price index if all goods were optimally sourced. Because all supply relationships are flexible in the long-run, i.e., when $h \to \infty$, changes in aggregate home expenditure shares and the long-run trade elasticity, thus, remain sufficient for long-run welfare analysis in our model, as in Eaton and S. Kortum (2002). However, staggered sourcing decisions spell additional welfare effects in the short-run, i.e., when not all goods can be sourced optimally.

Staggered adjustment of suppliers spells time-varying distortions in prices and terms-of-trade, captured by the terms $(\Xi_{dj,h})^{1/(\sigma_j-1)}$ in (3.24). Intuitively, these distortions manifest via expenditure allocations, and will vary across goods depending on when their current supplier was chosen. If a good was last optimally sourced

k periods ago, the resulting distortion in its price at horizon h can be informed by the difference between the share of domestic expenditures on all goods time h and on optimally sourced goods at time $h-k$, $(\lambda_{ddj,h}/\lambda^{k=0}_{ddj,h-k})^{(\sigma_j-1-\theta_j)/\theta_j}$. Intuitively, a decrease in $\lambda_{ddj,h}/\lambda^{k=0}_{ddj,h-k}$ indicates that suppliers that were chosen k periods ago are now, at horizon h, less competitive; the implied deterioration in a country's aggregate terms-of-trade is decreasing in the elasticity of substitution, σ_j, and increasing in the share of goods sourced from these suppliers, $\zeta_j \cdot (1-\zeta_j)^k$, is higher.

As an implication of Proposition 3, the trade elasticity relevant for welfare analysis varies over time. To further illustrate this point, it is useful to approximate changes in industry-level prices up to a first-order, which yields

$$\begin{aligned}\log\left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}}\right)^{-\frac{1}{\theta_j}}(\Xi_{dj,h})^{\frac{1}{\sigma_j-1}} \quad &\approx \quad -\frac{1}{\theta_j}[1-(1-\zeta_j)^{h+1}]\log\frac{\lambda^{k=0}_{ddj,h}}{\lambda_{ddj,-1}} \\ &- \quad \frac{1}{\sigma_j-1}(1-\zeta_j)^{h+1}\log\frac{\lambda^{k=h+1}_{ddj,h}}{\lambda_{ddj,-1}} - \mathcal{E}^h_{dj}\end{aligned}$$

where $\mathcal{E}^h_{dj} = \sum_{\varsigma=1}^{h+1}(1-\zeta)^{\varsigma}\zeta\left[\frac{1}{\sigma_j-1}\log\frac{\lambda^{k=\varsigma}_{ddj,h}}{\lambda^{k=0}_{ddj,h-\varsigma_j+1}} - \frac{1}{\theta_j}\log\frac{\lambda^{k=0}_{ddj,h+1-\varsigma}}{\lambda^{0}_{ddj,-1}}\right]$.

The first term on the right captures how changes in the prices of goods that were procured optimally at least once contribute to the overall change in prices at horizon h, assuming that past changes in factor prices were equal to those observed h periods after the shock. The second term, in contrast, captures changes in aggregate prices due to changes in the prices of goods whose suppliers have never been adjusted. The relative importance of these two effects varies over time, in tandem with the structural trade elasticity.

The last term, $\mathcal{E}^h_{dj}$, captures how suboptimal sourcing decisions from the past continue to distort prices at horizon h by distorting the equilibrium adjustment of factor prices relative to the long-run. Such distortions are reflected in price differences between goods whose suppliers were adjusted before and those that are procured optimally at horizon h.

Staggered sourcing decisions, hence, imply that the trade elasticity relevant for welfare analysis differs from the structural elasticity in (3.23) due the dynamic interaction of sourcing decisions and factor prices. Due to these interactions, the welfare effects of trade shocks may, then, vary both quantitatively and qualitatively over time, even conditional on the structural parameters underlying the time variation in the trade elasticity. Viewed through this lens, Proposition 3 is fortunate in that it allows us to summarize these dynamic effects in terms of a few statistics, which, as we will now describe, also enables us deploy familiar tools from the international trade literature to solve exactly for the equilibrium response of prices and wages to trade shocks implied by the model.

3.3.3 Characterization of Impulse Responses

We now show that solving for the responses of trade and production to shocks does not require knowledge of the economy's structural fundamentals (productivities, and trade costs). As an implication, the so-called "hat algebra" of Dekle, Eaton, and S. Kortum (2007) can be deployed to characterize impulse responses in our model.

Absent inter-sectoral linkages, trade flows at time t can be expressed in terms of succinct changes in trade costs and wages, as well as past changes in trade flows for optimally sourced goods, trade costs and wages:

$$\lambda_{sdi,t} = \frac{\left[1 + \left(\hat{\tau}_{sdi,t}\hat{w}_{s,t}/\hat{w}_{d,t}\right)^{1-\sigma_i+\theta_i}\omega_{sdi,t-1}\right]\lambda^{k=0}_{sdi,t-1}\left(\hat{\tau}_{sdi,t}\hat{w}_{s,t}\right)^{-\theta_i}}{\sum_{s'\in\mathcal{N}}\left[1 + \left(\hat{\tau}_{s'id,t}\hat{w}_{s',t}/\hat{w}_{d,t}\right)^{1-\sigma_i+\theta_i}\omega_{s'id,t-1}\right]\lambda^{k=0}_{s'id,t-1}\left(\hat{\tau}_{s'id,t}\hat{w}_{s',t}\right)^{-\theta_i}}, \tag{3.26}$$

where the wedges

$$\omega_{sdi,t-1} \equiv \frac{\mu_{i,t}(1)}{\mu_{i,t}(0)} + \sum_{k'=2}^{\infty} \frac{\mu_{i,t}(k')}{\mu_{i,t}(0)} \left(\frac{\lambda_{ddi,t-1}^{k=0}}{\lambda_{ddi,t-k'}^{k=0}} \right)^{\frac{\sigma_i - 1}{\theta_i}} \frac{\lambda_{sdi,t-1}^{k=k'}}{\lambda_{sdi,t-1}^{k=0}} \prod_{\varsigma=t-k''+1}^{t-1} \left(\hat{\tau}_{sid,\varsigma} \frac{\hat{w}_{s,t}}{\hat{w}_{d,\varsigma}} \right)^{1-\sigma_i}, \tag{3.27}$$

summarize how prior distortions in factor prices continue to impact trade flows at time t by distorting the terms of trade.

Now suppose that the economy was in steady state at some time prior to t. Then, given bilateral country-sector trade flows, industry-level consumption and intermediate good expenditure shares as well as per-capita GDP, the only additional industry-level parameters that are required to recursively compute changes in trade flows at increasing time horizons are given by $\{\zeta_i, \theta_i, \sigma_i\}$. Given this recursive formulation for trade flows, we can express the market clearing conditions (3.21) in terms of changes in trade costs and factor prices, as in Dekle, Eaton, and S. Kortum (2007), and, hence, solve for the period-by-period change in wages associated with (a sequence of) trade shocks.

3.4 Estimation

We now turn to exploring the quantitative implications of our theory for the response of production and welfare to trade shocks. In this section, we outline and implement our approach to estimating the structural parameters that govern the time variation of the trade elasticity. In the next section, we will use these estimates for a quantitative re-evaluation of how the 2018 US-China trade war impacted trade, production and welfare.

3.4.1 Approach

Proposition (8) implies that we can express the trade elasticity at varying time horizons h as a function of the set of structural parameters $\Theta_i \equiv \{\theta_i, \sigma_i, \zeta_i\}$:

$$f_i^h(\Theta_i) \equiv \varepsilon_i^h = \frac{\partial \log X_{sdi,t+h}}{\partial \log \tau_{sdi,t}} = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] + (1-\sigma_i)(1-\zeta_i)^{h+1}.$$

Our approach to recovering these structural involves, as a first step, obtaining reduced-form estimates of the trade elasticity over varying horizons. Such estimates can be obtained from the following specification using local projection methods:

$$\log\left(\frac{X_{sdi,t+h}}{X_{sdi,t-1}}\right) = \beta_i^h \log\left(\frac{\bar{\tau}_{sdi,t}}{\bar{\tau}_{sdi,t-1}}\right) + \delta_{si,t+h} + \delta_{di,t+h} + u_{sdi,t+h},$$

where $X_{sdi,t}$ denotes the exports of industry i goods from s to d at time t,and $t_{sdi,t}$ is the associated gross ad valorem tariff. The remaining terms denote source- or destination-industry-year-specific country fixed effects, and $u_{sdi,t}$ is an idiosyncratic error term. The coefficient β_X^h captures the change in trade flows h periods ahead that follows an initial one-period change in tariffs. Suppose that tariff changes were always one-time permanent shocks. Then a consistent estimate of β_i^h would yield an estimate of the structural trade elasticity at horizon h, ε_i^h. We now show how to recover the structural parameters governing the trade elasticity in our model, given a set of reduced-form estimates its behavior at varying time horizons h. With a slight abuse of notation, let $\{\hat{\beta}_i^h\}_{h=0}^H$ denote a set of such estimates ranging up to horizon $H > 0$.

Intuitively, the parameter σ_i governs the behavior of the trade elasticity in the short-run, while θ_i pins down its long-run value. The rate at which the trade elasticity converges to its long-run value, in turn, depends on how fast buyers form new supply relationships, ζ_i. More formally, we can use the structural expression for the trade elasticity to show that ζ_i, at any time $h > 0$, satisfies

$$\log(1-\zeta_i) = \frac{1}{h}\log\left(\frac{f_i^H(\Theta)-\theta_i}{f_i^0(\Theta)-\theta_i}\right), \tag{3.28}$$

which captures the rate at which the process governing the trade elasticity converges to its long-run limit. Given a set of reduced-form estimates $\hat{\boldsymbol{\beta}}_i \equiv \{\hat{\beta}_i^h\}_{h=0}^H$, we recover our structural parameters by minimum distance:

$$\hat{\Theta}_i(\hat{\boldsymbol{\beta}}_i) = \arg\min_{\Theta}(f_i^h(\Theta)-\hat{\beta}_i^h)_{i\in\mathcal{I}})^T W \left(f_i^h(\Theta)-\hat{\beta}_i^h\right)_{i\in\mathcal{I}}, \tag{3.29}$$

where W is a H-dimensional weighting matrix. Provided that the estimates of the trade elasticity are consistent, the continuous mapping theorem implies that $\hat{\Theta}_i(\hat{\boldsymbol{\beta}}_i)$ will provide a consistent estimate of Θ.

