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Macroeconomic Implications of Behavioral Agents

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Macroeconomic Implications of Behavioral Agents

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To my well of inspiration and motivation for everything, Jihyeon and Daon

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Macroeconomic Implications of Behavioral Agents

Abstract

Beliefs about the future self's financial conditions and preferences are important when deciding how much to consume and leave for tomorrow. This thesis seeks to answer how the perception regarding future behaviors affects consumption and savings decisions and how the heterogeneity in the perception can explain wealth inequality and excess sensitivity of consumption to temporary income shocks. In Chapter 1, I consider households with an imperfect perception of expenditure shocks. Households may underestimate the expenditure shocks in the future and then undersave than the case they had a belief consistent with truth. The model requires many households who underestimate future shocks to match the distribution of liquid wealth found in the data. Next, Chapter 2 (coauthored with Anujit Chakraborty, Claudia Cerrone, and Leonhard Lades) estimates degrees of present bias and sophistication in the quasi-hyperbolic discounting framework. We find a significant degree of present bias in the effort domain using an online experiment. Also, the results suggest a large dispersion of present bias and sophistication exists among the experiment participants. Finally, Chapter 3 incorporates distributions of present bias and sophistication elicited in Chapter 2 into a life-cycle framework to explain the dispersion of wealth.

Chapter 1 introduces a life cycle model of consumption and savings where households face exogenous expenditure shocks. Households are heterogeneous as they have different levels of perceptions of expenditure shocks. My model predicts that households who underestimate the expenditure shocks tend to spend more now and save less for the future. Using this model, I calibrate the distribution of the perception of future shocks to match the dispersion of liquid wealth in data. Based on a realistic level of liquid wealth over the life cycle, the model features many households underestimating the future expenditure shocks, which generates a high marginal propensity to consume overall. I also provide a policy recommendation that can enhance overall welfare.

Chapter 2, joint work with Anujit Chakraborty, Claudia Cerrone, and Leonhard Lades, estimates the degrees of present bias and sophistication over effort and money using a simple yet novel experimental design. We find a significant degree of present bias in the effort domain but not in the money domain. However, we find a significant correlation between the estimates of present bias across effort and money domains. Furthermore, we find that subjects are partially sophisticated in the effort domain, though it is insignificant at the aggregate level. Lastly, we find a severe dispersion of present bias and sophistication in both effort and money domains.

Chapter 3 investigates the role of heterogeneous present bias and sophistication in a life-cycle model with both liquid and illiquid wealth. Using numerical simulations, I confirm the findings of previous literature that it is hard to draw monotonic relationships between consumption and the level of sophistication. However, under some conditions, such as binding borrowing constraints, I show that agents with a higher degree of sophistication tend to prefer commitment and save more illiquid assets in a stylized setting. Finally, I incorporate the distribution of present bias and sophistication measured in Chapter 2 into a more realistic life-cycle setting and show that heterogeneity of present bias and sophistication can lead to a severe dispersion of wealth and high dependence on borrowing.

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CHAPTER 1

Consumption and Savings under Imperfect Perception of Expenditure Shocks

1.1. Introduction

Why do households consume a large quantity from additional income, and who spends the most? This question of magnitude and heterogeneity underlying the marginal propensity to consume (MPC) is essential to the design of a policy, like fiscal stimulus to boost economic activity. In practice, the magnitude is important when determining the size or duration of the fiscal stimulus. The heterogeneity is also important to identify the structure of the fiscal stimulus over diverse groups of people and induce the maximal consumption response. In this chapter, I investigate these two aspects of MPC by employing a model of heterogeneous households with different degrees of behavioral biases.

A large piece of empirical evidence documents high MPCs in various contexts. Moreover, much of the vast literature indicates the strong association between low liquidity and large consumption responses (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). Reflecting on the empirical evidence, influential approaches of Carroll (1992, 1997), Kaplan and Violante (2014), and Carroll et al. (2017) all have hand-to-mouth households with low liquidity at their heart. Hence, in this liquidity perspective of generating large overall MPC, it is crucial to explain why and who are liquidity constrained.

If a model depends on liquidity constraints to generate a high MPC, the model needs to exhibit a realistic distribution of liquid assets, and thus MPC in the model is high because of a real reason. How are liquid assets distributed? Carroll et al. (2017) demonstrate that liquid wealth is distributed more unequally than net worth. Then, what kind of factors can explain this high dispersion of liquid wealth? Age can be a crucial factor driving the dispersion of liquid wealth since households across various stages of life would have different demands. However, I demonstrate that when we pool households by different age groups, the inequality of liquid wealth is still high. Thus, a factor explaining the distribution of liquid wealth would be independent of age. I show that even in subgroups of homogeneous households in education groups, occupation, and income[1], the dispersion of liquid wealth is still extremely high. This result is robust when I restrict my analysis to highly liquid assets, that bring homogeneous returns.

There are several challenges in the current literature on high MPCs and the dispersion of liquid wealth. First, we need a model generating the dispersion of liquid wealth, even when income and asset returns are homogeneous in expectation. Carroll et al. (2017) successfully generate dispersion of wealth comparable to data with discount factor heterogeneity. However, they do not explicitly separate liquid and illiquid assets, and the dispersion of wealth depends on heterogeneity in education level, which drives additional dispersion of income. Second, it is rare for models to employ intertemporal elasticity of substitution (IES) significantly below one, as suggested by the meta-analysis by Havránek (2015) and other micro evidence, like that given in Best et al. (2020), when explaining the high MPC. The lower IES strengthens households' desire to smooth consumption, and the model will likely exhibit lower MPC, as noted by Aguiar et al. (2020).

In response to the challenge, I introduce a model that can explain the high MPC and dispersion of liquid wealth with a realistic choice of the IES. Households are heterogeneous in the perceptions of future expenditure shocks, whereas income is homogeneous in expectation. The consumption and savings of households are crucially dependent on how much they underestimate or overestimate their future expenditure shocks. Households that underestimate

[1]In the literature, earnings heterogeneity (Castañeda et al., 2003), heterogeneity of asset returns (Hubmer et al., 2020), or entrepreneurship (Quadrini, 1999) are mentioned as important factors.

the expenditure shocks feel less need for precautionary saving and tend to consume more than other households that overestimate. This consumption gap creates a gap of savings between households that relatively underestimate and overestimate than others.[2] Over time, households that underestimate the expenditure shock consume more than what they originally planned in the past, which quickly depletes their liquid savings. The channel leading to the dispersion of liquid wealth also drives the heterogeneity of the MPC. The households with low liquid wealth due to the underestimation of future expenditure shocks will be more likely to become hand-to-mouth, where households consume all their disposable wealth. This chapter also uses the mechanism of Kaplan and Violante (2014) where households hold a disproportionately small quantity of liquid assets compared to illiquid assets to explain a high MPC. However, the behavioral channel makes the mechanism robust to the low IES, as underestimating the future expenditure shock can lead to persistently low liquid wealth.

Following Carroll et al. (2017), this chapter calibrates the model to match the share of liquid wealth at various percentiles, while letting the model match the median liquid asset holdings over income. Simultaneously, it does not arbitrarily increase the overall MPC by introducing households with a low level of wealth in the economy. Targeting the level of median liquid wealth over income makes the overall level of liquid wealth in the model similar to the data. The result that consumption has a monotonic relationship with the perception of expenditure shock is a crucial source of identification in this chapter. If the data suggest that the share of wealth must be lower at the lower percentile, then the model will respond by introducing households that underestimate future expenditure shocks, and the inequality of liquid wealth will be higher.

When calibrated to match the distribution of liquid wealth, my model implies that there have to be many households that underestimate the future expenditure shocks. Households perceive approximately one-third of the true expenditure shock on average. The model can

[2]Specifically, if distributions regarding future expenditure shock perceived by household A first-order stochastically dominates distributions perceived by household B, then the consumption of household A will be lower than that of household B under the same level of wealth.

also capture levels of liquid wealth at different percentiles over the life cycle. Furthermore, the simulated model implies that most households do not exhibit consumption responses. Instead, a small group of hand-to-mouth households leads to a high overall MPC. This extensive margin of MPC is consistent with the pattern presented in Fuster et al. (2020).

This chapter provides two policy implications for two different goals of the government. Our first policy implication is regarding whom the government must target to boost economic activities during a recession: focusing on low-liquidity households would lead to an effective stimulus. As Carroll (1992) and Kaplan and Violante (2014) noted, households with low liquidity are more likely to be hand-to-mouth, thereby exhibiting a larger consumption response. This chapter introduces an additional channel that supports this policy. Since liquid wealth is a good proxy of whether or not households are optimistic, targeting low-liquidity households would effectively cover the households that are more willing to consume. Our second policy is that if the government wants to enhance the overall ex-post welfare in the economy, it must introduce incentives to rectify the bias introduced by the misperception of future shocks. Based on the calibrated distribution of perception with a large mass of households that underestimate the future expenditure shock, this chapter recommends a policy that taxes people with low liquid wealth and transfers back money when they have high liquid wealth. The average consumption utility of the households can be enhanced while raising additional funds from taxes. One caveat of the tax scheme is that it taxes people with a low amount of liquid wealth, which can be regressive. However, the tax scheme does not penalize the low income and rather tries to correct the low accumulation of liquidity. Also, the additional funds collected by the tax scheme can further enhance welfare and dampen regressiveness.

Literature Review Carroll (1992, 1997) was among the first to adopt a borrowing constraint in a one-asset model to explain hand-to-mouth households exhibiting high MPC.

Kaplan and Violante (2014) note that the proportion of households that become hand-to-mouth is not sufficiently large to generate high overall MPC under the one-asset framework in a realistic setting. Kaplan and Violante (2014) introduce a new workhorse model with liquid and illiquid assets, which separates saving for retirement and precautionary motives. Aguiar et al. (2020) show that the original models cannot generate a realistic level of wealth and proportion of hand-to-mouth households. They introduce preference heterogeneity to remedy this. Aguiar et al. (2020) also note that these models still cannot generate high MPC with the low intertemporal elasticity of substitution (IES) estimated from the micro evidence.[3]

Carroll et al. (2017) introduce heterogeneity in the discount factor and education levels (that determine income) to explain the distribution of wealth and the high MPC. In their model, the households with low discount factors end up with low wealth, and consequently, generate a high MPC. There are three differences between this chapter and Carroll et al. (2017). First, we consider liquid and illiquid assets separately, while Carroll et al. (2017) do not. Second, we consciously do not utilize income dispersion by education levels because education levels or other demographic factors do not contribute much to the dispersion of liquid wealth. Lastly, this chapter can generate a higher level of MPC which is similar to the upper ranges of MPC reported in Havranek and Sokolova (2020).

The quasi-hyperbolic discounting (Strotz, 1955; Pollak, 1968; Laibson, 1997) can also produce high marginal propensity to consume. Under this framework, Laibson et al. (2007) used a two-asset life cycle model combined with quasi-hyperbolic discounting and estimate the key parameters (β, δ)[4] to match the accumulation of wealth and credit card borrowing. The problem of using (β, δ) is that it is difficult to produce a sizeable difference between

[3]MPC of 13.2% drops to 1.5% as IES drops from 1.5 to 0.5. This pattern is confirmed by the Table D.I in the appendix of Kaplan and Violante (2014) where rebate coefficient, measured using the method of Johnson et al. (2006), drops from 20% to 9% when IES decreases from 2 to 1.05.

[4]Under the quasi-hyperbolic discounting framework, β implies the degree of present bias that discounts the utility from the future rewards compared to the current reward. δ is the long-run discount factor which measures how much the agent discount the utility of later rewards.

sophisticated and naive agents, which mutes the role of perception regarding futures agents.[5] Moreover, the (β, δ) model generally does not guarantee a unique consumption solution unless the modeler uses the perception of β close to one, which implies the naive agent. In this chapter, as mentioned earlier, there is a clear connection between the degree of perception and consumption, unlike the quasi-hyperbolic discounting model.[6]

Lian (2021) presents a model where a larger absolute size of mistakes in predicting future consumption leads to a higher MPC today. In this chapter, both the size and the direction of errors are crucial in determining the MPC. In this chapter, households that underestimate the future expenditure shocks will save less and are more likely to be hand-to-mouth, which is a different implication from Lian (2021).

Bianchi et al. (2021) adopt insights of the psychology literature regarding selective memory recall and introduce a model where surprises can shape the future expectation in a biased manner. In particular, a high-income shock today can generate optimistic beliefs regarding future liquidity and increase the MPC. The merit of Bianchi et al. (2021) is that they specify the learning behavior of agents, which is absent in this chapter. However, they do not apply their model in a general life-cycle setting, unlike this chapter.

The remainder of this chapter is organized in the following manner. In Section 1.2, I present two stylized facts. First, there is a severe distribution of liquid wealth even after controlling for demographic factors. Second, there is a large overall MPC, with the heterogeneity of consumption responses. Section 1.3 presents the model in which several theoretical properties are derived. Section 1.4 takes the model to data, calibrates the distribution of

[5]When simulating the life cycle model under quasi-hyperbolic discounting, Angeletos et al. (2001) show no significant difference between naive agents, who perceive future agents' present bias as $\beta = 1$, and sophisticated agents, who perceive the future present bias correctly.

[6]In a stylized three-period problem, Salanie and Treich (2006) show that the relationship between consumption and perception of β depends on the shape of the utility function. However, there is no guarantee that this result would hold in the longer horizon, and the existence or uniqueness of solutions can be difficult to characterize(Harris and Laibson, 2001, 2002).

perception, and examines the model's performance. Section 1.5 presents policy implications of this model. Finally, Section 1.6 concludes with possible extensions.

1.2. Stylized Facts

1.2.1. Severe Dispersion of Liquid Wealth. Liquid wealth is a crucial part of households' wealth and a major source of consumption. Ensuring that households have sufficient liquidity is important since households with too little liquid wealth may face a high borrowing cost in an emergency. Do households hold a sufficient amount of liquid wealth? To see the holdings of liquid wealth in data, I use the Survey of Consumer Finances from 1989 to 2019, one of the most comprehensive datasets regarding households' portfolios. I focus on the working-age households between the ages 25 and 64 that earned above the minimum wage.[7] I define the liquid asset as the sum of money market, checking accounts, savings accounts and call accounts, directly held stocks, and bonds.

Unfortunately, there is a large group of households with very low liquid wealth. The presence of households with low liquid assets drives the severe dispersion of liquid wealth. As seen from Table 1.1, the average Gini coefficient[8] of the liquid asset in all survey waves between 1989–2019 is 0.89.[9]

Why is the inequality of liquid wealth so high? I first examine the demographic factors that are known to contribute to the high dispersion of net worth. Notable demographic factors are different education levels employed in Carroll et al. (2017), dispersion of income (Castañeda et al., 2003; Kaplan et al., 2018), asset return heterogeneity (Hubmer et al.,

[7]I employ these sample selection criteria to make the data similar to the model environment in Section 1.3. I use households above the minimum wage, as the model of Section 1.3 has no unemployment. Households after retirement may have different timing for the withdrawal of illiquid assets, whereas the model assumes a fixed date.

[8]The Gini coefficient represents the degree of inequality where zero implies perfect equality, and one implies the opposite. To calculate the Gini coefficient using the sample weights given in the Survey of Consumer Finances, I use the methodology of Lerman and Yitzhaki (1989).

[9]In the same setting, the average Gini coefficient of the net worth is 0.74, which already indicates a high dispersion of wealth. The Gini coefficient of liquid wealth is consistently higher than the net worth in various situations this section considers.

2020), and entrepreneurship (Quadrini, 1999). To test if a certain demographic factor can explain the dispersion of liquid wealth, I check if the Gini coefficient among subgroups of households classified using the demographic factor diminishes. The demographic factors used in this chapter are education levels, income, occupation, and age. To isolate the dispersion of wealth from the asset return heterogeneity, I also present the results using *highly liquid asset* that comprises money market, checking, and savings accounts. Since the returns of items in the highly liquid assets are likely to be similar across households, their balances would represent households' desire to save while controlling for the differences in investment skills or luck.

	All	Education			Income Tertiles		
		Graduate	Tertiary	Secondary	1st (highest)	2nd	3rd (lowest)
Liquid	0.89	0.87	0.86	0.86	0.85	0.80	0.90
Highly liquid	0.82	0.78	0.77	0.81	0.76	0.72	0.83

Table 1.1. Average Gini coefficients of liquid assets among working-age households by education and income levels: 1989–2019

Table 1.1 indicates that there is high inequality of liquid assets within subgroups of households that have similar education levels or income. Compared to the Gini coefficient of all households, the Gini coefficients within subgroups tend to be smaller, but the differences are small. In particular, the Gini coefficient of the lowest income tertile is approximately 0.9, which suggests a high proportion of households with a near-zero level of wealth. The Gini coefficient using the highly liquid asset is smaller than the case using liquid assets. These differences in Gini coefficients imply that assets that are not highly liquid, such as stocks and bonds, bring additional dispersion to the distribution of the liquid assets. Nevertheless, we can still see a high degree of inequality across all demographic subgroups when using highly liquid assets.

The life cycle model of consumption and savings implies that households accumulate large assets near the end of retirement, which would lead to the dispersion of wealth across different

	Age groups			
	25–34	35–44	45–54	55–64
Liquid	0.80	0.86	0.89	0.90
Highly liquid	0.74	0.79	0.82	0.82

Table 1.2. Average Gini coefficients of liquid assets among working-age households by age groups: 1989–2019

stages of life. Households with a near-zero level of wealth will be mostly young households, since they face uphill income profiles and postpone saving. Households with low liquidity will vanish from the middle-age as they begin accumulating wealth. Hence, if we group people by age groups, wealth inequality is expected to go away. Surprisingly, grouping the households by different life stages does not contribute to lowering the dispersion of liquid wealth. Table 1.2 suggests a trend where the degree of dispersion increase as households reach the retirement age, where the asset accumulation is at its peak. However, there is a significant dispersion of liquid assets in all age groups, and I conclude that age is not a crucial factor leading to the severe dispersion of liquid assets.

	Self-employment status		Occupation group		
	Work for others	Self-employed	Managerial	Technical	Other
Liquid	0.87	0.89	0.87	0.88	0.81
Highly liquid	0.79	0.82	0.78	0.81	0.76

Table 1.3. Average Gini coefficients of liquid assets among working-age households by different occupation groups: 1989–2019
Note: "Managerial" refers to managerial and professional workers. "Technical" refers to technical, sales, and service workers.

Lastly, we check if different occupational characteristics can contribute to the dispersion of liquid wealth. Different occupational groups may have different occupational needs. For example, self-employed workers may have a less predictable income, which might increase the desire for precautionary savings. Moreover, households with higher income, such as managerial and professional workers, would have outliers that lead to greater liquid wealth dispersion. Table 1.3 gauges inequality of liquid wealth by two classifications of occupation groups. In the first two columns, I contrast the group working for others and the self-employed. Not only is the degree of inequality similar, but severe inequality also exists in both groups. The remaining three columns provide an alternative classification of occupation groups by their roles. This alternative method for classifying the occupation confirms that different occupations do not lead to different degrees of liquid wealth inequality. Moreover, this inequality remains even when measured with highly liquid assets.

We checked if potential demographic factors that explain the dispersion of total wealth can also explain the dispersion of liquid wealth. However, the analysis in this section shows that the high dispersion of liquid wealth persists within subgroups of people with similar education and income levels or stages of life. Hence, there is a force driving severe dispersion of life regardless of demographic or economic factors. In Section 1.2.2, I provide in-depth analysis of the role of demographic factors and income to the distribution of wealth with alternative methods, such as normalizing the wealth using permanent income (Section 1.2.2.1) and residual wealth out of various control variables (Section 1.2.2.2). The results in Section 1.2.2 are also in line with the results provided in this section: demographic factors and income cannot fully explain the severe dispersion of wealth.

1.2.2. Further Investigation on the Distribution of Wealth. I provide a more detailed analysis on investigating contributions of demographic factors or income to the dispersion of liquid wealth. Largely, there can be two approaches. First, I normalize the liquid wealth by permanent income, which can reduce the dispersion of liquid wealth due to differences in the long-term trend of income. Second, I directly regress liquid wealth to various observable factors, which possibly possess a nonlinear relationship with liquid wealth.

In all analyses, I focus on households above the minimum wage and between ages 25 to 65. I want to emphasize that these sample selection criteria dampen the wealth dispersion compared to the case using all samples. Furthermore, I want to focus on households with some income because otherwise, the income can be noisy and exhibit an even weaker relationship with wealth. By having working-age households with some amount of income, we reinforce the role of demographic groups and levels of income in explaining the distribution of liquid wealth. However, these efforts do not fully explain why we have such a high dispersion of liquid wealth.

1.2.2.1. *Normalizing Liquid Wealth by Permanent Income.* According to the modern life cycle theory of consumption and savings with transitory and permanent income, permanent income governs the accumulation of wealth and the trend of consumption. Especially, if permanent income follows a random walk process, wealth can be proportional to permanent income under some assumptions (Carroll, 1997; Carroll et al., 2017). I choose two of the popular ways to normalize wealth by income. The first method estimated the residual component of income after controlling age. This way of capturing permanent income is commonly employed in theoretical models such as Kaplan and Violante (2014), and Carroll et al. (2017). The second method is finding a proxy variable for the permanent income. Variables such as total or food expenditure have been proposed to measure permanent income. The logic is simple: consumption and permanent income will have a one-to-one relationship. This relationship is because a shock in the persistent component of income will have a long-lasting effect on income in the future, making consumption of all future periods increase at once. Using this logic, I focus on food expenditure available in the Survey of Consumer Finances.

Method 1: Normalization by Estimated Permanent Income I assume that income follows (1.4), which is one of the most commonly held income process in the household finance literature. Let i denote the household, and t denote the time. The residual component of income $\tilde{y}_{i,t}$ can be measured as $\tilde{y}_{i,t} = y_{i,t}/\Xi_{i,t}$ where $\Xi_{i,t}$ is estimated from data. Then, the $\tilde{y}_{i,t}$ measures the deviation of income of household i compared to the peers with noise $\varepsilon_{i,t}$. If we have long series of income, then we are able to filter out $\varepsilon_{i,t}$, which is impossible when using cross-sectional data such as the Survey of Consumer Finances.

I compute Gini indices using the normalized variable $w_{i,t}$ which is defined as $w_{i,t} = W_{i,t}/\tilde{y}_{i,t}$ where $W_{i,t}$ is the raw data on wealth. Then, $w_{i,t}$ normalizes the wealth by controlling income that is relatively lower or higher than others which is assumed to be a permanent characteristic over the life cycle.

Table 1.4 displays Gini indices of different subgroups of households at each survey year. Compared to the case that uses raw wealth in Table B.1, the normalization of wealth alleviates the wealth inequality. However, in almost all cases, the Gini indices of liquid wealth are over 0.8, which still implies severe inequality.

Method 2: Normalization by Food Expenditures Another measure of permanent income can be food expenditures. The permanent income hypothesis implies that a permanent increase in income is associated with an increase in consumption for all future periods for consumption smoothing households. In this perspective, food expenditure can be a good candidate for measuring permanent income. Food expenditures are always positive, whereas other expenditures can occasionally be zero. Also, it has a smaller variance than other parts of expenditures, which better fits the properties of items that obey consumption smoothing.

In the Survey of Consumer Finances, the size of food expenditure is reported from 2004, and the Gini index using the wealth normalized by the size of food expenditure is shown in Table 1.5. I exclude households with abnormal food expenditure of zero, which occurs in a few observations. In every survey year, normalization using food expenditure does not alleviate the dispersion of wealth by a great degree. Compared to the previous method, which uses an estimated permanent income, the Gini indices are even higher in this approach.

1.2.2.2. *Normalizing Liquid Wealth by Regressions.* Another way of normalization would be directly acquiring residual wealth out of demographic characteristics and income. To do this, I regress the wealth with following factors: income, age, education, occupation group, self-employment status, and education level. The regression specifically looks as

$$\ln W_{i,t} = \beta + \text{Controls}_{i,t} + \text{Time Fixed Effect}_t + \varepsilon_{i,t}. \tag{1.1}$$

		‘89	‘92	‘95	‘98	‘01	‘04	‘07	‘10	‘13	‘16	‘19	Avg.
All	NW	0.61	0.64	0.62	0.61	0.64	0.64	0.61	0.66	0.67	0.65	0.66	0.64
	LIQ	0.81	0.82	0.82	0.82	0.85	0.85	0.82	0.85	0.85	0.84	0.82	0.83
	HLIQ	0.75	0.75	0.73	0.74	0.75	0.78	0.73	0.78	0.78	0.75	0.72	0.75
	Illiquid	0.58	0.59	0.56	0.56	0.56	0.55	0.54	0.57	0.59	0.58	0.60	0.57
Graduate education	NW	0.61	0.64	0.63	0.58	0.64	0.63	0.62	0.64	0.66	0.63	0.64	0.63
	LIQ	0.77	0.80	0.83	0.80	0.85	0.84	0.81	0.82	0.83	0.77	0.79	0.81
	HLIQ	0.71	0.73	0.70	0.71	0.74	0.78	0.72	0.76	0.78	0.69	0.72	0.73
	Illiquid	0.56	0.59	0.55	0.52	0.55	0.53	0.55	0.54	0.57	0.57	0.58	0.56
Tertiary education	NW	0.58	0.61	0.61	0.61	0.60	0.60	0.54	0.63	0.62	0.66	0.63	0.61
	LIQ	0.78	0.81	0.78	0.81	0.81	0.82	0.77	0.83	0.80	0.87	0.77	0.80
	HLIQ	0.71	0.71	0.65	0.77	0.72	0.70	0.69	0.72	0.72	0.78	0.62	0.71
	Illiquid	0.54	0.56	0.56	0.62	0.53	0.52	0.49	0.53	0.53	0.55	0.59	0.55
Secondary education	NW	0.62	0.62	0.61	0.61	0.62	0.64	0.60	0.65	0.69	0.65	0.65	0.63
	LIQ	0.82	0.79	0.81	0.81	0.78	0.83	0.80	0.83	0.89	0.81	0.84	0.82
	HLIQ	0.78	0.78	0.78	0.74	0.75	0.79	0.74	0.81	0.76	0.78	0.76	0.77
	Illiquid	0.59	0.57	0.55	0.55	0.56	0.58	0.55	0.59	0.62	0.60	0.60	0.58
1st income tertile	NW	0.51	0.55	0.53	0.54	0.54	0.52	0.51	0.57	0.54	0.55	0.56	0.54
	LIQ	0.75	0.77	0.77	0.78	0.79	0.76	0.74	0.80	0.74	0.74	0.73	0.76
	HLIQ	0.70	0.68	0.67	0.66	0.69	0.67	0.65	0.73	0.64	0.64	0.63	0.67
	Illiquid	0.46	0.49	0.48	0.48	0.47	0.45	0.45	0.46	0.47	0.48	0.51	0.47
2nd income tertile	NW	0.58	0.60	0.58	0.58	0.61	0.61	0.59	0.61	0.68	0.63	0.63	0.61
	LIQ	0.77	0.80	0.82	0.79	0.81	0.78	0.77	0.76	0.85	0.81	0.81	0.80
	HLIQ	0.68	0.72	0.73	0.71	0.73	0.73	0.72	0.73	0.81	0.71	0.71	0.73
	Illiquid	0.54	0.54	0.51	0.50	0.52	0.52	0.50	0.52	0.58	0.55	0.55	0.53
3rd income tertile	NW	0.73	0.74	0.72	0.70	0.75	0.77	0.72	0.76	0.79	0.76	0.76	0.74
	LIQ	0.88	0.87	0.88	0.88	0.91	0.94	0.91	0.92	0.94	0.93	0.90	0.91
	HLIQ	0.84	0.83	0.79	0.83	0.82	0.90	0.83	0.85	0.88	0.87	0.83	0.84
	Illiquid	0.71	0.71	0.66	0.69	0.68	0.68	0.67	0.70	0.71	0.71	0.73	0.70
Managerial	NW	0.58	0.62	0.59	0.60	0.61	0.59	0.56	0.61	0.63	0.60	0.62	0.60
	LIQ	0.76	0.79	0.81	0.80	0.83	0.81	0.78	0.80	0.81	0.76	0.76	0.79
	HLIQ	0.65	0.71	0.67	0.74	0.71	0.70	0.69	0.74	0.78	0.68	0.65	0.70
	Illiquid	0.54	0.57	0.52	0.58	0.54	0.51	0.51	0.53	0.54	0.54	0.57	0.54
Technical	NW	0.62	0.60	0.64	0.59	0.63	0.63	0.63	0.68	0.65	0.63	0.64	0.63
	LIQ	0.80	0.78	0.83	0.82	0.82	0.81	0.85	0.86	0.82	0.74	0.81	0.81
	HLIQ	0.76	0.73	0.74	0.74	0.73	0.74	0.75	0.81	0.73	0.69	0.75	0.74
	Illiquid	0.57	0.55	0.57	0.53	0.55	0.54	0.55	0.57	0.57	0.57	0.62	0.56
Other occupation	NW	0.57	0.63	0.57	0.59	0.57	0.60	0.58	0.60	0.62	0.62	0.63	0.60
	LIQ	0.76	0.78	0.79	0.79	0.75	0.76	0.79	0.77	0.78	0.77	0.83	0.78
	HLIQ	0.75	0.76	0.75	0.72	0.73	0.74	0.70	0.74	0.75	0.73	0.74	0.74
	Illiquid	0.57	0.58	0.54	0.52	0.50	0.53	0.52	0.54	0.56	0.60	0.58	0.55

TABLE 1.4. Gini index of the United States: 1989–2019, using Wealth Normalized by Estimated Permanent Income

Notes: Wealth normalized by estimated permanent income refers to $W_{i,t}/\tilde{y}_{i,t}$ where $\tilde{y}_{i,t}$ is the estimated permanent income based on (1.4).

		‘04	‘07	‘10	‘13	‘16	‘19	Avg.
All	NW	0.69	0.68	0.71	0.71	0.72	0.72	0.70
	LIQ	0.86	0.86	0.86	0.87	0.86	0.85	0.86
	HLIQ	0.80	0.78	0.82	0.79	0.79	0.78	0.79
	Illiquid	0.62	0.64	0.64	0.66	0.67	0.67	0.65
Graduate education	NW	0.66	0.66	0.67	0.67	0.68	0.68	0.67
	LIQ	0.84	0.82	0.84	0.83	0.82	0.82	0.83
	HLIQ	0.78	0.74	0.78	0.76	0.75	0.75	0.76
	Illiquid	0.58	0.63	0.59	0.62	0.64	0.63	0.62
Tertiary education	NW	0.62	0.63	0.64	0.67	0.69	0.66	0.65
	LIQ	0.81	0.83	0.79	0.82	0.85	0.80	0.82
	HLIQ	0.73	0.73	0.74	0.73	0.76	0.70	0.73
	Illiquid	0.56	0.59	0.59	0.61	0.62	0.61	0.60
Secondary education	NW	0.69	0.65	0.70	0.71	0.71	0.73	0.70
	LIQ	0.85	0.81	0.86	0.88	0.84	0.89	0.86
	HLIQ	0.82	0.77	0.85	0.82	0.81	0.80	0.81
	Illiquid	0.63	0.60	0.65	0.65	0.68	0.69	0.65
1st income tertile	NW	0.60	0.61	0.63	0.62	0.65	0.65	0.63
	LIQ	0.82	0.82	0.82	0.81	0.82	0.79	0.81
	HLIQ	0.73	0.71	0.77	0.72	0.73	0.69	0.72
	Illiquid	0.54	0.57	0.57	0.57	0.60	0.61	0.58
2nd income tertile	NW	0.65	0.60	0.63	0.67	0.63	0.66	0.64
	LIQ	0.81	0.76	0.76	0.82	0.76	0.83	0.79
	HLIQ	0.77	0.71	0.73	0.77	0.72	0.74	0.74
	Illiquid	0.56	0.51	0.54	0.57	0.54	0.56	0.55
3rd income tertile	NW	0.75	0.76	0.75	0.79	0.77	0.74	0.76
	LIQ	0.92	0.88	0.88	0.91	0.91	0.86	0.89
	HLIQ	0.88	0.82	0.85	0.86	0.86	0.82	0.85
	Illiquid	0.68	0.74	0.70	0.74	0.72	0.71	0.71
Managerial	NW	0.64	0.65	0.68	0.67	0.68	0.68	0.67
	LIQ	0.81	0.84	0.83	0.83	0.82	0.81	0.83
	HLIQ	0.74	0.74	0.78	0.75	0.74	0.72	0.75
	Illiquid	0.58	0.61	0.62	0.62	0.63	0.64	0.62
Technical	NW	0.69	0.63	0.71	0.71	0.72	0.69	0.69
	LIQ	0.85	0.82	0.87	0.86	0.85	0.83	0.84
	HLIQ	0.81	0.76	0.82	0.77	0.80	0.77	0.79
	Illiquid	0.62	0.58	0.62	0.65	0.66	0.64	0.63
Other occupation	NW	0.67	0.70	0.67	0.66	0.68	0.72	0.68
	LIQ	0.81	0.80	0.80	0.84	0.79	0.88	0.82
	HLIQ	0.79	0.75	0.75	0.79	0.77	0.79	0.77
	Illiquid	0.60	0.65	0.60	0.61	0.65	0.67	0.63

Table 1.5. Gini index of the United States: 1989–2019, using Wealth Normalized by Food Expenditures

Notes: Wealth normalized by food expenditures refers to raw level of wealth divided by the amount of food expenditure.

