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"TRAJECTORY" -- AN ORBIT AND ION OPTIC
MATRIX PROGRAM FOR THE 184-INCH CYCLOTRON*

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September 1969

ABSTRACT

TRAJECTORY is a Fortran IV program for simultaneously integrating the equations of motion of a charged particle in a magnetic field and the differential equations for the ion optic matrix representing small motion about the paraxial trajectory. The orbit calculation is carried out in the median plane with the vertical transformation matrix calculated from a Taylor-series expansion of the magnetic field off the median plane. The matrix so calculated is readily used with the ion-optic transport codes TRANSPORT¹ or OPTIK.² The trajectory data can be used to obtain internal target positioning for a given beam line.

INTRODUCTION

TRAJECTORY tracks charged particles (protons, positive and negative pions, etc.) in either the forward or reverse direction (Fig. 1). The particles can be started individually from a specified R , θ , and ϕ and tracked forward to find the trajectory traced by a particle leaving a target, or can be run backward so that the beam line can be extended back into the cyclotron to find the required target location. The particles can, also, be started from a rectangular array of R , ϕ (Pr) values at a fixed azimuth θ so that a slit or aperture accep-

tance can be determined by reverse tracking.

The program output is shown in Fig. 2 with the trajectory printed in polar and rectangular coordinates, and can be used for any cyclotron with cylindrical or rectangular geometry. It is possible to determine the target locations and meson wheel angle required for a given external beam line by properly choosing the starting location of particles to be tracked (Fig. 3).

The orbit calculation can be stopped at any desired radius, azimuth, or value of trajectory length, so that the accuracy of the matrix can be checked by comparison with the matrix determined by actual ray tracing. Also, the orbit can be stopped at a given point, the integration step reversed, and the calculations continued backward to the starting point, where any deviation of the matrix from a unit matrix reflects the error produced by the finite size of

* Work done under the auspices of the U. S. Atomic Energy Commission.

1. C. H. Moore, S. K. Howry, and H. S. Butler, TRANSPORT, a Computer Program for Designing Beam Transport Systems, Stanford University Linear Accelerator Report, 1964.

2. T. J. Devlin. OPTIK, an IBM 709 Computer Program for the Optics of High Energy Particle Beams, UCRL-9727, Sept. 1961.

the integration step and by the numerical procedures used. The program requires 47200 octal words of memory for 10 000 decimal points in the field array.

EQUATIONS OF MOTION

The basic equations are integrated in rectangular coordinates, x, y. The median plane motion in a field B is described by

$$m \frac{d^2 x}{dt^2} = \frac{e}{c} v \times B,$$

$$m \frac{d^2 y}{dt^2} = -\frac{e}{c} v \times B.$$

The velocity v, trajectory length s, and time t are related by $\frac{ds}{dt} = v$; and momentum p, magnetic field B, and radius of curvature R are related by $eBR = p$. Then

$$\frac{d^2 y}{ds^2} = -\frac{B_z(x, y)}{BR},$$

$$\frac{d^2 x}{ds^2} = \frac{B_z(x, y)}{BR}$$

Reducing these second-order equations to first order by the substitution $u = dx/ds$ and

$w = dy/ds$ gives

$$\frac{dx}{ds} = u,$$

$$\frac{dy}{ds} = w,$$

$$\frac{du}{ds} = \frac{B_z(x, y)}{BR},$$

$$\frac{dw}{ds} = -\frac{B_z(x, y)}{BR},$$

where the magnetic rigidity of the particle BR is related to momentum by $BR = 1.31324 \times 10^6 p$ in units of kilogauss-inches and MeV/c.

MATRIX EQUATIONS

TRAJ simultaneously integrates the equations of motion and the differential equations for the first-order ion optic matrix elements.³ When the trajectory length, S, is

taken as independent variable, the horizontal matrix compatible with a particle vector (x, θ , $\delta p/p$) can be written as

$$M_h = \begin{pmatrix} \cos k_x s & \frac{1}{k_x} \sin k_x s & \frac{1}{Rk_x^2} (1 - \cos k_x s) \\ -k_x \sin k_x s & \cos k_x s & \frac{1}{k_x R} \sin k_x s \\ 0 & 0 & 1 \end{pmatrix}$$

and the vertical matrix compatible with a particle vector (y, ϕ) as

$$M_v = \begin{pmatrix} \cos k_y s & \frac{1}{k_y} \sin k_y s \\ -k_y \sin k_y s & \cos k_y s \end{pmatrix}$$

where $k_x \equiv \sqrt{1-n}/R$, $k_y \equiv \sqrt{n}/R$, n is the field index $-RdB/BdR$, R the radius of curvature CP/eB , p the momentum, B the magnetic field, C the velocity of light, and e the unit of electric charge.