3.4.2 Implementation and Results

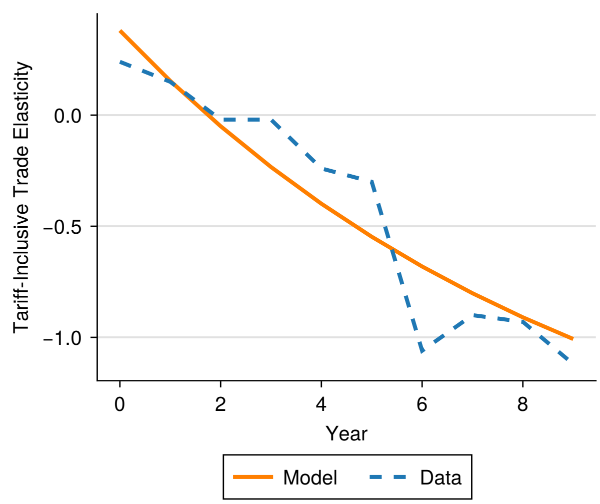


Figure 3.1. Horizon-Specific Trade Elasticity

To implement our estimation approach, we leverage a set of comprehensive reduced-form estimates of the trade elasticity at different time horizons by Boehm, Levchenko, and Pandalai-Nayar (2023). Following the reduced-form empirical ap-

Table 3.1. Trade Elasticity Parameter Estimates for the Manufacturing Industry

Parameter		Estimate
Supplier adjustment probability	ζ	0.10
Long-run Trade Elasticity	θ	1.89
Short-run Trade Elasticity	$\sigma - 1$	-0.63

proach outlined above, they find that arguably exogenous tariff changes in third countries predict a short-run trade elasticity that is substantially lower over shorter compared to longer horizons h, where $h = 0,1,...,10$. To recover our set of structural parameters, we focus on matching the implied empirical behavior of the trade elasticity within the first two years, as well as at horizons $h = \{8,9,10\}$. Specifically, we set the weighting matrix W so that our estimator targets the response of trade flows to an initial change in tariffs Table 3.1 presents the results.

We find that supply relationships reset at an annual rate of about 9 percent, indicating substantial stickiness in supply relationships. The long-run trade elasticity across manufacturing industries, on average, equals 3.2, consistent with estimates in the literature on gravity. Our estimate of the elasticity of substitution equals 1.145, suggesting that trade elasticity, in the short-run, will be substantially lower, given the stickiness of supply relationships.

Figure 1 graphs the structural trade elasticity implied by these parameter estimates, along with the reduced-form elasticity estimates by Boehm, Levchenko, and Pandalai-Nayar (2023). On impact $(h = 0)$, the structural trade elasticity is close to zero. Over time, it smoothly increases in absolute value, reflecting the gradual resetting of supply relationship and reaching a level of -2.2 after 10 years. Reassuringly, the structural trade elasticity matches the behavior of its empirical counterpart also at horizons that were not explicitly targeted by our estimator.

3.5 Quantitative Application: The 2018 US-China Trade War

We now apply our model to examine the general equilibrium responses of trade and production to the 2018 US-China trade war.

3.5.1 Calibration of the Initial Steady State

We assume that the world economy is in a steady state prior to the announcement of tariff changes due to the trade war. The remaining model parameters and initial levels of certain quantities are therefore calibrated so that trade activities implied by the model equilibrium in the absence of any shock match the data in 2017. For this purpose, we utilize the 2017 table from the 2023 edition of OECD Inter-Country Input-Output (ICIO) tables. Table (3.2) summarizes the parameters and initial levels obtained for the calibration.

Table 3.2. Model Parameters and Variable Levels for Initial Steady State

Parameters or Initial Levels	Notation	Level of Variation
Matching Input-Output Data Exactly		
Producer expenditure shares across inputs	α_{sij}, α_{sj}	Producer region-sector
Initial import shares by source region	λ_{sdi}	User region
Initial level of bilateral trade flows	X_{sdi}	Sector-specific bilateral pair
Derived from Model Equilibrium		
Household expenditure shares across sectors	η_{dj}	User region
Initial aggregate labor income	$w_s L_s$	Producer region
Deficit (difference between expenditure and income)	D_d	User region

The ICIO table covers 45 industries in 76 economies along with the constructed rest of the world (ROW). In the model, we allow 77 economies corresponding to each of these in the data. For China (mainland) and Mexico, the data additionally record the input-output relations for a subset of manufacturing activities only intended for goods to be exported separately.[4] To take advantage of these additional details for China and Mexico in the model, we view these two economies as consisting of two types of producers for each industry respectively. Namely, for each industry-specific good in these two economies, there is a set of regular producers delivering output for both domestic and foreign use; an additional set of producers produce special varieties that are only delivered abroad.[5] The technological parameters including those governing trade shares are allowed to be different across these two types of producers. However, the value added generated from all these producers are pooled together for computing the aggregate income in these two countries. Furthermore, labor inputs are assumed to be perfectly mobile across the two types of producers. These producers therefore face possibly different prices for intermediate inputs but identical wages.

Among the 45 industries, we exclude three of them that are primarily for public expenditure or services that are hard to classify. We further aggregate the remaining 42 industries into 32 sectors by combining certain non-manufacturing industries. A list of the sectors can be found in Appendix. Since we do not cover all industries in the ICIO table, the remaining data values no longer satisfy all restrictions imposed by accounting identities exactly.[6] For this reason, we need to take a stance on how we recover the identities. In other words, it is impossible to match the original ICIO table in every aspect. We choose certain dimensions of the data that we target exactly but use the model equilibrium conditions to derive those that cannot be targeted simultaneously.

We set the technological parameters $\{\{\alpha_{sij}\}_i, \alpha_{sj}\}_{sj}$ so that the expenditure shares

[4] These are available in the extended version of the ICIO tables for addressing heterogeneity of producers that do not directly deliver output in the domestic markets.

[5] Depending on the calibration procedure, any (residual) domestic final use generated for output from the second type of producers are treated as arising only from exogenous deficit but not labor income.

[6] The output from a region-sector pair must be identical to the sum of intermediate or final use of the region-sector good around the world.

across each production inputs match those in the data exactly.[7] We also match the initial import shares $\{\lambda_{sdi}\}_{sdi}$ exactly. Notice that these parameters already determine a complete input requirement matrix for the world economy. However, we still need to determine the relative levels of output across all region-sector pairs and there are alternative approaches. Since the exposure of each economy to the trade war depends on the initial levels of bilateral trade flows, we choose to target the levels of bilateral trade flows exactly, which include self trade.[8] From these bilateral trade flows, we immediately obtain the levels of output from each region-sector pair and the total expenditure on each sectoral good in each region.[9] From the levels of output and the technological parameters, we obtain the level of total expenditure on each intermediate input in each region. Subtracting these levels of intermediate use from total expenditures yield the levels of final use for each sector in each region that ensure all accounting identities hold. We set the household expenditure shares across goods from different sectors based on these derived final use. Lastly, with the region-sector specific value added shares, we compute the initial levels of aggregate labor income.[10] The discrepancy between total expenditure and total labor income in each country is treated as exogenous deficit that our model does not address.

3.5.2 Measuring the Tariff Changes

The tariff changes associated with the trade war are obtained from Fajgelbaum et al. (2020). Since the tariffs are determined at a detailed Harmonized System (HS) code level, we compute weighted averages of these tariff changes within each of the

[7]The ICIO tables contain the margins for taxes or subsidies. These margins are treated as special expenditures that are not contributing to any part of the disposable income. For this reason, the sum of the expenditure shares across inputs are smaller than one.

[8]Alternatives include targeting the levels of sectoral final use, or sectoral value added, etc.

[9]Again, because the input-output relations no longer hold after excluding some industries, these values implied by the subset of bilateral trade flows can be different from the original values in the ICIO table which cover all industries.

[10]Without dealing with other factor income, we abstract away from heterogeneity in the labor income shares within the value added components.

model sectors across different years. The tariff changes are aggregated both across different HS code and across months when they take into effect. For the aggregation across product categories, we determine the most relevant model sector based on their associated industry classifications and use the annual bilateral trade volume of each product in 2017 as weights. For the temporal aggregation across months, it has already been implemented by Fajgelbaum et al. (2020) using the shares of months within a year for which the tariff changes are in effect as weights. Table (3.3) collects the aggregated tariff changes on US imports from China along with the sectoral composition of US imports from China in 2017. Table (3.4) collects the aggregated retaliatory tariff changes on US exports to China. Notice that for model calibration, we have relied on the OECD ICIO table, which reconciles trade data with national accounts. However, for aggregating tariff changes, we require the 10-digit HS-code level data from US Census. It is therefore inevitable to see some discrepancies of the relative importance of sectoral imports or exports between the two types of data. Fortunately, for most of the sectors, the discrepancies seem to be small. For the tariff changes, we see that since many products were affected only after the second half of 2018, the aggregate changes at the annual level in 2018 are much smaller than those in 2019. By the end of 2019, all tariff changes associated with the trade war had been in place.

Table 3.3. US Tariff Increases on Imports from China

Affected Sector in Model	2017 Imports in Total (%) OECD ICIO	US Census	Cumulative Increases in Tariffs (%) 2018	2019	2020–
Agriculture, forestry and fishing	0.5	0.6	2.5	14.7	20.6
Mining and quarrying	0.0	0.1	1.0	5.6	7.4
Food products, beverages and tobacco	1.9	0.8	2.6	15.5	22.3
Textiles, textile products, leather and footwear	17.7	12.8	0.6	6.6	13.8
Wood and products of wood and cork	1.3	0.8	2.9	16.4	22.1
Paper products and printing	1.2	1.3	2.1	11.0	15.8
Coke and refined petroleum products	0.2	0.1	2.4	14.2	20.5
Chemical and chemical products	3.2	3.1	2.7	12.7	17.7
Pharmaceuticals, medicinal and botanical products	1.3	0.5	0.0	0.1	0.1
Rubber and plastics products	3.7	3.6	2.2	10.9	15.1
Other non-metallic mineral products	2.4	1.7	2.1	12.3	17.4
Basic metals	1.0	0.9	8.8	22.4	24.5
Fabricated metal products	3.8	4.1	3.4	15.0	20.0
Computer, electronic and optical equipment	29.0	36.3	2.0	8.1	11.2
Electrical equipment	9.1	8.9	3.9	14.9	18.8
Machinery and equipment, nec	6.9	7.3	6.1	18.3	22.3
Motor vehicles, trailers and semi-trailers	4.2	3.2	4.7	19.3	24.7
Other transport equipment	0.8	0.7	7.2	20.5	24.0
Furniture and other manufacturing	11.7	13.3	1.1	7.1	11.0

Notes: "Imports in Total" are the shares of industry-specific US imports in total imports from China. "OECD ICIO" refers to the input-output table used for calibrating the model. "US Census" refers to the HS-level bilateral trade data accessed via USA Trade Online. The tariff changes are aggregated based on weights derived from the US Census data. Tariff changes from Fajgelbaum et al. (2020).