The following regression lets us acquire the estimated residual term $\hat{\varepsilon}_{i,t}$ which is the unexplained part of the log wealth that cannot be attributed to control variables. Then, I compute the Gini index of $\exp(\hat{\varepsilon}_{i,t})$. This is based on the idea that the wealth can be decomposed as $W_{i,t} = \Theta_{i,t}\xi_{i,t}$ which is a multiplicative form, and $\xi_{i,t} = \exp(\varepsilon_{i,t})$ measures the residual wealth. Alternative way can be using $W_{i,t}$ instead of $\ln W_{i,t}$ in (1.1) and defining residual (plus the intercept) as the residual wealth. However, this produces a noisy measure of residual wealth, and the computation of Gini coefficients can be problematic with many negative values. When using $\ln W_{i,t}$, we have consistent measures over different survey years, and the level of residual wealth is kept positive.

	Net worth	Liquid	Highly liquid	Illiquid
Whole sample	0.76	0.90	0.82	0.72
Time fixed effect	0.77	0.90	0.82	0.73
Income	0.76	0.89	0.81	0.71
Age (4th order polynomial)	0.73	0.87	0.80	0.71
Education (14 groups)	0.73	0.86	0.80	0.70
Self employed or not (2 groups)	0.76	0.90	0.81	0.69
Occupation group (4 groups)	0.76	0.89	0.81	0.71
Income+Age	0.72	0.86	0.79	0.70
Income+Age+Education	0.69	0.83	0.77	0.68
Income+Age+Education+Employment status	0.68	0.82	0.76	0.64

TABLE 1.6. Gini indices with wealth normalized by regressing demographic characteristics and the level of income

Table 1.6 shows Gini indices measured with the residual wealth. The first two rows serve as a sensitivity check. In the first row, we have the Gini index based on the raw level of wealth without running any regression by pooling data over all survey years. The second row measures the Gini index based on residuals of (1.1) where control variables are constant term and time fixed effects. The Gini indices of the first two rows are similar, which implies that [1] pooling over different years does not dramatically change the result, and [2] the method of calculating the Gini index based on residuals of regression (1.1) provides a sensible result.

The next five rows of Table 1.6 are based on residuals where we employ different demographic characteristics and incomes one by one. None of the five factors significantly brings down inequality for all types of wealth. I use the polynomial of degree four for age, which provides the best result in reducing inequality out of one to four degrees of polynomials. We can investigate nonlinear relationships between income and wealth, but employing a polynomial degree more than one or logarithm of income worsens the result. Education, self-employment status, and occupation groups are included in the regression as dummy variables that indicate the group that each individual belongs. None of the factors alone can effectively explain the dispersion of wealth.

The last three rows of Table 1.6 use combinations of control variables. The Gini index of liquid wealth becomes 0.82 when using all control variables, but this still implies severe wealth inequality. Under the regression approach, all the control variables cannot fully explain the dispersion of wealth.

1.2.3. Large and Heterogeneous MPC. Numerous empirical studies report excess sensitivity of consumption: consumers' MPC of a windfall gain is large. For example, Souleles (1999) reports that consumers spend 34.4 to 64 percent of income tax refunds over a quarter. Agarwal and Qian (2014) measures that consumers spent 80 percent of unanticipated fiscal stimulus in Singapore over a 10-month period. In response to the social security tax reform, Parker (1999) finds that households spent approximately 20 percent of additional after-tax income over three-month periods on nondurable goods. In the income tax rebate of 2001, Johnson et al. (2006) find that households spent 20 percent to 40 percent of the tax rebate over a quarter on nondurable goods. Based on the fiscal stimulus of 2008, Parker et al. (2013) find that households' spent 50 percent to 90 percent of the stimulus payments over a quarter. Using predictable payments from the Alaska permanent fund, Kueng (2018) finds that households spent 20 percent for nondurable and services over a quarter. The magnitude of the MPC may differ, but they are usually over twenty percent when measuring the increase in consumption over a quarter. Havranek and Sokolova (2020) collect estimates of MPC in both micro and macro literature. In their data, when restricting the sample with micro-level evidence and MPC measured over a quarter, the median MPC is approximately 22.7 percent, whereas the mean MPC is 30.2 percent.

Heterogeneity of Consumption Responses Households exhibit heterogeneous consumption responses. Three notable patterns of the heterogeneous consumption responses are [1] association of low liquidity and high consumption responses, [2] extensive margin of consumption response, and [3] the role of sophistication and financial planning.

First, a large piece of evidence suggests a strong relationship between high consumption response and low liquidity, consistent with the buffer stock theory of saving (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). In addition, there is another piece of evidence that indicates the importance of persistent characteristics (Parker, 2017). The stance of this chapter is that the persistent characteristics drive low liquidity and thus, bring the high consumption response, as Gelman et al. (2019), Gelman (2021), and Carroll et al. (2017).

Second, there is evidence of an extensive margin of consumption responses. Not all households exhibit a large consumption response, and the high overall MPC is led by a small group of households in the economy. For example, based on two stimuli from U.S. in years 2001 and 2008, Misra and Surico (2014) show that approximately half of the households do not show a significant and positive response to the stimulus. Fuster et al. (2020) asked directly to survey respondents how much they would spend out of a windfall gain; approximately 70 percent of the respondents exhibited MPC of zero.

Third, households' attitudes and perceptions regarding the future are important. Parker (2017) finds strong spending responses among households that lack sophistication and financial planning and hints that persistent characteristics play an essential role. Motivated by this finding, this chapter adopts persistent behavioral friction, which can lead to different consumption responses.

1.2.4. Misprediction of Expenditures. Households generally save for two reasons. In the 2019 Survey of Consumer Finance, the most frequently selected reason for saving was to prepare for retirement (34.1 percent) among working-age households. The following big reason for saving was to prepare for the 'rainy days,' which was selected by 27.7 percent of the respondents.[10] The two main reasons for saving are consistent with implications of modern consumption savings models, such as Carroll (1992). Households accumulate a large amount of wealth to prepare for retirement, and simultaneously, some portion of the wealth is left to deal with the income fluctuations and other random events, such as liquidity or expenditure shocks.

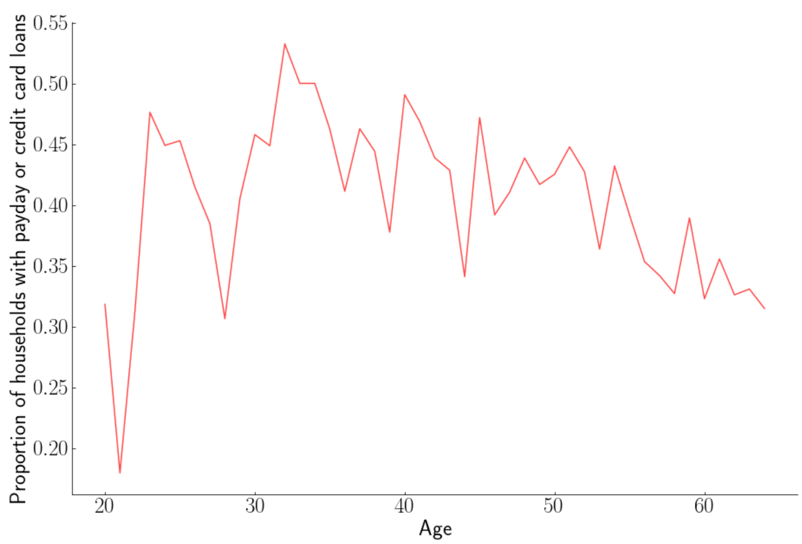

FIGURE 1.1. Proportion of having payday or credit card loans by age
Source: Reproduced from the Survey of Consumer Finances at 2019

However, households' portfolio choice in the Survey of Household Finances suggests a lack of preparation for rainy days. Among working-age households, three percent of households

[10]The third reason to save was to buy own house, selected by 5.8 percent of the sample.

even resort to payday loans, which typically accompany high interest. As a most frequently selected option, 38.3 percent of people answered that using a payday loan was because of an emergency. Furthermore, among subjects who currently borrow money, 38.9 percent answered that they had an experience being behind the payment by two months or more. These facts show that many households cannot avoid borrowing with high interest or paying penalties for being behind schedule, which brings additional costs. Thus, even though households save to prepare for the rainy days, the unpredicted expenses force households to use borrowing facilities that bring extra cost. Also, there is a prevalence of credit card borrowing where the median borrowing rate in the same survey is around 17 percent. Approximately 38.6 percent of households have credit card loans. The borrowing rate and the borrowing frequency are too high to be rationalized with the models without any behavioral frictions.[11]

The pattern of borrowing over the life cycle is at odds with the implications of the life cycle models without featuring a borrowing constraint. Such models would suggest that households would mainly borrow until the peak of the income profile and stop borrowing from then. However, Figure 1.1 does not indicate a sharp decrease in the frequency of borrowing near the middle age, and the high frequency of borrowing also persists throughout the life cycle. Hence, contributions from other factors must persistently lead households to borrow throughout the life cycle.

As a related study, Berman et al. (2016) provide an evidence that consumers tend to underweight the costs rather than income when predicting the future spare money. Howard et al. (2020) study an expenditure prediction bias which is a tendency to underestimate future expenses.

[11]For example, the current workhorse model of consumption and savings by Kaplan and Violante (2014) targets 26 percent of the households to borrow, under the nominal borrowing rate of 10 percent.

1.3. Model

Households face consumption and savings problems for T number of periods. Interest rates and income are exogenous. There are two types of consumption. The first part of consumption is determined in the long-run perspective, and its trajectory is smooth over the life cycle. The second part of the consumption, which we call an expenditure shock, is not a choice variable where short-run motives are more important than consumption smoothing in the long run. Examples of expenditure shocks are vehicle repair costs or sudden medical expenses. Extra spending on items in this category does not bring additional utility. However, such expenses are an unavoidable part of consumption, and determine the minimuum level of consumption at each period.

The expenditure shock at period t is drawn from a random variable $\mathbf{\Gamma}_t$, which maps a set of events to nonnegative real numbers. Households have an imperfect perception of expenditure shocks, where the perception of $\mathbf{\Gamma}_t$, denoted as $\widetilde{\mathbf{\Gamma}}_t$, can be different from the actual $\mathbf{\Gamma}_t$. Similarly, variables with tilde, such as $\widetilde{x}$, represents the perception of a variable x. Without the time subscript, $\widetilde{\mathbf{\Gamma}} = \{\widetilde{\mathbf{\Gamma}}_1, \cdots, \widetilde{\mathbf{\Gamma}}_T\}$ represents the sequence of perception of expenditure shocks. In the same way, $\mathbf{\Gamma}$ is the sequence of distribution of true expenditure shocks, $\mathbf{\Gamma} = \{\mathbf{\Gamma}_1, \cdots, \mathbf{\Gamma}_T\}$. Γ_t and $\widetilde{\Gamma}_t$ are particular realizations of random variables $\mathbf{\Gamma}_t$ and $\widetilde{\mathbf{\Gamma}}_t$.

Households retire at age T_r. During the working-age periods, households at time $t < T_r$ solve a following problem:

$$V_t^{\widetilde{\mathbf{\Gamma}}}(X_t, Z_t; \Gamma_t) = \max_{C_t, S_t, A_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\widetilde{V}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}, Z_{t+1}; \widetilde{\Gamma}_{t+1}) \right], \tag{1.2}$$

subject to

$$Y_t + R_t^S S_{t-1} = X_t \geq C_t + S_t + A_t, \quad Z_{t+1} = R_{t+1}^A (A_t + Z_t), \text{ and}$$

$$C_t \geq 0, A_t \geq 0 \text{ and } S_t \geq 0.$$

$V_t^{\widetilde{\Gamma}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ is the current self's value function conditional on the savings of liquid asset S_{t-1} and illiquid asset Z_{t-1} from the previous period, and a realization of the expenditure shock Γ_t. Note that S_t is a stock variable, and A_t is a flow variable. The stock of illiquid asset at t is written as Z_t. The disposable wealth X_t at time t is defined as the sum of current income Y_t and gross return of savings from liquid assets $R_t^S S_{t-1}$, where R_t^S is the gross interest rate on liquid assets.

The value function crucially depends on the perception $\widetilde{\mathbf{\Gamma}}$, which determines the continuation value $\widetilde{V}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(S_t, Z_t; \widetilde{\Gamma}_{t+1})$ defined as

$$\widetilde{V}_{t'}^{\widetilde{\mathbf{\Gamma}}}(X_{t'}, Z_{t'}; \widetilde{\Gamma}_{t'}) = \max_{C_{t'}, S_{t'}, A_{t'}} u(C_{t'} - \widetilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\widetilde{V}_{t'+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t'+1}, Z_{t'+1}; \widetilde{\Gamma}_{t'+1}) \right],$$

subject to

$$Y_t + R_t^S S_{t-1} = X_t \geq C_t + S_t + A_t,$$

$$Z_{t+1} = R_{t+1}^A (A_t + Z_t),$$

$$C_t \geq 0, A_t \geq 0 \text{ and } S_t \geq 0, \text{ and}$$

$$\begin{aligned} \widetilde{V}_{T_r-1}^{\widetilde{\mathbf{\Gamma}}}(X_{T_r-1}, Z_{T_r-1}; \widetilde{\Gamma}_{T_r-1}) = & \max_{C_{T_r-1}, S_{T_r-1}, A_{T_r-1}} u(C_{T_r-1} - \widetilde{\Gamma}_{T_r-1}) \\ & + \delta \mathbb{E}_{T_r-1} \left[\widetilde{W}_{T_r}^{\widetilde{\mathbf{\Gamma}}}(Z_{T_r} + R_{T_r}^S S_{T_r-1}) \right]. \end{aligned}$$

The constraints $S_t \geq 0$ and $A_t \geq 0$ eliminate borrowing. In particular, the constraint on the flow of illiquid asset $A_t \geq 0$ ensures that the withdrawal of illiquid assets before retirement is impossible. Since there are no costs of saving A_t, this model imposes an extreme case of asymmetric adjustment cost. An alternative means of imposing illiquidity of an asset can be requiring an adjustment cost, as in Kaplan and Violante (2014). In particular, Kaplan and Violante (2014) assume symmetric adjustment cost for illiquid assets. In reality, the withdrawal of illiquid assets should be possible, but paying back a mortgage or saving in a retirement account does not require any adjustment cost. Hence, the adjustment costs of illiquid assets in the real world will be somewhere between the extreme asymmetric cost of this model and the symmetric adjustment cost of Kaplan and Violante (2014). The assumption that this model makes is for theoretical tractability. In particular with adjustment costs, the continuation values can be locally convex due to the non-convexity of choice sets. To acquire a clear relationship between the degrees of perception and consumption, I do not impose adjustments that can add unnecessary difficulty in the theoretical analysis.

After retirement, $t \in \{T_r, \cdots, T\}$, households solve the following problems:

$$
\begin{aligned}
W_t^{\widetilde{\mathbf{\Gamma}}}(X_t;\Gamma_t) &= \max_{C_t,S_t} u(C_t-\Gamma_t)+\delta\mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1};\widetilde{\Gamma}_{t+1})\right], \quad \text{where} \\
\widetilde{W}_{t'}^{\widetilde{\mathbf{\Gamma}}}(X_{t'};\widetilde{\Gamma}_{t'}) &= \max_{C_{t'},S_{t'}} u(C_{t'}-\widetilde{\Gamma}_{t'})+\delta\mathbb{E}_{t'}\left[\widetilde{W}_{t'+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t'+1};\widetilde{\Gamma}_{t'+1})\right], \\
&\qquad \text{subject to} \\
X_{T_r} &= Y_{T_r}+R_{T_r}^{S}S_{T_r-1}+R_{T_r}^{A}Z_{T_r-1}, \\
Y_t+R_tS_{t-1} &= X_t \geq C_t+S_t \text{ if } t\neq T_r, \\
C_t &\geq 0, \text{ and } S_t\geq 0, \text{ and} \\
\widetilde{W}_T^{\widetilde{\mathbf{\Gamma}}}(X_T;\widetilde{\Gamma}_T) &= u(X_T-\widetilde{\Gamma}_T).
\end{aligned}
\tag{1.3}
$$

After retirement, the problem becomes simple with only two choice variables, C_t, and S_t. Also, at the period of retirement, T_r, households can access the illiquid assets Z_{T_r}, they accumulated thus far. In the final period, households consume all available wealth. I denote the value function after retirement using the letter W instead of V to distinguish value functions in two different regimes.

The optimal consumption and savings before retirement at t are functions solving (1.2) with inputs when X_t, Z_t, and Γ_t are based on the perception $\widetilde{\mathbf{\Gamma}}$. If there is no room for confusion, I will omit inputs of the functions to save space. Similarly, the optimal consumption and savings before retirement are functions that solve (1.3) with inputs X_t, Γ_t, and $\mathbf{\Gamma}$.

Following the literature, a household is hand-to-mouth if the level of optimal consumption C_t equals available wealth X_t, that is $C_t(X_t, Z_t, \Gamma_t; \widetilde{\mathbf{\Gamma}}) = X_t$.

The domain of the utility function $u : \mathbb{R} \to \mathbb{R}$ consists of real numbers, where $u' > 0$ and $u'' < 0$. An example of this kind of utility function is the exponential utility function with constant absolute risk aversion. The Inada condition cannot be incorporated in this model unless there is a guarantee that Γ_t is not too large. Alternatively, I can impose the Inada condition and assume that $\Gamma_t < Y_t$ and $\widetilde{\Gamma}_t < Y_t$ in every period. I also assume that R^A_t and R^S_t are deterministic, and $1/\delta > R^A_t > R^S_t$. This assumption eliminates the incentive to save liquid assets in the period just before retirement $T_r - 1$; hence $S_{T_r} = 0$.

Optimism and Pessimism. I call a household that correctly perceives the future random variable, where the perception $\widetilde{\mathbf{\Gamma}}$ is identical to $\mathbf{\Gamma}$, a *sophisticated* household. Considering two different perceptions $\widetilde{\mathbf{\Gamma}}^1$ and $\widetilde{\mathbf{\Gamma}}^2$, I define that $\widetilde{\mathbf{\Gamma}}^1$ is more *optimistic* (*pessimistic*) than $\widetilde{\mathbf{\Gamma}}^2$ if cumulative density functions (CDFs) in $\widetilde{\mathbf{\Gamma}}^2$ ($\widetilde{\mathbf{\Gamma}}^1$) first-order stochastically dominates CDFs in $\widetilde{\mathbf{\Gamma}}^1$ ($\widetilde{\mathbf{\Gamma}}^2$). From now on, I denote $\widetilde{\mathbf{\Gamma}}^2_t >_1 \widetilde{\mathbf{\Gamma}}^1_t$ if CDF, of $\widetilde{\mathbf{\Gamma}}^2_t$ first-order stochastically dominates CDF of $\widetilde{\mathbf{\Gamma}}^1_t$. Similarly, $\mathbf{\Gamma}^2 >_1 \mathbf{\Gamma}^1$ implies that $\mathbf{\Gamma}^2_t >_1 \mathbf{\Gamma}^1_t$ for any t.

Relationship between Consumption and Perception. How is consumption related to the perception of future expenditure shocks? I establish the relationship between the level of consumption, and the degree of optimism and pessimism. I first investigate the behavior of households after the retirement. In the first proposition, I show that when two households face the same disposable wealth X_t and the expenditure shock Γ_t, the household that are more optimistic will spend more.

Proposition 1.3.1. *For every* $t \geq T_r$*, and* $w_t \geq 0$*,* $C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$ *if* $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

The intuition underlying Proposition 1.3.1 is simple, and the proof is in Appendix A. The household with more pessimistic beliefs perceives that it will face a larger expenditure shock in the future. The presence of large expenditure shocks in the future decrease $C_t - \Gamma_t$

at any future period t. The marginal value of saving will also be higher, since the pessimistic households will perceive that the long-run part of the consumption will be lower. However, optimistic households do not value savings as much as pessimistic households. Note that Proposition 1.3.1 does not specify where Γ_t is drawn from. As long as the perceptions of households can be ordered, the comparative statics of consumption is possible under the same X_t and Γ_t.

If consumption at any point, under the same condition, is always higher among optimistic households, can we also claim a similar argument for assets? To answer this question, consider the following scenario. Fix a sequence of income $Y_t, Y_{t+1}, \cdots, Y_T$, interest rates $R_t^S, R_{t+1}^S, \cdots, R_T^S$, and expenditure shocks $\Gamma_t, \Gamma_{t+1}, \cdots, \Gamma_T$. Moreover, for household i, and when $t \geq T_r$ and $k \leq T-t$, I define the endogenous disposable income of i at time $t+k$ recursively as,

$$X_{t+k}^i(X_{t+k-1}^i; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^i) = Y_{t+k} + R_{t+k}^S \left[X_{t+k-1}^i - C_{t+k-1}(X_{t+k-1}; \Gamma_{t+k-1}, \widetilde{\mathbf{\Gamma}}^i)\right],$$

which depends on the endogenous choices of consumption, $C_t, C_{t+1}, \cdots, C_{t+k-1}$.

In this manner, $X_{t_k}^i(\cdot; \cdot, \widetilde{\mathbf{\Gamma}}^i)$ tracks the accumulation of assets of household i over the life cycle. Since we fix exogenous factors, such as income, interest rates, and expenditure shocks, the comparison between $X_{t_k}^i(\cdot; \cdot, \widetilde{\mathbf{\Gamma}}^i)$ and $X_{t_k}^i(\cdot; \cdot, \widetilde{\mathbf{\Gamma}}^j)$ lets us focus on how the different degree of perception can lead to a different path of asset accumulation.

Corollary 1.3.1. *Fix $t \geq T_r$ and $X_t \geq 0$. Then, for any $k \leq T-t$ and realization of $Y_t, \cdots, Y_{t+k}$ and $\Gamma_t, \cdots \Gamma_{t+k}$,*

$$X_{t+k}^2(X_{t+k-1}^1; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^2) \geq X_{t+k}^1(X_{t+k-1}^1; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^1)$$

as long as $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

This result reveals that for any two households where the perceptions are ordered as $\widetilde{\mathbf{\Gamma}}^2 <_1 \widetilde{\mathbf{\Gamma}}^1 <_1 \mathbf{\Gamma}$, the consumption profile of the household with $\widetilde{\mathbf{\Gamma}}^1$ would be more flatter than the other household with $\widetilde{\mathbf{\Gamma}}^2$. Moreover, when beginning with the same level of wealth X_t and facing the same income and expenditure shocks, the wealth of the household with more optimistic belief, $\widetilde{\mathbf{\Gamma}}^2$ would be depleted faster. Hence, at the end of the life cycle, the household with pessimistic beliefs will have more left to consume. Based on this result, this paper uses the stock of liquid assets as a tool to elicit the distribution of relative optimism and pessimism.

We obtain a similar result as Proposition 1.3.1 after the retirement, which is a two-asset model. The intuition underlying Proposition 1.3.2 is the same as that for Proposition 1.3.1.

PROPOSITION 1.3.2. *For every* $t < T_r$, $C_t(X_t, Z_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq C_t(X_t, Z_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$ *if* $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

However, Corollary 1.3.1 does not extend to the two-asset case. This is because even though household A is more pessimistic than household B, it does not necessarily imply that household A would save both liquid and illiquid assets more than B. For example, suppose imminent expenditure shock is larger than the expenditure shocks after retirement. In such a case, the savings of illiquid assets of a pessimistic household can be lower than that of the optimistic household. A numerical exercise in Figure 1.2 shows such a case. As the household moves along the horizontal axis from left to right, the household becomes more aware of the expenditure shock and becomes more pessimistic. The relatively optimistic households underestimate the need for precautionary saving in period two and mainly save using the illiquid assets, which brings a higher rate of return. However, the sophisticated households that perfectly know the large expenditure shock mainly save using the liquid asset. The relationship of the consumption and savings with the perceptions in the two-asset case needs to be verified using a simulation, and I show in Section 1.4 that the more pessimistic households tend to accumulate more liquid illiquid assets than their optimistic counterparts.

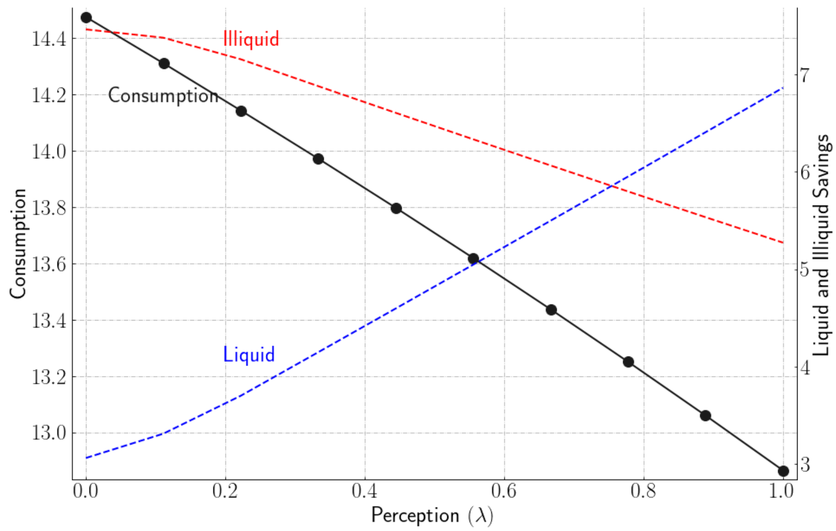

FIGURE 1.2. A case where pessimistic household have lower illiquid savings than an optimistic household
Notes: The figure presents the simulation result of a three-period model with the following parameter values. Utility function is defined as $u(c) = c^{1-\sigma}/(1-\sigma)$ where $\sigma = 0.5$. Income is $\{25, 10, 5\}$ for each period. Expenditure shocks of magnitudes 7 and 1 for periods two and three, respectively, occur with probability 0.5. The discount factor is 0.95, and the interest rates for liquid and illiquid assets are 0 and 2 percent, respectively. The x-axis shows the perception of households, where a value λ on the x-axis denotes the percentage of the shock magnitude perceived by the household.

Fluctuations of C_t and $C_t - \Gamma_t$. Next, we develop the dynamic properties of $C_t - \Gamma_t$ and C_t. To see this, I explicitly write all the random variables as inputs of consumption in this period, C_t, and the next period C_{t+1} as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. I omit $\widetilde{\mathbf{\Gamma}}$ because comparing the perception of the expenditure is not important here. The total expenditure $C_t(X_t(Y_t), \Gamma_t)$ can be decomposed into two parts. The core part of consumption $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ is what the household attempts to smooth in the lifetime. The rest, Γ_t, is the expenditure shock. The key difference between the two is that, they have very different relationships with total consumption next period, $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. Suppose that there is an increase in income Y_t or interest rate R^s_t. Then, the available wealth at this period X_t will increase and due to the globally concave continuation value, consumption and savings at period t will strictly increase as long as the household is not hand-to-mouth. This leads to increase in C_{t+1} as well since X_{t+1} increase. When there is an increase in the expenditure shock Γ_t, the total expenditure C_t in this period will increase. However, the MPC cannot exceed one, so $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ will decrease. Simultaneously, because of decrease in the available wealth next period, consumption next period will decrease in expectation. Hence, in any case, $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$ will exhibit a positive correlation. However, the same mechanism will lead to a negative correlation between Γ_t and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$). The next proposition formally proves the intuition given above.

PROPOSITION 1.3.3. *Fix* S_{t-1}. *Then* $cov(C_t - \Gamma_t, C_{t+1} - \Gamma_{t+1} | S_{t-1}) \geq cov(C_t, C_{t+1} | S_{t-1})$.

Proposition 1.3.3 suggests a method of separating the endogenous part of consumption $C_t - \Gamma_t$ from the expenditure shock, Γ_t. The categories of expenditure that exhibit significant serial correlation are likely to be smoothed over the life cycle. In contrast, the nonpersistent part of the expenditure would be motivated by short-run fluctuations in taste or urgent spending needs.

1.4. Calibration

In this section, we calibrate key elements of the model using data. First, we check model properties after calibrating the basic parameters, income process, and expenditure shocks. Second, we check the model's performance after calibrating the distribution of perception to match the dispersion of liquid wealth.

1.4.1. Income Process. Following Carroll (1997), the income process follows

$$Y_{i,a} = P_{i,a}\Xi_a\varepsilon_{i,a} \tag{1.4}$$

where

$$P_{i,a} = P_{i,a-1}\xi_{i,a}, \text{ and}$$

$$\Xi_a = \beta_0 + \beta_1 a + \beta_2 a^2 + \beta_3 a^3 + \beta_4 a^4.$$

The income of individual i at age a depends on the permanent level of income $P_{i,a}$, age-specific term Ξ_a, which shapes the overall income profile, and the transitory shock $\varepsilon_{i,a}$. Shocks in permanent and transitory income, $\ln \varepsilon_{i,a}$ and $\ln \xi_{i,a}$, follow normal distribution with standard deviations 0.013 and 0.043 respectively, following Carroll et al. (2017)[12]. The age-specific term Ξ_a follows fourth-order polynomials of age, which is also standard. This paper estimates parameters determining Ξ_a by using the Consumer Expenditure Survey from 1997 to 2013. The choice of modeling permanent income as a geometric random walk enables the normalization of every variable by the permanent level of income, which reduces the computational complexity.

The model of this paper lacks a few realistic features. First, I do not have unemployment in the model. Since occupational characteristics do not explain the dispersion of liquid wealth, I build a model with a single type of household that always earns labor income. This abstraction ensures that the dispersion of liquid wealth is not a result of the additional dispersion of income from unemployment. Second, there are no income or social security taxes. Introducing taxes can be useful to conduct realistic policy experiments, but this model has single representative households with homogeneous income in expectation. I assume that every household has similar trajectory of expected income over the life cycle, which controls the contribution of income dispersion towards the dispersion of liquid wealth.

[12]This paper has monthly time frequency, so the standard deviations given in Carroll et al. (2017), which use quarterly frequency, are divided by three.

1.4.2. Calibration of the Expenditure Shock. Separating the products in the household consumption basket into persistent and nonpersistent parts, we can find a stark difference between the two. For each product j, I measure the degree of persistence of expenditure using the following simple panel regression:

$$\Delta C_{i,j,t} = \rho_j \Delta C_{i,j,t-1} + \varepsilon_{i,j,t} + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,j,t}, \text{ where} \tag{1.5}$$

$$\Delta C_{i,j,t-1} = \gamma_j C_{i,j,t-2} + \text{Year FE}_{t-1} + \text{Month FE}_{t-1} + \xi_{i,j,t-1}.$$

To deal with endogeneity caused by adding lagged dependent variable, $C_{i,j,t-1}$, I employ the Anderson and Hsiao (1981) estimator by using a lag of level variable, $C_{i,j,t-2}$, as an instrument. I use a monthly panel since quarterly data only allows four data points per household, which is not long enough to estimate (1.5). Moreover, monthly frequency matches well with the time-frequency used in the model. Year FE_t and Month FE_t stand for year and month fixed effects, respectively. In this simple regression, ρ_j captures the persistence of $C_{i,j,t}$. The first difference cancels out individual fixed effects. I use the Consumer Expenditure Survey from 1997 to 2013 using 44 product classifications by Kueng (2015) to estimate the persistence based on the model above.[13][14]

The following households are excluded from the analysis, consistent with the sample selection criteria used in the literature. First, I exclude households living in college housing. Second, households with top-coded income, missing age and family size, and not having full 12-month survey responses, are excluded. Third, I select households between the ages of 25 and 81 years to be consistent with the model. Fourth, households must not have any variation in the family size to isolate the effects from the change in family composition. Fifth, I exclude the households with zero food expenditure from the sample.

[13]We can use a more detailed product classification, but if the category of products is too narrow, there can be redundancy when estimating the persistence. For example, I use utility payments as a category of consumption instead of treating gas and electricity separately, which might share similar characteristics. Using a broad category of products can also help us to reduce the size of the analysis.

[14]I use the sample only from 1997 to 2013, where the classification can be applied.

Table B.2 shows the estimated ρ_j in (1.5) for 44 products. Twenty-one products exhibit positive and significant persistence. In particular, alcohol, tobacco, food, and personal care expenditures are highly persistent. The remaining 23 products do not exhibit strong persistence, such as education service, home management, insurance, and vehicle-related payments. First, denote the set of products that are persistent as S_p and nonpersistent as S_n. Then, we construct variables $C^p_{i,t} = \sum_{j \in S_p} C_{i,j,t}$ and $C^n_{i,t} = \sum_{j \in S_n} C_{i,j,t}$, which aggregates expenditures over persistent and nonpersistent groups. Moreover, we define the total expenditure of household i at time t as $C_{i,t}$.

The persistence of aggregate variables Next, we check the distinctive properties of $C^p_{i,t}$ and $C^n_{i,t}$. Following Proposition 1.3.3, I constructed $C^p_{i,t}$ to include products that have a high serial correlation. However, this does not guarantee that $C^p_{i,t}$ as a whole would also have higher persistence than the total expenditure $C_{i,t}$. Using the regression in (1.5), I measure the persistence of aggregate variables $C^p_{i,t}$ and $C^n_{i,t}$ and their estimated persistence are 0.041 and 0.008, respectively. We can conclude that the high persistence of individual products are well transferred to the aggregate variable.

