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Publication Date

1991-10-01

Peer reviewed



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WORKING PAPER NO. 92-200

**ARM PREPAYMENT IN THEORY AND
PRACTICE: JUSTIFYING BACKWARD AND
FORWARD PATH DEPENDENCE
IN A HAZARD FUNCTION**

By

**GREGORY M. SCHWANN
NANCY E. WALLACE**

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ARM PREPAYMENT IN THEORY AND PRACTICE:
JUSTIFYING BACKWARD AND FORWARD PATH DEPENDENCE
IN A HAZARD FUNCTION

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WORKING PAPER NO. 92-200

First draft: December 1990
This Revision: October 1991

Abstract

This paper develops an empirical model of rational ARM prepayment. The theoretical model addresses the borrower heterogeneity problem directly. A central concern in the formulation of the theoretical model is to discover the potential limits of forward looking option valuation based empirical models, and to explore why empirical models with backward looking elements - the preceding path - may have considerable merit. The theoretical model leads to an empirical specification similar to standard random utility estimation models in a dynamic framework. The model is tested on a data set that includes detailed information about heterogenous borrowers and their wealth portfolios at the time of origination.

ARM PREPAYMENT IN THEORY AND PRACTICE: JUSTIFYING BACKWARD AND FORWARD PATH DEPENDENCE IN A HAZARD FUNCTION

Introduction

There is surprisingly little published research on adjustable rate mortgage (ARM) prepayment (Kau et al., 1991; McConnell and Singh, 1991; Cunningham and Capone, 1990; Bartholomew, et al. 1988; Berk and Roll, 1988) even though adjustable rate lending in this country has at times accounted for more than 50% of all residential lending and for some lenders accounts for nearly all their residential mortgage portfolio. Recent trends in adjustable rate mortgage lending and the securitization of pools of adjustable rate mortgages along with the current reappraisal of regulatory requirements highlight the need for appropriate pricing models for adjustable rate mortgages. Pricing models necessarily rely on accurate representations of the dynamics of prepayment.

Most recent work on exogenous prepayment has focussed on the prepayment dynamics of pools of mortgages for which there is little information about individual borrowers. Schwartz and Torous (1989), McConnell and Singh (1991; 1990), and Stanton (1991) have incorporated empirically estimated prepayment functions directly into the contingent claim valuation framework of Brennan and Schwartz (1985). The Schwartz and Torous (1989) model and most Wall Street models (Carlson and Singh, 1987; Berk and Roll, 1988; Youngblood, 1988; Altarescu et al., 1987; Asay and Sears, 1988) are purely empirical and cannot provide quantitative predictions about the effects of future economic regime shifts. The Stanton model (1991) extends the theoretical models of Timmis (1985) and Dunn and Spatt (1986) in which transaction costs and other frictions inhibit borrower prepayment. Although the structural model is derived for heterogenous borrowers, the empirical application uses GNMA pool data which is devoid of any detailed description of behavior at the individual level. The important contribution of this work is that it provides a theoretical foundation useful for the analysis of a wide variety of prepayment behavior.

The purpose of this paper is to develop a theoretical basis for an empirical model of rational ARM prepayment. The theoretical model addresses the borrower heterogeneity problem directly, in the spirit of Stanton (1991), Timmis (1985), and Dunn and Spatt (1986), however our notion of transactions cost is more broadly defined. A central concern in the formulation of the

theoretical model is to discover the potential limits of forward looking, option valuation based empirical models, and to explore why empirical models which have included a backward looking element (Bartholomew, et al., 1988) - the preceding path - may have considerable merit. In short, our model seeks to reveal why empirical models of prepayment which consider only the optimal exercise of the option, given transaction costs, uncover numerous households which make apparently irrational economic choices concerning prepayment.

Our model is intended for use by lenders in the primary market and issuers of mortgage backed securities who possess information on borrower characteristics and wish to determine the dynamics of prepayment by class of borrower. The model is tested on a data set that includes detailed information about heterogeneous borrowers and their wealth portfolios at the time of origination. An advantage of our model, as with those of Stanton (1991), Dunn and Spatt (1986), and Timmis (1985), is that it leads to a structural model of rational prepayment. The parameters obtained, therefore, can be interpreted to have economic rather than purely correlational significance. The theoretical model also leads to an empirical specification similar to standard random utility estimation models (McFadden, 1973) in a dynamic framework.

The paper is organized into 7 sections. In Section 2, the theoretical model is presented and the empirical implications of the model are summarized. The empirical model is presented in Section 3, with a brief description of the nonlinear least squares algorithm. The algorithm allows for the inclusion of time-dependent covariates making the model fully parametric. The option valuation methodology is discussed in Section 4. The construction of the data set is discussed in Section 5, and the data are summarized. The results and conclusions follow in Section 6 and 7.

I. Optimal Mortgage Prepayment

In this portion of the paper, we examine a household's decision to prepay an existing residential mortgage. We begin by formulating a multi-period model of consumption and portfolio allocation under uncertainty in which the household chooses a vector of current period consumption goods in spot markets, chooses its asset portfolio (excluding its mortgage) in the securities market, and chooses a prepayment level for its mortgage obligation based on rates in

the mortgage market. Our focus is on the uncertainty in the asset and mortgage markets. We assume that the returns on all assets, including mortgages, are uncertain at the beginning of each time period, when the asset and prepayment choices must be made. Future spot prices for consumption goods are also uncertain.

A. Modelling Prepayment

There are $T+1$ time periods in the household's allocation program. These periods are indexed by $t \in \{0, 1, \dots, T\}$. Period zero is the current period, which may or may not be the starting date of the mortgage. Period T is the date when the household sells its current dwelling, pays out its mortgage (if it is not fully amortized), and moves to a new dwelling. All of the period zero variables are known to the household, including the realized values of the household's assets and liabilities and the spot prices of all consumption goods. In future periods, these variables are uncertain.

The household is uncertain about its assets and liabilities because the rates of return on financial assets are random. We assume that the household can invest in m distinct financial assets with the random rates of return $r(t+1) = (r_1(t+1), \dots, r_m(t+1))^T$ bridging period t and $t+1$. In addition, we assume that the volatilities of the rates of return are separate stochastic variables, denoted by $\eta(t+1)$. We take η to be the vector of the variances of the rates of return (see: Hull and White, 1987, 1988), but we could expand our definition to include all $m(m+1)/2$ distinct variances and covariances. We regard mortgages as derivative securities, driven by the rates of return on the financial assets. Therefore, future mortgage interest rates and interest rate volatility are perfectly correlated with the portfolio of financial securities and its volatilities. We adopt the convention that the current contract rate on a mortgage is set at the end of the preceding period. Hence, all mortgage uncertainty is forced into the future. The household is also uncertain about the value of its home. Two sources of uncertainty are at play: first, there is randomness in the household's mortgage indebtedness, and second, there is randomness in the market price of its dwelling. Randomness in mortgage indebtedness is a manifestation of interest rate uncertainty, conjoined with the contractual rules of the mortgage instrument. Last, future spot prices for the n consumer goods the household demands are random. These prices are denoted by $p(t) = (p_1(t), \dots, p_n(t))^T$. We include the wage rates of household members in this vector.

The vector $(r(t), V(t), p(t), \eta(t))$ follows a stochastic process adapted to the natural time filtration. The household receives new information from the market at the end of period t , and uses it in making its period $t+1$ choices, but all future values are unknown. We do not need to specify the processes governing the evolution of housing prices, $V(t)$, and consumption good prices, $p(t)$, but for our empirical work, we need to specify the interest rate processes, $r(t)$, and the volatility processes, $\eta(t)$, in order to value the financial option to prepay. We leave the specification of the latter processes to section 4, where we set out the empirical prepayment model.

The household maximizes an intertemporal von Neuman-Morgenstern utility function which is additively separable in the time partition of commodity space. This function is represented by

$$U(x(0), \dots, x(T)) = \mathcal{E}_t \left[\sum_{t=0}^T u(x(t)) \rho^{-t} \right] \quad (1)$$

where $x(t) = (x_1(t), \dots, x_n(t))^T$ is the vector of period t consumption commodities and ρ is the household's subjective time discount rate. \mathcal{E}_t denotes the expectation operator, where the expectation is taken with respect to the period t information set. For the usual reasons, we assume that the elementary utility function, u , is a strictly quasi-concave function of $x(t)$.