$$\frac{dM_h}{ds} = \begin{pmatrix} -k_x \sin k_x s & \cos k_x s & \frac{1}{Rk_x} \sin k_x s \\ -k_x^2 \cos k_x s & -k_x \sin k_x s & \frac{1}{R} \cos k_x s \\ 0 & 0 & 1 \end{pmatrix}$$

$$\frac{dM_v}{ds} = \begin{pmatrix} -k_y \sin k_y s & \cos k_y s \\ -k_y^2 \cos k_y s & -k_y \sin k_y s \end{pmatrix}$$

The matrix equations to be integrated become

$$\frac{dM_h(1,1)}{ds} = M_h(2,1),$$

$$\frac{dM_h(1,2)}{ds} = M_h(2,2),$$

$$\frac{dM_h(1,3)}{ds} = M_h(2,3),$$

$$\frac{dM_h(2,1)}{ds} = -k_x^2 M_h(1,1),$$

$$\frac{dM_h(2,2)}{ds} = -k_x^2 M_h(1,2),$$

$$\frac{dM_h(2,3)}{ds} = \frac{1}{R} - k_x^2 (1,3).$$

3. This procedure is based on Program FOCUS, by Loren Meissner (unpublished).

$$\frac{dM_v(1,1)}{dS} = M_v(2,1),$$

$$\frac{dM_v(1,2)}{dS} = M_v(2,2),$$

$$\frac{dM_v(2,1)}{dS} = k_y^2 M_v(1,1),$$

$$\frac{dM_v(2,2)}{dS} = k_y^2 M_v(1,2).$$

The integration is carried from the initial conditions

$$x = 0,$$

$$y = 0,$$

$$\frac{dx}{dS} = \cos \phi,$$

$$\frac{dy}{dS} = \sin \phi,$$

where ϕ is the angle between the particle direction and the tangent to the circle at the particle location, and

$$M_h(1,1) = 1,$$

$$M_h(1,2) = 0,$$

$$M_h(2,1) = 0,$$

$$M_h(2,2) = 1,$$

$$M_v(1,1) = 1,$$

$$M_v(1,2) = 0,$$

$$M_v(2,1) = 0,$$

$$M_v(2,2) = 1,$$

i. e., the matrices are started from unit matrices.

The coordinate system used by the ion optic transport codes by TRANSPORT and OPTIK are different, as shown in Figs. 4 and 5. Programs TRAJ and OPTIK use a right-handed system such that a positive field (flux lines upward) deflects protons to the right with the x axis to the left.

The coordinate system of TRANSPORT is such that a positive field (flux lines upward) deflects electrons to the left with the x axis to the right. This sign convention requires a sign reversal of the dispersive matrix elements when the TRAJ matrix is to be used with TRANSPORT. The natural system of units is used by OPTIK and TRAJ for parti-

cle vectors (inches, radians, percent); whereas TRANSPORT uses inches, milliradians, and percent. Therefore, a scaling matrix is employed by program TRAJ to convert its output into the appropriate coordinate and unit system. This scaling matrix is introduced by a 3. 7. 1. or 3. 7. 2. data card, and has the value

$$UU = \begin{pmatrix} 1. & 1. & 0. & 0. & 1. \\ 1. & 1. & 0. & 0. & 1. \\ 0. & 0. & 1. & 1. & 0. \\ 0. & 0. & 1. & 1. & 0. \\ 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

for OPTIK (3. 7. 2.) and the value

$$UU = \begin{pmatrix} 1. & .001 & 0. & 0. & -.01 \\ 1000. & 1. & 0. & 0. & -10. \\ 0. & 0. & 1. & .001 & 0. \\ 0. & 0. & 1000. & 1. & 0. \\ 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

for TRANSPORT (3. 7. 1.).