Another manner of observing the persistence is to examine the relationship between the total expenditure, and its components $C^p_{i,t}$ and $C^n_{i,t}$ by the following regression:

$$\Delta C_{i,t} = \alpha \Delta C^p_{i,t-1} + \beta \Delta C^n_{i,t-1} + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,t}, \text{ where} \tag{1.6}$$

$$\begin{pmatrix} \Delta C^p_{i,t-1} \\ \Delta C^n_{i,t-1} \end{pmatrix} = (\eta^p, \eta^n) \begin{pmatrix} C^p_{i,t-2} \\ C^n_{i,t-2} \end{pmatrix} + \text{Year FE}_t + \text{Month FE}_t + \xi_{i,t}. \tag{1.7}$$

The estimated α and β are 0.033 and 0.006, respectively. Hence, an increase in the nonpersistent part leads to almost no change in the total expenditure next period.[15] When changing the dependent variable in (1.6) to $C^p_{i,t-1}$, the estimated α and β becomes 0.046 and -0.001, respectively. Although $C^n_{i,t}$ represents items that are individually nonpersistent, they are

[15]Similar results are found when changing the lag of instruments variables in (1.7) to 3 and 4 instead of 2.

also largely unrelated to the total expenditure $C_{i,t}$ next period. In this sense, I model $C^n_{i,t}$ as an exogenous part of the expenditure, which does not exhibit any autocorrelation and has a weak relationship with the total expenditure over time.

Variances of Persistent and Nonpersistent Expenditures If a product is a part of the consumption and savings problem in a long horizon, the consumption growth rate will be steady. In particular, in a short period, there will be small variations to the expenditure if income is the major source of the random shocks. Note that $C^n_{i,t}$ has nonpersistent components of the expenditure, but this does not imply that those components will also have high variance.[16] I show that $C^n_{i,t}$ actually has a larger variance than $C^p_{i,t}$, even though its size is much smaller, which indicates that $C^n_{i,t}$ as a whole violates consumption smoothing.

For each household i, over 12 months, I calculate the standard deviation of $C^n_{i,t}$ and $C^p_{i,t}$. In other words, I calculate $\text{sd}(C^m_{i,t}) = \sqrt{\sum_{t=1}^{12}(C^m_{i,t} - \bar{C}^m_i)/12}$ where $\bar{C}^m_i = \sum_{t=1}^{12} C^m_{i,t}/12$ for $m \in \{n, p\}$. The averages of $\text{sd}(C^n_{i,t})$ and $\text{sd}(C^p_{i,t})$ over all households are 1,271 and 570, respectively. Although the nonpersistent part constitutes approximately a quarter of the total expenditure, the standard deviation is twice as large as the persistent part. Moreover, the averages of $\text{sd}(C^n_{i,t})/\bar{C}^n_i$ and $\text{sd}(C^p_{i,t})/\bar{C}^p_i$ for overall households, which adjusts magnitudes of expenditures, are 1.56 and 0.29, respectively. When adjusted for the size, it is evident that the standard deviation of the nonpersistent part is approximately five times larger than the persistent part.

Distributions of Persistent and Nonpersistent Expenditures There is a stark difference between distributions of persistent and nonpersistent expenditures. Figure 1.3 depicts the distribution of the nonpersistent and persistent expenditures. First, the distribution of nonpersistent expenditure $C^n_{i,t}$ has a mode near zero, and the frequency decreases for higher expenditure levels. Moreover, it is heavily skewed to the right, and resembles the exponential

[16] Suppose that a random variable x_t follows $x_t = \rho x_{t-1} + e_t$. Other things being equal, while keeping the variance of e_t constant, an increase in $\rho \geq 0$ will make the unconditional variance of x_t increase as well. This property implies that the method of creating $C^n_{i,t}$ by choosing persistent component is not necessarily related to its low variance.

distribution. On the other hand, the persistent expenditure $C^p_{i,t}$ is centered around a positive

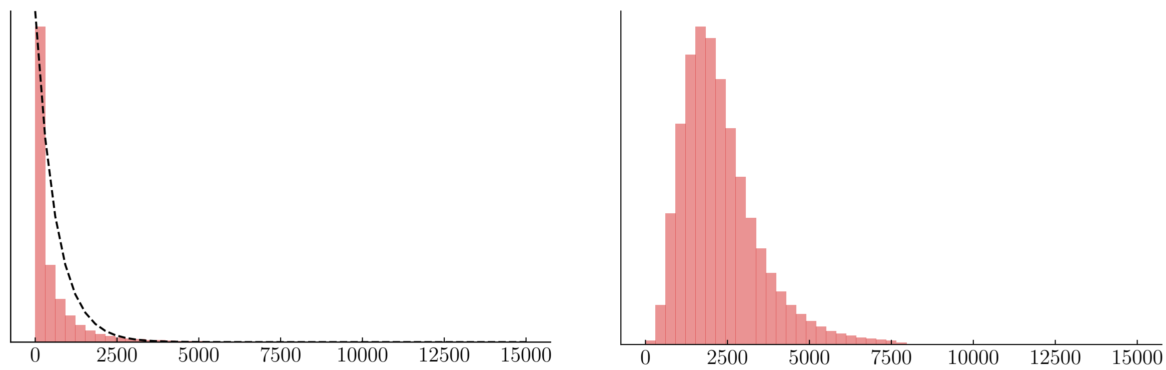

FIGURE 1.3. Distribution of the nonpersistent (left) and the persistent (right) expenditures

Note: The black dotted line on the left-hand side figure fits the probability density function of exponential distribution.

value and has a shape similar to the log-normal distribution. Thus, almost always, the persistent part is consumed strictly positive values.

Two different distributions represent the fundamental difference between persistent and nonpersistent expenditures. The persistent expenditure is a crucial part of the consumption and savings problem, and the consumer must spend a certain quantity. However, households do not spend any of the nonpersistent components in many cases, which cannot be rationalized with a utility function that satisfies the Inada condition. To make the nonpersistent component an endogenous part of the consumption and savings problem, we need a large preference shock and a utility function that violates the Inada condition.

1.4.3. Feeding the Expenditure Shocks to the Model. I calibrate the expenditure shocks in following steps. First, I estimate the individual persistent income P_i using the model in (1.4).[17] Then, I make a normalized expenditure shock, by $c^n_{i,t} = C^n_{i,t}/P_i$. Based

[17]The Consumer Expenditure Survey asks for the income twice at most. For each household i, I use the mean income Y_i over two different periods, which can help to isolate the transitory component. Then, I measure the persistent component of household i as $P_i = Y_i/\Xi_{i,a}$ based on the age a.

on the observation from Figure 1.3, I assume that the expenditure shocks follow an exponential distribution where the mean represents all information about the distribution. Of course, other distributions can represent the expenditure shocks better, but I avoid the use of distributions that may require complicated combinations of parameters.

Households may have different levels of expenditure shocks over the life cycle. To account for this, I estimate the trend of $\mu_a = \sum_{i=1}^{I(a)} c_{i,a}^n / I(a)$, where $I(a)$ is the number of households at age a. Specifically, I use the fourth-order polynomial of ages to filter the trend of μ_a. However, as evident from Figure 1.4, it is still relatively flat compared to income. In addition, the size of the expenditure shock is small, approximately a fifth of the income, which implies that arbitrarily large expenditure shocks do not drive the results of this paper.

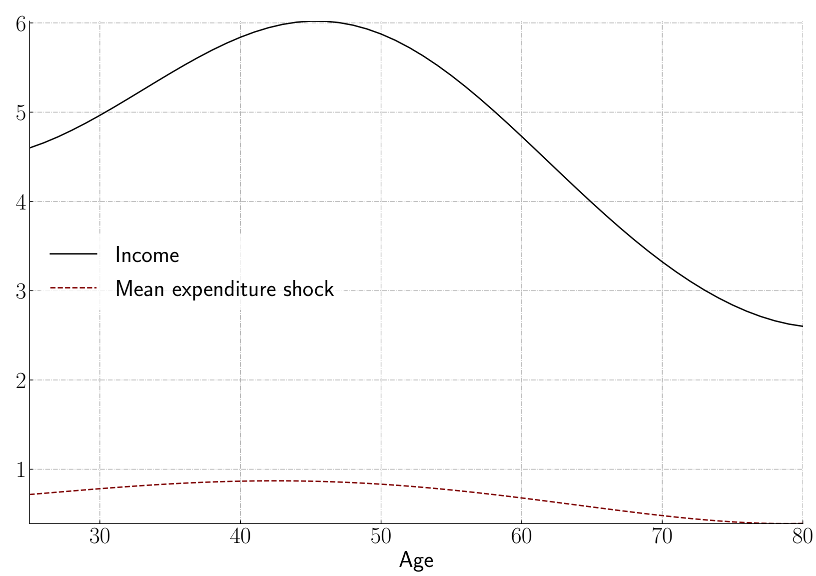

FIGURE 1.4. Trend of the income and calibrated expenditure shocks
Note: The unit of the y-axis is $1,000.

Endowing the Degree of Perception The household i is endowed with λ_i, representing the degree of perception of the expenditure shock. While the true distribution that draws the expenditure shock at age a is $\exp(\mu_a)$, the household i perceives that the expenditure shock will be drawn from $\exp(\lambda_i \mu_a)$. Hence, $\lambda_i = 1$ implies that the household i is sophisticated. If $\lambda < 1$ (or $\lambda > 1$), then the household is optimistic (or pessimistic) relative to the sophisticated households.

Why Households May Persistently Underestimate Expenditure Shocks Suppose that a household fits the nonpersistent part distribution based on the exponential distribution with past observations. Then, the efficient maximum likelihood estimator (MLE) for the parameter of exponential distribution would be the mean of the past observations. In a distribution, like an exponential distribution[18], the median is always less than the mean, which implies that the median household will observe a value that is less than the true mean. Thus, the MLE estimate based on the limited observations will be underestimating the true parameter.[19]

Discount Factor and Interest Rates The discount factor δ is 0.96 in annual terms, which is standard. The returns of liquid and illiquid assets are held constant. Following Kaplan and Violante (2014), the real returns on liquid and illiquid assets are -1.5 percent and 2.3 percent, respectively, in annual terms.

The Choice of IES I use the constant relative risk aversion (CRRA) utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$. One of the most important parameters when modeling the life cycle consumption and savings problem is the IES, which is $1/\sigma$ in this paper. Empirically, when examining micro evidence, IES is suggested to be 1/3 (Havranek and Sokolova, 2020), and I follow this recommendation.

Other Features of the Model Households enter the economy at age 25 and retire at age 65. All households die at age 80, with no bequest. Adding bequest can help explain the increasing trend of wealth even after retirement, but I attempt to keep the model as simple as

[18] The result generalizes to the case where $f(c)$ is a decreasing probability density function.

[19] With exponential distribution, the result can be generalized in the following manner. Fix n, and $\bar{x} = (x_1 + \cdots + x_n)/n$ where $x_1, \cdots, x_n$ are all drawn from the identical exponential distribution $\mathrm{Exp}(\lambda)$ with parameter λ. Then, the median of $\bar{x}$ is less than λ. In the limit, $n \to \infty$, $\bar{x}$ tends to λ by the central limit theorem.

possible to rule out other factors that can affect dispersion of liquid wealth. The model has a monthly frequency, which results in 660 periods, with 480 periods before retirement. To make the model solvable, I transform the utility function as $u(c) = (\max\{c, \underline{u}\})^{1-\sigma}/(1-\sigma)$, where $\underline{u} = 10^{-5}$ (1 cent).

1.4.4. Simulation of the Model. *Calculation of the MPC* This paper calculates the MPC by giving \$800 that are not subsequently taxed to all households. For household i at time t, I first compute the original level of consumption $c_t^i(w_t^i)$. Fixing the level of disposable income and illiquid assets, to create the same environment, I compute the counterfactual level of consumption using the rebate r, which is $\hat{c}_t^i(w_t^i + r)$. Then, the MPC of household i at time t can be calculated as $MPC_t^i = (\hat{c}_t^i(w_t^i + r) - c_t^i(w_t^i))/r$. This method of calculating MPC is similar to that of Carroll et al. (2017), Aguiar et al. (2020), and Fuster et al. (2020) who measure the pure increase in consumption by winning a lottery, which is effectively calculating the slope of the consumption function. The size of the stimulus is larger than that in some of previous literature such as Kaplan and Violante (2014), Aguiar et al. (2020), and Fuster et al. (2020), who give \$500 to households. The choice of giving \$800 is to give households a quantity approximately between \$500 (2001 tax rebate) and \$1,200 (the individual payments in CARES Act in one of the stimulus and relief packages for COVID-19). Choosing a lower quantity of rebate will induce higher consumption responses due to the concavity of the consumption function (Carroll and Kimball, 1996), and the existence of the liquidity constraint.

An alternative method to measure a consumption response that resembles the actual fiscal stimulus can be embedding the belief that the stimulus has to be funded by taxes in the future. After the government spreads the rebate, it can collect taxes k periods later, and households that are not liquidity constrained for $k-1$ periods will not react to the stimulus by Ricardian equivalence, as shown in Barro (1974). The policy experiment of Kaplan and Violante (2014) reproducing the 2001 tax rebate is an example of measuring consumption response in this manner where taxes were collected 10 years after the rebate was given. This paper abstracts from realistic components of tax rebates and does not specifically aim to replicate a fiscal stimulus. In this paper, the measurement of the MPC focuses on purely representing the slope of the consumption function.

Model Properties To investigate the consumption and savings over the life cycle under different perceptions of expenditure shocks, I simulate the model economy with 256,000 households with λ_i that is evenly distributed over $[0, 2]$. Every household begins with the same initial level of wealth, which is the median liquid and illiquid wealth of household heads aged 24–26 of the Survey of Consumer Finances.

Figure 1.5 tracks the median wealth, consumption, and MPC within every 20 groups of perceptions $[\lambda^j, \lambda^j + 0.1)$ where $\lambda^0 = 0$ and $\lambda^{20} = 1.9$. Then, I track the median wealth of households within each quintile of perceptions. As hinted by Propositions 1.3.1 and 1.3.2, Figure 1.5 indicates that a household with a higher level of λ_i accumulates a larger quantity of liquid and illiquid assets throughout their life. In particular at the age of 65, the wealth of the top group with $\lambda \in [1.9, 2.0)$ is approximately six times larger than the lowest group with $\lambda \in [0, 0.1)$.

Unlike liquid and illiquid wealth, different degrees of perception do not make remarkable differences in consumption except in the initial and terminal periods. Households with correct perception exhibit a flat consumption profile until they reach retirement, which implies successful consumption smoothing under the low IES of 1/3. Since they are worried about the possibility of high expenditure shocks, they retain liquid assets for precautionary motives, which makes the consumption rise near the terminal period. The precautionary saving motive is higher among relatively more pessimistic households. Hence, households with an extreme level of pessimism near $\lambda_i \simeq 2$ perceive that their initial liquid wealth is not enough and begin accumulating liquid assets right away, thereby making the initial consumption lower than that of the other groups. Optimistic households underestimate the size of the expenditure shock, which depletes liquid assets and high consumption at the beginning of life.

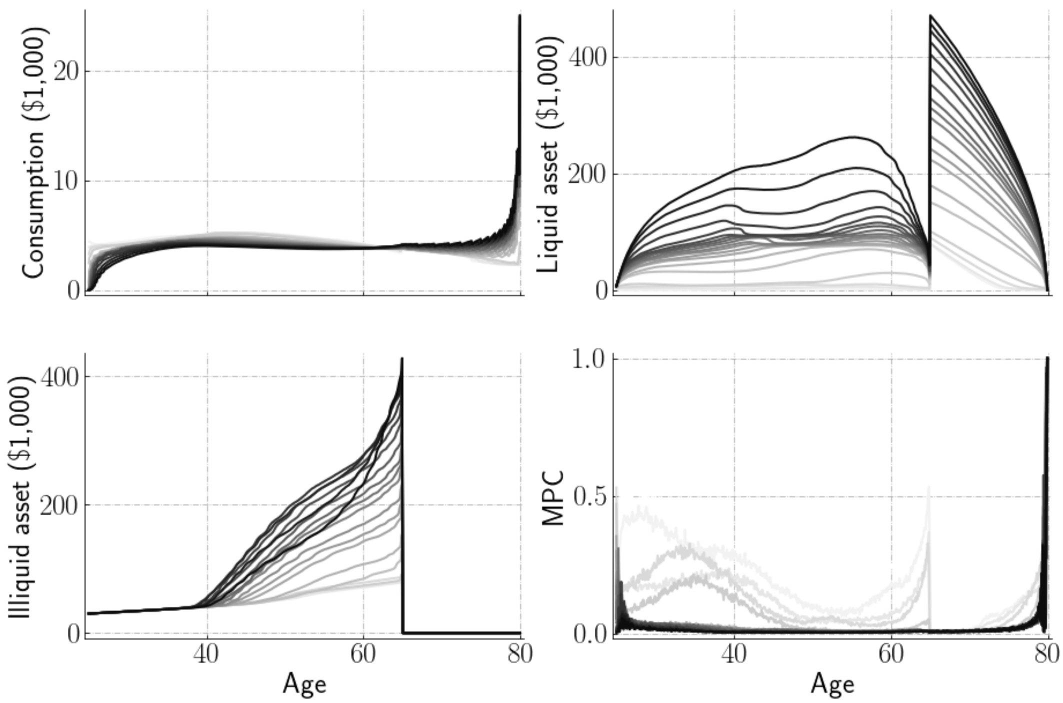

FIGURE 1.5. Trajectory of key variables by different levels of perception
Note: Twenty groups of households with perceptions $[\lambda^j, \lambda^j + 0.1)$, where $\lambda^1 = 0$ and $\lambda^{20} = 1.9$ are plotted and the darker line indicates more pessimistic households where λ^j is larger.

There are also large differences in the MPC over the life cycle by different levels of perceptions. First, the optimistic households exhibit higher MPC than the pessimistic households. Since optimistic households occasionally meet binding borrowing constraints, their MPC is also larger than that of pessimistic households. The average monthly MPC of the most optimistic households, $\lambda_i \in [0, 0.1)$, is 22.4, while the middle group, $\lambda_i \in [0.9, 1.0)$, exhibits 2.2 percent and the most pessimistic group, $\lambda_i \in [1.9, 2.0)$ exhibits 1.4 percent. Since the low liquidity drives the high MPCs, there will be negligible differences among households that accumulate sufficient wealth so that their borrowing constraint will be rarely binding.

By this reason, households with perceptions $\lambda_i \in [0, 0.5)$ show high MPCs, while the others do not.

The trajectory of the MPC out of a transitory income shock follows the implication from the buffer-stock theory of savings (Carroll, 1992, 1997). The MPC is generally high at young ages since the current income is lower than the future income levels, which makes the borrowing constraint binding. As households accumulate assets at the middle age, MPC falls and becomes almost zero when the income is at its peak. However, consumption responses of optimistic households versus rational expectations or pessimistic households differ starkly when they approach to retirement. The quantity of liquidity they have accumulated thus far sets the consumption limit for all households at retirement. However, optimistic households that underestimate the need for precautionary savings may face larger expenditure shock than they expected and become hand-to-mouth. On the other hand, households that have accumulated sufficient assets do not exhibit a large consumption response from a transitory income shock. The mismatch between expected and actual liquidity available at retirement serves as a "second terminal-period effect" and makes MPC rise as households retire.

Calibration Strategy and Empirical Target The key moment to match in the calibration is the distribution of wealth. This paper adopts the strategy of Castañeda et al. (2003) and Carroll et al. (2017) by targeting the share of liquid wealth held by 20th, 40th, 60th, 80th percentiles along with median liquid wealth among working-age households. There can be various reasons to hold illiquid assets, such as bequest motive and housing investment, which are not modeled in this paper. Moreover, the dispersion of liquid wealth in the model can be affected by the introduction of large illiquid wealth at retirement. By these reasons, I only focus on the dispersion of liquid wealth before retirement. By arbitrarily introducing numerous optimistic households, the model has a chance to highlight households with very low liquidity and perfectly explains the share of wealth held at different percentiles. To prevent this issue, I let the model match the median liquid wealth over income, similar to Carroll et al. (2017).

There have been various approaches to model the heterogeneity of consumers with different parameters. A commonly used approach is assuming a discrete number of households with different utility or discounting parameters such as Krusell and Smith (1998). There is also an approach by Carroll et al. (2017) that assume uniform distribution and compute the lower and upper bounds of the parameter. In reality, the parameter of interest is likely to be drawn from a continuous distribution, and we cannot have *a priori* guess how the distribution looks like. To employ a continuous and versatile distribution, I use a beta distribution that can have various shapes, nesting uniform distribution as a special case. Hence, the perception of households λ_i is assumed to be drawn from a beta distribution. I fit three parameters where each has a distinct role. The left-hand side panel of Figure 1.6 illustrates the role of first two parameters, α and β. They control the shape of the beta distribution. The theoretical mean is $\alpha/(\alpha+\beta)$; hence, a larger value of α implies more households with rational expectations in general. The size of both α and β controls the height of the peak. If $\alpha = \beta = 1$, then the distribution becomes a uniform distribution where the probability density function is flat. If α and β are high, the values drawn will be centered around a mean. To control the overall size of the values in the distribution, I also fit a scaling parameter γ representing the maximum value in the domain.

Calibration Result The set of parameters to generate the moments in Table 1.7 is $(\alpha, \beta, \gamma) = (1.53, 6.22, 1.58)$. Calibrated parameters imply that the distribution of perception would look like what is depicted in the right-hand side of Figure 1.6. Based on this result, the average household would realize 27 percent of the true mean of the expenditure shocks. This low level of perception is necessary to make the model generate a low level of liquid wealth among working-age households. The maximum value of the distribution $\gamma = 1.58$ implies

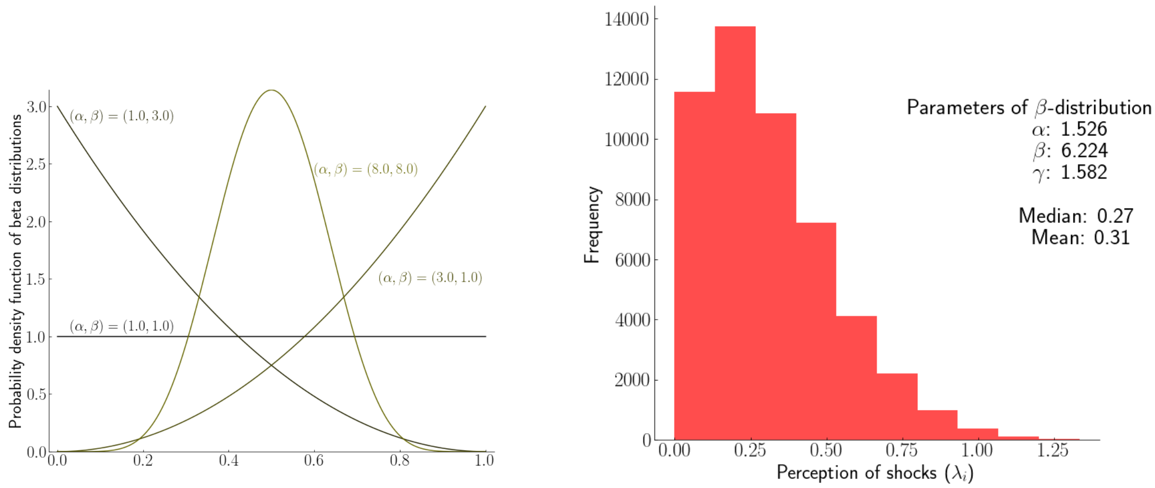

FIGURE 1.6. Examples of beta distribution (left) and the calibrated distribution (right)

the existence of a small group of households that are more pessimistic than households with rational expectations.

Comparison of key moments between the model and data Table 1.7 compares the moments of actual data versus simulated data from the calibrated model. The model-generated moments are fairly close to the data. In particular, the median liquid wealth over income is well aligned with the data. Controlling the median liquid wealth ensures that the calibration result is not driven by arbitrarily generating many households with very low liquidity. However, there is some gap between the share of wealth at the 60th and 80th percentile.

	Share of wealth held at percentiles				Median liquid wealth
	20th	40th	60th	80th	over income
Data	<0.01	0.01	0.05	0.2	0.89
Model	<0.01	0.02	0.08	0.23	0.89

TABLE 1.7. Calibration result

Unlike the simulated model, households with an exceptionally high level of wealth exist in data, which is difficult to generate using the life-cycle model. As a result, Figure 1.7 shows that the model-generated Lorenz curve is slightly flatter than the data.

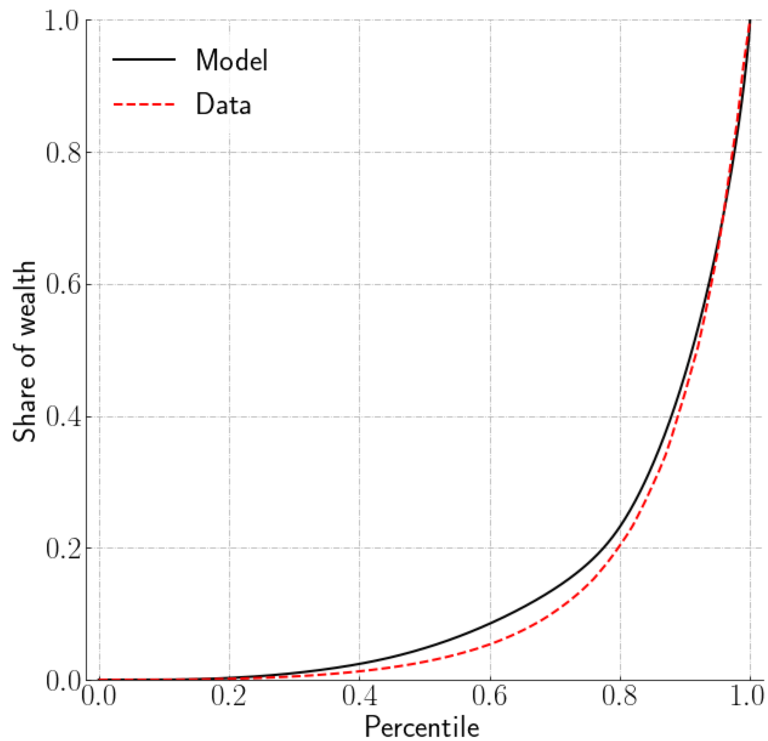

FIGURE 1.7. Lorenz curve: data versus model

Table 1.8 compares the trend of liquid wealth over income at different percentiles of data and the simulated model. The numbers in the table are not the targets of the calibration, but the simulated moments line up well with the actual data. In particular, it can explain that over 40 percent of the households in data do not have liquidity near their monthly income.

Controlling for the Dispersion of Income This model incorporates transitory and permanent income shocks for reality but does not depend on the income shocks to generate a

		Liquid wealth over income			
		20th	40th	60th	80th
25–34	Data	0.12	0.44	1.06	2.67
	Model	0.02	0.35	1.14	2.6
35–44	Data	0.15	0.51	1.18	3.14
	Model	0.14	0.36	1.17	2.6
45–54	Data	0.20	0.61	1.48	4.29
	Model	0.32	0.67	1.38	4.35
55–64	Data	0.28	0.86	2.31	7.23
	Model	0.16	0.57	1.48	8.03
Overall	Data	0.21	0.68	1.74	5.40
	Model	0.14	0.48	1.26	4.22

TABLE 1.8. Liquid wealth over income at 20th, 40th, 60th, and 80th percentiles

dispersion of wealth. Table 1.9 shows the dispersion of wealth under various ways of simulating the economy. Households' perceptions are all drawn from calibration according to Figure 1.6 to ensure that each scenario in Table 1.9 only differs by how the random shocks are drawn. Also, households perceive that all income shocks and expenditure shocks exist. By controlling the shocks drawn in the simulation, we can understand the contribution of each type of shock to the dispersion of liquid wealth.

In the first row of Table 1.9, we have the baseline result, which is identical to the one presented in Table 1.7. The second row is simulated with no transitory income shocks. It is not clear if the silence of transitory income shocks contributes to more or less dispersion of liquid wealth. The third row is simulated without permanent income shocks. Without the permanent income shock, households in each age group will have the same income level in expectation, which can alleviate the dispersion of wealth. The absence of permanent income shocks increases the share of wealth held at percentiles but by a small amount. The fourth row eliminates the expenditure shocks and assumes that all households face the mean level of the shocks. We can see that variations in the expenditure shocks do not lead to a significant difference in the dispersion of wealth compared to the baseline result. The last two rows shut down income and expenditure shocks simultaneously. When all the shocks are not present, where the only source of the heterogeneity is the level of perception according to Figure 1.6, the dispersion of wealth is not very different from the baseline result.

	Share of liquid wealth held at percentiles			
	20th	40th	60th	80th
Baseline	<0.01	0.02	0.08	0.23
No transitory income shocks	<0.01	0.02	0.08	0.22
No permanent income shocks	<0.01	0.03	0.10	0.25
No expenditure shocks	<0.01	0.02	0.08	0.22
No transitory and permanent income shocks	<0.01	0.02	0.09	0.24
No income and expenditure shocks	<0.01	0.02	0.08	0.23

Table 1.9. Dispersion of wealth without idiosyncratic shocks

MPC in the calibrated model The model exhibits high overall MPC. The average monthly MPC among all households is 12.4 percent. In quarterly and annual terms, it can be translated to 32.8 and 79.6 percent following the conversion formula used by Carroll et al. (2017).[20] Figure 1.8 shows the relationship between the MPC and size of the stimulus. As the size of stimulus becomes larger two forces contributes to decrease in the MPC. The concavity of the consumption function (Carroll and Kimball, 1996) by having a CRRA utility function and income shocks makes MPC to fall. Also, the stimulus itself can make liquidity constraint less binding.

It is evident from the left-hand side panel of Figure 1.9 where extensive margin plays a big role since most of the households do not show a positive consumption response to the rebate. However, households with large expenditure shocks would move to the region with a positive consumption response. This will be more effective among households with low levels of liquid wealth. The right-hand side panel suggests extensive margin in the individual consumption response. When a household faces a large expenditure shock, then they may exhibit a high

[20]The exact values for quarterly and annual MPC need to be examined by actually tracking the change in consumption over the quarter and year. I convert the monthly MPC of x to the MPC over k months, by $1-(1-x)^k$. I drop the households exhibiting MPCs less than -0.001 which constitutes only 0.8 percent of the total sample. When not excluding any household, the average monthly MPC is 12.7 percent.

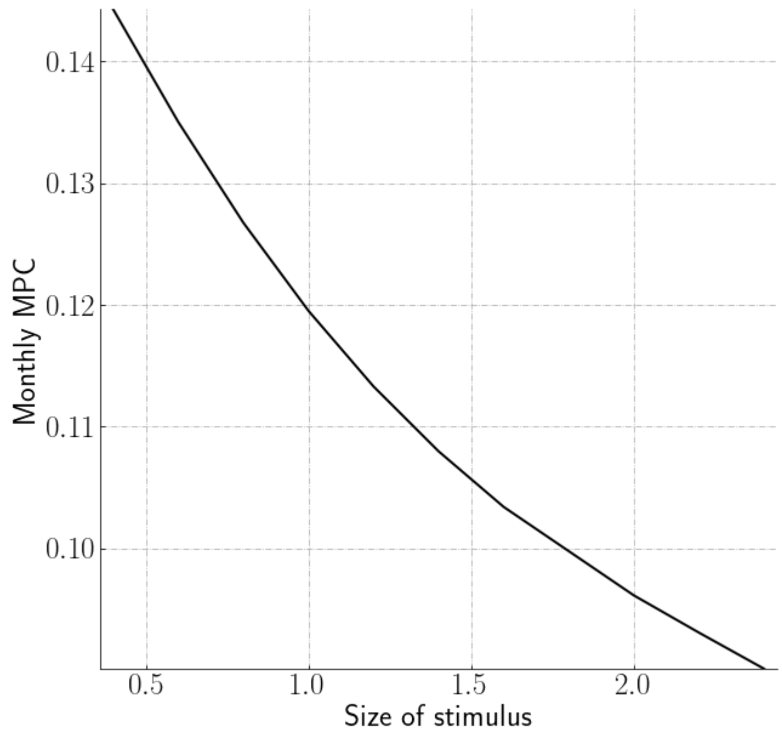

Figure 1.8. MPC by size of stimulus

MPC temporarily. However, in most cases where they face negligible expenditure shocks, they exhibit a low MPC to cope with the desire to smooth consumption.

The right-hand side panel of Figure 1.9 shows the individual response of a sample household that perceives 50 percent of the actual shock. Not only is there an extensive margin of consumption responses in the aggregate, but depending on the size of the expenditure shocks, each individual also exhibits infrequent but large consumption responses.

