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Block Theory Application to Unlined Rock Spillway Erodibility Assessment

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ABSTRACT: Scour of rock is a complex process and can be very problematic for dams when excessive scour threatens dam stability. Removal of individual rock blocks is one of the principal mechanisms by which scour can occur, particularly in unlined spillways and on dam abutments/foundations. To alleviate some of the complexity, commonly used methods for scour prediction tend to simplify the rock mass using rectangular block geometries or incorporate empirical relationships for the rock mass and do not actually model the physical scour process. Such simplifications can be problematic, particularly for block analysis, where the 3D orientation of discontinuities within the rock mass largely influence block removability. To better represent the 3D structure of the rock mass, block theory is used to evaluate stability of removable rock blocks subject to hydraulic forces. Block theory provides a rigorous methodology to identify removable blocks, determine potential failure modes, and assess block stability. An actively eroding unlined spillway at a dam site in Northern California is used to demonstrate the use of this approach for the evaluation of the scour potential at the site.

1. INTRODUCTION

Scour of rock is a complex process and can be very problematic for dams when excessive scour threatens dam stability. The principal challenge for the designer is to identify which blocks are most susceptible to hydraulic removal and under what conditions. To this end, a methodology using block theory [1] was developed and applied to assess scour potential of individual rock blocks in an unlined rock spillway at a dam site in Northern California.

The dam site is located in the Sierra Nevada Mountains along Interstate 80 near Donner Pass. The dam, originally constructed in 1919, has both a primary and secondary (emergency) spillway located on the northern side of the reservoir. The primary spillway consists of ten radial spillway gates that discharge directly into a rock lined valley comprised of jointed granodiorite (Figure 1). The design capacity of the primary spillway is approximately 1,560 m^3/s with an additional capacity of 210 m^3/s provided by the emergency spillway. Snowmelt from the Sierra fills the reservoir in the Spring, typically resulting in continued discharge from approximately April to July. Early winter rain or snow events (Nov. - Jan.) have also been known to produce periodic spills, including the flood of record which occurred in January of 1997 with a recorded discharge of approximately 570 m^3/s. Based on communication with site personnel the actual discharge was likely larger (~710 m^3/s) due to the failure of stream gauges downstream during the rising portion of the flood hydrograph.

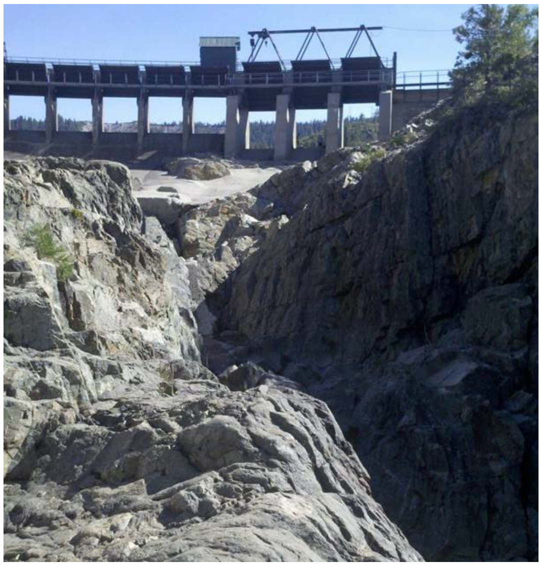

Figure 1. Canyon formed by scour showing spillway gates (background)

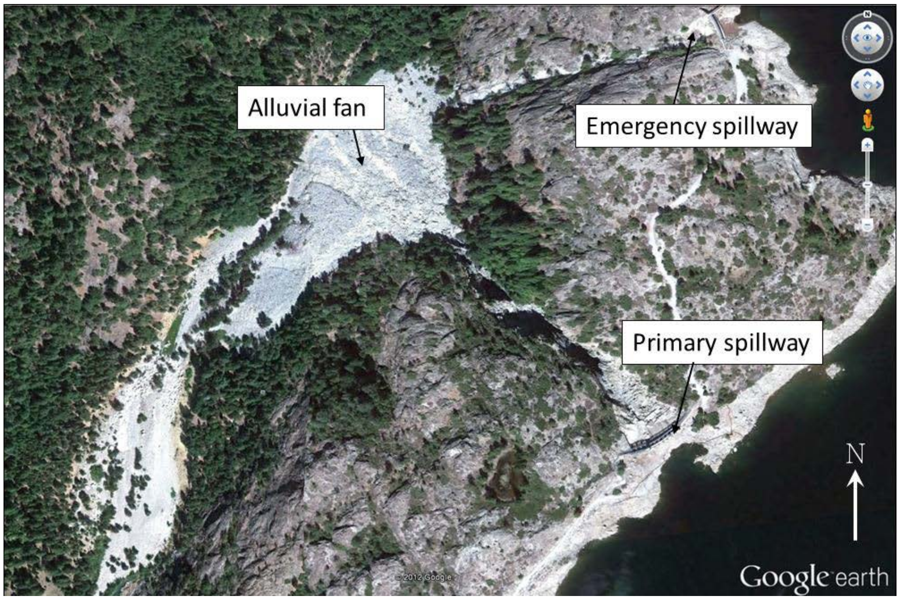

Figure 2. Aerial view of spillway showing alluvial fan of eroded material.

