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Uncertainty, Risk Aversion, and Optimal Defense Against Interruptions in Supply

by

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Abstract

Recent international conflicts have resurrected concerns about how to manage supply disruptions or sudden escalation of need for energy, and other critical imports such as vaccines or military components. Major proactive measures prominently include support of domestic production, and accumulation of reserves or maintenance of stand-by production. This paper develops a clear transparent method for comparing instruments and for identifying the optimum policy mix. We show how a country's risk aversion influences the best mix of policies, and interacts unexpectedly with the degree of risk itself. Specifically, high-risk aversion and low risk are shown to favor domestic production support as the better defense and disfavor stockpiling (and conversely). In clarifying a country's best policy response to risks of supply interruption, this analysis predicts how income level and risk aversion characteristics should shape arguments for and against interference with free trade on grounds of "national security."

Keywords: stockpiles, risk management, embargo protection, risk aversion, emergency preparations.

JEL Codes: D81, F11, H56

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Uncertainty, Risk Aversion, and Optimal Defense Against Interruptible Supply

1. INTRODUCTION:

Recurring volatility of world markets, storminess of political structures, and mounting terrorist capabilities have resurrected analytic interest in national security management of critical resources especially energy (e.g. MacAvoy , Ames, and Corridore, 2004) and bio-security items such as vaccines and drugs. . As global interdependence deepens, the political leverage from management of import and export policy grows apace. Inevitably many essential inputs to national welfare and security are purchased form foreigners. Earlier one might have thought that --- having been stimulated by the oil crises of the 1970's --- most of the most policy relevant work on this subject was behind us. No longer! And new challenges to the security of such resources, originating in accelerating interdependence due to globalization, extend the range of scenarios and goods properly considered at risk of disruption such as vaccines, exotic defense materials, and special high-tech items like nano-technologies (Ihori, 1999; Sandler and Hartley, 1995; Sandler, 2000; Zycher, Solomon, and Yager, 1991).

Central to this problem has always been how to compare alternative government responses to a threat of supply interruption or sudden escalation of need. Why governments should be more adept than the private sector at predicting/managing supply disruption is of course a crucial issue. Information asymmetries between public and private sectors may cut either way. Nevertheless, as clearly articulated by Zycher (2002), the private sector suffers from one major disadvantage: if private firms do prepare for emergency and the emergency arises, history suggests over and over that their own government will likely then tax away the "windfall excess" profits[1].

[1] If domestic dependence on supply of critical goods is more difficult to insure against, writes Zycher, than is foreign dependence, then "ironically , domestic dependence causes greater vulnerability....What could make insurance more difficult for domestic purchases than for foreign ones? One possibility is the expectation of price controls. Producers know that... prices can rise dramatically when a government at war or preparing for war increases its purchases..... These price increases serve an important function: they reward domestic producers for stockpiling goods, maintaining excess production capacity, and increasing production quickly. But domestic producers also know that governments, wanting goods on the cheap, often impose price controls on just such goods.... Taking anticipated price controls into account, domestic producers do not stockpile as much or maintain as much excess capacity." (p. 662). For more detail and alternative positions see Zycher, Solomon, and Yager(1991), also Williams and Wright (1991, pp. 410-51).

For purposes of this paper we will, therefore, assume that stockpiling is executed by the state. But even if it were implemented by private firms the issue of how a country should prepare for supply disruption becomes primarily an issue of public policy. When does one policy instrument dominate others, and when and how should different policies be mixed? Major anticipatory proactive measures available prominently include support or protection of domestic production[2], and accumulation of stockpiles (or its close substitute, maintenance of stand-by production)[3]. This paper extends our knowledge of what the maximization-of-expected-utility approach can inform us about comparing these two. In particular, I show how the degree of risk aversion[4] in a country's utility function influences the best mix of preferred policies, and on how that absolute risk aversion[5] interacts significantly and unexpectedly with the degree of risk itself. Underlying these effects is the essential connection between threat or risk of adversity (war) and distribution of its impact on welfare in both war and peace. When a new or newly recognized incremental threat arises it reduces utility in the bad contingency (war), but an optimizing policy response will transfer some of this loss to peacetime, thereby alleviating the wartime loss.

The analysis here will draw on our knowledge of adversity management from other contexts such as unemployment insurance, health and property loss, or income distribution across uncertain life outcomes (Becker, 1982; Atkinson and Stiglitz, 1980; McGuire, 1991; McGuire and Becker, 1994; McGuire, 2000). An omission from the paper concerns how the choice of proactive protection may exacerbate or even bring

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[2] Ideally stimulation of domestic production is effected by subsidy and not efficiency distorting intervention in trade. See Bhagwati, Panagariya, and Srinivasan 1998. Here we refer to such a superior policy instrument as "subsidy-protection."

[3] Stand-by production capabilities in the defense industrial base resemble stockpiling. There current resources, transformed into production capacity, are set aside for utilization in emergency in the expectation that supplies from alternative sources (including foreign sources) may be unavailable or slow. So ---as I thank a referee for pointing out --- the fact that rich countries support their defense industrial bases accords with our expectation.

[4] To characterize risk aversion we employ the extension to multi-commodity situations of the Arrow-Pratt (1963, 1964) definition of risk aversion as proposed by Kihlstrom and Mirman (1981) viz. <u>any concave</u> transformation of the common underlying utility function $U(x^j, y^j)$ representing the same ordinal preferences across individuals or contingencies. The "underlying" function, U, is not altered by the transformation say $V= f[U(\cdot)]$; the same ordinal rankings and indifference curves as in Figures. 1 - 5 apply, except now these are renumbered.

[5] All allusions to risk aversion here will refer to absolute risk aversion of the function $f[\cdot]$, though the same qualitative conclusions apply equally to relative risk aversion, since our arguments refer to/require only once-over, local changes in f''.

about the undesirable disruption. Dealing with the game theoretic aspects of managing against an intelligent adversary would require extensions of such papers as Nichols and Zeckhauser, 1977, or Bergstrom, Loury and Persson 1985, and is left for another day.

2. PREVIEW-PRELIMINARIES

Our analysis of optimal preparation for supply disruption will compare (1) policies that subsidize ongoing domestic provision during normal times so that there is less trade to be interrupted, with (2) policies that hoard supplies as stockpiles in anticipation of the evil day. Assuming standard state-independent expected utility we demonstrate that a correlation --- a rough equivalence --- exists between rational responses to higher risks of emergency and to lower absolute aversion to risk (RA) in the rational agent's utility function. Specifically, high risk and low RA both weight optimal policy in favor of stockpiling as the better anticipatory policy. Correspondingly, low risk and high RA both will weight optimal policy toward subsidizing domestic production and away from stockpiling.

This hitherto unrecognized equivalence is not absolute or exactly fixed. It is somewhat elastic because the price of stockpiling (to be defined) is an essential variable for comparing the efficacy of production subsidy vs. stockpiles and this price depends on the risk of interruption. Higher chance of adversity raises the price of stockpiling compared to subsidy-of-production, thereby generating a substitution effect against stockpiling. This tends to curtail its place in the optimal mix of policies and to offset the correlation just described. But opposed to this substitution effect, higher risk will push resources out of "peace" (ordinary times) and into "war" (emergency periods), and this give rise to a positive income effect which again favors stockpiling. The income effect dominates more and more if risk aversion in the underlying utility function declines. Table III in the conclusion of this paper presents a schematic summary of these findings and their consequences.

A corollary follows from this correspondence between low risk aversion and superiority of stockpiling, or conversely between high-risk aversion and superiority of production subsidy as an optimal policy. If, as ordinarily expected, risk aversion decreases with income, then the case for stockpiling increases with a country's income. Thus, somewhat perversely, poor countries, *ceteris paribus* being more risk averse, should favor production support/subsidy relatively, and rich countries should favor stockpiling. This means that, all else equal, a group of rich countries should not want risks of supply disruption to curtail trade in ordinary times — indicating that greater wealth and declining risk aversion support trading partnerships or alliances. Of course, even the richest of countries may be peculiarly risk averse. But whether strong risk aversion is due to ordinary lower income, or to idiosyncratic or cultural factors, in either case, countries in this category should find trading partnerships less beneficial.

These generalities can offer only the roughest guidelines for policy, so to lend intuitive support for them it helps to review one established property of the expected utility model that underlays these effects. I term this effect "the perfect insurance paradox" or "utility reversal paradox." (See McGuire and Becker, 1994). This paradox or reversal builds on the theorem that when perfect fair insurance can be purchased, an expected utility maximum will equate the marginal utility of numeraire income across contingencies (Ehrlich and Becker, 1972). If the state-independent utility function then is linear homogeneous (i.e. homothetic and CRS) it also follows that fair insurance yields an optimum of so much *ex ante* transfer across contingencies (from good to "bad" state of nature) that *ex post* utility is actually higher in the "unfavorable" contingency — hence the term "reversal." (Ehrlich and Becker 1972, Atkinson and Stiglitz 1980, Becker 1982, McGuire and Becker, 1994). This tendency obtains for any expected utility maximization (even if all our specific assumptions are not perfectly satisfied), and lies behind our conclusions as to preferred policies. That is, the incentives which produce the reversal outcome operate even though utility may be neither homothetic nor CRS, and perfectly fair insurance is only an ideal never realized.

3. NOTATION AND MODEL SET-UP

Because I want to focus especially on these two parameters (a) amount of risk, and (b) degree of risk aversion, I will make the production choices open to a country (being faced with risk of emergency interruption) as simple as possible. Thus I assume a Ricardian, constant average cost economy[6], producing two goods, where the import supply may be unexpectedly and we assume totally cut off with risk $(1-\beta)$. Ricardian constant cost is equivalent to an assumption that the country in question is "small" with respect to

[6] For an earlier preliminary analysis with increasing opportunity cost see McGuire and Shibata (1985).

the international system, and therefore is unable to influence world prices.

TABLE I HERE

The country is a unitary monolith; its expected utility depends only on its consumption of x and y. Its time horizon extends over two periods. First is the present -- denoted by superscript k-- just about to begin and during which no trade disruption can occur. The second time period is the future when the country faces two distinct contingencies; exports and imports permitted with probability β and forbidden with probability (1- β), contingencies represented by superscripts π (for "peace") and ω (for "war") respectively.

