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DUALITY ROTATIONS

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INTRODUCTION

The invariance of Maxwell's equations under "duality rotations" has been known for a long time. These are rotations of the electric and magnetic fields into each other or, in relativistic notation, rotations of the electromagnetic field strength $F_{\mu\nu}$ into its dual

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma} \tag{1.1}$$

This invariance can be easily extended to the case when the electromagnetic field is in interaction with the gravitational field, which does not transform under duality (Misner and Wheeler 1957). Minimal electromagnetic couplings violate duality invariance and it is also easy to see that the Yang-Mills equations do not admit an invariance of this type (Deser and Teitelboim 1976).

Non-minimal couplings of the magnetic moment type can be duality invariant and, in some cases, this invariance generalizes to a non-abelian group. This happens in extended supergravity theories without gauging of the SO(N) symmetry (Ferrara, Scherk and Zumino 1977). The assumption that the theory is invariant under duality rotations can be used to simplify the construction of the correct supersymmetric Lagrangian (Cremmer and Scherk 1977, Cremmer et al. 1977). For $N = 4$ supergravity the U(4) duality extends to a larger $SU(4) \times SU(1, 1)$ non-compact duality invariance (Cremmer et al. 1978) and a similar situation occurs for $N > 4$; in particular for $N = 8$ the theory is invariant under a non-compact E_7 duality (Cremmer and Julia 1979). A non compact duality invariance arises when there are scalar fields in the theory, which can transform non-linearly.

Irrespective of supersymmetry, it is interesting to understand the special properties of theories admitting duality rotations. As we shall see, the Lagrangian of such a theory is not invariant under the transformations, nor does it change by a total derivative, but it transforms in a particular way which implies that the system of the equations of motion is invariant and that observables, such as the energy momentum tensor and therefore the total energy and momentum, are invariant. In this lecture I describe the main results of a recent paper on the properties of theories admitting duality rotations written in collaboration with M. K. Gaillard (1981).

As an example, consider the Lagrangian (Ferrara et al. 1977)

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{i}{2} \bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\psi + \frac{a}{2} F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi + b(\bar{\psi}\sigma_{\mu\nu}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi), \quad (1.2)$$

where $F_{\mu\nu}$ is the curl of a vector potential and ψ is a massless Dirac spinor. (Our gamma matrices are real, $(\gamma_5)^2 = -1$, $\sigma_{\mu\nu}\gamma_5 = \tilde{\sigma}_{\mu\nu}$. The ordinary space-time derivative is denoted by ∂_μ.) The equations of motion for $F_{\mu\nu}$ are

$$\partial_\mu(F^{\mu\nu} - a\bar{\psi}\sigma^{\mu\nu}\psi) = 0 \quad (1.3)$$

together with the Bianchi identities

$$\partial_\mu\tilde{F}^{\mu\nu} = 0. \quad (1.4)$$

Clearly (1.3) and (1.4) transform into each other by the duality rotation

$$\delta F_{\mu\nu} = \lambda\tilde{F}_{\mu\nu} - \lambda a\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi \quad (1.5)$$

$$\delta\psi = -\frac{\lambda}{2}\gamma_5\psi \quad (1.6)$$

where λ is an infinitesimal real parameter. In order for the theory to be invariant under (1.5) and (1.6), one must check that the equation of motion for ψ is also invariant. It is easy to verify by explicit calculation that this is true if the coupling constant b is related to the magnetic moment coupling a by

$$b = \frac{1}{8} a^2. \quad (1.7)$$

Duality invariance gives relations among couplings. It is not an invariance of the Lagrangian, and it is also easy to see that the Lagrangian does not simply change by a divergence (except if one uses the equations of motion, but for the vector fields only). One can also verify quite easily (and it follows from the general argument below) that the energy momentum tensor (both the canonical and the symmetric) is invariant. Since the total energy and the equations of motion are invariant, it follows that the S-matrix is invariant under the transformation which operates on the "in" and "out" fields as in (1.5) (1.6) but with the coupling constant a set equal to zero. This is, of course, a purely formal statement, which ignores difficulties in the definition of the S-matrix, due to the vanishing masses and the non-renormalizability of the theory.

The duality transformations (1.5) (1.6) should not be confused with the chiral transformations

$$\delta F_{\mu\nu} = 0 \tag{1.8}$$

$$\delta\psi = \alpha\gamma_5\psi^* \tag{1.9}$$

(α infinitesimal constant parameter) under which the Lagrangian is actually invariant.