While significant erosion of the spillway occurred during the 1997 flood, erosion of the spillway has been occurring continuously throughout the life of the spillway at discharges much less than the flood of record. This continuous erosion resulted in the formation of an actively enlarging slot canyon. Based on measurements made of the alluvial fan at the mouth of the spillway canyon using aerial photography, it is estimated approximately 18,400 m^3 of intact rock material has been scoured away (Figure 2).

Remedial measures such as rock bolting and installation of a concrete apron near the spillway gates were recommended, based on studies from previous investigators, e.g. [2], and appear to have temporarily retarded scour migration.

2. METHODOLOGY

Block removability is strongly influenced by the 3D orientations of the discontinuities bounding the block and accordingly numerous kinematic failure modes exist [3] (Figure 3). Commonly used methods for scour prediction, however, tend to simplify the rock mass using rectangular block geometries and limit analysis to the lifting failure mode, e.g. [4], or incorporate empirical relationships for the rock mass and do not actually model the physical scour process, e.g. [5, 6]. To better represent the 3D structure of the rock mass, block theory has been applied to evaluate stability of removable rock blocks subject to hydraulic forces.

Block theory provides a rigorous methodology to identify removable blocks, determine potential failure modes, and assess block stability. For this analysis, only tetrahedral blocks were considered. These are blocks bound by three different discontinuity (joint) sets and one free face (e.g., spillway channel bottom, abutment face, etc.). For a rock mass with three joint sets and one free face, eight different block shapes exist, only one of which is removable. Only removable blocks are analyzed for scour potential as these blocks signify locations of scour initiation. Procedures for identifying removable blocks can be found in [1].

Once a removable block is identified, block stability can be assessed. Stability is dependent on the kinematic failure mode (Figure 3) which is subject to 3D geometric constraints (imposed by the orientations of the joint sets and free face) as well as the orientation of the resultant of all active forces applied to the block (which, for scour assessment, are hydraulic load and gravity). Details for determining the kinematic failure mode of a removable block with a known resultant orientation can be found in [1, 7, and 8].

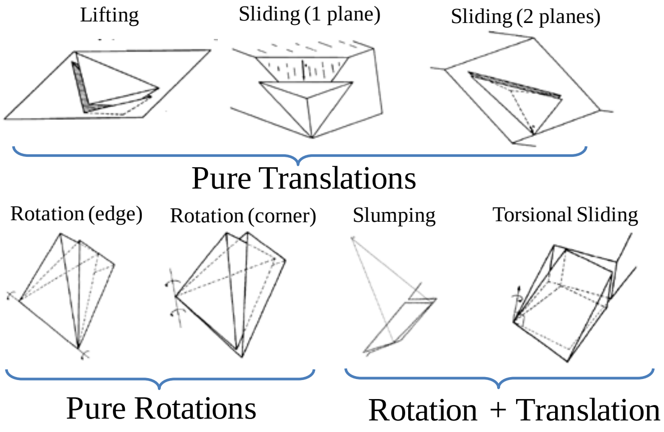

Figure 3. Kinematic block failure modes [3].

2.1. Characteristic Pressure & Distribution on Block Faces

For a given set of flow conditions, consider a corresponding characteristic dynamic pressure defined by an average dynamic pressure, P_m, a fluctuating dynamic pressure, P', and a frequency, ε. The characteristic dynamic pressure attempts to represent the main features of a flow field (as defined by the geometry, location, flow type, etc.) in a simplified manner, and may be expressed as:

$$P = P' \cdot \sin(2\pi\varepsilon t) + P_m \tag{1}$$

where t is the time. When the pressure fluctuations are relatively small (i.e., $P' << P_m$), the dynamic pressure is approximately equal to the mean pressure and accordingly the flow may be analyzed in a pseudo-static manner (Figure 4 – top). Therefore the characteristic dynamic pressure can be approximated by the pseudo-static pressure,

$$P \cong P_s = P_m - P' \cong P_m \tag{2}$$

When the magnitude of the pressure fluctuations comprise a significant portion of the characteristic dynamic pressure (i.e., $P' \sim P_m$), the dynamic nature of the flow field cannot be neglected. For this analysis, the characteristic dynamic pressure will be converted to a single dynamic impulse which is then applied to a rock block to access stability (Figure 4 – bottom). The reasoning for this is discussed in more detail later. The characteristic dynamic impulse can be expressed as:

$$P = P' \cdot \sin\left(2\pi\varepsilon t + \frac{3}{2}\pi\right) + P_s \tag{3}$$

