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A remark about the decoupling approximation of damped linear systems

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Abstract

A common approximation in the analysis of non-classically damped systems is to ignore the off-diagonal elements of the modal damping matrix. This procedure is termed the decoupling approximation. Contrary to widely accepted beliefs, it is shown numerically that over a finite range, errors due to the decoupling approximation can continuously increase at any specified rate while the modal damping matrix becomes more and more diagonal.

Keywords: Linear systems; Modal analysis; Coordinate coupling; Decoupling approximation

1. Introduction

It is well known that an undamped linear system possesses classical normal modes, and that in each mode different parts of the system vibrate in a synchronous manner. The normal modes constitute a modal matrix, which defines a linear coordinate transformation that decouples the undamped system. This process of decoupling the equation of motion of an undamped vibratory system is a time-honored procedure termed modal analysis. Upon decoupling, an undamped linear system can be treated as a series of independent single-degree-of-freedom systems.

In the presence of damping, a linear system cannot be decoupled by modal analysis unless it possesses a full set of classical normal modes, in which case the system is said to be classically damped (Caughey and O'Kelly, 1965). Practically speaking, classical damping means that energy dissipation is almost uniformly distributed throughout the system. This assumption is violated for systems consisting of two or more parts with significantly different levels of damping. Examples of such systems include soil-structure systems (Clough and Mojtahedi, 1976), base-isolated structures (Tsai and Kelly, 1988), and systems in which coupled vibrations of structures and fluids occur. In fact, experimental modal testing suggests that no real physical system is strictly classically damped (Sestieri and Ibrahim, 1994).

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A common approximation in the analysis of non-classically damped systems is to ignore the off-diagonal elements of the modal damping matrix. This procedure is termed the decoupling approximation, which could substantially streamline the solution of large-scale systems. It is generally believed that diagonal dominance of the modal damping matrix is a sufficient condition for the decoupling approximation. The purpose of this paper is to show numerically that, over a finite range, errors due to the decoupling approximation can continuously increase at any specified rate while the modal damping matrix becomes more diagonal with its off-diagonal elements decreasing continuously in magnitude. Thus small off-diagonal elements of the modal damping matrix may not be sufficient to neglect modal coupling by the decoupling approximation.

2. Errors due to the decoupling approximation

The equations of motion of an n-degree-of-freedom linear system can be written in the form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t), \tag{1}$$

where the generalized coordinate $\mathbf{x}$ and excitation $\mathbf{f}(t)$ are real n-dimensional column vectors. The mass matrix $\mathbf{M}$, the damping matrix $\mathbf{C}$, and the stiffness matrix $\mathbf{K}$ are real matrices of order $n \times n$. For passive systems, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are symmetric and positive definite. Associated with the undamped system is a generalized eigenvalue problem (Meirovitch, 1967)

$$\mathbf{K}\mathbf{u} = \lambda\mathbf{M}\mathbf{u}. \tag{2}$$

Owing to the definiteness of the coefficient matrices, the n eigenvalues $\lambda_i = \omega_i^2$ are real and positive. The corresponding eigenvectors are real and orthogonal with respect to $\mathbf{M}$ and $\mathbf{K}$ such that $\mathbf{u}_i^{\mathrm{T}}\mathbf{M}\mathbf{u}_j = 0$ and $\mathbf{u}_i^{\mathrm{T}}\mathbf{K}\mathbf{u}_j = 0$ for $i \neq j$. Define the modal and spectral matrices respectively by

$$\mathbf{U} = [\mathbf{u}_1|\mathbf{u}_2|\cdots|\mathbf{u}_n], \tag{3}$$

$$\mathbf{\Omega} = \mathrm{diag}[\omega_1^2, \omega_2^2, \ldots, \omega_n^2]. \tag{4}$$

Upon normalization, the orthogonality of the modes can be expressed in a compact form:

$$\mathbf{U}^{\mathrm{T}}\mathbf{M}\mathbf{U} = \mathbf{I}, \tag{5}$$

$$\mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \mathbf{\Omega}. \tag{6}$$