The TRANSPORT R(6,6) matrix and TRAJ M(5,5) matrix have the following correspondence:

$$R() = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & 0. & M_{15} \\ M_{21} & M_{22} & M_{23} & M_{24} & 0. & M_{25} \\ M_{31} & M_{32} & M_{33} & M_{34} & 0. & M_{35} \\ M_{41} & M_{42} & M_{43} & M_{44} & 0. & M_{45} \\ 0. & 0. & 0. & 0. & 1. & 0. \\ M_{51} & M_{52} & M_{53} & M_{54} & 0. & M_{55} \end{pmatrix}$$

The correspondence between the OPTIK T (6,6) matrix and the TRAJ M (5,5) matrix is

$$T() = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & 0 \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & 0 \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & 0 \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The TRAJ matrix M() transforms a particle vector $(x, \theta, y, \phi, \delta p/p)$ in the forward direction,

$$V = M V_0.$$

The reverse matrix, M_r , transforms a particle vector, V to V_0 ,

$$V_0 = M_r V.$$

Obviously, $MM_r = 1$, therefore $M_r = M^{-1}$;

then the following relationship must exist between the forward and reverse matrix:

$$M = \begin{pmatrix} A & B & C \\ D & E & F \\ 0 & 0 & 1 \end{pmatrix} \quad M^{-1} = \begin{pmatrix} E & -B & BF-EC \\ -D & A & DC-AF \\ 0 & 0 & 1 \end{pmatrix}$$

The effect of these matrices on a particle vector is shown in Fig. 6. Both these matrices are printed as output under the headings of forward and reverse matrix.

PARAMETER INPUT

The various parameters of the calculation are read by subroutine READIN. The parameters are coded by a floating-point number described below. Once a parameter is changed, it so remains until changed again, except for the particle coordinates R, θ, φ, or X, Y, φ.

1. R. θ. φ. E. DS. SMAX. SOUT.

Particle input is in polar coordinates (R, θ), where R is in inches and θ is in degrees. The outward angular direction φ in degrees (shown in Fig. 7) is the angle between the particle direction and the tangent to a circle at the particle location. E is the particle energy in GeV, positive for a positive particle, negative for a negative particle. DS is the orbit integration step size in inches, and is taken negative if the orbit is to be calculated backward. SMAX is the maximum trajectory length (inches) with the printout of the ion optic matrix every SOUT (inches) along the trajectory. The FORMAT is (8F10.5).

Example: a 730-MeV proton starting tangentially at 78-inch radius at 172 deg azimuth can be specified by either of the following:

- 1. 78. 172. 0. .730 1. 100. 10.
- 1. 78. 172. 0. .730

2. X. Y. φ. E. DS. SMAX. SOUT.

Particle input in rectangular coordinates is in (8F10.0) format. The other parameters are the same as for polar coordinates.

See Fig. 8.

Example: 2. 56.5 -56.5 0. .730 1. 100. 10.

3. PARAMETER VALUE

Parameter values used by TRAJ will be assumed standard unless changed by one or more of the following cards in (3F10.0) format.

- 3. 1. NC2. --- input of the order of Lagrangian interpolation of magnetic field values.
- 3. 2. EO. --- input of the rest mass of the particle in GeV.
- 3. 3. SMAX. --- maximum trajectory length in inches.
- 3. 4. SOUT. --- distance between matrix printouts in inches.
- 3. 5.
- 3. 6. SPRINT. --- SPRINT = 0 gives printout at each integration step. SPRINT ≠ 0 gives distance between orbit printouts if not for each integration step in steps of the integration size.
- 3. 7. MATRIX. --- = 1 matrix units for TRANSPORT.
= 2 matrix units for OPTIK.
- 3. 8. PUNCH. ---
- 3. 9. PHIIN. --- φ input unit degrees for PHIIN = 0,
PR/P for PHIIN = 1.
- 3. 10. DS. --- integration step size in inches.

The standard values assumed by the program for the above parameters are NC2 = 7, EO = .938213 GeV, SMAX = 50 inches, SOUT = 50 inches, SPRINT = 0, MATRIX = 1, PUNCH = 0, PHIIN = 0, and DS = 1. inch.

4. XFIELD. BSCALE. PSCALE.

Magnetic field input specification. XFIELD selects the appropriate input

subroutines. BSCALE scales the radial field data and PSCALE scales the perturbation data. Normally BSCALE = PSCALE = 1.

4. 1. 1. 1. Radial magnetic field of the 184-inch cyclotron.

4. 2. 1. 1. Radial field + perturbation over a given sector.

4. 3. 1. 1. Two-dimensional polar magnetic field over a given sector.

4. 4. 1. 1. Two-dimensional rectangular magnetic field.

The subroutines called by 4.3. and 4.4. data must be supplied by the user.

5. SELECT. VARIABLE.

The calculations can be interrupted by the use of this card to give the printout of the ion optic matrix at the specified value for either the particle radius, azimuth, or trajectory length. If the appropriate value does not lie on the integration step, the orbit is reversed one step and a linear interpolation is used to find the integration step required to produce the desired value of R, θ , or S. This step is then taken, the matrix printed, and the integration advanced to the original stopping point. The integration then continues as usual.