For stability assessment, the biggest unknown is the active resultant force due to uncertainties in quantifying the hydraulic load. As such, it is necessary to make an estimate the hydrodynamic forces applied to the block and their distribution on the block face and within the joints bounding the block. In doing so, it is important to consider the nature of the flow conditions, namely is flow turbulent with a large degree of variability causing significant pressure fluctuations such that a dynamic analysis is required or are the fluctuations small such that analyzing an average pressure in a pseudo-static sense may suffice. In either case, principles in block theory can be applied to give an estimate of block stability.

In situations where flow conditions are complex and turbulent (such as plunge pools, rough channel flows with complex geometries, hydraulic jumps, etc.) it is logical to think that dynamic pressures may be distributed around the block in many different combinations that continually change over time. Therefore, for dynamic analysis, hydrodynamic pressures are applied to all the different combinations of block faces assuming a uniform pressure distribution across the block face. For tetrahedral blocks there are 15 combinations of block faces to which pressure may be applied: joint set 1 (J1), joint set 2 (J2), joint set 3 (J3), the free face (*f*), J1-J2, J1-J3, J1-*f*, J2-J3, J2-*f*, J3-*f*, J1-J2-J3, J1-J2-*f*, J1-J3-*f*, J2-J3-*f*, and J1-J2-J3-*f*. These are referred to as "hydraulic load scenarios."

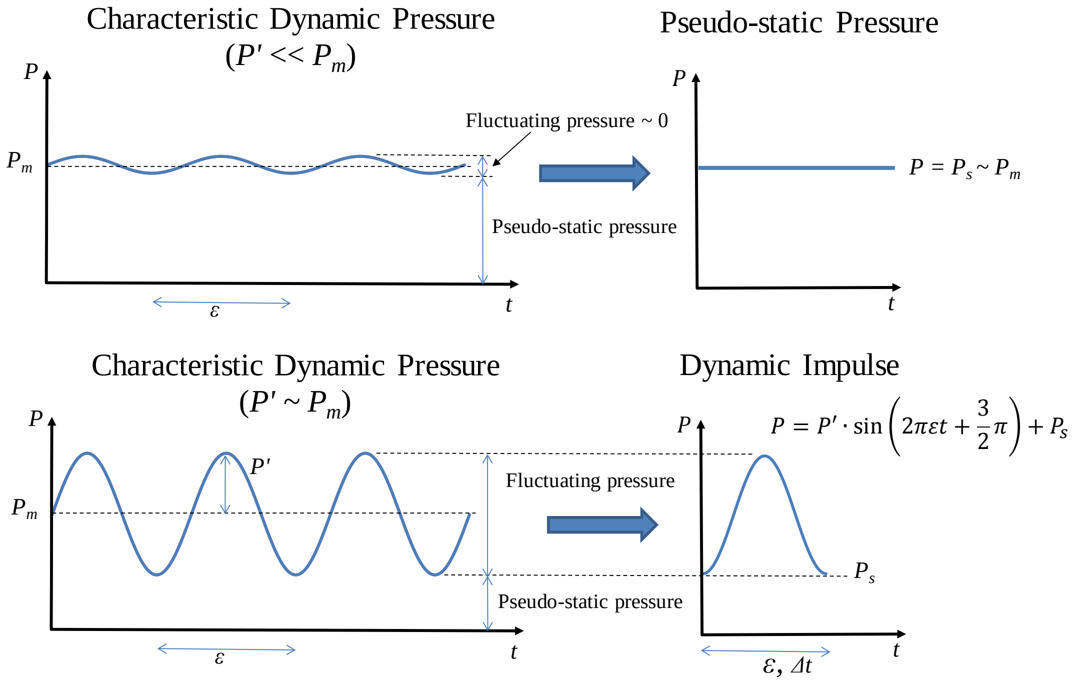

Figure 4. Simplification of characteristic dynamic pressure for pseudo-static (top) & dynamic (bottom) analyses.

This is a reasonable assumption as observations on the removal of rock blocks in laboratory studies by [9] and later by [10] have indicated that a single pressure pulse typically opens up one or two of the bounding joints (while subsequently closing the others) such that a large low frequency pressure fluctuation can cause significant pressure build-up in the open joints to eject the block. The assumption of uniform pressure distribution on block faces is likely valid only for blocks smaller than the characteristic length-scale of large-scale eddies within the flow. For larger blocks, this may be too conservative. A similar approach is adopted by [11] using BS3D code by [12].