Table 3.4. Retaliatory Tariff Increases on US Exports to China

Affected Sector in Model	2017 Exports in Total (%) OECD ICIO	US Census	Cumulative Increases in Tariffs (%) 2018	2019	2020–
Agriculture, forestry and fishing	14.7	15.1	11.9	31.1	31.3
Mining and quarrying	7.7	7.0	3.5	11.2	14.0
Food products, beverages and tobacco	4.2	2.8	10.4	19.9	21.0
Textiles, textile products, leather and footwear	0.4	0.9	2.5	12.1	15.3
Wood and products of wood and cork	1.5	1.5	2.7	12.9	16.3
Paper products and printing	2.0	2.5	2.1	7.7	8.8
Coke and refined petroleum products	2.6	1.0	10.2	26.0	26.0
Chemical and chemical products	10.6	9.6	3.7	12.5	14.3
Pharmaceuticals, medicinal and botanical products	2.5	2.8	0.3	1.6	2.7
Rubber and plastics products	1.4	1.3	2.3	10.0	12.4
Other non-metallic mineral products	0.6	0.8	4.0	13.7	15.8
Basic metals	10.7	1.9	4.1	15.4	18.9
Fabricated metal products	1.2	1.4	2.5	10.9	13.3
Computer, electronic and optical equipment	10.4	13.8	2.4	9.3	11.2
Electrical equipment	1.4	2.5	3.8	15.8	19.4
Machinery and equipment, nec	6.0	8.0	2.1	9.1	11.1
Motor vehicles, trailers and semi-trailers	8.6	11.0	10.5	21.5	21.7
Other transport equipment	12.4	13.3	0.0	0.1	0.1
Furniture and other manufacturing	1.1	2.8	4.1	12.8	14.2

Notes: The same notes for Table (3.3) apply.

3.5.3 The General Equilibrium Impact of the Trade War

We are interested in how the 2018 US-China trade war had affected the aggregate economic activities around the world and how its welfare impact had evolved over time. To that end, we conduct a general equilibrium counterfactual experiment in which we compute how the model outcomes evolve following the tariff changes we measure in (3.5.2) relative to a hypothetical scenario in which the trade war had not happened.

Trade Flows

Figure (3.2) plots the changes in the tariff-inclusive US (China) imports from China (US) among the 19 sectors listed in Tables (3.4), (3.3) that are directly affected by the trade war. With higher tariff payments and low short-run trade elasticity, the bilateral trade flows increase, as predicted by both the full GE model outcomes and the PE trade elasticity. The bilateral trade flows start to drop below the initial levels only since year 4 for US and year 2 for China. As time goes, the trade flows keep declining as the relevant trade elasticity shifts towards the long-run level. Notice that for US, the trade flows fall by less than what the structural trade elasticity predict due to the changes in factor prices. The discrepancies between what the full GE model predicts and what the PE trade elasticity predicts for US demonstrate the need of taking into account the GE effects.

Prices

The sluggish short-run response of US demand to the rise in trade costs induces a substantial rise in its domestic price level. As shown in Figure (3.3), aggregate price indices faced by US consumers and producers rise across all industries. Some industries, notably textiles, basic metals, and electrical equipment, see prices rise by over 4% as all retaliatory tariffs are in place. As sourcing decisions gradually adjust to the initial rise in trade cost, prices decline although they remain high. In contrast

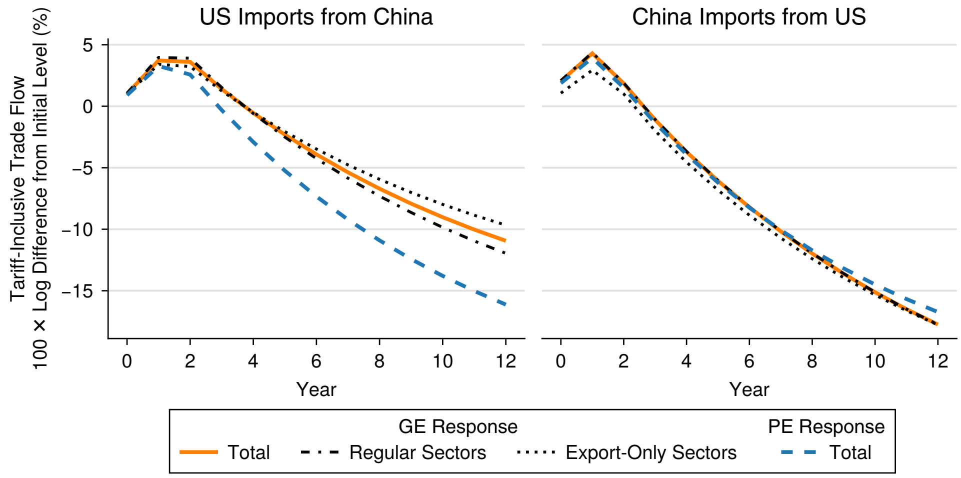


Figure 3.2. Changes in Tariff-Inclusive Trade Flows

Note: Tariff changes are gradually implemented over the first two years. The model determines changes in trade shares at the sector level. The country-level outcomes are based on aggregate trade flows summed across sectors. "GE Response" and "PE Response" refer to results generated from the full model involving factor price changes and results only based on the PE trade elasticity respectively. "Regular Sectors" and "Export-Only Sectors" are only relevant to China due to the feature of ICIO tables explained in Section (3.5.1).

to the substantial and uneven price hikes in US, domestic prices in China decline across all industries. Intuitively, a rise in trade barriers can temporarily improve a region's terms-of-trade when trade adjustment is not primarily driven by comparative advantage. Figure (3.4) shows the price impact on the remaining sectors that are not directly exposed to the tariff changes.

Real Wages and Welfare

Figure (3.5) traces the counterfactual response of real wages, as well as nominal wages and consumer prices in the US and China. In the long run, the trade war reduces real income in both countries by a similar magnitude. However, its short-run impact differs substantially between the US and China. In the US, the real wage responds gradually, with a moderate decline within the first two years of the trade war (-0.1%) that corresponds to about 50% of the overall effect (-0.22%). In contrast, while the long-run costs of the trade war in China are similar to those in the US, China also

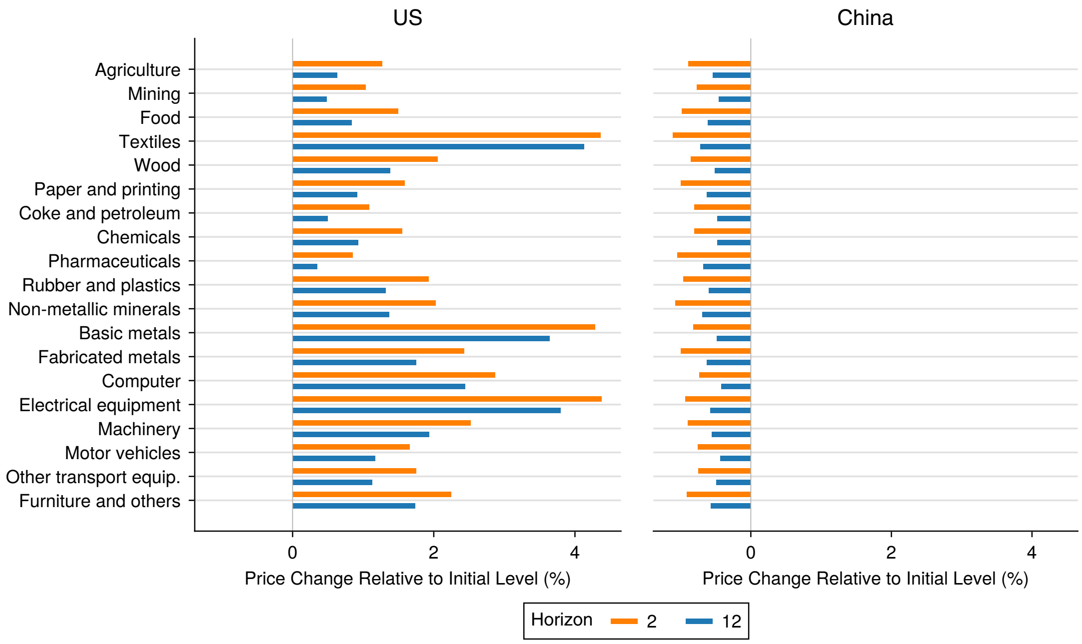


Figure 3.3. Changes in Price Indices Among Directly Affected Sectors

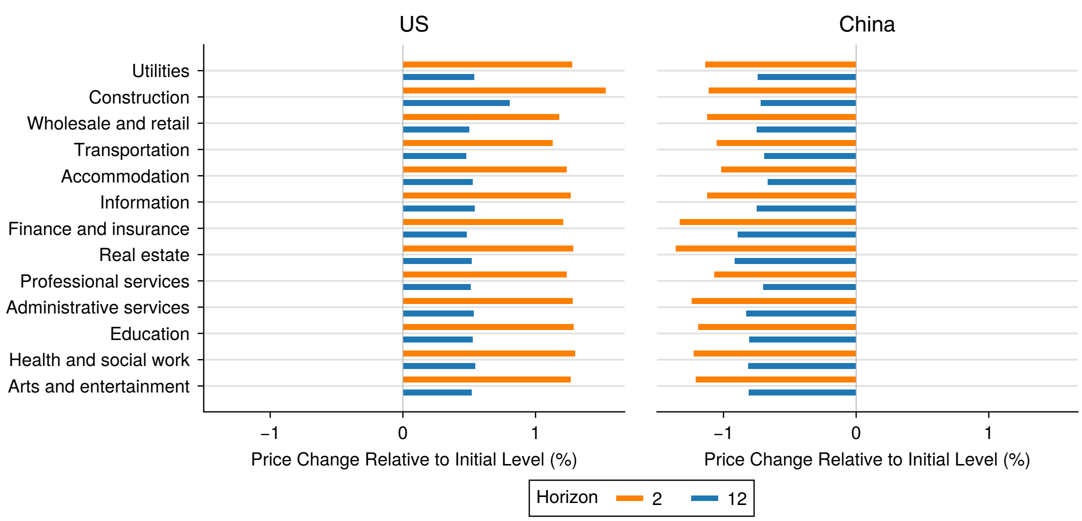


Figure 3.4. Changes in Price Indices Among Sectors Not Directly Affected

experiences a substantially larger decline in real income by the end of the second year (-0.3%).