The required data are in Format 3F10.4

5. 1. R. Prints matrix when radius = R.

5. 2. θ . Prints matrix when azimuth = θ .

5. 3. S. Prints matrix when trajectory length = S.

If the calculation is to be stopped at one of the above values so that other parameters can be changed, set STOP = SELECT by the following card:

5. 4. STOP. if STOP = 1. Calculations stop when radius is given on 5. 1. R, if STOP = 2. Calculations stop when azimuth is value given on 5. 2. θ .

if STOP = 3. Calculations stop when trajectory length is as given on

5. 3. S.

6. N.

Read N. comment cards.

7. R1. R2. DR. AZIM. PHI1. PHI2. DPHI.

Track particles starting at azimuth AZIM with

radius R1, PHI = PHI1,

PHI1 + DPHI, ... PHI2,

radius R1 + DR, PHI = PHI1,

PHI1 + DPHI, ... PHI2,

⋮

radius R2, PHI = PHI1,

PHI1 + DPHI, ... PHI2.

The total number of particles, N, tracked by this option is

$$N = \left(\frac{R2 - R1}{DR} + 1 \right) \left(\frac{PHI2 - PHI1}{DPHI} + 1 \right).$$

Radius is in inches, and PHI in units selected by the 3. 9. option.

The Format is (8F10.4).

8.

This card resumes the orbit and matrix calculation after it has been previously terminated without redefining the particle position. If one wishes to check the numerical accuracy produced by a given integration step size, he may calculate the trajectory out to some SMAX. and then reverse the calculation by reversing the sign of the integration step and calculate back to the original starting point by the following data:

1. R. θ . PHI. E. DS. SMAX. SOUT.

3. 10. -DS

8.

9.

This data card produces a page eject.

10.

This data card terminates the job.

Blank cards in the data deck are ignored except where they are required in the magnetic field input data sets.

MAGNETIC FIELD INPUT

One of four subroutines is used to read in the magnetic field. Subroutine B184 is used if XFIELD = 1. and reads the radially dependent, azimuthally independent field of the 184-inch cyclotron. Subroutines B184 and BP184 are called if XFIELD = 2. and store the radial field onto which are added field perturbations read from cards by BP184. These perturbations extend over the radial increment RMIN to RMAX and the azimuth sector defined by AMIN, AMAX. Subroutine B2DP is called if FIELD = 3. and reads a two-dimensional field in polar coordinates. Subroutine B2DP is called if XFIELD = 4. and reads a two-dimensional field in rectangular coordinates. Subroutines B2DP and B2DR are not supplied at present.

The sample data shown in Fig. 10 will track two protons of 730 MeV and three mesons in the forward direction, and then two protons in the reverse direction in the radial field of the 184-inch cyclotron. All data are in format (8F10.5) except comment card, whose format is (8A10).

SAMPLE DATA WITH FIELD PERTURBATIONS

The following example shows data for the radial magnetic field of the 184-inch cyclotron and the perturbations produced by the regenerator and magnetic channel (Fig. 9). These perturbations extend radially from 70 to 108 inches to 1-inch steps and azimuthally from 101 to 171 degrees in 1-degree steps. Within this boundary other perturbations may be added as shown. The format for the perturbation data is (I3, 2X15F5.0). The radius is the first number on the card (I3), and the data must be in radially increasing order. If more than one card is required per radius for the azimuthal field, the perturbations are in ascending azimuthal order at a given radius. A blank card signals the end of a perturbation set. Two blank cards in a row signal the end

of all perturbation sets. The first card of a perturbation set gives the azimuth at which the data begins. In the example, shown in Fig. 11, a second set of perturbations begins at 142 degrees and covers the radial range from 98 to 103 inches extending to 156 degrees.

COMPUTER PROGRAM

A listing of the Fortran IV computer program is given in the Appendix.

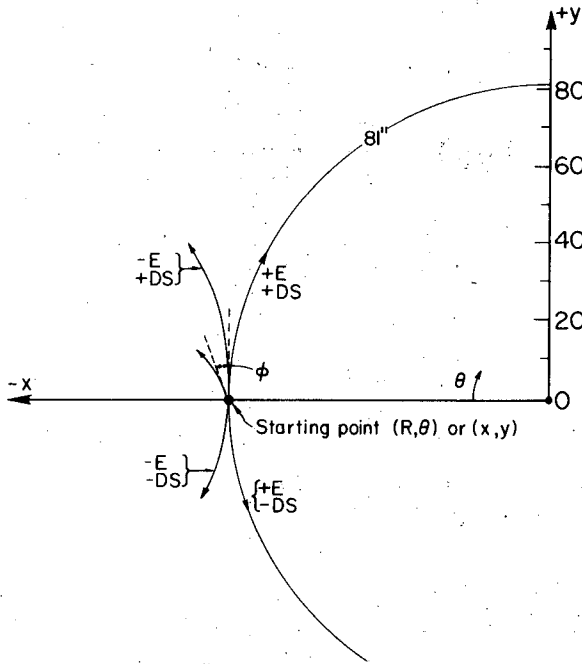


Fig. 1. Effect of sign of energy and integration step size.