If exports/imports are cut off, all home production must be consumed at home; no resource reallocations between sectors can be made[7]. In anticipation of embargo risks, therefore, the authorities will attempt to control. (y_M, y_D, x_D, x_E). Also authorities may set aside some exports during the present (normal times) to buy imports for future emergencies, which are then stockpiled rather than consumed

TABLE II HERE

The price of acquiring, maintaining, and dispensing stockpiles, i.e. p_S, may depend on the risks of disruption and, therefore, it may involve further costs beyond the purchase price (although parts of p_s may be fixed independent of β). Moreover, since stockpiling in reality takes place over many time periods, a positive rate of inter-temporal discount, ρ, will influence the value of p_S. However, we postpone until a later section all such considerations of interest and discounting and here assume that $p_S = p_S(\beta)$ remains a parameter since β is not chosen[8]. Now the two-time-period objective of a small country can be written by introducing a utility function U(y, x), assumed to be the same for all periods and contingencies.

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[7] As the duration of the emergency trade disruption lengthens, this assumption of complete factor immobility becomes less realistic. In fact the duration of an emergency to require advance preparation is just that over which factors of production are immobile! The private sector is assumed not to anticipate or make provision for trade interruption because (a) private producers have less adequate information about trade disruption than do governments and/or (b) domestic or foreign policy derived externalities of potential trade loss are not internalized by private markets.

[8] If β could be improved by deterrent or defensive measures, then the price of stockpiling would fall. This indicates interactions between preparations or protection from adversity and deterrence of it that go beyond our scope. See McGuire, Pratt and Zeckhauser (1991), also Ihori and McGuire (2006). An optimal regime would normally employ both security-deterrent expenditures plus stockpiling and/or protection instruments. The degree of protection/stockpiling should be lower with security-deterrent expenditures than without and if security expenditures become more efficient in preventing trade disruption, the feasibility of substituting security expenditures for protection or stockpiling of import-competing domestic industries increases. The greater the difference between international and domestic terms of trade, the greater the disadvantage of protection or subsidy against both stockpiling and defense.

$$
\begin{aligned}
&W = U^k[(x_H - p_D y_D - p_W y_M^k - p_S y_S),(y_D + y_M^k)] \\
&+\beta U^\pi[(x_H - p_D y_D - p_W y_M^\pi),(y_D + y_M^\pi + y_S)] \\
&+(1-\beta)U^\omega[(x_H - p_D y_D),(y_D + y_S)] \\
&s.t. \quad : \quad y_D \geq 0, \quad y_M^\pi \geq 0, \quad y_M^k \geq 0, \quad y_S \geq 0.
\end{aligned} \tag{1}
$$

Eq (1) assumes that stockpiles not used in an emergency are equally available in ordinary times, since today's stockpile $y_S = rx_S$ is available under both of the next period contingencies. We anticipate that any positive inter-temporal discount rate, ρ, may have various effects but for constructing the basic model at this stage, assuming a zero discount rate we ignore them. Eq. (1) then represents the complete two-period, two-contingency welfare function when both subsidy-protection and stockpiling are permitted [9].

4. Optimum Protection: Absent Stockpiling

The standard argument for trade intervention in the form of subsidy for importables is especially easy to make if we restrict the expected utility function to one period with two-contingencies. Therefore, for heuristic purposes, we will collapse the two-period or multi-period model to a single period with two contingencies. Eq (2) shows the objective function for this special case; x_S and y_S do not appear since only protection, no stockpiling is allowed. Now the two-contingency objective function of a small (Ricardian case, as mentioned above) country can be written [10]

$$
\begin{aligned}
&W = E(U) = \beta U^\pi[(x\wedge - p_D y_D - p_W y_M),(y_D + y_M)] \\
&+(1-\beta)U^\omega[(x\wedge - p_D y_D),(y_D)] \\
&s.t. \quad : \quad y_D \geq 0, \qquad y_M \geq 0.
\end{aligned} \tag{2}
$$

The superscripts on U (i.e. U^ω and U^π) serving to distinguish the two contingencies, indicate different realized values of the same utility function in adversity (ω) and regular times (π) respectively[11]. Necessary

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[9] As an alternative to (1), a repeated, "rolling", two-period decision context might be used to analyze stockpiling and protection. The two-period welfare function as in Eq. (1) is chosen instead for its simplicity.

[10] Objection might arise to the simplification to only two contingencies in only one time period, or for the implication that stockpiles not used in emergency are completely lost. The simplifications of constant opportunity cost, and zero interest are inessential, and the changes required if these assumptions were relaxed straightforward as shown below in section 9. The neglect of salvage value of stockpiles may be more troublesome, however, and the consequences of more realism there may not be so obvious. But in fact if we extend eq. (2) or eq. (7) to include multiple time periods, the change in outcome as we demonstrate in Section 9 is slight. See also McGuire (2000, 749-50).

[11] The ex post immobility of productive resources is captured by the assumption that y_D and $x_D = p_D y_D$ must have the same values under both outcomes, whereas freedom to trade in peacetime and the loss of this option in war follows from the entry

conditions for a maximum of Eq. (2) are given by (3) and (4) (where $U_X^\pi = \partial U_X^\pi / \partial X$).

$$U_X^\pi / U_Y^\pi = 1/p_W = t \qquad (3)$$

$$\frac{\beta U_X^\pi + (1-\beta)U_X^\omega}{\beta U_Y^\pi + (1-\beta)U_Y^\omega} \geq \frac{1}{p_D} \equiv q \qquad (4)$$

In peace, trade is optimally pursued, and MRS, the left hand side of (3) is equal to the world price. But, comparative advantage is not fully exploited; rather $(x_D < x_H \text{ or } y_D > 0)$ domestic production of y is subsidized to sustain an internal marginal cost equal to a weighted composite of wartime and peacetime marginal utilities. Evidently, if war is certain then $MRS^\omega = q$, and if war is impossible, then $MRS^\pi = t > q$. Therefore, at a cut-off $\beta = \beta^0 < 1$ a strict equality obtains in eq. (4). As β (probability of peace) declines to zero, x_D increases from zero to x_D^n. This "subsidy-only" optimum is pictured in Figure 1[12].

FIGURE 1 HERE

5. STOCKPILING AND STANDBY PRODUCTION

We can now consider the option of stockpiling --- first as an alternative to and then in conjunction with subsidy-protection. An ideal first best solution would be to transfer y from peace to war, and to transfer x from war to peace. If one could transfer x- and y-consumption independently between contingencies--- at fair insurance prices say for y -- the wartime surpluses and deficits would be curtailed individually [13]. And if v_X and v_Y reflected actuarially fair exchange rates $v_X = v_Y = \beta/(1-\beta)$, then such individually piecemeal optimization will equalize marginal utilities $U^\omega_X = U^\pi_X$ and $U^\omega_Y = U^\pi_Y$. Note also that this would actually equalize consumption across contingencies and therefore equalize utility also- - - a well-known conclusion (see Ehrlich and Becker 1972, or McGuire and Becker 1994).

Protection alone is partially effective in achieving such an ideal goal, and stockpiling alone is

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of y_M and $x_E = p_W y_M$ in peace <u>but not</u> war.

[12] For $\beta < \beta^0$ negative protection , i.e. $y_M <$ could be desired in this Ricardian world, but we eliminate this by assumption

[13] An insurance premium of v_Y is then paid for each unit of y transferred from peace to war, while a premium of v_X is received for each unit of x transferred from war to peace.

partially effective, but neither policy alone nor in combination actually supports a first best optimum. Consider subsidy or protection alone first: it reduces the x-surplus in war (reduces x^{ω}) and alleviates the y-deficit in wartime (increases y^{ω}) reallocating consumption in the same directions as would perfect insurance. But because subsidy supports inefficient production, it exacerbates the misallocations in peacetime -- reallocating consumption just the opposite of ideal insurance. To offset this inefficiency stockpiling will substitute imperfectly for the absence of ideal insurance, because in fact x- and y-consumption cannot be independently transferred and the ideally desired first best outcomes cannot be realized. Objective function (5) thus shows how stockpiling will partially substitute for insurance, allowing some of the intercontingency deficit/surplus to be corrected but not all of it. In other words stockpiling relieves the deficit in wartime consumption of good y but exacerbates the deficit in peacetime consumption of x that would be induced by protection, a qualitative result that continues to hold in the general case as represented by Eq (1)[14].

$$\begin{aligned} W &= \beta U^{\pi}[(x_H - p_D y_D - p_W y_M - p_S y_S),(y_D + y_M)] \\ &+(1-\beta)U^{\omega}[(x_H - p_D y_D),(y_D + y_S)] \\ &s.t. \quad : \quad y_D \geq 0, \quad y_M \geq 0, \quad y_S \geq 0. \end{aligned} \tag{5}$$

A crucial parameter in (5) is p_S, the unit price of "stockpiling," which determines (compared to other alternatives) the relative and absolute social benefits of acquiring stockpiles. These are incorporated in the social welfare function U which we take to be independent of the government or private execution of stockpiling or other preparations for adversity. (Here we assume that governments organize and execute stockpiling and other protective measures.) A satisfactory model of this cost, p_S, would allow for:

(1) a pure insurance or actuarial element in the price, where the amount of stockpile available in

[14] As demonstrated in Section 9, applied to the more complete, model of Eq (1) the two instruments have allocative consequences qualitatively similar to those of Eq (5). To see this consider subsidy-protection alone in Eq (1). First it reduces the x-surplus in war (reduces x^{ω}) and alleviates the y-deficit in wartime (increases y^{ω}). But in a two time period scenario the cost of this imperfect substitute for perfect insurance is greater because now protection-subsidy has a peacetime cost weighted by $(1+\beta)$, rather than merely β. And stockpiling has the same quantitative weight as before. A subsidy relieves the deficit in y^{ω}; but now the degree to which y exacerbates the deficit in peacetime consumption of x is reduced. The reason for this is that if war is avoided in the second period then the decision to stockpile merely has transferred y from peacetime consumption in period 1 to peacetime consumption in period 2. Aside from interest accruals (which we ignore) the excess-unnecessary stockpiling of y can be reversed in a peaceful second period by adjusting imports $y^{\pi}{}_M$ to off set the surplus of stockpiles. If p_S, and p_W differ the analysis is more involved but qualitatively the effect is unchanged.

an emergency depends on the chance of actually needing the stockpile, or the relative proportion of time spent depleting the stockpile to the time spent accumulating it (1-β)/β:

(2) a proportionate spoilage or storage cost, α and (3) a constant unit cost independent of frequency or risk of emergency, φ.