MPCs in other benchmark models Table 1.10 compares the MPCs across different modeling strategies and various values of IES. Following the calibration strategy thus far, I assume that $\delta = 0.96$ when IES is $1/3$. For all other values of IES, I convert the annual discount factor that makes the consumption choice approximately equal over different choices

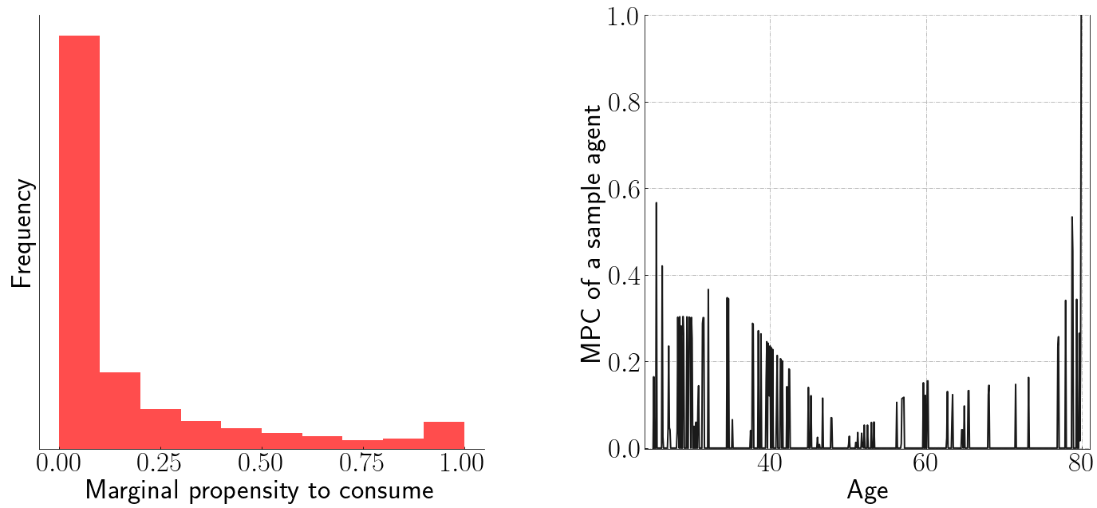

FIGURE 1.9. Distribution of MPC (left) and sample individual consumption response (right)

	MPC under different IES					
	1/3	0.5	1/1.5	1	1.5	2
No expenditure shocks with rational expectations						
One-asset	2.1	2.5	2.6	2.6	2.4	2.1
Two-asset	3.1	5.6	7.8	10.7	15.0	19.8
With expenditure shocks						
Two-asset						
Rational expectations	2.7	2.6	3.1	7.8	8.9	9.9
Fully optimistic	25.2	30.5	33.5	37.2	40.7	43.2
One-asset						
Rational expectations	2.0	2.1	2.1	2.3	2.2	1.9
Fully optimistic	17.7	20.7	21.7	22.0	21.1	19.1

TABLE 1.10. Comparison of MPCs across different models

Note: IES of 1/3 is the baseline case with annual discount factor of 0.96. The other cases with IES of α uses discount factor of $(0.96 \times R)^{\alpha/3}/R$ in annual terms. $R = 1.017$, which is the annual gross interest rate of the one-asset model in Kaplan and Violante (2014).

of IES.[21] The reason for applying this conversion is to compare MPCs across different values

[21]For two different pairs of (δ_1, σ_1) and (δ_2, σ_2) and common interest rate R, the consumption choice will be the same if $(\delta_1 R)^{1/\sigma_1} = (\delta_2 R)^{1/\sigma_2}$. Hence, to make the comparison between (δ_1, σ_1) and (δ_2, σ_2) fair, we can convert the δ_2 to satisfy $(\delta_1 R)^{\sigma_2/\sigma_1}/R$. In a stochastic model, this is not exact, and one needs to simulate the model by matching a particular statistic. However, the approximation appears to be accurate since MPCs of the one-asset model in Table 1.10 are stable across different values of IES.

of IES while maintaining the models to generate similar levels of wealth as in Kaplan and Violante (2014). An alternative approach would be recalibrating the discount factor each time for different values of IES, and different types of models, as in Kaplan and Violante (2014).[22]

A clear pattern is that, generally, low IES leads to lower MPC. Under the baseline case with IES of 1/3, the MPC without expenditure shock under the one-asset framework and rational expectations is only 2.1 percent. Under the two-asset model with rational expectations, the MPC becomes 3.1 percent, which is larger than the one-asset case, but the difference is negligible. The separation of liquid and illiquid assets does not dramatically affect the MPC. This result arises because the desire to smooth consumption is strong when using the small IES. When the IES is small, households prefer to maintain a flat consumption profile, as shown in Figure 1.5. Keeping the flat consumption profile is only possible if households accumulate enough liquid and illiquid assets. In this case, the borrowing constraints will rarely be binding, even with the low-interest rate on liquid assets. Moreover, low IES makes households insensitive to interest rate differences in liquid and illiquid assets.

However, the use of high IES can lead to higher MPC. Even among households with rational expectations, the MPC under the two-asset framework is approximately 20 percent with IES of two. Although there is a difference in the modeling strategy where Kaplan and Violante (2014) use symmetric adjustment cost, and I impose an asymmetric adjustment cost, the amplification channel of consumption responses by separating two different assets are present in both approaches.

Behavioral bias can let the model overcome having smaller consumption responses induced by small IES. Under the one-asset framework, fully optimistic households exhibit an MPC of 17.7 percent, significantly higher than the rational expectations benchmark. Using the two-asset model, there is an additional increase in the MPC, where MPC now becomes

[22] I use the same discount factor across different types of models if the IES is the same. This is to compare the role of different assumptions, such as introducing two assets, introducing expenditure shocks, and changing expectations.

25.2 percent. Even under the low IES, behavioral frictions lead to lower liquid assets among optimistic households. Separating the savings for retirement exacerbates the low accumulation of liquid assets, which serves as the only tool for precautionary saving.

We see a clear pattern between models with and without expenditure shocks when focusing only on households with rational expectations. In all cases, the models with expenditure shocks produce smaller MPCs than those without expenditure shocks for both one- and two-asset cases. This pattern can be attributed to the fact that households may require additional savings when adding an extra shock to the model for precautionary purposes. Analysis thus far hints that the presence of expenditure shock alone is not a factor that drives high MPCs. The behavioral bias, where households have a limited perception of expenditure shock, leads to high MPCs.

1.5. Policy Implications

This section investigates the role of perception $\widetilde{\mathbf{\Gamma}}$ in consumer welfare and provides policy recommendations.

1.5.1. Optimism and Welfare. For simplicity, I consider the one-asset model which features life after retirement. I define the *welfare function* $W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}})$ in a paternalistic view, which captures expected lifetime utility based on perception $\widetilde{\mathbf{\Gamma}}$ as

$$W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}) = u(C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}) - \Gamma_t) + \delta \mathbb{E}\left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}})\right],$$

where

$$C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}) = \arg\max_{c_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}; \widetilde{\Gamma}_{t+1})\right], \text{ and}$$

$$C_t \leq X_t = Y_t + R_t^s(X_{t-1} - C_{t-1}).$$

Both actual and perceived distributions of the expenditure shock are important to determine the welfare. When calculating the current utility $u(C_t - \Gamma_t)$, Γ_t is drawn from the actual distribution of the expenditure shock. On the other hand, the household makes a decision based on $\widetilde{\mathbf{\Gamma}}$ to gauge continuation value $\widetilde{W}^{\widetilde{\mathbf{\Gamma}}}_{t+1}$. An alternative and stronger means of defining the welfare would be using *ex-post* realizations of the expenditure shocks. However, there can be a possibility that the random draws of Γ_t can be close to the values drawn from $\widetilde{\mathbf{\Gamma}}_t$, and there is no guarantee that the households with correct perception will be better off. Defining the welfare in *ex-ante* perspective provides a chance to establish a monotonic relationship between the welfare function and the degree of perception, as indicated in the following proposition:

PROPOSITION 1.5.1. *Fix any $t \geq T_r$. Consider households 1 and 2 that are both optimistic such as $\mathbf{\Gamma} >_1 \widetilde{\mathbf{\Gamma}}^1 >_1 \widetilde{\mathbf{\Gamma}}^2$. Then it follows that $W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t; \Gamma_t \widetilde{\mathbf{\Gamma}}^2)$.*

Alternatively, $\mathbf{\Gamma} <_1 \widetilde{\mathbf{\Gamma}}^1 <_1 \widetilde{\mathbf{\Gamma}}^2$, implies that $W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t; \Gamma_t \widetilde{\mathbf{\Gamma}}^2)$.

Proposition 1.5.1 demonstrates that if we have two different households comparable with their relative optimism and pessimism, the household that is closest to the sophisticated case, $\mathbf{\Gamma}$, will have higher welfare.

The mechanism leading to this result is simple. An optimistic household will suffer welfare loss by the overconsumption now; when a large expenditure shock hits, the household will not have enough resources to spend money where the marginal utility will be greater than the current period. Since the loss of welfare accumulates over time, the fact that a household with the perception that is farther away from the reality is worse off holds for any period. A similar explanation applies to pessimistic households.

This result reveals that when choosing priorities of correction, the government should first target the households with abnormally low or high degrees of perception. For these households, the expected increase in welfare will be higher by government intervention. Then, it is important to design a policy based on the degree of perception elicited by the real data. In the following account, I suggest a policy to enhance the overall welfare by correcting the low liquid savings of optimistic households.

An Example of an Ex-Post Welfare Improving Policy What kind of policy can improve the welfare of households with behavioral frictions? This question can be important in reality since if the government can correct the behavior of households with simple policy tools, it can prevent households from facing high borrowing costs when they face large expenditure shocks.

To answer this question, I impose three criteria that a policy must satisfy. First, a policy should not depend on external funds and, preferably, the implementation should be possible without interpersonal transfers. Without this condition, there can be obvious welfare improving policy of spreading money to households. Moreover, if the government can make interpersonal transfers, it would be crucial to assign the utility weights to different households and discuss what type of households we should prioritize. Second, the policy instruments must follow observable variables. Hence, the government cannot design a policy conditional on the households' optimism and pessimism, which would also be impossible in the real world situation. Third, the government should implement dynamically consistent policies, and the households should form correct beliefs about them. Without this condition, the degree of freedom for the government is large, and the government will eventually lose credibility.

I suggest the following policy to penalize low liquidity and giving households sufficient buffer to prevent expenditure from depleting their liquid wealth. The government imposes a proportional tax τ_l up to the liquid wealth of $W_{l,t}$ at time t. For wealth that exceeds $W_{l,t}$ and less than $W_{r,t}$, the government gives a tax rebate proportional to τ_r. Hence, the tax $T_{i,t}(X_{i,t})$ that households i with the level of liquid wealth $X_{i,t}$ pay at time t would be

$$T_{i,t}(X_{i,t}) = \min\{W_{l,t}, X_{i,t}\}\tau_l - \min\{\max\{X_{i,t} - W_{l,t}, 0\}, W_{r,t} - W_{l,t}\}\tau_r, \tag{1.8}$$

which is also the tax revenue for the government. In simple terms, the government penalizes households with wealth less than $W_{l,t}$ which induces optimistic households to accumulate liquid wealth, at least $W_{l,t}$. However, such policy can make households with the rational expectation to undersave and keep the level of wealth between $W_{l,t}$ and $W_{r,t}$, so there is no guarantee for overall welfare improvement. However, if we keep $W_{r,t}$ sufficiently small, households that already accumulate large liquid wealth will not be affected by this policy. Simultaneously, $W_{r,t}$ must be sufficiently large to make low liquidity households prepare for the underestimated expenditure shocks.

If $W_{r,t} = 2W_{l,t}$ and $t_l = t_r$, then all the transfers $(X_{i,t} - W_{l,t})t_r$ can be covered by the taxes $X_{i,t}t_l$ collected earlier, so this policy does not need any interpersonal transfers. Hence, if households correctly form beliefs regarding (1.8), then such a policy will satisfy all three criteria mentioned above. I simulate the economy with the above setting under various t_l and $W_{l,t}$ based on the calibrated distribution of perception in Figure 1.6. I impose that $W_{l,t} = \bar{w}\Xi_t$, where Ξ_t is the age specific trend of income in (1.4). Rather than fixing $W_{l,t}$ and $W_{r,t}$, by making $W_{l,t}$ proportional to the trend of income, I can make the intended minimum savings $W_{l,t}$ vary with income, so optimistic households will be naturally inclined to save a larger quantity of liquid wealth during the middle age when the income peaks.

$T_{l,t}/\Xi_t$	$T_{r,t}/\Xi_t$	τ_l	τ_r	Avg. utility	Avg. revenue
1	2	0.01	0.01	-191.6	0.06
		0.02	0.02	-191.7	0.07
		0.03	0.03	-192.1	0.1
		0.04	0.04	-192.9	0.14
2	4	0.01	0.01	-189.7	0.13
		0.02	0.02	-188.8	0.23
		0.03	0.03	-188.8	0.29
		0.04	0.04	-189.5	0.53
No policy benchmark				-191.8	0

TABLE 1.11. Policy Simulation Based on the Calibrated Model
Note: 'Avg. utility' refers to mean level of utility, which is defined as $\frac{1}{I\times T}\sum_{i\in I}\sum_{t\in T} u(C_{i,t} - \Gamma_{i,t})$. where I is the number of the simulated households and T is the total number of periods. 'Avg. revenue' refers to the average revenue that the government raises, which is defined as $\frac{1}{I\times T}\sum_{i\in I}\sum_{t\in T} T_{i,t}(X_{i,t})$. The unit of revenue is \$1,000. τ_l and τ_r in the table are annualized, actual value τ_l^{actual} used in the simulation is $\tau^{*,\text{actual}} = 1 - (1+\tau^*)^{1/12}$ for $\tau^* = \{\tau_l, \tau_r\}$.

Table 1.11 presents the result of the calibration exercise. I selected four different tax (or transfer) rates, which are 1%, 2%, 3%, and 4% in annual terms. In some cases, the average utility is higher than the benchmark without any policy. In particular, when $T_{l,t}/\Xi_t = 2$ and $T_{r,t}/\Xi_t = 4$, the average utility is higher than the benchmark case. Hence, inducing households to save the liquid wealth of twice their income trend can be desirable. Simultaneously, the government can raise tax revenue by increasing the τ_l and τ_r as well.

The tax scheme in this example is admittedly regressive since it taxes households with a low accumulation of liquid wealth. However, the tax scheme does not target the absolute level of wealth and targets the ratio of wealth to permanent income. By considering the permanent income, I try to mitigate the regressive nature of the tax scheme and consider the innate ability to accumulate wealth. In the real application, we also need a similar proxy to control the ability to save, such as a history of income, which can normalize the size of wealth. Further, the additional funds collected in this tax scheme can help households vulnerable to expenditure shocks and create further welfare enhancement.

1.5.2. Policy to Boost Economic Activity. There can be cases where government would want to boost economic activity rather than enhance welfare when facing an economic recession. In this case, Propositions 1.3.1 and 1.3.2 remind us that the households with the more optimistic beliefs will likely show greater consumption responses. As in the previous welfare improving policy, the quantity of liquid assets is a good proxy for the degree of perception in this model. To achieve a greater consumption response from the stimulus, the government must spread the rebates to low liquid wealth households. First, they will spend more due to the concavity of the consumption function Carroll and Kimball (1996). Second, this paper allows an additional boost to the consumption response, as low liquidity households that are more likely to be optimistic will spend more by underestimating expenditure shocks.

Let us consider a simple policy experiment where a government gives rebates to households with a liquid asset to income ratio less than l^*. Table 1.12 presents MPCs for various values of l^*. Increase in l^* introduces households that are less likely to be hand-to-mouth and are pessimistic to the fiscal stimulus, which results in a decrease in the average MPC. In an extreme case where the households that have liquid wealth less than weekly income

	Values of l^*					
	0.25	0.5	1	2	4	All
MPC	30.8	26.0	22.0	19.6	18.2	12.7

Table 1.12. MPC of households with liquid asset to income ratio less than l^*

Note: households with calculated MPC greater than -0.1 percent are considered.

($l^* = 0.25$), the monthly MPC is 30.8, which is about 2.5 times larger than the average MPC in the whole economy.

In a real-world setting, the fact that fiscal stimulus has to be taxed later can limit the effectiveness of a fiscal policy. When households realize the increase in future tax burden and are not liquidity constrained, the fiscal policy would have no effect at all, as noted by (Barro, 1974). This wisdom also applies to this model, and the strength of the fiscal policy will depend on the size of hand-to-mouth households. However, in this realistic setting, the policy recommendation is the same, and the government must target households with low liquid wealth that are likely to have binding liquidity constraints.

1.6. Conclusion

Using a model of heterogeneous agents with different perceptions of expenditure shocks, this paper generates high overall MPC by matching the severe dispersion of liquid wealth found in the data. The calibration reveals that most households have a low perception of future expenditure shock, which leads them to have a very low level of liquidity and exhibit high MPCs. I conclude by discussing several extensions of this paper.

First, the model in this paper assumes a fixed retirement date with no early withdrawal of illiquid assets. Alternatively, we can impose adjustment costs like Kaplan and Violante (2014). This paper had to rely on a fixed retirement date to build a two-asset model with clean theoretical predictions. However, introducing adjustment costs can allow real-world behaviors such as early withdrawal of retirement accounts and lumpy adjustments of illiquid assets.

Second, this paper depends on the parsimonious model of expenditure shocks, which was assumed to be exogenous. In reality, all the categories of consumption can depend on the level of income and wealth. This assumption can be relieved by modeling the expenditure system where households have an imperfect perception of preferences. However, to rationalize the high variance of the non-persistent part and its stark difference distribution with the persistent part, it would be necessary to incorporate large and temporary preference shocks to make expenditure shocks endogenous.

Third, the learning of expenditure shocks was not allowed. This paper isolated the learning based on the calibrated distribution of the expenditure shocks. Since the distribution of the expenditure shock is extremely skewed, median households will underestimate the mean and mainly observe a negligible magnitude of shocks. Also, there are many categories of consumption that would make it difficult to learn all the aspects of the expenditure shocks. Therefore, I limited the model's scope and the degree of freedom by focusing on the pure role of optimism and pessimism. However, some extreme events can shape the expectation of households, and such learning behaviors can also lead to an interesting source of heterogeneity and consumption dynamics.

CHAPTER 2

Eliciting Present Bias and Sophistication over Effort and Money

2.1. Introduction

Evidence shows that people have a time inconsistent taste for immediate gratification known as *present bias*. Their relative preference for well-being at an earlier date over a later date gets stronger as the earlier date gets closer, which implies that the discount rate decreases with the time horizon (hyperbolic discounting). This can lead to preference reversals: preferences between two future rewards can then reverse in favour of the earlier reward as it gets closer (Thaler, 1981).

This evidence motivated the introduction of the quasi-hyperbolic discounting model, also known as (β, δ) model (Phelps and Pollak, 1968; Laibson, 1997). The key assumption of this model is a declining discount rate between the current date and the next one, but a constant discount rate thereafter. While being a simplification of hyperbolic discounting, this model captures the key feature of hyperbolic discounting: a time inconsistent preference for immediate gratification. O'Donoghue and Rabin (1999) show how people's awareness of their present bias, *sophistication*, can affect their behavior. In particular, they show that people who are fully aware of their time inconsistency (sophisticated) will perform onerous tasks weakly earlier than people who are fully unaware (naive). O'Donoghue and Rabin (2001) develop a model of partial naivete, where a person is aware of her time inconsistency but underestimate its magnitude. This (β, δ) discounting framework explains various problems related to self-control, such as portfolio choice (Laibson, 1997; Gelman, 2021; Kuchler and Pagel, 2021), health control (O'Donoghue and Rabin, 2006), and environmental regulations (Cropper et al., 1999; Karp, 2005).

The widespread use of the (β, δ) discounting led to empirical studies estimating the degree of present bias β in various experimental settings. For instance, Imai et al. (2021) conducts a meta-analysis of 28 articles that estimates present bias using the convex time budget protocol. Predominantly many studies tend to observe present bias using monetary rewards offered at various timings Andreoni and Sprenger (2012); Andersen et al. (2014); Andreoni et al. (2015); Augenblick et al. (2015) due to its simplicity. However, using money can strain the test of present bias in several ways. First, there is no guarantee that subjects will consume the money as soon as they receive the payment, which makes the estimation based on a stream of income instead of consumption. Second, choosing the timing of receiving money can be affected by borrowing and lending opportunities which are distinct from preferences Cubitt and Read (2007). Reflecting these difficulties, an increasing number of experimental studies try to elicit present bias in the consumption domain rather than money Augenblick et al. (2015); Augenblick and Rabin (2019); Cheung et al. (2022).

Our first research question is whether there is a distinction between the present bias in effort β_e and the present bias in the monetary domain β_m. We develop a novel survey design where subjects choose the amount of work they want to complete over three consecutive days to answer this question. Unlike previous approaches, which estimate the present bias for effort and money by asking separate multiple price lists or binary choices, we only ask the number of work participants wish to do. By extending the survey design of Augenblick and Rabin (2019), we can disentangle the present bias for effort and money by giving variations in the timing of exerting effort and receiving payments. Unlike Augenblick and Rabin (2019), we have a treatment where the decision day is the same as the payment day, which makes the separation of present bias of effort and money possible.

In a nutshell, we can identify the present bias for the effort by observing the extra work that the subjects wish to do in the future than in the present. If the agent exhibits a more severe degree of present bias, the agent will discount more of the future cost of exerting effort, which is discounted by $\beta_e\delta$. Hence, a present-biased agent in the effort domain will schedule more work in the future than in the present. Further, we can separate the present bias β_e from $\beta_e\delta$ by comparing the gap between effort choices of tomorrow and today (capturing $\beta_e\delta$) with the gap between effort choices of the day after tomorrow and tomorrow (capturing δ). In this way, we can disentangle β_e from the long-term discount factor δ.

Similarly, we can estimate β_m by observing jobs chosen when payments are given now, tomorrow, and the day after tomorrow. In the typical 'money now or later' experiments, subjects have the freedom to choose the payment date in multiple options. This freedom can make subjects facing borrowing constraints choose rewards at an earlier date or choose a favorable interest rate than what is available in the market, as noted by Cohen et al. (2020). We sidestep these possible confounds in money now or later experiments by eliciting the present bias for money by indirectly observing the effort choice. Subjects cannot merely choose to have maximum monetary rewards in the present because the tasks that subjects should finish are costly. Also, financial conditions external to the experiment environment are irrelevant to making choices in our design.

As a result, we find a significant present bias in the effort domain but not in the money domain in line with previous studies such as Andreoni and Sprenger (2012), Andersen et al. (2014),Andreoni et al. (2015), Augenblick et al. (2015). On average, subjects choose about five more (or 13.2 percent more) transcription tasks for later than now. However, we do not find a significant decrease in tasks when the payments are given later than now, which implies no present bias for money. When structurally estimating β_e and β_m on the aggregate level, the estimate for the β_e is 0.71 while β_m is 1.03. This result is confirmed by our nonparametric analysis, which shows that agents who allocate more work in the future tend to work less when the payment is in the future.

The second research question asks if a present-biased agent in the effort domain is also likely to be present-biased in the money domain. Previous literature gives a mixed answer.

Augenblick et al. (2015) finds that β_e and β_m are uncorrelated. In contrast, Cheung et al. (2022) estimates present bias for monetary and dietary rewards and finds a correlation between them. To analyze the relationship between the present bias in different domains, we estimate β_e and β_m at the individual level and find a strong and positive relationship.

The third research question is about subjects' understanding of the future present bias, which is an important element of intertemporal choice O'Donoghue and Rabin (1999, 2001). We adopt the framework of Augenblick and Rabin (2019) which asks subjects to predict their future choices. By asking to predict how many tasks they would choose to complete in the future, we can compare their prediction to the actual choice and reveal the degree of sophistication $\widehat{\beta}$. We find that on the aggregate level, subjects are partially sophisticated in the effort domain where the estimated $\widehat{\beta}_e$ lies between $(\beta_e, 1)$ but is not significant. On the other hand, in the money domain, estimated $\widehat{\beta}_m$ is close to one, as in the case of β_m.

Some novel features of this experiment facilitate the estimation of present bias and its sophistication. In a usual setting, the wage would be wage rate times the number of tasks that the subject should do as Augenblick et al. (2015), and Augenblick and Rabin (2019). Instead, we make that the number of additional tasks increases as the wage increases. This experiment feature essentially makes the marginal return from additional works decrease. By convexifying the effort choices, it would be more costly for subjects to make corner choices of choosing the maximum number of tasks. This feature can help the identification because if choices are at the corner, the changes in the effort will likely be unresponsive to changes in effort and treatments in the experiment.

The remaining part of the paper is structured as follows. In Section 2.2, we introduce the survey design. In Section 2.3, we introduce the model and explain the identification strategy. Section 2.4 checks the validity of the data and shows the evidence of present bias in both nonparametric and structural settings. Finally, Section 5 concludes.

2.2. Survey Design

We recruited 87 subjects from Prolific over three days. Out of 87 subjects who finished the first day survey, 79 subjects remained to complete the second day of the survey, and 72 subjects completed the survey on the final date. Out of 72 subjects who completed the subjects, we had 67 who responded to all questions without missing values. Our analysis mainly considers these 67 subjects who completed the whole survey without any missing values in their responses.

Subjects make two kinds of decisions. First, they choose how many jobs they wish to complete now and in the future. Table 2.1 summarizes the choices made on each date. J_{ij} is a decision to be made on day i about the number of jobs to complete on day j. For example, on day 3, a subject makes a choice J_{33}, which can matter for the jobs to be done on day 3. If i and j of J_{ij} satisfy $i > j$, the decision is made for the future.

Second, subjects also make *predictions* about their future decisions. As shown in Table 2.1, subjects make two predictions on Day 1 regarding their decisions on Day 2: J^p_{22} and J^p_{23}. They predict how many jobs they would wish to complete on Day 2 (J^p_{22}) and on Day 3 (J^p_{23}), respectively. On Day 2, they predict how many jobs they would wish to complete on Day 3, which is denoted as J^p_{33}.

		Work date		
		1	2	3
	1	J_{11}	J_{12}, J^p_{22}	J_{13}, J^p_{23}
Decision days	2		J_{22}	J_{23}, J^p_{33}
	3			J_{33}

TABLE 2.1. Structure of the survey

Note: J_{ij} refers to the decision made at day i about how many jobs to do on day j. J^p_{ij} refers to the *prediction* of the decision to be made on day i about how many jobs to complete on day j.

For each type of decision problem J_{ij}, subjects choose the number of jobs they would like to complete for five different wage rates. The five different wage rates are 1, 3, 5, 7, and 9 pence for each job done. Since the experiment recruits subjects from the United Kingdom, the payment is given in British pounds. In total, we observe $9 \times 5 = 45$ decisions per subject.

Decisions that Counts On each day j, subjects work for one randomly chosen 'decision that counts' from a pool of choices $\{J_{ij}\}_{i \leq j}$. For example, on day 3, one of the choices made from J_{13}, J_{23}, and J_{33} constitutes the pool of decisions to draw. Since subjects make choices for five different wages, one out of 15 decisions will be randomly chosen and will determine the workload for the subject.

Jobs vs. Tasks There is an important distinction we make in this experiment between *jobs* and *tasks*. For each decision that counts, the wage for the chosen decision is the wage rate times the number of jobs the subject has chosen. The number of jobs they choose is relevant to the payments subjects receive in this sense.

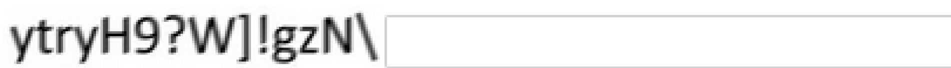

Figure 2.1. Example of a transcription task

When a subject chooses J number of jobs, she is required to finish $1 + 2 + \cdots + J = J(J+1)/2$ number of *tasks*. Each task look as Figure 2.1 where subjects are required to type what is shown into the given empty box.

Since the number of jobs relevant for payment differs from the number of tasks that subjects actually need to finish, subjects can be confused or distracted from converting the number of jobs to tasks. To minimize the unnecessary mental burden, we show both the number of jobs they choose and the corresponding number of tasks they need to finish simultaneously. Figure 2.2 shows the user interface for choosing the number of jobs/tasks. The user interface also shows the total wage they receive to confirm that the number of jobs is relevant to the payment.

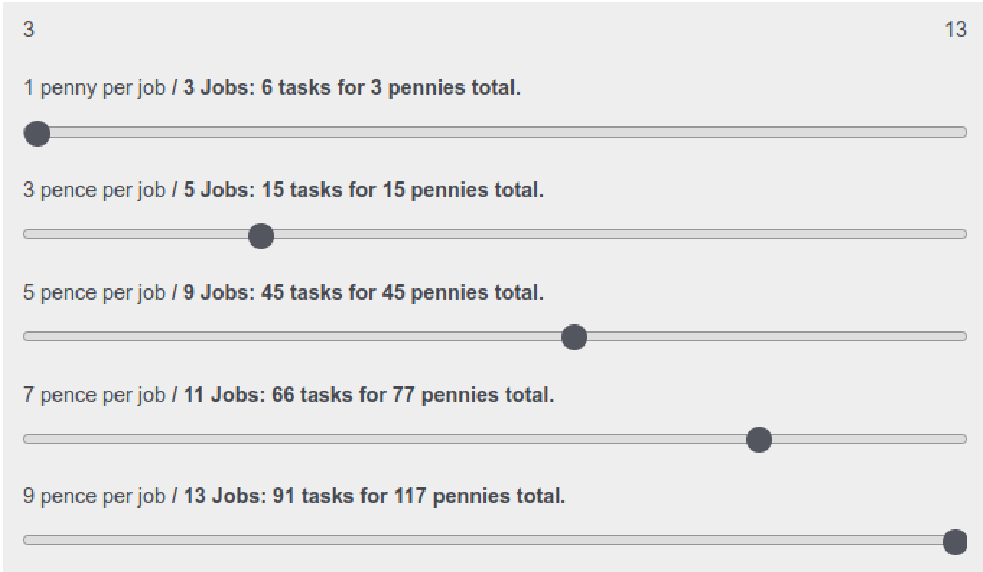

FIGURE 2.2. User interface for choosing jobs and tasks

Payments to Participants There are four sources of rewards for each participant. First, subjects receive a participation fee for each day. Subjects receive 2, 1, and 1 pound each for the three days, respectively. On the first day, subjects receive a higher participation fee than on other days since they need to make many decisions on day one compared to the other days; they need to go through mandatory tasks for training and go through instructions for the survey.

The second payment source is the completion fee of five pounds as subjects finish the survey for all three days. A high amount of completion fee incentivizes participants to finish all three days.

Third, participants are paid by the wage based on the randomly chosen decision that count for each day. This depends on the draw of wage rates, and the number of jobs they have chosen. The payment for transcription task each day can range from $1 \times 3 = 3$ pence to $9 \times 13 = 117$ pence.

Fourth, subjects have a chance to win an additional bonus payment conditional on a successful prediction. We compare the prediction J_{22}^p with J_{22}, prediction J_{23}^p with J_{23}, and prediction J^p_{33} with J_{33}. Out of $3 \times 5 = 15$ choices, we randomly draw a decision, and if the prediction is within two jobs of the actual decision, we give them a bonus of 20 pence. When making actual decisions for J_{22}, J_{23}, and J_{33}, we do not remind them of their predictions.

We make all the payments on the third day, and we ensure that subjects know this by making them go through a quiz about important features of our experiment, including the payment date. We paid around 33.6, 33.8, and 37.1 pence for transcription tasks on the three survey days. The bonus payment for a successful prediction was around 16.7 pence. Subjects received around 787.5 pence from the participation fees for each day and the completion fee. In total, they received 896.1 pence on average. A large portion of the payment was due to participation and completion fee, which do not depend on the decisions that subjects make. It can be worrying that disproportionately large payments are independent of effort choices. There is a possibility that subjects would not pay enough attention to the experiment, resulting in little variation of effort. Also, being complacent about the non-choice payments, subjects might choose a low level of effort only to complete the survey early on each day. In Section 2.4.1, we check that the data is sensible and that subjects do show enough variation in their choices.

2.3. Method

2.3.1. Model. Subjects choose the amount of effort by comparing the monetary reward and various costs accompanying the transcription task. We assume that the cost associated with transcription tasks can be captured by an exponential function as previous studies (Andreoni and Sprenger, 2012; Augenblick et al., 2015; Augenblick and Rabin, 2019), such as $C(h) = h^\gamma/\gamma$ where γ determines the convexity of the cost function.

In addition, our experimental design distinguishes the number of jobs chosen that are rewarded, and the number of tasks to be completed. When the subject i chooses J_i number of jobs, the subject has to actually complete $J_i(J_i + 1)/2$. The cost for choosing J_i amount of jobs in this experiment is

$$C(J_i(J_i+1)/2) = \frac{(J_i(J_i+1)/2)^\gamma}{\gamma}. \tag{2.1}$$

Under the (β, δ) discounting framework, the optimal amount of effort J_i^* is a solution to the following problem:

$$J_i^* = \arg\max_{J_i} \hat{\beta}_m^{1_i^{\text{predict } m}} \beta_m^{1_i^{\text{future } m}} \delta^{k_i^m} J_i w_i - \hat{\beta}_e^{1_i^{\text{predict } e}} \beta_e^{1_i^{\text{future } e}} \delta^{k_i^e} \frac{(J_i(J_i+1)/2)^\gamma}{\psi\gamma}. \tag{2.2}$$

The first term of (2.2) describes the reward from choosing J_i number of jobs. The wage rate is w_i, and the total wage at the observation i is $J_i w_i$. The second term is the cost of putting effort. ψ enables a comparison between the reward and cost which are in different units.

