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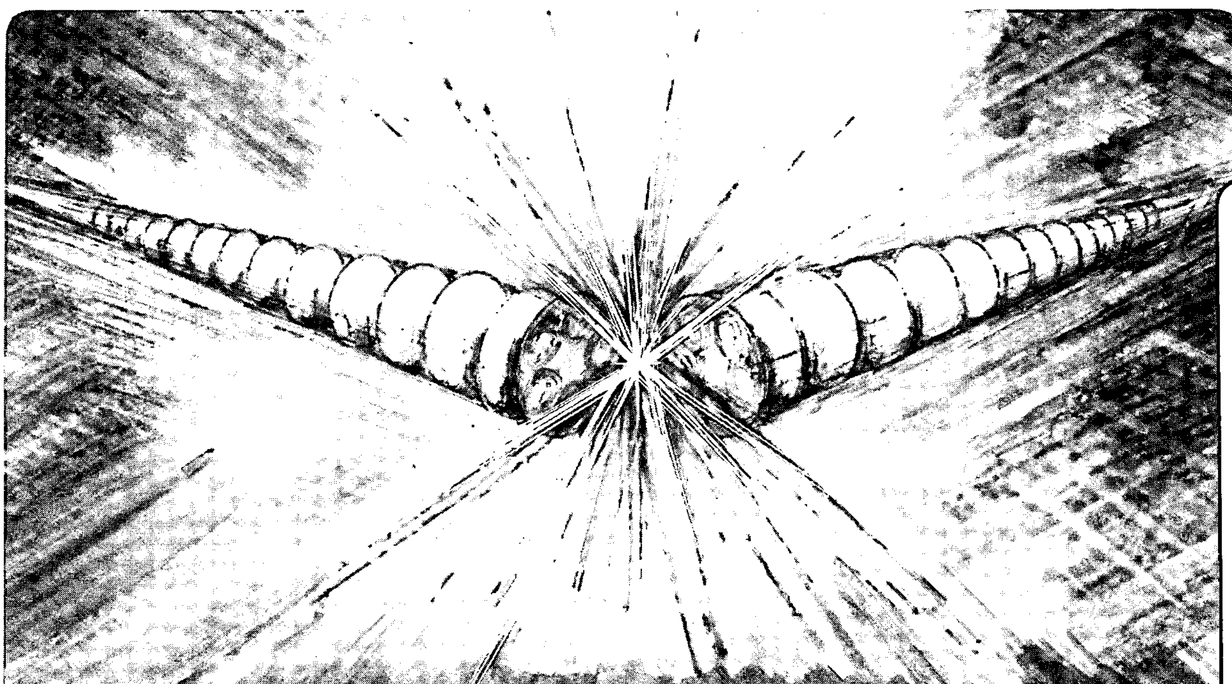

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ABSTRACT

In contrast to an electron induction accelerator, in which the particle velocity is virtually constant, the resistive and inductive components of accelerating module impedances can cause instability for an intense non-relativistic heavy ion beam accelerated in a similar structure. Since focusing requirements at the fusion pellet imply a momentum spread $\lesssim 3\times10^{-4}$ at the end of the accelerator, it is essential to understand and suppress this instability. There is also an economic issue involved for this application; selection of parameters to control the instability must not unduly affect the efficiency and cost of the accelerator. This paper will present the results of analytic and computational work on module impedances, growth rates and feed back (forward) systems.

1. INTRODUCTION

Inertial Confinement Fusion is based on the concept of creating a condition of very high density and temperature in a fuel pellet a few millimeters in size for a few nano-seconds. The material is not confined by external forces; the finite reaction time is due to the inertia of the fuel as it flies apart. For this process to work, an energy of several megajoules must be deposited in a thin surface layer in a few nano-seconds-an instantaneous power requirement of several hundred terawatts. The candidates for delivering the energy are lasers, which can deliver the required power but are as yet deficient in total energy and desired repetition rates of a few per second, and particle accelerators, which can deliver the required energy at a good repetition rate but as yet not at the required power level. Because the stopping distance is set by the required thickness of the surface layer, the choices in particle beams range from millions of amperes of light ions at several MeV kinetic energy to thousand of amperes of heavy ions at several GeV kinetic energy. At the heavy ion end of the scale the candidates are a set of R-F linacs plus accumulation and storage rings to reach the required level of joules and watts, and a linear induction accelerator, which is capable of accelerating kiloamperes of beam and would deliver the required energy and power in a single shot. The subject of this paper is a longitudinal instability which is certain to occur in an induction linac and is currently considered to be a major question as to the feasibility of the induction linac option.