While our analysis can be conducted in terms of the utility function (1), we find it convenient to fold all of the uncertainty in our problem back into the household's constraint set, where it originates. To do this, we define the household's period t net expenditure function

$$e(p(t), u(t)) = \min_{x(t)} \langle p(t)^T x(t) \mid u^*(t) \geq u(t), p(t) \gg 0_n, x(t) \in X_0 \rangle$$

relative to X_0 . The choice set for $x(t)$ is restricted to a subset of commodity space because the housing unit is fixed in our formulation and goods that are complementary with housing will also be fixed or choices for them limited. The expenditure function is a measurable function giving the state-contingent expenditure necessary to attain $u(t)$. It is dual to $u(t)$, and it is nondecreasing, concave and homogeneous of degree one in $p(t)$, and increasing in $u(t)$. One advantage of working with random expenditure, rather than random utility, is that it is measured

in monetary units and therefore fits neatly in a valuation context.

The household's maximization of (1) is constrained by the resources it inherits from the past. The household begins each period with the value of its financial assets, the equity in its dwelling, and its current full labor income. It uses these resources to purchase current consumption goods and invests any surplus resources in either financial assets or home equity. Therefore, the budget constraint for period t is

$$V(t) - B(t, C(t-1)) - \delta(t, C(t)) - k(t, C(t)) (B(t, C(t-1)) - \delta(t, C(t-1))) - \sum_{i=1}^m \langle A_i(t) + (1+r_i(t))A_i(t-1) \rangle = e(p(t), u(t)) \quad (2)$$

where

- $A_i(t)$ = the value of asset i held at the end of period t ;
- $V(t)$ = the value of the housing unit (land plus capital) at the beginning of period t ;
- $B(t, C(t-1))$ = the value of mortgage indebtedness at the beginning of period t , given the mortgage contract in force at the end of period $t-1$;
- $\delta(t, C(t))$ = the value of mortgage prepayment made at the beginning of period t , given the mortgage contract in force at the beginning of period t ;
- $k(t, C(t))$ = the mortgage constant for the mortgage contract in force at the beginning of period t ¹.

Because the assets yield stochastic returns, the resources available to the household in the next period are random with distribution that depends on the portfolio mix and the multivariate distribution generating function of the stochastic returns.

The first term in the budget constraint gives the equity position of the household in its home at the beginning of the time period. The portion $V(t) - B(t, C(t-1))$ of this term is determined

¹For example, if the contract is a fixed rate mortgage with amortization period S , then $k=i/(1-(1+i)^{-S})$.

by past choices and events, so the only way a household can increase its period t home equity is by making a prepayment. The second term gives the mortgage payment required under the contract rolled-over from the preceding period or under a new contract negotiated at the beginning of the period (i.e., the refinance option). Note that a one dollar prepayment reduces the mortgage payment by $k(t,C(t))$ so that the prepayment only costs the household $1 - k(t,C(t)) \leq 1$.² The third term in (2) equals the change in the value of the household's financial asset position between the beginning and end of period t . Obviously, this term can be either positive or negative. The sum of the three terms limits the income which can be spent on consumption, $e(p(t),u(t))$.

The household's choices are also limited by the amortization and prepayment rules in the mortgage contract. We assume that a prepayment reduces future monthly payments.³ Then, the value of the mortgage debt at the beginning of period $t+1$ equals the value of the debt at the beginning of period t less any prepayment in period t less debt amortization in period t . This is summarized by the mortgage balance equation

$$B(t+1,C(t)) = (1 + i(t,C(t)) - k(t,C(t))) \cdot (B(t,C(t-1)) - \delta(t,C(t))) \quad (3)$$

where the variable $i(t,C(t))$ is the contract mortgage interest rate.

In addition to the constraints (2) and (3), we need to link the decisions made by the household while in the current dwelling to those made in future dwellings. We do this by imposing the terminal wealth constraint

$$\mathcal{E}_t[W(T)] \geq W_T \quad (4)$$

where wealth in period T is

²Only in the case of balloon mortgages does $k=0$ and a \$1 prepayment cost the household \$1.

³Prepayment may shorten the amortization period of the mortgage instead of reducing the payment. We do not consider this case.

$$W(T)=V(T)-B(T,C(T-1))+\sum_{i=1}^m(1+r_i(T))A_i(T-1).$$

W_T may be interpreted as the amount of wealth the household believes it will need to meet its future consumption and housing objectives.

B. Optimal Prepayment in an Economy with an Unconstrained Securities Market.

The household's optimization problem is to maximize (1) subject to the period-by-period budget constraint (2), the mortgage balance equation (3), and the terminal wealth constraint (4). Because of the additive separability of the household utility function, the solution to the problem is easily characterized through dynamic programming. Let

$$\psi(t+1)=\psi(A_1(t+1),\dots,A_m(t+1),V(t+1),B(t+1)) \quad (5)$$

be the optimal value function for the household in period $t+1$. Then the household can be viewed as maximizing

$$u(t)+\frac{1}{\rho}\psi(t+1) \quad (6)$$

instead of (1).

Substituting (3) into (2) and (5), imposing (2) as a constraint, and differentiating (6) with respect to $u(\tau)$, $t \leq \tau \leq T$, gives the first order condition for the optimal utility value

$$1=\mathcal{E}_t[\lambda(\tau)\partial e(p(\tau),u(\tau))/\partial u(\tau)] \quad (7)$$

where $\lambda(\tau)$ is the marginal utility of wealth for period τ . Driving the expectation operator through (7), using the rule $\mathcal{E}[a \times b]=\mathcal{E}[a][\mathcal{E}(b)+\text{cov}(a,b)]$, and rearranging, yields

$$\mathcal{E}_t[\lambda(\tau)]=\left(\mathcal{E}_t\left[\frac{\partial e(p(\tau),u(\tau))}{\partial u(\tau)}\right]+\xi_e(t,\tau)\right)^{-1} \quad (8)$$

where

$$\xi_e(t,\tau) = cov_t \left[\frac{\lambda(\tau)}{\mathcal{E}_t[\lambda(\tau)]}, \frac{\partial e(p(\tau), u(\tau))}{\partial u(\tau)} \right]$$

The expectation operator and the covariance term $\xi_e(t,\tau)$ in (8) drop away for the current period because $p(t)$ is a known vector. Therefore, a household has the optimal current period utility when it sets the expected marginal utility of wealth equal to the inverse of the marginal cost of additional utility. The major difference between the current and future periods is the addition of the term covariance $\xi_e(t,\tau)$. This term can be interpreted as the risk premium associated with fluctuations in future real household wealth resulting from future price level changes. The risk premium depends on the correlation between $p(\tau)$ and $p(\tau')$, $\tau \leq \tau'$ -- the tighter the correlation the higher the risk premium. Hence, $\xi_e(t,\tau)$ is a measure of the permanence of future price level changes. Given this interpretation of $\xi_e(t,\tau)$, equation (8) requires that a household set its future utility levels to equate the marginal utility of wealth with the risk adjusted cost of additional utility.

Optimal financial planning on the part of the household involves choosing the amounts invested in the financial assets, $A_i(\tau)$, $\tau \geq t$, $i=1, \dots, m$, the value of the mortgage prepayments, $\delta(\tau)$, $\tau \geq t$, and selecting future mortgage contracts $C(\tau)$, $\tau \geq t$, from among the refinancing options available. We examine each of these choices in turn.

An optimal asset portfolio requires

$$\mathcal{E}[\lambda(\tau)] = \frac{1}{\rho} \mathcal{E}_t[\lambda(\tau+1)(1+r_i(\tau+1))] \quad \tau \geq t, \quad i=1, \dots, m \quad (9)$$

where the envelope theorem was used to obtain $\lambda(\tau+1) = \partial \psi(\tau+1) / \partial A_i(\tau+1)$, $i=1, \dots, m$. These

conditions can be stated in more familiar terms by using the expectation rule presented above to obtain

$$\mathcal{E}_i[r_i(\tau)] + \xi_i(t, \tau) = \mathcal{E}_i[r_j(\tau)] + \xi_j(t, \tau) \quad \tau \geq t, \forall i \neq j = 1, \dots, m \quad (10)$$

where

$$\xi_i(t, \tau) = \text{cov}_i \left[\frac{\lambda(\tau)}{\mathcal{E}_i[\lambda(\tau)]}, r_i(\tau) \right]$$

Since $\xi_i(t, \tau)$ is the risk premium associated with randomness in the return to asset i , equation (10) presents the well known requirement that the household equate the risk-adjusted rates of return for all financial assets.