In Figure (3.6), we leverage the ACR-style welfare formula shown in Proposition (9) to elucidate how the presence of adjustment frictions alter the transitory dynamics

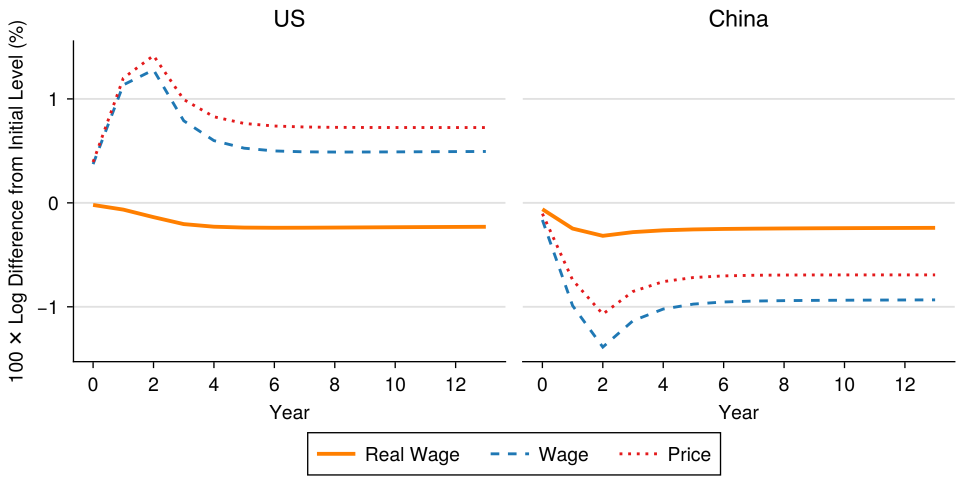


Figure 3.5. Changes in Real Wages, Wages and Consumer Prices

Note: Tariff changes are gradually implemented over the first two years. "Real Wage" in year t refers to real wage changes between t and the initial steady state generated by the full model. "Wage" refers to the corresponding change in nominal wage. "Price" is the change in aggregate consumer price index.

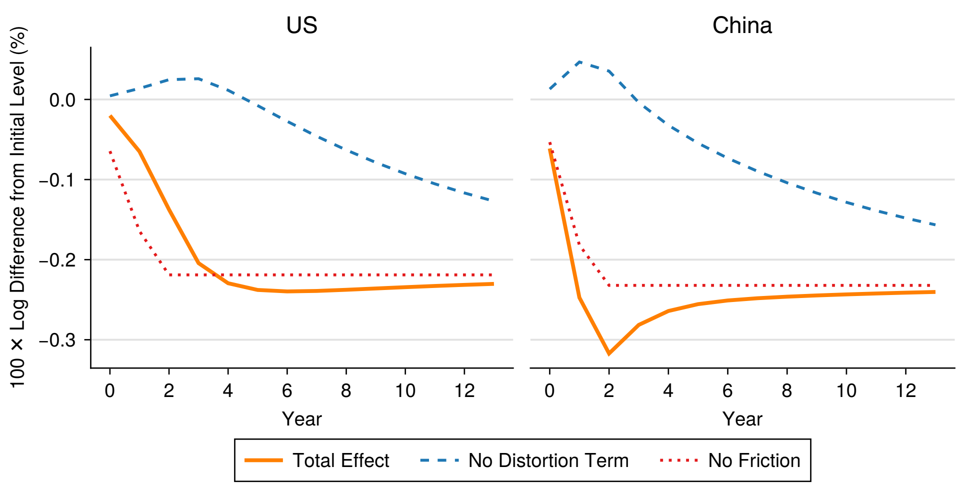


Figure 3.6. Horizon-Specific Welfare Impact

Note: Tariff changes are gradually implemented over the first two years. "Total Effect" in year t refers to the real wage changes between t and the initial steady state generated by the full model. "No Distortion Term" refers to the (partial) welfare impact when applying the ACR formula to the model-implied domestic trade shares while ignoring the distortion term Ξ. "No Friction" refers to the change in real wages when assuming that the economy reaches the long-run outcomes instantly ($\zeta = 1$).

of real wages. Interestingly, if one applies a multisector version of the original welfare formula from Arkolakis, Costinot, and Rodríguez-Clare (2012) to the economy with adjustment frictions, the results will be very misleading, as illustrated by the curve labeled as "No Distortion Term". The reason is that in addition to using the less appropriate long-run trade elasticity, the observed changes in aggregate domestic trade shares go in the opposite direction relative to the actual changes in real wages over the short run. With substantial adjustment frictions among trade partners, the changes in aggregate domestic trade shares over the initial years are not in line with the long-run Ricardian forces that govern the original welfare formula. In fact, the distortion term Ξ highlighted in Proposition (9) is quantitatively substantial and drives most of the short-run welfare impact.

As another illustration, we compare how the paths of welfare impact differ from the hypothetical scenario in which there is no adjustment friction ($\zeta = 1$). In this case, the economy jumps to the long-run outcomes instantly.[11] For the US, we see the model predicts short-run welfare impact that is smaller than the long-run outcome over the initial years. However, for China, the short-run welfare impact is noticeably larger than the long-run level. In particular, the much lower trade elasticity in the short run does not mechanically imply larger welfare impact over the short run.[12] The asymmetry of the responses of real income over time illustrates that sourcing frictions may mitigate or amplify the costs of trade disruptions in the short run. Note that over the initial years, prices and wages rise by a similar magnitude in the US; while in China, domestic wages fall substantially more than consumer prices. The intuition is that sluggishness in the response of trade flows helps smooth the transition for the US: It benefits not only from additional tariff revenues generated by the fact that producers continue to import goods from China but also from a limited response of its export demand to the rise in its export prices. In contrast, adjustment frictions pose additional short-run

[11]The smaller impact over the first two years are merely from the fact that the tariff changes are not fully implemented until the end of 2019.

[12]Again, with the adjustment friction, changes in aggregate domestic shares alone are not sufficient for accounting the welfare impact.

costs for China as they impede its ability to leverage the decline in domestic wages to increase exports, while only generating limited additional tariff revenue (due to the fact that it imports relatively little from the US to begin with). The gradual realignment of trade flows with comparative advantage over time therefore ameliorates the welfare loss for the US but exacerbates the real impact in China. Moving toward the future horizons, the welfare impact gets closer to the long-run levels while being slightly lower for both countries, due to the persistent effects of the distortions on the prices and allocations.

Effects on Third-Party Countries

We conclude by highlighting how accounting for short-run adjustment frictions affects the welfare implications of the US-China trade war for third-party countries. In Figure (**??**), we present the counterfactual responses of real income in Mexico and Vietnam. Notably, both countries experience benefits from the tariff increases in the long run, while also facing losses in the short run. For Mexico, this short-run income loss ranks among the largest for all third-party countries; however, it distinguishes itself as one of the few countries that benefit from the trade war in the long run.

Therefore, the welfare impact of trade disruptions can qualitatively differ over time. Intuitively, when trade adjustments are subject to frictions, disruptions negatively affect all countries in the short run. In the long run, however, realignments of supply relationships may benefit some countries. In the context of the US-China trade war, both Mexico and Vietnam experience a sustained increase in their domestic wages, reflecting both the reallocation of US and Chinese demand as well as their favorable positions in the international production network.

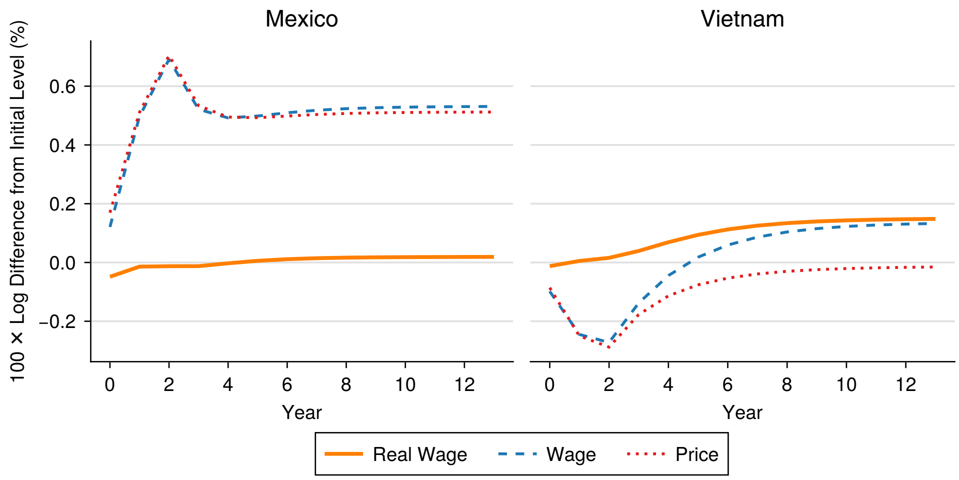


Figure 3.7. Changes in Prices and Wages in Mexico and Vietnam

Note: The same notes for (3.5) applies.