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II. THE PROBLEM

The instability in question is a close relative of the well-known microwave instability in circular accelerators but with distinctive features in this unfamiliar parameter range. That it should occur in a linear accelerator is due to the fact the ions are non-relativistic all the way (10 GeV at mass 200 $\rightarrow$ 50 MeV/nucleon $\rightarrow \beta = 0.3$) and regenerative bunching can occur at an unpleasant rate in a kilo-ampere beam subjected to the impedance of the accelerating modules themselves.

Figure 1 shows a typical module as presently conceived. The module is energized through the transmission line entering at the upper left; the characteristic impedance of this line is seen as a resistance by the beam. Analytic estimates, more or less corroborated by model measurements, indicate that in the frequency range of greatest concern, about five to fifty megahertz, the impedance is well represented by a parallel R-C circuit–the line impedance in parallel with the capacity of the narrow gap below the end of the feed line. At about 100 MHz, there are modes involving the space above the magnetic material to the right of the transmission line which can be represented by a parallel R,L,C circuit. Unfortunately, the Q of this mode depends on R-F properties of the magnetic

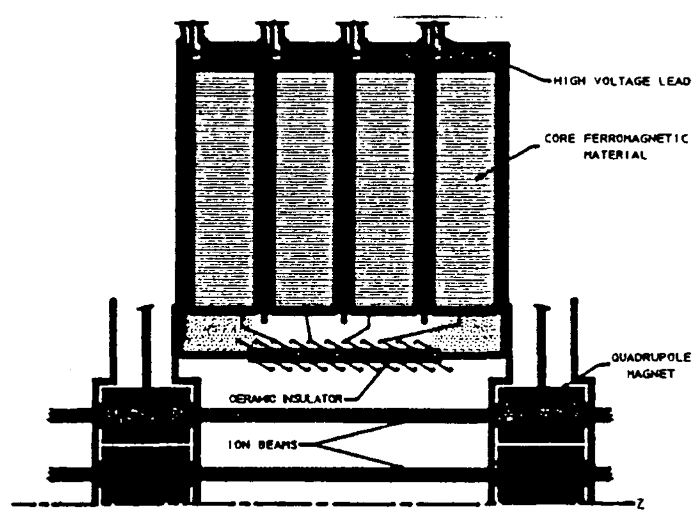

Fig. 1. Typical set of accelerating modules. The voltage is applied through the leads at the top and appears across an insulated gap below the leads. The insulating material can be seen projecting into the space below the magnetic material.

material, probably Metglas, which are not known at this time. Finally, there are many modes at much higher frequency associated with the pill-box like region surrounding the beam, but those are thought to be unimportant. In fact, PIC simulations indicate that even the 100 MHz mode is only marginally important.

The low frequency impedance is, however, quite important and here arises a conflict with the over-all performance and cost. To ease the instability, the line impedance should be as low as possible and the gap capacity as high as possible to shunt the R-F image currents away from the resistance. These desires are diametrically opposed to the demand for high efficiency and low cost. Maximum efficiency requires that the line impedance be equal to the module voltage divided by beam current to minimize reflected power. Also, the characteristic time τ = RC should be short compared to the beam passage time of 100-500 ns. in order to establish the required voltage without undue strain–i.e., the capacity should be low. As will be seen in the next section, best performance and cost lead to a predicted e-folding distance of about 100 meters in an accelerator five to ten kilometers long. In order to bring the theoretical problem into a range of control by momentum spread (less than $\sim 3\times10^{-4}$ because of chromatic effects in focusing on the fuel pellet) and/or feed-forward (see later section) we contemplate reducing line impedance by a factor of three (25% efficiency loss) and increasing the RC time by a factor of three (cost unknown).