The modal matrix $\mathbf{U}$ defines an invertible coordinate transformation

$$\mathbf{x} = \mathbf{U}\mathbf{q}. \tag{7}$$

In terms of the principal coordinate $\mathbf{q}$, the equation of motion takes the canonical form

$$\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{\Omega}\mathbf{q} = \mathbf{g}(t) \tag{8}$$

for which $\mathbf{g}(t) = \mathbf{U}^{\mathrm{T}}\mathbf{f}(t)$. The symmetric matrix

$$\mathbf{D} = \mathbf{U}^{\mathrm{T}}\mathbf{C}\mathbf{U} \tag{9}$$

is referred to as the modal damping matrix. While an undamped system can be decoupled entirely by modal analysis, a damped system is completely decoupled if and only if the modal damping matrix is diagonal.

Write $\mathbf{D}$ in the form

$$\mathbf{D} = \mathbf{D}_d + \mathbf{D}_o, \tag{10}$$

where $\mathbf{D}_d = \mathrm{diag}[d_{11}, d_{22}, \ldots, d_{nn}]$ is a diagonal matrix composed of the diagonal elements of $\mathbf{D}$, and $\mathbf{D}_o$ is a matrix with zero diagonal elements and whose off-diagonal elements coincide with those in $\mathbf{D}$. The decoupling approximation amounts to simply neglecting the off-diagonal elements of $\mathbf{D}$ and thus replacing $\mathbf{D}$ by $\mathbf{D}_d$. The system response by the decoupling approximation satisfies the decoupled equation

$$\ddot{\mathbf{q}}_a(t) + \mathbf{D}_d\dot{\mathbf{q}}_a(t) + \mathbf{\Omega}\mathbf{q}_a(t) = \mathbf{g}(t). \tag{11}$$

The error due to the decoupling approximation is equal to

$$\mathbf{e}(t) = \mathbf{q}(t) - \mathbf{q}_a(t). \tag{12}$$

It can be shown (Morzfeld et al., in press) that the relative steady-state error due to the decoupling approximation is given by

$$s(\mathrm{i}\omega) = \frac{\|\mathbf{E}(\mathrm{i}\omega)\|}{\|\mathbf{Q}(\mathrm{i}\omega)\|} = \frac{\|\omega \mathbf{H}_a(\mathrm{i}\omega)\mathbf{D}_o\mathbf{Q}(\mathrm{i}\omega)\|}{\|\mathbf{Q}(\mathrm{i}\omega)\|}, \tag{13}$$

where $\mathbf{E}(\mathrm{i}\omega)$ and $\mathbf{Q}(\mathrm{i}\omega)$ are, respectively, the Fourier transforms of $\mathbf{e}(t)$ and $\mathbf{q}(t)$, $\mathrm{i} = \sqrt{-1}$ and $\mathbf{H}_a(\mathrm{i}\omega) = (\mathbf{\Omega} - \omega^2\mathbf{I} + \mathrm{i}\omega\,\mathbf{D}_d)^{-1}$ is the frequency response matrix of the decoupled system (11). While any vector norm may be used in Eq. (13), the Euclidean norm is chosen in this paper. However, any other norm will yield similar results in subsequent analysis.

3. Indices of diagonality of the modal damping matrix

How can one quantify the property of being diagonal? When does a matrix become more diagonal than another? These issues will first be clarified in the present section.