Also equation (5) might include costs of stockpile extraction during the emergency, war, embargo etc. This last effect will be neglected, which leaves:

$$p_S = [\alpha(1-\beta)/\beta) + \phi]p_W \tag{6}$$

6. THE OPTIMAL STOCKPILE: COSTS INDEPENDENT OF RISKS OF ADVERSITY

Now we determine optimal $x_S = p_S\, y_S$. To start we make several simplifications.

(1) Stockpile cost is unrelated to β, i.e. α = 0 in eq. (6);

(2) Cost of stockpiling limited to the inherent resource cost of commodity purchase. Thus to establish an initial point of reference spoilage, storage, or retrieval cost are ignored.

(3) The stockpile is purchased at world price during peace for consumption in war so that β = 1, and $p_S = p_W$ in eq. (6) or equivalently r = t in the notation set up; and

(4) that all stockpiles become useless with zero utilization or salvage value if the bad event "war" is avoided.

These simplifying assumptions will serve to establish a baseline for analysis of optimum storage. In particular (4) may seem questionable, but we delay consideration of usage of stockpiles even if emergency fails to arise until section 9 of the paper which focuses precisely on this question.

First, if stockpiling , y_S, is the only allowed policy -- complete free trade, $y_M > 0$, being assumed in peace and protection prohibited $y_D = 0$ -- then necessary conditions for an optimum (using $p_S = p_W$) become:

$$-\beta p_W U_X^{\pi} + (1-\beta)U_Y^{\omega} = 0 \tag{7a}$$

$$-p_W U_X^{\pi} + U_Y^{\pi} = 0 \tag{7b}$$

The first term in (7a) represents marginal social cost of stockpiling (MSCS) and the second term represents the marginal social benefit (MSBS); so Equation (7a) just requires MSCS = MSBS, while (7b) gives the optimal trading condition during peace, "optimal" since U is assumed to adequately represent the entire social welfare of the entire society irrespective of the particulars of who executes protective programs. Combining these at an optimum then gives

$$\frac{1}{p_W} = t = \frac{U_x^{\pi}}{U_y^{\pi}} = \frac{\beta U_x^{\pi}}{(1-\beta)U_y^{\omega}} \tag{8}$$

A depiction of pure stockpiling decisions is given in Figure 2. Stockpiling essentially shifts Production Possibilities leftward by the amount, say, x^*_S during peace so that peacetime consumption is shown by point "a" along curve t*, with optimum utility U^{π}. Curve t* originates at $(x_H - x^*_S)$ because during peacetime free trade and complete specialization in producing good x is maintained.

FIGURE 2 HERE

This now calls attention to the first remarkable implication of the expected utility model for management of supply uncertainty. The return for reducing peacetime utility to $U^{\pi}*$ from initial ("i") $U^{\pi i}$ is that wartime utility rises from $U^{\omega i}$ to $U^{\omega}*$. The best U^{ω} - U^{π} combination follows from equation (8). Figure 2 shows this maximum (m) as $U^{\pi m}$ - $U^{\omega m}$. As β declines, greater peacetime sacrifices are warranted, and conceivably depending on a high enough likelihood of war, the optimum can require so much stockpiling that $U^{\omega}* > U^{\pi}*$. This is the "utility reversal paradox," identified in Section 2 above.

This result should be compared with the optimum under "pure protection"; rational trade interference/control alone can never proceed so far that utility in peacetime is pushed below utility in war. But with stockpiling as an instrument, a nation that plans rationally could quite conceivably prepare for an emergency so strenuously that it is worse off if the emergency fails to happen. If, in the theoretical and unlikely case, in fact *U is homothetic* and stockpiling can be purchased at an actuarially "fair price" as in Eq (6), then for even the smallest chance of war/emergency, optimal stockpiling for adversity will always entail sufficient preparation that a reversal of utility positions results. That is, the optimal stockpile being actuarially cheap will be so great that utility is actually higher in the "bad" event, the emergency. The utility reversal result requires fair insurance to equalize marginal utility of good y across contingencies; and homothetic utility, therefore, implies the stated reversal[15] (McGuire 1991; and McGuire and Becker 1994.) The geometric depiction of the optimal stockpile *with perfect insurance* is similar to Figure 2. In this case however, the price line for transforming x^{ω} into y^{π} is $p_S = p_W(1-\beta)/\beta$, which is very steep at low risks of adversity indicating how very cheap it is to stockpile for a very unlikely event when fair insurance is embedded in the prices[16].

Because the assumptions of homotheticity/linear-homogeneity and expected utility are so innocuous and widely accepted, this means that the *prima facie* policy case for truly serious stockpiling preparations is much stronger than casual observation might suggest. Even though fair, zero-loading, insurance prices are unrealistic, and therefore the perfect reversal result (i.e. that $U^{\omega}* > U^{\pi}*$) is suspect, the argument strongly supports much more serious attention to preparation for supply disruption than it now receives.

7. THE OPTIMAL MIX OF STOCKPILING AND TRADE CONTROL WHEN STOCKAGE COST IS INDEPENDENT OF RISK

Now combining these models of protection and stockpiling so that, $y_D \geq 0$ in eq. (5) we will see how degree of risk and the measure of risk aversion systematically influence the best mix of stockpiling and protection. This generates another condition relating marginal social benefits of protection (MSBP) to marginal social costs (MSCP) in addition to eqs. (7a), (7b), (8). As we shall see presently, it also yields another surprising discovery as to how the best composition of preparations depends on chances of adversity, and on the country's aversion to risk as embodied in its utility function. Differentiating (5) with respect to y_D gives (9a).

$$\beta U_Y^{\pi} + (1-\beta)U_X^{\omega} - [\beta p_D U_X^{\pi} + (1-\beta) p_D U_X^{\omega}] = 0 \qquad (9a)$$

Equation (9a) simply says that MSBP - MSCP = 0 at an optimum. Rearranging (9a) and using $p_W U^{\pi}_X = U^{\pi}_Y$

[15] A similar utility reversal paradox was noted by Atkinson-Stiglitz (1980), and Becker (1982) in the case of income and welfare distributions. The same reversal incentive is embedded in the structure of our best preparation for adversity problem.

[16] For example suppose an energy crisis occurs every 21 years, and that petroleum can be stocked with no costs of storage nor losses from deterioration. Then the annual costs of a stockpile of size R is 1/20 p_WR. That is, 5% of R is accumulated every year for 20 years, and the total is expended in the 21st year.

(from optimum peacetime trade) yields:

$$(1-\beta)U_Y^{\omega} \quad = (1-\beta)p_D U_X^{\omega} + \beta(p_D - p_W)U_X^{\pi} \qquad (9b)$$

whence the expected benefit from an increment of domestic y-production in "war" equals its expected marginal cost in war plus the expected extra domestic cost of home production in excess of import cost during peace. Comparing (9b) and (7a) at the same marginal benefit in war $(1-\beta)U^{\omega}_Y$, shows that stockpiling alone, or stockpiling in combination with protection, or protection alone is the optimal provision for adversity, according as MSCS $\lesseqgtr$ MSCP as shown in Equation (9c).

$$\beta p_W U_X^{\pi} \quad \lesseqgtr (1-\beta)p_D U_X^{\omega} + \beta(p_D - p_W)U_X^{\pi} \qquad (9c)$$

If both stockpiling and subsidy are used simultaneously, then combining all three necessary conditions Eqs. (7 - 8) give (10) to explain whether and to what degree to mix y_D and y_S

$$\frac{U_X^{\pi}}{U_Y^{\pi}} = t = \frac{1}{p_W} \qquad (10a)$$

$$\frac{U_X^{\pi}}{U_Y^{\omega}} = \frac{1-\beta}{\beta}t = \frac{(1-\beta)}{\beta p_W} \qquad (10b)$$

$$\frac{U_X^{\omega}}{U_Y^{\omega}} = 2q - t = \frac{(2p_W - p_D)}{p_W p_D} \qquad (10c)$$

Note first if all these conditions obtain, the optimum is "interior" and thus when p_S is independent of β, the <u>solution marginal rate of substitution (MRS) does not depend on chances of war or peace</u>. Second, wartime MRS, assuming it to be positive, can only satisfy Eq (10c), i.e. equal (2q - t),provided (2q - t) > 0, or q > t/2 (or $p_W > p_D/2$). For q < t/2 (i.e. a very severe comparative disadvantage in home production of good y compared to importation and stockpiling) only stockpiling, and no trade protection should be chosen.

