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Positron transport and acceleration in beam-driven plasma wakefield accelerators using plasma columns

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# High-Quality Positron Acceleration in Beam-Driven Plasma Accelerators

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Acceleration of positron beams in plasma-based accelerators is a highly-challenging task. To realize a plasma-based linear collider, acceleration of a positron bunch with high-efficiency is required, while maintaining both a low emittance and a sub-percent-level energy spread. Recently, a plasma-based positron acceleration scheme was proposed in which a wake suitable for the acceleration and transport of positrons is produced in a plasma column by means of an electron drive beam [Diederichs et al., Phys. Rev. Accel. Beams 22, 081301 (2019)]. In this Article, we present a study of beam loading for a positron beam in this type of wake. We demonstrate via particle-in-cell simulations that acceleration of high-quality positron beams is possible, and we discuss a possible path to achieve collider-relevant parameters.

## I. INTRODUCTION

Plasma-based particle accelerators potentially enable compact linear electron-positron colliders due to their large acceleration gradients [1]. In a plasma wakefield accelerator (PWFA), an ultra-relativistic, high-charge density particle beam expels all plasma electrons from its propagation axis and an ion cavity is formed [2, 3]. The cavity, also referred to as bubble or blowout, features a region with a large, longitudinally accelerating gradient and a transversely linear restoring force for relativistic electrons. Whereas high-energy gain, high-efficiency [4, 5], and stably beam-loaded [6] electron acceleration has been demonstrated experimentally in PWFAs, stable and quality preserving positron acceleration remains a challenge. Identifying a positron acceleration scheme that fulfills the requirements imposed by a particle collider, namely the stable and efficient acceleration of high-charge positron bunches, while maintaining both a low emittance and a low energy spread, has been an outstanding challenge, and previously proposed positron acceleration concepts were not able to meet all the necessary requirements. For instance, utilizing hollow core electron drive beams showed only a per-mille-level driver-to-witness energy conversion efficiency [7]. PWFAs driven by a positron beam have been investigated in Ref. [8]. While this scheme demonstrated high-efficiency acceleration of the positron witness beam, the nonlinear nature of the transverse focusing fields, and their variation as the drive beam evolves renders the preservation of the witness beam emittance challenging. Hollow core plasma channels have been proposed as potential plasma target candidates for positron acceleration [9, 10]. However, owing to the lack of any focusing field for the beam in a hollow channel, this scheme suffers from severe beam breakup instability [9, 11].

In a recent article, a novel method for positron acceleration was proposed that uses an electron beam as driver and a plasma column as the acceleration medium [12]. For a plasma column with a column radius smaller than

the blowout radius, the transverse wakefields are altered, resulting in an elongation of the background plasma trajectories returning towards the axis. This creates a long, high-density electron filament, leading to the formation of a wake phase region which is suitable for acceleration and transport of positron beams. Despite the nonlinear nature of the transverse wakefields, it was shown that quasi-matched propagation of positron beams was possible. Due to the non-uniformity of the accelerating field created in these structures, the energy-spread was found to be at the percent-level, which is too high for application in a plasma-based linear collider. Another study has investigated beam loading of simple Gaussian beams in these plasma structures [13]. Despite achieving higher efficiency than the one reported in Ref. [12], the emittance was not preserved.

In this Article, we investigate beam loading of a positron bunch in the nonlinear wake formed in a plasma column with the goal of minimizing the energy spread of the bunch, while maintaining both a low emittance and a high charge. Beam loading has been first described for linear wakes in Ref. [14]. In the nonlinear blowout regime, beam loading of electron beams was studied in Ref. [15], where an analytical expression for the longitudinal witness beam current profile that eliminates the energy spread was obtained. Owing to the different nature of the wakefield structure, this type of analytic result is not valid in the case of the nonlinear positron accelerating fields considered in this study. Here, beamloading is studied by means of a numerical algorithm that reconstructs, slice-by-slice and self-consistently, the longitudinal current profile of an optimal witness beam which flattens the accelerating fields within the bunch. We further discuss the transport of the positron witness bunch and its optimization with the goal of minimizing the energy spread and preserving the emittance, both crucial parameters for the employment of this acceleration scheme in a future plasma-based linear collider.

Lastly, we assess a possible path to achieve collider-relevant parameters.