For pseudo-static analysis, a similar approach is adapted except that some of the hydraulic load scenarios may be excluded to account for a preferential flow direction (such as in a channel) where it may not make sense to have block movements upstream.

2.2. Stability

Scour potential at the dam site in Northern California was assessed assuming the characteristic dynamic pressure may be approximated by a "constant" pressure and block stability could be evaluated in a pseudo-static manner using limit equilibrium analysis. Methodology for dynamic stability assessment can be found in [13].

For each applicable hydraulic load scenario, the critical hydraulic force required to bring the block to limit equilibrium for each of the kinematically feasible block failure modes is determined. The equilibrium expressions [1] are:

For lifting,

$$F = |\boldsymbol{r}| \tag{4}$$

For 1 – plane sliding,

$$F = |\boldsymbol{n_i} \times \boldsymbol{r}| - |\boldsymbol{n_i} \cdot \boldsymbol{r}| \cdot \tan\varphi_i \qquad for\ all\ i \tag{5}$$

For 2 – plane sliding,

$$F = \frac{1}{|\boldsymbol{n_i} \times \boldsymbol{n_j}|^2} \cdot \begin{bmatrix} |\boldsymbol{r} \cdot (\boldsymbol{n_i} \times \boldsymbol{n_j})| \cdot |\boldsymbol{n_i} \times \boldsymbol{n_j}| - \\ |(\boldsymbol{r} \times \boldsymbol{n_j}) \cdot (\boldsymbol{n_i} \times \boldsymbol{n_j})| \cdot \tan\varphi_i - \\ |(\boldsymbol{r} \times \boldsymbol{n_i}) \cdot (\boldsymbol{n_i} \times \boldsymbol{n_j})| \cdot \tan\varphi_j \end{bmatrix} for\ all\ i \neq j \tag{6}$$

where F = fictitious, required stabilizing force applied in the direction of movement to maintain equilibrium (N), φ_i and φ_j = friction angles (deg) on joints i and j, respectively, $\boldsymbol{n_i}$ and $\boldsymbol{n_j}$ = upward normal vectors on block face i and j, respectively (dimensionless), and $\boldsymbol{r}$ = active resultant force vector (N). When F is negative the block is considered stable, and when F is positive the block is unstable. When F is zero, the block is in equilibrium such that any further increase in the pressure will result in removal of the block.

The active resultant can be calculated as follows:

$$\boldsymbol{r} = \sum_{i=1}^{x} (P \cdot A_i \cdot \boldsymbol{v_i}) + m_b \cdot \boldsymbol{g} \tag{7}$$

where P = pseudo-static pressure which is varied until limit equilibrium is reached (N/m^2), A_i = area of i^{th} joint plane (m^2), $\boldsymbol{g}$ = acceleration due to gravity (m/s^2), $\boldsymbol{v_i}$ = the block-side normal vector (dimensionless) and x = number of block faces being analyzed (corresponding to the hydraulic load scenario being analyzed). The change in orientation of the active resultant due to increased pressure applied to the block can be shown graphically on a stereonet to help assist in selecting appropriate hydraulic load scenarios for pseudo-static analysis. This is referred to as the "active resultant path" and is discussed later on.

For channel flow conditions, the pressure, P, can be related to flow velocity by assuming a small protrusion of the removal block above the surrounding rock mass, which is likely reasonable for rough spillway channels. Flow impacting the protrusion causes development of stagnation pressure beneath the block (Figure 5). Relationships between the stagnation pressure (expressed in terms of pressure head) and upstream flow velocity were developed by [14].