FIGURE 3 HERE

For q > t/2 a mix of stockpiling and protection is optimal. Tolley and Wilman (1978) reached similar conclusions. Figure 3 illustrates this general result which is valid for any diminishing MRS utility function. As before, with neither stockpiling nor protection this country obtains $U^{\pi i}$ and $U^{\omega i}$ at initial outcomes in peace and adversity respectively. Now suppose the country is prepared to accept a lesser welfare, say $U^{\pi}*$, in peace. What is the maximum utility under adversity attainable as constrained by $U^{\pi}*$, domestic resources, technology, and world prices? The diagram pictures the answer to this question. To attain $U^{\pi}*$ in peace a country may produce $y*_D$ internally foregoing $x*_D$ to do so. This option would give $U^{\omega p}$ in war and $U^{\pi}*$ in peace. On the other hand the country might stockpile $y*_S$. To do this it should pay $x*_S$ as exports. Thus using a pure stockpile-only option, its peacetime consumption opportunities via trade are shown by the line t* with the same slope as t, which also allows $U^{\pi}*$ in peace, but $U^{\omega s}$ in war. Consequently, either $U^{\omega p}$ or $U^{\omega s}$ are feasible in war at the same peacetime sacrifice of $U^{\pi i}$ - $U^{\pi}*$. But linear combinations of $x*_S$ and $x*_D$ will give wartime consumption at any point on the line "ab" connecting these two "pure" outcomes, while maintaining peacetime consumption and utility $U^{\pi}*$. For example combination $x^C{}_S$ and $x^C{}_D$ gives point c on line ab[17]. The slope of line ab is (2q-t). As shown in the diagram, the optimum mix between x_D and x_S -- i.e., between protection and stockpiling -- occurs where the indifference curve is just tangent to opportunity line ab. Again, to derive this optimal allocation no restriction on the utility function (other than diminishing MRS) is required. Now we can use this construction to analyze the interdependence between (1) chance of adversity as represented by β, (2) risk aversion (RA) as given by the effect of changes in scale of U, and (3) the optimal mix between protection and stockpiling. Assume for these purposes that U is homothetic, though not necessarily 1st degree homogeneous.

FIGURE 4 HERE

Figure 4 then demonstrates unambiguously that *if the optimally chosen peacetime utility* $U^{\pi}*$ *declines, the optimal wartime utility* $U^{\omega}*$ *increases*. Moreover, the figure shows how the mix between stockpiling and protection varies systematically as chosen peacetime welfare $U^{\pi}*$ changes. Let $U^{\pi}*$ vary as $U^{\pi a}$, $U^{\pi b}$, $U^{\pi c}$, which allows corresponding $U^{\omega a}$, $U^{\omega b}$, and $U^{\omega c}$. As shown with this progression the relative

[17] The explanation of this slope is as follows: when one unit less of x is "stockpiled" $\Delta x_S = -1$, and two more units of x are allocated to internal production $\Delta x_1 = +2$, the net effect is one less unit of x-consumption available during war. This generates in turn +2q units of y from the internal reallocation, but -t units of y from the stockpile; thus the slope $\Delta y/\Delta x$ becomes (2q-t).

importance of stockpiling necessarily increases. (In the figure at $U^{\omega a}$, no reliance is placed on stockpiling, whereas at $U^{\omega c}$ no reliance is placed on protection).

7.1 Effects of Risk of Adversity and of Risk Aversion on Optimal Mix of Subsidy and Stockpile

Now we ask, how does this progression relate to β, and to Risk Aversion[18]? To answer this, first suppose RA is given and optimal allocations are made. From this point let β decrease: with higher risk of war, the optimal U^{π}* declines and more reliance is place on stockpiling. Next suppose β is given and from the optimum let RA decline approaching risk neutrality: as declining returns to scale in the utility function vanish, the utility function approaches linear homogeneity, and the "perfect insurance reversal" result is approached, viz., all costs of protection against adversity are borne in peacetime. Consequently as RA declines (becomes more risk neutral), so does the best choice of U^{π}* decline as well, and the best mix shifts toward stockpiling, just as when β declines. Conversely, the more risk averse the utility function (i.e. the more curvature with changes in scale) the lesser is the optimal utility loss during peace and the greater the utility loss during war for any given hazard (1-β). Correspondingly, greater RA ceteris paribus raises optimal U^{π}* and shifts the best mix of policy toward protection. Again the same unexpected conclusion: higher RA produces the same result as lower risk (1- β), lower RA produces the same result as higher risk (1- β) ! (This conclusion depends on independence of stockpile costs from chance of war)[19].

8. Optimal Policy under Perfect Insurance

[18] I adopt the extension of conventional one dimensional risk aversion to utility functions with many commodities as proposed by Kihlstrom and Mirman (1981), essentially that more concave utility shells represent greater risk aversion.

[19] For a sufficiently low probability of war, the optimal values of x_1 and x_S may be negative at an interior solution. In a Richardian economy with a linear production possibility curve, negative x_1 and x_S imply negative production and/or negative consumption of one commodity. Since neither of these outcomes makes economic sense, both are excluded by assumption. However, if we drop Ricardo, and allow a curved PPC with increasing opportunity costs, then although expected welfare still cannot be improved by negative protection -- i.e. by artificial discouragement of domestic production and encouragement of domestic production of the export good which will then, of course, be in excess supply in the event of war. Nevertheless negative stockpiling may be welfare enhancing at low probabilities of embargo/war. Negative stockpiling amounts to gambling. In the event "peace" the country accepts a fixed amount of x_S -- i.e. of its export good -- from abroad; it sells part of its total availability of good x (domestic production plus its receipt of $-x_S$) on peace time world markets at world prices for good y and consumes the y so obtained. In the event "war," it delivers $-rx_S = y_S$. This amount of y_S is provided from the country's already wartime-insufficient domestic production of good y. But if β is sufficiently high ((1-β) sufficiently low) the gain in peace from this gamble can outweigh the loss in war.

In the previous section the price of stockpiles was assumed not to functionally depend on β; it assumed $\alpha = 0$, $\lambda = 1$ in Eq. (6). Now we consider the opposite pure case letting $\alpha = 1$. Expectedly, the trade off between the two options changes significantly when the price of stockpiling reflects a fair insurance cost. For now the superior option depends on MSCS $\gtrless$ MSCP as shown now by (11) rather than (9d), and a mix of both instruments calls for equality in (11).

$$(1-\beta)p_W U_Y^{\pi} \gtrless \beta(p_D - p_W)U_X^{\pi} + (1-\beta)p_D U_X^{\omega} \qquad (11)$$

At such an interior optimum the full set of necessary conditions is:

$$\frac{U_X^{\pi}}{U_Y^{\pi}} = t = \frac{1}{p_W} \qquad (12a)$$

$$\frac{U_X^{\pi}}{U_Y^{\omega}} = t = \frac{1}{p_W} \qquad (12b)$$

$$\frac{U_X^{\omega}}{U_Y^{\omega}} = 2q - t = \frac{(p_W - \beta p_D)}{(1-\beta)p_W p_D} \qquad (12c)$$

The expressions in (12a)-(12c) are to be compared, equation by equation with (10a)-(10c). The first expression (12a) shows no change from (10a), merely allowing free peacetime trade even if domestic production costs are subsidized. The second and third expressions in (12) differ from (10) where costs of stockpiling were independent of risk of adversity. Thus when stockpiling is priced as perfect insurance, the first order-interior optimum significantly changes. The greatest effect of making the cost of stockpiling depend on risk is thus to introduce that risk into the necessary conditions for an interior optimum as in (12c).

8.1 Income and Substitution Effects of Risk on Price of stockpiles

The influence of β on p_S and therefore on the choice between protection and stockpiling consists of an income and a substitution effect. We illustrate this influence with an example. When $\beta = ½$ (12c) and (10c) are equivalent, the solution MRS in both cases being equal to (2q-t). But for lesser risks ($\beta > ½$) introducing β lowers (the absolute value of) the solution value MRS, while for higher risks (for $\beta < ½$)

introducing β raises that value of MRS[20]. Geometrically, low values of β (greater risk) make the opportunity set from protection/stockpiling combinations steeper. Thus, fairly priced insurance against loss of access to imports of y should cause the optimal mix of protection/stockpiling to substitute away from stockpiling and toward protection at greater risks via a "substitution effect," because greater risk raises the price of insurance. However, greater risk of the bad event (trade loss) will tend to shift resources between contingencies *toward adversity*, and this "income effect" favors stockpiling as in the Figure 4 move from A to C. So while in the earlier sections of this paper's first model (of a fixed trade-off between protection and stockpiling) greater risk of emergency unambiguously shifts the optimal mix toward stockpiling, in the second model (where the trade-off depends on β), the effect is ambiguous; substitution effects favor protection and income effects favor stockpiling.

8.2 Influence of Risk Aversion on Choice of Stockpiling vs. Protection

When stockpile price is influenced by risk, our analysis again and surprisingly uncovers an unexpected relation between risk aversion and relative preference for stockpiling versus protection; and this relation is not beset by any ambiguity dependent on the relation between stockpile costs and risks of emergency.

Consider again a small Ricardian economy; let β be given such that a mix of protection and stockpiling (an interior solution) is best. That is assume stockpiling does not dominate protection --- i.e. assume (2q - t) > 0 in the first model or (q - β t) > 0 in the second model. Then as between two countries identical except for risk aversion, the less risk averse should place greater reliance on stockpiling and less on protection than would the more risk averse country. This effect follows from the fact that the less risk averse country will ceteris paribus insure more against trade disruption, and this in turn follows from the perfect insurance paradox. In the limit perfect risk neutrality corresponding to CRS utility entails complete insurance via stockpiling such that all the costs of a hazard of war are absorbed in peacetime. Short of this

[20] That is

$$(2q-t)\ \substack{\geq \\ <}\ \left[\frac{(q-\beta t)}{(1-\beta)}\right] \quad as \quad \beta\ \substack{\geq \\ <}\ \frac{1}{2} \tag{13}$$

limit because peacetime utility losses are less when the utility function is closer to linear homogeneous (less risk aversion), more peacetime costs will be accepted the lower is RA. The effect is illustrated by Figure 4. Although Figure 4 was derived especially for the first of our two models, the same illustration applies when the price of stockpiling p_S depends on β risk as in eq. (6). For any given value of β, the options of mixing stockpiling and protection consist of a set of linear opportunity curves, similar to lines A, B, and C in Figure 4. Ceteris paribus, the less risk averse a country, the greater its optimal of utility under adversity, $U^{\omega}*$ and the lower its optimal value of peacetime utility $U^{\pi}*$. And as demonstrated above, when $U^{\pi}*$ declines, the stockpile-protection opportunity curve shifts out from A to B to C, and reliance on stockpiling increases[21].

