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RADIATIVE DECAYS OF η AND η' AS PROBES OF QUARK CHARGES*

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ABSTRACT

Radiative decays of η and η' allow us to distinguish between fractional and integral charge quark models. Accurate measurements of $\eta' \to \gamma\gamma$ and $\eta' \to \pi^+\pi^-\gamma$ are essential to test the analysis and the preliminary conclusion, which favors the assignment of fractional charges.

Since the original formulation of the quark model, several compelling considerations have motivated the introduction of a "color" SU(3) degree of freedom, with nine quarks instead of three. This has been accomplished in two classes of models, distinguished by whether the quarks are assigned fractional[1] or integral charges.[2] Theoretical fashions may at a given moment prefer one model or the other, but on the basis of direct experimental evidence it is very difficult to decide between the two possibilities. As long as non-singlet color degrees of freedom are not excited, the spectroscopic and deep inelastic predictions of the two models are indistinguishable. Since it is conjectured that color may never be excited[3] or that the threshold is at ultra-high energies,[4] it is not clear when--or even if--the issue can be resolved experimentally.

The purpose of this note is to emphasize the unique[5] role of the $\eta(549)$ and $\eta'(958)$ mesons in providing experimental evidence of the quark charges.[6] We find that the measured rates $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta \to \pi^+\pi^-\gamma)$ and the ratio $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma)$ are most consistent with the assignment of fractional charges. The rates $\Gamma(\eta' \to \gamma\gamma)$ and $\Gamma(\eta' \to \pi^+\pi^-\gamma)$ are not yet known. Careful measurement of these rates is crucial for assessing the reliability of the analysis presented here and for choosing between the two quark models.

In the limit $p_\pi, p_\eta, p_\gamma \to 0$, the decays $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$ are determined by the low energy theorems of chiral symmetry which are exact to any finite order in perturbation theory.[7] The analagous theorem for $\pi^0 \to \gamma\gamma$ provides a principal motivation for the introduction of a color SU(3). Both color models predict the same rate for $\pi^0 \to \gamma\gamma$, but because of the SU(3) singlet component of $\eta(549)$, the prediction for $\eta \to \gamma\gamma$ is different in the two models (for the SU(3) singlet component the predictions of the two models differ by a factor of two in the amplitude). We will see that the measured rates for $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$ require a small negative mixing angle θ, in agreement with the mass mixing formula which gives $\theta = -11^\circ$. Even though the angle is small, the rates for $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$ are quite sensitive to the singlet component of the η. However, the analysis of the η decays which follows is not very sensitive to the precise value of θ. In particular, <u>for any value of θ</u>, the integral charge model predictions disagree with the measured rates.

The largest source of uncertainty is the application of the PCAC smoothness hypothesis. There do exist successful low energy applications of vector dominance, which require extrapolations over comparable ranges of mass.[8] Here this problem can only be approached empirically: the η and η' poles may or may not dominate the continuum in these applications. The PCAC mass extrapolations, the validity of PCAC for the SU(3) singlet current,[9] and the assumed η-η' mixing are best tested in this context by testing the predictions of the models against the experimental values of the four decay rates being considered.

The amplitudes for the η decays are

$$M(\eta \to \gamma\gamma) = \epsilon_{\mu\nu\alpha\beta}\, k_1^{\mu}\, k_2^{\nu}\, \epsilon_1^{\alpha}\, \epsilon_2^{\beta}\, \mathcal{F}_\eta(k_1 \cdot k_2) \qquad (1)$$

$$M(\eta \to \pi^+\pi^-\gamma) = \epsilon_{\mu\alpha\beta\gamma}\, \epsilon^{\mu}\, k^{\alpha}\, p_+^{\beta}\, p_-^{\gamma}\, \mathcal{G}_\eta(p_+ \cdot k, p_- \cdot k, \ldots) \qquad (2)$$

where ϵ and k are photon polarizations and momenta and $p_\pm$ are $\pi^\pm$ momenta. Taking $|\eta\rangle = \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle$, the low energy theorems[10] are

$$\mathcal{F}_\eta(0) = -\frac{e^2}{4\sqrt{3}\,\pi^2}\,\frac{\cos\theta}{F_8}\left(1 - 2\sqrt{2}\,\xi\,\frac{F_8}{F_1}\tan\theta\right) \qquad (3)$$

$$\mathcal{G}_\eta(0,0,\ldots) = -\frac{e}{4\sqrt{3}\,\pi^2}\,\frac{\cos\theta}{F_\pi^2\, F_8}\left(1 - \sqrt{2}\,\frac{F_8}{F_1}\tan\theta\right) \qquad (4)$$

where $F_\pi = 95$ MeV is the PCAC constant for the pion, F_8 and F_1 are the analagous constants for η_8 and η_1, and $\xi = 1(2)$ for quarks of fractional (integral) charge. We use SU(3) symmetry to relate $F_8 = F_\pi$; the experimental validity of the SU(3) relation

$F_\pi = F_k$ suggests that this is a reliable application of SU(3).