Additional convexity introduced to the experiment design facilitates identification of optimal efforts even with the case where the cost function is linear in tasks, or when γ is near one. If $C(\cdot)$ is linear, then the solution of (2.2) can happen in corners of the range of J_i, or there is no guarantee for a unique solution. In this paper, the convex task structure helps to pin down a unique solution to the problem in (2.2) due to extra convexity in $J_i(J_i+1)/2$. To see this, note that the second derivative of the cost function with respect to J_i in this experiment is

$$\frac{d^2C(J_i(J_i+1)/2)}{dJ_i} = C''(J_i(J_i+1)/2)) \left[\frac{2J_i+1}{2}\right]^2 + C'(J_i(J_i+1)/2)).$$

Even if $C''(J_i(J_i+1)/2))$ is near zero, the cost of exerting effort J_i can be still convex if $C'(J_i(J_i+1)/2)) > 0$.

Table 2.2 shows the type of decision problems that are related to the key variables: $1_i^{\text{future } e}$, $1_i^{\text{predict } e}$, $1_i^{\text{future } m}$ and $1_i^{\text{predict } m}$. If observation i indicates a problem where the payment date is later than the decision date which is not a prediction task, then $1_i^{\text{future } m} = 1$ and the subject discounts the future reward $J_i w_i$ by β_m. If i was about a prediction task instead, then $1_i^{\text{predict } m} = 1$ and the subject discounts the future reward by $\widehat{\beta}_m$.

Subjects discount the cost associated with the effort when the work date is later than the decision date. Within such observations, if observation i is not a prediction task, then $1_i^{\text{future } e} = 1$ which makes the subject discounts the cost of exerting effort in the future by β_e. If the observation was a prediction task, then $1_i^{\text{predict } e} = 1$, and the cost of effort is discounted by $\widehat{\beta}_e$.

	Timing	Prediction task?	In the experiment
$1_i^{\text{future } m}$	Payment date is later than decision date	Not prediction task	J_{11}, J_{12}, J_{13}, J_{22}, J_{23}
$1_i^{\text{predict } m}$		Prediction task	$\widehat{J}_{22}$, $\widehat{J}_{23}$
$1_i^{\text{future } e}$	Work date is later than the decision date	Not prediction task	J_{12}, J_{13}, J_{23}
$1_i^{\text{predict } e}$		Prediction task	$\widehat{J}_{23}$

TABLE 2.2. Classification of variables in the experiment

The long-run discount factor δ discounts the reward or cost of effort in farther future. k_i^m measures the distance between the payment date and the decision date. k_i^e represents the distance between the work date and the decision date. For example, if observation i is about a decision task J_{12}, $k_i^m = 3 - 1 = 2$ since the terminal date is 3, and $k_i^e = 2 - 1 = 1$.

The optimality of the solution to (2.2) requires

$$\left(\frac{\hat{\beta}_m^{1_i^{\text{predict } m}} \beta_m^{1_i^{\text{future } m}} w_i \psi}{\hat{\beta}_e^{1_i^{\text{predict } e}} \beta_e^{1_i^{\text{future } e}} \delta^{k_i^e - k_i^m}} \right) - \left(\frac{J_i(J_i+1)}{2} \right)^{\gamma-1} \left(\frac{2J_i+1}{2} \right) = 0. \tag{2.3}$$

The more the subject is present-biased in the money domain, the subject discounts the future rewards more and will work less, which implies that β_m and J_i^* will have a positive relationship. If the subject is sophisticated and understands the true level of present bias in the future (i.e., $\widehat{\beta}_m$ is low), then the subject will predict a lower level of effort that the subject is going to choose in the future. Similarly, stronger present-bias in the effort domain makes agent discount or underestimate future costs, which makes the agents to choose larger amount of jobs. If the subject is sophisticated in the effort domain, and $\widehat{\beta}_e$ is low, then the subject will predict lower amount of jobs to choose in the future.

2.3.2. Identification. We employ the maximum likelihood estimation (MLE) to estimate the parameters of the model $\theta = (\beta_e, \widehat{\beta}_e, \beta_m, \widehat{\beta}_m, \delta, \psi, \gamma)$. Since subjects may make errors when solving (2.2), we allow some error $\varepsilon_i = J_i^*(\theta) - J_i$ when making decisions so that the optimal level of effort implied in the model and choices that the subjects make can differ. We assume that this error term follows a normal distribution with a standard deviation σ. Specifically, our parameter estimate $\widehat{\theta}_{\text{MLE}}$ according to MLE with N data points is defined as

$$\widehat{\theta}_{\text{MLE}} = \arg\max_{\theta} \sum_{i=1}^{N} \ln \mathcal{L}_i(\theta), \quad \text{where}$$

$$\ln \mathcal{L}_i(\theta) = -\ln \sigma - 0.5 \ln 2\pi - 0.5 \left(\frac{J_i^*(\theta) - J_i}{\sigma} \right)^2. \tag{2.4}$$

We construct the asymptotic variance-covariance matrix of θ, $\text{Cov}(\theta)$, as

$$\text{Cov}(\theta) = N^{-1} \left[H(\theta)^{-1} S(\theta) H(\theta)^{-1} \right], \quad \text{where}$$

$$H(\theta) = \sum_{i=1}^{N} \frac{\partial^2 \ln \mathcal{L}_i(\theta)}{\partial\theta\partial\theta'} \quad \text{and } S(\theta) = \sum_{i=1}^{N} \left(\frac{\partial \ln \mathcal{L}_i(\theta)}{\partial\theta} \right) \left(\frac{\partial \ln \mathcal{L}_i(\theta)}{\partial\theta} \right)'.$$

Role of Parameters Parameters in θ have distinct roles. γ controls the responsiveness of effort to changes in wages. To see this, suppose that we have two data points that are identical except the wage rates. If the jobs chosen increase from 3 to 4, when the wage rate increases from 1 to 3, then (2.3) implies that the γ should satisfy

$$\frac{3}{1} = \frac{2 \times 4 + 1}{2 \times 3 + 1} \left(\frac{4 \times 5/2}{3 \times 4/2} \right)^{\gamma - 1} \qquad \Rightarrow \qquad \gamma \simeq 2.659.$$

If the jobs are unresponsive to change in wages, the estimated γ will be higher.

The identification of the rest of parameters depends on γ. We can identify δ comparing choices of effort across future dates. For example, let us consider two choices $e_{12} = 10$ and

$e_{13} = 11$ at the same wage rate. If $\gamma = 2$, δ can be identified as

$$\frac{1}{\delta_e} = \frac{2 \times 11 + 1}{2 \times 10 + 1} \left(\frac{11 \times 12/2}{10 \times 11/2} \right)^{2-1} \qquad \Rightarrow \qquad \delta \simeq 0.76.$$

Present bias of effort (or money) reflect increase (or decrease) in the effort by underestimating the future cost (or reward). Suppose that we have two choices $e_{12} = 11$, and $e_{11} = 10$ at the same wage rates. If $\gamma = 2$ and $\delta = 1$, then two choices imply that

$$\frac{1}{\beta_e \delta} = \frac{2 \times 11 + 1}{2 \times 10 + 1} \left(\frac{11 \times 12/2}{10 \times 11/2} \right)^{2-1} \qquad \Rightarrow \qquad \beta_e \simeq 0.76.$$

We can also identify β_m similarly by comparing e_{22} and e_{33} at the same wage rates.

The last parameter of interest ψ scales the overall level of effort. Higher ψ means higher marginal utility from effort, making the subject choose higher level of effort.

2.4. Results

2.4.1. Monotonicity and Corner Choices. We check the basic properties of the data regarding the frequency of violating monotonicity and corner choices. The convexification of the number of tasks aims to reduce the extreme choices made at the corner solution of choosing the maximum task. Also, one unit change in the number of jobs gives a large change in the number of tasks, making subjects choose their actions seriously.

First, we measure the degree of monotonicity violation by counting the violation's frequency out of the nine choice problems. Each choice problem asks to choose effort for five different wages. We define that monotonicity violation happens in a choice problem if a subject chooses a strictly lesser effort at a higher wage rate in any cross-comparisons of effort choices at five different wage rates. Since our experiment features nine different choice problems, the measure of monotonicity violation ranges from zero to nine.

As shown in Table 2.3, about 58 percent of subjects do not show any monotonicity violations. Most subjects do not exhibit monotonicity violations. The number of subjects

	Number of problems violating monotonicity										
	0	1	2	3	4	5	6	7	8	9	Total
Number of Subjects	39	18	3	2	1	1	1	1	1	0	67

TABLE 2.3. Frequency of monotonicity violation

violating monotonicity in more than one choice problem is about 15 percent of the sample. Since each subject needs to make 45 decisions, it can be possible to make a mistake at least once. We have $9 \times 67 = 603$ choice problems in the sample. There are 60 incidences of monotonicity violations which are less than 10 percent of the total number of choice problems.

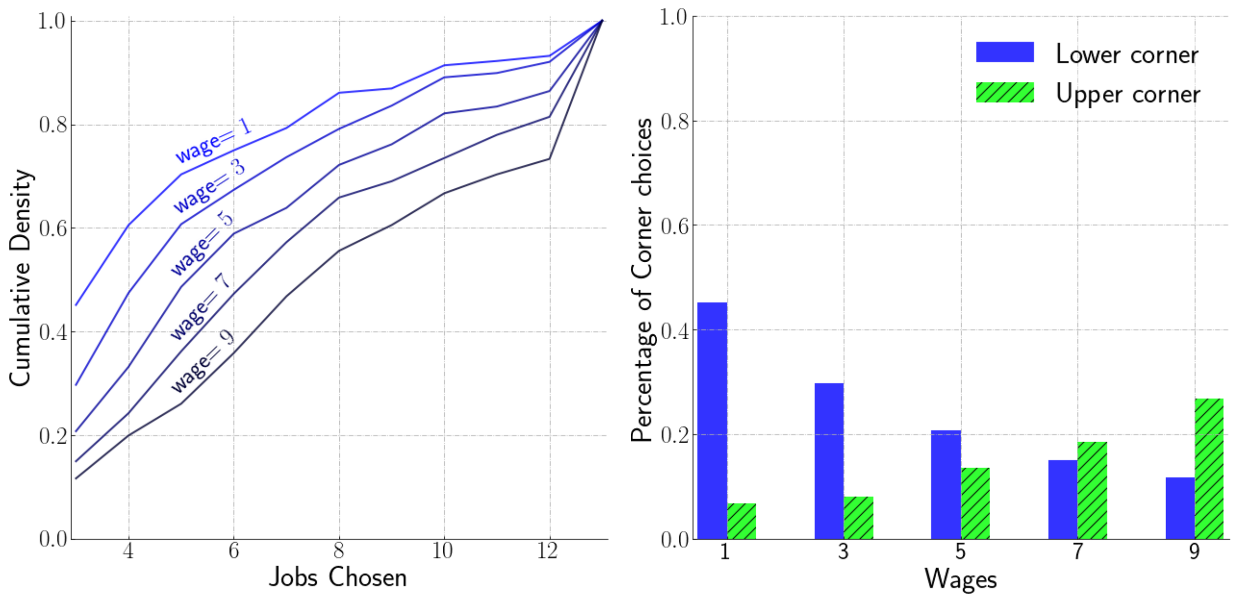

FIGURE 2.3. Distribution of effort choices and frequency of corner choices

The left panel of Figure 2.3 displays the cumulative density function (CDF) of jobs chosen for all wage rates, $\{1, 3, 5, 7, 9\}$. First, we can see a strictly monotonic increase in the jobs chosen to wage rates. The CDF of higher wage rate first-order stochastically dominates the CDF of lower wage. Though the portion of wage from the transcription task is small compared to other types of rewards, individuals respond to the change in wages, and there is a significant variation in the jobs chosen.

Out of all 3,015 choices, about 736 (24.4 percent) occur in the left corner with minimum jobs. Corner choices on the right happen among 444 choices (14.7 percent) where the maximum number of jobs are chosen. These add up to 1,180 (39.1 percent) choices. The right panel of Figure 2.3 shows the corner choice at all wage rates. When the wage rate is one, the fraction of choices made at the lower corner is about half. However, this fraction drops to about 10 percent at the highest wage. This pattern shows that subjects are not merely choosing a lower level of effort to end the experiment early and solely focusing on the high completion fee.

2.4.2. Nonparametric Analysis. Before estimating time discounting parameters in a structural framework, we measure the change in effort choices when facing decisions for the future and predictions for them. Following Augenblick and Rabin (2019), we employ the following specification at the aggregate level to measure the response of effort choices:

$$
\begin{aligned}
e_{i,j} = &\phi_0 + \phi_e^{\text{future}} 1_{i,j}^{\text{future } e} + \phi_e^{\text{predict}} 1_{i,j}^{\text{predict } e} + \phi_m^{\text{future}} 1_{i,j}^{\text{future } m} + \phi_m^{\text{predict}} 1_{i,j}^{\text{predict } m} \\
&+ \phi_\gamma w_{i,j} + \phi_\delta \left(k_{i,j}^m - k_{i,j}^e \right) + \mu_i + \varepsilon_{i,j}.
\end{aligned}
\tag{2.5}
$$

Indices i and j represent the individual and the decision problem, respectively. The parameter ϕ_e^{future} measures the decrease of effort when facing jobs to do in the future that is not a prediction task. Agents that are present biased will underestimate the future costs of efforts due to low β_e and choose to do more work with $\phi_e^{\text{future}} > 0$. In this sense, we call $-\phi_e^{\text{future}}$ the measure of present bias in the effort domain. For example, if ϕ_e^{future} is close to zero, the agent would be neither present nor future biased since the chosen effort levels are independent of whether the task should be done now or later. Similarly, agents who perceive present bias well (lower $\widehat{\beta}_e$) will predict choosing more work in the future. This prediction of choosing more work will result in larger estimates of ϕ_e^{predict}. We call $-\phi_e^{\text{predict}}$ the measure of sophistication in the effort domain based on this idea.

The estimate of ϕ_m^{future} shows the amount of extra work that agents would like to do when facing future rewards compared to the case of receiving rewards now. We call ϕ_m^{future} the nonparametric measure of present bias since a higher β_m corresponds to more works chosen when facing future rewards. ϕ_m^{predict} captures the perception of present bias in the money domain. If agents realize their present bias in the money domain, they will predict that their future selves would choose less work to do when receiving later rewards.

The other terms in (2.5) contain potentially important factors to explain the choice of effort. The distance between the day of receiving rewards and doing the tasks is $k_{i,j}^m - k_{i,j}^e$, which is related to the δ in the structural version, and $w_{i,j}$ is the wage rate. To allow different base levels of efforts chosen for different individuals i, we include fixed effect terms μ_i as dummy variables.

2.4.2.1. *Aggregate Analysis.* Table 2.4 displays nonparametric measures of present bias and sophistication over the effort and money domain. We have two types of the dependent variable $e_{i,j}$ in our analysis. First, we use the number of jobs chosen on the four left columns in Table 2.4, which is directly related to the amount of wage the subjects receive. The four columns on the right-hand side of the table use the number of tasks as the dependent variable, which is more relevant to the actual amount of work that the subjects should do. Furthermore, we also separately examine both the level and log of the dependent variable, which distinguishes the change in the number and percentage of jobs/tasks.

There are two kinds of samples covered in Table 2.4. First, in the full sample, we examine all 67 subjects who completed all three days of the experiment. There are nine kinds of decisions that asks choice of efforts over five different wage rates. In total, we have $67 \times 9 \times 5 = 3,015$ observations. Second, we exclude subjects with irregular behaviors. There are four subjects with no variation of efforts to wages for all nine kinds of decisions. Identifying present bias parameters crucially depends on their usual sensitiveness to wages, and it would be problematic to conduct structural analysis for these participants. This type of sample is labeled 'Regular' in the last row of the table.

	Jobs				Tasks			
Panel A: Effort domain								
Present bias	-0.543	-0.578	-0.070	-0.075	-4.881	-5.190	-0.132	-0.140
	(0.243)	(0.258)	(0.038)	(0.040)	(2.001)	(2.123)	(0.070)	(0.075)
Sophistication	-0.200	-0.212	-0.018	-0.019	-2.189	-2.328	-0.034	-0.036
	(0.169)	(0.180)	(0.025)	(0.027)	(1.426)	(1.516)	(0.047)	(0.050)
Panel B: Money domain								
Present bias	0.010	0.010	0.011	0.012	-0.399	-0.424	0.020	0.021
	(0.180)	(0.192)	(0.028)	(0.030)	(1.472)	(1.566)	(0.051)	(0.055)
Sophistication	0.226	0.240	0.036	0.038	1.645	1.749	0.067	0.071
	(0.154)	(0.164)	(0.023)	(0.024)	(1.364)	(1.451)	(0.042)	(0.045)
N	3015	2835	3015	2835	3015	2835	3015	2835
Dependent var.	Level	Level	Log	Log	Level	Level	Log	Log
Sample	Full	Regular	Full	Regular	Full	Regular	Full	Regular

TABLE 2.4. Nonparametric measures of present bias and sophistication in the aggregate level

Note: 'Present bias' and 'Sophistication' in Panel A corresponds to $-\phi_e^{\text{future}}$ and $-\phi_e^{\text{predict}}$ in (2.5). 'Present bias' and 'Sophistication' in Panel B corresponds to ϕ_m^{future} and $-\phi_m^{\text{predict}}$ in (2.5). The numbers in the parentheses are standard errors, which are clustered at the individual level.

In Panel A of Table 2.4, we mostly see significant estimates of present bias in the effort domain across all specifications. In detail, subjects choose 0.543 more jobs and 4.881 more tasks in the full sample when facing a job in the future than now. When using the log dependent variable, subjects choose about 7 percent more jobs or 13.2 percent more tasks in the full sample when facing future costs. We also find a similar pattern of results in the alternative sample selection scheme. We find some degree of sophistication on present bias in the effort domain. The absolute size of the measure of sophistication is smaller than the measure of present bias, which indicates partial sophistication. However, the nonparametric measures of sophistication are generally insignificant in all specifications.

Panel B of Table 2.4 measures present bias and sophistication in the money domain. We do not observe a significant degree of present bias, unlike in the effort domain. Also, subjects tend to be future biased, but the estimates are insignificant. This result is consistent with previous works by Andreoni and Sprenger (2012) and Augenblick et al. (2015) where they do not find evidence of present bias in the money domain. We also do not find significant estimates of perception of present bias in the money domain.

2.4.2.2. *Individual Analysis.* We apply the regression (2.5) at the individual level to derive nonparametric measures of present bias and sophistication for each subject. For the individual analysis, we only report the results using the level of jobs. However, the results are consistent when using tasks instead of jobs, as we did in the aggregate analysis. In the individual-level analysis, we drop four subjects who do not vary the level of jobs chosen for different wages for all nine types of decisions.

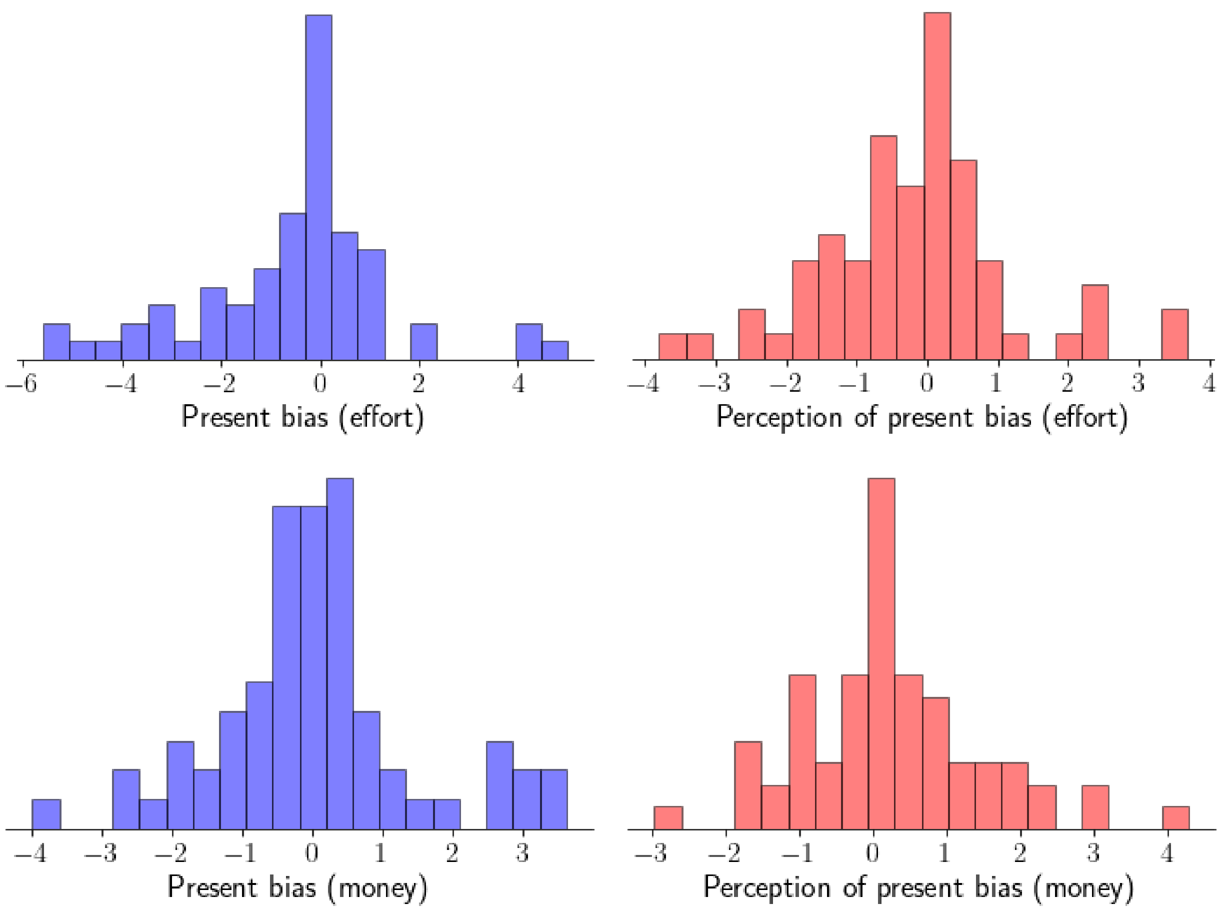

FIGURE 2.4. Nonparametric measures of present bias and its perception

Figure 2.4 displays the nonparametric estimates of present bias and sophistication in both effort and money domains. The top-left corner of the figure shows the distribution of nonparametric measures of present bias in the effort domain. We can see that estimates of most subjects lie in the region below zero, which indicates present bias. The median of the estimates is -0.2, where the mean is around -0.5. The histogram of the perception of present bias in the effort domain, in the top-right corner, shows that most subjects perceive present bias. The median and mean of the distribution are -0.14 and -0.2, respectively, where the sizes are smaller than the estimates for the present bias. In the money domain, present bias and sophistication measures are centered around zero. Following the shape of the distribution, medians of both present bias and sophistication in the money domain are near zero.

Figure 2.5 compares present bias and sophistication across effort and money domains. Individual estimates under zero represent the presence of present bias in the respective domain. In the top-left corner of Figure 2.5, we compare the nonparametric estimates of present bias in effort and money domains. We can see a strong positive relationship between the two, supported by Spearman rank correlation coefficient of 0.75 with a p-value less than 10^{-3}. Our results indicate that individuals with a present bias in the effort domain also tend to be present biased in the money domain.

The level of present bias and its perception are also highly correlated in both effort and money domains. We compare the present bias and its perception in the money domain in the top-right corner. The rank correlation between the two is 0.41 with a p-value less than 10^{-3}. Our analysis suggests that an agent who is present biased in the effort domain and chooses a higher level of effort when facing future costs tends to work less when the rewards are in the future. Though we do not observe a significant degree of present bias in the money domain at the aggregate level, as previous literature suggests, subjects who exhibit present bias in the money domain tend to be strongly present biased in the effort domain as well.

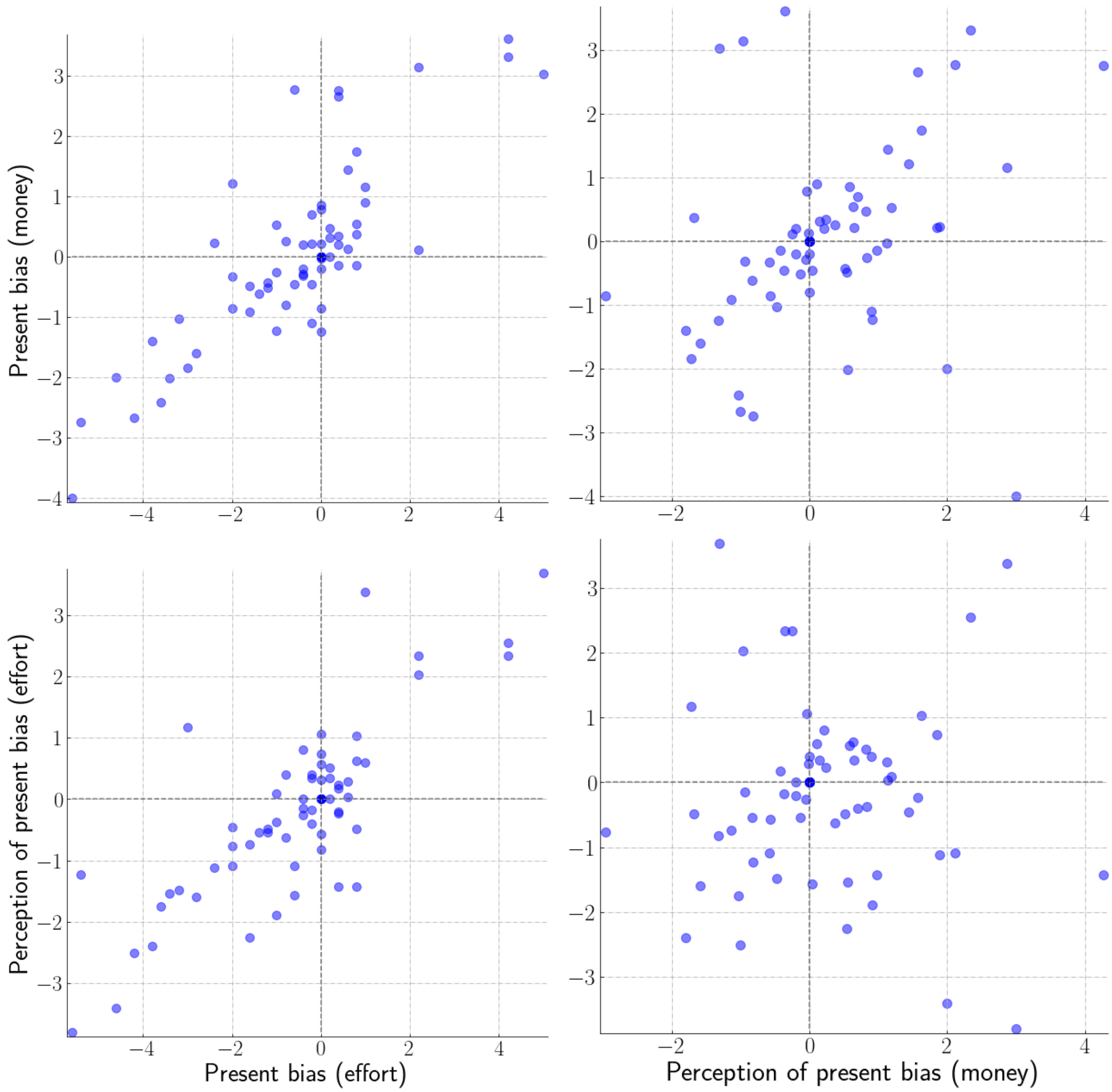

FIGURE 2.5. Comparison of monparametric measures of time discounting

Two figures on the top-right and bottom-left compare the present bias and sophistication in money and effort domains. In both cases, present bias and sophistication have a strong positive relationship. This positive correlation is natural since the perception of present bias needs the existence of present bias.

The bottom-right corner of Figure 2.5 shows a weak relationship between the perception of present bias in the effort and money domains. This weak relationship implies that the agent who is sophisticated in the effort domain and makes correct predictions about their future decisions regarding work choices would not necessarily do the same when the rewards are in the future. The correlation coefficient between the two estimates is 0.1 with a p-value of 0.44.

2.4.3. Structural Estimation.

2.4.3.1. *Aggregate Results.* Table 2.5 shows the estimates of the structural parameters. The first column named 'Symmetric' serves as the baseline where the present bias and sophistication is assumed to be the same, which is similar to Augenblick and Rabin (2019). The second column ('Separate') allows asymmetry in the present bias and sophistication in effort and money domains. In the last column, we exclude the data regarding predictions and only estimate the present bias of effort and money.

We observe a significant degree of present bias for effort (0.713) in the aggregate level from the specification [2]. However, we do not find evidence of present bias in the money domain (1.034), where the standard error is large. Considering the similar estimates of present bias in 'Symmetric' to present bias for effort in 'Separate,' we can see that the present bias in the 'Symmetric' model is mainly driven by present biased behaviors in the effort domain. The present bias for effort estimated in 'No prediction' is close to the 'Separate' which implies that we can find the present biased behaviors in the effort domain when only comparing the choices of actual tasks ($\{e_{11}, e_{22}, e_{33}\}$ vs. $\{e_{12}, e_{13}, e_{23}\}$) and not relying on the data regarding predictions ($\{p_{22}, p_{23}, p_{33}\}$).

We do not find significantly sophisticated behaviors in both the effort and money domain. The estimated parameter of sophistication for effort is less than one and larger than the estimated present bias in the effort domain, implying partial sophistication. However, the standard error is too large to conclude that the estimated parameter is significantly away

		[1] Symmetric	[2] Separate	[3] No prediction
Present bias	Effort	0.721 (0.517, 0.925)	0.713 (0.513, 0.913)	0.718 (0.514, 0.921)
	Money		1.034 (0.686, 1.382)	1.055 (0.659, 1.452)
Sophistication	Effort	0.858 (0.638, 1.077)	0.884 (0.582, 1.187)	
	Money		1.181 (0.814, 1.547)	
Discount factor δ		1.244 (1.03, 1.458)	1.131 (0.925, 1.337)	1.124 (0.917, 1.332)
Convexity γ		2.465 (2.369, 2.561)	2.542 (2.42, 2.664)	2.55 (2.383, 2.716)
Scale of effort ψ		200.041 (137.576, 262.507)	208.093 (128.102, 288.085)	211.427 (100.132, 322.721)
Size of error σ		3.265 (3.196, 3.335)	3.257 (3.188, 3.326)	3.259 (3.176, 3.342)
$\ln \mathcal{L}$		-4772.705	-4764.802	-3177.989
N		2835	2835	1890

TABLE 2.5. Structural estimates of the model

Note: 'Symmetric' assumes that present bias for effort and money is the same. 'Separate' generalizes 'Symmetric' by allowing the distinction between present bias for effort and money. 'No prediction' excludes the data related to predictions. Numbers in parentheses indicate the lower and upper bound of the 95% confidence interval.

from one. The sophistication parameter is larger than one in the monetary domain, but the magnitude is also not significantly away from one. The sophistication in the aggregate level, under the symmetry in the effort and money domain, is 0.858, but this estimate is not significantly away from one.

The discount factor is above one in all three specifications, and this is an odd result since the experiment covers only three days, and it does not imply discounting delayed rewards and costs. However, in our main specification, the estimated discount factor is noisy and not significantly far away from one. Furthermore, since our experiment employs a compact three-day design, we only have zero to two days of variation, making it difficult to capture the long-term discounting behavior.

Other estimated parameters are relatively stable across all designs. For example, the convexity parameter γ is well above one, implying that the cost function is significantly convex. This convexity of the cost function guarantees that our problem in (2.2) will allow for a unique solution if we have an interior solution. The scale of effort ψ is also similar across all columns of Table 2.5, implying that the baseline level of effort did not vary with the type of model chosen or with the range of the selected sample.

Since 'Symmetric' is a nested model of 'Separate,' we can test if two models are significantly different based on the likelihood ratio test under the same sample. The $\chi^2(2)$ statistic is over 15, which implies that the general model is preferred in terms of model fit.

2.4.3.2. *Individual Results.* We extend our analysis to estimate preference parameters in a structural setting individually. We exclude subjects whose structural estimates are outliers compared to other subjects in the individual analysis. We employ Grubb's test to identify the outliers under the significance level of 10^{-4}. This sample selection criterion excludes 8 out of 63 subjects with exceptionally high estimates of present bias and sophistication.

Figure 2.6 plots the distribution of present bias and sophistication across the effort and money domain. Many estimates for the present bias in the effort domain are under one. This is similar to the result in Table 2.5. The mean and median of present bias for effort are 0.84 and 0.85, respectively.

Several subjects have the estimated present bias parameter in the effort domain near zero. Five subjects have a present bias parameter of less than 0.2. These behaviors reflect their insensitivity to changes in wage, but a relatively large increase of tasks chosen when facing tasks to be done in the future than now. When ranking the ratio $\phi_e^{\text{future}}/\phi_w$, the five subjects with β_e less than 0.2 are all ranked from one to five. This sensitivity to works contributes to abnormally low estimates of β_e.