The modal damping matrix $\mathbf{D}$ is said to be diagonally dominant (Horn and Johnson, 1985) if

$$|d_{ii}| \geqslant \sum_{\substack{j=1 \\ j\neq i}}^{n} |d_{ij}| \tag{14}$$

for all $i = 1, \ldots, n$. The matrix $\mathbf{D}$ is diagonally dominant in a generalized sense (Berman and Plemmons, 1994) if there exist scalars α_i such that

$$|d_{ii}| \geqslant \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{\alpha_i}{\alpha_j}|d_{ij}|, \quad |d_{ii}| \geqslant \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{\alpha_j}{\alpha_i}|d_{ij}| \tag{15}$$

for all $i = 1, \ldots, n$. These definitions of diagonal dominance have solid footing in linear algebra and many important properties of diagonally dominant matrices have been established. Clearly, a diagonally dominant matrix is diagonally dominant in the generalized sense. Recall the definitions of $\mathbf{D}_d$ and $\mathbf{D}_o$ in Eq. (10). Let $|\mathbf{D}_d| = \mathrm{diag}[|d_{11}|, |d_{22}|, \ldots, |d_{nn}|]$ and similarly let $|\mathbf{D}_o|$ be a matrix whose elements are the absolute values of those in $\mathbf{D}_o$. It can be shown (Graham, 1987) that if the spectral radius (largest absolute value of any eigenvalue) of $|\mathbf{D}_d^{-1}||\mathbf{D}_o|$ satisfies

$$\sigma(|\mathbf{D}_d^{-1}||\mathbf{D}_o|) < 1, \tag{16}$$

then $\mathbf{D}$ is diagonally dominant in the generalized sense.

Based upon Eq. (14), an index of diagonality of modal damping may be readily defined as

$$\rho(\mathbf{D}) = \sum_{j=1}^{n} |d_{jj}| \Bigg/ \sum_{\substack{j,i=1 \\ j\neq i}}^{n} |d_{ji}|. \tag{17}$$

Clearly, $0 \leqslant \rho(\mathbf{D}) \leqslant \infty$ for any modal damping matrix $\mathbf{D}$. If $\mathbf{D}$ is diagonally dominant, then $\rho(\mathbf{D}) \geqslant 1$. A large value of $\rho(\mathbf{D})$ indicates a more diagonal matrix and, for a diagonal matrix, $\rho(\mathbf{D}) = \infty$. Another index of diagonality may be based upon the spectral radius of $|\mathbf{D}_d^{-1}||\mathbf{D}_o|$ in Eq. (16) and defined as

$$\rho_1(\mathbf{D}) = \sigma(|\mathbf{D}_d^{-1}||\mathbf{D}_o|). \tag{18}$$

If $\mathbf{D}$ is diagonally dominant in the generalized sense, $0 \leqslant \rho_1(\mathbf{D}) \leqslant 1$. When $\mathbf{D}$ is diagonal, $\rho_1(\mathbf{D}) = 0$. A small value of $\rho_1(\mathbf{D})$ indicates a more diagonal matrix and the two indices $\rho(\mathbf{D})$ and $\rho_1(\mathbf{D})$ have opposite trends. An advantage of using $\rho_1(\mathbf{D})$ is that it lies within a finite range. On the other hand, $\rho(\mathbf{D})$ can be computed more readily.

It is certainly possible to define other indices of diagonality. However, it will become evident that the choice of an index of diagonality of $\mathbf{D}$ is of minor significance in the characterization of modal coupling (Morzfeld et al., in press).

4. Numerical examples

It is generally accepted that errors due to the decoupling approximation must be small if the off-diagonal elements of the modal damping matrix $\mathbf{D}$ are small. In addition, the errors should decrease as $\mathbf{D}$ becomes more and more diagonal. Numerical examples are constructed to yield contradictory results: diagonal dominance may continuously increase while errors due to the decoupling approximation also continuously increase. Moreover, the errors can increase at any specified rate.

