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Abstract

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Doctor of Philosophy in Engineering – Civil and Environmental Engineering

University of California, Berkeley

Professor Mark Hansen, Chair

This dissertation presents dynamic stochastic optimization models for Air Traffic Flow Management (ATFM) that enables decisions to adapt to new information on evolving capacities of National Airspace System (NAS) resources. Uncertainty is represented by a set of capacity scenarios, each depicting a particular time-varying capacity profile of NAS resources. We use the concept of a scenario tree in which multiple scenarios are possible initially. Scenarios are eliminated as possibilities in a succession of branching points, until the specific scenario that will be realized on a particular day is known. Thus the scenario tree branching provides updated information on evolving scenarios, and allows ATFM decisions to be re-addressed and revised.

First, we propose a dynamic stochastic model for a single airport ground holding problem (SAGHP) that can be used for planning Ground Delay Programs (GDPs) when there is uncertainty about future airport arrival capacities. Ground delays of non-departed flights can be revised based on updated information from scenario tree branching. The problem is formulated so that a wide range of objective functions, including non-linear delay cost functions and functions that reflect equity concerns can be optimized. Furthermore, the

model improves on existing practice by ensuring efficient use of available capacity without necessarily exempting long-haul flights.

Following this, we present a methodology and optimization models that can be used for decentralized decision making by individual airlines in the GDP planning process, using the solutions from the stochastic dynamic SAGHP. Airlines are allowed to perform cancellations, and re-allocate slots to remaining flights by substitutions. We also present an optimization model that can be used by the FAA, after the airlines perform cancellation and substitutions, to re-utilize vacant arrival slots that are created due to cancellations.

Finally, we present three stochastic integer programming models for managing inbound air traffic flow of an airport, when there is adverse weather impacting the arrival capacity of the airport along with its arrival fixes. These are the first models, for optimizing ATFM decisions, which address uncertainty of future capacities of multiple NAS resources.

Mark Hansen, Chair

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CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1. INTRODUCTION

The National Airspace System (NAS[1]) can be viewed as a set of resources used by aircraft to fly from their origin airport to their destination. The resources consist of airports, fixes[2], and the airspace. The latter is subdivided into 3-dimensional polyhedrons called enroute sectors, each of which is controlled by an Air Traffic Control (ATC) facility. For example, the arrival and departure operations at an airport are controlled by a terminal area radar approach control (TRACON) facility. Traffic in enroute sectors is handled by controllers at an air route traffic control centers (ARTCCs), commonly known as "Centers".

The control and coordination of aircraft in the NAS is provided by the Air Traffic Management System, which has two main components: air traffic control (ATC) and air traffic flow management (ATFM). The main purpose of ATC is to ensure safe operations in the system by maintaining required separation between aircraft. The goal of ATFM is to coordinate air traffic so that demands for various resources don't exceed capacities. Demand is characterized by the time-varying need for various NAS resources by the aircraft flying through the system. Capacity of the NAS resources is also time-varying, and depends on factors such as physical infrastructure, staffing, and weather. For

[1] List of abbreviations is given in Appendix 2
[2] a geographical position determined by the presence of navigational equipment(s)

example, the number of runways, their configuration, and weather conditions determine the operational capacity of an airport.

Adverse weather – such as convective weather, fog, snow/ice, low cloud ceilings and visibility – can substantially reduce the capacity of NAS resources. Regions where such conditions occur may become non-flyable or require greater separation between flights. The resulting capacity reduction creates congestion and delays. Approximately 70% of the total delay in the NAS is attributable to weather (Robinson et al, 2004).

In the event of adverse weather, ATFM becomes necessary to reduce airborne holding of flights and overloading ATC facilities. Typical ATFM actions include ground delay programs in which certain flights are assigned later departure times, rerouting to avoid weather-affected regions, and imposing airborne delays through miles-in-trail restriction or other means. Among these, assigning pre-departure delays to flights (commonly known as ground delay) is a key strategy in ATFM. The underlying motivation is that it is cheaper and safer to delay a flight before its departure than while it is airborne.

Imperfect information on future conditions poses a major challenge to ATFM decision making. It is impossible to reliably forecast weather with the lead time and levels of temporal and spatial resolution necessary to perform ATFM deterministically (Evans, 2001). ATFM is therefore performed under uncertainty about the future capacities of NAS resources such as airports, sectors, fixes, and airways. There is also uncertainty in the demand for NAS resources which has been addressed in some recent studies (Ball et al, 2003; Vossen, 2004). However, in practice ATFM decisions are made by assuming deterministic time varying capacities and demand, but with certain ad hoc procedures such as exemptions of long-haul flights, that take the underlying uncertainty into account.

The research literature includes several optimization models for ATFM. In almost all such models, the objective is to minimize system-wide delay cost, which has two components – ground and airborne delays. Most of the optimization models in the literature are formulated as linear and/or integer programming models. Both deterministic and stochastic optimization methods have been applied to solve problems in ATFM. Much of the research has focused on ATFM when there is a single bottleneck – the destination airport. Work that considers multiple capacity constraints, including those for enroute sectors and origin airports, has been confined to the deterministic problem in which it is assumed that there is perfect information about future capacities. In light of the limits in weather forecasting capabilities discussed earlier, this is a significant limitation.

Stochastic optimization models have been developed mainly in context of a single airport ground holding problem (SAGHP) in which flights destined for a particular airport are assigned ground delays in order to mitigate arrival demand-capacity imbalance at the destination. Research on SAGHP is motivated by the requirement of suitable decision support models for planning and implementing a ground delay program (GDP) at an airport (GDP planning is discussed in details later in this chapter). The existing stochastic optimization models for SAGHP are mostly *static*, in the sense that delays are assigned at the beginning of a planning period, and not revised later. There has been little research on dynamic stochastic optimization models for SAGHP that utilize updated information on the evolving capacity of an airport, as the time of day progresses, to revise ground delay decisions of flights.

As research on ATFM optimization has progressed, there has been a paradigm shift from highly centralized ATFM to a system that combines centralized and decentralized decisions. Under this new paradigm, users (airlines) are given more autonomy in adapting their schedules to capacity shortfalls, while abiding by certain constraints imposed by the controlling entity (the FAA) in order to maintain safe operations, avoid overload in the NAS, and allocate scarce resources equitably. The FAA assumes the role of a service provider instead of a supreme controller as in the old paradigm.

1.2. RESEARCH CONTRIBUTIONS

This dissertation presents stochastic optimization models for ATFM that can utilize dynamic updates of information on evolving capacities of NAS resources. First we propose a dynamic stochastic model for the SAGHP that can be used for planning GDPs under uncertainty in future arrival capacities of an airport. Following this, we present a methodology and optimization models that can be used for decentralized decision making in the GDP planning process, using the solution from the dynamic stochastic SAGHP as a starting point. Finally we present stochastic optimization models that account for uncertainty in both airport and enroute capacities. The remainder of the dissertation is organized as follows:

In Section 3 of this chapter we describe the key elements of ATFM in the United States, and how it is performed in practice. We focus on planning and implementation, and the process of collaborative decision making (CDM) in GDPs. In Section 4 we highlight the existing literature on optimization models for ATFM.

In Chapter 2 we propose a dynamic stochastic model for SAGHP that accounts for uncertainty in airport arrival capacity, and is adaptive to the updated information as time progresses. We compare our model performance with that of the existing stochastic models for SAGHP, using an experimental setup with a realistic problem. We propose alternative objective functions that address equity issues in GDP planning.

In Chapter 3 we propose a methodology for using the solutions of the dynamic stochastic model for SAGHP for GDP planning in the context of CDM. We present an optimization model that can be used by individual airlines to perform substitutions to achieve their business goals, based on their original schedule or a reduced one with cancelled flights. Then we propose an optimization model that can be used by the FAA to re-allocate unutilized capacity created by cancellations, to other airlines.

In Chapter 4 we propose three optimization models that account for uncertainty in future capacities of both the destination airport and enroute airspace. We confined our formulations to a small problem that includes one airport and its arrival fixes. We propose static, two-stage, and multi-stage (dynamic) stochastic optimization models for making ground delay and rerouting decisions of flights that are inbound towards the airport.

We present the concluding remarks and recommendations for future research in Chapter 5, followed by the references in the Bibliography section at the end.

1.3. OVERVIEW OF ATFM IN THE UNITED STATES

The ATFM is performed on a national basis at the Air Traffic Control System Command Center (ATCSCC), commonly referred to as the "Command Center", which is located in Herndon, Virginia. The primary duty of the ATCSCC is to monitor the traffic situation in the NAS, and impose control measures in case of demand-capacity imbalance. (See FAA, 2004, for a detailed description of ATFM in the United States.) Figure 1.3.1 briefly explains the overall ATFM procedure. The information on time varying demand and capacity of various NAS components is stored in a computerized system known as Enhanced Traffic Management System (ETMS). ETMS includes several tools that are used to monitor the current and projected traffic situation. Whenever it is predicted that the demand exceeds capacity during some time period (usually 15 minutes or more) at some resource(s) ATCSCC imposes control measures to mitigate the problem. The major control measures are described below:

- *Ground Delay Program (GDP):* A GDP is issued when the arrival capacity of an airport falls below the demand for some time duration. Typically this is caused by the effect of bad weather impact on arrival capacity. In a GDP, some of the flights bound towards the airport whose scheduled arrival time lies within the congested period are delayed at their origin airport and assigned a later time slot of arrival. The underlying motivation is that it is cheaper and safer to delay a flight before take off than subject it to airborne delay. Sometimes GDPs are also issued in order to resolve congestion at enroute airspace in the vicinity of an airport.

Closely related to GDPs are Ground Stops, which are implemented in case of unexpected and severe congestion at an airport. When a Ground Stop is implemented, all flights destined towards an airport that haven't yet departed are hold until the congestion problem is resolved.

- *Rerouting:* Whenever a region in the NAS is affected by bad weather, flights whose primary/filed route passes through the affected region are rerouted through alternative routes. Rerouting can cause congestion in some of the weather-free parts of the NAS due to increased traffic in those regions, and often require miles-in-trail restrictions.

- *Metering Flow at Enroute Fixes (Miles-in-Trail Restriction):* In order to ensure that the flow of traffic in the enroute sectors or congested regions in the NAS doesn't exceed the capacities, distance based metering such as Miles-in-Trail (MIT) restrictions are issued at different fixes. MIT restrictions keep the flow of

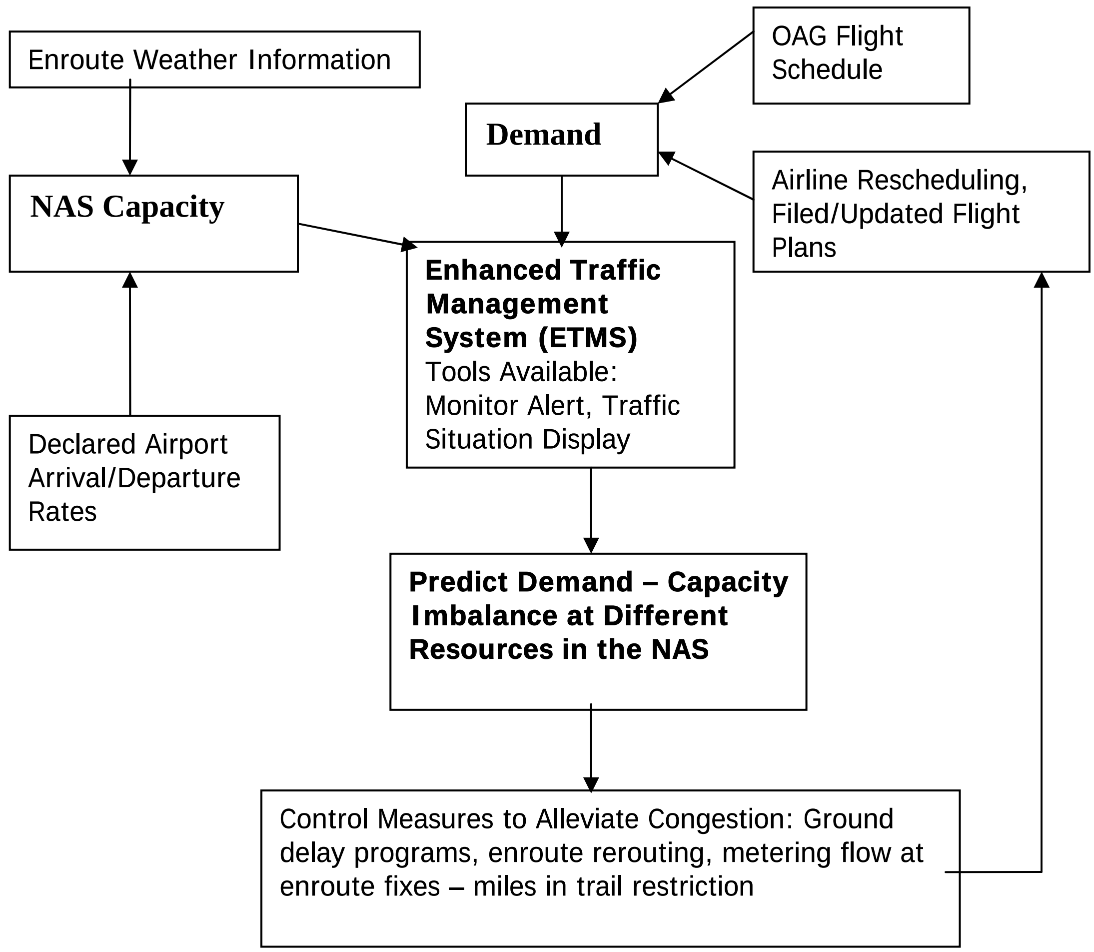

Figure 1.3.1 Overview of Air Traffic Flow Management Procedure

traffic below a certain level by specifying the minimum separation between two consecutive aircraft flying across the same fix. MIT restrictions often cause airborne delay, but in a manner that is less expensive and disruptive than airborne holding.

Until the early 1990s, ATFM was performed in a centralized setting in which the ATCSCC assumed the authority to impose restrictions on departure times and routes of individual flights at its own discretion. This paradigm has changed in the mid-1990s, with the development of Collaborative Decision Making (CDM). CDM allows increased participation of airlines in ATFM decision making (FAA, 1999). A joint initiative of the FAA and airlines, CDM is designed to resolve day-to-day NAS congestion problems through collaboration and information sharing. The key objectives of CDM are to create a common situational awareness among the FAA and airlines about NAS capacity and demand, facilitate information exchange between them, and to provide airlines with greater flexibility to make their own decisions.