DUALITY ROTATIONS

Consider a Lagrangian which is a function of n real field strength $F^a_{\mu\nu}$ and of some other fields χ^i and their derivatives $\chi^i_\mu = \partial_\mu\chi^i$

$$L = L(F^a, \chi^i, \chi^i_\mu). \tag{2.1}$$

Since

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\mu A^a_\mu, \tag{2.2}$$

we have the Bianchi identities

$$\partial^\mu \tilde{F}^a_{\mu\nu} = 0. \tag{2.3}$$

On the other hand, if we define

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}G^{a\lambda\sigma} \equiv 2\frac{\partial L}{\partial F^{a\mu\nu}}, \tag{2.4}$$

we have the equations of motion

$$\partial^\mu \tilde{G}^a_{\mu\nu} = 0. \tag{2.5}$$

We consider an infinitesimal transformation of the form

$$\delta\binom{F}{G} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\binom{F}{G}, \tag{2.6}$$

$$\delta\chi^i = \xi^i(\chi), \tag{2.7}$$

where A, B, C, D are real $n \times n$ constant infinitesimal matrices and $\xi^i(\chi)$ functions of the fields χ^i (but not of their derivatives), and ask under what circumstances the system of the equations of motion (2.3) (2.5) as well as the equations of motion for the fields χ^i are invariant. The analysis of Gaillard and myself (1981) shows that this is true if the matrices satisfy

$$A^T = - D,\ B^T = B,\ C^T = C, \tag{2.8}$$

where the superscript T denotes the transposed matrix, and the Lagrangian changes under (2.6) and (2.7) as

$$\delta L = \frac{1}{4}(FC\tilde{F} + GB\tilde{G}). \tag{2.9}$$

One can also see that this is essentially the most general possibility. The relations (2.8) show that (2.6) is an infinitesimal transformation of the real non compact symplectic group Sp(2n, R) which has U(n) as maximal compact subgroup. Clearly, particular theories may be only invariant under subgroups of the above. This is the case, for instance, for N = 8 supergr ity, where n = 28 (number of vector fields) but where the additional equirement of supersymmetry not only determines the particle spectru but also restricts Sp(56, R) and U(28) to their subgroups E_7 and SU(8).

Observe that the equations of motion (2.5) imply the existence of vector potentials such that

$$G^a_{\mu\nu} = \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu. \tag{2.10}$$

Using (2.2) and (2.10) one can write the right hand side of (2.9) as a divergence

$$\delta L = \frac{1}{2}\partial_\mu (A_\nu C\tilde{F}^{\mu\nu} + \mathcal{B}_\nu B\tilde{G}^{\mu\nu}), \tag{2.11}$$

but this is true only in virtue of the equations of motion for $F^a_{\mu\nu}$. Now the variation of the Lagrangian induced by a variation of the fields F^a only is, by (2.6),

$$\delta_F L = \delta F^a \frac{\partial L}{\partial F^a} = \frac{1}{2}(FA^T + GB)\tilde{G} \tag{2.12}$$

which, by using again the equations for $F^a_{\mu\nu}$, can be written as

$$\delta_F L = \partial_\mu (A_\nu A^T \tilde{G}^{\mu\nu} + \mathcal{B}_\nu B\tilde{G}^{\mu\nu}). \tag{2.13}$$

Therefore, using (2.8),

$$\begin{aligned}\delta_\chi L = (\delta - \delta_F)L &= \partial_\mu (\tfrac{1}{2} A_\nu C\tilde{F}^{\mu\nu} - \tfrac{1}{2}\mathcal{B}_\nu B\tilde{G}^{\mu\nu} + A_\nu D\tilde{G}^{\mu\nu}). \\ &= \frac{1}{2}\partial_\mu (A_\nu C\tilde{F}^{\mu\nu} - \mathcal{B}_\nu B\tilde{G}^{\mu\nu} + A_\nu D\tilde{G}^{\mu\nu} - \mathcal{B}_\nu A\tilde{F}^{\mu\nu}) \\ &= -\frac{1}{2}\partial_\mu \hat{J}^\mu.\end{aligned} \tag{2.14}$$

where

$$\hat{J}^\mu \equiv - A_\nu C\tilde{F}^{\mu\nu} + \mathcal{B}_\nu B\tilde{G}^{\mu\nu} - A_\nu D\tilde{G}^{\mu\nu} + \mathcal{B}_\nu A\tilde{F}^{\mu\nu}. \tag{2.15}$$

So far we have used the equations of motion for $F^a_{\mu\nu}$. Now, by the standard argument due to Emmy Noether, we know that the equations of motion for χ^i imply

$$\partial_\mu \left(\delta\chi^i \frac{\partial L}{\partial \chi^i_\mu}\right) = \delta_\chi L \tag{2.16}$$