Example 1. Consider two four-degree-of-freedom systems (Ajavakom, 2005) of the form (8). System 1 is governed by $\ddot{\mathbf{q}} + \mathbf{D}_1\dot{\mathbf{q}} + \mathbf{\Omega}_1\mathbf{q} = \mathbf{g}(t)$ where the spectral matrix, the modal damping matrix, and the excitation are given by

$$\mathbf{\Omega}_1 = \text{diag}[\,3.95^2 \quad 3.98^2 \quad 4.00^2 \quad 4.10^2\,], \tag{19}$$

$$\mathbf{D}_1 = \begin{bmatrix} 1.61 & -0.1865 & -0.1742 & 0.3838 \\ -0.1865 & 1.7 & 0.3792 & -0.1773 \\ -0.1742 & 0.3792 & 1.8 & -0.1742 \\ 0.3838 & -0.1773 & -0.1742 & 1.75 \end{bmatrix}, \tag{20}$$

$$\mathbf{g}(t) = \hat{\mathbf{g}}\exp(\mathrm{i}\omega t) = [\,1, \quad 1, \quad 1, \quad 1\,]^{\mathrm{T}}\exp(\mathrm{i}4.16t). \tag{21}$$

The equation of motion of System 2 has the form $\ddot{\mathbf{q}} + \mathbf{D}_2\dot{\mathbf{q}} + \mathbf{\Omega}_1\mathbf{q} = \mathbf{g}(t)$ which differs from System 1 only in the modal damping matrix:

$$\mathbf{D}_2 = \begin{bmatrix} 1.61 & 0.0009 & 0.04 & 0.041 \\ 0.0009 & 1.7 & 0.0227 & 0.0376 \\ 0.04 & 0.0227 & 1.8 & 0.04 \\ 0.041 & 0.0376 & 0.04 & 1.75 \end{bmatrix}. \tag{22}$$

It can be observed that both $\mathbf{D}_1$ and $\mathbf{D}_2$ satisfy Eq. (14) and are therefore diagonally dominant. Utilizing the proposed indices of diagonality, it is found that

$$\rho(\mathbf{D}_1) = 2.3 \ll 18.8 = \rho(\mathbf{D}_2), \tag{23}$$

$$\rho_1(\mathbf{D}_1) = 0.43 \gg 0.055 = \rho_1(\mathbf{D}_2). \tag{24}$$

Thus $\mathbf{D}_2$ is more diagonal than $\mathbf{D}_1$. This is perhaps obvious by inspection since each off-diagonal elements of $\mathbf{D}_2$ is at least an-order-of-magnitude smaller than the corresponding element of $\mathbf{D}_1$. Intuitively, one would expect System 2 to yield a smaller error in the decoupling approximation than System 1. However, calculation of the steady-state error in the decoupling approximation yields an opposite result:

$$s_1(\mathrm{i}\omega) = 2.76\% < 5.31\% = s_2(\mathrm{i}\omega). \tag{25}$$

Hence, errors in the decoupling approximation can be larger for systems whose modal damping matrix is more diagonal.

This example can be extended. Consider a series of systems $\ddot{\mathbf{q}} + \mathbf{D}_\alpha\dot{\mathbf{q}} + \mathbf{\Omega}_1\mathbf{q} = \mathbf{g}(t)$ indexed by a parameter α in such a way that $\mathbf{D}_\alpha$ is linearly interpolated between $\mathbf{D}_1$ and $\mathbf{D}_2$:

$$\mathbf{D}_\alpha = (1-\alpha)\mathbf{D}_1 + \alpha\mathbf{D}_2, \quad 0 \leqslant \alpha \leqslant 1. \tag{26}$$

As α increases from 0 to 1, the diagonal entries of $\mathbf{D}_\alpha$ remain constant while the index of diagonality $\rho(\mathbf{D}_\alpha)$ increases continuously from $\rho(\mathbf{D}_1) = 2.3$ to $\rho(\mathbf{D}_2) = 18.8$. At the same time, the second index of diagonality $\rho_1(\mathbf{D}_\alpha)$ decreases continuously from $\rho_1(\mathbf{D}_1) = 0.43$ to $\rho_1(\mathbf{D}_2) = 0.06$. In Fig. 1, the steady-state error $s(\mathrm{i}\omega)$