ATFM is performed in a de-centralized setting under CDM. FAA (or ATCSCC) is responsible for monitoring the demand-capacity status of the NAS, and for allocating constrained resources among the users in an equitable and efficient way. Airlines are encouraged to apprise the FAA with their operational plans and strategies; in return the FAA provides them with flexibility to make use of their share of resource capacities according to their individual business goals.

Initially, CDM focused on procedural changes in implementing a GDP. Benefits realized from CDM in the context of GDPs (Ball et al. 2000) have motivated recent efforts on Collaborative Routing (CR), aimed at mitigating enroute airspace congestion through increased collaboration among airlines and the FAA (Burke, 2002). In the following paragraphs we describe existing practice in GDP planning, followed by a discussion on CR initiatives.

1.3.1. GDP Planning in Practice

The time varying demand of flights inbound to an airport is derived from the published schedule in the Official Airlines Guide (OAG). Airlines report their day-to-day variations in individual flight schedules, delays and cancellations to the ATCSCC. The latter uses Flight Schedule Monitor (FSM), which is a decision support tool, to monitor arrival demand-capacity status of an airport. Whenever demand exceeds capacity a GDP is implemented.

GDPs are planned several hours in advance of the actual time periods when demand-capacity imbalances occur. As mentioned before, demand-capacity imbalances occur mostly due to bad weather, which is difficult to forecast accurately even 1-2 hours in advance. Therefore at the time of implementing a GDP, the ATCSCC faces uncertainty in airport arrival capacities later in the day. Although the airport capacity profile is uncertain at the time of GDP implementation, the ATCSCC assumes a deterministic capacity profile and plans the GDP accordingly.

The ATCSCC exempts certain flights from GDP, and allows them to arrive at their scheduled times. Normally exemptions are allowed for flights that originate from airports located beyond a certain distance from the destination airport. The distance is set based on the predicted severity of capacity shortfall. Exempting flights from GDP is the primary technique by which the FAA manages uncertainty in forecasts. The underlying motivation for exemptions is that delays assigned to long haul flights will be unrecoverable, and cause underutilization if airport capacity improves later. On the other hand, ground delays of flights with shorter stage length can be adjusted tactically to meet updated realizations of airport capacity as the day progresses. Flights which are already airborne at the time of GDP implementation cannot be assigned ground delays and are also exempted.

Exemptions in a GDP raise equity issues. Airlines that operate mostly long haul flights at an airport benefit from exemptions, while the short haul carriers incur higher delays. Recent studies addressing this problem are discussed later in this chapter. Exempting long haul flights may be reasonable when there is a high chance that airport capacity will improve later (i.e. weather will clear off). When the chances of capacity improvement are low, such a policy may be may be inefficient as well as unfair. One of the objectives of this research is to develop decision models that have the capability to optimally assign *de facto* flight exemptions depending on capacity forecasts.

After the exempt flights are assigned arrival slots, the remaining arrival capacity of the airport is distributed among the non-exempt flights by mechanism known as Ration-by-

Schedule (RBS). RBS is based on a first-scheduled-first serve policy. The non-exempt flights are ordered according to scheduled arrival times, and allocated arrival slots according to their position in that order. Hoffman (1997) provides a detailed discussion of the actual RBS algorithm.

Under CDM, airlines assume ownership of the arrival slots assigned to their flights. Airlines can utilize the slots they own according to their business goals. They are allowed to make internal schedule changes by substituting arrival slots among flights and canceling flights. For example, an airline may decide to swap the arrival times of two flights, or substitute a flight to a slot that belongs to a flight it has cancelled. After a round of substitutions and cancellations, airlines apprise the ATCSCC about their updated schedule.

Flight cancellations create slots to which no flight is assigned. This can happen if the airline canceling a flight has no other flights that can use the vacant slot. The ATCSCC runs the Compression algorithm to reallocate arrival slots to all non-exempt flights that are not cancelled. The basic principle behind this algorithm is that whenever possible a delayed flight is assigned an earlier arrival slot, so that the unused capacity during any time period is reduced (or eliminated if possible). While reallocating a flight to an open slot, preference is given to flights that belong to the airline which has released the slot.

The Compression algorithm works as follows. All flights are time-ordered based on their rescheduled (as a result of cancellations/substitutions) arrival slots. Each open slot has an owner airline, which has released the slot. At first a search is performed on the ordered list of flights to find a flight belonging to the same airline, whose assigned arrival slot is later than the open slot, which can be substituted to the open slot. If no such flight is found a similar search is performed on the remaining flights (belonging to different airlines). If a flight is found from either search process, it is assigned to the open slot. The flight's original slot now becomes open, and its ownership is transferred to the airline which owns the slot to which the flight is now assigned (i.e. the previously open slot). The algorithm is repeated for all open slots, until no flights can be found to fill an open slot.

1.3.2. Collaborative Routing (CR)

Unlike in GDP, ATFM decisions on alleviating enroute congestion are still very much centralized, and are taken by the ATCSCC without much involvement of NAS users. Developing a resource allocation mechanism for such problems is a major research challenge. Recently, some decision support tools and CR procedures have been deployed to allow collaboration among the FAA and airlines when developing strategies for rerouting flights to avoid weather affected regions in the NAS.

The National Playbook (FAA, 2004) provides guidelines to develop and implement rerouting strategies to avoid enroute convective weather. It contains the most common scenarios of convective weather activity that occur in the NAS, and proposes alternative routing options under such conditions. The Coded Departure Routes (CDR) is a database that provides a set of alternative routes between various airports. Both National Playbook and CDR provide a basis for developing rerouting strategies and facilitate collaboration between the ATCSCC and airlines during a weather event. The Collaborative Route Coordination Tool (CRCT) provides a platform for CR.

Low Altitude Arrival and Departure Routes (LAADR) provide alternative routes for air traffic flow at those airports which are located in regions with high volume of enroute traffic. LAADR are implemented to avoid or mitigate congestion in the enroute stream.

1.4. REVIEW OF LITERATURE ON OPTIMIZATION MODELS FOR ATFM

ATFM optimization has been a topic of research for more than a decade. There are two main categories of published research in this area: (1) optimization models that accounts for airport arrival and/or departure capacities, but ignores enroute capacity constraints, and (2) those that accounts for both airport and enroute capacity constraints. The former class of problems is commonly known as ground holding problem (GHP), while the latter is sometimes termed the multi-airport air traffic management problem. In this section, we present an overview of the published literature on optimization models for ATFM. In the subsequent chapters of this dissertation, we discuss some limitations of the models that our formulations overcome.

There is a substantial research literature on the ground holding problem (GHP). The objective of this class of problem is to minimize the sum of airborne and ground delay costs in the face of anticipated demand-capacity imbalances at destination airports, by assigning ground delays to flights. Within the domain of GHP, there are two sub-problems: the single airport ground holding problem (SAGHP) and the multi-airport ground holding problem (MAGHP). In SAGHP, the problem is solved for one destination airport at a time. In the MAGHP, a network of airports is considered, so that delay on a given flight segment can propagate to downline segments flown by the same aircraft. Some treatments of the MAGHP also consider crew and passenger connectivity effects.

The GHP was first systematically described by Odoni (1987). Following this, several deterministic models were developed, in which time varying airport capacities were assumed to be fully known beforehand. Terrab and Odoni (1993) proposed a deterministic model for SAGHP, in which the objective function minimizes the total cost of ground holding a set of flights. The cost of delaying each flight is represented through a linear cost function with flight-specific parameters, which is supplied as input. Hoffman (1997), and Hoffman and Ball (2000) proposed a deterministic model for the SAGHP with banking constraints, which imposes the condition that one or more groups of flights must arrive within pre-specified time windows. Adding such constraints is useful to model hub-spoke operations at major airports; see Hoffman (1997) for discussion on hub-spoke operations. Vossen (2002) proposed an optimization model for mitigating bias from exempting flights from a GDP.

Deterministic optimization, formulated as an integer program (IP), for MAGHP was first proposed by Vranas et al (1994). Their computational study showed exorbitant computing times for solving the IP optimally under realistic cases. Bertsimas and Stock

(1998) provided a *stronger* formulation (Nemhauser and Wolsey, 1998) to the deterministic MAGHP. Navazio and Romanin-Jacur (1998) proposed a deterministic optimization model for MAGHP with banking constraints.

Uncertainty in airport capacities has been addressed mainly in context of SAGHP; although Vranas et al (1994) provides some treatment of stochastic version of MAGHP. Richetta and Odoni (1993) proposed a static stochastic IP formulation for solving the SAGHP under uncertainty in airport arrival capacities. Hoffman (1997) and Ball et al (2003) proposed a modified version of the static stochastic optimization for SAGHP, which solves for optimal number of planned arrivals of aircraft during different time intervals. In the static models, decisions related to departure delays of flights are taken once at the beginning of planning horizon, and not revised later. This limitation was addressed by Richetta and Odoni (1994), who formulated a multistage stochastic IP with recourse for SAGHP. In their model, the ground delays of flights are not decided once at the beginning, rather they are decided at the scheduled departure time of the flights. However, ground delays once assigned cannot be revised later in their mode. In Chapter 2 we present a detailed discussion on some of the limitations of existing stochastic optimization models for SAGHP.

Deterministic optimization models addressing enroute capacity constraints were formulated as multi-commodity network flow problem by Helme et al (1992), and more recently by Bertsimas and Stock (2000). Unlike single-commodity flow network formulations, these models are computationally harder and do not guarantee integer solutions from LP relaxations. One of the assumptions made by Helme et al (1992) was that each aircraft route is pre-determined before its departure. Bertsimas and Stock (2000) model addresses routing as well as scheduling decisions, but it produces non-integer solutions for even small scale problems. Therefore the authors suggested heuristics to achieve integer solutions.

Disaggregate deterministic integer programming models for deciding departure time and route of individual flights were formulated by Lindsay et al (1993), and more recently by Bertsimas and Stock (1998). Although both formulations produce non-integer solutions from LP relaxation, the latter model achieves integrality in many more instances compared to the former, by virtue of its formulation (the details are discussed in Chapter 4). Goodhart (2000) formulated disaggregate deterministic models for ATFM, in which airlines priorities on various flights were accommodated. Such formulations are useful for applying optimization techniques in ATFM under the paradigm of CDM.

Weather related uncertainty in enroute airspace congestion was addressed by Nilim et al (2002). Their work focuses on dynamically rerouting an aircraft across a weather impacted region.

In summary, stochastic optimization methods for ATFM have been applied to solve the SAGHP, while there is much left to be done. One unexplored area is dynamic models that can adapt to updated information as time progresses, to revise the ground delay decisions of flights. Another is models that address both enroute airspace and airport capacity in a stochastic setting. Finally, to implement dynamic decision making in CDM environment, we must develop models that accommodate both decentralized and centralized decision making. These are the gaps filled by this dissertation.

CHAPTER 2: DYNAMIC STOCHASTIC MODEL FOR A SINGLE AIRPORT GROUND HOLDING PROBLEM

2.1. INTRODUCTION

In the single airport ground holding problem (SAGHP), flights inbound to an airport are allocated arrival slots, based on arrival capacity forecasts for the airport. The goal is to efficiently use the available capacity while absorbing necessary delays by ground-holding flights. If the forecast is accurate, then the ground delays will be such that the number of aircraft arriving at any time interval equals the airport “acceptance rate” (the vernacular for the maximum number of arrivals that the airport can accommodate) during that time. But in reality forecasts are rarely perfectly accurate, because it is very difficult to predict the operating conditions of an airport several hours in advance. Decisions made under uncertainty can cause airborne delays when the number of planned arrivals exceeds airport capacity during a time period. On the other hand, unnecessary ground delays will result if the capacity forecast proves pessimistic. In this chapter, we will present an optimization model to solve the SAGHP that features an improved method for handling uncertainty in the future capacity of the airport, and a formulation that can handle a broad spectrum of objective functions.

There is a substantial amount of literature on optimization models for the SAGHP. Most of the models assign ground delays to individual flights, with the objective of minimizing the total cost of delay. The optimization models are of mainly three types: deterministic, static stochastic, and dynamic stochastic. Deterministic models assume perfect knowledge of future capacity, whereas stochastic models address uncertainty in the forecasts. Static (or single stage) stochastic models account for uncertainty, but do not utilize updated information on the evolution of airport capacity. Dynamic stochastic models revise ground delay decisions based on evolving forecasts. To date, efficient deterministic and static stochastic models have been developed. A limited research literature exists on dynamic stochastic models, but as discussed in Section 2.2, existing dynamic stochastic models have significant limitations that the model presented in this chapter overcomes.

As theoretical research on ground holding problems has progressed, air traffic flow management has evolved from a completely centralized system to one in which users have more autonomy about how to adapt their schedules when adverse conditions reduce airport capacity. The FAA allocates arrival slots to airlines based on their original flight schedules and the first-come/first-served principle. This is known as ration-by-schedule (RBS); see Vossen (2002) and Hoffman (1997) for a detailed discussion. Users have accepted this as an equitable allocation method. However, to increase efficiency, airlines are then allowed to re-allocate their slots so as to better realize their own internal objectives – for example, by giving priority to flights that are the most time-critical. Thus the solution to the SAGHP, although assigning slots to flights, may in practice be used as a means of allocating slots to airlines, and the objective of the assignment includes equity as well as efficiency. We will show that our model can optimize metrics related to either efficiency or equity, or some mixture of the two.

In the remainder of this chapter, we will proceed as follows. In Section 2.2 we review the existing literature on the SAGHP. Section 2.3 summarizes the major contributions of our research on the SAGHP. In Section 2.4 we present our SAGHP model, followed by its application in a real world problem in Section 2.5. In Section 2.6 we propose alternative objective functions that take equity into account. We provide a discussion and concluding remarks in Section 2.7.

2.2. EXISTING LITERATURE

The uncertainty issue in the SAGHP was addressed by Richetta and Odoni (1993) using a static (single stage) stochastic integer programming (IP) model. Hoffman (1997) and Ball et al. (2003) formulated a closely related model, but with higher level of aggregation of flights in the decision variables. Also, they provided a formal proof of the underlying network structure of the model formulation. Total unimodularity of the constraint matrix guarantees integer solutions directly from the LP relaxation of the problem. Solutions of their model specify optimal planned arrival rates, by time period, based on a finite set of alternative arrival capacity scenarios. The rates can further be converted into arrival slots and different allocation schemes (such as RBS) can be applied to allocate slots to individual flights.