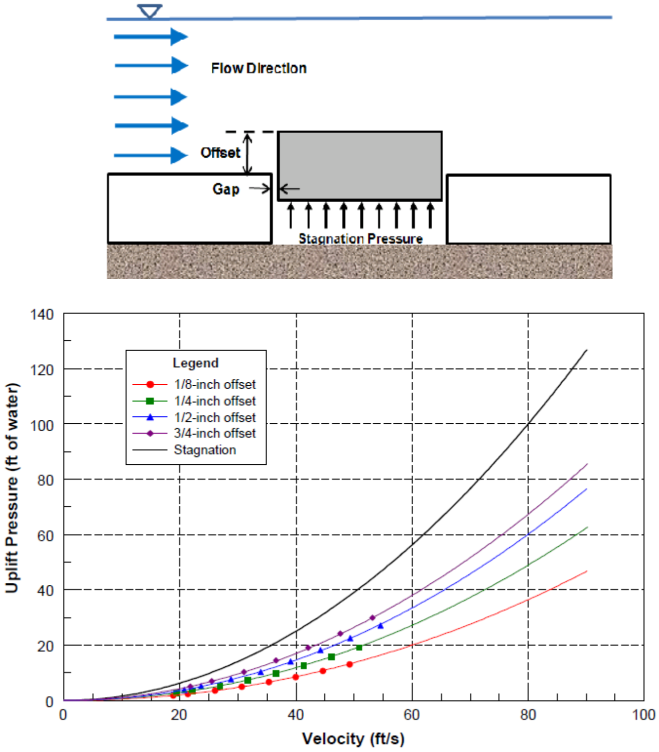

Figure 5. Schematic for stagnation pressure due to block offset (top) and relation between stagnation pressure and flow velocity (bottom, after [14]).

The research, conducted on hydraulic jacking of concrete spillway slabs, was adopted here to determine the critical flow velocity resulting in block removal. For this analysis, stagnation pressure was determined for a block with vented (open) joints, radius edge geometry, and an offset and gap of approximately 3 mm each (Figure 5).

As it is assumed that any of the pressure combinations being analyzed are plausible, the one yielding the lowest hydraulic force to result in block failure is considered to be the most critical.

Note that only equilibrium expressions for the pure translations are provided. Because the orientation of the active resultant is assumed to potentially vary in all directions, the most critical mode will almost always be one of the translations unless the friction angle of the rock joint is very high. Furthermore, the probability that a block is removable and rotatable is fairly low (~16%) [15]. As such, only 1-plane sliding, 2-plane sliding and lifting are considered.

2.3. Field Investigations

Field investigations were carried out to determine pertinent rock mass parameters (namely joint orientations and spacing). To do this, scan-line surveys were performed within the spillway area using a tape measure and Brunton geologic compass. Additional joint orientations were obtained by measuring discontinuities bounding block molds (i.e., locations where blocks had previously been removed).

Aerial Light Detection and Ranging (LiDAR) was also provided by the dam owner. Spatial values from the LiDAR data set were extracted and input into Meshlab [16], an open source software for processing and editing 3D triangular meshes. Normal vectors to the mesh, relating to the normal orientations of the joint faces on rock mass could be output such that the orientations of the joint sets could be obtained.

3. RESULTS

For this analysis, erodibility assessment of the spillway was limited to a single free rock face, although a more thorough analysis would consider all pertinent locations / faces. The free face in question is that directly downstream of the spillway gates. Based on field measurement, the spillway face has an orientation of 320 / 10 (dip direction / dip) in degrees. A schematic of the simplified scenario being analyzed is shown in Figure 6.

Joint data from field investigations used for subsequent block theory analysis are summarized in Table 1. Due to the presence of numerous steeply dipping joint sets at the spillway site that could not be adequately capture by aerial LiDAR measurements, the data were biased toward the more horizontally dipping joints. Because of this, priority was given to orientations obtained from hand measurements. In all, five joint sets were identified with average spacing ranging between approximately 0.5 m to 1.3 m.

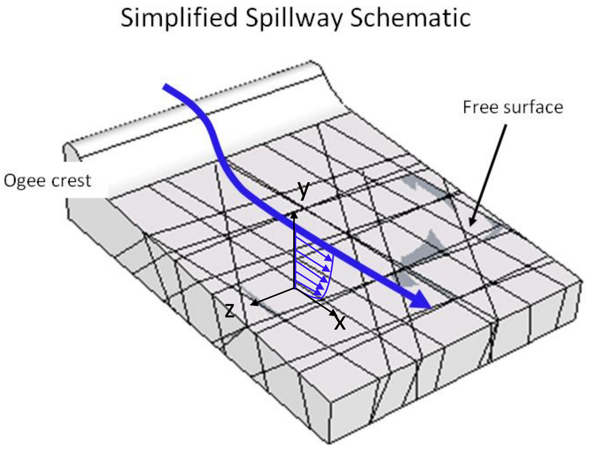

Figure 6. Simplified spillway schematic for analysis.

Table 1. Summary of joint data used for block theory analysis.

Joint Set	Orientation (deg)			Avg. Spacing (m)
	Strike	Dip Direction	Dip	
1	230	320	69	1.04
2	55	145	45	0.49
3	309	39	82	1.04
4	132	222	83	1.31
5	168	258	70	0.46

Since only tetrahedral blocks were considered, the five joint sets above were broken down into groups of three that, when combined with the free spillway face, yield a four-sided (tetrahedral) block with no repeated joint sets. In doing so, there are ten different combinations (joint groups) that require analysis, each of which will produce one removable block. Each of the ten blocks are identified by a “joint pyramid (JP)” code [1] relating to which side of the joint plane the block resides in space. A “0” indicates the block is above the joint plane while a “1” indicates the block is below the joint plane. For example, the JP code 001 indicates the block in question is above joint 1 (J1), above joint 2 (J2), and below joint 3 (J3).