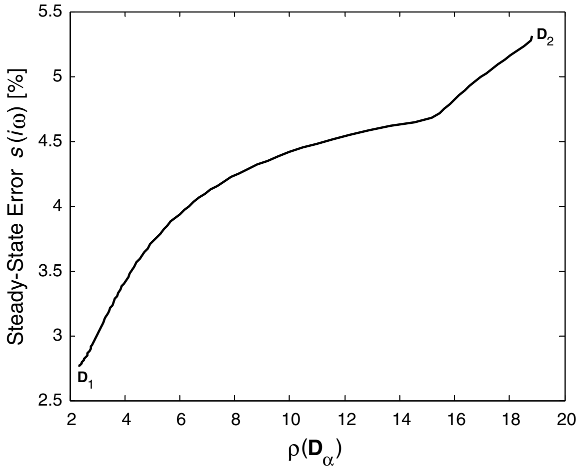

Fig. 1. Steady-state error due to the decoupling approximation vs. diagonality ρ of the damping matrix.

due to the decoupling approximation is plotted against the index of diagonality $\rho(\mathbf{D})$. It can be observed that as the modal damping matrix becomes more diagonal, the error due to the decoupling approximation increases continuously.

If the choice of an index of diagonality is of minor importance, one should be able to obtain consistent results using the second index of diagonality $\rho_1(\mathbf{D}_\alpha)$. As a measure of diagonality, $\rho_1(\mathbf{D}_\alpha)$ and $\rho(\mathbf{D}_\alpha)$ have opposite trends. For this reason, the steady-state error due to the decoupling approximation is plotted against the reciprocal of $\rho_1(\mathbf{D})$ in Fig. 2. As expected, the error curves in Figs. 1 and 2 are very similar. Both demonstrate that as diagonality of the modal damping matrix continuously increases, errors in the decoupling approximation continuously increase as well.

It is certainly possible to define a non-linear interpolation between the end-states $\mathbf{D}_1$ and $\mathbf{D}_2$. Instead of the linear interpolation represented by Eq. (26), one could use

$$\mathbf{D}_\alpha = \mathbf{D}_1 + \alpha^n(\mathbf{D}_2 - \mathbf{D}_1) = (1 - \alpha^n)\mathbf{D}_1 + \alpha^n\mathbf{D}_2, \quad 0 \leqslant \alpha \leqslant 1, \tag{27}$$

with any $n \geqslant 1$ to define a series of systems whose end-state are Systems 1 and 2. However, Eq. (27) also leads to Figs. 1 and 2 because all interpolations between $\mathbf{D}_1$ and $\mathbf{D}_2$ result in the same intermediate states as α increases from 0 to 1.

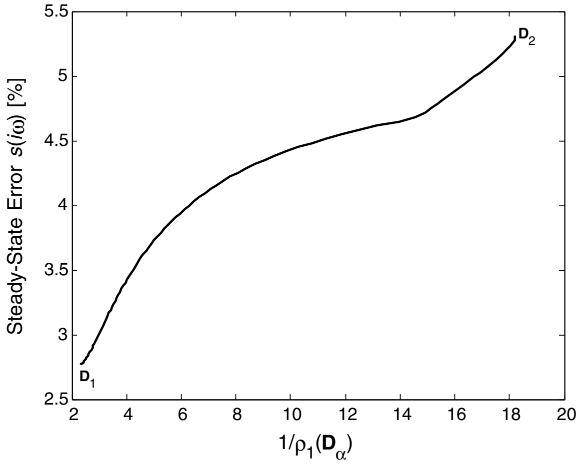

Fig. 2. Steady-state error due to the decoupling approximation vs. diagonality $1/\rho_1$ of the damping matrix.

