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A FAST MOM SOLVER (GIFFT) FOR LARGE ARRAYS OF MICROSTRIP AND CAVITY-BACKED ANTENNAS

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INTRODUCTION AND SUMMARY OF THE GIFFT METHOD

A straightforward numerical analysis of large arrays of arbitrary contour (and possibly missing elements) requires large memory storage and long computation times. Several techniques are currently under development to reduce this cost. One such technique is the GIFFT (Green's function interpolation and FFT) method discussed here that belongs to the class of fast solvers for large structures. This method uses a modification of the standard AIM approach [1] that takes into account the reusability properties of matrices that arise from identical array elements. If the array consists of planar conducting bodies, the array elements are meshed using standard subdomain basis functions, such as the RWG basis. The Green's function is then projected onto a sparse regular grid of separable interpolating polynomials. This grid can then be used in a 2D or 3D FFT to accelerate the matrix-vector product used in an iterative solver [2]. The method has been proven to greatly reduce solve time by speeding up the matrix-vector product computation. The GIFFT approach also reduces fill time and memory requirements, since only the near element interactions need to be calculated exactly. The present work extends GIFFT to layered material Green's functions and multiregion interactions via slots in ground planes. In addition, a preconditioner is implemented to greatly reduce the number of iterations required for a solution. The general scheme of the GIFFT method is reported in [2]; this contribution is limited to presenting new results for array antennas made of slot-excited patches and cavity-backed patch antennas.

FEED REGION AND RADIATION REGION: DEFINITION OF INTERPOLATION DOMAIN

The antenna structure is shown in Fig.1. In particular, the region above the ground plane may include a multilayered substrate with N conducting patches fed by a slot. Below the slot, the feed of each antenna is assumed not to interfere with the feed networks of other antennas. Mutual coupling between the patches and the slots is considered in the region above the ground plane. Therefore, the only approximation used in this approach is to neglect coupling between the microstrips and slots in the region below the lower ground plane. The multiport analysis obtained from this approach may be subsequently used as a multiport equivalent network for designing (or refining) the actual feeding network. Array scan blindness, grating lobe, and array edge effects are correctly taken into account since they are produced by the mutual coupling above the ground plane. Voltage generators $V_g^{\mathbf{p}}$, with $\mathbf{p}=(p_1,p_2)$ a generic double index, are defined on the microstrips below every slot (see Fig. 1a). The array is decomposed into blocks of elements with each element denoted by the two-component multi-index $\mathbf{p}$; a prime is added to distinguish source from observation element locations ($\mathbf{p}'=(p_1',p_2')$). Within each block representing an element, the electric and magnetic currents are expressed in terms of the usual basis functions. For example, for the patch antennas in Fig.1 (a,b) the vanishing of the tangential electric field (EFIE) is imposed on every patch element and on the microstrip lines, while on the slots we impose continuity of both the electric and the magnetic field (MFIE). Electric current unknowns are defined on the conducting patches $\left[\mathrm{I}_n^{\mathbf{p}'}\right]$ and microstrip ($\left[\bar{\mathrm{I}}_n^{\mathbf{p}'}\right]$), while magnetic current unknowns $\left[V_n^{\mathbf{p}'}\right]$ are placed on the slots, resulting in the system equation

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$$\begin{pmatrix} \left[Z_{mn}^{pp'+}\right] & 0 & \left[-\beta_{mn}^{pp'+}\right] \\ 0 & 0 & 0 \\ \left[\beta_{mn}^{pp'+}\right] & 0 & \left[Y_{mn}^{pp'+}\right] \end{pmatrix} \begin{pmatrix} \left[I_n^{p'}\right] \\ \left[\bar{I}_n^{p'}\right] \\ \left[V_n^{p'}\right] \end{pmatrix} + \delta_{p,p'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \left[Z_{mn}^{-}\right] & \left[-\beta_{mn}^{-}\right] \\ 0 & \left[\beta_{mn}^{-}\right] & \left[Y_{mn}^{-}\right] \end{pmatrix} \begin{pmatrix} \left[I_n^{p'}\right] \\ \left[\bar{I}_n^{p'}\right] \\ \left[V_n^{p'}\right] \end{pmatrix} = \begin{pmatrix} 0 \\ \left[V_{g,m}^{p}\right] \\ 0 \end{pmatrix} \quad (1)$$