We assume that the household can choose any prepayment amount between zero and the outstanding balance of the mortgage. Reverse annuity mortgages (RAMs) are not allowed, neither can the home owner intentionally increase his or her indebtedness through the mortgage instrument.

The mortgage contract dictates when prepayments can be made. But, when prepayment is allowed, the optimal prepayment level is obtained by differentiating (6) with respect to $\delta(\tau)$, subject to the constraints. Assuming for the moment that $\delta(\tau)$ has an interior solution, the necessary condition is

$$\mathcal{E}_i[\lambda(\tau)(1-k(\tau))] = \frac{1}{\rho} \mathcal{E}_i \left[\frac{\partial \psi(\tau+1)}{\partial B(\tau+1)} \frac{\partial B(\tau+1)}{\delta(\tau)} \right] \quad \tau \geq t. \quad (11)$$

The left hand side term in (11) gives the marginal cost of increasing prepayment in terms of foregone current period utility, while the right hand side is the discounted value of the future benefits from prepayment in terms of utility. The latter benefits arise because prepayment reduces the outstanding balance of the mortgage debt for future periods.

By repeatedly applying the envelope theorem, using the terminal wealth constraint, and using the asset equilibrium condition (9), we can write (11) as

$$\mathcal{E}_t[\lambda(\tau)(1-k(\tau))] = \frac{1}{\rho} \mathcal{E}_t \left[\sum_{v=\tau}^T \lambda(v+1)(1+k(v+1))H(\tau,v)\rho^{-(v-\tau)} \right] \quad (12)$$

where

$$H(\tau,v) = \prod_{v'=\tau}^{v-1} (1+i(v')-k(v'))$$

$$\lambda(T+1) = \lambda^*(T+1)\rho^{(T-\tau+1)}$$

$$k(T+1) = 0$$

The right hand side of (12) is the expected discounted value of the reductions in mortgage payments over the household's remaining tenure in its dwelling which result from a \$1 increase in prepayment. This interpretation follows immediately from noting that the term $1+k(\tau)$ is the reduction in the mortgage payment in period τ from a \$1 decrease in the mortgage debt at the beginning of the period, and that $H(\tau,v)$ gives the decrease in mortgage debt at the beginning of period v from a \$1 prepayment at the beginning of period τ . The variable $\lambda^*(T+1)$ is the multiplier for the terminal wealth constraint. The standard necessary condition (e.g., Fiacco and McCormick, 1976; Diewert, 1983) requires that $\lambda^*(T+1) \geq 0$, thus $\mathcal{E}[\lambda^*(T+1)] > 0$, except in the case of a degenerate distribution.

In order to investigate capital market equilibrium, we convert (12) into a risk-return

relation parallel to (10). This is done by eliminating the multipliers $\lambda(\tau)$, $t \leq \tau \leq T+1$, invoking equation (9) recursively to get

$$\mathcal{E}_i[\lambda(v)] = \frac{\rho^{v-\tau}}{D(\tau,v)} \mathcal{E}_i[\lambda(\tau)] \quad (13)$$

where

$$D(\tau,v) = \prod_{v'=\tau+1}^v (1+r_i(t,v'))$$

is the discount function over the period τ to v , and

$$r_i(t,v) = \mathcal{E}_i[r_i(v)] + \text{cov}_i \left[\frac{\lambda(v)}{\mathcal{E}_i[\lambda(v)]}, r_i(v) \right] = \mathcal{E}_i[r_i(v)] + \xi_i(t,v)$$

is the risk-adjusted forward rate for the i th financial asset. The values $r_i(t,\tau)$ for $\tau \geq t+1$ generate the yield curve for the asset, and $D(t,\tau)^{1/\tau} - 1$ as a function of τ gives the term structure (Vasicek, 1977).

Substituting (13) into (12), and using the properties of the expectation operator, we arrive at

$$(1 - \mathcal{E}_i[k(\tau)]) + \xi_k(t,\tau) = \frac{1}{\rho} \sum_{v=\tau}^T [1 + h(t,v) + \xi_h(t,v)] D(\tau,v)^{-1} \quad (14)$$

where

$$1+h(t,v)=\mathcal{E}_t[H(\tau,v)]\left[(1+\mathcal{E}_t[k(v)])+\text{cov}_t\left(\frac{H(\tau,v)}{\mathcal{E}_t[H(\tau,v)]},k(v)\right)\right]$$

$$\xi_k(t,\tau)=\text{cov}_t\left[\frac{\lambda(\tau)}{\mathcal{E}_t[\lambda(\tau)]},k(\tau)\right]$$

$$\xi_h(t,v)=\text{cov}_t\left[\frac{\lambda(v)}{\mathcal{E}_t[\lambda(v)]},(1+k(v))H(\tau,v)\right]$$

The left hand side of (14) gives the expected cost of a \$1 prepayment in period $\tau \geq t$. Notice that this cost includes a risk premium valuing unexpected changes in the mortgage constant. When applying (14) to period t , the expectation operator and the risk premium vanish, leaving just the cost of prepayment $1-k(t)$. The right hand side of (14) again equals the expected discounted value of the savings in future mortgage payments, but now the discounting is done in terms of the equilibrium risk-adjusted interest rates along the yield curve rather than the household's constant subjective discount rate. Equation (14) identifies two sources of household prepayment risk. The first arises because of variability in the mortgage constant. Specifically, the benefit from prepaying may be smaller than expected because the mortgage constant $k(v)$ is smaller than expected or because the mortgage balance is smaller than expected. The second risk arises because of variability in the marginal utility of wealth in period v . The prepayment may be worth less or more to the household simply because the household portfolio performed better or worse than the household expected.

To restate this result in terms of rates or return, we rearrange (14) and use the asset equilibrium conditions (10) to get

$$r_i(t, \tau+1) = \frac{1}{\rho} \frac{\sum_{v=\tau}^T [1+h(t,v) + \xi_h(t,v)] D(\tau, v)^{-1} - \xi_k(t, \tau)}{1 - \xi_l[k(t)]} \quad (15)$$

which states that the expected return from investing in home equity through prepayment must equal the risk-adjusted return on other financial assets. The novelty of (15) is that we have established how to compute the appropriate rate of return for a prepayment.

Equations (14) and (15) do not depend on the amount of the prepayment. Therefore, whether these equations have an interior solution depends on whether the distribution generating function for the joint stochastic process underlying these functions is independent from the prepayment amount. If they are independent, (14) and (15) do not have an interior solution and if the left hand side of (15) is less than the risk-adjusted equilibrium interest rate, $\delta(\tau)=0$; otherwise, $\delta(\tau)=B(\tau)$. This switching or bang-bang control has been presumed in most of the prepayment literature (Green and Shoven, 1986; Kau *et al.*, 1991; McConnell and Singh, 1991; Bartholomew *et al.*, 1988). However, the literature has not heeded the fact that this result depends on the distribution generating function assumed in the analysis.

When the bang-bang property of the control holds, the optimal prepayment problem is transformed into the prepayment option problem of finding the first hitting time on which to prepay. To construct the option, note that the right hand side of (15) gives the expected marginal yield on a prepayment made at the beginning of period τ . Let $\chi(\tau)$ denote this yield. The decision confronting the homeowner is whether to invest $B(\tau)$ in the mortgage and earn the expected return $\chi(\tau)$ or to invest the same funds in the securities market and earn the expected return $r_i(t, \tau+1)$. The household will exercise its option to prepay if $\chi(\tau) \geq r_i(t, \tau+1)$. It follows that the value of this yield option is

$$\max [\chi(\tau) - r_i(t, \tau+1), 0] \quad (16)$$

The prepayment option presented in (16) involves borrowing in the securities market to pay out the mortgage debt. This is just a refinancing scheme, and not a common one at that. Typically refinancing mortgage debt entails swapping one mortgage contract for another. In our formulation, this means exchanging the old sequence of mortgage payments for a new sequence

of mortgage payments. Thus, if a household changes mortgage contracts at the beginning of period τ , equation (15) gives the expected yield on the new contract, once the appropriate contract payments are substituted into the equation. Let $\chi^1(\tau)$ and $\chi^2(\tau)$ be the expected yields under the old and new contracts, respectively. Then, the option value of this refinancing scheme is

$$\max [\chi^1(\tau) - \chi^2(\tau), 0]$$

which just indicates that the household will refinance if the expected yield to prepaying the old contract exceeds the expected yield on the new contract.