## II. NONLINEAR WAKEFIELDS FOR POSITRON ACCELERATION

96  
97

98 The generation of positron beam focusing and acceler-  
 99 ating wakes using plasma columns was first described in  
 100 [12]. Using an electron drive beam and a plasma column  
 101 with a radius smaller than the blowout radius leads to  
 102 the formation of a wide longitudinal electron filament  
 103 behind the blowout bubble. This elongated region of  
 104 high electron density provides accelerating and focusing  
 105 fields for positron beams. This is illustrated in Figs. 1 and  
 106 2, which show two-dimensional maps of the accelerating  
 107 field  $E_z/E_0$  and focusing field  $(E_x - B_y)/E_0$ , respectively.  
 108 The fields are normalized to the cold, non-relativistic  
 109 wave-breaking limit  $E_0 = \omega_p mc/e$ , where  $c$  denotes the  
 110 speed of light in vacuum,  $\omega_p = (4\pi n_0 e^2/m)^{1/2}$  the plasma  
 111 frequency,  $n_0$  the background plasma density, and  $e$  and  
 112  $m$  the electron charge and mass, respectively. In this  
 113 example, we consider a plasma column with a radius  
 114  $k_p R_p = 2.5$  and a Gaussian electron drive beam with sizes  
 115  $k_p \sigma_{x,y}^{(d)} = 0.1$ ,  $k_p \sigma_\zeta^{(d)} = \sqrt{2}$ , and peak current  $I_b^{(d)}/I_A =$   
 116  $1$ , where  $I_A = mc^3/e \simeq 17$  kA is the Alfvén current. The  
 117 modeling was performed using the quasi-static Particle-  
 118 In-Cell (PIC) code HiPACE [16]. To reduce the high  
 119 computational cost of the modeling imposed by the re-  
 120 quired numerical resolution, the wakefields were com-  
 121 puted using an axisymmetric cylindrical solver based on  
 122 the one implemented in the quasi-static version of the  
 123 code INF&RNO [17], while the particles are advanced in  
 124 full 3D. Denoting by  $k_p = \omega_p/c$  the plasma wavenum-  
 125 ber, the dimensions of the computational domain are  
 126  $12 \times 12 \times 20 k_p^{-3}$  in the coordinates  $x \times y \times \zeta$ , where  
 127  $x$ , and  $y$  are the transverse coordinates, and  $\zeta = z - ct$   
 128 is the longitudinal co-moving coordinate, with  $z$  and  $t$   
 129 being the longitudinal coordinate and the time, respec-  
 130 tively. The resolution is  $0.0056 \times 0.0056 \times 0.0075 k_p^{-3}$ .  
 131 The background electron plasma was modeled with 25  
 132 constant weight particles per cell. The drive beam was  
 133 sampled with  $10^6$  constant-weight particles.

135 The positron focusing and accelerating phase is lo-  
 136 cated between  $-14 \lesssim k_p \zeta \lesssim -9$ . The accelerating field  
 137 has its peak at  $k_p \zeta \approx -11.5$ . Unlike in the blowout  
 138 regime case,  $E_z$  has a transverse dependence. The inset  
 139 of Fig. 1 shows  $E_z/E_0$  along the transverse coordinate  $x$   
 140 at three different longitudinal locations denoted by the  
 141 dashed ( $k_p \zeta = -12.5$ ), solid ( $k_p \zeta = -11.5$ ), and dot-  
 142 ted ( $k_p \zeta = -10.5$ ) lines in Fig. 1. In all three locations,  
 143  $E_z(x)$  has an on-axis maximum and decays for increasing  
 144 distances from the propagation axis. Notably, the trans-  
 145 verse gradient of the accelerating field is smaller further  
 146 behind the driver. The non-uniformity of  $E_z$  will lead  
 147 to a  $\zeta$ -dependent uncorrelated slice energy spread since  
 148 particles that remain closer to the axis will experience a  
 149 larger accelerating gradient compared to the ones further  
 150 off axis. This effect will be investigated more thoroughly  
 151 in section III C.

152 The transverse behavior of the focusing field,  $(E_x - B_y)/E_0$

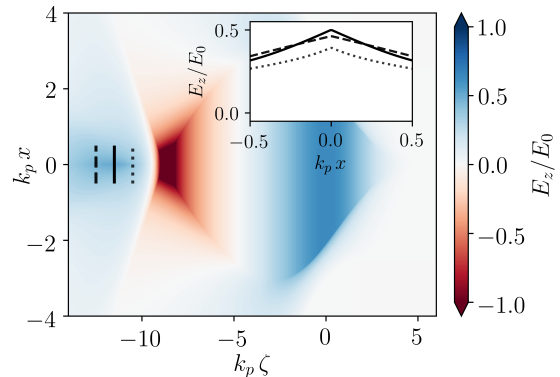


FIG. 1. Two-dimensional  $(\zeta, x)$  map of the accelerating wakefield,  $E_z/E_0$ . Positrons can be accelerated in the region  $-14 \lesssim k_p \zeta \lesssim -10$ . Inset: transverse dependence of accelerating field in the positron accelerating region at three different longitudinal locations denoted by the dashed ( $k_p \zeta = -12.5$ ), solid ( $k_p \zeta = -11.5$ ), and dotted ( $k_p \zeta = -10.5$ ) lines. The accelerating field falls off for increasing distance from the propagation axis. The gradient of the transverse field decreases further behind the driver.