Because flow in the spillway channel is predominantly unidirectional, particularly right below the spillway gates, it was decided to limit the orientation of the active result paths to an approximately 60 degree window shown in the Northwest quadrant of the stereonet (Figure 7). This is a reasonable assumption as a block moving against the direction of flow seems unlikely unless very large pressure fluctuations are present. This greatly reduced the number of stability analyses to be performed from 150 (15 hydraulic load scenarios for 10 removable blocks) to 35. Note, the angle for the window was arbitrarily selected and may be a topic for further research.

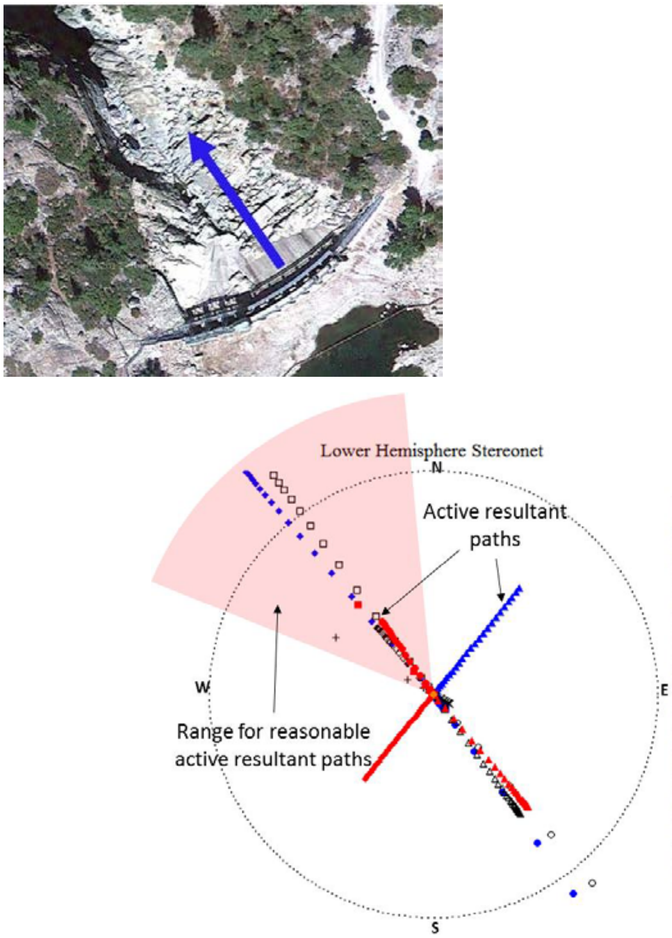

Figure 7. Stereonet showing limitation of active resultant paths for removable blocks (bottom) due to dominant flow direction in the spillway channel (top).

Block stability was assessed in vector format using Eqs. (4), (5) and (6), but can also be done stereographically. For the vector solution, the required stabilizing force, *F*, was plotted as a function of flow velocity to determine the critical velocity resulting in removal of the block.

Figure 8 shows block stability (vector and stereographic solution) for the most critical removable block originating from the joint group containing J1, J2, and J5. As indicated, at a flow velocity of 4.4 m/s the block will fail by 2-plane sliding (on J2 and J5) for a hydraulic pressure that is distributed uniformly across J1, J2, and J5. At a slightly higher flow velocity, a pressure distribution on J1 and J2 will also cause 2-plane sliding on J2 and J5. It is interesting to follow the active resultant path for these two hydraulic load scenarios as the velocity is increased. At approximately 4.9 m/s, 2-plane sliding is no longer kinematically feasible and in both scenarios, the mode changes to 1-plane sliding on J5. For the scenario when pressure is applied to J1 and J5, the 2-plane sliding on J2 and J5 is feasible at low velocities but does not become critical until flow velocity is approximately 20 m/s (not shown on the plot). Finally, if pressure is applied to J1, 2-plane sliding on J2 and J5 is also feasible at low velocities, however, increased flow velocity only provides more stability to the block.

The stability results for all the removable blocks are listed in Table 2. The data consist of the corresponding JP codes, the applicable hydraulic load scenarios (i.e., the load scenarios yielding an active resultant path that fits within the window shown in Figure 7), the kinematically feasible failure mode for each hydraulic load, the critical load scenario, the critical failure mode and finally the critical flow velocity.