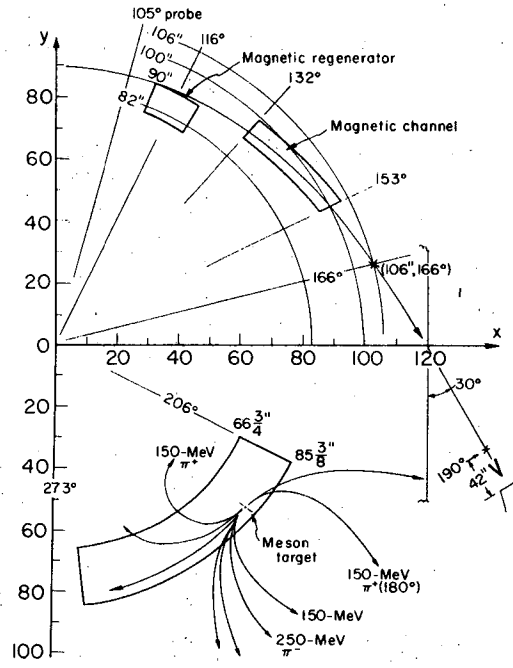


Fig. 3. Pion orbit tracking in the 184-inch cyclotron

INPUT DATA = 1.000000 90.096800 106.000000 3.112000 760728 1.000000 200.000000 10.000000

P(EGV/C) = 1.343831A
 DS(INCH/MS) = 1.000
 R(FC) (IN) = 1.7648F406
 C(EGV) = 7.07280

J	S(INCH)	X(INCH)	Y(INCH)	R(INCH)	A(DEG)	INDEX	R(GAUSS)	DPHI(DEG)
1	1.000	24.834	86.607	90.0968	106.0000	4.427E+00	192234.9	3.1120
2	1.000	24.408	86.378	90.1512	106.6348	5.767E+00	19402.2	3.1190
3	2.000	26.779	86.139	90.2056	107.2692	6.793E+00	19602.4	3.1211
4	3.000	27.747	85.889	90.2600	107.9032	7.753E+00	19783.1	3.1158
5	4.000	28.712	85.629	90.3143	108.5369	8.658E+00	19942.5	3.1059
6	5.000	29.675	85.357	90.3687	109.1702	9.455E+00	20057.9	3.0885
7	6.000	30.634	85.075	90.4220	109.8031	1.012E+01	20152.5	3.0685
8	7.000	31.590	84.781	90.4754	110.4357	1.066E+01	20221.3	3.0460
9	8.000	32.543	84.477	90.5283	111.0670	1.099E+01	20177.8	3.0220
10	9.000	33.492	84.162	90.5808	111.6967	1.093E+01	20057.0	3.0006
11	10.000	34.437	83.836	90.6330	112.3212	1.074E+01	19895.1	2.9844

----- FORWARD MATRIX -----

1.04254	.01017	0.	0.	-.00565
0.97552	1.05660	0.	0.	-1.14927
0.	0.	.95195	.00981	0.
0.	0.	-10.90245	.91801	0.

RADIAL DETM = 1.000095 VERTICAL DETM = .999897

J	S(INCH)	X(INCH)	Y(INCH)	R(INCH)	A(DEG)	INDEX	R(GAUSS)	DPHI(DEG)
12	11.000	35.379	83.499	90.6850	112.9423	1.051E+01	19723.0	2.9722
13	12.000	36.316	83.152	90.7367	113.5931	1.022E+01	19540.0	2.9452
14	13.000	37.250	82.795	90.7885	114.2236	9.978E+00	19357.1	2.9143
15	14.000	38.180	82.427	90.8407	114.8536	9.672E+00	19180.4	2.8808
16	15.000	39.106	82.049	90.8921	115.4833	9.407E+00	19011.0	2.8455
17	16.000	40.028	81.662	90.9442	116.1127	9.278E+00	18876.6	2.8092
18	17.000	40.946	81.264	90.9965	116.7416	9.362E+00	18789.9	3.0107
19	18.000	41.859	80.857	91.0492	117.3702	9.495E+00	18713.1	3.0299
20	19.000	42.768	80.440	91.1023	117.9984	9.590E+00	18634.8	3.0521
21	20.000	43.672	80.013	91.1557	118.6263	9.653E+00	18558.4	3.0764

----- FORWARD MATRIX -----

1.20005	.02147	0.	0.	-.02317
21.58199	1.21859	0.	0.	-2.36894
0.	0.	.78830	.01843	0.
0.	0.	-21.15531	.77397	0.