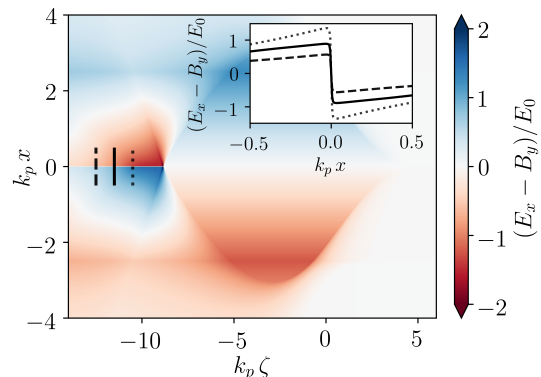


FIG. 2. Two-dimensional  $(\zeta, x)$  map of the focusing wakefield,  $(E_x - B_y)/E_0$ . Positrons can be focused in the region  $-14 \lesssim k_p \zeta \lesssim -9$ , which widely overlaps with the positron accelerating region. Inset: transverse dependence of focusing field at three different longitudinal locations denoted by the dashed ( $k_p \zeta = -12.5$ ), solid ( $k_p \zeta = -11.5$ ), and dotted ( $k_p \zeta = -10.5$ ) lines. The focusing field decays almost linearly for increasing distances from the propagation axis. The field decreases further behind the driver.

$B_y)/E_0$ , is depicted in the inset of Fig. 2, where we show transverse lineouts of the focusing wakefields for the same three longitudinal locations used in Fig. 1. We see that the transverse wakefield decays almost linearly for increasing distances from the propagation axis. The field decrease is smaller further behind the driver. As shown in Ref. [12], the field becomes almost a step-function when sufficiently loaded by a positron bunch.

### 161 III. SELF-CONSISTENT BEAM LOADING TO 162 MINIMIZE THE ENERGY-SPREAD

163 In many beam-driven plasma wakefield accelerator applications, both the driver and the witness beams are  
164 usually highly relativistic and evolve on a much longer time scale than the background plasma. In this case the  
165 quasi-static approximation [18], which allows treatment of the plasma and the relativistic beams in a separate  
166 manner, can be used. In the quasi-static approximation, the wakefields generated by a given beam are determined  
167 by initializing a slice of unperturbed plasma ahead of the beam and then follow its evolution as the slice is pushed  
168 through the beam from head to tail along the negative  $\zeta$  direction (here  $\zeta$  can be interpreted as a fast “time”  
169 that parametrizes plasma-related quantities), while the beam is assumed to be frozen. This implies that to calculate  
170 the fields at some longitudinal position  $\zeta$ , only the information upstream of this point is required.

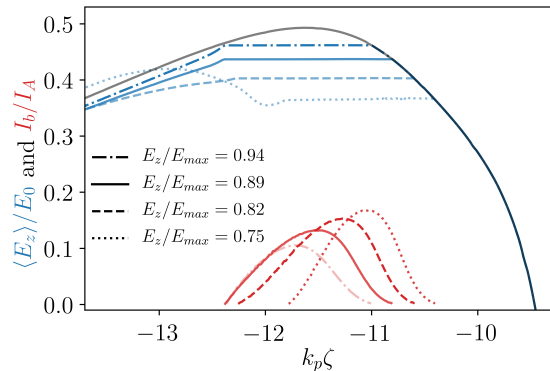
171 We used this feature of the quasi-static solution to design an algorithm that recursively constructs, slice-by-  
172 slice and starting from the head, the optimal current profile of a witness bunch such that the accelerating field  
173 along the bunch is constant and equal to a set value. This leads to a reduced energy spread of the accelerated  
174 particles. The algorithm is described in detail in the Appendix. We considered a (radially symmetric) bunch  
175 initially described as

$$176 \quad n_b(\zeta, r) = g_{\parallel}(\zeta)g_{\perp}(\zeta, r), \quad (1)_{214}$$

177 where  $g_{\parallel}(\zeta)$  and  $g_{\perp}(\zeta, r)$  denote the longitudinal and transverse density profiles, respectively. We require that,  
178 for any  $\zeta$ ,  $\int g_{\perp}(\zeta, r)rdr = \int g_{\perp}(\zeta = \zeta_{head}, r)rdr$ , where  $\zeta_{head}$  is the location of the bunch head, so that the bunch  
179 current density profile only depends on  $g_{\parallel}(\zeta)$ . For simplicity, we first consider bunches that are transversally  
180 Gaussian and longitudinally uniform, i.e.,  $g_{\perp}(\zeta, r) = \exp[-r^2/(2\sigma_r^2)]$ , where  $\sigma_r$  is the (longitudinally constant)  
181 rms bunch size. At every longitudinal location  $\zeta$  (bunch slice), the algorithm performs an iterative search for the  
182 optimal bunch current, determined via  $g_{\parallel}(\zeta)$ , that flattens the accelerating field in that particular slice. The  
183 procedure is repeated recursively for all the slices going from the head to the tail of the bunch. Note that, besides  
184 a constant accelerating field along the bunch, other field configurations yielding an energy chirp during acceleration  
185 are possible. In order for a solution to be found, the positron bunch has to be located in a phase of the wake  
186 where  $\partial_{\zeta}E_z < 0$ . To take into account the fact that in general,  $E_z$  varies in the transverse plane across the  
187 beam, the figure of merit considered by the algorithm is a transversally weighted accelerating field  $\langle E_z \rangle$ , defined  
188 as