9. MULTI TIME PERIOD ANALYSIS:

SALVAGE VALUES AND INTEREST RATES

With the major theorems of the paper now established, we can next turn to venues where the foregoing models lack realism. The preceding analysis may appear objectionable for two reasons: (a) the reduction of Eq. (1) to only two contingencies in only one time period, with the implication that stockpiles not used in an emergency are completely lost, and (b) the absence of a positive rate of interest/discount. The simplification of constant opportunity cost seems minor and the changes required if this assumption were relaxed straightforward. But the neglect of salvage value of stockpiles and of positive time discounting may be more troublesome, and the consequences of more realism not so obvious.

9.1 When Stockpiles Are Available in and Peace if War is Avoided

Nevertheless when Eq. (2) or Eq. (5) is altered back to (1) to incorporate a non-zero peacetime salvage value of stockpiled reserves, the change in outcome of the model is slight. If I can show this the heuristics of the single period, two contingency analyses above will be adequate to illuminate the more complete and realistic model.

Re-written the simplest two period welfare function becomes Eq. (14), where in contrast to the

[21] Linear homogeneous risk-neutral utility, to repeat, causes all the burden of preparation for adversity to fall on the peacetime contingency (if fair insurance is available); greater risk aversion and therefore diseconomies of scale in the utility function shift more of the burden of managing uncertainty to the adverse contingency (McGuire and Becker,1994).

earlier assumptions it is now assumed that stockpiles of y_S accumulated in the first period are 100% available (except for storage and deterioration) in the second period <u>under both contingencies</u>

$$\begin{aligned} W = {} & U^k[(x_H - x_D - x_S - x_E^k),(qx_D + tx_E^k) + \beta U^\pi[(x_H - x_D - x_E^\pi),(qx_D + tx_E^\pi + rx_S)] \\ & + (1-\beta)U^\omega[(x_H - x_D),(qx_D + rx_S) \\ s.t. \quad & x_D \geq 0, \quad x_S \geq 0, \quad x_E^k \geq 0, \quad x_E^\pi \geq 0 \end{aligned} \tag{14}$$

Thus our outcome-indices now become "k" for period 1 assumed to be peaceful, and for period 2, π if it is peaceful and ω under adversity. Since the referenced time period for k, π, or ω will be unambiguous we can omit time-period indices 1 and 2 unless otherwise required. An alternative and logically equivalent formulation of this welfare objective can be written to maximize welfare in the second period for any given welfare in the first. The constrained optimization then becomes

Maximize:
$$\begin{aligned} & \beta U^\pi[(x_H - x_D - x_E^\pi),(qx_D + tx_E^\pi + rx_S)] + (1-\beta)U^\omega[(x_H - x_D),(qx_D + rx_S)] \\ & -\lambda\{U^{kJ} - U^k[(x_H - x_D - x_E^k - x_S),(qx_D + tx_E^k)] \end{aligned} \tag{15}$$

where λ is the Lagrangian multiplier on the first period utility constraint U^{kJ}, with its value chosen depending on risk (1-β) The necessary conditions for this constrained maximization are:

$$\begin{aligned} & U_X^k - tU_Y^k = 0 \\ & U_X^\pi - tU_Y^\pi = 0 \\ & \beta r U_Y^\pi + (1-\beta) r U_Y^\omega - \lambda U_X^k = 0 \\ & -\beta U_X^\pi + \beta q U_Y^\pi - (1-\beta)U_X^\omega + (1-\beta)U_Y^\omega - \lambda U_X^k + \lambda q U_Y^k = 0 \end{aligned} \tag{16}$$

Rearrangement gives a relation between expected marginal benefits and costs in the second, uncertain, period at the optimum.

$$\underline{\qquad\qquad\text{MC}\qquad\qquad} + \underline{\qquad\qquad\text{MB}\qquad\qquad} = 0$$

$$-\beta U_X^\pi\left(\frac{t-q}{t}\right)\left(\frac{t+r}{t}\right) - (1-\beta)U_X^\omega + (1-\beta)U_Y^\omega\{q - [r\frac{(t-q)}{t}]\} = 0 \tag{17a}$$

The interpretation of these conditions, eq. (16-17) shown in Figure 5 is qualitatively very similar to Eq. (12-13). First, a map of <u>second period</u>, <u>wartime</u> transformation curves, TT, is derived from various combinations of stockpiling with return r, and protection with return q. With q > r each unit of good x foregone in the second period generates rx or qx of good y depending on whether the instrument is protection or stockage. Each curve TT is associated with one best value of first period utility. For example Figure. 5 shows one such curve as T^aT^a. which is associated with $U^{kJ} = U^{ka}$. The slope of TT is - [q-{(t-q)(r/t)}]. Evidently this slope cannot be positive if an interior solution obtains[22]. Then rearranging Eq. (17a) gives the relation between MRT and MRS in the second period. A mix of stockpiling and protection along T^aT^a short of tangency (shown by $\Omega > 0$ in (17b)) is optimal when Eq. (17b) indicates an interior solution. This should be compared with Eq (10c) of the one time period case.

$$\frac{U_X^{\omega}}{U_Y^{\omega}} = \{q - [r\frac{(t-q)}{t}]\} - \frac{\beta U_X^{\pi}}{(1-\beta)U_Y^{\omega}}(\frac{t-q}{t})(\frac{t+r}{t})$$

i.e. (17b)

$$MRS^{\omega} = \quad MRT^{\omega} \quad - \quad \Omega$$

Because peace prevails throughout period-1 (by assumption) as well as with positive probability in period-2 there is a higher weight on the loss from or cost of protection and therefore a greater reliance on stockpiling (in the optimal protection/stockpile mix) than in the single period two contingency tangency solution of Figs. 3 or 4. Note that at the optimum, the peaceful outcome in the second uncertain period may be superior to that in period 1 --- shown in Figure 5 by $U^{ki} < U^{\pi}*$ --- because of the retrieval of stockpiles from period-1.

FIGURE 5 HERE

Thus, as claimed above, introducing greater reality by way of multiple time periods and allowing utilization of (partially depleted) stockpiles (if the emergency does not materialize) does not alter the general qualitative character of the optimum balance; only the quantitative specifies are changed. Extension of the simpler model to a more realistic two periods causes a greater weight to be attached to non-emergency states-of-nature (both U^k and U^{π}) and, therefore, means that less reliance should be placed on protection and more on stockpiling than in the one period heuristics. Still as in the simpler one period analysis, the relationship between higher chance of disruption and higher dependence on stockpiling persists. A family of curves T^iT^i

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[22] This requires in turn that q >((rt/(r+t)). A reallocation of Δx=-1 along T^aT^a reduces income measured in terms of x along the international terms of trade line t by the amount [((t-q)/t)((t+r)/t)]. Thus one unit less of x consumption in wartime <u>reduces</u> peacetime income (measured in x as numeraire) by [((t-q)/t)((t+r)/t)] and, therefore, <u>reduces</u> expected utility by βU^{π}_X times this amount. Also at the same time, for Δx=-1, wartime utility is <u>reduced</u> by $(1-\beta)U^{\omega}_X$; however because less x allows for an increase in y, [Δy=q-{(t-q)(r/t)}], the utility <u>gain</u> is this amount, Δy, times $(1-\beta)U^{\omega}_Y$. An optimum allocation is reached when the first two marginal losses just balance the last marginal gain. Eq. (17) shows this balance

would show this in the same way Figure 4 does for the simpler case. For very small risk of emergency $(1-\beta) < (1-\beta^0)$ no action is warranted. For somewhat higher risks of emergency $(1-\beta^0)$ trade control and no stockpiling is indicated. At a still higher risk of war $(1-\beta^1)$ stockpiling is introduced and both instruments should be employed. And for a sufficiently high risk of emergency, after $(1-\beta^*)$, a pure stockpile strategy is optimal (where $\beta^0 > \beta^1 > \beta^*$)

9.2 Positive Discount Rate

Next, consider the implications of a positive interest rate. Although these are straight forward and consistent with preceding results, they add complexity and merit attention. First, introduction of a positive discount rate directly raises the current, out of pocket cost/price of stockpiling, causing an *intra-temporal* substitution effect against it. This effect shows up even in a single period two contingency model Eq. (5), where p_S would increase with ρ. But more naturally the effect appears in the multi-period framework of Eq. (14) where there is an inherent rationale for positive interest, and greater ρ will raise the value of p_S or lower the value of "r." Of course, as with any price change, higher ρ and higher p_S has an inter-temporal income effect of reducing the current period income. Assuming normal goods all around, this changes the optimal protection-stockpile mix reducing desired stockage. --- an intra-temporal income effect, which in addition to the substitution effect also disfavors stockpiling and favors subsidy-protection.

Next considering the fact that stockpiling in reality takes place over many time periods we require Eq. (14) to identify another effect of a positive rate of inter-temporal discount ρ. This is because any positive inter-temporal discount rate, ρ, requires that both U^{π} and U^{ω} be discounted by the factor $1/(1+\rho)$, and this has a damping influence on the incentive to shift the burden of risk from bad to good states. Because it is the future that is uncertain, a higher discount rate ρ lowers the evaluation at the present time of future risky loss (not because it is risky, but rather because it is in the future). And this leads in turn to an inter-temporal income-effect shifting burden to the risky future, and thus causing an inward shift in the opportunity sets of Figure 4 from C to A.--- a shift which disfavors stockpiling (assuming normality). In summary then a positive discount rate has ordinary income and a substitution effects and plus an inter-period burden shifting effect all of which --- under goods normality --- disfavor stockpiling..

9.3 Optimal Mix of Subsidy and Stockpiling

We can now summarize all these factors and their implications for a policy of preparations for trade interruption or embargo in our two period model with salvage value. As the risk of war in period-2 rises above $1\text{-}\beta^0$ (its minimum critical value below which no preparations are made) and some of the burden of this increased risk is shifted back to period 1, the optimal first period peacetime utility declines. The optimal response initially is to introduce protection but not to stockpile, the effect of which is to raise second period wartime utility and to reduce both U^k and U^π. As the risk of war increases beyond $(1\text{-}\beta^1)$, and period-1 peacetime utility U^k declines further, stockpiling is introduced raising both wartime and peacetime utilities in second period. Finally as $(1\text{-}\beta)$ approaches $(1\text{-}\beta^*)$ and U^k declines still further, until at $(1\text{-}\beta^*)$, protection drops out and only stockpiling is employed. Positive interest has price, income, and inter-period effects that damp this progression.