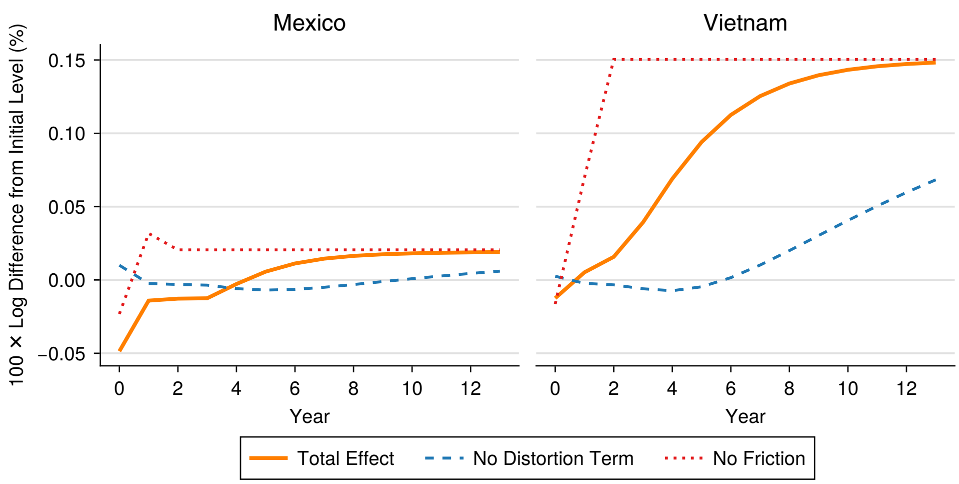


Figure 3.8. Welfare Impact in Mexico and Vietnam

Note: The same notes for Figure (3.6) applies.

3.6 Concluding Remarks

To account for imperfect adjustment to global supply chain shocks, we develop a Ricardian trade framework with frictions that result from infrequent decisions of producers to change global suppliers. We obtain novel formulas for accounting welfare changes to trade openness and trade shocks, derive novel estimation equations for trade elasticity estimation at varying time horizons, and quantify the model. Counterfactual experiments of the US-China trade war suggests that rich sectoral dynamics ensue, resulting in considerable short-term reallocations and substantive welfare fluctuations that are not captured by a standard welfare formula as in Arkolakis, Costinot, and Rodríguez-Clare (2012).

3.7 Acknowledgments

This chapter is in preparation for future publication. It has not yet been submitted to any journal, but it has been presented in the following conferences or institutions: ASSA Annual Meetings, 2025; World Bank Trade and Investment Research Seminar; NBER's Trade and Macroeconomics Summer Institute; CESifo Area Conference on Global Economy 2024; FREIT's Empirical Investigations in Trade and Investment (EITI), 15th Meeting; UCSD Faculty Seminar; and the European Central Bank's Trade Seminar. The author of the dissertation was an equal contributor to this paper.

3.8 Appendix

3.8.1 Ideal Price Indexes and Generic Trade Shares

The composite good in industry j is

$$Y_{dj,t} \equiv \left(\int_{[0,1]} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}.$$

Product space $\Omega_j = [0,1]$ can be partitioned into disjoint sets with $\Omega_j = \bigcup_{k=0}^{\infty} \Omega_{j,t}^k$, so we can rewrite the composite good as

$$Y_{dj,t} \equiv \left(\sum_{k=0}^{\infty} \int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}. \tag{3.30}$$

The assembler's associated cost minimization problem is

$$\begin{aligned}
\min_{\{y_{dj,t}(\bar{\omega})\}_{\bar{\omega} \in \Omega_{j,t}}, \{Y_{dj,t}^k\}} P_{dj,t} Y_{dj,t} \quad &= \quad \sum_{k=0}^{\infty} P_{dj,t}^k Y_{dj,t}^k \\
s.t. \quad & \quad Y_{dj,t} = \left(\sum_{k=0}^{\infty} \left(Y_{dj,t}^k \right)^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \\
& \quad Y_{dj,t}^k \equiv \left(\int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \\
& \quad P_{dj,t}^k Y_{dj,t}^k = \int_{\Omega_{j,t}^k} p_{dj,t}(\bar{\omega}) y_{dj,t}(\bar{\omega}) \mathrm{d}\bar{\omega},
\end{aligned}$$

where we define the partial composite good $Y_{dj,t}^k \equiv \left(\int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}$ for each partition k as a helpful construct for derivations and implicity define the associated partial ideal price index $P^k_{dj,t}$ that satisfies $P^k_{dj,t}Y^k_{dj,t} = \int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})y_{dj,t}(\bar{\omega})\mathrm{d}\bar{\omega}$.

Under homotheticity of the assembler's production, this problem can be solved in two steps. First, the assembler decides which share of cost it allocates to each partial composite good $Y^k_{dj,t}$. Given those choices, the assembler then decides the optimal cost for each intermediate good $y_{dj,t}(\bar{\omega})$. Optimal demand satisfies

$$Y^k_{dj,t} = \left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{-\sigma_j} Y_{dj,t} \quad \text{and} \tag{3.31}$$

$$y^k_{dj,t}(\bar{\omega}) = \left(\frac{p_{dj,t}(\bar{\omega})}{P^k_{dj,t}}\right)^{-\sigma_j} Y^k_{dj,t} = \left(\frac{p_{dj,t}(\bar{\omega})}{P_{dj,t}}\right)^{-\sigma_j} Y_{dj,t} \quad \text{for each } \bar{\omega} \in \Omega^k_{j,t}, \tag{3.32}$$

where the last equality also shows that the partitioned solution equals the standard solution under a constant elasticity of substitution. Replacing the demand functions above in the definition of the budget constraint results in the expressions for the ideal price indices:

$$P_{dj,t} = \left(\int_{[0,1]} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega}\right)^{\frac{1}{1-\sigma_j}}, \qquad P^k_{dj,t} = \left(\int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega}\right)^{\frac{1}{1-\sigma_j}}. \tag{3.33}$$

We have now established that partitioning the product space into disjoint sets results in well-behaved demand functions such that, given optimal choices within each set, we can analyze demand for each intermediate good independently and then aggregate. In subsequent derivations, expenditure shares within each partition k will play a crucial role, so we state a general definition here:

$$\lambda^k_{sdj,t} \equiv \frac{X^k_{sdj,t}}{X^k_{dj,t}} \equiv \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\int_{\Omega^k_{j,t}} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega} \tag{3.34}$$

$$= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}.$$

3.8.2 Trade Shares When Firms Are Sourcing Optimally ($k = 0$)

Under perfect competition, the destination price for intermediate good $\omega \in \Omega^0_{j,t}$ offered by country s to country d is $p_{sdj,t}(\omega) = c_{sdj,t}/z_{sj}(\omega)$ for the common unit cost component $c_{sdj,t}$ by (3.3) and supplier ω's productivity $z_{si}(\omega)$. Under the EK assumptions, the cumulative distribution function of prices is therefore

$$\tilde{F}_{sdj,t}(p) = \mathbb{P}\left[p_{sdj,t}(\omega) < p\right] = 1 - F_{sj}\left(\frac{c_{sdj,t}}{p}\right) = 1 - \exp\left\{-A_{sj}(c_{sdj,t})^{-\theta_j} p^{\theta_j}\right\}. \tag{3.35}$$

The resulting probability that country d sources an intermediate good $\omega \in \Omega^0_{j,t}$ from country s is

$$\mathbb{P}\left[s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] = \int_0^\infty \prod_{n \neq s} \left[1 - \tilde{F}_{ndj,t}\left(p\right)\right] \mathrm{d}\tilde{F}_{sdj,t}(p) = \frac{A_{sj}(c_{sdj,t})^{-\theta_j}}{\Phi_{dj,t}}, \tag{3.36}$$

where $\Phi_{dj,t} \equiv \sum_n A_{sj}(c_{sdj,t})^{-\theta_j}$.

For products in $\Omega^0_{j,t}$, the distribution of prices $G^0_{sdj,t}(p)$ paid in country d on products sourced from country s equals the overall distribution of prices paid in country d: $G^0_{dj,t}(p)$. For any given source country s:

$$G^0_{sdj,t}(p) = \mathbb{P}\left[p_{dj,t}(\omega) \leq p \middle| s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] = 1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}.$$

The unconditional distribution is the same as the distribution conditional on each source country, so

$$
\begin{aligned}
G^0_{dj,t}(p) &= \sum_s \mathbb{P}\left[p_{dj,t}(\omega) \le p \middle| s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] \mathbb{P}\left[s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] \\
&= \sum_s \left(1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}\right) \lambda^0_{sdj,t} = 1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}, \qquad (3.37)
\end{aligned}
$$

where the last equality follows from the fact that $\sum_s \lambda^0_{sdj,t} = 1$.

Putting these results together, we can now solve for the expenditure share within partition 0. Starting from the definition of expenditure shares,

$$
\begin{aligned}
\lambda^0_{sdj,t} &\equiv \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \left(p_{sdj,t}(\omega)\right)^{1-\sigma_j} \mathrm{d}\omega}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \left(p_{ndj,t}(\omega)\right)^{1-\sigma_j} \mathrm{d}\omega} \\
&= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{sdj,t} \mathrm{d}\omega}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{ndj,t} \mathrm{d}\omega} \\
&= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}} \\
&= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega}{\int_{[0,1]} \mathbf{1}\left\{\omega \in \Omega^0_{j,t}\right\} \mathrm{d}\omega} \\
&= \frac{\mu_{j,t}(0) \mathbb{P}\left[s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right]}{\mu_{j,t}(0)} \\
&= \frac{A_{sj}(c_{sdj,t})^{-\theta_j}}{\Phi_{dj,t}}, \qquad (3.38)
\end{aligned}
$$

where $\mu_{i,t}(0)$ is the measure of the set $\Omega^0_{i,t}$. The third line uses the fact again that the distribution of prices conditional on the source country is the same as the unconditional distribution of prices, and the last equality uses the probability that a given source country hosts the lowest-cost supplier.