RADIAL DETM = 1.000018 VERTICAL DETM = .999978

J	S(INCH)	X(INCH)	Y(INCH)	R(INCH)	A(DEG)	INDEX	R(GAUSS)	DPHI(DEG)
22	21.000	44.572	79.577	91.2096	119.2537	9.558E+00	18479.5	3.1025
23	22.000	45.467	79.132	91.2640	119.8808	9.588E+00	18394.2	3.1309
24	23.000	46.358	78.677	91.3189	120.5074	9.435E+00	18299.5	3.1619
25	24.000	47.244	78.213	91.3743	121.1337	9.127E+00	18195.4	3.1958
26	25.000	48.125	77.740	91.4304	121.7596	8.664E+00	18084.3	3.2327
27	26.000	49.001	77.259	91.4871	122.3850	8.141E+00	17961.1	3.2729
28	27.000	49.872	76.767	91.5446	123.0101	7.673E+00	17820.2	3.3171
29	28.000	50.738	76.267	91.6029	123.6347	7.226E+00	17659.4	3.3657
30	29.000	51.600	75.759	91.6620	124.2589	6.817E+00	17473.3	3.4195
31	30.000	52.456	75.242	91.7222	124.8826	6.446E+00	17260.5	3.4793

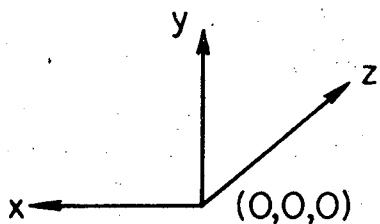
----- FORWARD MATRIX -----

1.47529	.03482	0.	0.	-.05347
---------	--------	----	----	---------

----- REVERSE MATRIX -----

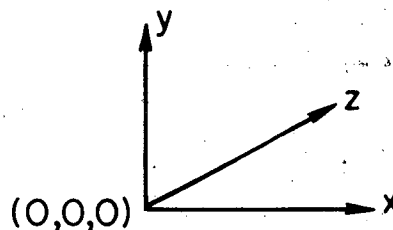
1.44813	-.03482	0.	0.	-.05111
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Fig. 2. Example of program output.



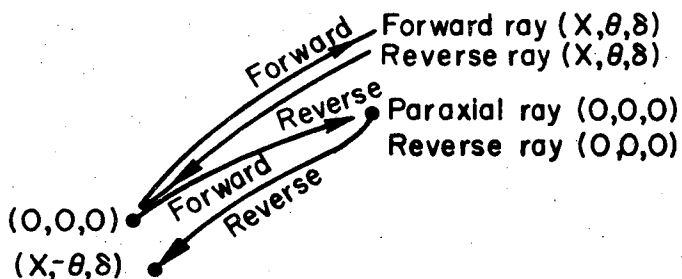
XBL 702-2381

Fig. 4. TRAJ and OPTIK coordinate system.



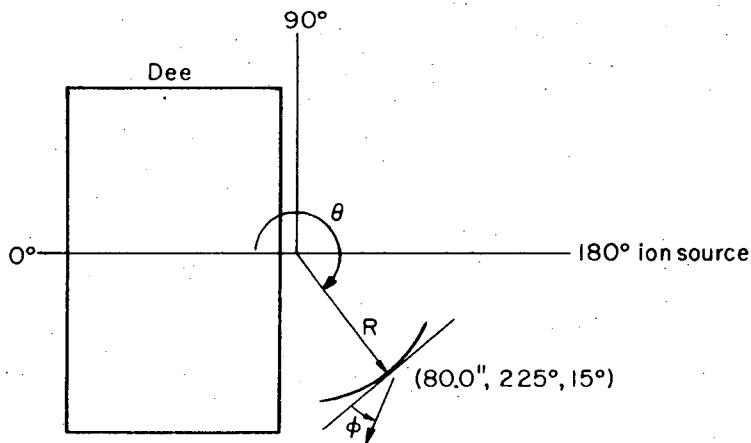
XBL 701-2271

Fig. 5. TRANSPORT coordinate system.