10. CONCLUSIONS

Economists, while generally prescribing free trade for its benefits, have long recognized validity in the argument for trade protection of domestic industries as insurance against possible sudden disruptions in international trade. The benefits of protecting import-competing domestic industries arise from reducing adjustments in domestic production in the event of trade disruptions or of having goods available at all if they are critical to national welfare or security.

Using deliberately frugal assumptions, this paper integrates the classic security argument for protection of domestic industries against international competition with a prominent alternative/complement --- stockpiling and/or maintenance of stand-by production capacity. In exploring the anatomy of this problem we show how the likelihood of interruption has an income effect which favors stockpiling, and (if operative) a substitution effect which favors protection as policy responses. The substitution effect follows whenever costs of stockpiling bear some positive relation to chances of disruption. Expectedly, the optimum relative mix of protection and stockpiling will depend on all of the three cost parameters considered in the analysis (i.e., internal production costs, world terms of trade, and stockpiling costs). But unexpectedly, even when stockpiles can be accumulated at world prices and stored at zero interest with no loss or deterioration, some degree of protection may still be justified. Moreover, the risk aversion of the subject country has been shown to systematically influence the optimal mix; paradoxically *lower risk aversion ceteris paribus* entails stronger preparation and increased importance for stockpiling. Schematically we can summarize/illustrate this conclusion with the following two tables, III and IV, where nature chooses either box H or L and independently either box M or N.

[TABLE III and IV HERE]

The analysis is easily generalized for multiple time periods of peace followed by one period with risk of disruption. Under that assumed scenario, there is a higher weight on the loss from or cost of protection and, therefore, a greater reliance on stockpiling in the optimal protection/stockpile mix than in the single-period two-contingency tangency solution of Figures 3 or 4. The analysis thus has constructed a method capable of shedding light into an important area of interdependence between economic policy and national security an area deserving more attention as globalization accelerates.

Of course this exercise has overlooked several factors appropriate to a comprehensive study of preparation for adversity. In addition to further elaborating the relationships between risk of war and stockpile costs, such dynamic elements as stockpile inventory management, or the deterrent value of stockpiles (see Bhagwati and Srinivasan, 1976; Nichols and Zeckhauser, 1977) as a discouragement to others against trade embargoes in the first place deserve investigation. And interactions between reserve accumulations, subsidy-protection of trade and risk reduction of via defense merit examination. Most important protection stockpiling and defense in a multi-country setting deserves exploration. Collaborative policies such preferential trading agreements, economic unions, exchanges of factors of production, and formation of military alliances deserve analysis for their implications for multi contingency risk management.

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Explanatory Note for Diagrams 1-5.

A "home" country is supposed to have a linear production possibility curve with slope q stretching between x_H and y_H and in Figures 1-5. The world trade line with slope t is steeper than q to indicate that the home country has a comparative advantage in production of X. Comparative advantage, therefore, leads to complete specialization in production at x_H and trade from that point to the tangency at $U^{\pi i}$. Emergency is represented by complete elimination of imports and, therefore, consumption at x_H with utility $U^{\omega i}$ unless preparations have been made in advance to cushion the loss.

Such preparations allow some of the burden of trade interruption to be shifted to "peace" from "war." Peacetime welfare contracts along the income expansion path through $U^{\pi i}$ - $U^{\pi n}$ etc. (not drawn). Wartime welfare increases --- depending variously on the policies chosen --- from $U^{\omega i}$ to $U^{\omega n}$ or $U^{\omega *}$.

Figure 1

Optimal Protection Without Stockpiling
One Time Period - Two Contingencies

X_H	Maximum domestic production of X.
Y_H	Maximum domestic production of Y.
1/q	Unit cost of Y produced at home.
1/t	Unit cost of Y bought on world markets.
$U^{\omega\, i}$	Wartime utility with no protection.
$U^{\pi\, i}$	Peacetime utility with no protection.
$X_D{}^n$	Choice that maximizes wartime utility.
$U^{\omega\, n}$	Greatest possible wartime utility.
$U^{\pi\, n}$	Peacetime utility if wartime utility is maximized.
*	Index for example of optimal choice when $\beta < 1$.
U^{ω} *	Example of wartime utility at optimum choice. *.
U^{π} *	Example of peacetime utility at optimum choice. *.

Figure 2

Optimal Stockpile Without Protection:
One Period - Two Contingencies

$1/q$	Unit cost of Y produced at home.
$1/t$	Unit cost of Y bought on world markets Same as unit cost of stockpiling and retrieval.
$U^{\omega\,i}$	Wartime utility with no stockpiling. X_S=0.
$U^{\pi\,i}$	Peacetime utility with no stockpiling. X_S=0.
X_S^m, X_S*	Alternative amounts of X used top buy Y-stockpiles.
Y_S^m, Y_S*	Corresponding amounts of Y purchased and available during wartime.
$U^{\omega\,m}$,U^{ω}*	Wartime utilities corresponding to X_S^m, X_S*.
$U^{\pi\,m}$,U^{π}*	Peacetime utilities corresponding to X_S^m, X_S*.

Figure 3

**The Optimal Mix Between Stockpiling and Protection:
One Period Two Contingencies**

1/q	Unit cost of Y produced at home.
1/t	Unit price of Y bought on world markets. Assumed to be the same as the cost of storing and retrieving one unit of Y
$U^{\pi\, i}$, $U^{\omega\, i}$	Peacetime-Wartime utilities with no protection and no stockpiling. X_D=0 and X_S=0.
$U^{\omega\, p}$	Wartime utility with protection only and no stockpiling. $X_D = X_D^*$ and $X_S = 0$..
$U^{\omega s}$	Wartime utility with stockpiling only and no protection. $X_D = 0$ and $X_S = X_S^*$.
U^{π} *	Peacetime utility resulting from either pure stockpiling or pure protection. $X_D = X_D^*$ <u>OR</u> $X_S = X_S^*$
ab	Set of wartime consumption possibilities, derived from weighted combinations of pure stockpiling and pure protection, and consistent with peacetime utility of U^{π} *
$U^{\omega\, c}$	Wartime optimum as constrained by consumption possibilities "ab."
X_S^c , X_D^c	Optimum stockpile and domestic production to achieve $U^{\omega\, c}$.

Figure 4

The Shift Toward Stockpiling in the Optimum Mix Between Protection and Stockpiling as the Risk of War Increases: One Time Period - Two Contingencies

A,B,C = Different wartime consumption opportunity sets (S) available at greater sacrifice of peacetime utility.

Each S is derived from linear combinations of protection and stockpiling.

As risk of war increases, lower peacetime utility U^{π} is acceptable and higher S-curve is available.

As S shifts out the optimum moves rightward, stockpiling increases.

Figure 5

Optimum Mix of Stockpiling and Protection in the Two Time-Period Two Contingency Case

1/r	Unit cost of acquiring a stockpile of Y enough to recover one unit.
$U^{k\,i}$	Initial 1st or 2nd Period peacetime utility: no protection and no stockpiling. X_D=0, X_S=0.
$U^{\omega\,i}$	Initial Second Period wartime utility: no protection and no stockpiling. $X_D = X_D$*, X_S = 0..
U^{ka}=U^{π} *	First period, peacetime utility consistent with either pure stockpiling or pure protection. $X_D = X_D$* OR $X_S = X_S$*, or linear combinations.
T^aT^a	Attainable wartime consumption possibilities, derived from combinations of stockpiling and protection, and consistent with peacetime utility of U^{ka}
U^{ω} * U^{π} *	Wartime and peacetime components of maximum 2nd period expected utility subject to first period constraint U^{ka} .
X_S^c ,X_D^c	Optimum stockpile and domestic production to achieve $U^{\omega\,c}$.