C. Optimal Prepayment with Capital Market Imperfections

In an unconstrained capital market, the household can finance its prepayment either by selling part of its financial portfolio, borrowing freely against future income, or reducing consumption expenditures. In this situation, it is not surprising that the household would liquidate its mortgage debt on the first date it is beneficial to do so. In reality, a household's ability to borrow is limited.

In this section, we consider two limitations on borrowing and investigate their impact on the value of the option to prepay. The first limitation is a simple short selling constraint. The second is a collateralization constraint. We show that the short selling constraint reduces the value of the option and, as a result, a household facing this constraint must be substantially in-the-money on the unconstrained option in order to exercise it. When our collateralization constraint is effective, the household's option to prepay may be worth more or less than the unconstrained option to prepay. In this case, both in-the-money and out-of-the-money unconstrained options may be exercised.

D. A Constraint of Short Sales

A restriction on short selling an asset means that the household can not hold a negative amount of the asset. For example, the restriction $\delta(\tau) \leq B(\tau)$ on the prepayment level is a natural short sales constraint on the mortgage debt. In addition to this restriction, we add the aggregate

short sales constraint

$$\sum_{i=1}^m A_i(\tau) \geq 0, \quad \tau \geq t$$

to the above optimization problem. It is easy to verify that adding this constraint alters only the asset equilibrium conditions, A non-negative multiplier, $\mu(\tau)$, $\tau \geq t$, is added to the equilibrium conditions, with the result that the future marginal utilities of wealth become

$$\mathcal{E}_t[\lambda(v)] = \frac{\rho^{v-\tau}}{D(\tau, v)} M(\tau, v) \mathcal{E}_t[\lambda(\tau)] \quad (17)$$

where

$$M(\tau, v) = \prod_{v'=\tau}^v (1 - \mu(t, v'))$$

$$\mu(t, v) = \mathcal{E}_t[\mu(v)/\lambda(v)] + cov_t \left[\frac{\lambda(v)}{\mathcal{E}_t[\lambda(v)]}, \frac{\mu(v)}{\lambda(v)} \right]$$

The variable $\mu(t, v)$ is the expected monetary value of the short sales constraint at time v , and $M(\tau, v)$ gives the cumulative effect of the short sales constraint from period τ to period v . Substituting the expression for $\lambda(v)$ into (17) and rearranging yields

$$\chi^{short}(\delta(\tau)) = \frac{1}{\rho} \frac{\sum_{v=\tau}^T [1 + h(t, v) + \xi_h(t, v)] M(\tau, v) D(\tau, v)^{-1} - \xi_k(t, \tau)}{1 - \mathcal{E}_t[k(\tau)]} \quad (18)$$

Since $\mu(\tau)$ is non-negative, $M(\tau, v) \leq 1$, implying that $\chi^{short}(\tau) \leq \chi(\tau)$. Hence the value of the option to prepay is reduced by the short sale constraint.

This result has a direct implication for empirical work. The value of the unconstrained option potentially overstates the value of the prepayment option to the household. Thus, if the value of the financial option to prepay is included in a regression, with no controls for household liquidity, the coefficients on the option value may be biased downwards to compensate for the

overvaluation. The degree of this bias depends directly on the importance of liquidity constraints in the household's lifetime allocation program. This result indicates a possible reason for the poor showing of the option variable in some prepayment studies.

E. A Collateralization Constraint

Our short sale constraint does not allow the household to borrow against the latent equity in its dwelling. This constraint is unduly restrictive. It eliminates the possibility of junior mortgages and obviates one of the reasons households cite for home ownership. In this section, we loosen our asset market constraint to

$$\sum_{i=1}^m A_i(\tau) \geq -[V(\tau) - B(\tau)] \quad \tau \geq t$$

so the household can tap the latent equity in its dwelling.

Loosening the asset market constraint alters the financial asset equilibrium conditions in the manner described in the preceding section. In addition, the constraint multiplier now appears in the prepayment equation. Some algebra shows that

$$\chi^{coll}(\delta(\tau)) = \frac{1}{\rho} \frac{\sum_{v=\tau}^T [1 + h^c(t, v) + \xi_h(t, v)] M(\tau, v) D(t, v)^{-1} - \xi_k(t, \tau)}{1 - \mathcal{E}_t[k(\tau)]} \quad (19)$$

where

$$1 + h^c(t, v) = \mathcal{E}_t[H(\tau, v)] \left[1 + \mathcal{E}_t[k(v)] + \mathcal{E}_t[\mu(v)/\lambda(v)] + cov_t \left(\frac{H(\tau, v)}{\mathcal{E}_t[H(\tau, v)]}, \frac{\mu(v)}{\lambda(v)} \right) \right]$$

The value of the constraint enters (19) in three places. The first is in $M(\tau, v)$, as it does in (18). Its effect is to reduce the value of the option to prepay. The second place is in the expected monetary value of the constraint, $\mathcal{E}_t[\mu(v)/\lambda(v)]$, which augments the return to prepayment. This second term raises the possibility that the option to prepay may be exercised sooner than in the

case of the unconstrained option. Indeed, if this term is large enough, some out-of-the-money financial options may be exercised. Last, the constraint adds another risk term to the return expression. This is the risk of being liquidity constrained at some point in the future. The term can have either a positive or negative sign, so its impact on the exercise of the option is unknown.

II. An Empirical Model of ARM Prepayment

Following the assumptions of the preceding section, the returns on assets, including future mortgage rates, are uncertain at the beginning of each time period, when the asset and prepayment choices must be made. However, households do have access to the current yield curve and to the profile of interest rate volatility along the yield curve. They also know the form of the stochastic process.

Uncertainty about future interest rates drives two sources of household prepayment risk. The first occurs because of the variability in the mortgage constant and the second occurs because of the variability in the marginal utility of wealth in period τ . The household's marginal valuation of these risks changes as new information is received from the market. The information is contained in the realization $(r(t), \eta(t))$, $(0 < t; 0 \leq T)$ of the stochastic processes. Once revealed, this information conditions the household's optimal value function $\Psi(t)$ since it conditions the household's expectations.

Define $\gamma(\zeta|\Psi_j(t))$, $\zeta \geq t$, as the rate at which a household with utility $\Psi(t)$ receives market information, and let $g(\Psi|\zeta, \Psi_j(t), \delta(t))$ be the density of potential utility levels. A household will prepay if $\Psi(\dots, B(t)-\delta(t)) > \Psi(\dots, B(t))$, so the rate $\gamma(\zeta|\Psi_j(t))$ at which households shift out of currently held mortgages can correctly be written as

$$\gamma(\zeta|\Psi(t))|_1 = \gamma(\zeta|\Psi(t))Pr[I = 1|T = \tau, \Psi(t)] \quad (20)$$

where $I=1$, if the household prepays and is zero otherwise. This says that the rate of prepayment equals the overall rate at which a utility level "offer" arrives, times the probability that the offer leads to optimal prepayment, given the receipt of the offer. We specify the overall rate of prepayment as:

$$\gamma(t|z(t)) = Pr[\tilde{T}=t | z(t)], \quad (21)$$

where T is a random variable representing the duration in the current mortgage and $z(t)$ is a vector of factors which affect the rate γ . The $z(t)$ vector contains factors which condition the consumer's utility $\Psi(t)$ and may also include factors summarizing the household's history. $Pr[.]$ denotes a probability.

As is well known, equation (21) is an instantaneous transition rate. It can be interpreted, roughly, as the probability of prepaying a mortgage within the next month, given the covariates $z(t)$ and given that the consumer has spent t months holding the mortgage without prepaying.

The overall prepayment rate is specified as a proportional hazards model of the log-logistic form:

$$\gamma(t|x(t)) = \{(\eta+1)t^\eta \exp(\alpha) / [1 + \exp(\alpha)t^{\eta+1}]\} \exp[\beta x(t)], \quad (22)$$

where $\eta > -1$, the parameters β , η , and α are to be estimated, and t is the number of months over which the mortgage has been held. The rationale for choosing this specification for the hazard model is that it guarantees a unimodally shaped hazard. We presume that in the initial periods of the mortgage, consumers are unlikely to prepay, so the effect of duration is positive. This is particularly so in our data set which has a high percentage of mortgages with teasers. Once the teasers are eliminated at the first interest rate adjustment, the prepayment level increases, reaches a peak, and then begins to fall as the holding period approaches the terminal state. Thus we expect the hazard for prepayment to be bell-shaped. This is the case for the log-logistic model with $\eta > 0$.