A major limitation of static stochastic models is the inability to revise ground holding decisions, based on updated information. Decisions are taken at the start of the planning horizon, and cannot be changed thereafter. Therefore, they do not enable decision makers to “plan to re-plan” by assigning ground delays with the recognition that departure times can be adjusted later, as better information about future capacity becomes available.

Richetta and Odoni (1994) formulated a (partially) dynamic multi-stage stochastic IP model. Rather than assigning delays to all flights at once, the Richetta-Odoni model assigns delay as the scheduled departure time approaches, so that decisions can be made with the most up-to-date forecast information. Uncertainty in airport arrival capacities is represented through a finite number of scenarios arranged in a probabilistic binary decision tree. As the day progresses, branches of the tree are realized, resulting in better information about future capacities. In Richetta-Odoni model, however, ground delays, once assigned, cannot be revised, even though this is technically possible so long as the flight has not yet departed. Moreover, their formulation requires that the scenario tree braches unveil one at a time.

In both of the above models -- Richetta-Odoni (1994), and Ball et al. (2003) -- decision variables correspond to ground delays assigned to groups of flights. For example, Ball et al. defines auxiliary variables G_t, which are the number of aircraft scheduled to arrive during time period t that are delayed by one or more time periods. The aggregate form of the decision variables restricts the objective function to measure linear delay costs. Use of non-linear functions of ground delay in the objective function leads to a non-linear programming problem. In the Richetta-Odoni model, non-linear measures of ground delays can be considered. However their objective function cannot measure non-linear deviation of the assigned ground delays from any standard allocation. For example, measuring squared deviation of flight-specific ground delays from their ration-by-schedule allocated delays is not allowed in the Richetta-Odoni model. Although traditionally GHP optimization models focused on minimizing overall cost of ground and airborne delays, where the cost functions are linear, recent studies (Vossen, 2002) have illustrated the use of alternative non-linear measures of delay costs to optimize slot allocation during a GDP. Such objective functions are more consistent with current slot allocation policies such as RBS.

Equity as well as efficiency must be considered when addressing the SAGHP. Although current CDM algorithms are accepted as equitable mechanisms for slot allocation, in practice they may be inequitable because of exemptions for long-haul flights. Equity issues in slot allocation during GDPs were discussed extensively by Vossen (2002). Vossen (2002) present an integer programming model to mitigate exemption bias during GDPs. The objective of the model is to assign available slots to the non-exempt flights in such as way that minimizes the expected deviation from the ideal slot allocation for different airlines, taking both exempt and non-exempt flights into account. The ideal allocation is based on the RBS procedure.

2.3. RESEARCH CONTRIBUTIONS

In this chapter, we present a dynamic stochastic IP model for the SAGHP, in which ground delays assigned to various flights can be revised during different decision stages, based on updated information on airport operating conditions. In our model, we specify constraints that can capture any generalized scenario tree representing evolving information about airport operating conditions, as described later. For all instances of the problem, we obtained integer solutions directly from the LP relaxation of the model; hence the computational times were in order of a few seconds even for large scale problems.

An additional advantage of our formulation is that it handles a wide range of objective functions. In addition to the standard linear delay cost function with different weights for airborne and ground delay, we can minimize expected squared arrival delay against schedule, and expected squared deviation between the assigned arrival slot and the ideal RBS arrival slot. Multiple objective functions can be considered through weighting schemes that either balance objectives or introduce subsidiary objectives as "tie-breakers" when the original model has multiple optima.

2.4. STOCHASTIC DYNAMIC OPTIMIZATION MODEL FOR SAGHP

In this section we present a dynamic stochastic optimization model that assigns ground delays to individual flights so as to optimize some objective related to quantities of airborne and ground delay, and that allows for the revision of ground delays for flights that have not yet departed in response to updated information. From this point onwards, we refer to this model as the Dynamic Revisable Ground Holding (DRGH) model.

Following Richeta and Odoni (1994), we represent the evolution of airport arrival capacity by a scenario tree. Each branch of the tree represents a capacity scenario or a group of scenarios realized as the time of day progresses. A capacity scenario corresponds to a possible time- varying arrival capacity profile. For example, Figure 2.4.1 shows a scenario tree when there are four possible scenarios of airport arrival capacities. The probabilities of occurrence are shown in parentheses. The time of day is represented by the horizontal axis, which is divided into T equal time periods. All scenarios are possible at the beginning of the day, but at time period τ_1, either scenarios 1 & 2 or scenarios 3 & 4 are eliminated as possibilities. Similarly at time τ_2, scenario 1 or 2 is realized if the upper branch evolves. If scenarios 3 or 4 evolve, no new capacity information becomes available until time τ_3.

The marginal probabilities of the scenarios change with time, as specific branches of the scenario tree are realized, but the conditional probabilities remain the same. For example, the bold line in Figure 2.4.1 represents evolution of the Scenario 2. Prior to τ_1, the marginal probability is 0.2. At time τ_1, this probability changes to 0.4 if the upper branch of the tree is realized, and goes to 0 if the lower branch is realized. Similarly, depending on the branch at if the upper branch at τ_2, the probability of Scenario 2 goes to 1 or 0.

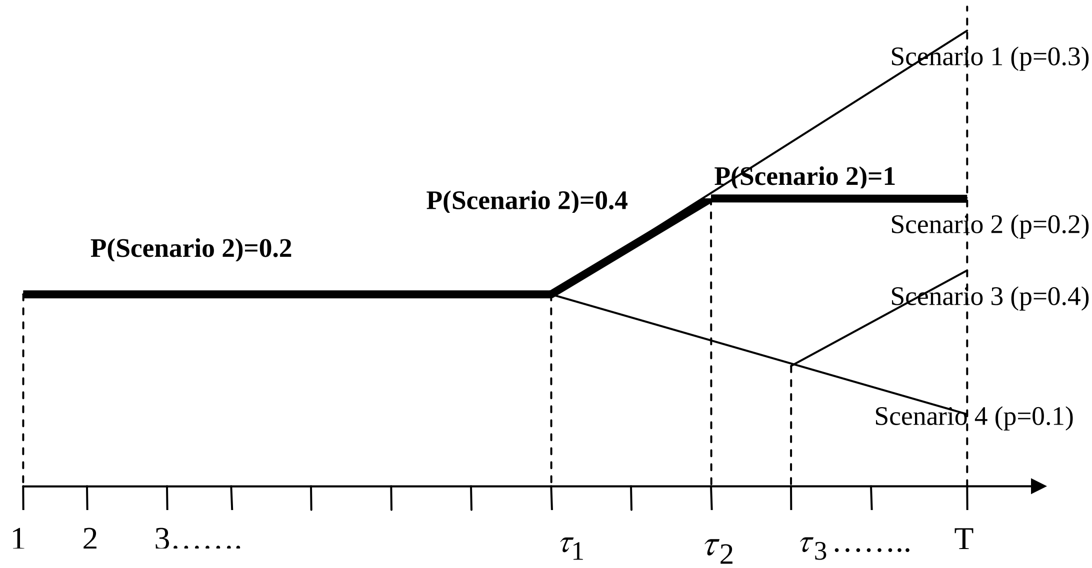

Figure 2.4.1 Scenario Tree of Evolving Airport Arrival Capacity

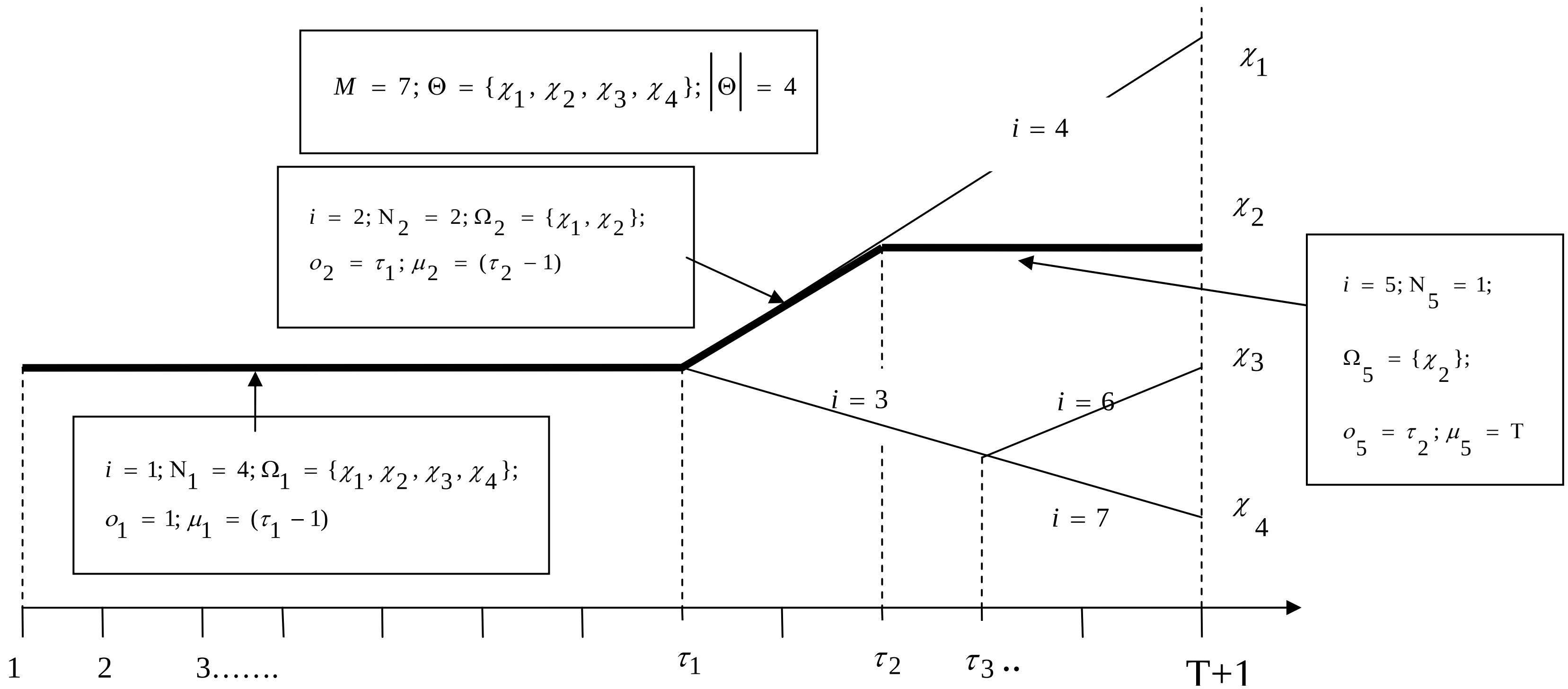

Figure 2.4.2 Representing the Scenario Tree in the Model

The scenario tree reveals the availability of information on airport operating conditions. The information may be based on forecasts, so that capacity changes are anticipated before they occur. If branching points in the scenario tree occur only when the operating conditions may change physically (for example at possible fog burn-off times) the active branch of the scenario tree will reflect the actual capacity at any instant. At the other extreme, perfect information on the future capacity may be available at the beginning of day; in this case all branching in the scenario tree occurs at the beginning of planning horizon, and different branches may correspond to the same instantaneous operating condition. In between these limiting cases are those in which branching occurs over the course of the planning period, but prior to when capacity scenarios actually diverge.

2.2.1. Model Formulation

Notation

Consider a set of flights $\Phi = \{1,...,F\}$ that are scheduled to fly to an airport for which a ground-hold program may be necessary. The time of day is divided into a finite set of time periods of equal duration, and is denoted by the set $\Gamma = \{1,..,T+1\}$. For instance, Γ might be a set of 48 time periods of 15 minutes each, amounting to a planning horizon of 12 hours. The scheduled departure and arrival times of a flight $f \in \Phi$ are denoted by $Dep_f \in \Gamma$ and $Arr_f \in \Gamma$ respectively. Let λ be the unit cost ratio for airborne and ground delays. We assume that both unit costs, and therefore λ, is the same for all flights, and that $\lambda > 1$.

Let Θ denote the set of capacity profile scenarios, and the unconditional probability of occurrence of a scenario $q \in \Theta$ be given by $P\{q\}$. The scenario-specific, time-varying arrival capacities of the airport -- M_t^q; $t \in \Gamma$, $q \in \Theta$ -- set upper bounds on the number of flights that can land during each time period. In order to allow all flights to land within the planning horizon, the arrival capacity of the last time period M_{T+1}^q is set to a very high value.

The scenario tree is input to the model through the following variables. Let M be the total number of branches of the scenario tree; $M \geq |\Theta|$. Each branch corresponds to a set of scenarios. The N_i scenarios corresponding to branch $i \in \{1,..,M\}$ are denoted by the set $\Omega_i = \{S_1^i,..,S_k^i,..,S_{\mathrm{N}_i}^i\}; S_k^i \in \Theta$. We assume that each branch has start and end nodes corresponding with the beginning of time periods. The time periods corresponding to start and end nodes of a branch are denoted by o_i and μ_i; $i \in \{1,..,M\}$. Figure 2.4.2 illustrates our notation using the same scenario tree that was shown in Figure 2.4.1.

Decision Variables

The decision variables in the model are binary variables defined as follows:

$$X_{f,t}^q = \begin{cases} 1 & \text{if flight f is planned to arrive by the end of} \\ & \text{time period t under scenario q;} \\ 0 & \text{otherwise} \end{cases} \qquad \begin{matrix} q \in \Theta, f \in \Phi, \\ t \in \{Arr_f,..,T+1\} \end{matrix}$$

The planned arrival time of a flight depends on its actual departure time and scheduled en route time, which we assume to be deterministic. This is the earliest time interval in which the flight can land given its assigned ground delay. Depending on airport arrival capacities, an aircraft may not be able to land during its planned arrival time period, and hence face airborne holding. The process is similar to single server queuing system, where the airport is a finite-capacity server with time varying capacities. Flights either land or enter the arrival queue at the planned arrival time. The decision variable $X_{f,t}^{q}$ corresponding to a flight is 0 before its scenario-specific planned arrival time, and 1 thereafter. Although the $X_{f,t}^{q}$ are scenario-specific, they are subject to coupling constraints in order to capture limitations in our knowledge about which scenario will occur, as explained below.