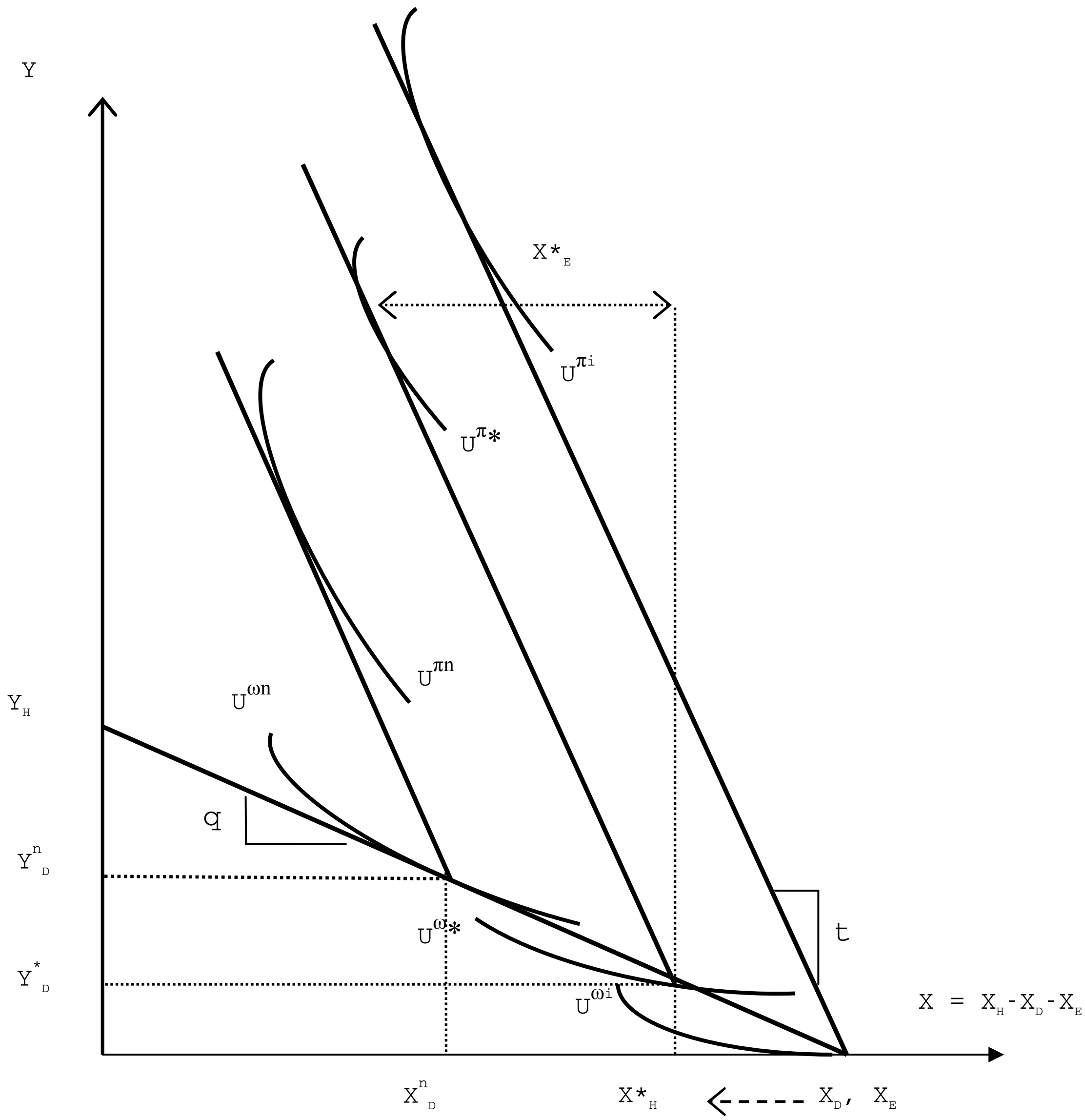
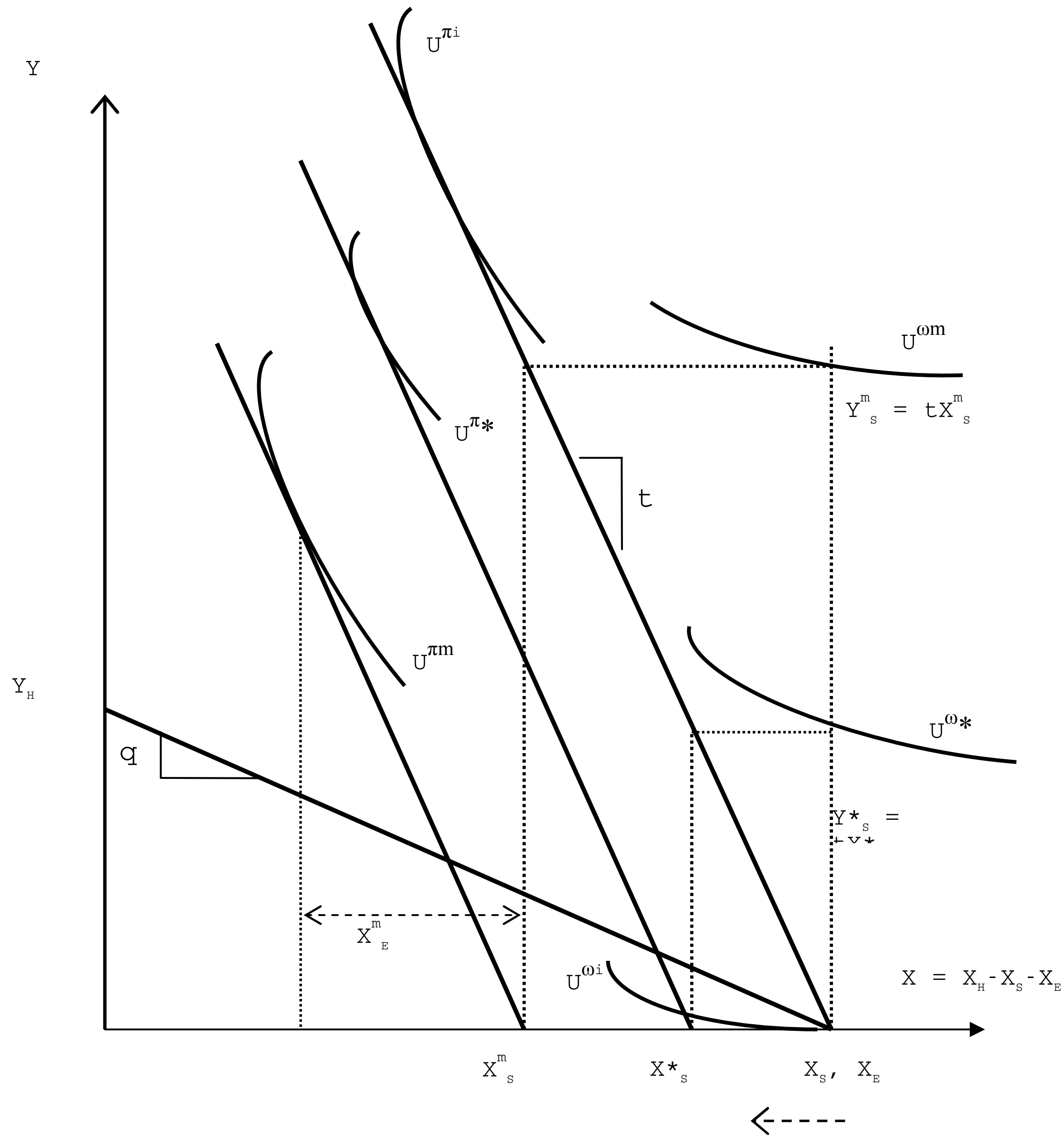
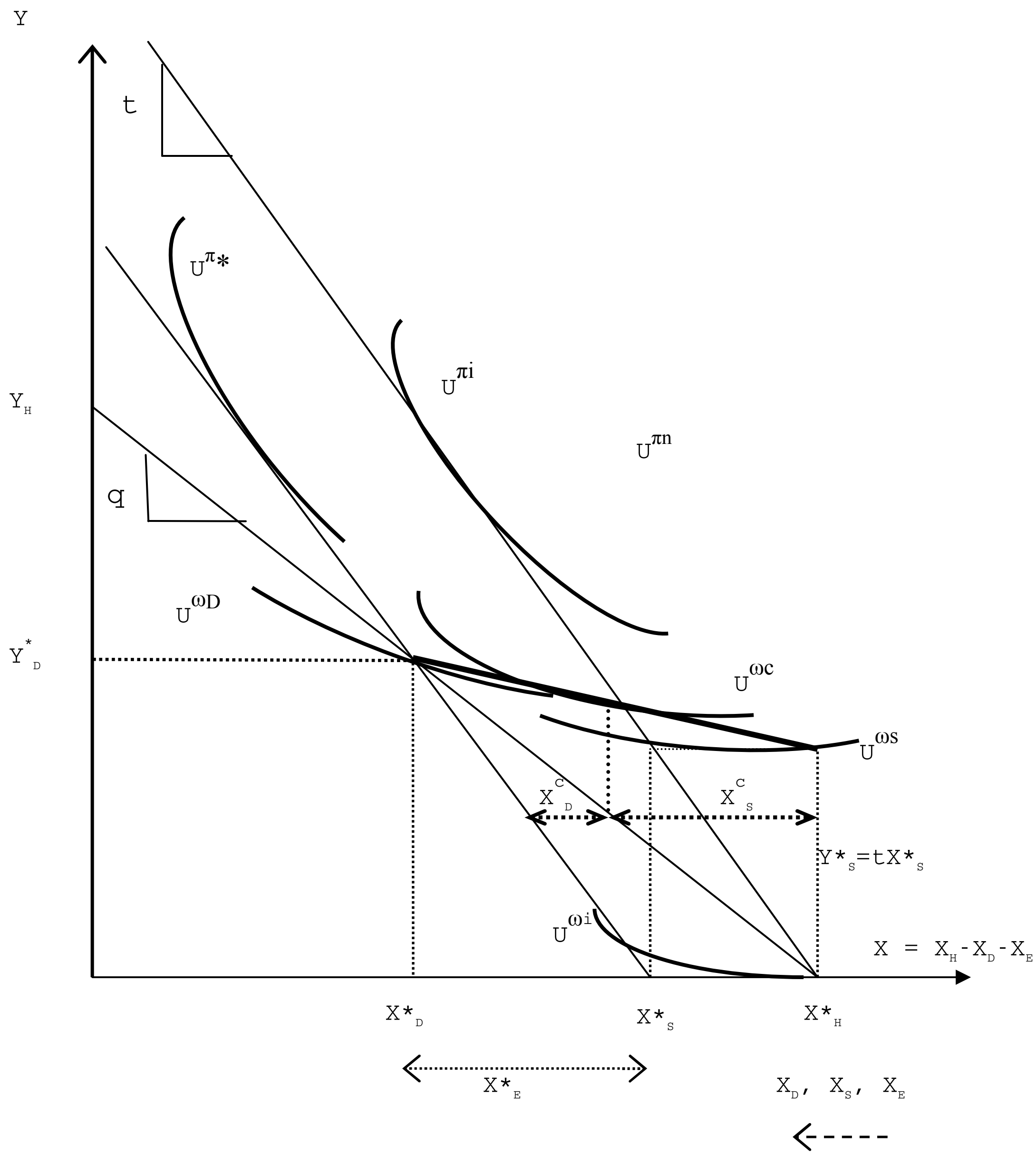