We can derive the corresponding ideal price indices using

$$
\begin{aligned}
\left(P^0_{dj,t}\right)^{1-\sigma_j} &= \int_{\Omega^0_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} = \int_{\Omega^*_{j,t}0} \int_0^{\infty} (p)^{1-\sigma_j} \mathrm{d}G_{dj,t} \mathrm{d}\bar{\omega} \\
&= \int_{\Omega^0_{j,t}} \int_0^{\infty} (p)^{1-\sigma_j} \theta_j \Phi_{dj,t} p^{\theta_j - 1} \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\} \mathrm{d}p \, \mathrm{d}\bar{\omega}.
\end{aligned}
$$

For a change of variables, define $x \equiv p_j^{\theta} \Phi_{dj,t}$, which implies that $\mathrm{d}x = \theta_j \Phi_{dj,t} p^{\theta_j - 1} \mathrm{d}p$ and $p = \left(x/\Phi_{dj,t}\right)^{1/\theta_j}$. Denoting $\gamma_j \equiv \Gamma\left([\theta_j + 1 - \sigma_j]/\theta_j\right)$, we can then rewrite the integral above as

$$
\left(P^0_{dj,t}\right)^{1-\sigma_j} = \int_{\Omega^0_{j,t}} \int_0^{\infty} \left(\frac{x}{\Phi_{dj,t}}\right)^{\frac{1-\sigma_j}{\theta_j}} \exp\{-x\} \, \mathrm{d}x \, \mathrm{d}\bar{\omega} = \gamma_j \mu_{j,t}(0) \left(\Phi_{dj,t}\right)^{-\frac{1-\sigma_j}{\theta_j}}, \qquad (3.39)
$$

$\mu_{j,t}(0)$ denotes the measure of the set $\Omega^0_{j,t}$. The results show that, when firms are adjusting, trade shares operate as in the frictionless economy of EK.

Using standard hat algebra for changes in the common unit cost component $\hat{c}_{sdj,t} \equiv c_{sdj,t}/c_{sdj,t-1}$, we can express trade shares and price levels within partition $k = 0$ as:

$$
\lambda^0_{sdj,t} = \frac{\lambda^0_{sdj,t-1} \hat{c}_{sdj,t}^{-\theta_j}}{\sum_n \lambda^0_{ndj,t-1} (\hat{c}_{ndj,t})^{-\theta_j}} \qquad (3.40)
$$

$$
P^0_{dj,t} = P^0_{dj,t-1} \left[\sum_s \lambda^0_{sdj,t-1} (\hat{c}_{sdj,t})^{-\theta_j}\right]^{-\frac{1}{\theta_j}}. \qquad (3.41)
$$

We next derive an analogous result for partitions $k > 0$ when firms are not adjusting their extensive margin of suppliers.

3.8.3 Trade Shares When Firms Are Not Adjusting ($k > 0$)

For intermediate goods $\omega \in \Omega^k_{j,t}$, assemblers last adjusted the least-cost supplier $t - k$ periods ago. In order to account for changes in trade shares and price levels, we therefore need to recall optimal sourcing choices at period $t - k$ and trace changes in parameters and prices since $t - k$.

Suppose that in period $t - k$ intermediate good ω was optimally sourced from country s to country d in industry j. Then the destination price in period t for this intermediate good will be:

$$p_{sdj,t}(\omega) = \frac{c_{sdj,t}}{z_{sj}(\omega)} = \frac{\prod_{\varsigma=t-k+1}^{t} c_{sdj,t-k}\,\hat{c}_{sdj,\varsigma}}{z_{sj}(\omega)} = p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{sdj,\varsigma}\right), \tag{3.42}$$

which is the initial destination price adjusted for the cumulative changes in trade costs and factor costs. Using this result, we can derive country d's expenditure share by source country across intermediate goods $\omega \in \Omega^k_{j,t}$

$$\begin{aligned}
\lambda^k_{sdj,t} &\equiv \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{ndj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j}\, \mathrm{d}G_{sdj,t-k} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j}\, \mathrm{d}G_{ndj,t-k} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j}\, \mathrm{d}G_{dj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j}\, \mathrm{d}G_{dj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\mu_{j,t}(k) \lambda_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \mu_{j,t}(k) \lambda_{ndj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \lambda^0_{ndj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}},
\end{aligned} \tag{3.43}$$

where $\mu_{i,t}(k)$ is the measure of the set $\Omega^k_{i,t}$. The third line again uses the fact that, at $t - k$, the distribution of prices conditional on the source is the same as the unconditional distribution; and the last line uses the result from the previous section that $\lambda^0_{sdj,t-k} =$

$\mathbb{P}\left[s = \arg\min_s \left\{ p_{sdj,t-k}(\omega) \right\}\right]$.

We can derive the corresponding ideal price indices using

$$\begin{aligned}
\left(P^k_{dj,t}\right)^{1-\sigma_j} &= \int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} \\
&= \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \in \check{G}\right\} \left(p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega \\
&= \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \in \check{G}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{sdj,t-k} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \\
&= \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t-k} \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \in \check{G}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \\
&= \frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)} \left(P^0_{dj,t-k}\right)^{1-\sigma_j} \sum_s \lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}
\end{aligned} \tag{3.45}$$

where $\check{G} \equiv \arg\min_m \left\{ p_{mdj,t-k}(\omega) \right\}$. The price level change in partition 0 satisfies $P^0_{dj,t} = P^0_{dj,t-1} \left[\sum_s \lambda^0_{sdj,t-1} (\hat{c}_{sdj,t})^{-\theta_j}\right]^{-\frac{1}{\theta_j}}$ by (3.39), so we can rewrite the ideal price for composite goods with the last supplier selection k periods ago

$$\left(P^k_{dj,t}\right)^{1-\sigma_j} \propto \left[\sum_n \lambda^0_{ndj,t-k-1} \hat{c}^{-\theta_j}_{ndj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}} \sum_s \lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}$$

Denoting $\gamma_j \equiv \Gamma\left([\theta_j + 1 - \sigma_j]/\theta_j\right)$ and using the fact that

$\left(P^0_{dj,t}\right)^{1-\sigma_j} = \mu_{j,t}(0)\left(\Phi_{dj,t}\right)^{-\frac{1-\sigma_j}{\theta_j}}\gamma_j$, we can rewrite the expression above as:

$$\left(P^k_{dj,t}\right)^{1-\sigma_j} = \gamma_j \mu_{j,t}(k)\left(\Phi_{dj,t-k}\right)^{-\frac{1-\sigma_j}{\theta_j}} \sum_s \left[\lambda^0_{sdj,t-k-1}\hat{c}^{-\theta_j}_{sdj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \tag{3.46}$$

after expressing $\lambda^0_{sdj,t-k}$ recursively.

3.8.4 Aggregation Over Partitions

The aggregate ideal price level of the final good can be rewritten as a combination of the price levels of the partial price indices for the composites of intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago:

$$\left(P_{dj,t}\right)^{1-\sigma_j} = \int_{[0,1]} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega} = \sum_{k=0}^{\infty}\int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega} = \sum_{k=0}^{\infty}\left(P^k_{dj,t}\right)^{1-\sigma_j}.$$

Using the price index expressions (3.39) and (3.46) from the preceding subsections yields

$$\begin{aligned}
\left(P_{dj,t}\right)^{1-\sigma_j} &= \gamma_j \sum_{k=0}^{\infty} \mu_{j,t}(k)\left(\Phi_{dj,t-k}\right)^{-\frac{1-\sigma_j}{\theta_j}} \sum_s \left[\lambda^0_{sdj,t-k-1}\,\hat{c}^{-\theta_j}_{sdj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}} \\
&\qquad \times \exp\left\{\mathbf{1}\{k>0\}\log\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right\} \\
&= \sum_{k=0}^{\infty}\frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)}\left(P^0_{dj,t-k-1}\right)^{1-\sigma_j}\sum_n\left[\lambda^0_{ndj,t-k-1}\hat{c}^{-\theta_j}_{ndj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}} \\
&\qquad \times \exp\left\{\mathbf{1}\{k>0\}\log\left[\sum_s \lambda^0_{sdj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right]\right\}.
\end{aligned} \tag{3.47}$$

Recall that, by optimal demand, expenditure shares of each partition relative to total expenditures are

$$\frac{P^k_{dj,t}Y^k_{dj,t}}{P_{dj,t}Y_{dj,t}} = \left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j}$$

Total expenditure shares are therefore simply the weighted average of trade shares across partitions

$$\lambda_{sdj,t} \equiv \sum_{k=0}^{\infty} \frac{P^k_{dj,t}Y^k_{dj,t}}{P_{dj,t}Y_{dj,t}}\lambda^k_{sdj,t} = \sum_{k=0}^{\infty}\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j} \lambda^k_{sdj,t}, \tag{3.48}$$

which can also be stated as

$$\begin{aligned} \lambda_{sdj,t} &= \left(\frac{P^0_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j} \frac{\lambda^0_{sdj,t-1}\hat{c}^{-\theta_j}_{sdj,t}}{\sum_n \lambda^0_{ndj,t-1}\hat{c}^{-\theta_j}_{ndj,t}} \\ &+ \sum_{k=1}^{\infty}\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j} \frac{\lambda^0_{sdj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \lambda^0_{ndj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}}. \end{aligned}$$

$$(3.50)$$

Writing $\lambda^0_{sdj,t-k}$ and $\lambda^0_{ndj,t-k}$ recursively, we can express trade shares compactly as

$$\lambda_{sdj,t} = \sum_{k=0}^{\infty}\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j} \frac{\lambda^0_{sdj,t-k-1}\hat{c}^{-\theta_j}_{sdj,t-k}\exp\left\{\mathbf{1}\{k>0\}\log\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right\}}{\sum_n \lambda^0_{ndj,t-k-1}\hat{c}^{-\theta_j}_{ndj,t-k}\exp\left\{\mathbf{1}\{k>0\}\log\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}\right\}}. \tag{3.51}$$

3.8.5 Convergence

Results in the preceding subsection imply that trade shares can be expressed a sum over infinitely many partitions. We now establish regularity conditions for convergence.

Lemma 2 (Convergence). *If cumulative changes in trade costs are finite-valued* $\lim_{k\to\infty} |\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, *then price levels* $P^k_{dj,t} < \infty$ *and trade shares* $0 < \lambda_{dj,t} < 1$ *are finite-valued.*

Proof. Note that $\left(\Phi_{dj,t-k}\right)^{(\sigma_j-1)/\theta_j} < \infty$ and $\sum_s \left[\lambda^0_{sdj,t-k-1} A_{sj} \hat{c}^{-\theta_j}_{sdj,t-k}\right]^{(\sigma_j-1)/\theta_j} < \infty$ are both finite-valued, because they are equilibrium objects of a static equilibrium of the model. Also note that, for any $k > m$, if $|\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, then $|\prod_{\varsigma=t-m+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, since the product up to k includes every term in the product up to m. Therefore, if $\lim_{k\to\infty} |\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, then, for every $k < \infty$, the product will also be finite. It follows that $P^k_{dj,t} < \infty$ is finite valued for every k. Given that $\lim_{k\to\infty} \mu_{j,t}(k) = \lim_{k\to\infty} (1-\zeta_j)^k \zeta_j = 0$. These findings also guarantee that $P_{dj,t} < \infty$. □

3.8.6 Proofs

Proof of Proposition 1.