Example 2. Consider a collection of four-degree-of-freedom linear systems. System 1 is the same as in Example 1, with its spectral matrix, modal damping matrix and excitation given, respectively, by (19)–(21). The equations of motion of Systems 3–5 have the form $\ddot{\mathbf{q}} + \mathbf{D}_i\mathbf{q} + \mathbf{\Omega}_1\mathbf{q} = \mathbf{g}(t)$, $i = 3, 4, 5$, which differ from System 1 only in the off-diagonal elements of the modal damping matrices

$$\mathbf{D}_3 = \begin{bmatrix} 1.61 & -0.0947 & -0.0140 & 0.3911 \\ -0.0947 & 1.7 & 0.3367 & -0.0125 \\ -0.0140 & 0.3367 & 1.8 & -0.0140 \\ 0.3911 & -0.0125 & -0.0140 & 1.75 \end{bmatrix}, \tag{28}$$

$$\mathbf{D}_4 = \begin{bmatrix} 1.61 & 0.0762 & 0.0008 & 0.1142 \\ 0.0762 & 1.7 & 0.2090 & 0.0006 \\ 0.0008 & 0.2090 & 1.8 & 0.0388 \\ 0.1142 & 0.0006 & 0.0388 & 1.75 \end{bmatrix}, \tag{29}$$

$$\mathbf{D}_5 = \begin{bmatrix} 1.61 & -0.0008 & 0.0863 & 0.1047 \\ -0.000 & 1.7 & 0.0380 & 0.0006 \\ 0.0863 & 0.0380 & 1.8 & 0.0863 \\ 0.1047 & 0.0006 & 0.0863 & 1.75 \end{bmatrix}. \tag{30}$$

It can be checked that

$$\rho(\mathbf{D}_1) = 2.32 < \rho(\mathbf{D}_3) = 3.97 < \rho(\mathbf{D}_4) = 7.80 < \rho(\mathbf{D}_5) = 10.83, \tag{31}$$

$$\rho_1(\mathbf{D}_1) = 0.43 > \rho_1(\mathbf{D}_3) = 0.26 > \rho_1(\mathbf{D}_4) = 0.14 > \rho_1(\mathbf{D}_5) = 0.11. \tag{32}$$

Thus $\mathbf{D}_5$ is more diagonal than $\mathbf{D}_4$, $\mathbf{D}_4$ is more diagonal than $\mathbf{D}_3$, and $\mathbf{D}_3$ is more diagonal than $\mathbf{D}_1$. Intuitively, one would expect that System 1 has the largest steady-state error due to the decoupling approximation among these four systems. However, calculation of the steady-state error due to the decoupling approximation yields the opposite result:

$$s_1(\mathrm{i}\omega) = 2.76\% < s_5(\mathrm{i}\omega) = 10.83\% < s_4(\mathrm{i}\omega) = 12.76\% < s_3(\mathrm{i}\omega) = 23.04\%. \tag{33}$$

The steady-state error is significantly smaller in System 1 than in Systems 3–5.

In analogy to Example 1, define series of systems, doubly indexed by $i = 3, 4, 5$ and the parameter α, such that the modal damping matrix $\mathbf{D}^i_\alpha$ is linearly interpolated between $\mathbf{D}_1$ and $\mathbf{D}_i$:

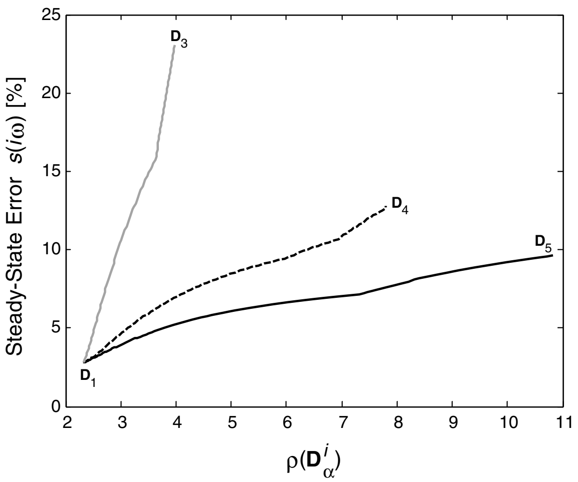

Fig. 3. Steady-state error due to the decoupling approximation vs. diagonality ρ of the damping matrix.