Therefore, using all equations of motion we see that the current

$$J^\mu = \xi^i \frac{\partial L}{\partial \chi^i_\mu} + \hat{J}^\mu \tag{2.17}$$

is conserved

$$\partial_\mu J^\mu = 0. \tag{2.18}$$

The current (2.17) is not invariant under the gauge transformations

$$A^a_\mu \to A^a_\mu + \partial_\mu \alpha^a \tag{2.19}$$

$$B^a_\mu \to B^a_\mu + \partial_\mu \beta^a$$

which leave invariant (2.2) and (2.10). Instead, it changes as

$$J_\mu \to J_\mu - \frac{1}{2}\partial_\nu(\alpha C\tilde{F}^{\mu\nu} - \beta B\tilde{G}^{\mu\nu} + \alpha D\tilde{G}^{\mu\nu} - \beta A\tilde{F}^{\mu\nu}) \tag{2.20}$$

The corresponding integrated charge $\int J^0 d^3x$ is gauge invariant. One can see that it is actually the generator of the duality transformations, by using a Coulomb-like gauge and developing the appropriate canonical formalism.

The fact that the duality currents are not gauge invariant and therefore, as operators, are not true Lorentz vectors, shows that one cannot apply here the usual arguments (Colemen and Witten 1980, Weinberg and Witten 1980) according to which massless spin one states carrying the associated charge cannot exist. This is relevant for some recent attempts to connect supergravity with particle phenomenology (Ellis et al. 1980) in which one postulates that such massless spin one bound states arise dynamically.

Although the Lagrangian is not invariant under the transformations (2.6) (2.7), the derivative of the Lagrangian with respect to an invariant parameter is invariant. Assume that L depends upon an invariant parameter λ. If $\xi^i(\chi)$ is independent of λ, we differentiate (2.9) with respect to λ and obtain

$$\frac{\partial \delta L}{\partial \lambda} = \frac{1}{2}\tilde{G}B\frac{\partial G}{\partial \lambda} = \frac{\partial L}{\partial F}B\frac{\partial G}{\partial \lambda} \quad . \tag{2.21}$$

On the other hand, since

$$\delta L = \left(\xi^i \frac{\partial}{\partial \chi^i} + \chi^j_\mu \frac{\partial \xi^i}{\partial \chi^j}\frac{\partial}{\partial \chi^i_\mu} + (FA^T + GB^T)\frac{\partial}{\partial F}\right)L, \tag{2.22}$$

it follows that

$$\frac{\partial \delta L}{\partial \lambda} = \delta\frac{\partial L}{\partial \lambda} + \frac{\partial G}{\partial \lambda}B^T\frac{\partial L}{\partial F} \; . \tag{2.23}$$

Comparing (2.21) and (2.23), we find

$$\delta\frac{\partial L}{\partial \lambda} = 0. \tag{2.24}$$

The parameter λ could be a coupling constant. For instance, in the example of the introduction, differentiate L given by (1.2) with respect to a, with the condition (1.7),

$$\frac{\partial L}{\partial a} = \frac{1}{2}F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi + \frac{1}{4}a(\bar{\psi}\sigma_{\mu\nu}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi) \, . \tag{2.25}$$

It is easy to check that this expression is invariant under (1.5) (1.6). The result (2.24) provides a way of checking that a theory admits duality rotations or of constructing the Lagrangian for such a theory, by switching on couplings in an invariant way. The case when the ξ^i depend on λ is a little more delicate (see Gaillard and Zumino 1981).

If λ represents an external gravitational field, (2.23) implies that the energy momentum tensor, which is the variational derivative of the Lagrangian with respect to the graviational field, is invariant under duality rotations.

A Lagrangian satisfying (2.9) can be constructed by observing that, from (2.6) (2.8),

$$\frac{1}{4}\delta(F\tilde{G}) = \frac{1}{4}(FC\tilde{F} + GB\tilde{G}) \, . \tag{2.26}$$

Therefore

$$L = \frac{1}{4}FG + L_{inv} \, , \tag{2.27}$$

where L_{inv} is actually invariant under (2.6) (2.7). For instance, one can easily check that (1.2), with (1.7), is of this form, with

$$\tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}} = -F_{\mu\nu} + a\bar{\psi}\sigma_{\mu\nu}\psi \tag{2.28}$$

and

$$L_{inv} = -\frac{i}{2}\bar{\psi}\gamma^\mu\overleftrightarrow{\partial}_\mu\psi + \frac{a}{4}F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi + \frac{a^2}{8}(\bar{\psi}\sigma_{\mu\nu}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi) \, . \tag{2.29}$$