When the economy is in steady state, then for any $t <$ changes must satisfy $\hat{\mathbf{F}}_t = \hat{\mathbf{F}}_{\mathbb{1}}$ and $\hat{\mathbf{w}}_t = \hat{\mathbf{w}}_{\mathbb{1}}$ so that $\hat{c}_{s,t} = 1$ for all $s \in \mathcal{D}$. For the firms that are adjusting at t ($k = 0$), evaluating eq:trade-shares at those values, $\lambda^0_{sdj,t} = \lambda^0_{sdj,t-1} = \cdots = \lambda^0_{sdj,0}$ for all t. For the firms that are not adjusting at t ($k > 0$), we have $t - k > 0$ in equilibrium as long as the partition exists and can evaluate eq:trade-shares using the same logic as above: $\lambda^k_{sdj,t} = \lambda^0_{sdj,t-k} = \lambda^0_{sdj,0}$ for all t. From eq:trade-shares, it is easy to see that $\lambda_{sdj,t} = \lambda^0_{sdj,t}$, which shows that $\boldsymbol{\lambda}_t = \boldsymbol{\lambda}^{EK}$ in steady state.

To derive the stationary distribution of contract lengths, begin by noting that the case $k = 0$ is trivial, since $\mu(0) = \mathbb{P}[K_t = 0] = \zeta_j$ does not vary. Now consider the case $k > 0$. Note that:

$$\begin{aligned}\mathbb{P}[K_t = k, k > 0] &= \sum_{l=0}^{\infty} \mathbb{P}[K_t = k, k > 0 | K_{t-1} = l]\, \mathbb{P}[K_{t-1} = l] \\ &= (1-\zeta_j)\mathbb{P}[K_{t-1} = k-1]\end{aligned}$$

The remaining proof for $k > 0$ then follows by induction. For $K_t = 1$, $\mathbb{P}[K_t = 1] = (1-\zeta_j)\zeta_j$, and for $K_t = 2$, $\mathbb{P}[K_t = 2] = (1-\zeta_j)\mathbb{P}[K_{t-1} = 1] = (1-\zeta_j)^2\zeta_j$, and so forth recursively, for an arbitrary $K_t = k$ we must have $\mathbb{P}[K_t = k] = (1-\zeta_j)^k\zeta_j$. This is the probability density function of a geometric distribution with mean $(1-\zeta_j)/\zeta_j$ and standard deviation $\sqrt{1-\zeta_j}/\zeta_j$.

Finally, using the definition of the measure μ, $\mu_{j,t}(k) = \mathbb{P}[K_t = k]$ for $t \geq k$. Given the Markov property of K_t, the following distribution will be stationary for all $k \in \mathbb{N}_0$:

Proof of Proposition 2.

For ease of notation, we suppress sector subscripts throughout the derivations. Consider a one-time permanent change in trade costs such that $\hat{\tau}_{sd,t} \neq 1$ and $\hat{\tau}_{sd,t+h} = 1$ $\forall h > 0$. To characterize the partial trade elasticity at horizon h, we first characterize the elasticity for trade shares of each partition, then aggregate them up using the consumption shares derived from the CES preferences over partitions. The change in expenditure shares on intermediate goods in the kth partition in period $t+h$, relative to period $t-1$ is given by

$$\log\frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases} -(\sigma-1)\log\hat{\tau}_{sd,t} + \log\frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}}\left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})}\right)^{1-\sigma} & ,k\geq h \\ \log\frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}}\left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})}\right)^{1-\sigma} & ,1\leq k<h \\ \log\frac{\lambda^0_{sd,t+h-1}}{\lambda^k_{sd,t-1}}\left(\frac{(c_{s,t+h}/P^0_{d,t+h})}{(c_{s,t-1}/P^0_{d,t-1})}\right)^{\theta} & ,k=0 \end{cases}$$

The first line denotes intermediate goods that have not updated suppliers since the shock arrived. For such intermediate goods, changes in expenditure shares still explicitly depend on the shock to trade costs. The remaining intermediate goods have updated at least once, and a "new" optimal sourcing share $\lambda^0_{sd,t+h-k}$ from a time period between t and $t+h$ encodes the "initial price index" relative to which changes in expenditure shares are updated as well as the effect of the shock in trade costs. Unit costs are the relevant GE variables.

Denote

$$\Delta \boldsymbol{G}^{EK}_{sd,t,t+h} = -\theta\log\prod_{k=1}^{h}\frac{\hat{c}_{sd,t+k}}{\hat{P}^0_{sd,t+k}}$$

and

$$\Delta \boldsymbol{G}^{k}_{sd,\varsigma,t+h} = (1-\sigma)\log\prod_{\varsigma'=\varsigma+1}^{t+h}\frac{\hat{c}_{sd,\varsigma'}}{\hat{P}^k_{sd,\varsigma'}}$$

Then we can solve backwards to express all changes in trade shares above in terms of $\lambda^0_{sd,t-1}$, if possible:

$$\log\frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases} -(\sigma-1)\log\hat{\tau}_{sd,t} + \log\frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} + \Delta \boldsymbol{G}^k_{sd,t,t+h} & ,k\geq h \\ -\theta\log\hat{\tau}_{sd,t} + \log\frac{\lambda^0_{sd,t-1}}{\lambda^k_{sd,t-1}} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k} + \Delta \boldsymbol{G}^k_{sd,t+h-k,t+h} & ,1\leq k<h \\ -\theta\log\hat{\tau}_{sd,t} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h} & ,k=0 \end{cases}$$

Use the fact that outcomes determined at t and earlier do not respond to the change in trade costs. Hence, the elasticity of $\lambda^k_{sd,t+h}$ with respect to a change in trade costs at t, is hence given by,

$$\frac{\mathrm{d}\log(\lambda^k_{sd,t+h}/\lambda^k_{sd,t})}{\mathrm{d}\log\tau_{sd,t}} = \begin{cases} -(\sigma-1)+\frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} & ,k\geq h \\ -\theta+\frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k}}{\mathrm{d}\log\tau_{sd,t}}+\frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t+h-k,t+h}}{\mathrm{d}\log\tau_{sd,t}} & ,1\leq k<h \\ -\theta+\frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} & ,k=0 \end{cases}$$

To a first order, the change in overall expenditures at time $t+h$ caused by a one-time permanent shock to trade costs at t is given by

$$\begin{aligned}
\frac{\mathrm{d}\log(\lambda_{sd,t+h}/\lambda_{sd,t})}{\mathrm{d}\log\tau_{sd,t}} = {} & \textstyle\sum_{k=0}^{\infty}\omega_k\left\{\frac{\mathrm{d}\log\lambda^k_{sd,t+h}/\lambda^k_{sd,t}}{\mathrm{d}\log\tau_{sd,t}}+(1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
= {} & \textstyle\sum_{k=0}^{h-1}\omega_k\left\{-\theta+\frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}}+\frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t+h-k,t+h}}{\mathrm{d}\log\tau_{sd,t}}+(1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
& \textstyle+\sum_{k=h}^{\infty}\omega_k\left\{(1-\sigma)+\frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}}+(1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
= {} & \textstyle-\theta\sum_{k=0}^{h-1}\omega_k+(1-\sigma)\sum_{k=h}^{\infty}\omega_k \\
& \textstyle+\sum_{k=0}^{h-1}\omega_k\frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} \\
& \textstyle+\sum_{k=0}^{h-1}\omega_k(1-\sigma)\left\{\frac{\sum_{\varsigma=t+h-k+1}^{t+h}\mathrm{d}\log c_{sd,\varsigma}}{\mathrm{d}\log\tau_{sd,t}}+\frac{\sum_{\varsigma=t}^{t+h-k}\mathrm{d}\log P^k_{sd,\varsigma}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
& \textstyle+\sum_{k=h}^{\infty}\omega_k(1-\sigma)\left\{\frac{\sum_{i=0}^{t+h}\mathrm{d}\log c_{sd,t+i}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
& \textstyle-(1-\sigma)\frac{\sum_{i=0}^{h}\mathrm{d}\log P_{sd,t+i}}{\mathrm{d}\log\tau_{sd,t}}
\end{aligned}$$

where $\omega_k \equiv \frac{\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma}\lambda^k_{sdj,t}}{\sum_k\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma}\lambda^k_{sdj,t}} = \frac{\mu_t(k)\lambda^k_{sdj,t}}{\sum_k \mu_t(k)\lambda^k_{sdj,t}}$. If t was a steady state, then $\omega_k = \mu(k)$, and the partial horizon-h trade elasticity equals:

$$\varepsilon^{t+h}_{sd} \equiv \frac{\partial \log \lambda_{sdj,t+h}}{\partial \log \tau_{sd,t}} = -\theta \sum_{k=0}^{h-1} \mu(k) + (1-\sigma) \sum_{k=h}^{\infty} \mu(k).$$

Using the stationary distribution of $\mu_t(k)$ to substitute for $\mu(k)$, we obtain the expression stated in the main text.