Corresponding to the $X_{f,t}^{q}$ is a set of corresponding auxiliary variables for the departure time period. Specifically we define:

$$Y_{f,t}^{q} = \begin{cases} X_{f,t+Arr_f-Dep_f}^{q} & if \quad t + Arr_f - Dep_f \le T \\ 1 \quad otherwise \end{cases} \quad ; \begin{matrix} q \in \Theta, f \in \Phi, \\ t \in \{Dep_f,..,T+1\} \end{matrix}$$

The departure release variables track the planned arrival times but are displaced earlier in time by the amount $Arr_f - Dep_f$. The ground delay for a particular flight is the difference between when its departure release variable is turned on and its original scheduled departure time Dep_f. A second set of auxiliary variables, denoted by W_t^q, specify the number of aircraft in the arrival queue at the end of time period *t* under scenario *q*. The W_t^q are determined by the capacity profile of scenario *q* and planned arrival schedule specified by the $X_{f,t}^{q}$. We assume that the system is empty at the beginning of the planning period and that all flights arrive by the end of period T+1, so that $W_0^q = W_{T+1}^q = 0$.

Objective Function

The objective function minimizes the expected sum of ground and airborne delay costs for all flights, and is given as follows.

$$Min \sum_{q \in \Theta} P\{q\} \times \left\{ \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - Arr_f \right) \times (X_{f,t}^q - X_{f,t-1}^q) \right] + \lambda \times \sum_{t=1}^{T} W_t^q \right\} \quad (2.4.1)$$

The expression $(X_{f,t}^q - X_{f,t-1}^q)$ is 1 only if flight f is planned to arrive at time t. If a flight is planned to arrive later than its scheduled time, it is assumed that the required delay occurs at its origin airport, and the first component of (2.4.1) measures this amount. The second component of the objective function measures the total airborne delay arising from planned arrivals exceeding arrival capacity. The second component is multiplied by the delay cost ratio to account for differences in airborne and ground delay unit costs.

Constraints

The set of constraints are given by the Expressions (2.4.2) – (2.4.6) below.

$$X_{f,t}^q - X_{f,t-1}^q \geq 0; \quad \forall f \in \Phi, q \in \Theta, t \in \{Arr_f ..T+1\} \quad (2.4.2)$$

$$W_{t-1}^q - W_t^q + \sum_{f \in \Phi} \left(X_{f,t}^q - X_{f,t-1}^q \right) \leq M_t^q; \quad t \in \{1..T+1\}, q \in \Theta \quad (2.4.3)$$

$$W_0^q = W_{T+1}^q = 0 \quad (2.4.4)$$

$$X^{q}_{f,T+1} = 1 \qquad \forall f \in \Phi, q \in \Theta; \qquad (2.4.5)$$

$$Y^{S^i_1}_{f,t} = ... = Y^{S^i_k}_{f,t} = ... = Y^{S^i_{N_i}}_{f,t}; \qquad f \in \Phi, t \in \{1..T\}; S^i_k \in \Omega_i : \mathrm{N}_i \geq 2 \ \ and \ o_i \leq t \leq \mu_i$$

(2.4.6)

Constraints (2.4.2) reflect the requirement that the decision variables $X^{q}_{f,t}$ be non-decreasing in *t*. Thus if a flight is planned to arrive by the end of time period τ, under capacity scenario q, then $X^{q}_{f,t}$ has to be 1 for all $t \geq \tau$. Constraint set (2.4.3) specifies that the number of arrivals during any time period is limited by the scenario-specific airport arrival capacity for that time period. The number of arrivals in a time period *t* is the sum of the reduction in the size of the arrival queue between the end of *t* and the end of the previous time period *t-1*, and the number of flights whose planned arrival time is in *t*. If the number of planned arrivals during a time period exceeds arrival capacity, then the excess flights are subject to airborne delay and added to the arrival demand for the next time period.

Constraint (2.4.4), as mentioned earlier, ensures that the system is empty at the beginning and end of the planning period. Constraints (2.4.5) ensure that all flights arrive at the destination airport by the latest time period in the planning horizon.

Constraint set (2.4.6) is a set of coupling constraints (or sometimes referred as non-anticipativity constraints in the literature) on the ground-holding decision variables $Y^{q}_{f,t}$.

These constraints force ground delay decisions to be made solely on information available at time *t*. For a given time period *t*, we require the decisions to be the same for all scenarios associated with the same scenario tree branch *i* (in other words the scenarios belonging to the set Ω_i) in that time period. For example in Figure 2.4.1 (or 2.4.2) all four scenarios are in set Ω_1, which extends from time period 1 through time period $\tau_1 - 1$. Therefore $Y^1_{f,t} = Y^2_{f,t} = Y^3_{f,t} = Y^4_{f,t} \quad \forall f \in \Phi, 1 \le t \le \tau_1 - 1$. Similarly we require that $Y^1_{f,t} = Y^2_{f,t}, \quad \forall f \in \Phi, \tau_1 \le t \le \tau_2 - 1$ and $Y^3_{f,t} = Y^4_{f,t}, \quad \forall f \in \Phi, \tau_1 \le t \le \tau_3 - 1$. $Y^1_{f,t}$ and $Y^2_{f,t}$ are decoupled from τ_2 onward, as are $Y^3_{f,t}$ and $Y^4_{f,t}$ beginning in time period τ_3.

Coupling constraints equate the scenario-specific planned arrivals of individual flights under different scenarios. For example, consider a flight scheduled to depart during time period $(\tau_2 - 1)$, and assume destination airport conditions evolve according to scenario χ_2 (see Figure 2.4.1). The planned arrival time for this flight must be the same under χ_2 and χ_1 if the flight is released on time, because both of these scenarios are still possible until the next period, τ_2. If, on the other hand, the flight is ground delayed by 1 time period, further ground delay decisions can be based solely on which scenario is realized at τ_2, which in this case is χ_2.

The DRGH model always produces solutions with expected delay costs less than or equal to those of Ball et al. (2003) and Richetta-Odoni (1994). This is because the latter models are equivalent to our model except that they impose additional constraints, the removal of which can only improve the value of the objective function at optimum. We demonstrate this in Appendix 1.

2.2.2. Example

To demonstrate the properties of the stochastic dynamic model presented in the last section, we apply it to a small problem and compare the results with those obtained from the Richetta-Odoni (1994) model for the same problem. There are a total of 13 aircraft, 13 time periods, and 4 capacity scenarios: $\Theta = \{\xi_1, \xi_2, \xi_3, \xi_4\}$. By using an example with a small number of flights we are able to clearly demonstrate differences between the two models.

The capacity scenarios for this example are shown in Figure 2.4.3. All scenarios have a capacity of 1 arrival per time period through the end of period 6. Under the most favorable scenario, ξ_1, the capacity increases to 2 for period 7, and then to 3 for all subsequent periods. Under ξ_2 and ξ_3, the capacity increases follow the same pattern of ξ_1 but occur 1 and 2 time periods later, respectively. ξ_4 resembles ξ_3 except that the capacity remains at 2 for one additional period. Such a set of scenarios might correspond to a situation where arrival capacity is reduced by morning fog, with the scenarios corresponding to different fog burn-off times and durations.

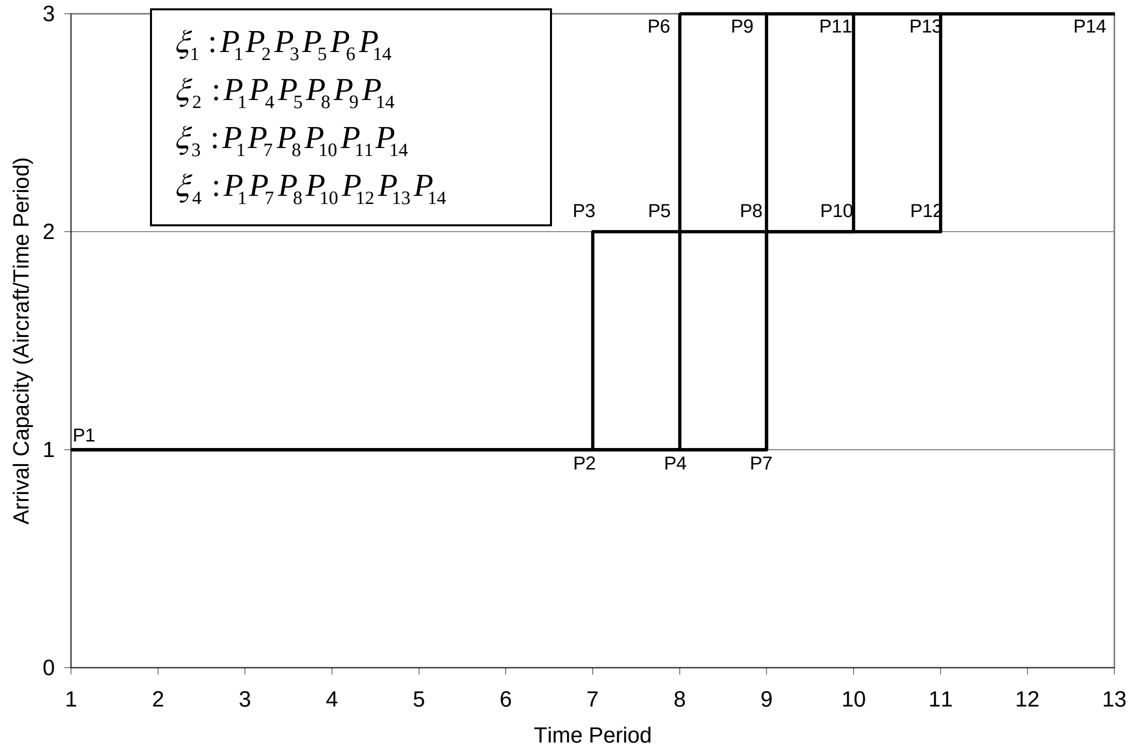

Figure 2.4.3 Arrival Capacities Corresponding to Different Scenarios

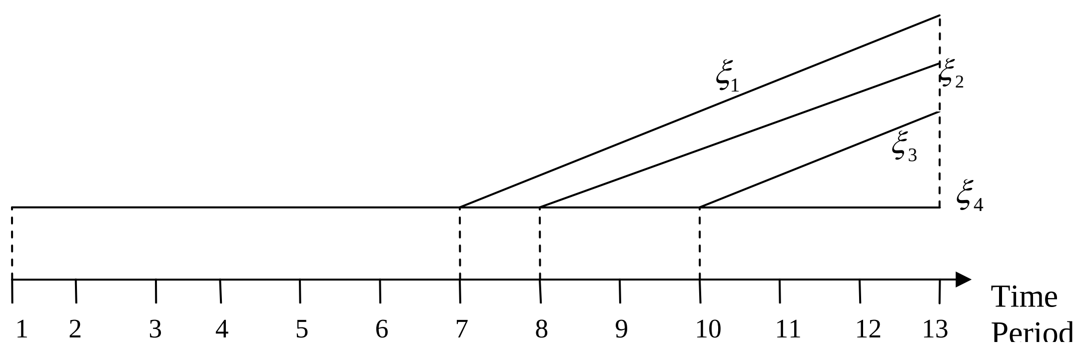

Figure 2.4.4 Scenario Tree

The scenario tree assumed for this problem is shown in Figure 2.4.4. In this figure branches occur when additional information about which capacity scenarios will evolve becomes available. For purposes of this example, we assume that the scenario tree branches when capacity profiles diverge. Thus we know at the beginning of time period 7 if ξ_1 is the realized scenario. If it is not, then at the beginning of period 8 we know whether ξ_2 is realized, and so on. We again emphasize that contemporaneous branching of the capacity profiles and the scenario tree in this example is not necessary for our model to be applicable. For example, the scenario tree could branch at time period 6, allowing us to know whether the capacity will increase one period later.

Two flight schedules are considered in our example. These are shown in Tables 2.4.1 and 2.4.2. The flight schedules are identical except for Flight 2, which has an earlier departure time, but the same arrival time, in Table 2.4.2. The consequences of this difference will be discussed below. Comparing either schedule with Figure 2.4.3 we see that under the most favorable scenario, ξ_1, scheduled arrival demand never exceeds arrival capacity, while under ξ_2, ξ_3, and ξ_4 there are capacity shortfalls that become successively more severe. Further, by comparing the flight arrival schedules with the scenario tree, we are able to define decision stage of each flight, as required for the Richetta-Odoni model. The decision stage specifies the time period when the planned arrival and departure times for a flight are frozen, and is based upon the branching times of the scenario tree.

To complete our model inputs we must specify scenario probabilities and the delay cost ratio. For the former we set $P\{\xi_1\} = 0.5; P\{\xi_2\} = 0.3; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1$. This implies there is a high chance that fair operating conditions will prevail. The cost ratio parameter λ is set to 5, which means a unit of airborne delay is 5 times more expensive than a unit of ground delay.

We first compare results from the Richetta-Odoni and the DRGH models assuming the flight schedule in Table 2.4.1. Expected delay costs are shown in Figure 2.4.5, while Table 2.4.1 presents the scenario-specific ground delays for each flight obtained from the two models. Both models use updated information in assigning ground delays. In the Richetta-Odoni model, however, ground delays, once assigned, are not revised. In contrast, our model has the ability to revise ground delays for flights that have not yet departed. This allows a significant reduction in expected ground delay cost, as can be seen from Figure 2.4.5.

Consider Flight 2 which is scheduled to depart during time period 6, which belongs to decision stage 1 in the Richetta-Odoni model, since time period 6 is precedes any of the branching nodes in the decision tree. That model assigns Flight 2 a ground delay of 3 time periods, which may not be revised (see Table 2.4.1). Solution 1 of our model assigns 1 time period of delay to this flight and thus lets it depart at time 8 if scenario ξ_1 is realized. Otherwise if scenario ξ_2 occurs, the flight departs during time period 9 with a delay of 2 periods. If ξ_2 is not realized, the ground delay of Flight 2 is increased to 4 periods. Next, consider Flight 3 which is scheduled to depart at time period 2. Either solution of our model assigns it a ground delay of 1 period and therefore the flight departs at time period 3, before any branching nodes. Therefore its ground delay cannot be subsequently revised. Flight 8, which is scheduled to depart during time period 7 (2nd decision stage in Richetta-Odoni Model) is released at its scheduled time by either model if ξ_1 is realized. If not, the Richetta-Odoni model assigns 3 time periods of delay to this flight, and cannot revise this later if ξ_2 occurs. Our model, in contrast, assigns Flight 8 just 1 time period of delay under ξ_2, increasing it to 3 time periods only under ξ_3 and ξ_4.