III. THEORY

Instead of working with the Vlasov equation, it is simpler to work with the linearized fluid equations with zero pressure (no momentum spread), keeping in mind that the criterion for stabilization by a momentum spread is the same as for the microwave unstability of a coasting beam. Furthermore, the situation is formally similar to the transverse beam break-up instability without an external focusing force and will be analyzed in a similar fashion. The fluid equations are:

$$\frac{\partial}{\partial t}\frac{I}{v} + \frac{\partial I}{\partial z} = 0 \qquad \text{continuity equation} \qquad (1)$$

$$\frac{\partial v}{\partial t} + v_0\frac{\partial v}{\partial z} = \frac{eE}{m} - \frac{eg}{4\pi\varepsilon_0 mN}\frac{\partial}{\partial z}\frac{I}{v} \qquad \text{Force equation} \qquad (2)$$

$$\frac{\partial E}{\partial t} + \frac{E}{\tau} = -\frac{1}{C}(I-I_0) \qquad \text{Circuit equation} \qquad (3)$$

where I is current, v is velocity, and I_0 and v_0 are the unperturbed values.* C is the capacity, averaged in z, in Farad-meters of the low-frequency equivalent circuit, R is the smoothed resistance in ohms/meter, τ=RC, and N is the number of beamlets simultaneously accelerated.

The second term in (2) is the space charge force per beamlet since the beamlets are isolated transversely by closely spaced conducting sheets inserted in the beam region of Fig. 1. It has the effect of producing space charge waves running forward and backward along the beamlets. As the accelerator was first conceived, there was to be a single beam (N=1) and stability seemed to be insured by the mechanism that backward moving waves growing amplitude would reflect from the back end of the bunch and damp out as they moved forward. In the present concept there would be a dozen or more separate beamlets. The space charge force is then negligible compared to the induced module force in the low frequency range and will be omitted hereafter.

If time is replaced by t - z/v_0, the time after the head of the bunch passes a particular point along the accelerator, the equations become:

$$v_0\frac{\partial}{\partial z}\frac{\delta I}{I_0} = \frac{\partial}{\partial t}\left(\frac{\delta v}{v_0}\right) \qquad (1')$$

$$\frac{\partial}{\partial z}\frac{\delta v}{v_0} = \mathcal{E} \qquad (2')$$

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\mathcal{E}}{\tau} = -K^2 v_0\frac{\delta I}{I_0} \qquad (3')$$

where $\mathcal{E} = \frac{e\delta E}{mv_0^2}$ and $K^2 = \frac{eI_0}{mv_0^3 C} = \frac{eI_0 R}{mv_0^3\tau}$

The most likely way for a perturbation to be launched is by a voltage error on a module at (say) z=0. The initial conditions for the set of equations are then:

$$\frac{\delta v}{v_0}(0,t) = f(t)$$
$$\frac{\delta I}{I_0}(0,t) = 0 \qquad (4)$$
$$\mathcal{E}(0,t) = \mathcal{E}(z,0) = 0$$

Using a Laplace transform in z:

$$\tilde{y}(k) = \int_0^\infty dz e^{-kz} y(z) \ , \qquad (5)$$

the equations lead to

$$\hat{\mathcal{E}} = -\frac{K^2}{k^7+K^2}\int_0^t dt' \frac{d}{dt'} f(t')\exp\left[-\frac{k^2}{k^2+K^2}\frac{t-t'}{\tau}\right] \qquad (6)$$

* This analysis neglects acceleration; the qualitative features are not seriously affected by this omission.

If the initial velocity perturbation is $\frac{\delta v}{v_0} = \Delta e^{i\omega t}$, the integral in (6) is easily done and the Laplace transform can be inverted analytically or by a numerical integration around the poles. The analytic result is:

$$\mathcal{E} = -\frac{K\Delta e^{-t/\tau}}{1+i\omega\tau}\left\{\sum_{n=0}^{\infty}\frac{(Kz)^{n+1}j_n(Kz)}{n!^2}\left(\frac{t}{2\tau}\right)^n + i\omega\tau\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{t}{\tau}\right)^n\sum_{k=0}^{n}\frac{(Kz)^{k+1}j_k(Kz)}{2^k k!}(1+i\omega\tau)^{n-k}\right\} \quad (7)$$

where $j_n(Kz)$ is the n^{th} order spherical Bessel function.

Figure 2 shows two examples of the corresponding $\frac{\delta v}{v_0}$ as a function of Kz and t/τ. The most violent growth occurs for $\omega\tau = \frac{1}{\sqrt{3}}$ but becomes relatively benign for higher frequencies; the case, $\omega\tau=3$, is also shown for comparison.