$$189 \quad \langle E_z \rangle = \frac{\int_0^{\infty} E_z(r)g_{\perp}(r)r dr}{\int_0^{\infty} g_{\perp}(r)r dr} \quad (2)_{217}$$

$$190 \quad = \frac{\int_0^{\infty} E_z(r) \exp[-r^2/(2\sigma_r^2)]rdr}{\int_0^{\infty} \exp[-r^2/(2\sigma_r^2)]rdr} \quad (2)_{218}$$



191 FIG. 3. Current profiles for an optimally loaded wake  $I_b/I_A$   
(red), and corresponding lineouts of the transversally averaged  
192 accelerating field,  $\langle E_z \rangle / E_0$  (blue). The current profiles  
193 and their corresponding accelerating gradient is given for four  
194 values of the witness bunch head position,  $k_p \zeta_{head} = -11.0$   
(dotted dashed),  $-10.8$  (solid),  $-10.6$  (dashed),  $-10.4$  (dotted).  
195 Their relative averaged accelerating gradients within the beam are  
(in the same order as the starting position) 0.462, 0.438, 0.403,  
196 0.367 with respect to the maximum accelerating gradient of the  
197 unloaded wake  $E_{max}/E_0 = 0.49$ . The same line style marks the  
198 current profile and its corresponding field lineout. The presented  
199 current profiles optimally load the wake, resulting in a flattened  
200  $\langle E_z \rangle / E_0$ .

201 In case of a transversally uniform accelerating field, e.g. as in the  
202 blowout regime, the averaged accelerating field simply reduces to the  
203 on-axis accelerating field.

#### 204 A. Optimization of the witness bunch position

205 The choice of the location of the witness bunch head,  $\zeta_{head}$ , sets  
206 the amplitude of the accelerating gradient and determines the shape  
207 of bunch current profile. In the following, we study the effect of  
208 different witness head positions for a bunch in the wake described  
209 in Section II. To fulfill the requirement that the bunch head has  
210 to be located in a wake phase such that  $\partial_{\zeta}E_z < 0$ , and to  
211 achieve a reasonable acceleration gradient, we chose  $-11.5 \lesssim k_p \zeta_{head} \lesssim -10$ .  
212 Also, we consider a witness bunch an emittance such that  $k_p \epsilon_x = 0.05$ ,  
213 and a bunch size  $k_p \sigma_r = 0.0163$ . Numerical results for the current  
214 profiles and their corresponding loaded averaged accelerating fields,  
215  $\langle E_z \rangle$ , for four values of the witness bunch head position are  
216 depicted in Fig. 3. Interestingly, placing the bunch head in a  
217 more forward position in the wake, corresponding to a lower  
218 accelerating gradient, does not necessarily increase the charge of  
219 the witness bunch. This can be seen in Tab. I, where we show the  
220 witness charge,  $Q_w$ , as a function of the bunch head position.  
221 Values of the charge have been computed assuming a background  
222 density of  $n_0 = 5 \times 10^{17} \text{ cm}^{-3}$ . For this density the charge of  
223 the drive beam is  $Q_d = 1.5 \text{ nC}$ .

The driver-to-beam efficiency,  $\eta$ , can be calculated from the charge of the witness beam, its energy gain rate,  $E_w^+$ , the charge of the drive beam, and its energy loss rate,  $E_d^-$ , via

$$\eta = \frac{Q_w E_w^+}{Q_d E_d^-}. \quad (3)$$

For the chosen density the driver energy loss rate is 34 GeV/m. Values of the energy gain for the witness bunch and the efficiency as a function of the witness head position are given in Tab. I.

The results show that the efficiency peaks around the position  $-10.8 \lesssim k_p \zeta_{head} \lesssim -10.6$ . As shown in section II, the accelerating field is transversely flatter for more negative head positions, therefore the case  $k_p \zeta_{head} = -10.8$  is preferable since the choice of this witness position will result in a smaller energy-spread, while maintaining close to maximum efficiency. We recall that for this witness position the charge of the bunch is 52 pC and the efficiency  $\eta \approx 3\%$ . This is less than what was achieved with a simple Gaussian density profile in [12], which featured a witness bunch charge of  $Q_w = 84$  pC and an efficiency of  $\eta \approx 4.8\%$ . However, energy-spread minimization was not taken into consideration in that study, which lead to an energy-spread on the few-percent-level.