Given the specification for the hazard, we can rewrite equation (22) as a nonlinear least squares model. Following Petersen (1986a, 1986b), we factor the log likelihood into the sum of the log likelihood contributions for each segment of time wherein the step-function covariates remain constant. Taking appropriate derivatives, the gradient vector obtained is equivalent to those in standard scoring algorithms, such as BHHH algorithm (Berndt *et al.*, 1974). Petersen (1986a, p 285) has shown that the approximate variance-covariance matrix of the estimator is equivalent to that derived from scoring algorithms.

The data structure for this reformulation of the hazard problem requires a separate record

for each time interval over which the covariates remain constant. The time interval for the records is monthly, corresponding to the time interval for the interest rate resets on the adjustable rate mortgages. The household's forward expectations concerning the interest rate process are represented in the month-by-month value of the prepayment option. This value is obtained using a generalization of the binomial Ho and Lee (1986) model. The path of previous interest changes is summarized in the month by month outstanding balance and payment on the contract. Other factors conditioning the prepayment rate include the wealth portfolio of the household and its income. These factors will be discussed in Section 5.

III. Evaluation of the Prepayment Option

At the end of each duration interval, the consumer must be able to evaluate the prepayment option, given knowledge of the yield curve and its stochastic process. The Ho and Lee (HL 1986) model for valuing interest rate contingent claims is based exactly on this primitive. HL assume that bond prices are given by discounted expected values, where the discounting is based on the rolled over short rate. Assuming that the time interval is of length one, the price $P(s)$ at time s of an asset paying $P(t)$ at $t > s$ is given by

$$P(s) = E_s \left[P(t) e^{-\sum_{\tau=t}^{t-1} r(\tau)} \right] \quad (23)$$

where $r(\tau)$ is the instantaneous riskless rate for time τ . In our application of HL, we further assume that the expectations hypothesis holds so that actual and risk neutral probabilities are equivalent.

An advantage of the HL model for our application is that it takes the vector of discount bond prices, $d_{i,n} = (d(i,i+1), \dots, d(i,i+n))$, as exogenous, where $d_{i,n}$ denotes the value at time i of receiving \$1 with certainty at time n in the future. The stochastic process for the bond prices is parameterized by a risk-neutral probability θ of bond prices increasing, an "upstate," and a complementary risk neutral probability $(1 - \theta)$ of bond prices decreasing, a "downstate." Upward and downward shifts in the yield curve are obtained by

specifying perturbation functions: $f(j)$ for the upward shift in the yield curve and $f^*(j)$ for the downward shift. Given this structure, bond prices in the upstate are

$$d(i,n) = f(n-i) \frac{d(i-1,n)}{d(i-1,i)} \quad (24)$$

while in the downstate at time i , they are

$$d(i,n) = f^*(n-i) \frac{d(i-1,n)}{d(i-1,i)} . \quad (25)$$

By assumption, the functions f and f^* satisfy

$$\theta f(j) + (1-\theta) f^*(j) = 1 \quad (26)$$

which means that θ and $(1 - \theta)$ can be interpreted as risk-neutral probabilities, even without the local expectations hypothesis. Thus, the perturbation functions satisfy

$$f(0) = f^*(0) = 1 \quad (27)$$

so that bond price will always equal \$1 at maturity, as required.

Under the further assumption of path independence, the f and f^* functions are determined up to a parameter η . They are given by

$$f(t) = \frac{1}{(\theta + (1-\theta)\eta^t)} \quad (27)$$

and

$$f^*(t) = \frac{\eta^t}{(\theta + (1-\theta)\eta^t)} \quad (28)$$

where η is state-time independent.

As shown, by Dybvig (1988), Pedersen, Shiu, and Thorlacius (1989) and Cooley, LeRoy, and Parke (1990), Wallace and Wang (1989), the original formulation of the HL model suffers from some drawbacks which impede its usefulness for valuing contingent claims on long bonds such as mortgages. The most problematic feature is the state independence of perturbation

functions. This implies that the volatility term η is state independent and therefore independent from the level of the interest rate. Dybvig (1989, p. 5) and Pedersen *et al.* (1989, p. 23) note that the volatility assumption implies

$$\lim_{n \rightarrow \infty} D_{i,n} = \infty.$$

This leads to rate paths with negative values, which can dominate the expectation in equation (23) for large values of t . Cooley, LeRoy, and Parke (1990, p. 8) further note that the variance assumption implies interest rates have conditional variances with no empirical validity.

The generalizations of the HL which have been suggested have focused on making the perturbations $f(j)$ and $f^*(j)$ functions of state and time, while at the same time maintaining the binomial path independent process set out by HL. Redefining the perturbation function, of course, implies that $\eta(j)$ is similarly redefined. Cooley, LeRoy, and Parke (1989, p. 20) propose defining $\eta_m(j)$ to assure that conditional interest rate variances rise at a decreasing rate with j and that long rates have lower conditional variances than short rates. Pedersen *et al.* (1989) suggest restricting the estimated forward rate volatilities such that negative interest rates cannot occur and suggest functional forms which ensure this. Both strategies were attempted, with mixed results. The results obtained using a version of the Pedersen *et al.* (1989) method are reported here.

A function of implied conditional variances was estimated using the implied volatility from options on Eurodollar futures and T-bond futures as data. The function was constrained such that negative interest rates were not permitted. $\eta_m(j)$ was then solved from

$$\ln(\eta_m(t)) = \sqrt{\frac{V^*(r(t))}{n\theta(1 - \theta)}} \quad (29)$$

where $V^*(r(t))$ is the estimated conditional variance of interest rates corresponding to the t^{th} maturity (Pedersen, 1989, Hull, 1989).

In summary, the exogenously determined factors needed to implement the generalized HL model are an interval specific estimate of the yield curve and estimates of the implied interest rate volatilities conditioned on the observed term structure of interest rates. Given these

estimates, and assuming a risk-neutral probability of .5, a 360 period HL path independent lattice is derived. Although the interest rate is path independent, it is well known that the cash flows of ARMs with payment and interest caps are path dependent with realizations that expand at the rate 2¹. Given the impossibility of accounting for all these paths, an antithetic Monte Carlo strategy was used to randomly draw paths of implied forward rates. The option value was computed as the difference between expected value of the noncallable ARM and the callable ARM at that time period. Both values are conditioned on the elements of the mortgage contract. The ARM is assumed to be prepaid when the present value of the existing mortgage minus the book value is greater than the prepayment penalty. No other transaction costs were assumed.

IV. Data Description

The data for this analysis consisted of a random sample of ARM contracts held in the portfolio of a large California Savings and Loan Institution. The sampling frame consisted of new originations across the four years extending from March, 1985 to July, 1988. These contracts reflect "modern" ARM contracts. The sample was constructed such that only right censoring would be present; that is, a mortgage could leave the pool only because of prepayment. Only one mortgage in the 1000 contracts drawn from the S&L left the pool because of default and it was excluded from the analysis.

Once the contracts were identified, the month by month cash flows for forty-two months were calculated given the year end balances, the contract elements (caps, etc.), and the reset rates used by the institution over the period. The origination documents were then coded so each mortgage in the sample was linked to detailed information about the borrower's wealth, income, and socio-economic characteristics at the time of origination. The final sample consisted of 653 observations, of which 123 prepaid. As discussed in Section 3, the observational unit for the proportional hazard model is a month long time interval. A separate record is generated for each time interval the mortgage remained extant; thus the records for a mortgage extend from its origination date, standardized to the 15th of the month, to the lesser of its payoff date or the end of the analysis period, August, 1988. Sample means for the data are reported in Table 1 based on all time intervals.

The variables reported in Table 1 have been categorized by their time dependent characteristics. The borrowers in the sample are all from California; they have a mean age of 42 years, tend to be married, and have a mean household income of \$5,351.88 per month. On average, they have three years of education after high school and have one dependent. Their asset wealth is computed from their self-reported values and does not include the value of the real estate collateralizing the ARM. Their liabilities do not include the ARM mortgage balance. As is clear from Table 1, there is considerable variability in the contract elements found in these mortgages. Most of the ARMs have a 12 month payment adjustment frequency. The contract interest rate is adjusted every month, on average, with an average initial adjustment occurring after the 6th month. All of the loans allow for negative amortization and on average, the loans have floor and ceiling payment caps depending on whether or not there has been negative amortization.