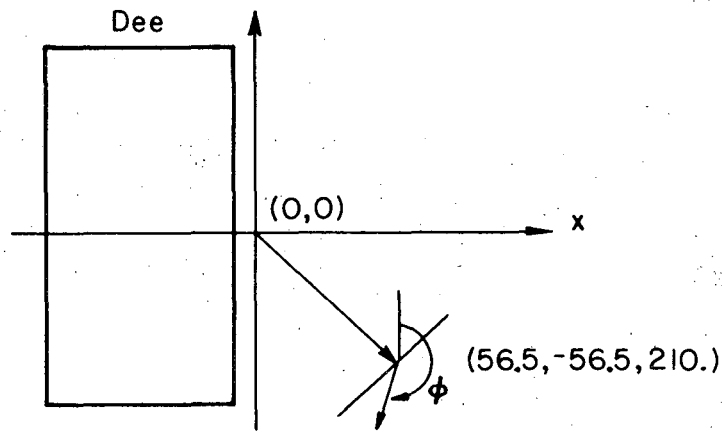
XBL 701-2272

Fig. 6. Beginning and end points for particle vectors tracked by the forward and reverse matrices.



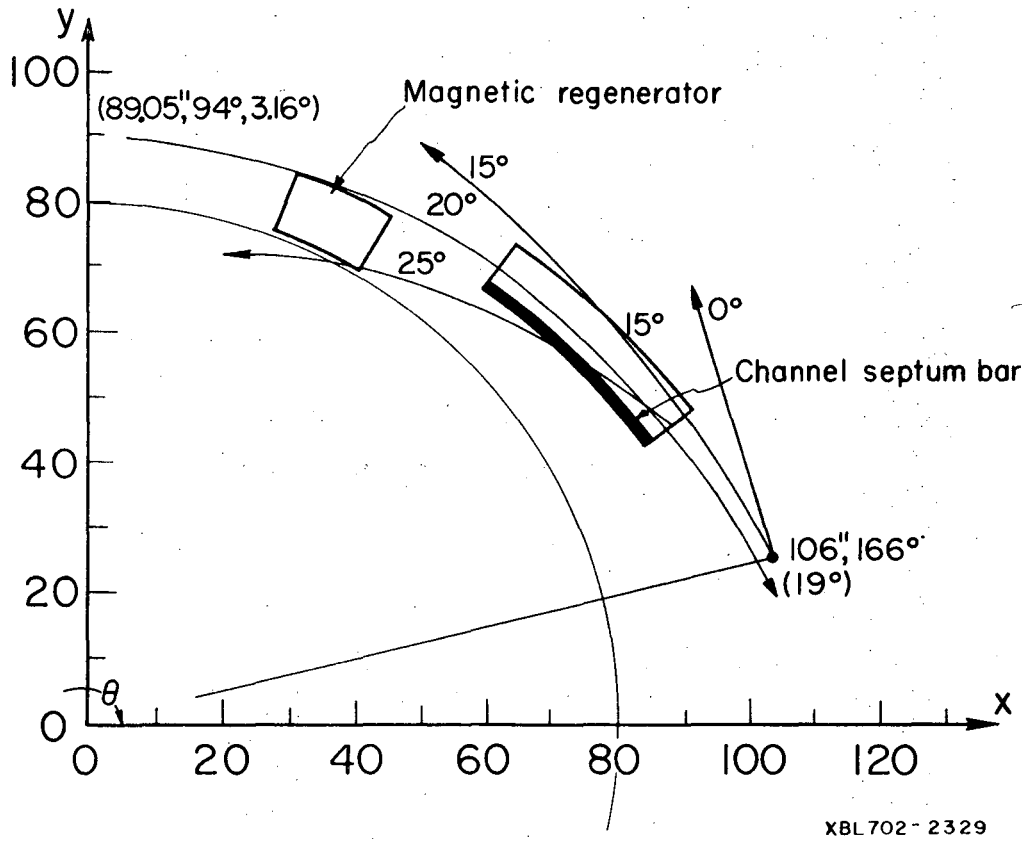
XBL 701-2273

Fig. 7. Polar geometry of 184-inch cyclotron; ϕ is taken positive for positive radial momentum (outward).



XBL701-2274

Fig. 8. Rectangular geometry for 184-inch cyclotron for same input conditions shown above;
 $\phi_{\text{rect}} = \theta_{\text{polar}} - \phi_{\text{polar}}$



XBL702-2329

Fig. 9. Backward tracking into magnetic channel.