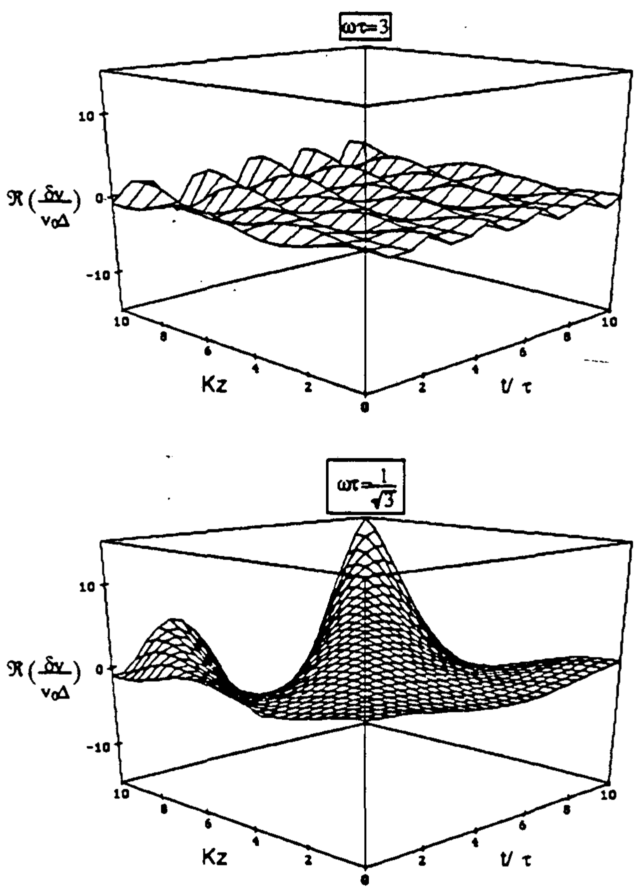

Fig. 2. $\frac{\delta v}{v_0\Delta}$ as a function of Kz and t/τ for $\omega\tau = 3$(top) and $\frac{1}{\sqrt{3}}$ (bottom). Kz = 10 corresponds typically to a distance of one to three kilometers.

Equation (7) is not very revealing of essential features, which can be seen better by setting $\frac{df}{dt} = \delta(t)$ in equation (6) and using a saddle point analysis.[1].

The inversion integral becomes:

$$\mathcal{E}(z,t) = \frac{K^2}{2\pi i}\int\frac{dk}{k^2+K^2}\exp\left[kz - \frac{k^2}{k^2+K^2}\frac{t}{\tau}\right] \quad (8)$$

The method of steepest descents consists in finding the stationary points of the quantity in the exponent by setting its derivative equal to zero and then integrating across these saddle points to approximate the contour integral. Our interest, however, is only in the real part of the exponential at the saddle points; that is, the real parts of the solutions to the equation:

$$\frac{d}{du}\left[uKz - \frac{u^2}{u^2+1}\frac{t}{\tau}\right] = Kz - \frac{2u}{(u^2+1)^2}\frac{t}{\tau} = 0 \quad (9)$$

The roots of the quartic depend on the ratio $\frac{t}{\tau Kz}$. A case of particular interest [1] is when $\frac{t}{\tau} < \frac{8}{3\sqrt{3}}Kz$. The real part of the exponent is then given by:

$$Kz\left\{u-\Delta + \frac{2u\Delta^2}{\Delta^2+16\mu^4\left[1+\mu^2-2\mu\sqrt{1+\mu^2}\right]}\right\} \quad (10)$$

where $\Delta = t/\tau Kz$ and μ is defined by $\Delta = 4\mu^2\sqrt{1+\mu^2}$. The bracket is a maximum at $\Delta = \frac{3}{4\sqrt{2}}$, where (10) becomes $\frac{Kz}{2\sqrt{2}}$. That e-folding rate is the same as the worst case $\omega\tau = \frac{1}{\sqrt{3}}$ shown in Fig. 2. The point of maximum growth should move backward along the bunch at $\frac{t}{\tau} = \frac{3}{4\sqrt{3}}Kz$, in rough agreement with Fig. 2.

<u>Numerical Values</u>

For $\omega\tau = \frac{1}{\sqrt{3}}$, the frequency is $f = \frac{1}{2\pi\sqrt{3}\tau} = \frac{1}{2\pi\sqrt{3}\,t_p}\frac{t_p}{\tau}$, where t_p is the pulse duration. For the desirable value, $\frac{t_p}{\tau} \sim 20$, f = 4-20 MHz for t_p = 500-100 n.s. and f = 1-7 MHz for τ increased by a factor of three. These frequencies are well in the range of the R-C model. The growth rate is $\frac{K}{2\sqrt{2}} = \sqrt{\frac{eI_oR}{16W\ell_p}\frac{t_p}{\tau}}$, where W is the kinetic energy and $\ell_p \sim$ 10 meters, is the bunch length. If the module feed line is matched, eI_oR = 1 MeV/meter, the intended acceleration rate.