The time variant factors reported in Table 1 include the observed contract rates, estimates for the implied volatility, the spread between long and short rates, and a calculated prepayment option value. The average contract rate (the interest rate plus margin) over the sample period of 42 months fell 64 basis points, on average, from an initial average rate of 10.44% to an average rate of 9.8%. The observed origination rates were in the range of 10% to 12% for the ARMS originated in 1985, observed rates began to decrease by mid 1986 though August of 1988. There were no periodic rate caps in the ARM contracts, the index was 11th district Cost of Funds Index, and the binding constraint for these contracts was the floor and ceiling payment caps.

The values for the observed spread between the one Year T-Bill rate and the 20 Year T-Bond rate were obtained from the *Wall Street Journal* using a trading day on or close to the 15th of every month, beginning March 15, 1985 and ending August 15, 1986. The term structure estimates used a data set of zero coupon Treasury bonds yields which was obtained from Drexel Burnham Lambert. The implied volatilities were calculated using data on Eurodollar futures and equivalent maturity bond T-Bill yields, again collected from the *Wall Street Journal*. Options on T-Bond futures were obtained from the *Wall Street Journal* for the same dates. The 15th of the month is the benchmark date used for the adjustments on the ARM contract rates, mortgage payments and outstanding balances on mortgages, etc. Estimates of the term structure of interest rates were obtained by applying Nelson and Seigel (1987) to the zero coupon bond data for the periods 3/86 through 8/88 and extrapolating forward from month 132. The functional

form

$$r(j) = B_0 + B_1 \exp\left(-\frac{\sqrt{n^*}}{\zeta}\right) + B_2 \left[\left(\frac{\sqrt{n^*}}{\zeta}\right) \exp\left(-\frac{\sqrt{n^*}}{\zeta}\right) \right] \quad (32)$$

was estimated by performing a grid search on ζ over $\zeta = .1(1.2)^k$ for $k = 1, \dots, 50$.⁴ Selected estimated yield curves for the sample period are reported in Figure 1. As is clear, the estimated functional form allows for considerable flexibility in the term structure. All of the reported yield curves are for the 15th of the month or the nearest preceding trading day.

Unfortunately, the literature provides no definitive evidence of the linkage between interest rate volatility and the shape of the yield curve (Litterman, *et al.*, 1988; Bookstaber, 1988; Dybvig, 1989; Hull and White, 1990; Melville and Overdahl, 1989). As previously discussed, the generalization of the HL model requires a parameterized linkage. Our strategy to empirically establish this relationship should be viewed as crude at best. A Black-Scholes model (1973) for valuing bond options is used to derive the interest rate volatility implied by the price of 3-month Eurodollar futures options and to derive the bond price volatility implied by the prices of 240 month T-bond futures options. The price volatility is converted to an interest volatility through a second order Taylor series approximation. The evident inconsistencies in this strategy are that the Black-Scholes pricing model assumes the options are European and that the interest rate volatility is constant. A term structure of these fitted interest-rate volatilities is then estimated using the Nelson and Siegel (1987) in a manner similar to that for the yield curve. These fitted, volatility curves tend to have curvatures similar to those for the estimated yield curves and for about two thirds of the forty two periods led to paths with non-negative interest rates. Figure 2 provides some examples of the yield and volatility curves.

The option values are calculated using 200 draws over the tree at each reset period. The mean of these draws is reported in Table 1. Given the evolution of interest rates over the period and the relatively low volatility estimates (excluding October and November, 1987), it is not

⁴ This strategy was suggested by Cooley, LeRoy, and Parks (1990); the modification of Nelson and Siegel allows the long end of the yield curve to lie flatter.

surprising that the option is not very valuable. The binding constraint for these mortgages is the floor payment cap, if there had been negative amortization. Usually some negative amortization occurred after the teaser period because of the lagging payment adjustment. The estimates of the option values are used as exogenous variables in the estimation of the hazard function. The month-by-month values summarize the market information on the yield curve and volatilities, given the elements of each mortgage contract. Two other time varying covariates are included as alternate proxies for this information and are commonly used by other authors (Follain, 1988).

VI. Estimation Results

Four versions of the model, Equation (22), are estimated. The hazard model is first estimated only as a function of time and the value of the option or proxies for the option. These estimates are presented in Table 2. Neither the option value nor the proxies are statistically significant. Additionally, the proxies have the wrong signs, which is surprising because it implies that the more valuable the option, the lower the prepayment rate. Equivalently, the higher the volatility or the greater the long/short rate differential the lower the prepayment rate. The prepayment option parameter has the correct sign but is insignificant.

Table 2 illustrates that models that rely exclusively on forward looking measures to predict the probability of prepayment for ARMs have little explanatory power. They do not capture the data generating process underlying the prepayment observations. Our results suggest that prepayment can be better explained on the basis of the age of the mortgage than on the value of the option. This result suggests that estimating prepayment rates solely on the value of the option and its optimal prepayment is too restrictive. However this could be an artifact produced by the low interest rate variability of our data.

In an effort to uncover why a purely option focussed specification of a hazard model performs so poorly we expanded the model to include a more complete set of covariates. These covariates include measures of the wealth, liabilities, and income of the households in our sample. Such measures follow from our theoretical model presented in Section 2. In particular, the liabilities variable is intended to capture possible liquidity constraints faced by the household. It is more likely that a liquidity constraint is binding shortly after origination because households have just mobilized their financial resources to make the downpayment on their dwelling. Since

this lowers the value of the option, it is less likely that it will be exercised. Accordingly, the time of exercise will be pushed further into the future. Additionally, if real estate values appreciate rapidly during the initial period of the mortgage, the households latent equity grows rapidly and as a result the expected future value of the liquidity constraint decreases with time. This too, pushes exercise into the future.

We include two socio-economic factors as covariates: the number of dependents and length of job tenure. The number of dependents proxies potential household liquidity problems. Two arguments can be made for the job tenure variable. First it measures income and wealth variability. A more variable income stream increases the equilibrium risk adjusted rates of return to the household (see Equation 12) which reduces the value of the option to prepay. Second, a short tenure in the job may indicate future liquidity problems. This also reduces the value of the option.

The final variable, outstanding balance at time t , represents the end state of the amortization path. The mortgage balance may be higher either because of negative amortization or because the initial loan amount is high. If the household is risk averse a higher mortgage balance implies greater benefits to prepayment because it increases near term consumption levels.

The results of the hazard function estimation with the more complete vector of $z(t)$ are reported in Table 3. In Model III, the proxies for the option remain insignificant with the wrong signs. Comparing Model I with Model III, the peak payoff⁵ ranges are similar: 36.8 months for the Model I and 36.6 months for the Model III. In fact the hazard structure is almost identical. The important differences lie with the extended set of covariates present in Model III. To begin, a quick examination of the loglikelihoods for the two models show that Model III vastly outperforms Model I. In terms of individual covariates, job tenure and household liabilities are statistically significant at the .05 level and have negative signs. This implies that the rate of prepayment slows for households with greater liabilities, or greater liquidity constraints. Years on the job is also statistically significant at .05, the longer the consumer's job tenure the slower the rate of prepayment. Both these effects conform to our expectations. The

⁵ The peak in the prepayment rate can be calculated from $t^* = \exp\{(1/\alpha + 1)\ln[\alpha \exp(-\gamma)]\}$.

unpaid balance variable has a positive sign and is also statistically significant. This also meets our expectations.

We now compare models II and IV. It is interesting to note that the gap between the peak prepayment dates is larger than the gap between the peak prepayments above. The peak prepayment period is 37.55 months for Model II and 39.985 months for Model IV, presently we don't have an explanation for this. The only covariates that are statistically significant in Model IV are the household liabilities and household assets. In both cases the parameter estimates are negative. The results for household assets are puzzling.

VI. Conclusions

This paper is developed along two lines. First, we have sought to develop a theoretical model of ARM prepayment and to plumb this model to try to explain apparent irrationalities in ARM prepayment. The model suggests that empirical prepayment models should include a backward looking element and points to the importance of household liquidity in the value of the option to prepay. Second, we have incorporated some of our insights in the development of our empirical model. Our results indicate that there may be considerable merit to empirical prepayment models that account for more than the option valuation. Whether the increased cost of building such models is warranted of course remains to be seen.