In general (2.27) can be used to construct Lagrangians, as described in

Gaillard and Zumino (1981). What we need is the expression for G as a function of F and χ. The algebra is considerably simplified if one introduces the operator j which changes an antisymmetric tensor into its dual (see Misner and Wheeler 1967, Cremmer and Julia 1979)

$$jT_{\mu\nu} = \tilde{T}_{\mu\nu}$$
$$(j)^2 = -1. \tag{2.30}$$

For many purposes this operator can be used in much the same way as the usual imaginary unit i. Let us assume that G is linear in F and write

$$G = jKF + X \tag{2.31}$$

where the matrix $K(\chi)$ and $X(\chi)$ are functions of the fields χ^i and may contain j. Now (2.6) and (2.7) imply that

$$\delta K = -jC - jKBK + DK - KA, \tag{2.32}$$
$$\delta X = DX - jKBX. \tag{2.33}$$

Introducing two antisymmetric Lorentz tensors $(H_{\mu\nu}(\chi), I_{\mu\nu}(\chi))$ which transform under (2.6) (2.7) like $(F_{\mu\nu}, G_{\mu\nu})$, the quantity

$$X = -jI - KH \tag{2.34}$$

satisfies (2.33). The Lagrangian (2.27) becomes

$$L = -\frac{1}{4} FKF + \frac{1}{4} jFX + L_{inv}$$
$$= -\frac{1}{4} FKF + \frac{1}{4} F(I - jKH) + L_{inv}. \tag{2.35}$$

We cannot assume that L_{inv} depends only upon the fields χ^i because (2.35), using (2.4), must reproduce (2.31), Therefore L_{inv} must contain the invariant, linear in F,

$$\frac{1}{4}(FI - GH) = \frac{1}{4} F(I - jKH) + \frac{1}{4} jH(I - jKH). \tag{2.36}$$

The result is finally

$$L = -\frac{1}{4} FKF + \frac{1}{2} F(I - jKH) + \frac{1}{4} jH(I - jKH) + L_{inv}(\chi), \tag{2.37}$$

where the last term depends only on the fields χ^i and their derivatives.

From the transformation property (2.32) and from (2.8) we see that the matrix K can be taken to be symmetric. As we shall see in the next section, it can be taken to be a function of scalar fields only and it has the form

$$K = 1 + \ldots \tag{2.39}$$

where the dots represent terms which vanish with the scalar fields, so that the first term in the right hand side of (2.37) contains the kinetic term for the vector fields. When there are no scalars and the duality rotations are restricted to the compact subgroup, the matrix K is just equal to the unit matrix, as in the simple example described in the introduction.

SCALAR FIELDS

Scalar fields valued in the quotient (coset) space Sp(2n, R)/U(n) can be described by a group element of Sp(2n, R) represented by the matrix

$$g = \begin{pmatrix} \phi_o & \phi_1^* \\ \phi_1 & \phi_o^* \end{pmatrix}, \qquad (3.1)$$

where ϕ_o and ϕ_1 are complex n × n matrices satisfying

$$\phi_o^\dagger \phi_o - \phi_1^\dagger \phi_1 = 1, \qquad (3.2)$$

$$\phi_o^T \phi_1 = \phi_1^T \phi_o. \qquad (3.3)$$

One can insure that the scalars are in the quotient space by requiring the theory to be invariant under the gauge transformation

$$g(x) \rightarrow g(x)[k(x)]^{-1}, \qquad (3.4)$$

where k(x) is an element of U(n) represented by the matrix

$$k = \begin{pmatrix} U & o \\ o & U^* \end{pmatrix}, \qquad (3.5)$$

$$U^\dagger U = 1. \qquad (3.6)$$

Alternatively, one can parameterize the quotient space by using the n × n matrix (symmetric in virtue of (3.3))

$$Z = \phi_1 \phi_o^{-1}, \quad Z^T = Z \qquad (3.7)$$

which is invariant under the gauge transformation (3.3).

The effect of Sp(2n, R) on the quotient space is described by the rigid transformation

$$g(x) \rightarrow g_o g(x), \qquad (3.8)$$

where g_o belongs to Sp(2n, R). Therefore we require the Lagrangian to be invariant under (3.8) also. It is sometimes convenient to use (3.4) to go to a special gauge, in other words to choose a representative for the equivalence class. In order to reestablish the special gauge, (3.8) must then be accompanied by a suitable transformation of the type (3.4). This gives rise to a non-linear realization of Sp(2n, R). Here we shall work in an arbitrary gauge and require separate invariance under (3.4) and (3.8).