3.8.7 Proof of Proposition 3.

We begin by rearranging eq:trade-shares to express the prices of composite goods in terms of home expenditure shares

$$\begin{aligned}
\lambda_{ddi,t}P^{1-\sigma_i}_{di,t} &= \gamma_i\mu_i(0)\left(\Phi^0_{di,t}\right)^{-\frac{1-\sigma_i}{\theta_i}}\lambda^0_{ddi,t} + \textstyle\sum_{k\geq 1}\gamma_i\mu_i(k)\left(\Phi^0_{di,t-k}\right)^{-\frac{1-\sigma_i}{\theta_i}}\Phi^k_{di,t}\lambda^k_{ddi,t} \\
&= \gamma_i\mu_i(0)\left(\Phi^0_{di,t}\right)^{-\frac{1-\sigma_i}{\theta_i}}\lambda^0_{ddi,t} + \textstyle\sum_{k\geq 1}\gamma_i\mu_i(k)\left(\Phi^0_{di,t-k}\right)^{-\frac{1-\sigma_i}{\theta_i}}\lambda^0_{ddi,t-k}\left(\frac{c_{dd,t}}{c_{dd,t-k}}\right)^{1-\sigma_i} \\
&= \gamma_i\mu_i(0)\left(\frac{c^{-\theta_i}_{dd,t}}{\lambda^0_{ddi,t}}\right)^{-\frac{1-\sigma_i}{\theta_i}}\lambda^0_{ddi,t} \\
&\quad + \textstyle\sum_{k\geq 1}\gamma_i\mu_i(k)\left(\frac{c^{-\theta_i}_{dd,t-k}}{\lambda^0_{ddi,t-k}}\right)^{-\frac{1-\sigma_i}{\theta_i}}\lambda^0_{ddi,t-k}\left(\frac{c_{dd,t}}{c_{dd,t-k}}\right)^{1-\sigma_i}.
\end{aligned}$$

It follows that

$$P^{1-\sigma_i}_{di,t} = c^{1-\sigma_i}_{dd,t}\left(\lambda^0_{ddi,t}\right)^{\frac{1-\sigma_i}{\theta_i}}\frac{1}{\lambda_{ddi,t}}\gamma_i\left[\mu_i(0)\lambda^0_{ddi,t} + \sum_{k\geq 1}\mu_i(k)\left(\frac{\lambda^0_{ddi,t}}{\lambda^0_{ddi,t-k}}\right)^{-\frac{1-\sigma_i}{\theta_i}}\lambda^0_{ddi,t-k}\right] \tag{3.52}$$

where the price index is expressed in terms of unit cost and domestic trade shares. With the unit cost under Cobb-Douglas technology, the above equation can be rewritten as

$$\frac{P_{di,t}}{w_{d,t}} = \left(\lambda^0_{ddi,t}\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_i - 1)} \left(\gamma_i \xi_{di,t}\right)^{1/(1-\sigma_i)} \alpha_{di}^{-\alpha_{di}} \prod_j \left(\frac{P_{dj,t}}{\alpha_{dji} w_{d,t}}\right)^{\alpha_{dji}}$$

where

$$\xi_{di,t} \equiv \mu_i(0)\lambda^0_{ddi,t} + \sum_{k\geq 1} \mu_i(k) \left(\frac{\lambda^0_{ddi,t}}{\lambda^0_{ddi,t-k}}\right)^{\frac{\sigma_i - 1}{\theta_i}} \lambda^0_{ddi,t-k}.$$

Taking logs yields

$$\log \frac{P_{di,t}}{w_{d,t}} = \log B_{si,t} + \sum_j \alpha_{sji} \log \frac{P_{sj,t}}{w_{s,t}}$$

where $B_{di,t} \equiv \alpha_{di}^{-\alpha_{di}} \left(\prod_j \alpha_{dji}^{-\alpha_{dji}}\right) \left(\lambda^0_{ddi,t}\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_i-1)} \left(\gamma_i \xi_{di,t}\right)^{1/(1-\sigma_i)}$. In matrix notation, this leads to

$$(\mathbf{I} - A_d) \log \hat{\boldsymbol{P}}_{d,t} = \log \boldsymbol{B}_{d,t},$$

where $A_d = \left\{\alpha_{dji}\right\}$ and $\log \hat{\boldsymbol{P}}_{d,t}$ and $\log \boldsymbol{B}_{d,t}$ are $I \times 1$ vectors. Inverting this system of equations, we obtain

$$\frac{P_{di,t}}{w_{d,t}} = \prod_j B_{dj,t}^{\bar{a}_{dji}},$$

where $\bar{a}_{dji}$ is the (j, i) entry of the Leontief matrix $(\mathbf{I} - A_d)^{-1}$. The consumer price index in country d can be written as

$$P_{d,t} = \prod_i \left(P_{di,t}\right)^{\eta_i} = w_{d,t} \prod_{i,j} B_{dj,t}^{\bar{a}_{dji}\eta_i} = w_{d,t} \prod_j B_{dj,t}^{\sum_i \bar{a}_{dji}\eta_i}$$

It follows that the real wage is given by

$$W_{d,t} \equiv \frac{w_{d,t}}{P_{d,t}} = \prod_j B_{dj,t}^{-\sum_i \bar{a}_{dji}\eta_i}.$$

Taking the ratio between real wages in $t-1$ and $t+h$ yields

$$\frac{W_{d,t+h}}{W_{d,t-1}} = \prod_j \left[\left(\frac{\lambda^0_{ddj,t+h}}{\lambda^0_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}} \right)^{-\frac{1}{\sigma_j - 1}} \left(\frac{\xi_{dj,t+h}}{\xi_{dj,t-1}} \right)^{\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji} \eta_i},$$

If $t-1$ is a steady state, then $\lambda^k_{ddj,t-1} = \lambda_{ddj,t-1}$ for all $k \in \{0,1,2,...\}$ and the above expression simplifies to

$$\frac{W_{d,t+h}}{W_{d,t-1}} = \prod_j \left[\left(\frac{\lambda^0_{ddj,t+h}}{\lambda^0_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}} \right)^{-\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji} \eta_i} \tag{3.53}$$

$$= \prod_j \left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda^0_{ddj,t+h}}{\lambda_{ddj,t+h}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}} \right)^{-\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji} \eta_i} \tag{3.54}$$

$$= \prod_j \left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h} \right)^{\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji} \eta_i}, \tag{3.55}$$

where

$$\Xi_{dj,t} \equiv \zeta_j \left(\frac{\lambda_{ddj,t+h}}{\lambda^0_{ddj,t+h}} \right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} + \sum_{k=1}^{h} \zeta_j (1-\zeta_j)^k \left(\frac{\lambda_{ddj,t+h}}{\lambda^0_{ddj,t+h-k}} \right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} + (1-\zeta_j)^{h+1} \left(\frac{\lambda_{ddi,t+h}}{\lambda_{ddj,t-1}} \right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} \tag{3.56}$$

is obtained after combining the last two factors in eq:acrl2.

Works cited in Chapter 3

Anderson, **James** and **Yotov**, **Yoto** (Dec. 2022). *Estimating Gravity from the Short to the Long Run: A Simple Solution to the 'International Elasticity Puzzle'*. en. Tech. rep. w30809. Cambridge, MA: National Bureau of Economic Research, w30809. DOI: 10.3386/w30809. URL: http://www.nber.org/papers/w30809.pdf (visited on 05/27/2024).

Arkolakis, **Costas**, **Costinot**, **Arnaud**, and **Rodríguez-Clare**, **Andrés** (Feb. 2012). New Trade Models, Same Old Gains? *American Economic Review*. 2012. 102, pp. 94–130.

Arkolakis, **Costas**, **Eaton**, **Jonathan**, and **Kortum**, **Samuel S** (2011). Staggered Adjustment and Trade Dynamics (preliminary and incomplete). *Unpublished Manuscript, Yale University*. 2011.

Armington, **Paul S.** (Mar. 1969). A Theory of Demand for Products Distinguished by Place of Production (Une theorie de la demande de produits differencies d'apres leur origine) (Una teoria de la demanda de productos distinguiendolos segun el lugar de produccion). *Staff Papers - International Monetary Fund*. 1969. 16, p. 159.

Boehm, **Christoph E.**, **Levchenko**, **Andrei A.**, and **Pandalai-Nayar**, **Nitya** (Apr. 2023). The Long and Short (Run) of Trade Elasticities. *American Economic Review*. 2023. 113, pp. 861–905.

Calvo, **Guillermo A.** (Sept. 1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*. 1983. 12, pp. 383–398.

Dekle, **Robert**, **Eaton**, **Jonathan**, and **Kortum**, **Samuel** (May 2007). Unbalanced Trade. *American Economic Review*. 2007. 97, pp. 351–355.

Dekle, **Robert**, **Eaton**, **Jonathan**, and **Kortum**, **Samuel** (Mar. 2008). Global Rebalancing with Gravity: Measuring the Burden of Adjustment. Working Paper. DOI: 10.3386/w13846. URL: https://www.nber.org/papers/w13846 (visited on 08/05/2022).

Eaton, **Jonathan** and **Kortum**, **Samuel** (2002). Technology, Geography, and Trade. *Econometrica*. 2002. 70, pp. 1741–1779.

– (May 2012). Putting Ricardo to Work. *Journal of Economic Perspectives*. 2012. 26, pp. 65–90.

Fajgelbaum, **Pablo D**, **Goldberg**, **Pinelopi K**, **Kennedy**, **Patrick J**, and **Khandelwal**, **Amit K** (Feb. 2020). The Return to Protectionism*. *The Quarterly Journal of Economics*. 2020. 135, pp. 1–55.

Fontagné, **Lionel**, **Guimbard**, **Houssein**, and **Orefice**, **Gianluca** (July 2022). Tariff-based product-level trade elasticities. *Journal of International Economics*. 2022. 137, p. 103593.

Fontagné, **Lionel**, **Martin**, **Philippe**, and **Orefice**, **Gianluca** (Nov. 2018). The international elasticity puzzle is worse than you think. *Journal of International Economics*. 2018. 115, pp. 115–129.

Kollintzas, **Tryphon** and **Zhou**, **Ruilin** (Apr. 1992). Import price adjustments with staggered import contracts. *Journal of Economic Dynamics and Control*. 1992. 16, pp. 289–315.

Lucas, **Robert E.** and **Prescott**, **Edward C.** (Sept. 1971). Investment Under Uncertainty. *Econometrica*. 1971. 39, p. 659.

Montero, **Miquel** and **Villarroel**, **Javier** (Sept. 2016). Directed random walk with random restarts: The Sisyphus random walk. *Physical Review E*. 2016. 94, p. 032132.

Ruhl, **Kim J** (n.d.). The International Elasticity Puzzle. Working Paper 08-30.

Souza, **Gustavo de**, **Hu**, **Naiyuan**, **Li**, **Haishi**, and **Mei**, **Yuan** (Mar. 2024). (Trade) War and peace: How to impose international trade sanctions. *Journal of Monetary Economics*. 2024, p. 103572.

Yilmazkuday, **Hakan** (Oct. 2019). Estimating the trade elasticity over time. *Economics Letters*. 2019. 183, p. 108579.