Then for W = 1 GeV and $\frac{t_p}{\tau} = 20$, $\frac{K}{2\sqrt{2}} \sim (100 \text{ meters})^{-1}$. If R is reduced by a factor of three and τ increased by a factor of three, $\frac{K}{2\sqrt{2}} \sim (300 \text{ meters})^{-1}$.

Feed Forward

Since the ions are non-relativistic ($\beta < 0.3$), the possibility arises of observing some aspect of the unstable growth at one point along the accelerator and preparing a down stream module to apply a correction. Probably the easiest thing to observe is the perturbed electric field, which would appear as an error in module voltage as a function of time during bunch passage. The negative of this error could be applied to the bunch a hundred meters or less down stream. To see how this would work, replace eqn. (2') by:

$$\frac{\partial}{\partial z}\frac{\delta v}{v_0} = \mathcal{E}(z) - \mathcal{E}(z-\mathcal{L}) \sim \mathcal{L}\frac{\partial \mathcal{E}}{\partial z} \qquad (2")$$

where $\mathcal{L}$ is the feed-forward distance.

A simple dispersion analysis of (1'), (2") and (3') leads to a damping factor, $\exp\left[-\frac{\omega^7\tau^7}{1+\omega^2\tau^2}K^2\mathcal{L}z\right]$, which is indicative but misleading in that the apparent advantage of making $\mathcal{L}$ large is due to the Taylor expansion approximation in (2"). Figure 3 shows the results of a simulation in which the perturbation grows from noise and in the right hand plot is fed forward at a distance of 100 meters. Much remains to be studied about this scheme, but it looks promising.

IV. CONCLUSION

This instability can be overcome by manipulating R and C of the modules, but at the price of reduced efficiency and increased cost. We shall continue to look for a compromise by involving a feed-forward system and a tolerable momentum spread. We have to rely heavily on theoretical analysis for now since there is no possibility of realistic experimental study without an accelerator capable of handling some hundreds of amperes. A group at the University of Maryland is planning a model experiment using low-energy electrons [2], from which we hope to gain considerable information.

V. REFERENCES

[1] For more details, see E.P. Lee and L. Smith, "Asymptotic Analysis of the Longitudinal Instability of a Heavy Ion Induction Linac," Proc. 1990 Linac Conf. p. 716-718, September 1990. See also E.P. Lee and L. Smith, "Analysis of Resonant Longitudinal Instability in a Heavy Ion Induction Linac," paper LRA62, this conference.

[2] J.G. Wang et al., "Resistive Wall Instability Experiment at the University of Maryland," paper LTP17, this conference.

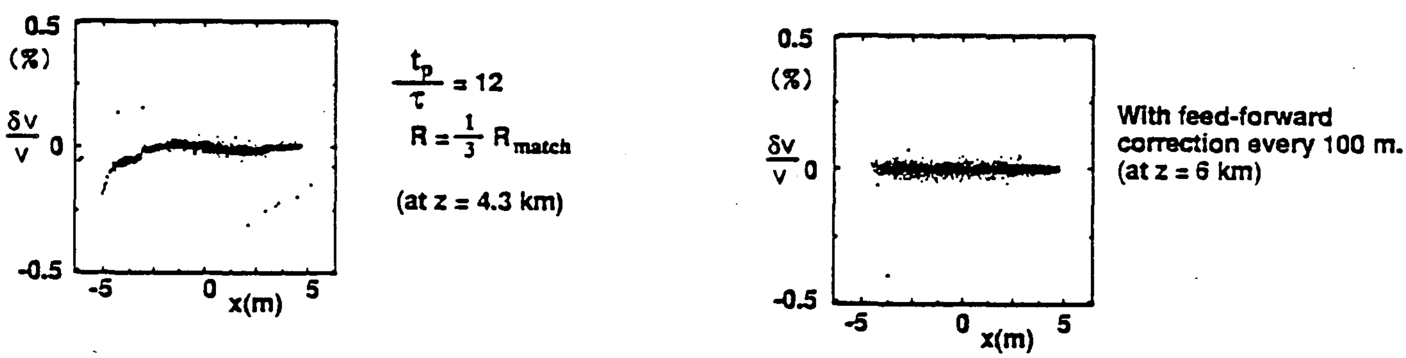

Fig. 3. Phase space plots with and without feed-forward correction. External impedance is assumed to be represented by parallel R and C.