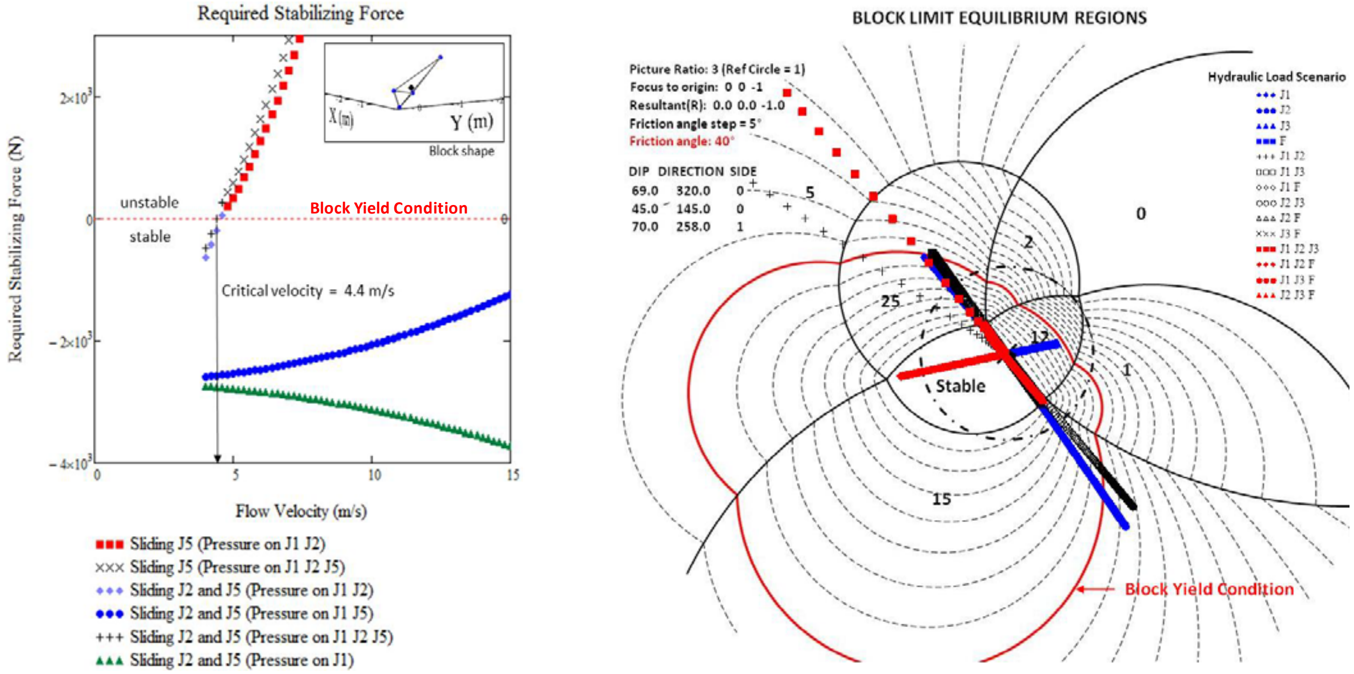

Figure 8. Vector (left) and stereographic (right) limit equilibrium solution for most critical removable block.

As indicated in Table 2, critical velocities resulting in block removal range from 4.4 m/s to 11.9 m/s. It should be noted that three joint groups did not yield any block that could kinematically be removed. Field and laboratory investigations are currently planned to verify predicted critical values.

4. CONCLUSIONS

A block theory based approach has been developed that can be applied to evaluate the scour potential of 3D rock blocks for an unlined rock spillway. The main considerations for using block theory to evaluate erodibility are 1) rock mass geometry (namely discontinuity orientations and spacing to determine block shape, size and removability), 2) flow conditions (pseudo-static or dynamic characteristic pressure), 3) hydrodynamic pressure distribution on rock block, and 4) block stability.

In the example application presented here, a pseudo-static analysis and block theory were used to evaluate the influence of hydraulic load on ten removable tetrahedral block types. The results showed that block removal could be triggered in the range of critical flow velocities from 4.4 m/s to 11.9 m/s. The analysis also showed that three joint groups did not yield any block that could kinematically be removed for the entire range of flow anticipated in the spillway. This highlights the influence of discontinuity orientation on block stability, and the need to incorporate 3D rock mass structure in determining scour potential.

The use of block theory in scour assessment has several other implications. With detailed field mapping, blocks most susceptible to scour can be targeted such that more efficient remediation measures can be implemented thus potentially reducing costs. Finally, analyses may be used as a planning tool for future projects to determine the most optimal layout of new spillways that are least susceptible to scour.

Table 2. Block stability results summary.