Previous authors[6] used Eq. (3) with the assumption that $F_8 = F_1$; in dynamical terms, this amounts to assuming that the singlet and octet $q\bar{q}$ wave functions have equal values at the origin. Since η and η' are far from being ideally mixed, the binding energies in the singlet and octet are very different, casting some doubt on the assumption $F_8 = F_1$. Of course, it is conceivable that F_8/F_1 is less sensitive to singlet-octet differences than the binding energy: it would be interesting to study this question in dynamical models.

Here we sidestep our ignorance of F_8/F_1 and (to some extent) of θ by using $\Gamma(\eta \to \gamma\gamma)/\Gamma(\eta \to \pi^+\pi^-\gamma) = 7.60 \pm 0.25$,[11] which is the most reliable of all the experimental quantities we consider, to determine $\frac{F_8}{F_1}\tan\theta$. The result[12] is $\frac{F_8}{F_1}\tan\theta = -0.12 \pm 0.016$ for fractional charges and $\frac{F_8}{F_1}\tan\theta = -0.05 \pm 0.007$ for integral charges. Notice that these results agree qualitatively with the usual quark model η-η' mixing hypothesis, i.e., for $F_8/F_1 \sim O(1) > 0$, θ must be small and negative.

The remaining dependence on $\cos\theta$ in Eqs.(3) and (4) is moderate provided that θ is small. For instance, for $\theta = -11^\circ$ we have for the fractional (integral) charge model $\Gamma(\eta \to \gamma\gamma) = 283 \pm 21$ eV (258 ± 16 eV) and $\Gamma(\eta \to \pi^+\pi^-\gamma) = 38 \pm 1$ eV (32 ± 0.6 eV). The fractional charge model is in better agreement with the experimental values[13] $\Gamma(\eta \to \gamma\gamma) = 324 \pm 46$ eV and $\Gamma(\eta \to \pi^+\pi^-\gamma) = 43 \pm 6$ eV. For any value of θ we have the upper bounds $\Gamma(\eta \to \gamma\gamma) < 294 \pm 22$ eV (268 ± 17 eV) and $\Gamma(\eta \to \pi^+\pi^-\gamma) < 39.3 \pm 1.3$ eV (32.9 ± 0.6 eV) for fractional (integral) quark charges. For integral quark charges the upper bound for $\eta \to \pi^+\pi^-\gamma$ is $\sim 1\frac{1}{2}$ standard deviations below the experimental value.

If the η were a pure octet state, $\theta = 0$, then we would have predicted, for both fractional and integral quark charges, that $\Gamma(\eta \to \gamma\gamma) = 164$ eV and $\Gamma(\eta \to \pi^+\pi^-\gamma) = 28.7$ eV. Thus we see that singlet-octet mixing has a major effect on these decays even if θ is small.

As we acquire additional experimental and theoretical understanding of the parameters θ and F_8/F_1, it will be possible to use the η decay rates to distinguish more sharply between the two quark models. For instance, if $\theta = -11^\circ$ were established, then it would follow from the above analysis that $\frac{F_8}{F_1} = 0.62 \pm 0.08$ (0.26 ± 0.03) for the fractional (integral) charge model. A value like $\frac{F_8}{F_1} = 0.26 \pm 0.03$ differs so drastically from the naive expectation $\frac{F_8}{F_1} = 1$ that it might imply dramatic differences in the singlet and octet wave functions. Dynamical bound state models (e.g., bags) could provide insight into the plausibility of such a value. If, for instance, it were established that $F_8 = F_1$ and $\theta = -11^\circ$, then we would have $\Gamma(\eta \to \pi^+\pi^-\gamma) = 45$ eV for both models but $\Gamma(\eta \to \gamma\gamma) = 379$ eV and $= 696$ eV for fractional and integral charges respectively. It is very suggestive that the naive assumptions, $F_1 = F_8$ and $\theta = -11^\circ$, provide such a good description of the data if we assign the quarks fractional charges.