(An analogous equation holds for the array of cavity-backed antennas.) The + or – superscripts denote operators for regions above or below the ground plane. The matrix $Z_{mn}^{pp'}$ is the EFIE operator connecting blocks $\mathbf{p}$ and $\mathbf{p}'$, and $Y_{mn}^{pp'}$ is its dual, representing the magnetic field due to magnetic current sources; $\beta_{mn}^{pp'}$ is the corresponding MFIE operator. Subscripts *m* and *n* denote unknowns within the p and $\mathbf{p}'$ cells, respectively. The corresponding matrices Z_{mn}^{-}, Y_{mn}^{-}, and β_{mn}^{-} that appear on the diagonal blocks represent the coupling below the ground plane on?? each element; they affect only the $\mathbf{p}=\mathbf{p}'$ self blocks because the Kronecker delta $\delta_{p,p'}=1$ for $\mathbf{p}=\mathbf{p}'$, and $\delta_{p,p'}=0$ for $\mathbf{p}\neq\mathbf{p}'$. Note that the number of subblocks in the first matrix in (1) grows as the square of the number of array elements while the second matrix is the same for all the array elements. In standard MoM, for large arrays the first matrix requires huge resources of memory, fill time, and solution time. The computational difficulty arises from the upper region because of the coupling between all the array elements. The numerical burden is reduced by applying the GIFFT method to this region.

The GIFFT method begins by setting out a regular grid of Green's function interpolation points across the entire array. The points are typically chosen so there are five or six points per half-wavelength array cell. The points are used as equi-spaced interpolation nodes for Lagrange interpolating polynomials that approximate the Green's function as

$$\mathbf{G}(\mathbf{r}-\mathbf{r}') \cong \sum_{\mathbf{i},\mathbf{i}'} L_{\mathbf{i}}(\mathbf{r})\ \mathbf{G}_{\mathbf{i}-\mathbf{i}'}\ L_{\mathbf{i}'}(\mathbf{r}')$$

where $\mathbf{i},\mathbf{i}'$ are double indices representing interpolation point locations overlaying the observation and source cells, respectively. Every necessary component of the various scalar and dyadic Green's functions required for the problem is sampled for every separation of source and observation interpolation point. It can be seen from the above that the Green's function approximation is of convolutional form, and a matrix-vector product involving it may utilize an FFT. After the Green's function is sampled, the basis functions in the upper region (where coupling between array elements is assumed) are projected onto the interpolating polynomials. A correction is performed for neighboring elements by computing the nine interactions of a cell with its neighbors exactly. An iterative solver is then used that employs the FFT to perform the discrete convolution associated with the computation of matrix/vector products. Because there exist several non-zero components of the Green's function dyads for both electric and magnetic vector potentials, several multiplications must be carried out in the FFT domain, combining each vector component of the transformed currents with associated components of the dyadic Green's functions.

For the cavity-backed antenna the unknowns are placed on the various slots of the cavity (and thus not on the patch itself), and on the feeding microstrip below the slot (see Fig.1). A cavity Green's function is used, accelerated with the Ewald method.

When using an iterative solver such as BiCGStab on a very large matrix system, the solution may converge very slowly if conditioning is poor. For this reason, a *block diagonal preconditioner* is implemented to improve the solution time. The preconditioner used here simply inverts the self-matrix block and uses this as a preconditioner. Physically, this is equivalent to using the no-coupling solution for a single array cell as the preconditioner, and has been found to be highly effective in practice.

RESULTS

Two test arrays are simulated with different structures and the results of the GIFFT method both with and without preconditioning are compared to an "exact" MoM solution of these arrays. The "exact" solution does not use interpolation or fast multiplication, but utilizes an iterative solution (without preconditioning) and the Toeplitz nature of the matrices to speed fill time and storage.

The first case considered is an array of 25×25 square patches as shown in Fig.1, arranged on a rectangular lattice with periods 30[mm], and placed on a grounded dielectric substrate. The feed slot has dimensions $10[\text{mm}]\times1[\text{mm}]$ and is located 7[mm] off the center of the patch. The microstrip under the ground plane has width of 1.6[mm], and a length of 11.92[mm] that includes an open stub of length 11.67[mm]. Each patch, slot and microstrip is meshed using quadrilaterals, creating 128 unknowns per array element. The GIFFT method uses fifth-order interpolating polynomials in both planar directions. Table 1 shows the run times for the standard MoM and GIFFT solutions of the two arrays. It can be clearly seen that the GIFFT method offers a dramatic savings in both setup and solve times. It can also be seen that use of the preconditioner drastically reduces the number of BiCGstab iterations needed for a solution, further reducing solution time.