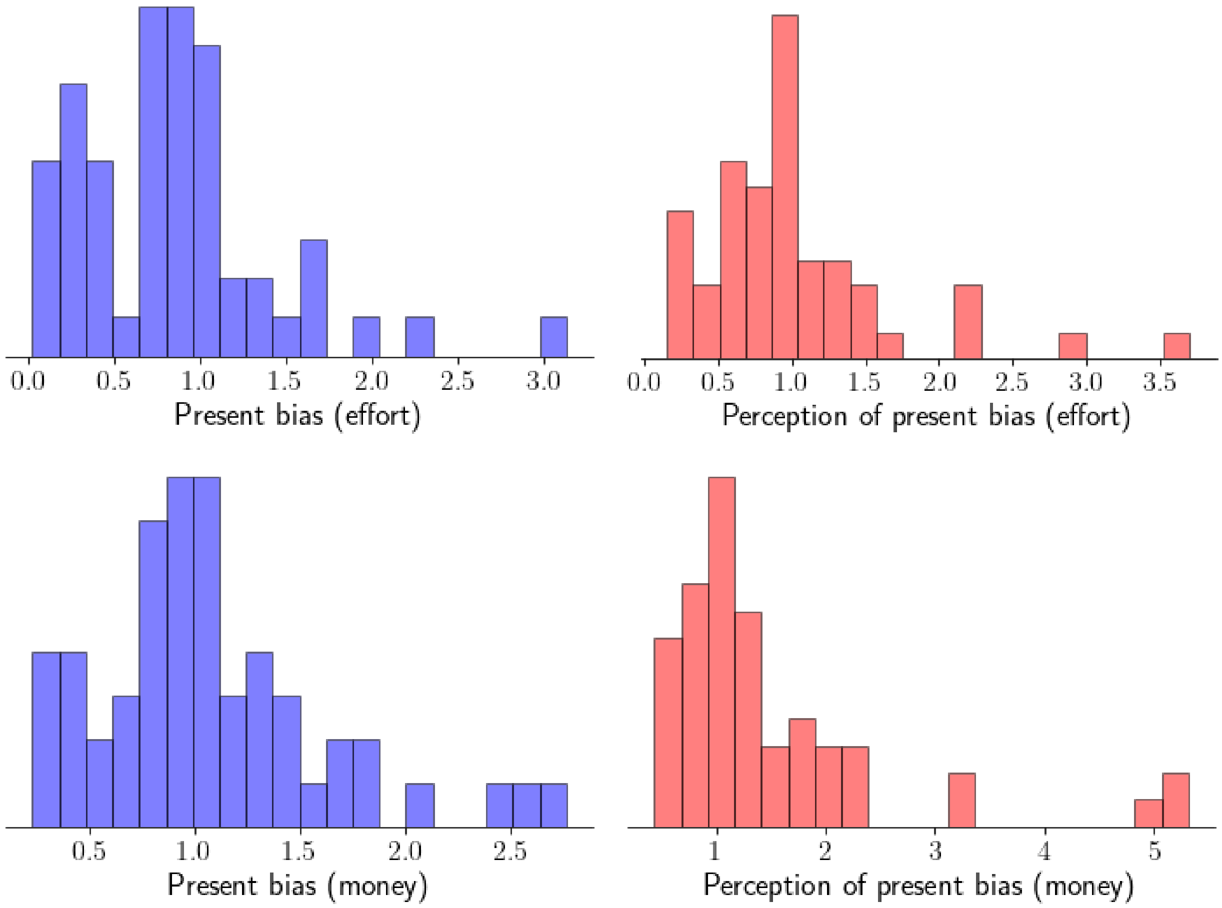

FIGURE 2.6. Structural estimates of present bias and its perception in effort and money domain

The distributions of present bias and sophistication in the effort domain look similar. However, the median and the mean of $\widehat{\beta}_e$ are 0.9 and 1.0, respectively, which is larger than the respective statistics for β_e.

Confirming Table 2.5 where we do not find a significant decrease of tasks when facing future rewards, individual estimates of β_m are centered around one, which indicates time-consistent behaviors. The median and mean of β_m are 0.98 and 1.07, respectively. Individual estimates of $\hat{\beta}_m$ are highly skewed to the left, and the median and mean are 1.16 and 1.47, respectively. The estimates of sophistication tend to be less noisy than the present bias since there are relatively fewer observations that help the identification than other present bias parameters.

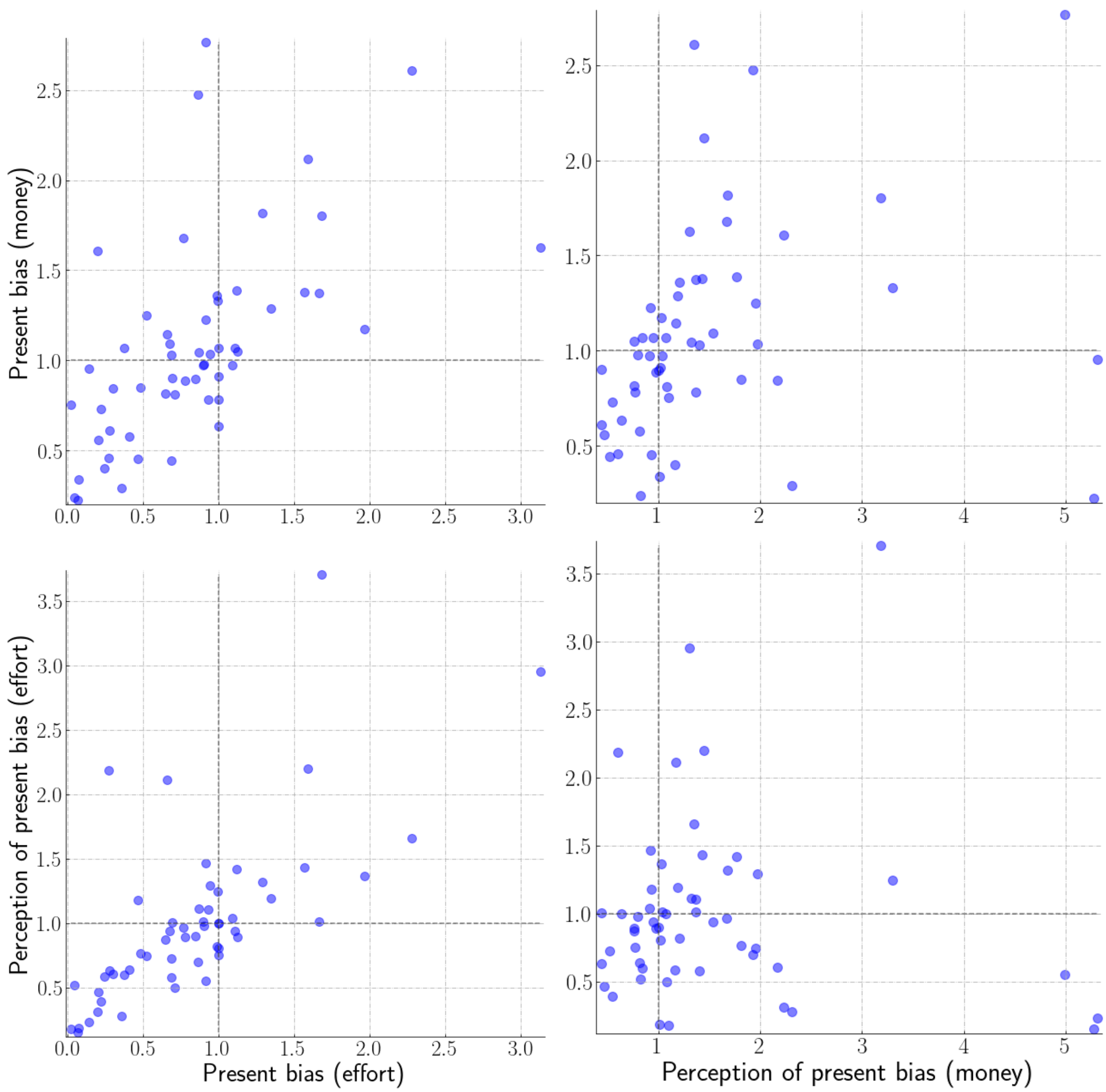

FIGURE 2.7. Comparison of time discounting factors

Figure 2.7 compares the estimated level of present bias and sophistication across effort and money domains. The top-left corner shows strong and positive relationship between β_e

and β_m. Supporting this observation, the correlation between β_e and β_m is 0.67, significantly away from zero.

The present bias and sophistication in the respective domains are also significantly and positively related. The top-right corner of Figure 2.7 compares β_m and $\widehat{\beta}_m$, and their correlation is 0.48. β_e and $\widehat{\beta}_e$ which are plotted at the bottom-left corner, show significant correlation of 0.74. Similar to Figure 2.5, the correlation between $\widehat{\beta}_e$ and $\widehat{\beta}_m$ is 0.02 with a large p-value of 0.87.

If the subjects in the analysis do not show monotonicity of effort choices to wage, information from those subjects will contain larger noise and make inference about preference parameters difficult. I further exclude from 55 subjects in the individual analysis that violates monotonicity more than three times out of nine decision problems.

If the subjects in the analysis do not show monotonicity of effort choices to wage, information from those subjects will contain larger noise and make inference about preference parameters difficult. I further excluded subjects that violated monotonicity more than three times out of nine decision problems to account for this issue. Figures B.1 and B.2 show the histogram of estimates and scatter plots that show relationships among estimates. This alternative sample selection scheme does not change the results of this section.

2.5. Conclusion

We developed a novel survey design to estimate the degree of present bias in effort and monetary domains. Subjects chose a significantly higher amount of work when facing future tasks, which resulted in significant estimates of present bias in the effort domain. Also, subjects were partially sophisticated, but estimates for sophistication were not significantly away from one. On the contrary, we do not find present bias in the money domain, thus making sophistication in the money domain close to one. Moreover, our approach does not depend on binary choices asking for money now or later, which lets us circumvent the confounds of estimating present bias in the money domain.

Our simple three-day design can accompany studies applying the quasi-hyperbolic discounting. Researchers can save the number of questions required to elicit the present bias and sophistication when studying the implications of quasi-hyperbolic discounting and the role of sophistication. Building upon the simple three-day design, researchers can emphasize treatments other than temporal discounting in their experiments.

CHAPTER 3

Portfolio Choice under Heterogeneity of Present Bias and Sophistication

3.1. Introduction

This chapter introduces present bias and sophistication heterogeneity under the influential (β, δ) discounting framework. Under the (β, δ) discounting scheme, agents discounts future reward that is k period away from now by $\beta\delta^k$. For a delay each period, the rewards are additionally discounted by δ. Whenever comparing rewards involving now, the agent discounts future rewards by the degree of present bias, β, which makes it different from the conventional exponential discounting.

When modeling an agent's behavior using the (β, δ) discounting, it is important to specify the agent's belief about his future preferences. Denoting the perception of the present bias as $\widehat{\beta}$, a naivete believes that $\widehat{\beta} = 1$. In this case, the agent believes that all future selves will behave as an exponentially discounting agent. When the agent correctly perceives his future preferences, $\widehat{\beta} = \beta$, then we say that the agent is sophisticated. If $\widehat{\beta}$ is in between β and one, then the agent is partially sophisticated.

The role of sophistication is important when making decisions since it shapes future rewards. Under binary choices, the seminal work by O'Donoghue and Rabin (1999) shows how sophisticated agents may procrastinate compared to the naive agents. O'Donoghue and Rabin (2001), in a multinomial choice setting, formally define the equilibrium under partially sophisticated agents.

Incorporating (β, δ) discounting into the consumption and savings problem has a long history. Strotz (1955) and Pollak (1968) were among the first to explore consumption and savings behavior under naive and sophisticated agents. Laibson (1997) investigates how the sophisticated agents can exhibit excess sensitivity to income and violations of the Ricardian equivalence. Angeletos et al. (2001) calibrate the life-cycle model of a sophisticated agent to show that present-biased agent can accumulate higher illiquid assets than the exponentially discounting agents. Laibson et al. (2007) estimate the time discounting parameters under (β, δ) framework using moments from credit card debt, and asset accumulation over the life cycle. Recently, Gelman (2021) applies the quasi-hyperbolic discounting to explain excess sensitivity of consumption to the arrival of predictable income. Kuchler and Pagel (2021) explain the mismatch between the planned debt pay-down schedule and consumers' actual actions using a quasi-hyperbolic discounting model. They infer the distribution of sophisticated and naive agents by comparing the sensitivity to paycheck receipt. Laibson et al. (2021) show that adding present bias in a model where households are heterogeneous in terms of income can result in amplification of the household balance-sheet channels of macroeconomic policy.

The previous literature that shares the closest goal with this chapter is Nighswander (2021) whose goal is investigating the role of time discounting heterogeneity in explaining the distribution of wealth. Nighswander (2021) incorporates long-run discount factor and present bias heterogeneity into the life-cycle and general equilibrium frameworks to explain the wealth distribution. However, there are several differences between this chapter and Nighswander (2021). First, the work by Nighswander (2021) focuses on the naive agent, which mutes the role of perception regarding present bias in the future. However, this chapter also allows variations in the level of sophistication, which can be potentially important when characterizing the demand for commitment. Second, this chapter incorporates the present bias and sophistication that are estimated in an experiment that is independent of the model environment, whereas Nighswander (2021) uses several agents with a present bias parameter over 0.9 where the distribution is calibrated to match the wealth distribution in data.

Literature on present bias and sophistication in the consumption and savings problem generally faces a serious issue on the stability of the solution. Harris and Laibson (2001) show that the consumption function under the quasi-hyperbolic discounting function tends not to be smooth among partially sophisticated agents unless $\widehat{\beta}$ is sufficiently close to one. This difficulty led to the invention of alternative discounting schemes, such as instantaneous gratification preferences introduced by Harris and Laibson (2013), which approximates the (β, δ) discounting in a continuous-time framework. They show that instantaneous gratification preferences may provide a unique equilibrium. When applying the instantaneous gratification preferences to the consumption and savings problem, Maxted (2021) shows that there can be an equilibrium where present bias and sophistication do not matter to the level of illiquid savings.

Reflecting the difficulty of finding a clear theoretical relationship between time discounting parameters and consumption, I numerically seek to investigate the role of present bias and sophistication in a life-cycle framework. I find that whether the borrowing constraints or adjustment of illiquid assets happens or not can be important. Although the response of consumption and savings to sophistication is generally noisy, we can still draw a clear relationship between consumption and savings with households at the binding borrowing constraint. Households with low liquidity levels are likely to face binding liquidity constraints, which can shut down a source of discontinuity. In this case, I generally find a positive relationship between the degree of sophistication and the savings of illiquid assets.

When considering the distribution of present biased and sophisticated agents estimated from Chapter 2, I show that the heterogeneity of present bias and sophistication can lead to great dispersion of liquid and illiquid wealth. Also, the simulated model generates numerous households in a borrowing status. Finally, I document a pattern in the data that shows a great dispersion of wealth even when considering households near the median. Unlike conventional approaches in modeling the dispersion of wealth by creating income differences and capital adjustment costs, such as Kaplan et al. (2018), the empirical finding in this chapter calls for alternative explanations, which might explain the dispersion of wealth independent of income. The result from Chapter 2 can provide another angle of looking at the dispersion of wealth and different commitment demands through a heterogeneity of perception regarding future preferences.

The remaining chapter has the following structure. Section 3.2 presents the model with liquid and illiquid assets, under quasi-hyperbolic discounting framework. Also, I numerically conduct a comparative statics analysis. Section 3.3 documents the high dispersion of wealth in the data and show the implications of present bias and sophistication heterogeneity in a life-cycle framework. Section 3.4 concludes.

3.2. Model

The model features both liquid and illiquid assets. Households enjoy utility from consumption C_t and the stock of illiquid assets Z_t. In each period, households decide the level of consumption C_t, the flow of illiquid asset A_t, and liquid savings S_t that maximizes their expected lifetime utility based on the (β, δ) framework. Their lifetime utility is defined as,

$$u(C_t) + v(Z_t) + \beta \mathbb{E}_t \left[\sum_{k=1}^{T-t} \delta^k \left(u(C_{t+k}) + v(Z_{t+k}) \right) \right]. \tag{3.1}$$

The parameter β discounts all future utility. δ is a long-term discount rate which discounts the utility in the future $t+k$ evaluated at time t by δ^k.

The resource constraint states that the sum of consumption C_t, liquid savings S_t, and adjusted flow of illiquid asset $\phi_t(A_t)$ equals to the total available liquid wealth X_t as

$$X_t = Y_t + R_t^X S_{t-1} = C_t + S_t + \phi_t(A_t), \tag{3.2}$$

where R_t^X is the gross interest rate on the liquid account. The available asset in period t, X_t, is a sum of income Y_t, value of liquid account starting at period t, $R_t^X S_{t-1}$. Market is incomplete due to a borrowing constraint on S_{t-1} as

$$S_t \geq -\bar{b}.$$

When the agent is borrowing, $S_t < 0$, there is a premium on the gross interest rate as,

$$R_t^X = \begin{cases} R_t^s & \text{if } S_t \geq 0 \text{ and} \\ R_t^b & \text{otherwise.} \end{cases}$$

We assume that $R_t^b > R_t^s$, so borrowing is relatively more costly compared to the reward from savings.

The function $\phi_t(\cdot)$ adjusts the illiquid asset flow, making Z_t illiquid. Withdrawing an illiquid asset is possible, but agents may pay a cost that depends on the size of withdrawal $A_t < 0$. The adjustment function $\phi_t(\cdot)$ is time-varying to allow a realistic feature with no withdrawal penalty of retirement accounts after the retirement age. Specifically, $\phi_t(\cdot)$ is defined as

$$\phi_t(A_t) = \begin{cases} A_t & \text{if } A_t \geq 0 \\ \phi_t^-(A_t) & \text{otherwise.} \end{cases}$$

$\phi_t^-(A_t)$ is an increasing function which satisfies $\phi_t^-(|x|) < x$ for any x. The interpretation of this adjustment function is that lower level of A_t would incur higher adjustment costs. When $A_t \geq 0$, then the room for $C_t + S_t$ is bounded by $X_t - A_t$. However, if agent decides to withdraw, $A_t < 0$, then $C_t + S_t$ is bounded by $X_t - \phi_t(A_t) < X_t - A_t$. Instead of $X_t - A_t$, the agent is effectively paying an adjustment cost, which makes available assets for consumption and liquid assets decrease.

The flow of illiquid asset A_t contributes to the accumulation of stock of illiquid asset Z_t as

$$Z_t = R_t^z Z_{t-1} + A_t \geq 0, \tag{3.3}$$

where R_t^z is the gross interest on the illiquid assets. The stock of illiquid asset depends on the previous contribution $R_t^Z Z_{t-1}$ plus A_t. There is a constraint $Z_t \geq 0$, implying net borrowing using the illiquid is impossible.

I define continuation value $W_t^{\widetilde{\beta}}(X_t, Z_t)$ as,

$$W_t^{\beta}(X_t, Z_{t-1}) = u(C_t^*) + v(Z_t^*) + \delta \mathbb{E}_t \left[W_{t+1}^{\beta}(X_{t+1}, Z_t) \right], \tag{3.4}$$

where

$$\{C_t^*, S_t^*, A_t^*\} = \arg \max_{C_t, S_t, A_t} u(C_t) + v(Z_t) + \widetilde{\beta}\delta \mathbb{E}_t \left[W_{t+1}^{\beta}(X_{t+1}, Z_t) \right] \tag{3.5}$$

subject to (3.2) and (3.3).

The continuation value in (3.4) accumulates the current utility to the continuation value $\mathbb{E}_t \left[W_{t+1}^{\widetilde{\beta}}(X_{t+1}, Z_t) \right]$ which is discounted by δ instead of $\widetilde{\beta}\delta$. When making choices as described in (3.5), agents compare the current utility with the continuation value which is discounted by $\widetilde{\beta}\delta$.

The continuation value $W_t^{\widetilde{\beta}}(X_t, Z_{t-1})$ has an input $\widetilde{\beta}$. This $\widetilde{\beta}$ is crucial to determining the continuation value since the choice of agents in the future depends on the value of $\widetilde{\beta}$. If $\beta = \widetilde{\beta}$, the plan of the agents will be the same as the actual decisions. In this case, I call that the agent is sophisticated. If $\widetilde{\beta} = 1$, I call that the agent is naive, since the agent do not realize his present-biased behavior.

Next, I describe how agents in this model choose their actions. Each agent in this model is endowed $(\beta, \widehat{\beta}, \delta)$. I assume that δ is homogeneous across all agents. Agents are *ex ante* heterogeneous with respect to $(\beta, \widehat{\beta})$. In short, their perception of the continuation value is at any point of time is consistent to their beliefs about their future actions based on $\widehat{\beta}$, but they evaluate the continuation value by the actual present bias parameter β.

Similar to O'Donoghue and Rabin (2001), I formally introduce the optimal actions of the agent as the following.

Definition 3.2.1. *A perception-perfect strategy of an agent with* $(\beta, \widehat{\beta})$ *at* t *is composed of consumption* $C_t(X_t, Z_{t-1}; \beta, \widehat{\beta})$, $A_t(X_t, Z_{t-1}; \beta, \widehat{\beta})$, *and* $S_t(X_t, Z_{t-1}; \beta, \widehat{\beta})$ *that are solutions to*

$$\max_{C_t, A_t, S_t} u(C_t) + v(Z_t) + \beta\delta\mathbb{E}_t \left[W_{t+1}^{\widehat{\beta}} (X_{t+1}, Z_t) \right] \tag{3.6}$$

$$\textit{subject to } (3.2) \textit{ and } (3.3), \textit{ and } W_{t+1}^{\widehat{\beta}} (X_{t+1}, Z_t) \textit{ defined in } (3.4). \tag{3.7}$$

Under this perception-perfect strategy, there are distinct roles of β and $\widehat{\beta}$. β compares the utility today to all future expected lifetime utility. If the value β is lower (more present biased), the agent will consume more today and save less where C_t increase and S_t decrease. However, it is unclear how it would lead to changes to A_t. Two forces towards A_t are at work. If the agent already has a large stock of Z_t, an agent with low β may withdraw Z_t and use them for a higher level of consumption. However, if the agent has a small stock of Z_t, the agent will now choose a higher amount of A_t to enjoy the utility from illiquid savings.

On the other hand, the role of $\widehat{\beta}$ is related to the ability to smooth the consumption of future agents. It can affect the level of consumption and savings in two different ways. First, if agents correctly perceive the β in the future, low $\widehat{\beta}$ implies that agents know that the level of future consumption is low. In this case, extra dollars passed to the future can bring higher marginal utility, incentivizing savings. Second, low $\widehat{\beta}$ worsens the ability to smooth consumption in the future. The poor consumption smoothing makes extra savings spread inefficiently across future periods. This inefficiency will decrease the marginal utility of savings, and agents can lower savings when $\widehat{\beta}$ is low.

3.2.1. Properties of the Model in a Stylized Setting.

Since β and $\widehat{\beta}$ do not have a clear relationship with consumption and savings, in theory, I need to investigate the properties of the model using numerical methods. Using a stylized setting, I investigate the properties of the model from various angles.

I study an economy with $T = 15$ periods where the income process follows

$$y_t = \mu_t \exp(\varepsilon_{y,t}).$$

Transitory income shock $\varepsilon_{y,t}$ is drawn from a normal distribution $N(0, \sigma_y^2)$. μ_t is defined as $\mu_t = 5 + \frac{5(15-t)}{14}$ which makes expected income to decrease over time in an interval $[5, 10]$. The decreasing sequence is chosen to invoke the need for saving and induce consumption smoothing. If the income sequence is flat or upward sloping over time, we will have a trivial solution where the agent chooses the maximum amount of consumption at each time. I set $\sigma_y = 0.1$ where the one standard deviation shock translates to ten percent deviation from the trend income.

I set interest rates as $R_t^s = \exp(\varepsilon_{s,t})$, $R_t^b = 1.03 \times \exp(\varepsilon_{s,t})$, and $R_t^z = 1.03 \times \exp(\varepsilon_{z,t})$ to ensure that the borrowing rate is always higher than the interest rate for liquid savings. $\varepsilon_{s,t}$ is drawn from the normal distribution $N(0, \sigma_s^2)$ where $\sigma_s = 0.001$ so that the interest rate for the liquid asset is small. Note that the underlying shocks for R_t^s and R_t^b is the same to make R_t^b is higher than R_t^s for any state of the world. R_t^z has higher mean in expectation than the R_t^s which compensates the illiquidity of Z_t, where $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$. I set $\sigma_z = 0.015$ since in reality, illiquid asset tends to be riskier than the liquid assets.

I assume constant relative risk aversion (CRRA) utility function for $u(C_t)$ and $v(Z_t)$ where $u(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$ and $v(Z_t) = \frac{Z_t^{1-\rho}}{1-\rho}$. The curvature of the utility function is $\rho = 1/3$. The long-run discount rate δ is 0.99. The adjustment function is defined as $\phi_t^-(Z_t) = \theta_t Z_t$ where $\theta_t = 0.85 \times \frac{T-t}{14}$ so that the withdrawal penalty decreases as the agent approaches the final period. The borrowing constraint is set as $\bar{b} = 3$ so that the liquid savings cannot be lower than -3.

Throughout the simulation, I will consider three cases of time preference. In the first case, I fix $\beta = 0.5$, and vary the value of $\widehat{\beta} \in [0.1, 1.0]$. This case allows us to observe the role of sophistication while fixing the degree of present bias. This kind of agent is also called a *partially sophisticated agent* in the literature. In the second case, I observe characteristics of a *naive agent* by fixing $\widehat{\beta} = 1$, and varying $\beta \in [0.1, 1.0]$ where agents incorrectly perceives that he can commit to future actions. In the third case, I analyze a *sophisticated agent* where $\beta = \widehat{\beta} \in [0.1, 1.0]$.

3.2.2. Choice of Partially Sophisticated Agents. Figure 3.1 displays the choices of a partially sophisticated agent with a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a low stock of illiquid wealth $Z_0 = 0$. The agent at the beginning of the life cycle fits to this case where he only has liquid asset from precautionary motives, but holds no illiquid assets yet.

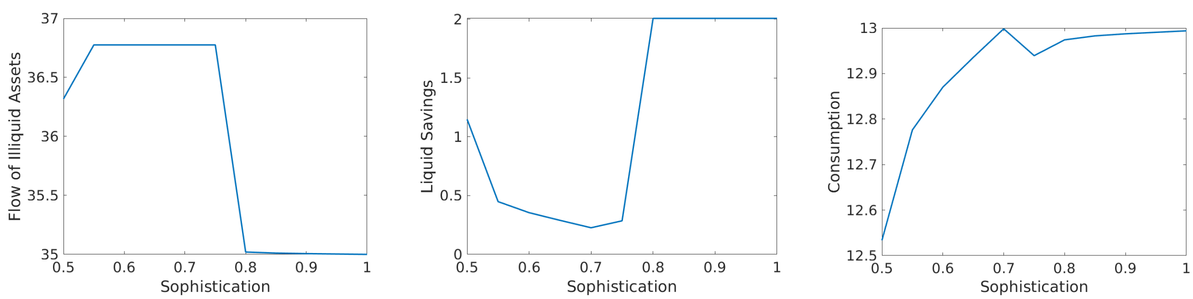

FIGURE 3.1. Relationship between key control variables and sophistication of a partially sophisticated agent with a high disposable wealth and low illiquid wealth

Note: x-axis is the sophistication which has the range $\widehat{\beta} \in [0.1, 1.0]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a low stock of illiquid wealth $R_1^z Z_0 = 0$.

As the agent becomes more sophisticated, $\widehat{\beta}$ becomes close to true $\beta = 0.5$, the level of consumption decreases, and the agent allocate more assets to illiquid and liquid savings. When $\widehat{\beta}$ is close to 0.5, the level of an illiquid asset tends to be higher than the case where $\widehat{\beta}$ is close to 1. This pattern implies that sophisticated agents value the commitment device more than naive agents do.

The portfolio composition between the sophisticated and the naive agent has a stark difference. The liquid savings of the sophisticated agent is smaller than those of the naive agent. Naive agents appreciate the flexibility of the liquid asset since they perceive that they will act as a time-consistent agent and have a chance to save illiquid assets later.

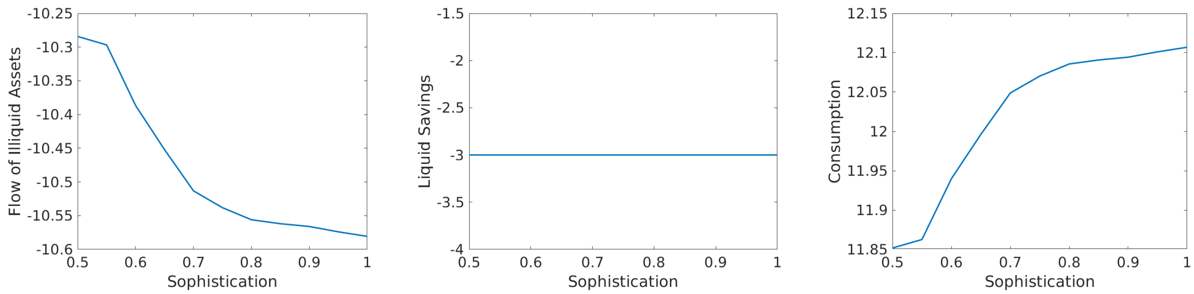

FIGURE 3.2. Relationship between key control variables and sophistication of a partially sophisticated agent with a low disposable wealth and a high illiquid wealth

Note: x-axis is the sophistication which has the range $\widehat{\beta} \in [0.1, 1.0]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a low disposable wealth $Y_1 + R_1^s S_0 = 0$ and a high stock of illiquid wealth $R_1^z Z_0 = 50$.

Analysis of the role of time preference requires can change by different assumptions of initial wealth. Retaining the same partially sophisticated agent, I investigate an agent with low disposable income $Y_1 + R_1^s S_0 = 0$ and high illiquid wealth $R_1^z Z_0 = 50$. As we will see in Section 3.3, this type of agent is the most prevalent throughout the main simulation. This type of agent fits a typical agent in Kaplan and Violante (2014) during the middle age, where the level of liquid net worth is low when accumulating the illiquid assets.

Figure 3.2 shows how a partially sophisticated agent allocates consumption, the flow of illiquid assets, and liquid savings with low disposable wealth and high illiquid wealth. The consumption is an increasing function of $\widehat{\beta}$, which implies that the sophisticated agents save more in general.

The flat line in the middle panel of Figure 3.2 shows that the borrowing constraint binds at $S_1 = \bar{b} = -3$, which can be resulting from the low endowment of liquid wealth. Since the borrowing constraint is binding, one source of discontinuity has been eliminated, which contributes to a smooth consumption function and a smooth flow of illiquid assets.

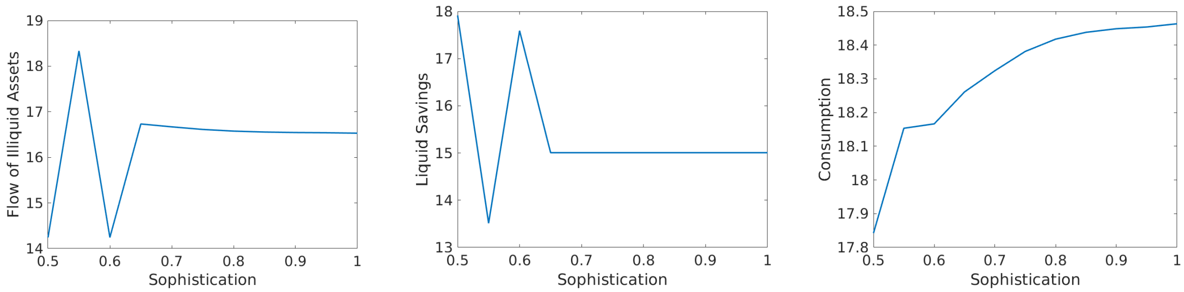

FIGURE 3.3. Relationship between key control variables and sophistication $(\widehat{\beta})$ of a partially sophisticated agent with a high disposable wealth and a high illiquid wealth

Note: x-axis is the sophistication which has the range $\widehat{\beta} \in [0.1, 1.0]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a high stock of illiquid wealth $R_1^z Z_0 = 50$.

Figure 3.3 investigates the role of present bias in the key control variables under a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and high illiquid assets $R_1^z Z_0 = 50$. Giving high liquid and illiquid wealth can be useful to see the desired portfolio composition of agents across different levels of sophistication. Out of the disposable wealth of 50, agents use about one-third to save liquid wealth and another one-third to accumulate illiquid assets. At the end of period 1, agents will have about 15 for liquid savings S_1 and $R_1^z Z_0 + Z_1 \simeq 50 + 16 = 66$ as the stock of illiquid assets, which makes the stock of illiquid assets about four times larger than the liquid savings. We can conclude that agents in this economy prefer to hold substantially more illiquid assets than liquid ones due to higher returns from illiquid assets and utility value from them. In Figure 3.3 I do not see clear preference for illiquid assets among sophisticated agents. However, when combining the flow of illiquid assets and liquid savings, households generally save more as they become more sophisticated.