Choosing bunch head positions that are closer to the driver, i.e.,  $k_p \zeta_{head} \geq -10.2$ , yields complex (e.g., multi-peaked) bunch current profiles. In this case, the positron beam significantly alters the background plasma electron trajectories, resulting in the formation of a second on-axis electron density peak behind the blowout region. This, in principle, allows for the loading of a second positron beam or an increase of the length of the first. This can be seen in Fig. 3. In fact, for  $k_p \zeta_{head} = -10.4$  (dotted line) we see that  $\langle E_z \rangle$  has a local maximum behind the bunch which is higher than the value within the bunch, allowing for further beam loading. We did not investigate further such forward starting positions because we consider the resulting complex bunch structures difficult to realize experimentally.

TABLE I. Charge and energy gain of the witness bunch, and driver-to-witness efficiency as a function of the witness head position. Values are computed assuming a background plasma density  $n_0 = 5 \times 10^{17} \text{cm}^{-3}$ . The driver parameters are the same as in Section II, yielding  $Q_d = 1.5$  nC and  $E_d^- = 34$  GeV/m.

$k_p \zeta_{head}$	$Q_w$ [pC]	$E_w^+$ [GeV/m]	$\eta$ [%]
-10.4	54	25.0	2.7
-10.6	57	27.4	3.1
-10.8	52	29.8	3.0
-11.0	36	31.4	2.2

## B. Minimizing the correlated energy-spread

Using the weighted accelerating field  $\langle E_z \rangle$  from Eq. 2 as the figure of merit in the proposed algorithm yields a bunch current profile that eliminates the correlated energy-spread only under the assumption that the bunch size does not change during acceleration. However, this assumption is generally not true. First, if the spot size is not matched to the focusing field at some position along the bunch due to, e.g., the slice-dependent nature of the transverse wakefields, it will evolve until it is matched. Second, due to the acceleration, the matched spot size adiabatically decreases with increased particle energy. Both effects must be taken into account in order to eliminate the correlated energy spread entirely. Eliminating the mismatch requires performing a slice-by-slice matching of the beam, i.e., introducing a slice-dependent bunch size,  $\sigma_r(\zeta)$ . Note that this also leads to a  $\zeta$ -dependence of  $g_\perp$ . In our algorithm, calculation of the self-consistent, slice-dependent bunch size can be done numerically while the optimal bunch is generated. As a desirable side effect, the slice-by-slice matching also minimizes the emittance growth [19]. To take into account the change of  $\sigma_r(\zeta)$  due to acceleration, the averaged spot size over the acceleration distance should be used in calculating  $\langle E_z \rangle$  in Eq. 2. We recall that for the here considered step-like wakes with a field strength of  $\alpha$  the matching condition for a given emittance  $\epsilon_x$  is  $\sigma_{r,matched}^3 \simeq 1.72 \epsilon_x^2 / (k_p \alpha \gamma)$  and so the matched spot size is expected to scale with the energy as  $\sigma_{r,matched} \propto \sqrt[3]{1/\gamma}$ , where  $\gamma$  is the bunch relativistic factor [12].

The averaged bunch size over the acceleration distance can then be estimated as

$$\begin{aligned} \bar{\sigma}_r(\zeta) &= \frac{\sigma_r(\zeta)}{\gamma_{final} - \gamma_{init}} \int_{\gamma_{init}}^{\gamma_{final}} \sqrt[3]{\frac{\gamma_{init}}{\gamma}} d\gamma \\ &= \frac{3}{2} \sigma_r(\zeta) \gamma_{init}^{1/3} \frac{\gamma_{final}^{2/3} - \gamma_{init}^{2/3}}{\gamma_{final} - \gamma_{init}}, \end{aligned} \quad (4)$$

where  $\gamma_{init}$  and  $\gamma_{final}$  refer to the initial and final bunch energy, respectively. Note that to calculate  $\bar{\sigma}_r(\zeta)$ , the final beam energy  $\gamma_{final}$  is required. We also notice that the inclusion of the slice-by-slice matching and of the energy-averaged bunch size when computing the optimal bunch profiles do not significantly alter the current profiles and charges discussed in Section III A. Changes to the optimal beamloading algorithm including slice-by-slice matching and averaged spot size are described in the Appendix.