Table 2.4.1 also shows that there our model has multiple solutions. For example, the planned arrival times for Flights 2 and 8 can be exchanged without affecting the value of the objective function. The non-uniqueness can be resolved by changing the objective function itself or by augmenting it in a multi-criteria optimization framework. This will be elaborated below in Section 6 of this chapter.

Finally, in Table 2.4.2, we compare results with the alternative departure time for Flight 2. The solution of the Richetta-Odoni model is unchanged. On the other hand, with the revised schedule there is less opportunity to revise the ground delay of Flight 2, because its departure time is much earlier than any of the branching node times. Thus our model assigns a ground delay of 4 time periods, irrespective of the scenario: in effect under this schedule Flight 2 has become an “exempt” long-haul flight. Furthermore, because the early release of Flight 2 foregoes the opportunity to revise its departure time later, the delay cost advantage of our solution over that of Richetta-Odoni is correspondingly less (as evident from Figure 2.4.5).

Table 2.4.1 Ground Delays for Flights, Original Flight Schedule, by Model and Scenario

Flt No.	Sched. Dep. Time Period	Sched. Arr. Time Period	Decision Stage	**Time Periods of Assigned Ground Delay, Richetta-Odoni Model**				**Time Periods of Assigned Ground Delay, DRGH Model (Optimal Solution 1)**				**Time Periods of Assigned Ground Delay, DRGH Model (Optimal Solution 2)**			
				ξ_1	ξ_2	ξ_3	ξ_4	ξ_1	ξ_2	ξ_3	ξ_4	ξ_1	ξ_2	ξ_3	ξ_4
1	1	7	1	0	0	0	0	0	0	0	0	*0*	*0*	*0*	*0*
2	***6***	***7***	***1***	***3***	***3***	***3***	***3***	***1***	***2***	***5***	***5***	***1***	***2***	***4***	***4***
3	***2***	***8***	***1***	***0***	***0***	***0***	***0***	***1***	***1***	***1***	***1***	***1***	***1***	***1***	***1***
4	5	8	1	0	0	0	0	0	0	0	0	0	0	0	0
5	4	8	1	2	2	2	2	0	0	0	0	0	0	0	0
6	3	9	1	0	0	0	0	1	1	1	1	1	1	1	1
7	5	9	1	0	0	0	0	0	0	0	0	0	0	0	0
8	***7***	***9***	***2***	***0***	***3***	***3***	***3***	***0***	***1***	***2***	***2***	***0***	***1***	***3***	***3***
9	7	10	2	1	1	1	1	0	1	2	2	0	1	2	2
10	8	10	3	0	0	2	2	0	0	1	1	0	0	1	1
11	7	11	2	0	0	0	0	0	0	0	0	0	0	0	0
12	9	11	3	0	0	1	1	0	0	1	1	0	0	1	1
13	10	12	4	0	0	1	1	0	0	1	1	0	0	1	1

Table 2.4.2 Ground Delay for Individual Flights, Alternative Flight Schedule, by Model and Scenario

Flight #	Dep.	Arr.	Decision Stage	DRGH Model			
				ξ_1	ξ_2	ξ_3	ξ_4
1	1	7	1	0	0	0	0
2	***4***	***7***	***1***	4	4	4	4
3	2	8	1	***1***	***1***	***1***	***1***
4	5	8	1	0	0	0	0
5	4	8	1	0	0	0	0
6	3	9	1	1	1	1	1
7	5	9	1	0	0	0	0
8	7	9	2	***0***	***1***	***3***	***3***
9	7	10	2	0	1	2	2
10	8	10	3	0	0	1	1
11	7	11	2	0	0	0	0
12	9	11	3	0	1	1	1
13	10	12	4	0	0	1	1

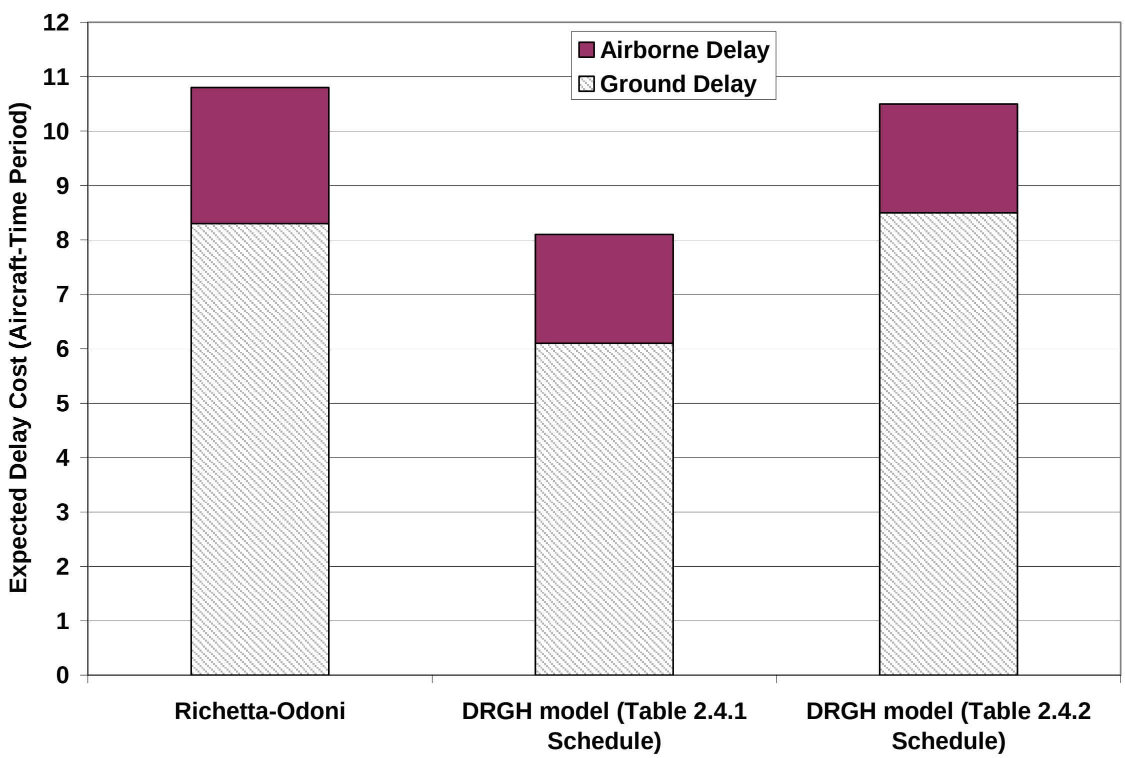

Figure 2.4.5 Expected Delay Costs

2.3. EXPERIMENTS WITH A LARGE SCALE PROBLEM

2.3.1. Experimental Setup

We now consider a large-scale problem with many more flights and decision periods. Our goal is to compare the performance of our model with that of existing models--Richetta-Odoni (1994) and Ball et al (2003)--for a realistic case. We also investigate the sensitivity of our results by comparing a baseline case with a set of alternative cases in which particular model inputs are varied.

Baseline Definition (Case 1)

We consider 351 flights scheduled to arrive before 12:15PM at Dallas Fort Worth Intl. Airport (DFW) on July 14, 2003. The cumulative scheduled arrival demand is shown in Figure 2.5.1. The FAA Aviation Systems Performance Metrics (ASPM) database is the source of data on scheduled departure and arrival times of individual flights.

Our analysis period consists of 50 quarter-hour (qhr) periods beginning with 12 midnight-12:15am and extending through 12:15pm-12:30pm. (The last of these is the *"T+1"* period by the end of which it is assumed all flights have landed.) Scheduled arrival demand during each time interval is derived from the flight schedules. We consider six capacity scenarios: $\Theta = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6\}$. In each of them, there are two levels of arrival capacity: low (15 arrivals/qhr) and high (35 arrivals/qhr). In all scenarios, capacity begins at the low value and at some point switches to the high one. When this jump occurs depends on the scenario. As shown in Figure 2.5.2, in the best case--ξ_1--the

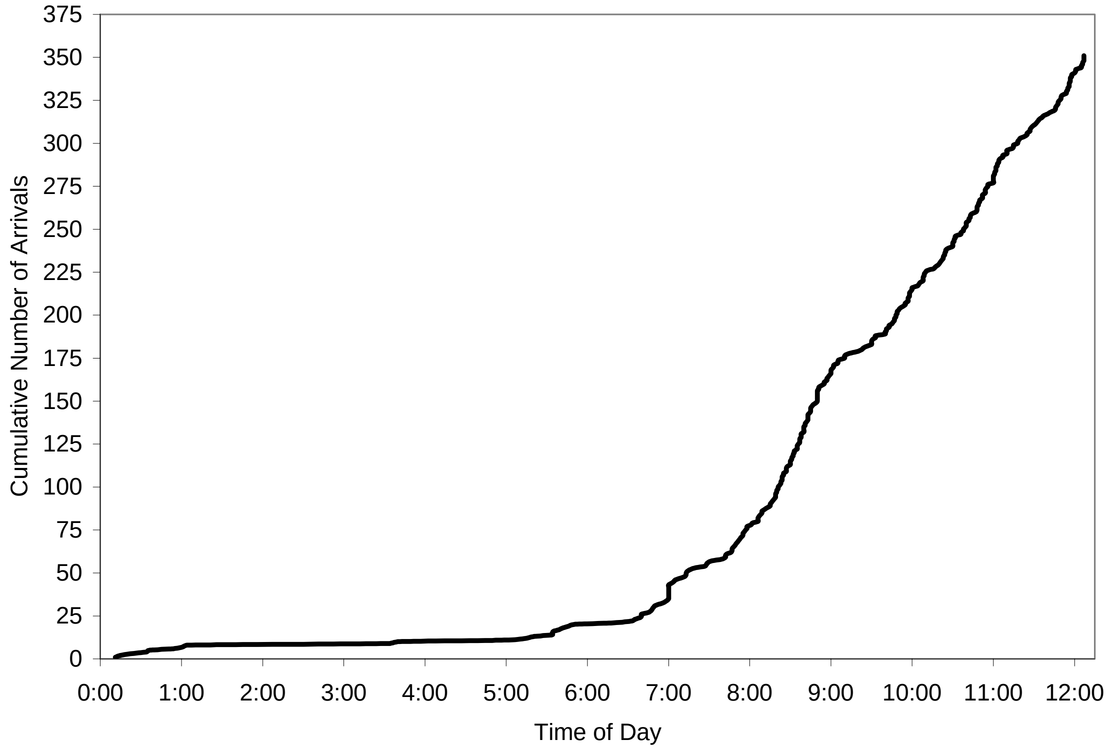

Figure 2.5.1 Arrival Demand at Until 12:15 PM at DFW on July 14, 2003

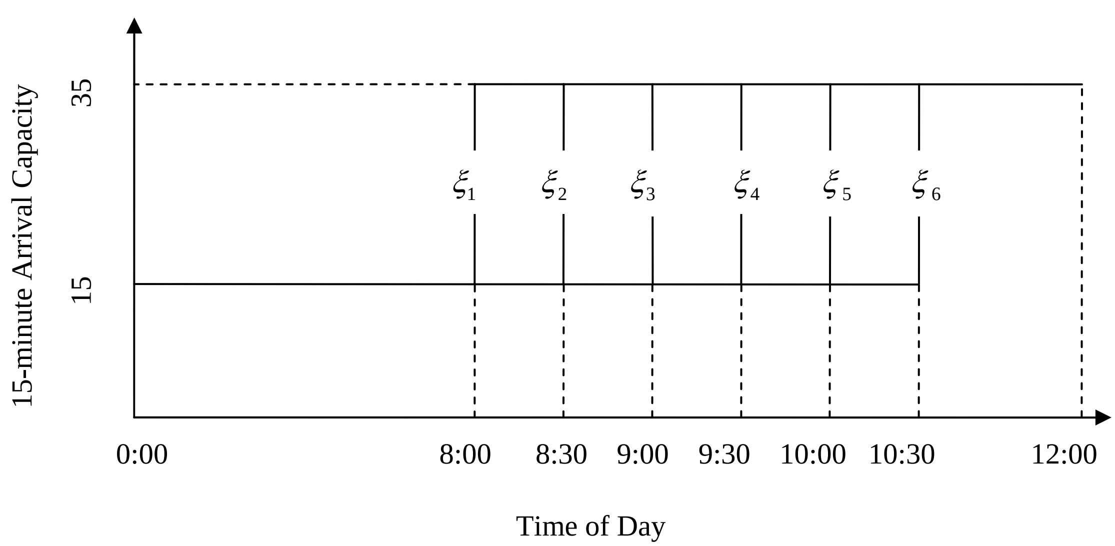

Figure 2.5.2 15-MinuteArrival Capacities of Different Scenarios

capacity goes up at 8am, while in the worst--ξ_6--this happens at 10:30am. The baseline scenario tree, shown in Figure 2.5.2a, assumes no advance information, so that branching nodes are concurrent with possible capacity changes.

The scenario probabilities assumed in the baseline make the favorable scenarios the most likely. Their probabilities are:

$P\{\xi_1\} = 0.4; P\{\xi_2\} = 0.2; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1; P\{\xi_5\} = 0.1; P\{\xi_6\} = 0.1$. The cost ratio between airborne and ground delays - λ - is set to 3.

In addition to this baseline case, we define 4 alternative cases. Each of these alternatives is different from the baseline in just one respect. Below we explain each alternative.

Case 2: Change in Delay Cost Ratio. In this case we vastly increase the ratio of airborne to ground delay cost, from $\lambda = 3$ to $\lambda = 25$.

Case 3: Change in Probability Mass Function of Scenarios. In this case we change the scenario probabilities so that the least favorable ones are the most likely. The probabilities are set to:

$P\{\xi_1\} = 0.1; P\{\xi_2\} = 0.1; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1; P\{\xi_5\} = 0.2; P\{\xi_6\} = 0.4$. The cost ratio is reset to that in baseline case, i.e. $\lambda = 3$.