Table 1: Matrix setup (fill) and solve times for GIFFT and standard MoM

	Setup Time	Solve Time	Number Iterations
25x25 array of patches			
MoM w/ Toeplitz fill	≈ 9 hr	≈ 11 min per single BiCGstab iteration	>100 Program stopped earlier
FAIM w/ preconditioner	≈ 25 min	≈ 2 min (all iterations)	8
4x4 array of cavity-backed antennas			
MoM w/ Toeplitz fill	≈ 43 min	≈ 9 hr	13000 (BiCGstab err =5%)
GIFFT w/ preconditioner	≈ 13 min	20 s	24
10x10 array of cavity-backed antennas			
GIFFT w/ preconditioner	≈ 16 min	≈ 3 min	34 (BiCGstab err = 1%)

The GIFFT method also drastically reduces memory storage requirements. For example, for the $625 = 25\times25$ square patch array, each element is discretized using $n = 128$ basis functions on the patch and the 38 on the slot and microstrip, requiring storage of 16,384 complex numbers for each $\mathbf{p},\mathbf{p}'$ block $\left[Z_{mn}^{\mathbf{pp}'}\right]$ of the impedance matrix. Instead, using GIFFT with a fifth-order interpolation scheme, only 25 Green's function samples per cell are stored. For layered media, this number must be multiplied by the number of unique dyadic and scalar potential terms used in the mixed-potential formulation. The GIFFT storage advantage is further amplified by the fact that if there are M = 625 array elements in the square array, there are M^2 = 390,625 matrix blocks in the complete matrix, which is why Toeplitz storage is used. For the 25×25 array, this means that the system matrix for a standard solution would contain about $117\times117\times M^2 + 16\times16\times M = 5.3\times10^9$ complex entries (a Toeplitz implementation would reduce the number to 1.7×10^7), while there are only $25\times4\times6\times M = 3.7\times10^5$ entries in the sampled Green's function array (the factor 4 accounting for padding to make the Green's function sampling matrix circulant) in addition to near-interaction blocks [2].

The second example consists of an array of cavity-backed patch antennas, as shown in Fig. 1. Each patch antenna is suspended on a thin dielectric substrate layered on top of a cavity, which is in turn fed by a slot excited by a microstrip line with stubs. For each array element there are 276 unknowns for a total of 27,600 unknowns. As can be seen in table, the GIFFT set up and solution times are reduced, even for this small 4×4 array antenna. After 13,000 BiCGstab iterations, the relative error of the MoM solution, without preconditioner, was still of the order of 1%-5%, while the GIFFT solution reached the target 10^{-4} relative error. The average error between the MoM and the GIFFT solutions, counting all the unknowns, is 3.8%, with the high error likely due to comparison with the non-converged MoM results.

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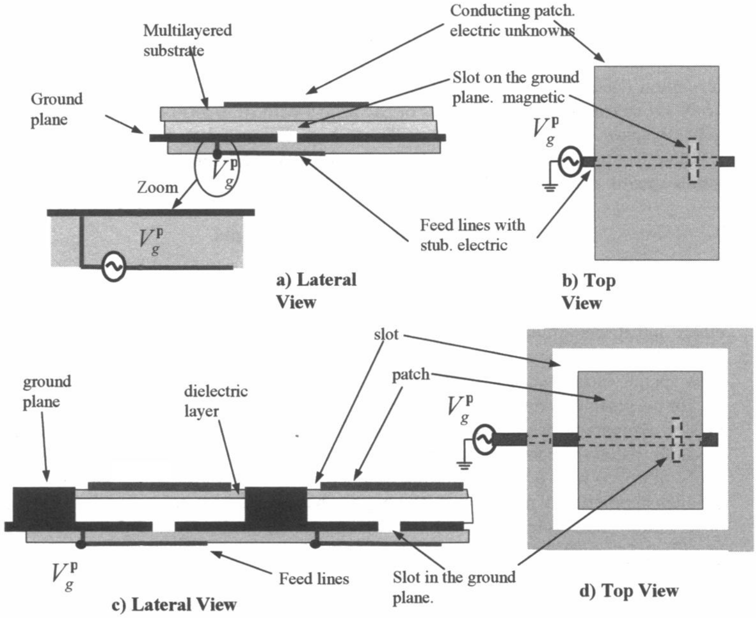

Fig. 1. Two types of array elements: patch antennas, and cavity backed patch antennas. Lateral views (a), (c). Top views (b), (d). In both cases GIFFT may take into account a multilayer environment. Each array element ***p*** is fed by an independent microstrip line excited by a $V_g^{\mathbf{p}}$ voltage generator (here $\mathbf{p}=(p_1, p2)$ denotes a double index). The array elements are coupled via the radiation region, i.e., the region above the lower ground plane.