The efficacy of the slice-by-slice matching and inclusion of the average spot size is demonstrated in Fig. 4, where we show the mean energy of each slice for a positron witness bunch that accelerates from 1 GeV to  $\approx 5.5$  GeV in a distance of 15 cm. The blue line refers to algorithm flattening  $\langle E_z \rangle$  with a longitudinal uniform  $\sigma_r$ . The red line and the green line refer to additionally applying slice-by-slice matching and averaging of the bunch spot size over the acceleration distance, respectively. In this example,

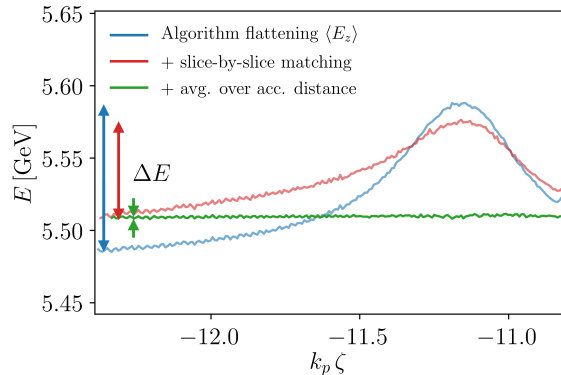


FIG. 4. Mean energy of each slice vs. longitudinal position in the positron bunch after acceleration. The blue line refers to algorithm flattening  $\langle E_z \rangle$  with a longitudinal uniform  $\sigma_r$ . The red line and the green line refer to additionally applying slice-by-slice matching and averaging of the bunch spot size over the acceleration distance, respectively. That way, the correlated energy spread can be reduced to the noise level.

the location of the bunch head was  $k_p \zeta_{head} = -10.8$  and the bunch had an initial emittance such that  $k_p \epsilon_x = 0.05$ . All the other parameters were as before (see Section II). In order to mitigate the computational cost, these results were obtained with a frozen field approximation (i.e., the particles of the witness bunch are pushed in a non-evolving wakefield). This approach has shown both reasonable agreement of the energy spread and the emittance of the witness bunch with full quasi-static PIC simulations. The agreement is facilitated by the slice-by-slice matching, which mitigates the witness beam evolution. We see that the bunch obtained without slice-by-slice matching and using the initial longitudinal uniform  $\sigma_r$  (blue line in Fig. 4) shows a range of mean energy variation of  $\Delta E \approx 100$  MeV. Using the slice-by-slice matching (red line) reduces the amplitude of the variation to  $\Delta E \approx 60$  MeV. Finally, by using the energy-averaged bunch size  $\bar{\sigma}_r(\zeta)$  in the calculation of the optimal bunch (green line) the correlated energy spread is essentially removed ( $\Delta E \approx 3$  MeV).

### C. Minimizing the uncorrelated energy-spread

Whereas the correlated energy-spread can be completely eliminated, the uncorrelated energy-spread can only be reduced, as it arises from the transverse non-uniformity of  $E_z$ . We did not identify a strategy to reduce the transverse gradient of  $E_z$  by loading the wake with a positron bunch. However, we explored two possible solutions to minimize the impact of such gradient and reduce the uncorrelated energy spread. First, one can position the witness bunch in a region of the wake where  $E_z$  is transversally as flat as possible, and second, one can use a transversally smaller witness beam.

As described in Sec. II,  $E_z$  flattens transversally further behind the driver (i.e., for more negative  $\zeta$ ). Therefore, it is favorable to choose the starting position of the bunch furthest behind the driver, which still has a reasonable efficiency. According to this criterion and the results from Sec. III A, the optimal starting position is  $k_p \zeta_{head} = -10.8$ .

In the following, we study the dependence of the uncorrelated energy-spread on the witness bunch emittance. Since the smaller the emittance the smaller the bunch is, a bunch with a smaller transverse extent will sample a smaller domain of  $E_z$  and, hence, it will acquire a smaller uncorrelated energy spread. For a flat beam, and assuming that in the vicinity of the axis the accelerating field can be modeled as  $E_z(x) = E_{z,0} - \beta|x|$ , where  $\beta$  describes the transverse gradient of  $E_z$  (see inset of Fig. 1), then, from geometric considerations, we expect the relative slice (uncorrelated) energy spread at saturation to scale as  $\sigma_\gamma/\gamma \sim \beta\sigma_r/E_{z,0}$ , and so  $\sigma_\gamma/\gamma \rightarrow 0$  in the limit of a small bunch. We note that it is not possible with our current numerical tools to model collider-relevant low-emittance witness beams, owing to the required high resolution and associated computational costs. To overcome this limitation, we use a reduced model to assess the scaling of the energy-spread for these conditions. Since the correlated energy-spread can be eliminated with the procedure discussed in the previous section, we consider a single slice of the beam in the reduced model. We chose the slice of the peak current of the positron bunch, which we have found to reasonably represent the total energy-spread of the bunch. Using the previous example with a starting position of  $k_p \zeta_{head} = -10.8$ , the peak of the current is located at  $k_p \zeta_{peak} = -11.45$ . We reuse the simulation, which included the slice-by-slice matching and the averaging over the acceleration distance. Assuming the same density of  $n_0 = 5 \times 10^{17} \text{cm}^{-3}$  as for the efficiency consideration, the emittance of the beam is  $\epsilon_x = 0.05 k_p^{-1} = 0.38 \mu\text{m}$ . In the reduced model, we generate test particles, which we advance with a second-order-accurate particle pusher in the radial fields provided by the simulation. High-resolution simulations with the cylindrically symmetric PIC code INF&RNO indicate that the focusing field converges towards a step function [12]. Likewise, we model the focusing field in the reduced model with a piecewise constant function,  $(E_x - B_y)/E_0 = -\alpha \text{sign}(x)$ , where  $\alpha = 0.6$  for our example. We have found the model in reasonable agreement with HiPACE simulations in terms of energy-spread, emittance, and bunch size evolution. This is shown for the energy-spread in Fig. 5. The black dashed line and the blue solid line describe the energy-spread at an emittance of  $\epsilon_x = 0.38 \mu\text{m}$  obtained from the HiPACE simulation and the reduced model, respectively. Under the assumption that a smaller emittance beam with the same charge does not significantly change the wake structure, we can decrease the beam emittance in the reduced model to previously numerically inaccessible values. The results are shown in Fig. 5. The energy-spread of the