3.2.3. Choice of Naive Agents. Next, I analyze the choices of naive agents where $\widehat{\beta}$ is one, and I vary the level of present bias $\beta \in [0, 1]$. To make a clean comparison and deal with the case that frequently appears in the main simulation, I pay attention to the case when agents have low disposable income and high illiquid wealth.[1]

[1]In Appendix B, I show the policy functions under alternative assumption about endowments.

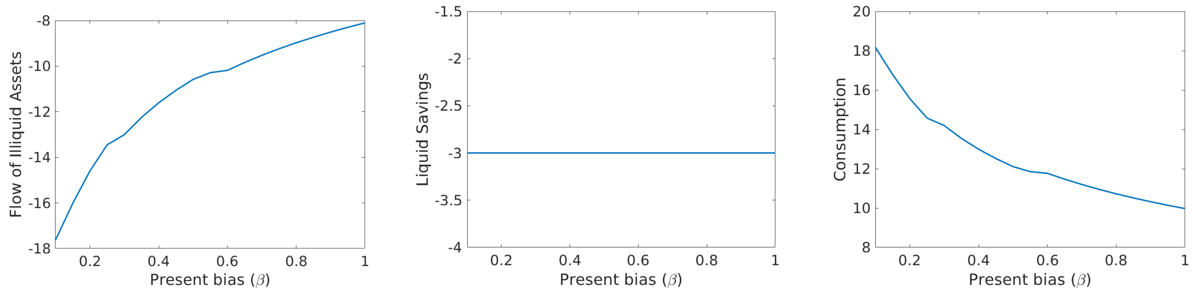

Figure 3.4. Relationship between key control variables and present bias (β) of a naive agent with a low disposable wealth and a high illiquid wealth

Note: x-axis is the degree of present bias $\beta \in [0.1, 1]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a low disposable wealth $Y_1 + R_1^s S_0 = 0$ and a high stock of illiquid wealth $R_1^z Z_0 = 50$.

Figure 3.4 shows how the flow of illiquid assets, liquid savings, and consumption changes by varying the level of present bias. There is a monotonic relationship between the level of present bias and consumption. As the agent becomes more present biased, he is willing to save less since he discounts the marginal value of saving more.

In this analysis, agents endowing low disposable wealth wish to consume more than the given disposable income but face a limit due to a borrowing constraint. This pattern is shown in the middle panel of Figure 3.4. Eventually, they need to withdraw the illiquid assets and pay the withdrawal penalty, which leads to the negative flow of illiquid assets on the left panel of the figure.

3.2.4. Choice of Sophisticated Agents. In the last case, I analyze behaviors of the sophisticated agents where $\beta = \widehat{\beta}$. Varying β effectively changes the level of present bias and the perception of β simultaneously, and the effect of this change on consumption and savings is not obvious. When analyzing partially sophisticated agents, as in Section 3.2.2, decreasing $\widehat{\beta}$ leads to an increase in the flow of illiquid assets by improving the perception of the future level of present bias. However, in Subsection 3.2.3, decreasing β while fixing $\widehat{\beta}$ led to a decrease in the overall level of savings.

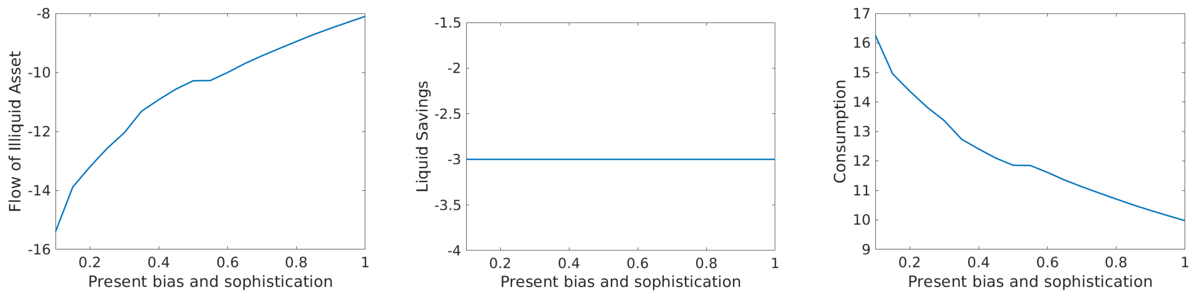

FIGURE 3.5. Relationship between key control variables and present bias (β) of a sophisticated agent with a low disposable wealth and a high illiquid wealth

Note: x-axis is the degree of present bias, which equals to the sophistication ($\beta = \widehat{\beta}$), which has the range $\beta \in [0.1, 1.0]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a low disposable wealth $Y_1 + R_1^s S_0 = 0$ and a high stock of illiquid wealth $R_1^z Z_0 = 50$.

Figure 3.5 shows how these two competing forces ultimately shape consumption and savings. I endow low disposable wealth as in Subsection 3.2.3, which makes the borrowing constraint binding for all values of $\beta = \widehat{\beta}$. We can see a clear pattern that consumption is a decreasing function of β in this case. When comparing the greater desire to increase consumption (due to β) and increase saving to compensate low level of consumption in the future periods (due to $\widehat{\beta}$), the desire to increase consumption today prevails. Reflecting this mechanism, we can see that the agent with lower β withdraws the stock of illiquid assets more than the agents with high β.

3.3. Simulation

3.3.1. Dispersion of Wealth in Data. We investigate the distribution of liquid and illiquid wealth using the Survey of Consumer Finances from 1989 to 2019. I consider households satisfying the following criteria. First, I consider households between the ages of 25 and 80. Second, after selecting households based on age, I further chose households with income between 20 and 80th percentiles in each age cohort.[2] By choosing the households around the median income, I show that there still exists a great deal of heterogeneity in liquid and

[2]Choosing a narrower band of income can make the households with zero income included in the analysis at older age groups.

illiquid net worth holdings. Third, I choose households that have a nonnegative net worth. Otherwise, many households have a negative illiquid net worth. Current workhorse models that feature illiquid assets such as Kaplan and Violante (2014); Kaplan et al. (2018) assume positive holdings of illiquid net worth in general. I also assume that agents in the model can only hold nonnegative illiquid wealth as others.

I define the *liquid net worth* as liquid asset minus the liquid debt. Liquid asset comprises transactions account, certificate of deposit, pooled investment funds, directly held stocks and bonds, savings bonds, and other managed assets.[3] Liquid debt is composed of lines of credit, and credit card balances.

Illiquid net worth (or illiquid net wealth) is defined similarly which subtracts illiquid debts from the illiquid assets. Similar to Kaplan et al. (2018), illiquid assets and debts incur adjustment costs. An illiquid asset is a sum of the cash value of whole life insurance, other financial assets[4], the total value of vehicles, value of the primary residence, value of other residential estates, net equity in nonresidential real estate, the value of business, and other non-financial assets. Illiquid debt covers the value of mortgages and home equity loans secured by the primary residence held by the household, home equity lines of credit, debt for residential property, installment loans, and other debts. The main difference between Kaplan et al. (2018) and this chapter is the coverage of assets and debts and the way of treating stocks. This chapter has broader coverage of assets and debts that are not considered in Kaplan et al. (2018). I do not exclude any financial and non-financial assets and debts from the analysis and use all available data. This chapter treats directly held stock as a part of liquid assets. In Kaplan et al. (2018), stocks were classified as illiquid since they tend to be parts of retirement accounts or other illiquid assets. This chapter separates the directly held

[3] I retrieve the data using the 'Survey Documentation and Analysis.' The detailed definition of variables are also available from the website at https://sda.berkeley.edu/sdaweb/analysis/?dataset=scfcomb.

[4] The definition of other financial assets according to the 'Survey Documentation and Analysis' tool of the Survey of Consumer Finances includes loans from the household to someone else, future proceed from lawsuits, royalties, futures, non-public stock, deferred compensation, oil, gas, and mineral investments.

stocks from other stocks in the retirement account. In this way, we can treat directly held stocks that are usually not subject to high adjustment costs as a part of liquid assets.

	Liquid net worth over income	Illiquid net worth over income
0.005	-1.056	-0.028
0.01	-0.659	0.000
0.05	-0.173	0.068
0.1	-0.071	0.173
0.25	0.000	0.606
0.5	0.063	1.883
0.75	0.356	4.728
0.9	1.523	11.482
0.95	3.881	23.985
0.99	33.954	145.885
0.995	83.329	355.862

TABLE 3.1. Liquid and illiquid net worth over income at different percentiles

One surprising feature of the data in Table 3.1 is that even though I have chosen households specifically around the median level of income, there is a great dispersion of net worth in both liquid and illiquid domains. For example, about 22 percent of the households are net borrowers in the liquid asset domain. Conversely, one percent of households have negative illiquid net worth, which the model cannot capture.

Tables 3.2 and 3.3, displays the percentile of liquid and illiquid net worth over income ratios in the Survey of Consumer Finances. The values of the liquid and illiquid net worth ratios are relatively stable near the median. However, there are fluctuations in the values at the extremes. In the last rows of two tables show the fraction of households with negative liquid and illiquid net worth. Unlike the net borrowing status in the liquid wealth domain, less than two percent of households have larger illiquid debt than assets every year.

Table 3.4 presents the fraction of households with negative liquid and illiquid net worth in different age groups. The fraction of households who are net borrowers in the liquid wealth domain is consistently over 20 percent below age 55. At the same time, a small number of households have negative illiquid net worth, which indicates that households generally

	'89	'92	'95	'98	'01	'04	'07	'10	'13	'16	'19
0.005	-0.45	-0.52	-0.79	-0.86	-0.70	-1.06	-1.74	-1.63	-1.22	-1.21	-1.16
0.01	-0.29	-0.37	-0.41	-0.52	-0.49	-0.76	-0.88	-0.94	-0.76	-0.70	-0.76
0.05	-0.08	-0.13	-0.16	-0.17	-0.14	-0.23	-0.26	-0.24	-0.16	-0.16	-0.16
0.1	-0.03	-0.05	-0.07	-0.07	-0.06	-0.10	-0.12	-0.10	-0.06	-0.07	-0.07
0.25	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.08	0.07	0.05	0.08	0.08	0.06	0.06	0.05	0.06	0.06	0.07
0.75	0.33	0.34	0.31	0.47	0.43	0.44	0.35	0.30	0.34	0.35	0.32
0.9	1.41	1.30	1.27	1.74	1.95	1.76	1.46	1.39	1.46	1.61	1.47
0.95	3.49	2.98	3.22	3.81	4.69	4.33	3.53	3.39	4.48	4.71	4.59
0.99	25.20	19.65	18.36	22.74	50.06	28.80	32.47	44.47	51.73	46.90	64.55
0.995	55.57	64.67	38.63	38.48	125.19	61.34	73.90	108.21	98.93	99.81	130.04
< 0	0.15	0.19	0.24	0.19	0.19	0.23	0.24	0.21	0.19	0.20	0.21

TABLE 3.2. Percentile of liquid net worth over income: 1989-2019
Note: the last row labeled '< 0' refers fraction of households with net liquid worth less than zero.

	'89	'92	'95	'98	'01	'04	'07	'10	'13	'16	'19
0.005	-0.03	-0.02	-0.02	0.00	0.00	0.00	-0.02	-0.03	-0.06	-0.08	-0.05
0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.04	0.06	0.11	0.08	0.07	0.07	0.09	0.07	0.07	0.04	0.05
0.1	0.14	0.14	0.22	0.16	0.18	0.18	0.20	0.16	0.18	0.16	0.18
0.25	0.55	0.49	0.63	0.62	0.65	0.68	0.82	0.59	0.52	0.58	0.60
0.5	1.72	1.53	1.66	1.83	1.84	2.04	2.43	1.83	1.86	1.91	1.99
0.75	4.46	3.88	3.81	4.33	4.74	5.02	5.66	4.71	5.00	5.11	5.02
0.9	10.15	9.26	8.75	9.49	11.21	12.76	13.41	12.16	12.53	13.94	13.22
0.95	18.88	17.65	17.68	17.32	21.92	28.43	29.61	27.56	28.38	29.69	30.36
0.99	113.38	102.72	75.05	89.92	150.74	140.00	138.90	203.48	164.38	226.81	206.94
0.995	220.37	221.10	145.88	228.18	423.29	441.86	344.07	410.82	397.42	477.38	422.00
< 0	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01

TABLE 3.3. Percentile of illiquid net worth over income: 1989-2019
Note: the last row labeled '< 0' refers fraction of households with net liquid worth less than zero.

depend on the illiquid asset to accumulate wealth. The conventional life-cycle model implies that households generally borrow when the income level is low at the early stage when facing a borrowing constraint. In general, the fraction of households decreases as mean income increases in each age group, following the life-cycle model's wisdom. However, at least around 15 percent of households have a negative liquid net worth in all age groups, where their behaviors cannot be explained by the income trend in the life cycle.

	[25, 30)	[30, 35)	[35, 40)	[40, 45)	[45, 50)	[50, 55)	[55, 60)	[65, 70)	[70, 75)	[75, 100)
Liquid	0.20	0.23	0.24	0.22	0.22	0.20	0.18	0.15	0.16	0.15
Illiquid	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Mean Income	44.6	58.4	66.9	74.3	80.0	83.6	67.4	38.4	15.8	5.5

TABLE 3.4. Fraction of households with negative liquid and illiquid net worth at different age groups

Note: $[a, b)$ at the columns include households where the household head is between ages a and b. Mean income denotes the average labor income of each age group weighted by the sample weight. The unit of mean income is \$10,000.

3.3.2. Dispersion of Wealth in the Simulated Economy. I simulate the economy in a richer realistic setting than in Subsection 3.2.1. I simulate in an annual freuqnecy where the initial age is 25, and the terminal age is 80. Hence, we have a total of $T = 56$ periods. Similar to Kaplan and Violante (2014), I set deterministic interest rates for liquid asset and borrowing as $R_t^s = 0.985$ and $R_t^b = R_t^s + 0.075$. I assume that the interest rate for the illiquid rate is drawn from $R_t^z = 1.023 \exp(\varepsilon_{z,t})$ where $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$ and $\sigma_z = 0.015$. The income of agent i follows

$$Y_t = \mu_0 + \mu_1 \text{age}_{i,t} + \mu_2 \text{age}_{i,t}^2 + \mu_3 \text{age}_{i,t}^3 + +\mu_4 \text{age}_{i,t}^4 + \varepsilon_{y,t}$$

where $\varepsilon_{y,t} \sim N(0, \sigma_y^2)$ and $\sigma_y = 0.1$. The trend of annual labor income is estimated from the Survey of Consumer Finances based on households between ages 25 and 80. Parameters μ_0, μ_1, μ_2,μ_3, and μ_4 are estimated from the Survey of Consumer Finances based on households within 20 to 80th percentiles.

The adjustment cost is proportional to the amount they withdraw from the stock of illiquid assets. Before the retirement age of 65, they pay ten percent as the adjustment cost, and no adjustment cost is required after retirement. I assume that households have no previously saved liquid savings and illiquid wealth. Hence, the only source for consumption and savings at age 25 is their initially drawn income at the age 25.

The only source of heterogeneity ex-ante is the degree of present bias and sophistication. I use the distribution of present bias and sophistication estimated in Chapter 2, which has

55 observations of $(\beta_i, \widehat{\beta}_i)_{i=1}^{55}$. I further draw income and interest rate shocks 500 times for each of i in the subjects in Chapter 2 which makes the total number of simulated data at each period $55 \times 500 = 27,500$.

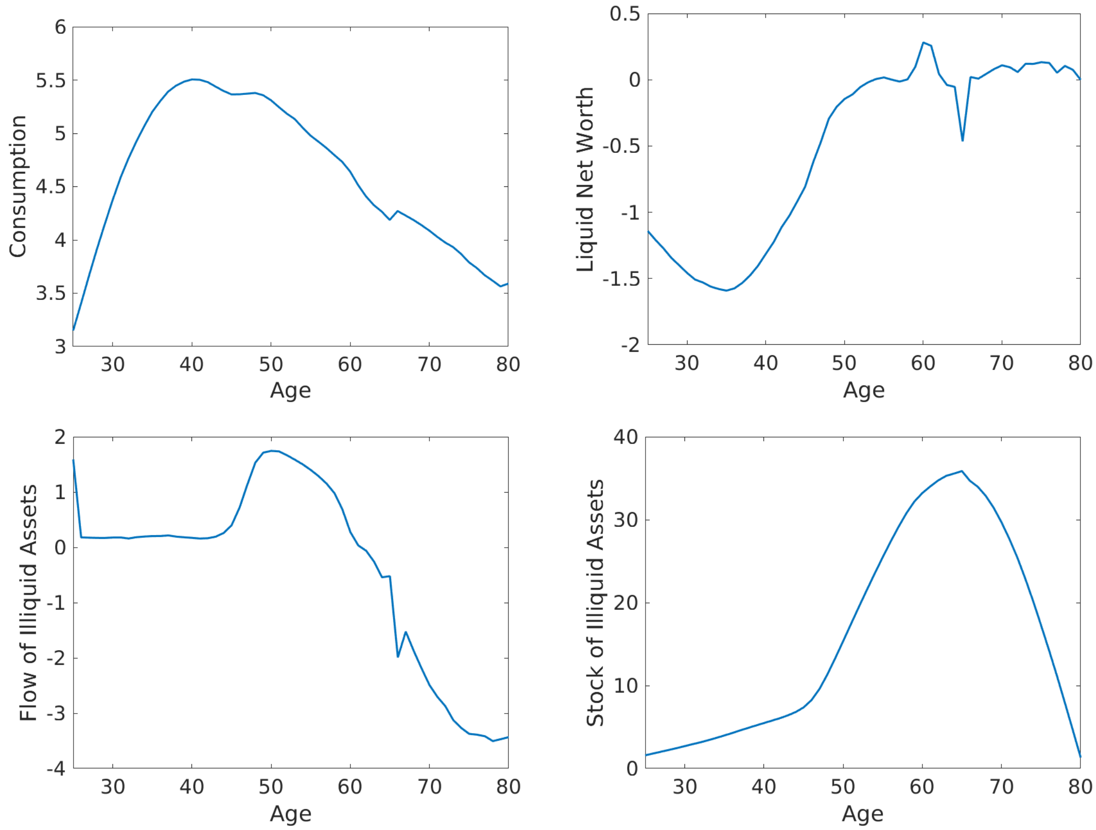

FIGURE 3.6. Average profile of consumption, liquid savings, flow of illiquid assets, and stock of illiquid assets in the simulated economy

Note: unit of y-axis is \$10,000. Results are based on $500 \times 55 = 27,500$ agents over 56 periods.

Figure 3.6 shows the trend of mean consumption, liquid net worth, stock, and flow of illiquid assets. Consumption exhibits a humped-shaped profile over the life cycle. A slight downward jump in the consumption profile happens at age 65, the retirement age. Due to a large overall degree of present bias, households generally do not save liquid assets until age 50. The stock of illiquid assets follows a smooth trend over the life cycle and peaks around the retirement age. After retirement, the flow of illiquid assets becomes negative, and the stock of illiquid assets slowly decreases until the terminal period.

Next, I analyze the relationship between present bias and sophistication over the life cycle. To observe how β and $\widehat{\beta}$ affects the choice variables in the overall level, I employ a following regression

$$\left\{\begin{array}{c} C_{i,t} \\ A_{i,t} \\ S_{i,t} \end{array}\right\} = \theta_0 + \theta_\beta \beta_i + \theta_{\widehat{\beta}} \widehat{\beta}_i + \theta_Y Y_{i,t}$$

$$+ \gamma_1 \text{age}_{i,t} + \gamma_2 \text{age}_{i,t}^2 + \gamma_3 \text{age}_{i,t}^3 + \gamma_4 \text{age}_{i,t}^4 + u_{i,t} \tag{3.8}$$

based on the simulated data of 27,500 agents over 41 periods before the retirement.[5] By controlling the variation of the choice variables by age and income, which is partially reflected in Figure 3.6, I investigate how β_i and $\widehat{\beta}_i$ are related to the choice of consumption, the flow of illiquid assets, and liquid savings. Since the trend of choice variables are already shown in Figure 3.6, I specifically focus on θ_β, $\theta_{\widehat{\beta}}$, and θ_Y rather than the constant and coefficients for age trend.

Dependent variable	Present bias	Sophistication	Income
Consumption	(-0.847, -0.842)	(-0.082, -0.078)	(0.002, 0.006)
Flow of Illiquid asset	(0.314, 0.32)	(-0.066, -0.061)	(0.019, 0.023)
Liquid Savings	(0.526, 0.53)	(0.139, 0.143)	(0.001, 0.004)

TABLE 3.5. Estimated relationship between choice variables and time discounting parameters

Note: numbers in the parenthesis are 95% confidence intervals for each estimated parameters θ_β, $\theta_{\widehat{\beta}}$, and θ_Y in (3.8).

Table 3.5 shows the estimated confidence intervals for θ_β, $\theta_{\widehat{\beta}}$, and θ_Y over three different dependent variables. As households become less present biased, the consumption and flow of illiquid assets decrease. In this model, agents enjoy utility from the stock of illiquid assets,

[5]Including the agents after the retirement can give misleading information regarding the asset accumulation. Agents who accumulated a large amount of assets when young has to withdraw more than others after the retirement. Hence, saving more when young would automatically imply larger negative flow of savings which can give two contrasting movements at the same time.

making illiquid assets similar to durable goods. Present biased agents may choose to increase current period utility by having a higher level of consumption but lower levels of the flow of illiquid assets and liquid savings. As $\widehat{\beta}$ becomes close to one, consumption tends to increase as opposed to the analysis in Section 3.2. In general, there is no guarantee that consumption is lower among sophisticated agents. For example, as pointed out by Salanie and Treich (2006), depending on the curvature of the utility function, consumption and the degree of sophistication can be in an inverse relationship.

In the simulated economy, the flow of illiquid assets decreases as agents $\widehat{\beta}$ is close to one. Overall, sophisticated agents in the simulated economy value the commitment device more than the naive agents. As shown in Section 3.2, there is some substitution between the flow of illiquid assets and liquid savings. Naive agents tend to save less for the illiquid asset and allocate them to liquid assets that provide flexibility.

	Liquid net worth over income	Illiquid net worth over income
0.005	-0.409	0
0.01	-0.373	0
0.05	-0.341	0.362
0.1	-0.322	0.428
0.25	-0.275	0.753
0.5	0	2.436
0.75	0	12.392
0.9	0	40.359
0.95	0.116	68.858
0.99	1.896	118.908
0.995	2.928	149.48

Table 3.6. Liquid and illiquid net worth over income at different percentiles in the simulated model

Next, I investigate the distribution of liquid and illiquid net worth. Table 3.6 shows the liquid and illiquid net worth over income at different percentile values, which can be compared with the data in Table 3.1. Since the model is not calibrated to match the median liquid or illiquid net worth, some discrepancies exist between the model and the data. Especially, the liquid net worth over income in the model is far too small compared to the actual data, while agents in the hold larger illiquid wealth than data. For a better fit with data, several parameters can be calibrated to match the distribution of asset holdings at different percentiles. For example, the parameter that compares the utility from consumption and illiquid wealth can help match the relative holdings of liquid net worth and illiquid net worth. Also, matching the long-run discount rate can help match the overall asset holdings.

A promising feature of the heterogeneity of present bias and sophistication is that it does generate large dispersion of liquid and illiquid wealth. The model's Gini index of liquid net worth is around 0.96, while the Gini index of illiquid wealth is around 0.51. In the actual data, the Gini index of liquid net worth is 0.95, which is close to the simulated data, and the Gini index of illiquid wealth is 0.72. Though the model has less dispersion of illiquid assets than the actual data, introducing the option to rent durable goods instead of solely depending on the stock of illiquid assets, the fraction of households with zero illiquid assets can increase and improve the fit with data.

3.4. Conclusion

This chapter introduced present bias and sophistication heterogeneity to the life-cycle model with liquid and illiquid assets. First, I examined the role of present bias and sophistication in portfolio choice. By comparing choices of naive, partially sophisticated, and sophisticated agents, I find that a higher degree of present bias generally contributes to an increase in consumption. Also, greater sophistication generally leads to a higher flow of illiquid assets. Second, by simulating the life-cycle model under the heterogeneity of present bias and sophistication found in the experimental evidence, I find a possibility of generating sufficiently many households with low liquid net worth and creating a severe dispersion of liquid wealth.

Several enhancements to the calibration strategy can improve the model fit with data. First, I did not calibrate the model based on target moments such as the percentile of liquid and illiquid wealth. Instead, I can calibrate the long-run discount factor to match the overall liquid and illiquid wealth level in the economy. Also, using a richer utility function of durable goods can control the relative illiquid wealth holdings to liquid wealth.

APPENDIX A

Proofs

A.1. Proof of Proposition 1.3.1

Proof. Write the cumulative density functions of $\widetilde{\mathbf{\Gamma}}_t^1$ and $\widetilde{\mathbf{\Gamma}}_t^2$ as $F_t^1 : \mathbb{R}_+ \to [0,1]$ and $F_t^2 : \mathbb{R}_+ \to [0,1]$, respectively. I prove the proposition using backward induction. In the final period, consumers spend all available wealth, hence $C_T(X_T, \Gamma_T; \widetilde{\mathbf{\Gamma}}) = X_T$ irrespective of what Γ_T and $\widetilde{\mathbf{\Gamma}}$ is. Since $F_t^1 >_1 F_t^2$, it follows that

$$\mathbb{E}_{T-1}\left[\frac{\partial W_T^{\widetilde{\mathbf{\Gamma}}^1}(X_T;\Gamma_T)}{\partial X_T}\right] = \int u'(X_T - \Gamma_T)dF_T^1(\Gamma_T) > \int u'(X_T - \Gamma_T)dF_T^2(\Gamma_T)$$
$$= \mathbb{E}_{T-1}\left[\frac{\partial W_T^{\widetilde{\mathbf{\Gamma}}^2}(X_T;\Gamma_T)}{\partial X_T}\right].$$

It is obvious that $\mathbb{E}_{T-1}\left[W_T^{\widetilde{\mathbf{\Gamma}}^1}(X_T;\Gamma_T)\right]$ is concave to X_T. Now, we proceed to general periods. Let the following properties [T1] and [T2] hold at period $t+1$:

$$\text{[T1]}\quad 0 < \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right] < \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right],\ \text{and}$$

$$\text{[T2]}\quad \mathbb{E}_t\left[\frac{\partial^2 W_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}^2}\right] < 0 \text{ and } \mathbb{E}_t\left[\frac{\partial^2 W_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}^2}\right] < 0.$$

Then, I show that properties [T1] and [T2] also hold at period t; moreover, $C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) \leq C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^2)$ where the inequality is strict when the consumer saves a positive amount.

For any $t \geq T_r$, fix a shock Γ_t. By [T1] and [T2], if the agent is hand-to-mouth under $\widetilde{\mathbf{\Gamma}}^1$, then the agent is also hand-to-mouth under $\widetilde{\mathbf{\Gamma}}^2$ since $u'(C_t - \Gamma_t) > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right] > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right]$ for any $C_t \in (0, X_t]$. It follows that $C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) = C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^2) = X_t$

by [T2]. If the agent saves under $\widetilde{\mathbf{\Gamma}}^1$, where $C_t(X_T; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) < X_T$, then

$$u'(C_t(X_T; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1)) = \mathbb{E}_t \left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(w_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial w_{t+1}} \right] > \mathbb{E}_t \left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(w_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial w_{t+1}} \right]$$

by [T1]. This leads to $C_t(X_T; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) < C_t(X_T; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$. Irrespective of whether the agent is hand-to-mouth under $\widetilde{\mathbf{\Gamma}}^1$, this leads to

$$\frac{\partial \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}^1}(X_t; \widetilde{\Gamma}_t)}{\partial X_t} = u'(C_t(X_T; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1) - \widetilde{\Gamma}_t) \geq `u'(C_t(X_T; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2) - \widetilde{\Gamma}_t) = \frac{\partial \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}^2}(X_t; \widetilde{\Gamma}_t)}{\partial X_t}.$$

Using the fact that $F^1 >_1 F^2$, we can conclude that [T1] holds at time t. [T2] is obvious because consumption is an increasing function of wealth. □

A.2. Proof of Proposition 1.3.2

Before stating the main proof, let me state a related result.

LEMMA A.2.1. *Suppose that $u(c)$ is strictly concave, and $F(s,a)$ is concave. For a convex set $B(\bar{x})$,*

$$V(\bar{x}, \bar{A}) = \max_{s,a} u(\bar{x} - s - a) + F(s, Z)$$

$$s.t. \;\; s + a < \bar{x}, Z = \bar{A} + a, s \geq 0 \text{ and } a \geq 0.$$

- *only allows a unique solution for $c = \bar{x} - s - a$,*
- *value function $V(\bar{x}, \bar{A})$ is concave, and*
- *if $u(\cdot)$ and $F(s,a)$ are concave and differentiable, then $V(\bar{x})$ is also differentiable at $\bar{x}$ and A.*

1. I first show the uniqueness of c. Suppose that another triplet c', s', and a' where $c \neq c'$ exists. Then, for a $\lambda \in (0,1)$,

$$u(\lambda c + (1-\lambda)c') + F(\lambda s + (1-\lambda)s', \lambda Z + (1-\lambda)Z') > u(c) + F(s, Z)$$

is a contradiction.

For concavity of V, let $S' = (s', a')$ and $c' = \bar{x}' - s' - a'$ denote the solution that maximizes the objective function, given $\bar{x}'$ and $\bar{A}'$. Similarly, let $S'' = (s'', a'')$ and $c'' = \bar{x}'' - s'' - a''$ be the maximizers given $\bar{x}''$ and $\bar{A}''$.

For any $\lambda \in [0,1]$, $\lambda S' + (1-\lambda)S''$ is feasible when given $\bar{x}^* = \lambda\bar{x}' + (1-\lambda)\bar{x}''$ and $\bar{A}^* = \lambda\bar{A}' + (1-\lambda)\bar{A}''$. Define $S^* = \lambda S' + (1-\lambda)S''$ and $c^* = \lambda c' + (1-\lambda)c''$. Then,

$$\begin{aligned}
V(\bar{x}^*, \bar{A}^*) &\geq u(c^*) + F(s^*, Z^*) \\
&\geq \lambda\left[u(c') + F(s', Z')\right] + (1-\lambda)\left[u(c'') + F(s'', Z'')\right] \\
&> \lambda V(\bar{x}', \bar{A}') + (1-\lambda)V(\bar{x}'', \bar{A}'').
\end{aligned}$$

For differentiabilty, Lemma Benveniste and Scheinkman (1979) show that if $W(x, A|s, a) = u(x - s - a) + F(s, a + A)$ is a concave function on a convex set B, then a concave function $V(x, A)$ where $V(x^*, A^*) = W(x^*, A^*)$ and $V(x, A) \geq W(x, A)$ for all other $(x, A) \in B$, then V is differentiable at (x^*, A^*). □

Now we begin the proof of Proposition 1.3.2

Proof. I provide the proof using backward induction. In each step, I show that [T1] following two inequalities

$$\frac{\partial \widetilde{V}^1_{t+1}(X_{t+1}, Z_{t+1})}{\partial X_{t+1}} > \frac{\partial \widetilde{V}^2_{t+1}(X_{t+1}, Z_{t+1})}{\partial X_{t+1}}, \text{ and } \frac{\partial \widetilde{V}^1_{t+1}(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}} > \frac{\partial \widetilde{V}^2_{t+1}(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}}$$

hold, and [T2] $\widetilde{V}^i_{t+1}(X_t, Z_t)$ is a concave function.

Starting from period $T_r - 1$, agent i solves

$$\text{(A.1)} \qquad \max_{C,S,A} u(C - \Gamma_{T_r-1}) + \delta\mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}(R^S_{T_r}S + R^A_{T_r}(A + Z_{T_r-1})\right]$$

$$\text{(A.2)} \qquad \text{subject to } C + S + A \leq X_{T_r-1}, C \geq 0, S \geq 0, \text{ and } A \geq 0.$$

Trivially, $S = 0$ since it is an inferior asset compared to A with $R^A_{T_r} > R^S_{T_r}$. Hence, this is effectively a one-asset case. We can apply the same steps as those in Proposition 1.3.1 and claim that $C^2_{T_r-1}(X_{T_r-1}, Z_{T_r-1}) \geq C^1_{T_r-1}(X_{T_r-1}, Z_{T_r-1})$. Moreover,

$$\begin{aligned}\frac{\partial \widetilde{V}^i_{T_r-1}}{\partial Z_{T_r-1}} &= u'(C^i_{T_r-1} - \widetilde{\Gamma}^i_t)\frac{\partial C^i_{T_r-1}}{\partial Z_{T_r-1}} + \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r}\left[1 + \frac{\partial A_{T_r-1}}{\partial Z_{T_r-1}}\right]\right] \\ &= \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r}\right]\end{aligned}$$

whether or not the constraint on $C^i_{T_r}$ is binding. If agents 1 and 2 are both hand-to-mouth, then the wealth at period T_r is the same; this yields

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^2_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^1_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

If the agent 2 is hand-to-mouth but agent 1 saves, then

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < u'(C^2_{T_r-1} - \widetilde{\Gamma}^2_{T_r-1}) < u'(C^1_{T_r-1} - \widetilde{\Gamma}^1_{T_r-1}) = \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

If both agents save, then

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = u'(C^2_{T_r-1} - \widetilde{\Gamma}^2_{T_r-1}) < u'(C^1_{T_r-1} - \widetilde{\Gamma}^1_{T_r-1}) = \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

A similar argument can show us that $\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial X_{T_r-1}} < \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial X_{T_r-1}}$ as we have done in Proposition 1.3.1, but I do not repeat it here. Moreover, $\widetilde{V}^i_{T_r-1}(X_{T_r-1}, Z_{T_r} - 1)$ is concave by Lemma A.2.1.