Case 4: Early Branching. In this case we seek to evaluate the value of earlier information on how capacity will evolve. We change the scenario tree so that branch

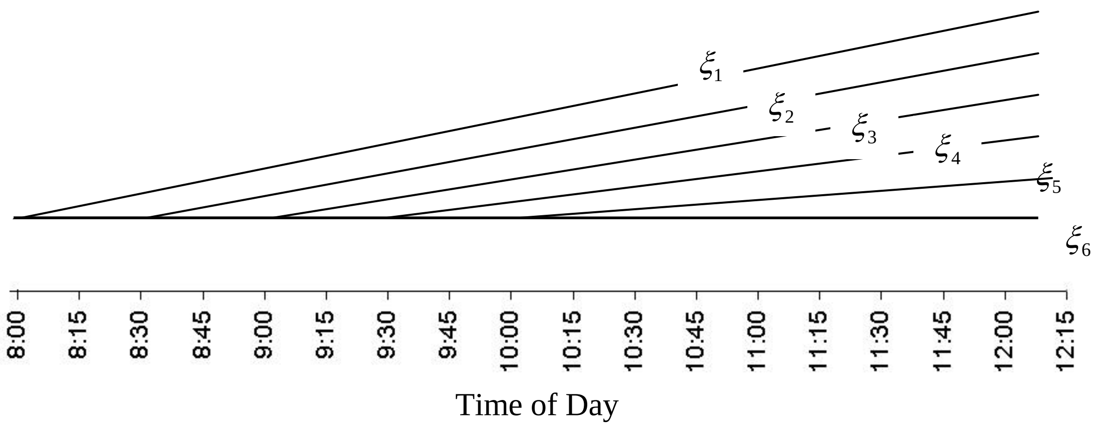

Figure 2.5.2a Scenario Tree for Baseline Case

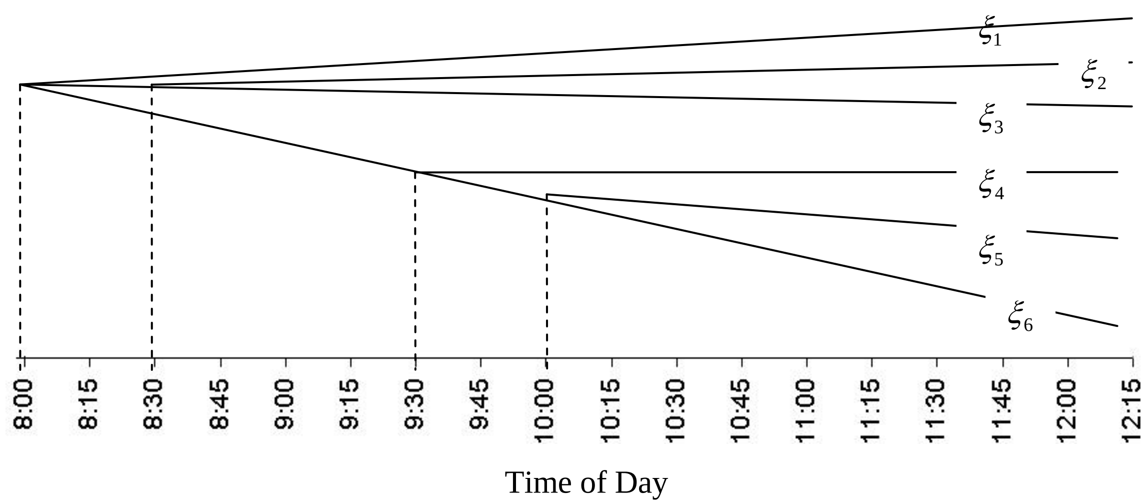

Figure 2.5.2b Different Scenario Tree

nodes occur 30 minutes prior to associate capacity changes. Therefore the scenario tree looks similar to the one in Figure 2.5.2a, but ξ_1 is realized at 7:30AM, ξ_2 at 8:00 AM, and so on. Note that the scenario-specific capacity profiles are identical to the baseline; only the information about the change becomes available earlier.

Case 5: Different Scenario Tree. As mentioned earlier, Richetta-Odoni model can only handle specific type of scenario tree in which scenarios unveil one by one. To demonstrate the ability of our model to capture any generalized branching process, we consider the scenario tree shown in Figure 2.5.2b. In contrast to the baseline tree, this one has three branches extending from the 8am branching node, with additional sub-branching for scenarios 2 and 3 at 8:30

2.3.2. Results

We applied three models—the Ball et al. static stochastic model (Ball et al. 2003), the Richetatta-Odoni dynamic stochastic model and the DRGH model that we presented in Section 2.4 – to the five problem cases described above. All models in all cases yielded integer optimal solutions from their LP relaxations. Computational times for our model, while slightly longer than the others because of the number of variables involved, were on the order of 2-5 seconds on a 2.2 GHz personal computer.

Expected delay costs from the three models are summarized in Table 2.5.1. Figure 2.5.3 shows the total expected costs as a percentage of expected delay cost under perfect

Table 2.5.1 Expected Delay Costs* (in Aircraft-Hours) from Three Models

	Static Stochastic Model (Ball et al. 2003)			Dynamic Stochastic Model (Richetta and Odoni 1994)			Dynamic Stochastic Model (DRGH model)		
	Ground Delay	Airborne Delay	**Total**	Ground Delay	Airborne Delay	**Total**	Ground Delay	Airborne Delay	**Total**
Case 1: Baseline	*19.5*	*16.95*	***36.45***	*20.8*	*15.08*	***35.88***	*27.75*	*5.25*	***33***
Case 2	47.25	0.00	**47.25**	39.33	0.00	**39.33**	33.38	0.00	**33.38**
Case 3	47.25	0.00	**47.25**	44.75	0.00	**44.75**	42.40	0.00	**42.40**
Case 4	19.50	16.95	**36.45**	22.93	8.85	**31.78**	24.98	0.15	**25.13**
Case 5	19.5	16.95	**36.45**	**	**	******	24.85	4.8	**29.65**

*Costs measured is units of ground delay, with airborne delay and extra flight time multiplied by the delay cost ratio λ.

** Richetta-Odoni model not applicable to scenario tree assumed in Case 5.

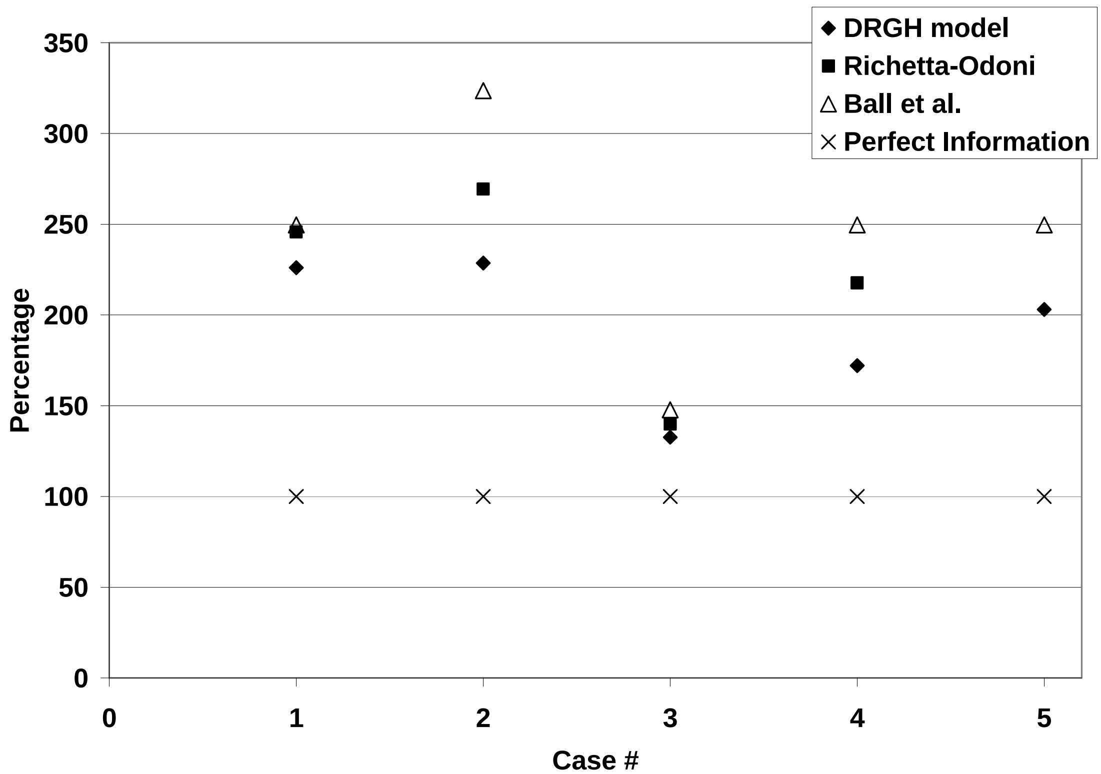

Figure 2.5.3 Expected Delay Cost from Three Models vs. Expected Cost under Perfect Information

information. The latter metric is calculated assuming that with perfect information all delays are taken on the ground and that delay is minimized subject to the scenario-specific capacity constraint. Delay with perfect information is thus the deterministic delay calculated from the arrival schedule and the scenario-specific capacity profile, taking into account the associated scenario probabilities. Figure 2.5.3 therefore exhibits the performance loss due to imperfect information.

In Case 1, the baseline, our model – the DRGH model – produces 10% lower total expected delay cost than the Ball et al. static stochastic model, whereas the Richetta – Odoni model saves only 2%. While all three models yield expected delay costs over twice that for the perfect information case, our model reduces this gap by about one-sixth. With a high chance that good operating conditions will prevail, and a moderate cost ratio (λ=3), the Ball model assigns less ground delay to flights, facing a possibility of high airborne delays if the lower capacity scenarios prevail. This is an optimal strategy because in the static model ground delays, once imposed, cannot be revised, should favorable conditions make the ground delays unnecessary.

Although the Richetta-Odoni dynamic model utilizes updated information on evolving conditions, cannot subsequently revise them. In the baseline case, 219 out of 351 flights—62% --are scheduled to depart before the first branch in the scenario tree. All these flights belong to the first decision stage in the Richetta-Odoni model, and are thus assigned ground delays without any updated capacity information. Thus, as in the Ball et al. model, many flights are assigned low ground delays, resulting in higher expected airborne delays. Our model, in contrast, can impose higher ground delays initially with recourse to reduce them if capacity permits. In the baseline case, many flights that are scheduled to depart after 7:15AM are delayed until 8:00AM and assigned revised departure times based on updated information about the occurrence of ξ_1. Therefore we see higher expected ground delay from solutions of our model, but lower expected airborne delay. Expected delay cost is lower, but only moderately because of the low λ value.

This interpretation is confirmed from "planned" arrival demand during different time intervals, given in Table 2.5.2. The numbers reflect arrival demand during different 15-minute time periods for each capacity scenario, based on scheduled arrival time and assigned ground delays. For example if ξ_1 occurs, 33 flights arrive between 9:00-9:15AM in our model, whereas 25 flights arrive if scenarios ξ_2-ξ_6 occur. The difference is achieved by assigning higher ground delays to some flights if ξ_1 is not realized at 8:00AM. Similarly if ξ_2 is realized at 8:30AM some flights are allowed to depart with reduced ground delays. Otherwise the ground delays of these flights are either unchanged or increased. Thus we see 26 planned arrivals during 9:30-9:45AM if ξ_2 is realized, while this number is 18 if ξ_3-ξ_6 prevail. Planned arrival numbers are more aggressive (due to lower ground delays) in the Richetta-Odoni model during 9:00-9:30AM. 35 flights are planned to arrive during 9:00-9:15AM, whereas only one of these flights is delayed in response to the realization at 8:00AM that ξ_1 will not occur. That one flight has a scheduled departure time after 8:00AM and hence it is assigned delay

Table 2.5.2 Planned Arrival Rates

Time Period	DRGH Model						Richetta-Odoni Model					
	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
9:00AM-9:15AM	33	25	25	25	25	25	35	34	34	34	34	34
9:15 AM-9:30 AM	16	5	5	5	5	5	14	14	14	14	14	14
9:30AM-9:45AM	12	26	18	18	18	18	12	12	12	12	12	12
9:45AM-10:00AM	20	25	12	12	12	12	20	21	13	13	13	13
10:00AM-10:15AM	12	12	33	33	33	33	12	12	20	20	20	20

Table 2.5.3 Planned Arrival Rates in Case 2 (λ =25)

Time Period	DRGH Model						Richetta-Odoni Model					
	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
9:00AM-9:15AM	23	15	15	15	15	15	16	15	15	15	15	15
9:15 AM-9:30 AM	26	15	15	15	15	15	15	15	15	15	15	15
9:30AM-9:45AM	12	23	15	15	15	15	19	15	15	15	15	15
9:45AM-10:00AM	20	28	15	15	15	15	29	23	15	15	15	15
10:00AM-10:15AM	12	12	29	15	15	15	14	17	16	15	15	15

during the 2nd decision stage. The Richetta-Odoni model must assign ground delays to the other 34 flights during stage 1, and these cannot be revised later. If less favorable capacity scenarios occur, our model plans for more arrivals during later time periods of the day when high capacity is almost certain.

Under Case 2, in which airborne delay unit cost is much higher than that of ground delay (λ =25), we can expect aggressive ground delays by all three models in order to keep expected airborne delay as low as possible. This is confirmed from expected delay values for Case 2 given in Table 2.5.1. No airborne delay is produced by any of the models. The Ball et al. static model assigns ground delays based on worst capacity scenario ξ_6 . At any time, the dynamic models assigns ground delays based on ξ_6 , unless some other scenario has been realized.

Table 2.5.3 presents the scenario-specific planned arrival demand for Case 2, from the dynamic models. Comparing the number of planned arrivals during 9:00AM-9:15AM in Richetta-Odoni model with the corresponding numbers in Table 2.5.2, we see that ground delays are more severe, and the planned arrivals are based on the low capacity profile (15 arrivals/qhr). If ξ_1 is realized at 8:00 AM, one extra flight is allowed to arrive during this time interval, and higher numbers during later time periods. Our model efficiently utilizes the realization of ξ_1 by revising ground delay levels to lower amounts, and allows more aircraft to arrive during earlier time periods. These revisions enable us to reduce delay cost by 15% compared to Richetta-Odoni, and 30% compared to Ball et. al

(Figure 2.5.3). Our model yields greater savings in Case 2 than in Case 1 because the high λ value forces plans to be based on the worst case capacity scenario.