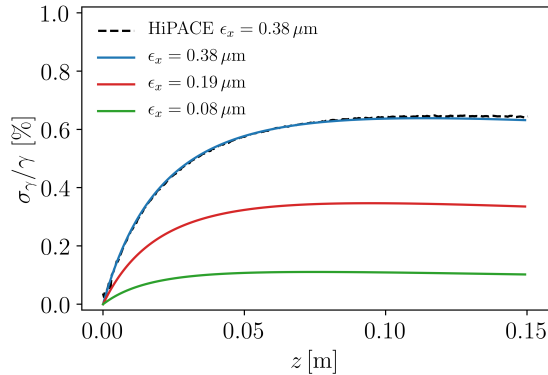


FIG. 5. Relative slice energy spread vs. acceleration distance. Advancing test particles in an approximated step function yields similar energy-spread in comparison with the HiPACE simulation. The results indicate that emittances smaller than  $0.1 \mu\text{m}$  induce an energy spread below 0.1%.

423 peak-current slice of the beam is  $\approx 0.65\%$  for both the  
 424 simulation (dashed line) and the reduced model (blue  
 425 line). The final energy-spread and the emittance growth  
 426 of the whole bunch in the PIC simulation are  $\approx 0.7\%$   
 427 and  $\approx 2\%$  (both not shown in Fig. 5), respectively. The  
 428 results of the reduced model indicate that for emittances  
 429 well below  $0.1 \mu\text{m}$ , we can achieve energy spreads below  
 430 0.1%. The red and green line denote the energy spreads  
 431 for initial emittances of  $\epsilon_x = 0.19 \mu\text{m}$  and  $\epsilon_x = 0.08 \mu\text{m}$ ,  
 432 respectively. Their corresponding final energy spreads  
 433 are 0.3% and 0.1%. This indicates the path to possible  
 434 collider-relevant parameters. However, this model does  
 435 not capture the change of the wake structure due to a  
 436 reduced witness bunch spot size. Eventually, when the  
 437 on-axis density of the positron bunch exceeds the density  
 438 of the background electrons, we expect a significant dis-  
 439 ruption of the positron accelerating wake structure. Ad-  
 440 ditionally, a finite initial background plasma temperature  
 441 can smooth the piecewise constant focusing field, possi-  
 442 bly affecting the results presented here. These effects will  
 443 be the topic of further research and require extensive de-  
 444 velopment of simulation tools to enable detailed studies.

#### 445 IV. CONCLUSION

446 High-quality positron acceleration with sub-percent-  
 447 level energy spread is possible in beam driven plasma  
 448 wakefield accelerators. Utilizing an electron drive beam  
 449 and a narrow plasma column allows for high-charge, and  
 450 low-emittance positron beams. By shaping the longitu-  
 451 dinal density profile of a transversally Gaussian witness  
 452 beam, the energy-spread can be controlled and kept at  
 453 the sub-percent-level. Thereby, correlated energy spread  
 454 can be completely eliminated. The uncorrelated energy  
 455 spread scales with the transverse beam spot size. Our

456 results indicate that using collider-relevant beam emit-  
 457 tances might yield energy spreads as low as 0.1%. Fur-  
 458 ther research will aim to strengthen this result. Addi-  
 459 tionally, the efficiency might be increased by proper shap-  
 460 ing the drive beam [20, 21], by optimizing the transverse  
 461 plasma profile [12] or by using the here proposed tech-  
 462 nique to generate longitudinally chirped bunches. Ex-  
 463 tending these results to higher efficiencies will pave the  
 464 path to a plasma-based collider.

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#### Appendix: Algorithm for determining the bunch profile with optimized beam loading

The algorithm used in this study calculates, by ex-  
 ploiting the quasi-static approximation, the longitudinal  
 current profile of a witness bunch that maintains the av-  
 erage accelerating gradient over the full bunch length.  
 The average accelerating gradient is set at the bunch  
 head,  $\langle E_{z,head} \rangle$ . The bunch is constructed recursively by  
 stacking infinitesimal longitudinal slices of charge, one af-  
 ter the other, starting from the head and going towards  
 the tail of the bunch. For each slice, the calculation of  
 the optimal current is done using an optimized bisection  
 procedure.