Most of the assumptions underlying the preceding analysis of η decays are best tested by comparing the analagous results for the η' decays with experimental data. The η' decays are also of great interest because they are more sensitive to the quark charges, since η' is taken to be primarily an SU(3) singlet. A detailed discussion

will be presented elsewhere; here we indicate the difficulties and state some of the results.

Only experimental upper bounds are established for $\Gamma(\eta' \to \gamma\gamma)$ and $\Gamma(\eta' \to \pi^+\pi^-\gamma)$.[11] The branching ratio for $\eta' \to \gamma\gamma$ is consistently determined by different experiments but there is considerable disagreement about the branching ratio for $\eta' \to \pi^+\pi^-\gamma$. Taking the range of values reported[14] for $\eta' \to \pi^+\pi^-\gamma$ we find that $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma)$ varies between 9.1 ± 2.2 and 17 ± 3.

Given these experimental uncertainties, it is best to use the values of $\frac{F_8}{F_1}\tan\theta$ determined above from the reliably measured ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma)$. Then the remaining dependence of $\Gamma(\eta' \to \gamma\gamma)$ and $\Gamma(\eta' \to \pi^+\pi^-\gamma)$ on θ includes (for $\theta \neq 0$) a strongly varying factor $\sin^{-2}\theta$. But in the ratio $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma)$ this factor cancels and for $-\theta \lesssim 17^\circ$ the remaining dependence on θ is far less severe. For $0 < -\theta \lesssim 17^\circ$ we have

$$7.1 > \frac{\Gamma(\eta' \to \pi^+\pi^-\gamma)}{\Gamma(\eta' \to \gamma\gamma)} \gtrsim 2.7 \pm 0.5 \quad \text{for fractional charges and}$$

$$1.8 > \frac{\Gamma(\eta' \to \pi^+\pi^-\gamma)}{\Gamma(\eta' \to \gamma\gamma)} \gtrsim 0 \quad \text{for integral charges.}$$

At $\theta = -11^\circ$ the ratio is 5.4 ± 0.2 for fractional charges and 0.5 ± 0.1 for integral charges. So for this range of θ the prediction of the fractional charge model is much nearer to the range of the data. Better fits to $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma)$ can be obtained for large values of θ, but then the predictions for the more reliably measured η decay rates fail by two standard deviations or more.

CONCLUSION

For the integral charge quark model there is no good fit to the available data, regardless of the values of θ and F_8/F_1. For fractional quark charges the available data is fitted with the intuitively plausible choices of $F_8/F_1 \cong O(1)$ and θ small and negative. For instance, in the fractional charge model taking $\frac{F_8}{F_1} \tan\theta = -0.12 \pm 0.016$ to fit the ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma)$, we find that the choice $\theta = -6^0$ implies that $\frac{F_8}{F_1} = 1.14 \pm 0.15$. For the measured quantities we then compute $\Gamma(\eta \to \gamma\gamma) = 291 \pm 22$ eV, $\Gamma(\eta \to \pi^+\pi^-\gamma) = 38.9 \pm 1.3$ eV, and $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma) = 6.7$. The predictions for the η decays are in excellent agreement with experiment and the prediction for the ratio of η' decays is one standard deviation below the lower range of the available data. Accurate data for the ratio of the η' decays and for their absolute rates will allow us to test the validity of this analysis and to decide whether the quark model with fractional charges can consistently describe these radiative decays.

A detailed discussion will be presented elsewhere.

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FOOTNOTES AND REFERENCES

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10. Both square and triangle anomalies contribute to the low energy theorem for $\eta \to \pi\pi\gamma$. For a derivation see M. Chanowitz, M-S. Chen, and L-F. Li, Phys. Rev. <u>D7</u>, 3104 (1973).

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12. It is well established experimentally that in $\eta \to \pi^+\pi^-\gamma$ and $\eta' \to \pi^+\pi^-\gamma$, the dipion is dominated by the rho meson, cf., A. Cnops, et al., Phys. Letters <u>26B</u>, 398 (1968) and Ref. 11. Thus in computing the phase space integrals for these decays we include a Breit-Wigner final-state interaction factor for the rho, normalized to unity at zero dipion mass as required by the low energy theorem.

13. For $\Gamma(\eta \to \gamma\gamma)$ I use the most recent value obtained by A. Browman et al., Phys. Rev. Letters <u>32</u>, 1067 (1974). These authors made a more careful subtraction of the coherent background to the Primakoff peak than was possible in previous experiments. For $\Gamma(\eta \to \pi^+\pi^-\gamma)$ I use the branching ratios of Ref. 11.

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