In Case 3, which features higher probabilities of low-capacity scenarios, the planned arrival numbers are exactly same as in Case 2 (Table 2.5.3), again ensuring no airborne delay. Under the dynamic models, as well as the perfect information hypothesis, Case 3 has higher expected delay costs than Case 2 because of the higher probabilities of low capacities. In contrast the Ball model yields the same expected cost as in Case 2, because it bases ground delays on the worst case. In sum, there is less opportunity in this case for the dynamic models to leverage updated information into reduced delays, resulting in little improvement over the static model. Thus we can see that the percentage loss from imperfect information is less (Figure 2.5.3).

Results from Case 4 reveal the value of earlier information on airport operating conditions. Expected delay cost of the static model (Ball et al.) solution remains the same as in Case 1 because there is no change in the scenario specific capacity profiles. The Richetta-Odoni and DRGH models exploit this advanced information by adapting ground delay decisions. The early information vastly decreases the number of flights in the first decision stage for the Richtetta-Odoni model. Our model exploits the information to virtually eliminate airborne delays while increasing ground delays only slightly compared with the baseline case. This greatly reduces the performance loss from imperfect information.

Finally, with the scenario tree in Figure 2.5.2b (Case 5), our model can exploit the availability of better information on evolving capacity conditions, whereas the Richetta-Odoni model cannot handle the sub-branching included in this case. If at 8:00AM scenario ξ_1 is not realized and the upper branch of scenario tree (scenarios ξ_2 *or* ξ_3) evolves, then it is known with certainty that after 9:00AM the capacity profile will be high (35 arrivals/qhr). Our model makes efficient use of this information by revising ground delays in order to utilize the airport capacities to the maximum possible extent. Table 2.5.4 contains the scenario based planned arrivals at different time intervals. If the upper branch of scenario tree is realized at 8:00AM, ground delays are less severe than if the lower branch is realized. Therefore we see higher planned arrivals during 9:00-9:30 if the upper branch evolves (ξ_2 *or* ξ_3 based planned arrivals). Comparing the scenario based planned arrivals to the corresponding numbers of the baseline case; we see a higher utilization of arrival capacities after 9:00AM. Thus our model results in a loss from imperfect information that is 1/3 less than that under the static model (see Figure 2.5.3).

Table 2.5.4 Planned Arrival Rates from the DRGH Model Applied to Case 5

Time Period	Scenario					
	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
9:00AM-9:15AM	33	33	33	25	25	25
9:15 AM-9:30 AM	16	16	16	5	5	5
9:30AM-9:45AM	12	12	12	15	15	15
9:45AM-10:00AM	20	20	20	15	15	15
10:00AM-10:15AM	12	12	12	33	33	33

2.2. ALTERNATIVE OBJECTIVE FUNCTIONS

The dynamic stochastic model – DRGH model – presented in Section 2.4 assigns ground delays to a subset of flights, such that the expected total cost of delay, assumed to be a linear function of expected airborne delay and ground delay, is minimized. In practice, it is doubtful whether delay cost is a simple linear function, and whether expected delay cost is an adequate optimization objective. In this section, we demonstrate the ability of our model to incorporate other objective functions and use that capability to perform trade-off analyses between different objectives.

We consider two alternative objective functions. The first replaces the ground delay component of the original objective function with the square of the ground delay. Thus the objective function becomes:

$$Min \sum_{q \in \Theta} P\{q\} \times \left\{ \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - ARR_f\right)^2 \times (X_{f,t}^q - X_{f,t-1}^q) \right] + \lambda \times \sum_{t=1}^{T} W_t^q \right\}$$

(2.6.1a)

Although quadratic in ground delay, the function remains linear in our decision variable and thus is easily incorporated in our model. Compared to the original objective function (2.4.1), objective function (2.6.1a) penalizes long ground delays more heavily. One consequence of this is that, all else equal, flights with earlier scheduled arrival times will be assigned earlier arrival slots. This is consistent with the "Ration-by-Schedule" (RBS) concept that is used in the present air traffic management system. While the first-scheduled first-served policy is often justified on equity grounds it has also been conjectured that delay costs are non-linear, in which (2.6.1a) would align with an efficiency objective (Vossen 2002).

The second alternative objective function considers the expected squared deviation of the assigned ground delay from the ground delay that would be assigned using the present RBS algorithm and assuming perfect information about the capacity scenario. This objective function has the form:

$$Min \sum_{q \in \Theta} P\{q\} \times \left\{ \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - RBS_f^q \right)^2 \times (X_{f,t}^q - X_{f,t-1}^q) \right] + \lambda \times \sum_{t=1}^{T} W_t^q \right\}$$

(2.6.1b)

where RBS_f^q is the time period during which flight f can arrive based on RBS allocation if scenario $q \in \{1..Q\}$ occurs in reality. This objective function embraces RBS slot allocation as the "gold standard" and seeks to get as close to that slot allocation as possible in circumstances where future capacity is uncertain.

We incorporate these alternative objective functions into our model in three different ways. First, use them as tie-breakers to choose among non-unique optimal solutions in the original model. This involves introducing the ground delay components of (2.6.1a) and (2.6.1b) into the original objective function, but with very small weights relative to the linear expected delay terms. Expression (2.6.1c) forms the objective function with multiple measures of ground delay, each associated with some weight (δ_1 and δ_2).

Second, we replace the original objective function with these alternatives. Finally, we explore trades between the original objective function and these alternatives by solving the model with a range of relative weightings.

$$Min \sum_{q \in \Theta} P\{q\} \times \left\{ \begin{array}{l} \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - Arr_f\right) \times (X_{f,t}^{q} - X_{f,t-1}^{q}) \right] \\ + \delta_1 \times \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - RBS_f^{q}\right)^2 \times (X_{f,t}^{q} - X_{f,t-1}^{q}) \right] \\ + \delta_2 \times \left[\sum_{f \in \Phi} \sum_{t = Arr_f}^{T+1} \left(t - ARR_f\right)^2 \times (X_{f,t}^{q} - X_{f,t-1}^{q}) \right] \\ + \lambda \times \sum_{t=1}^{T} W_t^{q} \end{array} \right\} \qquad (2.6.1c)$$

Table 2.6.1 summarizes our results when objectives (2.6.1a) and (2.6.1b) are introduced as tiebreakers into three variants of the large-scale problem discussed in Section 2.5. For each case, we compare the results when these additional objectives are ignored - i.e. when expression (2.4.1) is used in the objective function, when only (2.6.1a) introduced (i.e. $\delta_1 = 0;\ 0 < \delta_2 << 1$ in 2.6.1c), when only (2.6.1b) introduced ($0 < \delta_1 << 1;\ \delta_2 = 0$), and when both are introduced ($0 < \delta_1 << 1;\ 0 < \delta_2 << 1$). The optimum value of the original objective function is not changed because of the low weights given to the additional terms, but, as expected, there is a substantial reduction in objectives (2.6.1a) and (2.6.1b) when they are introduced. The expected squared ground delay is reduced 15-20% and the expected squared deviation from the RBS allocation 25-30%, simply by taking these objectives into account in choosing among the non-unique optimal solutions to the original problem. These reductions are slightly higher in Cases 2 and 3, which respectively feature higher airborne delay costs and higher probabilities of low capacity scenarios than Case 1. Virtually all of this gain can be realized by only adding objective (2.6.1b), whereas the results from including (2.6.1a) only are slightly less favorable. In either case, by introducing the additional terms in the objective function, we arrive at a solution that yields the same value of total expected delay cost, but a different distribution of delay among flights that causes reduction in the quadratic delay measures.

Figures 2.6.1 and 2.6.2 depict the trade-offs between the original objective function (2.4.1) and the alternatives (2.6.1a) and (2.6.1b). The figures were constructed by assigning varying weights to pairs of objectives consisting of the original one and either (2.6.1a) or (2.6.1b). When (2.6.1a) is paired with the original objective (Figure 2.6.1), the nature of the trade-off depends on the case. In the baseline, Case 1, the trade-off is fairly linear, implying that the optimum obtained is highly sensitive to the relative importance of linear ground delay and squared ground delay. Cases 2 and 3 display more pronounced ”knees”; in these cases similar solutions will be obtained over a wide range of relative weightings. When objective function (2.6.1b) is traded against the linear objective function (Figure 2.6.2), the trade-off curves exhibit fairly uniform levels of curvature for all three cases, although the Case 3 (high probability of low capacity) results reveal a steeper slope, implying that delay reduction carries a stronger penalty in terms of increased deviation from the RBS allocation.

Finally Table 2.6.2 and Figures 2.6.3-2.6.5 compare results from optimizing the original objective function (2.4.1) or objective function (2.6.1b) for Cases 2 and 3. These cases are particularly comparable because, for either objective function, in both cases the optimal solution features no expected airborne delay, but for different reasons. In Case 2 the cost of airborne delay is too large, while in Case 3 the probability of a low-capacity scenario is too great. Table 2.6.2 summarizes results in terms of the expected and worst-case ground delays, averaged over all flights and flights categorized by length-of-haul. The length-of-haul is categorized, based on scheduled flight time, as short (less than 1 hour), medium (between 1 and 2 hours), or long (more than 2 hours). In each case, the choice of objective function strongly affects the allocation of ground delay. With the original objective function, short-haul flights incur much higher expected and worst-case ground delays than the long-haul flights. In this case, the optimal solution involves releasing long-haul flights at or near their scheduled departure times and using the short-haul flights to absorb delays if low-capacity scenarios eventuate. When the objective function 2.6.1b is optimized, overall delays increase slightly, but the allocation of delays shifts markedly, with long-haul flights incurring the greatest expected ground delays, and worst-case delays only slightly less than the shorter flights. By attaching a high penalty to large deviations between the actual allocation and the RBS allocation, we obtain a solution that is much more similar to the first-scheduled/first-served philosophy that RBS incorporates.

Figures 2.6.3-2.6.5 provide additional interpretation of selected optimization results by showing the cumulative curves for scheduled arrivals along with assigned arrival delays for a given objective function and case. Each data point corresponds to a flight. The abscissa corresponds to the assigned arrival time, while the ordinate gives the position of the flight in the original flight sequence. Points that fall on the schedule curve have no assigned ground delay. The horizontal displacement between the scheduled arrival curve and a point corresponds to the amount of ground delay assigned to the flight the point represents. The figures focus on flights that are assigned ground delays--those scheduled to arrive between 8 am and 11 am.

Comparing Figures 2.6.3 and 2.6.4 reveals the influence of the objective function on the delay allocation under Case 2. As shown in Figure 2.6.3, under the linear delay objective function, the bulk of the delays are absorbed by a handful of short- and medium-haul flights whose delayed arrival times falls between 9:30 and 10 am. These are flights that can be released after the branching points at 8:00 AM and 8:30 AM, in the event that the higher capacity branches are realized (see Figure 2.5.2a). When squared deviation from the RBS allocation is the objective, delays are more evenly distributed and hence assigned arrival times are more strongly correlated with scheduled arrival times. Figure 2.6.5 shows the results for the squared deviation from RBS allocation objective under Case 3. The solution is quite similar to Figure 2.6.4 except that the overall delay is slightly higher due to the higher probability of low capacity scenarios in this case. This difference is particularly notable for flights scheduled to arrive between 9:45 and 10:30. Several long-haul flights in this period are released on schedule in Case 2, on the bet that high capacity scenarios will be realized. This solution resembles the policy exempting long-haul flights from a GDP. It is not optimal to take this bet in Case 3, because the odds of the high capacity scenarios are considerably less. Thus, releasing long haul flights on-time may lead to severe ground holdings for short haul flights, causing high expected deviation from RBS allocation.

Table 2.6.1 Expected Values of Different Delay Measures* when Alternative Objective Functions are Used

	Objective Function (2.4.1)			Ground Delay Measure of (2.6.1b) Introduced, i.e. Objective Function (1c) with $0 < \delta_1 << 1;\ \delta_2 = 0$			Ground Delay Measure of (2.6.1a) Introduced, i.e. Objective Function (1c) with $\delta_1 = 0;\ 0 < \delta_2 << 1$			Both (2.6.1a) and (2.6.1b) Introduced, i.e. $\delta_1 << 1;\ \delta_2 << 1$		
Case	Total Delay	Squared Deviation from RBS	Squared Ground Delay	Total Delay	Squared Deviation from RBS	Squared Ground Delay	Total Delay	Squared Deviation from RBS	Squared Ground Delay	Total Delay	Squared Deviation from RBS	Squared Ground Delay
1	132	405	406	132	309	354	132	335	347	132	310	347
2	133	511	545	133	371	441	133	396	441	133	371	441
3	169	771	840	169	551	680	169	587	680	169	551	680

* Delays are measured 15-minitute units; airborne delays are multiplied by the cost ratio λ.