4.	1.	1.						
RADIAL FIELD								
1.	10A.	1.						
23347	23327	23306	23284	23261	23238	23217	23198	
23181	23164	23147	23130	23112	23094	23075	23056	
23037	23020	23004	22990	22977	22964	22952	22940	
22928	22915	22902	22888	22874	22860	22845	22830	
22816	22802	22789	22776	22764	22752	22741	22730	
22720	22709	22698	22687	22676	22665	22654	22642	
22630	22618	22606	22594	22583	22572	22561	22551	
22541	22531	22520	22510	22503	22498	22493	22487	
22479	22470	22459	22447	22435	22423	22412	22403	
22395	22387	22379	22371	22362	22354	22346	22338	
22326	22298	22236	22116	21916	21618	21209	20685	
20043	19265	18347	17327	16256	15180	14136	13133	
12169	11250	10380	9560	8792	8070	7389	6745	
6135	5560	5022	4517					
BLANK CARD								
1.	81.	92.	3.	.730	1.	100.	10.	
2.	81.	3.	3.					
3.	2.	.139579						
1.	77.	220.	0.	-.12				
1.	78.	220.	0.	-.18				
1.	79.	220.	0.	-.20				
3.	2.	.938213						
2.	0.	90.	0.	.7	-1.			
2.	0.	90.	2.					
BLANK CARD								

Fig. 10. Example of radial magnetic field input data.

4.	2.	1.	1.													
RADIAL MAGNETIC FIELD AND REGENERATOR AND CHANNEL PERTURBATIONS OF 184																
1.	10A.	1.	101.	171.	1.											
23347	23327	23306	23284	23261	23238	23217	23198									
23181	23164	23147	23130	23112	23094	23075	23056									
23037	23020	23004	22990	22977	22964	22952	22940									
22928	22915	22902	22888	22874	22860	22845	22830									
22816	22802	22789	22776	22764	22752	22741	22730									
22720	22709	22698	22687	22676	22665	22654	22642									
22630	22618	22606	22594	22583	22572	22561	22551									
22541	22531	22520	22510	22503	22498	22493	22487									
22479	22470	22459	22447	22435	22423	22412	22403									
22395	22387	22379	22371	22362	22354	22346	22338									
22326	22298	22236	22116	21916	21618	21209	20685									
20043	19265	18347	17327	16256	15180	14136	13133									
12169	11250	10380	9560	8792	8070	7389	6745									
6135	5560	5022	4517													
BLANK CARD																
70.	10A.	1.	101.	171.	1.											
101																
71	-32	-36	-38	-36	-30	-21	-5	12	28	42	50	48	41	29	14	
71	-2	-14	-25	-36	-48	-60	-77	-92	-104	-110	-110	-95	-77	-57	-39	
71	-27	-31	-41	-52	-61	-65	-52	-33	-11	11	29	34	34	32	29	
71	28	37	49	62	76	87	91	92	91	88	85	84	84	86	89	
71	92	98	102	101	94	79	49	16	-18	-46	-66					
72	-3	-8	-12	-14	-13	-8	6	21	37	50	58	54	45	32	16	
72	-1	-12	-23	-33	-44	-56	-75	-92	-106	-114	-115	-99	-78	-55	-36	
72	-23	-30	-42	-57	-69	-76	-63	-44	-20	3	24	30	31	30	28	
72	28	38	51	65	78	89	92	91	88	84	81	81	82	86	91	
72	97	106	112	113	107	92	61	25	-11	-43	-65					
73	25	18	9	1	-4	-4	8	24	41	54	66	63	54	41	24	
73	7	-6	-14	-25	-37	-50	-71	-92	-108	-118	-120	-102	-79	-54	-32	
73	-17	-24	-37	-53	-68	-78	-68	-50	-29	-6	14	20	22	22	21	
73	22	33	47	62	76	86	87	85	80	75	71	72	75	82	90	
73	99	111	119	122	117	102	69	31	-7	-41	-65					
107	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	
107	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	
107	-0	-0	-0	-0	-0	-0	-0	2	4	4	3	-0	-10	-18	-21	-16
107	-0	36	79	124	166	200	213	214	202	180	150	117	82	50	22	
107	-0	-10	-14	-12	-6	-0	2	3	3	2	-0					
108	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	
108	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	
108	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
108	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	
108	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	
BLANK CARD																
142																
98	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
99	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
100	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	
101	375	375	375	375	375	375	375	375	375	375	375	375	375	375	375	
102	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	
103	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
BLANK CARD																
BLANK CARD																
1.	89.05	94.	3.14	.730	1.	100.	10.									
BLANK CARD																

Fig. 11 Example of radial and azimuthal perturbation magnetic field data input.

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