The steps of the algorithm are as follows. First, for  
 any generic longitudinal slice  $i$  ( $i = 0$  represents the  
 bunch head, slices are counted starting from the head),  
 the algorithm computes the weighted accelerating field  
 right behind the current slice assuming zero charge in  
 the slice. We denote this quantity by  $\langle E_{z,i} \rangle$ . We then  
 check if  $|\langle E_{z,i} \rangle| > |\langle E_{z,head} \rangle|$ . The absolute value is used  
 so the algorithm works for both electron and positron  
 witness bunches. If this condition is not fulfilled then  
 no further beamloading is possible and the recursive pro-  
 cedure terminates (i.e., the bunch tail is reached). On  
 then other hand, if the condition is satisfied, then beam-  
 loading is possible and the algorithm initializes the op-  
 timized bisection procedure to determine the current in  
 the slice. We recall that the current is set via the  $g_{\parallel}$   
 function in Eq. 1. In order for the bisection procedure to

converge, values of the current lower ( $g_{\parallel,min}$ ) and higher ( $g_{\parallel,max}$ ) than the optimal one need to be determined. Since we know that with no charge in the  $i$ -th slice we have  $|\langle E_{z,i} \rangle| > |\langle E_{z,head} \rangle|$ , then we can set  $g_{\parallel,min} = 0$ . Determining  $g_{\parallel,max}$  requires a trial and error procedure where, starting from, e.g.,  $g_{\parallel,max} = 1.2g_{\parallel,i-1}$ , the value of the current in the slice is progressively increased in a geometric way (i.e., typically multiplying the current by a factor 10) until overloading of the wake is reached, i.e., until the condition  $|\langle E_{z,i} \rangle| < |\langle E_{z,head} \rangle|$  is satisfied. Note that every time the value of the current in the slice is changed, a solution of the quasi-static field equations for the slice is required in order to determine the current value of the weighted accelerating field behind the slice. Once  $g_{\parallel,min}$  and  $g_{\parallel,max}$  are known, the optimized bisection procedure begins. A new value of the current is computed according to

$$g_{\parallel} = w_g g_{\parallel,min} + (1 - w_g) g_{\parallel,max}, \quad (\text{A.1})$$

where  $w_g = (|\langle E_{z,head} \rangle| - |\langle E_{z,min} \rangle|) / (|\langle E_{z,max} \rangle| - |\langle E_{z,min} \rangle|)$ , and where  $\langle E_{z,min} \rangle$  and  $\langle E_{z,max} \rangle$  are the averaged field values behind the slice correspond to  $g_{\parallel,max}$  and  $g_{\parallel,min}$ , respectively. The bisection procedure iterates, and the algorithm advances to the next slice ( $i + 1$ ), if the averaged field computed with  $g_{\parallel}$  converges to  $\langle E_{z,head} \rangle$  within a predetermined tolerance, otherwise  $g_{\parallel,min}$  and  $g_{\parallel,max}$  are updated and a new optimized bisection is performed. We note that by using Eq. A.1 instead of the classical bisection procedure, i.e.,  $g_{\parallel} = 0.5(g_{\parallel,min} + g_{\parallel,max})$ , the number of iterations required to reach convergence is significantly reduced.

### Algorithm modifications for slice-by-slice matching and average bunch size

Incorporating the slice-by-slice matching procedure into the algorithm requires the following modification. At each slice  $i$ , the matched spot size  $\sigma_{r,matched}$  needs to be determined. This is done by exploiting a fixed-point method. We generate a Gaussian test particle distribution with some rms size,  $\sigma_r(i)$ . As an initial guess, the spot size from the previous slice,  $\sigma_r(i-1)$ , is used. Then, the test particles are evolved in time without acceleration in the focusing field given by  $(E_x - B_y)(i-1)$  by using a second order accurate particle pusher until the second order spatial moment of the distribution has saturated. The value of the moment is used to set a new value for  $\sigma_r(i)$ , and the whole process is repeated until the sequence of values of  $\sigma_r(i)$  has converged. Note that the focusing field of the slice  $i-1$  is used to compute  $\sigma_r(i)$  under the assumption that the longitudinal resolution is high enough that the focusing field changes only marginally between two adjacent slices. To further take into account the spot size reduction due to acceleration of the particles, the averaged matched spot size  $\bar{\sigma}_{r,matched}$  can be calculated via equation 4. Finally,  $\sigma_{r,matched}$  or  $\bar{\sigma}_{r,matched}$  can be used to calculate the average accelerating field  $\langle E_z \rangle$  by equation 2. It should be noted that  $\bar{\sigma}_{r,matched}$  is only used to calculate  $\langle E_z \rangle$ , the bunch is still generated with a spot size of  $\sigma_{r,matched}$ .

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