Figure 1



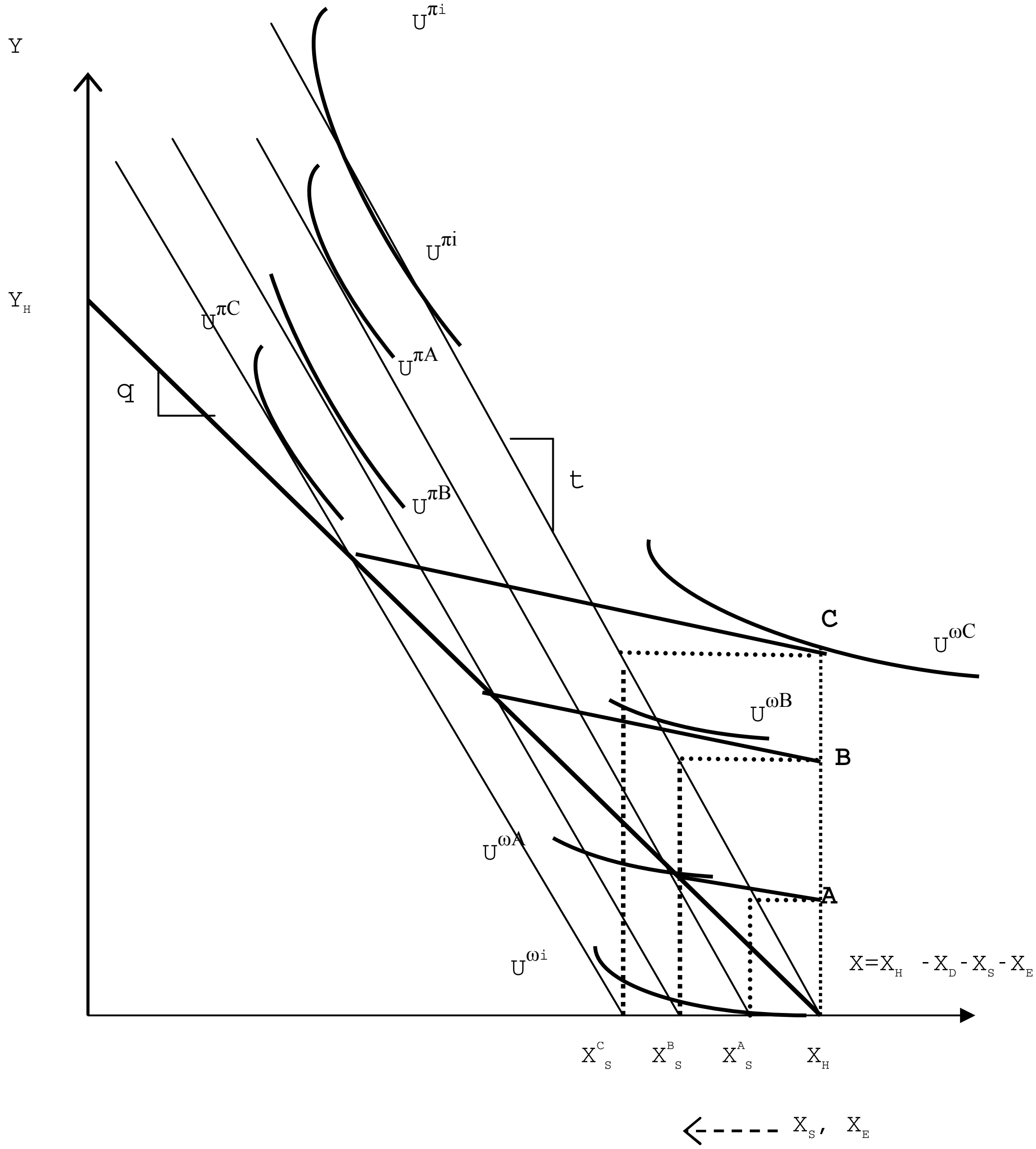
$U^{\pi i}$
Y
$U^{\omega m}$
$Y^m_S = tX^m_S$
$U^{\pi *}$
t
$U^{\pi m}$
Y_H
$U^{\omega *}$
q
$Y^*_S =$
X^m_E
$U^{\omega i}$
$X = X_H - X_S - X_E$
X^m_S
X^*_S
$X_S,\ X_E$

Figure 2


Y
t
$U^{\pi *}$
$U^{\pi i}$
Y_H
$U^{\pi n}$
q
$U^{\omega D}$
Y^*_D
$U^{\omega c}$
$U^{\omega s}$
X^C_D
X^C_S
$Y^*_S = tX^*_S$
$U^{\omega i}$
$X = X_H - X_D - X_E$
X^*_D
X^*_S
X^*_H
X^*_E
X_D, X_S, X_E

t

Figure 3



Y
$U^{\pi i}$
$U^{\pi i}$
Y_H
$U^{\pi C}$
$U^{\pi A}$
q
t
$U^{\pi B}$
C
$U^{\omega C}$
$U^{\omega B}$
B
$U^{\omega A}$
A
$U^{\omega i}$
$X=X_H - X_D - X_S - X_E$
X^C_S
X^B_S
X^A_S
X_H
X_S, X_E

Figure 4

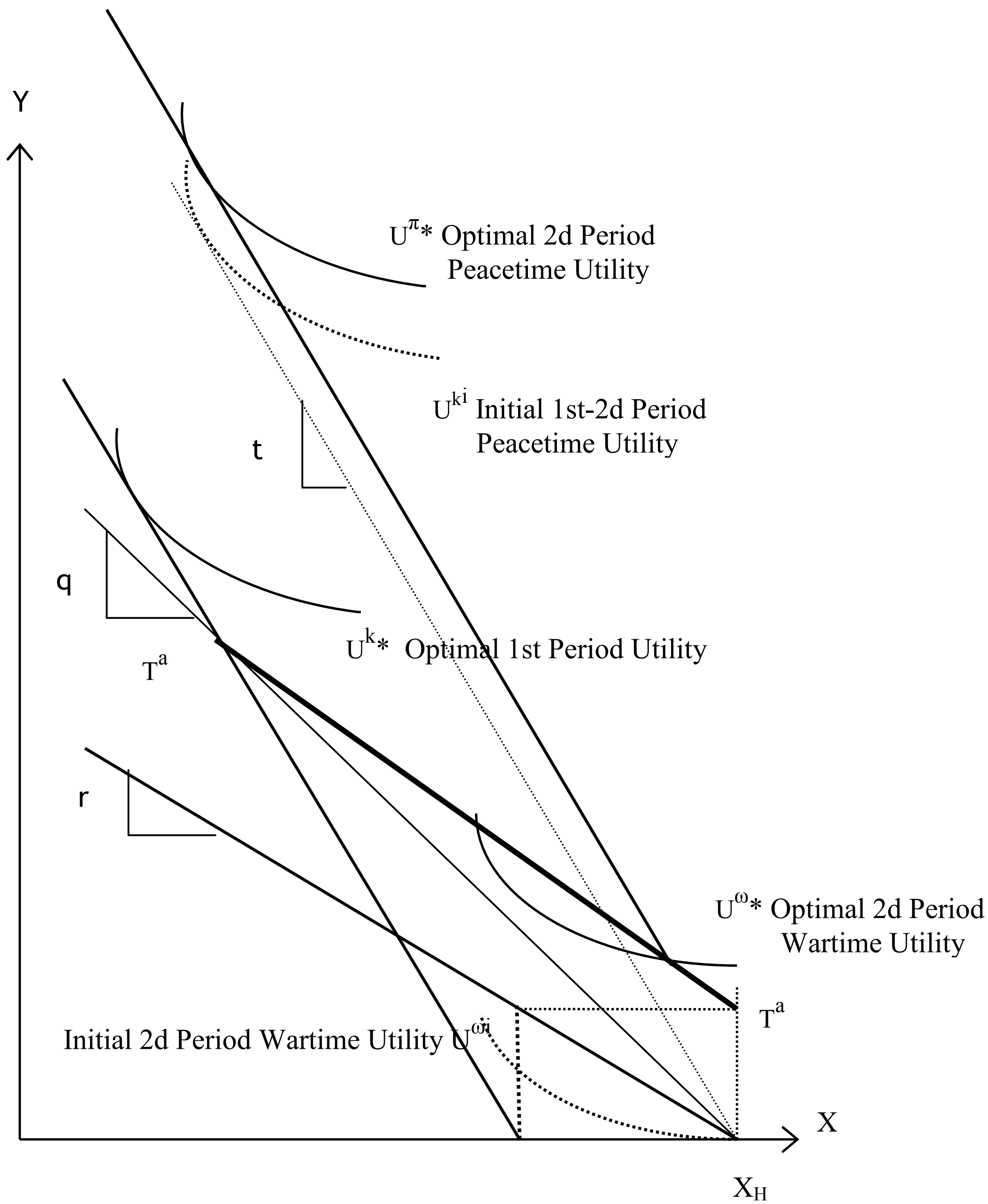


Figure 5
Two Time Periods, Stockpiles Recovered in Second Period

Table I
Notation

Variable	Description	Comment
x	export good	numeraire good is x
y	import good	
p_D q	unit domestic production cost of y in terms of good x. slope of home PPC	$p_D = 1/q$
p_W t	world trading price of y in terms of good x. slope of world trading line	$p_W = 1/t$
x_H	maximum production of x at home, determined by overall resource constraints.	
y_D	amount of y produced at home	$y_D = qx_D$
x_D	amount of home production of x <u>foregone</u> to produce y_D	$x_D = p_D y_D$
y_M	amount of y imported	$y_M = tx_E$
x_E	amount of x exported to purchase y_M on world markets.	$x_E = p_W y_M$
$y_M + y_D$	home consumption of y	
$x_H - x_D - x_E$	home consumption of x.	

Table II
Notation

Variable	Description	Comment
x_S	the amount of exports used to purchase a stockpile of good y to be set aside for emergency use and not consumed except in emergency	
y_S r	the amount of imports so obtained and stockpiled. the quantity of stockpile that one unit of x_S buys net of spoilage and transaction costs.	$y_S = r\, x_S$
p_S	unit cost of y-stockpiles in terms of good x including spoilage and transaction cost of delivering to and retrieving stockpiles.	$p_S = 1/r$

Table III
Effect of Risk Aversion on Optimal Policy

	Risk Aversion	
	High	Low
Superior Policy Weighted Toward	Production Subsidy H	Stockpile L

Table IV
Effect of Risk of Adversity on Optimal Policy

	Risk of Import Interruption (1-β)	
	High	Low
Income Effect: Weights Superior Policy Toward	Stockpile M	Production Subsidy N
Price Effect Weights Superior Policy Toward [assuming $p_s = p_s\{(1-\beta)/\beta\}$]*	Production Subsidy M	Stockpile N
* p_s = price of stockpiling; (1-β) = likelihood of interruption; β = likelihood of no interruption		