Now, we go to any arbitrary period $t < T_r - 1$ given that [T1] and [T2] hold at period $t + 1$. Let us denote $\widetilde{F}^i_t(S^i_t, A^i_t) = \mathbb{E}_t\left[\widetilde{V}^i_{t+1}(X^i_{t+1}(S^i_t), Z^i_{t+1}(A^i_t))\right]$. Note that the objective function $u(X_t - S^i_t - A^i_t) + \widetilde{F}_t(S^i_t, A^i_t)$ is differentiable and concave to S^i_t and A^i_t with linear constraints by [T1] and [T2], hence, Kuhn-Tucker optimality conditions are necessary and sufficient.

We deal with the following four cases.

- Case 1: Both agents 1 and 2 are hand-to-mouth.

In this case, there is nothing to prove since the consumption of two agents is identical. Moreover, [T1] holds trivially.

- Case 2: Only agent 2 is hand-to-mouth.

  Also in this case, the assumption already implies that [T1] holds.

- Case 3: Only agent 1 is hand-to-mouth.

  I show the impossibility of this case. Assume that agent 1 is only saving illiquid asset A_t^1. The assumption implies that

$$u'(C_t^2 - \Gamma_t) = \frac{\partial \widetilde{F}_t^2(S_t^2, A_t^2)}{\partial S_t^2} \leq \frac{\partial \widetilde{F}_t^1(S_t^2, A_t^2)}{\partial S_t^2}.$$

  Hence, by the Inada condition on $u(\cdot)$ and continuity and concavity of $\widetilde{F}_t(\cdot,\cdot)$, there must exist certain levels of $\Delta > 0$ and $\Delta' > 0$ such that

$$u'(C_t^2 - \Delta - \Delta' - \Gamma_t) = \frac{\partial \widetilde{F}_t^1(S_t^2 + \Delta, A_t^2 + \Delta')}{\partial S_t^2}.$$

  However, this cannot be true because $C_t^2 - \Delta - \Delta'$, $S_t^2 + \Delta$, and A_t^2 is also a solution for agent 1's problem, and we only allow for unique optimal consumption by Lemma A.2.1. Similar arguments can be made when agent 1 is only saving liquid asset S_t^1.

- Case 4: All agents are not hand-to-mouth.

  The argument is the same as that for case 3.

Hence, the consumption is always higher for agent 2. Concavity and differentiability of the value function are guaranteed by Lemma A.2.1. Then, the continuation value $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i) = \mathbb{E}_t\left[V_{t+1}^i(X_{t+1}^i, Z_{t+1}^i)\right]$ is also concave since $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i)$ is the sum of concave functions.

Finally, we need to show that [T1] holds at period t. To see this, $C_t^2(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2) \geq C_t^1(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1)$ implies that

$$
\begin{aligned}
& u'\left(C_t^1(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1)) - \widetilde{\Gamma}_t\right) \geq u'\left(C_t^2(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2)) - \widetilde{\Gamma}_t\right) \\
& \Rightarrow \mathbb{E}_{t-1}\left[\frac{\partial \widetilde{V}_t^1}{\partial X_t}\right] \geq \mathbb{E}_{t-1}\left[\frac{\partial \widetilde{V}_t^2}{\partial X_t}\right],
\end{aligned}
$$

where the last line follows from the fact that $\widetilde{\Gamma}_t^1 >_1 \widetilde{\Gamma}_t^2$. □

A.3. Proof of Proposition 1.3.3

PROOF. When fixing S_{t-1}, the consumption function in terms of random variables Y_t, Y_{t+1}, Γ_t and Γ_{t+1} can be written as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)$ for periods t and $t+1$.

We need to show that $\text{cov}(\Gamma_t, C_{t+1}X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)) < 0$. Define $\bar{\Gamma}_t = \mathbb{E}\left[\Gamma_t\right]$. Fix Y_t, Y_{t+1}, and Γ_{t+1} Then,

$$
\left[\Gamma_t - \bar{\Gamma}_t\right]\left[C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) - C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \bar{\Gamma}_t)\right] \leq 0.
$$

Taking expectation with respect to Γ_t conditional on Y_t, Y_{t+1}, and Γ_{t+1} yields,

$$
\mathbb{E}\left[\left[\Gamma_t - \bar{\Gamma}_t\right] C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) \middle| Y_t, Y_{t+1}, \Gamma_{t+1}\right] \leq 0.
$$

Now taking expectation over Y_t, Y_{t+1}, and Γ_{t+1} yields,

$$
\mathbb{E}\left[\Gamma_t C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\right] - \bar{\Gamma}_t \mathbb{E}\left[C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\right] \leq 0.
$$

This shows that $\text{cov}(C_t - \Gamma_t, C_{t+1}) \geq \text{cov}(C_t, C_{t+1})$. □

A.4. Proof of Proposition 1.5.1

PROOF. I only show the proof for the case where $\mathbf{\Gamma} >_1 \widetilde{\mathbf{\Gamma}}^1 >_1 \widetilde{\mathbf{\Gamma}}^2$. The proof for the other case is similar to that for this this case. Assume that following holds for continuation value at $t+1$:

$$
\begin{aligned}
[a] \quad & \mathbb{E}_t \left[W^*_{t+1}(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1) \right] \geq \mathbb{E}_t \left[W^*_{t+1}(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^2) \right] \\
[b] \quad & \mathbb{E}_t \left[W^*_{t+1}(Y_{t+1} + R^S_{t+1}(\bar{S} + \Delta); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}) - W^*_{t+1}(Y_{t+1} + R^S_{t+1}\bar{S}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}) \right] \\
& \quad \geq \mathbb{E}_t \left[\widetilde{W}^{\widetilde{\mathbf{\Gamma}}}_{t+1}(Y_{t+1} + R^S_{t+1}(\bar{S} + \Delta); \widetilde{\Gamma}_{t+1}) - \widetilde{W}^{\widetilde{\mathbf{\Gamma}}}_{t+1}(Y_{t+1} + R^S_{t+1}\bar{S}; \widetilde{\Gamma}_{t+1}) \right] \text{ for all } \Delta \geq 0.
\end{aligned}
$$

As the first step, I show that using [a] and [b], we have $W^*_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq W^*_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$. Fix $X_t > 0$ and denote consumption under $\widetilde{\mathbf{\Gamma}}^1$ as C^1. Then, consumption under $\widetilde{\mathbf{\Gamma}}^2$ can be written as $C^1 + \Delta_C$ for some $\Delta_C \geq 0$. Then we will compare the following two terms:

$$
U^1 = u(C^1) + \delta \mathbb{E}_t \left[W^*_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1) \right], \text{ and} \tag{A.3}
$$

$$
U^2 = u(C^1 + \Delta_C) + \delta \mathbb{E}_t \left[W^*_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1 - \Delta_C); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1) \right]. \tag{A.4}
$$

Then, we can deduce

$$
\begin{aligned}
u(C^1 + \Delta_C) - u(C^1) \leq\ & \delta \mathbb{E}_t \big[\widetilde{W}^{\widetilde{\mathbf{\Gamma}}^1}_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1); \widetilde{\Gamma}_{t+1}) \\
& \quad - \widetilde{W}^{\widetilde{\mathbf{\Gamma}}^1}_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1 - \Delta_C); \widetilde{\Gamma}_{t+1}) \big] \\
\leq\ & \delta \mathbb{E}_t \big[W^*_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1) \\
& \quad - W^*_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1 - \Delta_C); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1) \big] \\
\leq\ & \delta \mathbb{E}_t \big[W^*_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1) \\
& \quad - W^*_{t+1}(Y_{t+1} + R^S_{t+1}(X_t - C^1 - \Delta_C); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^2) \big].
\end{aligned}
$$

Rearranging the last row shows that $U^1 \geq U^2$.

Now, to continue the backward induction, I show that [a] and [b] holds with the continuation value at t. [a] is obvious, which is just taking the expectation on the above inequality with the same random variable.

To show [b], note that $F(x) >_1 G(x)$ between two CDFs $F(x)$ amd $G(x)$ implies that

$$\int v(x)dF(x) \geq \int w(x)dG(x)$$

if $v(x) \geq w(x)$ for all x and if $w(x)$ is increasing in x. Using this strategy, we need to show that

$$\begin{aligned} v(\Gamma_t) =& W_t^*(Y_t + R_t^S(\bar{S}+\Delta); \Gamma_t, \widetilde{\mathbf{\Gamma}}) - W_t^*(Y_t + R_t^S\bar{S}; \Gamma_t, \widetilde{\mathbf{\Gamma}}) \\ &\geq \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t + R_t^S(\bar{S}+\Delta); \Gamma_t) - \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t + R_t^S\bar{S}; \Gamma_t) = w(\Gamma_t) \end{aligned}$$

for all Δ and Γ_t, and the terms on the right-hand side, $w(\Gamma_t)$ is increasing in Γ. To show the former, I denote the $C_t(\Delta)$ as the level of consumption under available wealth $Y_t + R_t^S(\bar{S}+\Delta)$, and $X_{t+1}(\Delta)$ to denote $X_{t+1}(\Delta) = Y_{t+1} + R_{t+1}^S(Y_t + R_t^S(\bar{S}+\Delta) - C_t(\Delta))$. Note that $C_t(\Delta)$ and $X_{t+1}(\Delta)$ are increasing functions of Δ. We can rearrange $v(\Gamma_t) - w(\Gamma_t)$ as,

$$\begin{aligned} v(\Gamma_t) - w(\Gamma_t) =& u(C_t(\Delta)) + \delta\mathbb{E}_t\left[W_{t+1}^*(X_{t+1}(\Delta); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}})\right] \\ &- u(C_t(0)) - \delta\mathbb{E}_t\left[W_{t+1}^*(X_{t+1}(0); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}})\right] \\ &- u(C_t(\Delta)) - \delta\mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}(\Delta); \widetilde{\Gamma}_{t+1})\right] \\ &+ u(C_t(0)) + \delta\mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}(0); \widetilde{\Gamma}_{t+1})\right] \\ =& \delta\mathbb{E}_t\left[W_{t+1}^*(X_{t+1}(\Delta); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}})\right] - \delta\mathbb{E}_t\left[W_{t+1}^*(X_{t+1}(0); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}})\right] \\ &- \left[\delta\mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}(0); \widetilde{\Gamma}_{t+1})\right] - \delta\mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}(\Delta); \widetilde{\Gamma}_{t+1})\right]\right]. \end{aligned}$$

By the fact that [b] holds at period $t+1$, this term is positive.

Now, we turn to the fact that $w(\Gamma_t)$ is an increasing function. Using the differentiability of $w(\Gamma_t)$[1], it is sufficient to show that $\partial\widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(X_t; \Gamma_t)/\partial X_t = u'(C_t(X_t; \Gamma_t) - \Gamma_t)$ is increasing in Γ_t. For any $\Delta_\Gamma > 0$, $C_t(X_t; \Gamma_t + \Delta_\Gamma) - C_t(X_t; \Gamma_t) < \Delta_\Gamma$, hence $w(\Gamma_t)$ is increasing in Γ_t by the concavity of $u(\cdot)$. □

[1]As in Propositions 1.3.1 and 1.3.2, differentiability is guaranteed by Lemma A.2.1.

APPENDIX B

Other Figures and Tables

B.1. Additional Tables for Chapter 1

		‘89	‘92	‘95	‘98	‘01	‘04	‘07	‘10	‘13	‘16	‘19	Avg.
All	NW	0.71	0.71	0.70	0.72	0.74	0.74	0.74	0.77	0.77	0.79	0.78	0.74
	LIQ	0.88	0.87	0.89	0.89	0.90	0.89	0.90	0.91	0.90	0.91	0.89	0.89
	HLIQ	0.81	0.79	0.79	0.79	0.81	0.82	0.82	0.85	0.83	0.84	0.82	0.82
	Illiquid	0.68	0.68	0.66	0.68	0.70	0.70	0.70	0.72	0.73	0.76	0.76	0.71
Graduate education	NW	0.70	0.68	0.68	0.68	0.72	0.71	0.70	0.73	0.73	0.75	0.75	0.71
	LIQ	0.86	0.86	0.86	0.86	0.89	0.87	0.88	0.89	0.88	0.87	0.87	0.87
	HLIQ	0.76	0.76	0.77	0.75	0.78	0.80	0.79	0.82	0.81	0.79	0.78	0.78
	Illiquid	0.66	0.66	0.64	0.64	0.68	0.66	0.67	0.67	0.70	0.73	0.73	0.68
Tertiary education	NW	0.69	0.67	0.67	0.69	0.70	0.68	0.71	0.70	0.72	0.76	0.72	0.70
	LIQ	0.85	0.83	0.86	0.87	0.87	0.86	0.87	0.85	0.85	0.89	0.85	0.86
	HLIQ	0.79	0.73	0.72	0.79	0.78	0.77	0.78	0.78	0.75	0.82	0.76	0.77
	Illiquid	0.64	0.64	0.65	0.66	0.66	0.65	0.68	0.67	0.68	0.72	0.69	0.67
Secondary education	NW	0.67	0.67	0.64	0.65	0.67	0.69	0.66	0.71	0.73	0.74	0.75	0.69
	LIQ	0.85	0.82	0.87	0.83	0.83	0.85	0.85	0.88	0.90	0.88	0.89	0.86
	HLIQ	0.81	0.80	0.81	0.75	0.78	0.81	0.79	0.86	0.83	0.83	0.82	0.81
	Illiquid	0.66	0.64	0.60	0.61	0.63	0.64	0.64	0.67	0.69	0.72	0.73	0.66
1st income tertile	NW	0.64	0.65	0.64	0.66	0.66	0.65	0.66	0.68	0.67	0.71	0.70	0.67
	LIQ	0.85	0.84	0.86	0.86	0.87	0.85	0.86	0.86	0.84	0.86	0.83	0.85
	HLIQ	0.78	0.73	0.74	0.75	0.77	0.76	0.75	0.80	0.74	0.76	0.73	0.76
	Illiquid	0.61	0.62	0.61	0.62	0.63	0.62	0.63	0.64	0.64	0.67	0.68	0.63
2nd income tertile	NW	0.58	0.60	0.58	0.58	0.62	0.60	0.58	0.60	0.67	0.63	0.64	0.61
	LIQ	0.76	0.80	0.82	0.79	0.81	0.78	0.78	0.77	0.85	0.81	0.82	0.80
	HLIQ	0.67	0.72	0.74	0.70	0.71	0.73	0.71	0.74	0.79	0.71	0.71	0.72
	Illiquid	0.53	0.53	0.50	0.49	0.52	0.51	0.49	0.51	0.57	0.54	0.55	0.52
3rd income tertile	NW	0.72	0.72	0.70	0.69	0.74	0.75	0.71	0.75	0.78	0.76	0.74	0.73
	LIQ	0.88	0.86	0.87	0.86	0.90	0.93	0.91	0.92	0.93	0.92	0.89	0.90
	HLIQ	0.84	0.82	0.78	0.81	0.81	0.87	0.82	0.85	0.88	0.86	0.82	0.83
	Illiquid	0.71	0.69	0.64	0.66	0.67	0.67	0.67	0.69	0.70	0.71	0.71	0.68
Managerial	NW	0.70	0.69	0.69	0.71	0.71	0.71	0.72	0.74	0.73	0.76	0.75	0.72
	LIQ	0.85	0.85	0.87	0.88	0.88	0.86	0.89	0.88	0.87	0.88	0.86	0.87
	HLIQ	0.75	0.75	0.75	0.77	0.79	0.78	0.79	0.82	0.79	0.79	0.77	0.78
	Illiquid	0.67	0.67	0.65	0.68	0.68	0.68	0.69	0.71	0.70	0.73	0.73	0.69
Technical	NW	0.72	0.68	0.70	0.70	0.69	0.72	0.68	0.77	0.75	0.78	0.77	0.72
	LIQ	0.89	0.85	0.88	0.88	0.87	0.85	0.88	0.92	0.89	0.90	0.89	0.88
	HLIQ	0.86	0.78	0.80	0.80	0.76	0.79	0.80	0.87	0.81	0.84	0.82	0.81
	Illiquid	0.68	0.65	0.66	0.68	0.64	0.66	0.64	0.67	0.72	0.72	0.75	0.68
Other occupation	NW	0.62	0.63	0.60	0.61	0.63	0.64	0.64	0.68	0.65	0.66	0.70	0.64
	LIQ	0.80	0.80	0.81	0.81	0.80	0.78	0.79	0.82	0.82	0.79	0.86	0.81
	HLIQ	0.78	0.76	0.76	0.73	0.77	0.76	0.75	0.76	0.77	0.76	0.76	0.76
	Illiquid	0.60	0.59	0.57	0.54	0.56	0.56	0.59	0.59	0.61	0.64	0.67	0.59

TABLE B.1. Gini index of the United States: 1989–2019

Notes: ‘NW’ refers to total net worth. ‘LIQ’ and ‘Illiquid’ represent liquid and illiquid assets respectively. ‘HLIQ’ refers to the highly liquid assets.

	ρ	p-value	Expenditure shock
Adult care	0.338	0.008	
Alcohol away from home	0.190	<0.001	
Alcohol at home	1.000	<0.001	
Child care	0.118	0.026	O
Clothes	-0.015	0.009	O
Clothing services	0.205	<0.001	
Domestic services	0.257	0.136	O
Education durables	0.034	0.063	O
Education services	0.003	0.731	O
Entertainment durables	-0.003	0.588	O
Entertainment services	0.054	0.005	
Fees and charges	0.023	0.215	O
Food away from home	0.064	<0.001	
Food at home	0.883	<0.001	
Furniture rental	1.000	<0.001	
Gasoline expenses	0.447	<0.001	
Health care durable	0.010	0.397	O
Health insurance	0.242	<0.001	
Health care service	0.026	<0.001	
Other household expenditures	0.066	0.002	
Home insurance	0.002	0.933	O
Home management	-0.042	0.004	O
Home maintenance and repairs	0.036	<0.001	
Home-related equipment and supplies	0.093	0.093	O
Household furnishings and equipment	0.013	0.282	O
Household textiles and linens	0.041	0.298	O
Jewelry	0.005	0.657	O
Life insurance	-0.084	0.041	O
Occupational expenses	0.115	<0.001	
Parking expenses	0.001	0.941	O
Public transportation	0.012	0.078	O
Personal care products	-0.036	0.217	O
Telephone services	0.246	<0.001	
Personal care services	0.999	<0.001	
Reading material	1.000	<0.001	
Rent	0.063	<0.001	
Rented vehicles	0.048	<0.001	
Tobacco	1.000	<0.001	
Utilities	0.056	<0.001	
Vehicle	0.002	0.172	O
Other vehicle-related durables	0.021	0.277	O
Vehicle insurance	-0.050	<0.001	O
Vehicle service	-0.005	0.430	O
Water and other public services	-0.142	<0.001	O

Table B.2. Classification of expenditure shocks

B.2. Additional Figures for Chapter 2

Following figures show how the alternative sample selection scheme of choosing subjects that exhibit fewer monotonicity violations affect the distribution of present bias and sophistication.

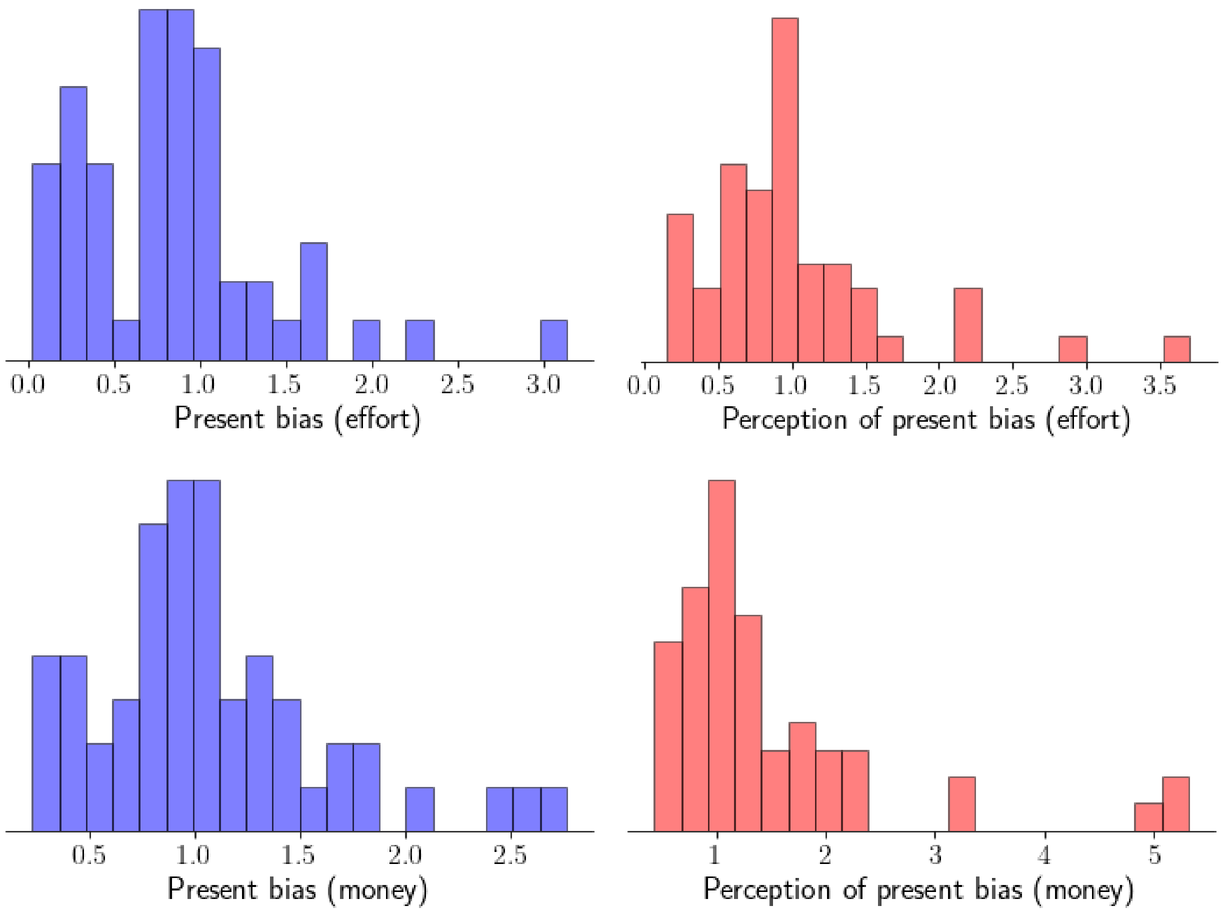

FIGURE B.1. Structural estimates of present bias and its perception in effort and money domain

Note: based on subjects exhibiting monotonicity violations less than three times out of nine type of decision problems.

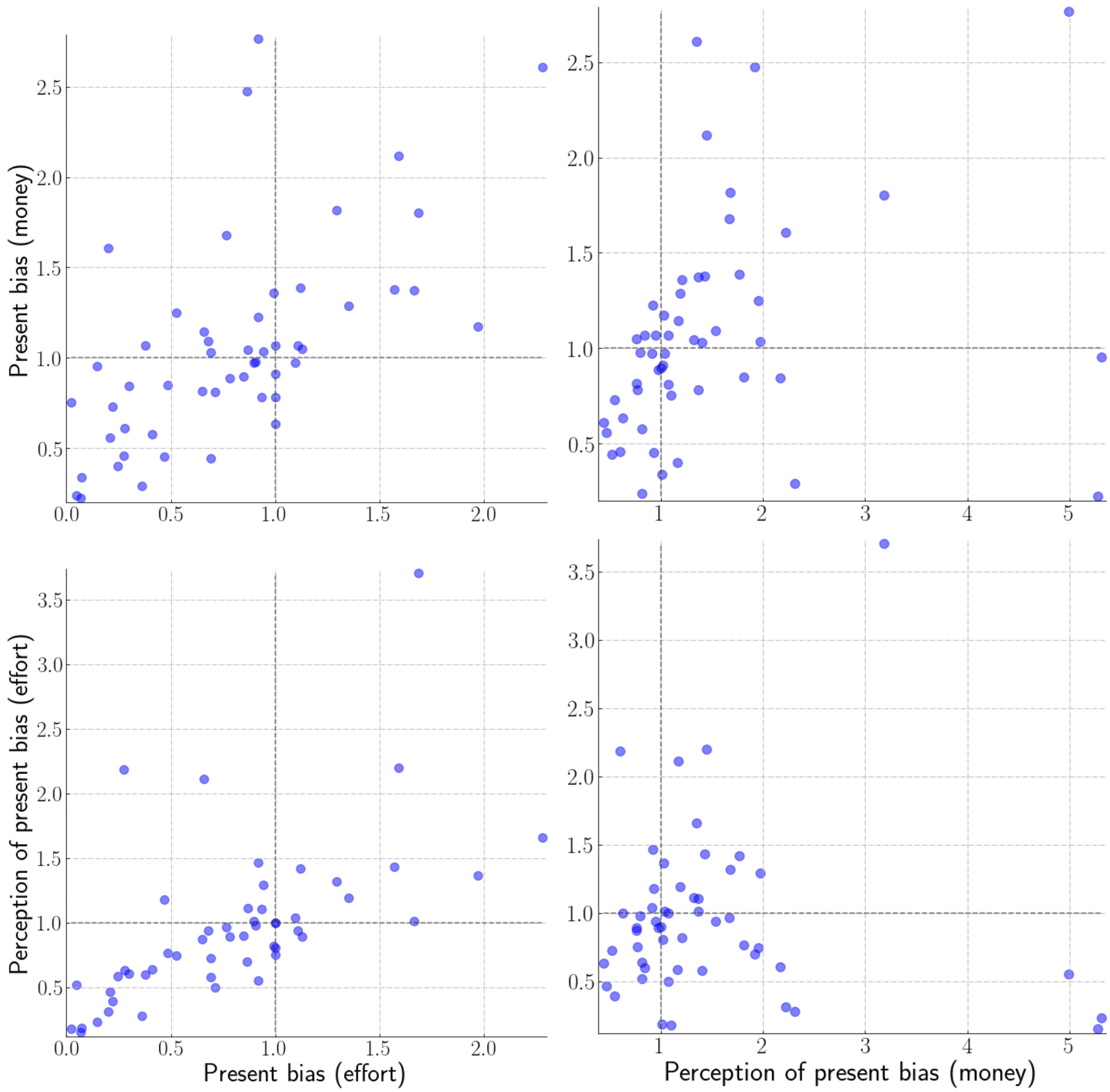

Figure B.2. Comparison of time discounting factors
Note: based on subjects exhibiting monotonicity violations less than three times out of nine type of decision problems.

B.3. Additional Figures for Chapter 3

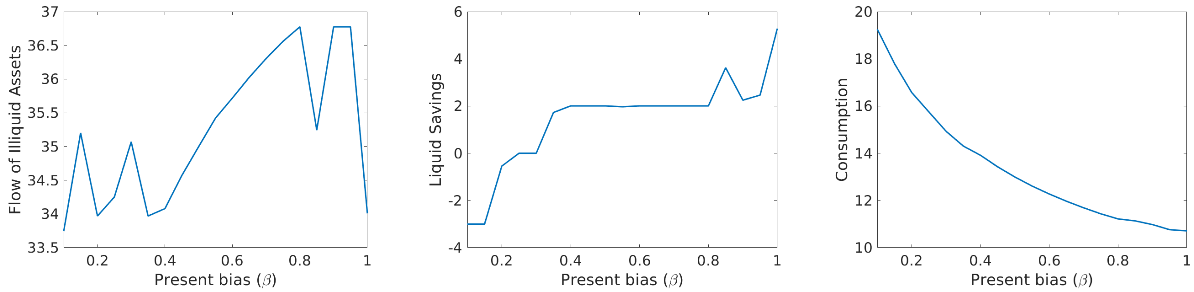

FIGURE B.3. Relationship between key control variables and present bias (β) of a naive agent with a high disposable wealth and a low illiquid wealth

Note: x-axis is the degree of present bias $\beta \in [0.1, 1]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a low stock of illiquid wealth $R_1^z Z_0 = 0$.

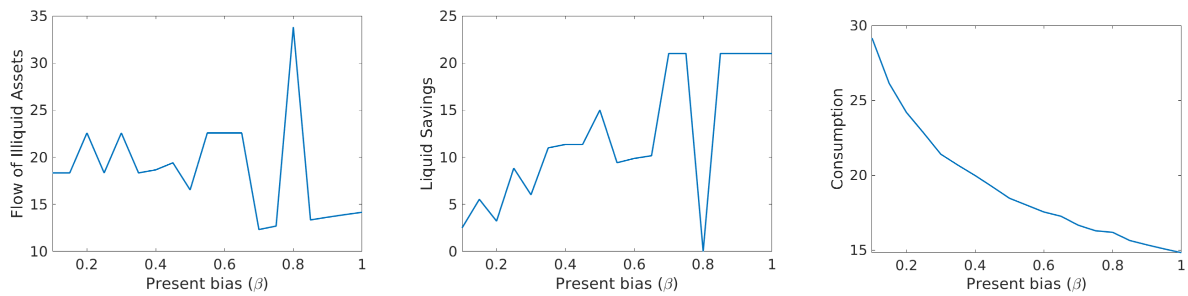

FIGURE B.4. Relationship between key control variables and present bias (β) of a naive agent with a high disposable wealth and a high illiquid wealth

Note: x-axis is the degree of present bias $\beta \in [0.1, 1]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a high stock of illiquid wealth $R_1^z Z_0 = 50$.

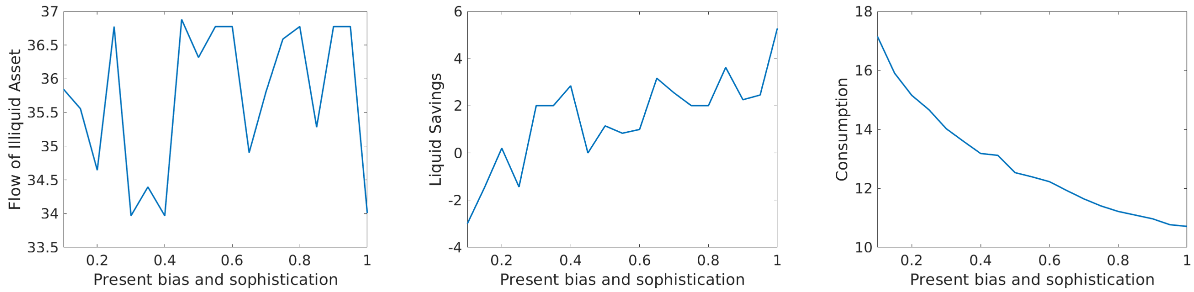

FIGURE B.5. Relationship between key control variables and present bias (β) of a sophisticated agent with a high disposable wealth and a low illiquid wealth

Note: x-axis is the degree of present bias, which equals to the sophistication ($\beta = \widehat{\beta}$), which has the range $\beta \in [0.1, 1.0]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a low stock of illiquid wealth $R_1^z Z_0 = 0$.

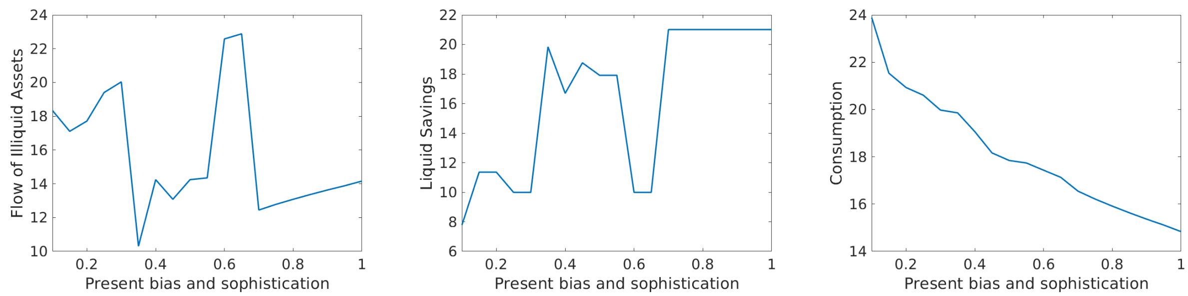

FIGURE B.6. Relationship between key control variables and present bias (β) of a sophisticated agent with a high disposable wealth and a high illiquid wealth

Note: x-axis is the degree of present bias, which equals to the sophistication ($\beta = \widehat{\beta}$), which has the range $\beta \in [0.1, 1.0]$. y-axis is the flow of illiquid assets, liquid savings, and consumption at the initial period when given a high disposable wealth $Y_1 + R_1^s S_0 = 50$ and a high stock of illiquid wealth $R_1^z Z_0 = 50$.

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