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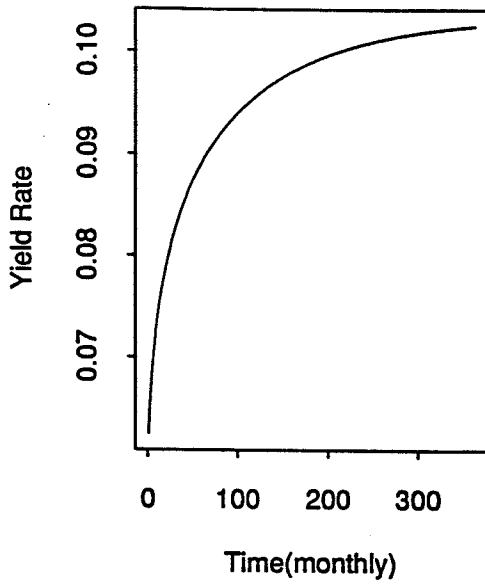
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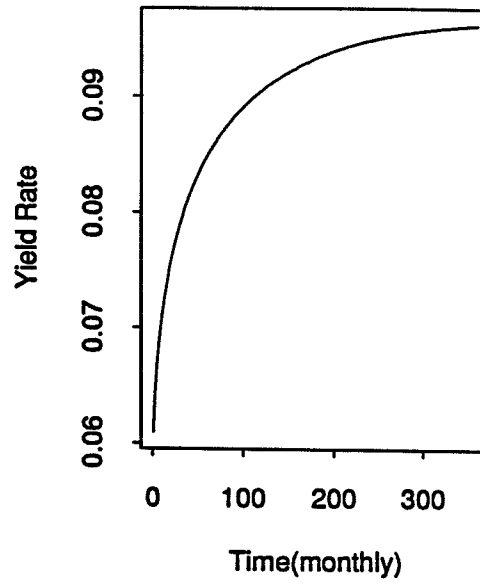
Figure 1

Estimated Term Structure

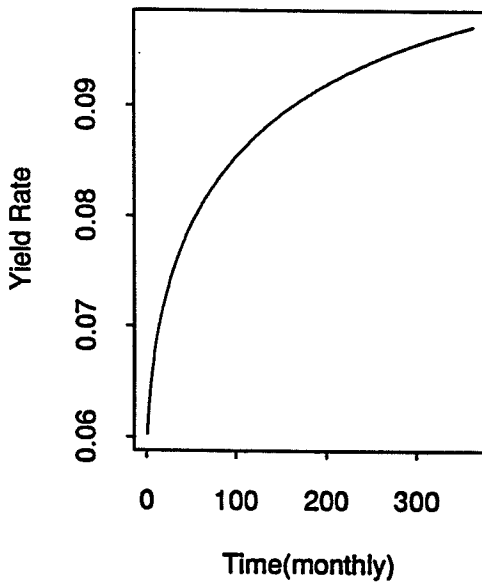
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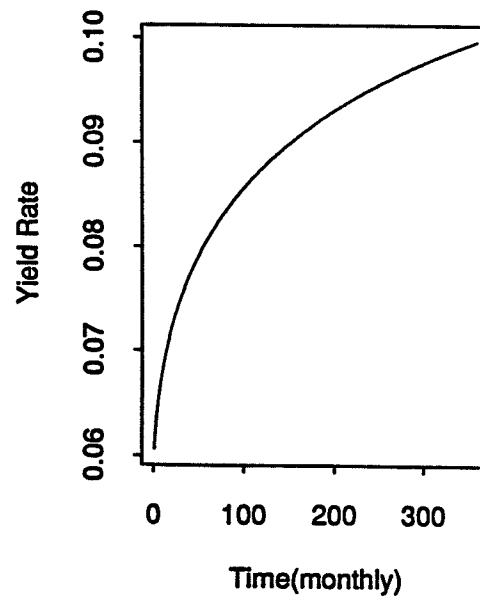
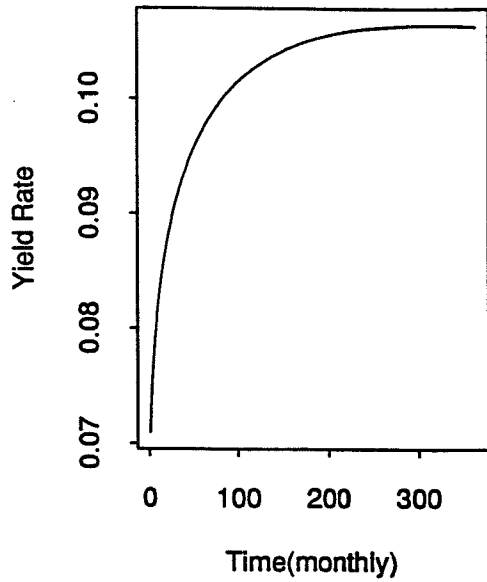
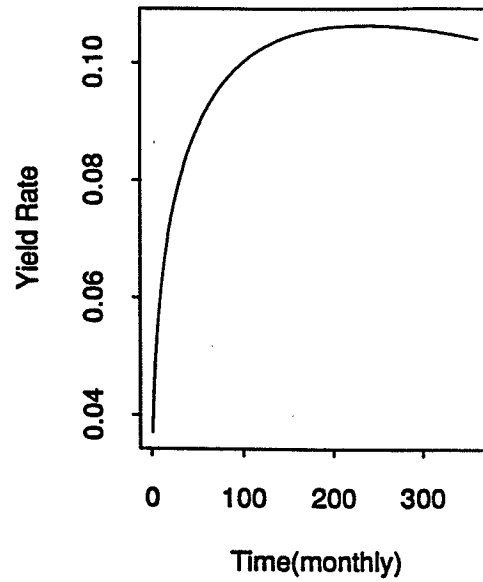