In order to construct the invariant Lagrangian (see Gaillard and Zumino 1981 and references therein) we note that $g^{-1}\partial_\mu g$ belongs to the Lie algebra of Sp(2n, R) and can be split into a part Q_μ which is in the Lie algebra of U(n) and a part P_μ perpendicular to it

$$g^{-1}\partial_\mu g = Q_\mu + P_\mu . \tag{3.9}$$

Under (3.8) this expression is invariant, while under (3.4) it transforms as

$$g^{-1}\partial_\mu g \to k(g^{-1}\partial_\mu g - k^{-1}\partial_\mu k)k^{-1}, \tag{3.10}$$

so that

$$Q_\mu \to kQ_\mu k^{-1} - \partial_\mu k\, k^{-1}, \tag{3.11}$$

$$P_\mu \to kP_\mu k^{-1}. \tag{3.12}$$

Consequently the Lagrangian

$$L = -\frac{1}{2}\,\mathrm{Tr}P_\mu^2 \tag{3.13}$$

is invariant and contains the kinetic term for the scalar fields. This formula can be made more explicit by observing that, form (3.2)(3.3),

$$g^{-1} = \begin{pmatrix} \phi_o^\dagger & -\phi_1^\dagger \\ -\phi_1^T & \phi_o^T \end{pmatrix}. \tag{3.14}$$

One then finds

$$Q_\mu = \begin{pmatrix} \phi_o^\dagger\partial_\mu\phi_o - \phi_1^\dagger\partial_\mu\phi_1 & 0 \\ 0 & -\phi_1^T\partial_\mu\phi_1^* + \phi_o^T\partial_\mu\phi_o^* \end{pmatrix}, \tag{3.15}$$

$$P_\mu = \begin{pmatrix} 0 & \phi_o^\dagger\partial_\mu\phi_1^* - \phi_1^\dagger\partial_\mu\phi_o^* \\ -\phi_1^T\partial_\mu\phi_o + \phi_o^T\partial_\mu\phi_1 & 0 \end{pmatrix}. \tag{3.16}$$

Using (3.16) in (3.13) one finds, with a little algebra,

$$L = -\,\mathrm{tr}\{\partial_\mu Z(1 - Z^\dagger Z)^{-1}\partial_\mu Z^\dagger(1 - ZZ^\dagger)^{-1}\} . \tag{3.17}$$

The infinitesimal form of (3.8) is

$$\delta g = \begin{pmatrix} T & V^* \\ V & T^* \end{pmatrix} g , \tag{3.18}$$

where

$$T = -T^\dagger = M - iN$$

$$V = V^T = R - iS \tag{3.19}$$

and the real matrices M, N, R, S are related to those of equation (2.6) by

$$A = M + R,\ B = S + N,\ C = S - N,\ D = M - R. \tag{3.20}$$

This corresponds to using the complex basis $F \pm iG$

$$\delta\begin{pmatrix} F + iG \\ F - iG \end{pmatrix} = \begin{pmatrix} T & V^* \\ V & T^* \end{pmatrix} \begin{pmatrix} F + iG \\ F - iG \end{pmatrix}. \tag{3.21}$$

The transformation on Z induced by (3.18) is easily worked out to be

$$\delta Z = R - ZRZ - iS - iZSZ + [M, Z] + i\{N, Z\}. \tag{3.22}$$

Note that the matrix Z^{-1} transforms exactly like $Z^* = Z^\dagger$. From the definition (3.7) and from (3.2) it follows that

$$1 - Z^\dagger Z = (\phi_o \phi_o^\dagger)^{-1}. \tag{3.23}$$

Since the right hand side is a positive matrix, this means that the eigenvalues of $Z^\dagger Z$ are smaller than one. Note that, if one introduces

$$K = \frac{1 - Z^*}{1 + Z^*} \tag{3.24}$$

one finds, from (3.22),

$$\delta K = [M, K] - \{R, K\} - iK(S + N)K - i(S - N). \tag{3.25}$$

This is the same as (2.32), if one uses (3.20) and replaces $i \to j$. Therefore (3.24), with the replacement $i \to j$, gives the matrix K which is to be used in the Lagrangian (2.37).

It is not difficult to introduce other fields besides the scalars. Let the field ψ be invariant under (3.8) and let it transform as

$$\psi(x) \to k(x)\psi(x) \tag{3.26}$$

under (3.4). From (3.11) we see that

$$D_\mu \psi = \partial_\mu \psi + Q_\mu \psi \tag{3.27}$$

is a covariant derivative (here k and Q_μ are matrices in the appropriate representation). Using (3.27) one can construct invariant Lagrangians for fields other than the scalars, e.g. spinors or Rarita - Schwinger fields.

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