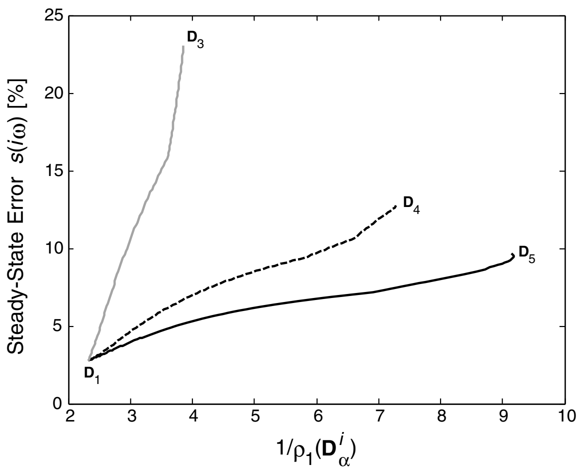

Fig. 4. Steady-state error due to the decoupling approximation vs. diagonality $1/\rho_1$ of the damping matrix.

$$\mathbf{D}^i_\alpha = (1-\alpha)\mathbf{D}_1 + \alpha\mathbf{D}_i, \quad i = 3,4,5, \quad 0 \leqslant \alpha \leqslant 1. \tag{34}$$

As α increases from 0 to 1, the diagonal entries of $\mathbf{D}^i_\alpha$ remain constant while the index of diagonality $\rho(\mathbf{D}^i_\alpha)$ increases continuously for $i = 3$, 4, 5. The steady-state error due to the decoupling approximation $s(\mathrm{i}\omega)$ can be computed for the three series of systems and the results are plotted against the index of diagonality $\rho(\mathbf{D}^i_\alpha)$ in Fig. 3. A qualitatively identical plot is obtained in Fig. 4, where $\rho_1(\mathbf{D}^i_\alpha)$ is used to measure the diagonality of the modal damping matrix. It can be observed from Figs. 3 and 4 that as the modal damping matrix becomes more diagonal continuously, the errors due to the decoupling approximation increase continuously for all three series of systems. While this observation has already been made in Example 1, *this example shows that the error curves can be constructed to have any gradient*, regardless of the initial modal damping matrix.

As previously explained, any non-linear interpolation between $\mathbf{D}_1$ and $\mathbf{D}_i$ for $i = 3,4,5$ also leads to Figs. 3 and 4 because all interpolations between $\mathbf{D}_1$ and $\mathbf{D}_i$ result in the same intermediate states as α increases from 0 to 1. Although a limited set of data is presented herein, extensive calculations have been performed by the authors, and all numerical simulations have yielded qualitatively identical results on the characteristics of modal coupling.

5. Conclusions

The principal coordinates of a non-classically damped linear system are coupled by the off-diagonal elements of the modal damping matrix. A common approximation is to ignore the off-diagonal elements in the modal damping matrix. This procedure is termed the decoupling approximation and amounts to neglecting coupling in the principal coordinates. The errors due to the decoupling approximation have been examined numerically in this paper. Two indices of diagonality have been introduced to quantitatively measure how diagonal the modal damping matrix is. These indices are non-negative and monotonic functions of the off-diagonal elements in the modal damping matrix.

Numerical examples have been constructed to demonstrate that, over a finite range, the errors due to the decoupling approximation can increase continuously at any specified rate while the modal damping matrix becomes more and more diagonal with its off-diagonal elements decreasing in magnitude continuously. Thus diagonal dominance of the modal damping matrix may not be sufficient to neglect modal coupling by decoupling approximation.

Using complex algebra, an explanation for the unexpected behavior exhibited in the numerical examples is provided by Morzfeld et al. (in press). Coupling effect is not as intuitive and simplistic as is usually thought. As a necessary procedure to treat large-scale systems, the decoupling approximation will be used more frequently in the next decade as more complex structures are built. Through research into the characteristics of coordinate coupling, it is hoped that the decoupling approximation can be used by practicing engineers with increased confidence and discretion.

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