Joint Group	JP Code	Applicable Hydraulic Load Scenarios	Failure Mode	Critical Load Scenario	Critical Mode	Critical Velocity (m/s)
J1 J2 J3 f	000	J1	S2	J1 J2 J3	L	4.7
		J1 J2	L			
		J1 J3	S2			
		J1 f	-			
		J1 J2 J3	L			
		J1 J3 f	-			
J1 J2 J4 f	001	J1	S24	J1 J2 J4 & J1 J2	S4	4.6
		J2	S4			
		J1 J4	S2			
		J1 f	-			
		J1 J2 J4	S4			
		J1 J4 f	-			
J1 J2 J5 f	001	J1	S25	J1 J2 J5	S25	4.4
		J1 J2	S25, S5			
		J1 J5	S25			
		J1 f	-			
		J1 J2 J5	S25, S5			
		J1 J5 F	-			
J1 J3 J4 f	100	J3 J4	-	-	-	-
		J1 J3 J4	-			
J1 J3 J5 f	100	J3 J5	S1	J1 J3 J5	S1	8.0
		J1 J3 J5	S1			
		J3 J5 f	-			
J1 J4 J5 f	110	J4 J5	-	-	-	-
		J4 J5 f	-			
J2 J3 J4 f	000	J3 J4	S2	J2 J3 J4	S2	11.9
		J2 J3 J4	S2			
J2 J3 J5 f	000	J3 J5	S2	J2 J3 J5	S2	5.0
		J2 J3 J5	L, S2			
		J3 J5 f	-			
J2 J4 J5 f	010	J4 J5	S24,S2	J2 J4 J5	S24	4.4
		J2 J4 J5	S24, S2, L, S4			
		J4 J5 f	-			
J3 J4 J5 f	001	J3 J4	-	-	-	-
		J3 J4 J5	-			

Notes: L – lifting, SX – 1-plane sliding on Joint X, SXY – 2-plane sliding on Joint X and Joint Y

5. ACKNOWLEDGMENTS

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REFERENCES

1. Goodman, R.E. and G. Shi. 1985. *Block Theory and Its Application to Rock Engineering.* Englewood Cliffs, N.J.: Prentice Hall.
2. Goodman, R.E., and C. Powell. 2003. Investigations of blocks in foundations and abutments of concrete dams. *J. Geotech. and Geoenvir. Eng.* 129(2): 105-116.
3. Goodman, R.E. 1995. Block theory and its application. *Geotechnique* 45(3): 383-423.
4. Bollaert, E. 2002. Transient Water Pressures in Joints and Formation of Rock Scour due to High-Velocity Jet Impact, *Communication No. 13.* Laboratory of Hydr. Constructions, Ecole Polytechnique Federale de Lausanne, Switzerland.
5. Annandale, G.W. 1995. Erodibility. *J. of Hyd. Research.* 33: 471-494.
6. Annandale, G.W. 2006. *Scour Technology*. 1st ed. New York: McGraw-Hill.
7. Mauldon, M. and R.E. Goodman. 1996. Vector analysis of keyblock rotations. *J. of Geotech. Eng.*, 122(12): 976-987.
8. Tonon, F. 1998. Generalization of Mauldon's and Goodman's vector analysis of keyblock rotations. *J. of Geotech. and Geoenv. Eng.* 124(10): 913-922.
9. Yuditskii, G.A. 1967. Actual pressure on the channel bottom below ski-jump spillways. *Izvestiya Vsesoyuznogo Nauchno-Issledovatel-Skogo Instuta Gidrotekhiki*, 67: 231-240 (in Russian).
10. Melo, J.F., A.N. Pinheiro, and C.M. Ramos. 2006. Forces on plunge pool slabs: influence of joints location and width, *J. of Hyd. Eng.*, 132(1): 49-60.
11. Asadollahi, P. 2009. Stability analysis of a single three dimensional rock block: effect of dilatancy and high-velocity water jet impact. *Ph.D. Thesis*, Dept. of Civil Engineering, Univ. of Texas, Austin.
12. Tonon, F. 2007. Analysis of single rock blocks for general failure modes under conservative and non-conservative forces. *Int. J. for Num. and Analytical Methods in Geomech.*, 31:1567-1608.
13. George, M.F. and N. Sitar. 2012. Block Theory Application to Scour Assessment of Unlined Rock Spillways. University of California Report No. UCB GT 12-02.
14. USBR (2007). Uplift and crack flow resulting from high velocity discharges over open offset joints. *Report DSO-07-07*, United States Bureau of Reclamation, Denver.
15. Mauldon, M. 1990. Technical note: Probability aspects of the removability and rotatability of tetrahedral blocks, *Int. J. of Rock Mech, Min. Sci. and Geomech. Abst*, 27(4): 303-307.
16. Meshlab. 2011. Meshlab v1.3.0a. Open source software for 3D mesh processing and editing. Meshlab.sourceforge.net.