Figure 1 (Continued)
Estimated Term Structure

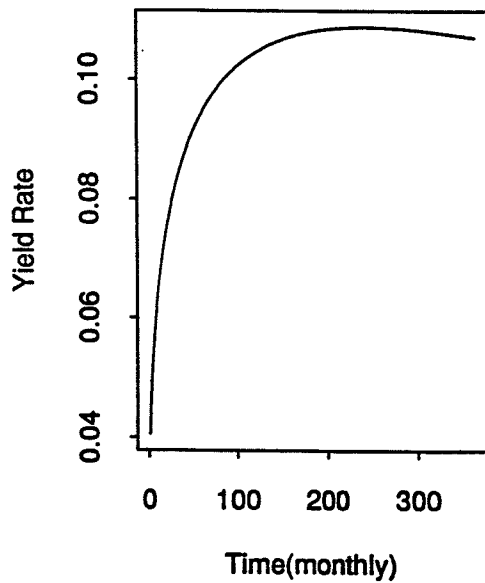
June 1985



July 1985



August 1985



September 1985

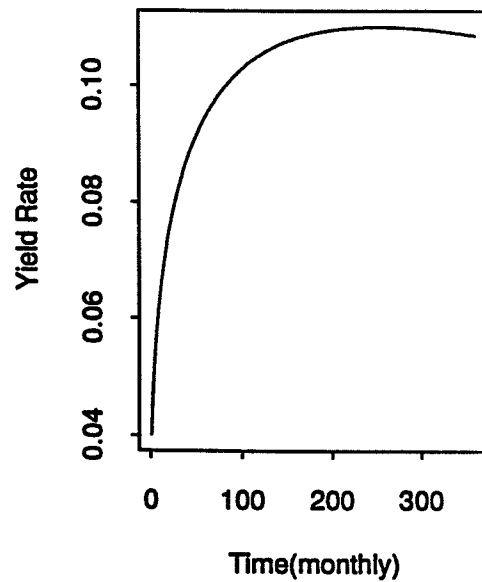


Figure 2

Estimated Term Structure/ Implied Volatilities

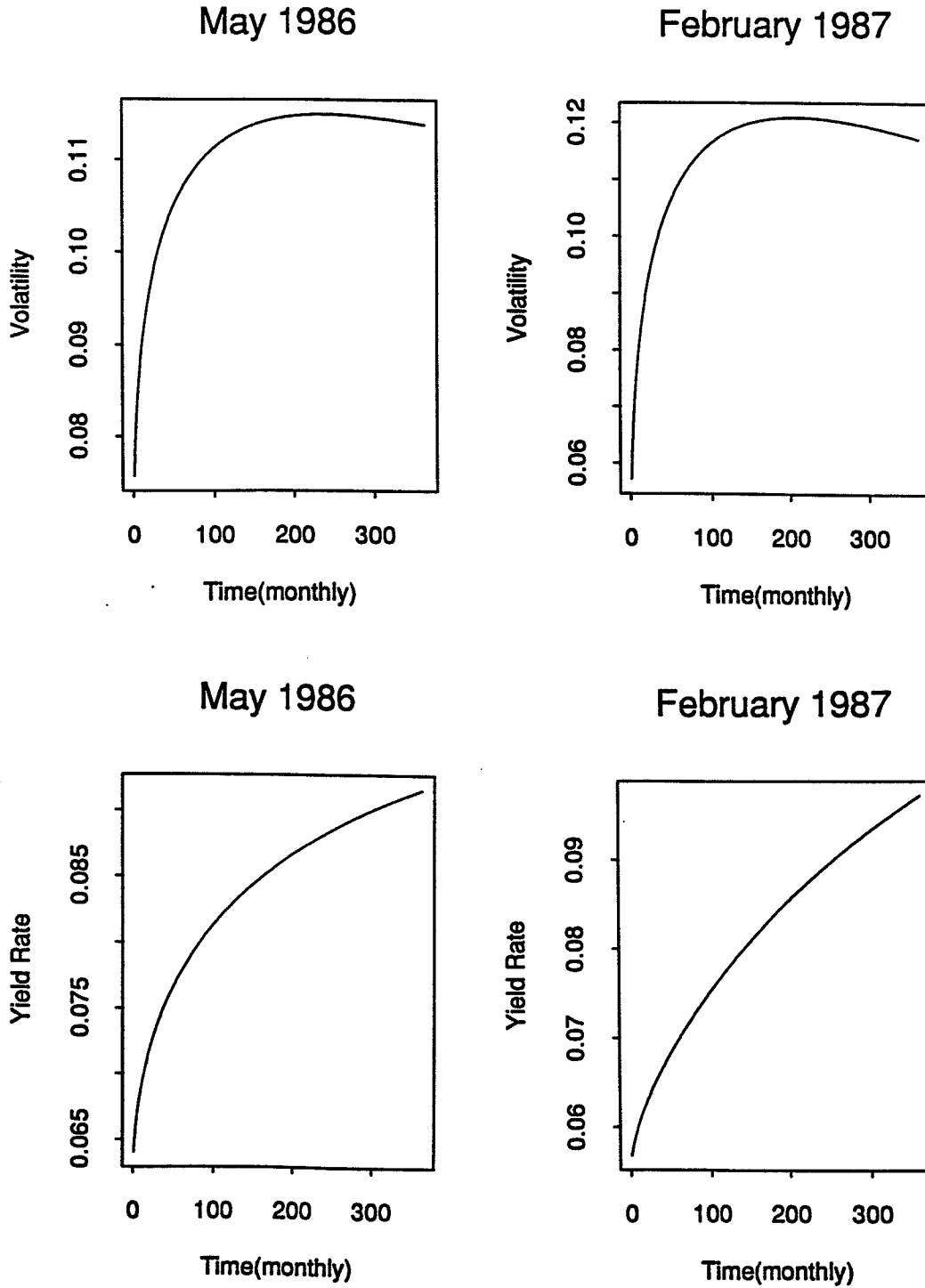


Table 1

DESCRIPTIVE STATISTICS FOR THE MORTGAGE SAMPLE

VARIABLE NAME	MEAN	STANDARD DEVIATION
TIME INVARIANT:		
Borrower Characteristics		
Tenure in Home (Years)	5.93	5.96
Number of Dependents	.92	1.15
Married ¹	.69	.42
Borrower's Education (Years)	15.16	3.00
Household Assets	470,458.65	1,190,826.00
Household Liabilities	135,799.16	601,403.78
Household Monthly Income	5,351.88	5,063.68
Borrower's Age	42.00	10.65
Borrower's Years on Job	8.36	7.85
Contract Characteristics		
Interest Adjustment Frequency (Months) ²	2.162	1.77
Payment Adjustment Frequency (Months)	11.77	1.27
Life of Loan Rate Cap (%)	5.00	.00
Payment Cap ³	1.89	.44
Teaser Rate	.70	.73
Teaser Period (Months)	10.30	14.30
Discount Points Paid (%)	1.0	1.00
Prepayment Penalty ⁴	.23	.42
Loan to value Ratio (%)	70.70	11.93
Initial Balance	123,819.00	63,889.15
Initial Contract Rate (%)	10.44	.99
Initial Payment	1,061.00	544.57
Margin (%)	2.20	.30

Table 1 (Continued)

DESCRIPTIVE STATISTICS FOR THE MORTGAGE SAMPLE

VARIABLE NAME	MEAN	STANDARD DEVIATION
TIME VARIANT:		
Contract Characteristics		
Average Contract Rate Over Sample (%)	9.8	.8
Average Payment Over Sample	1,022.88	552.29
Option Characteristics		
Implied Volatility on Options on Eurodollar Futures (Average maturity 160 days) (%)	3.5	1.5
Observed Spread between One Year T-Bill rate and the 20 Year T_Bond rate (%)	4.2	1.6
Calculated Option Value	201.85	750.83

1. Dummy variable: 1 = married, 0 = not married.
2. Scale variable: 1 = Monthly beginning in Month 3, 2 = Monthly beginning Month 6, 3 = Monthly beginning Month 12, 4 = Monthly beginning Month 24, 4 = Monthly beginning Month 36, 6 = Every 6 months.
3. Scale variable: 1 = No payment cap, 2 = Maximum payment increase limited to 7.5% over previous payment limit amount. Decrease limited to 7.25% only if there is unpaid deferred interest, 3 = Maximum payment increase limited to 7.5% over previous payment amount, 4 = Maximum payment increase limited to 7.5% over previous payment amount. Decrease limited to 7.5% only if there is unpaid deferred interest.
4. Scale variable: 0 = none, 1 = 6 months interest.

Table 2

Estimates of the Effects of the Option Value or Proxies
for the Option Value on the Rate of Prepayment

VARIABLE NAME	PARAMETER ESTIMATE	STANDARD ERRORS
Model I:		
Constant	42.2595*	.38491
Constant of Hazard ¹	-40.2058*	.00493
Duration (Months) ²	.0863*	.00147
Volatility	-3.6608	3.21564
Long/Short rate Diff.	-.8071	2.93583
-Loglikelihood	21050.23	
Model II:		
Constant	40.1328*	.38491
Constant of Hazard ¹	-40.9227*	.00493
Duration (Months) ²	.0889*	.00147
Prepayment Option	2.4937	1.84839
-Loglikelihood	18236.53	

1. This is the α parameter in the log-logistic model in Equation (22).
 2. This is the γ parameter in the log-logistic model in Equation (22).
 It measures months since the mortgage was originated.
 * Significant at .05% level

Table 3

Estimates of the Effects Household Wealth and the Option Value or Proxies
for the Option Value on the Rate of Prepayment

VARIABLE NAME	PARAMETER ESTIMATE	STANDARD ERRORS
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Model III:

Constant	40.1462*	.49978
Constant of Hazard ¹ .	-40.0771*	.04662
Duration (Months) ² .	.0798*	.01398
Volatility	-4.2636	3.14089
Long/Short rate Diff.	-3.2469	6.02593
Number of Dependents	.0149	.01132
Years on Job	-.5384*	.18307
Household Assets	-.0081	.02139
Household Liabilities	-.4015*	.11091
House Income	-.0331	.15485
Unpaid Balance at t	.4910*	.16294
-Log likelihood	25687.62	

Model IV:

Constant	49.4369*	.09425
Constant of Hazard ¹ .	-40.3871*	.05438
Duration (Months) ² .	1.8710*	.03482
Prepayment Option	1.3490	.94583
Number of Dependents	.0040	.00345
Years on Job	-.0505	.02831
Household Assets	-.0541*	.02139
Household Liabilities	-.0115*	.00411
Household Income	-.0131	.01549
Unpaid Balance at t	-.0452	.03629
-Log likelihood	20687.62	

1. This is the α parameter in the log-logistic model in Equation (22).

2. This is the γ parameter in the log-logistic model in Equation (22).

It measures months since the mortgage was originated.

* Significant at .05% level