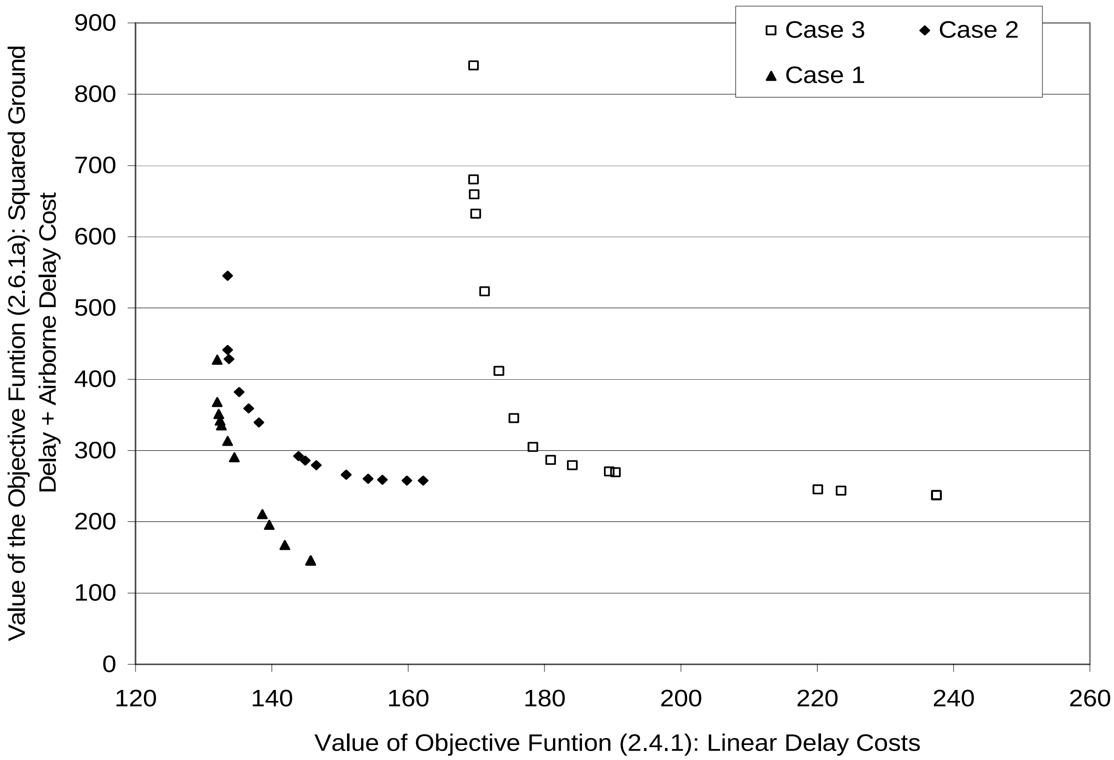

Figure 2.6.1 Tradeoff Curves between Objective Functions (2.4.1) and (2.6.1a)

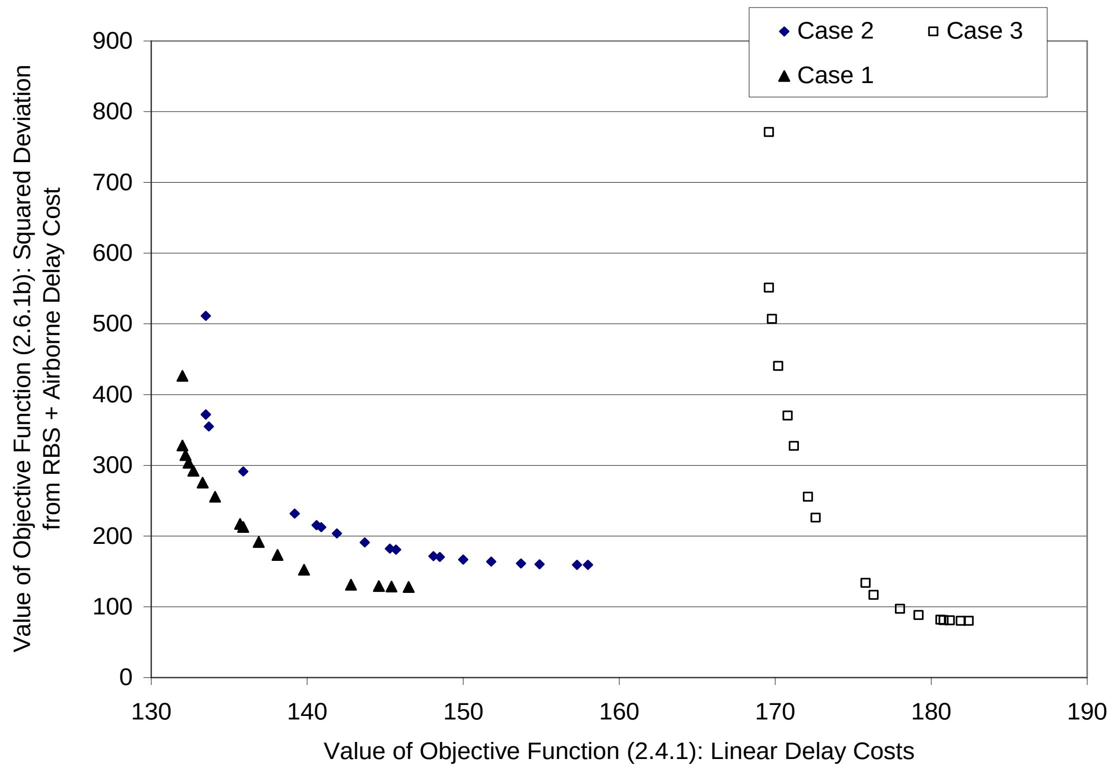

Figure 2.6.2 Tradeoff Curves between Objective Functions (2.4.1) and (2.6.1b)

Table 2.6.2 Expected and Worst Scenario Delays across Flights

Case	Objective Function	Average Expected Delays Across Flights (minutes)				Average Worst Scenario Delays Across Flights (minutes)			
		Overall	Short Haul	Medium Haul	Long Haul	Overall	Short Haul	Medium Haul	Long Haul
2	(2.4.1)	5.7	15.3	6.9	2.2	8.1	28.3	8.8	2.2
	(2.6.1b)	6.8	3.9	7.8	6.7	8.1	8.0	9.6	6.8
3	(2.4.1)	7.3	23.5	8.5	1.9	8.1	28.3	9.2	1.9
	(2.6.1b)	7.8	5.4	8.6	7.7	8.1	6.6	8.9	7.7

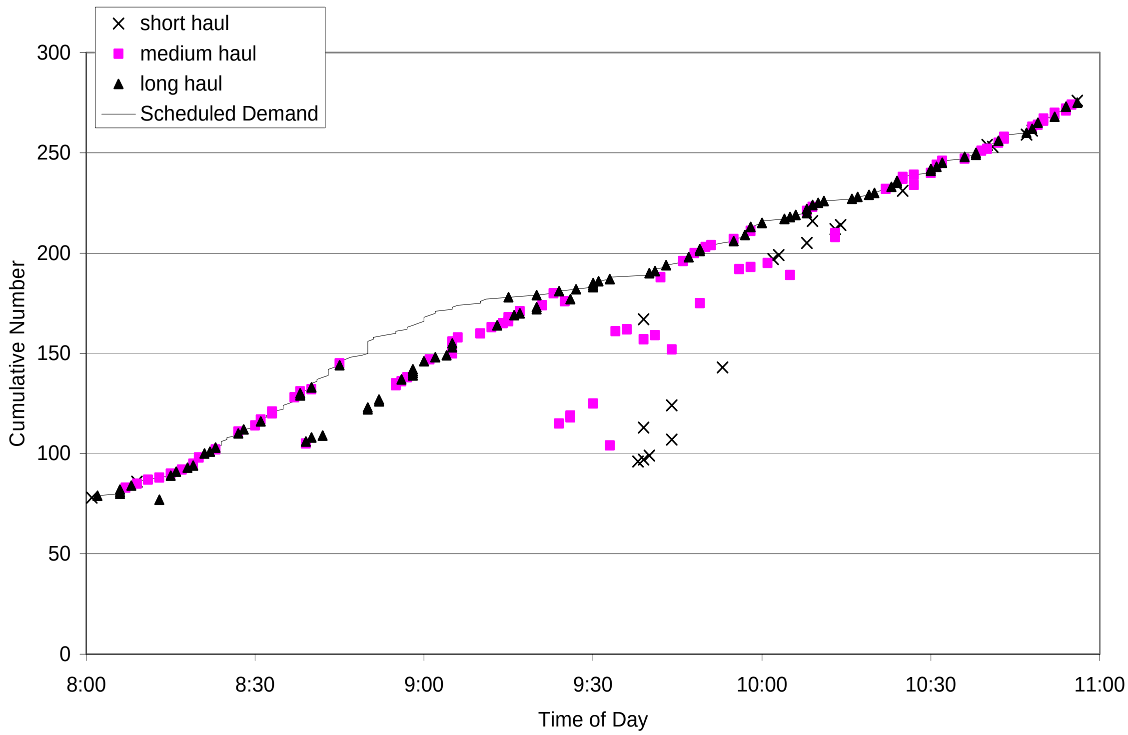

Figure 2.6.3 Expected Delays of Flights in Case 2, with Objective Function (2.4.1)

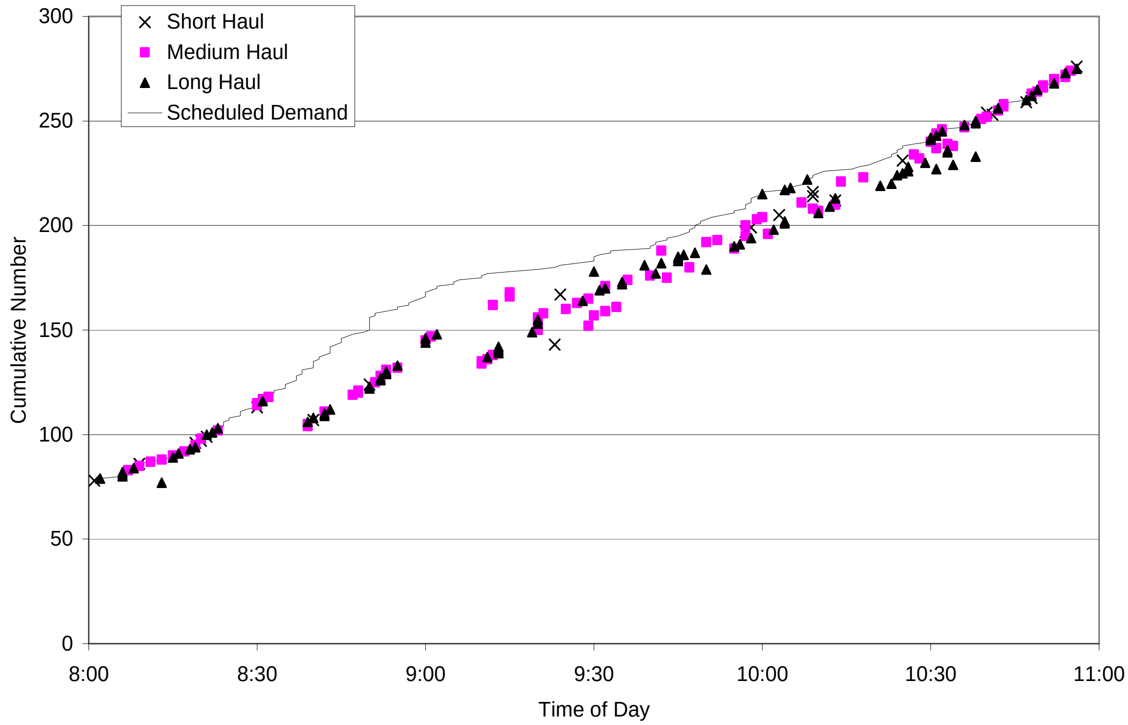

Figure 2.6.4 Expected Delays of Flights in Case 2, with Objective Function (2.6.1b)

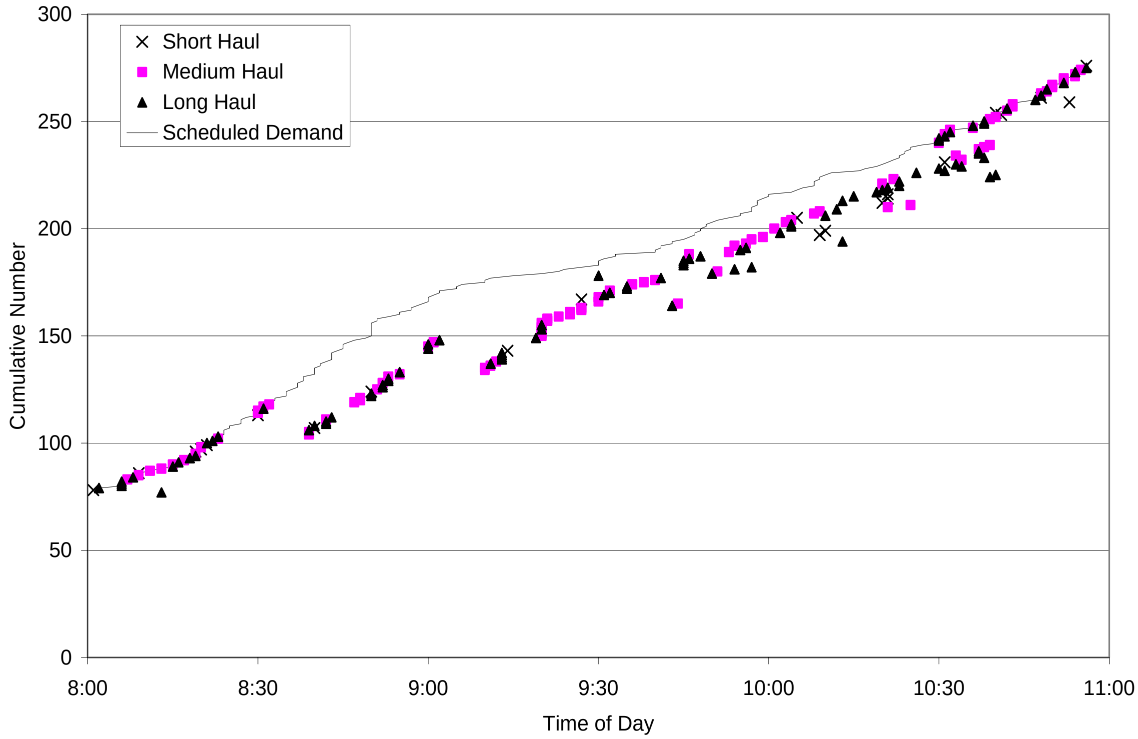

Figure 2.6.5 Expected Delays of Flights in Case 3, with Objective Function (2.6.1b)

2.2. DISCUSSIONS

In this chapter, we have presented a dynamic stochastic IP model for the single airport ground holding problem (SAGHP). Unlike previous models, ours allows assigned ground delays to be revised based on updated information on airport operating conditions. The model overcomes some of the major limitations of existing dynamic models for SAGHP (Richetta-Odoni, 1994), by allowing revisions to previously assigned ground delays for unreleased flights, handling a larger class of capacity scenario trees, and permitting non-linear ground delay measures in the objective function. Uncertainty in airport operating conditions is addressed by considering a finite set of possible scenarios of how the airport arrival capacity may evolve. The marginal probabilities of scenarios change over time, based on updated information or forecasts. In all instances, our model produces integer solutions directly from the LP relaxation, and hence the computational times were in the order of few seconds even for realistic-scale problems.

The ability to revise ground delays of flights that haven't departed, based on improved information on airport capacities, makes our model more adaptive to changing conditions. The model assigns ground delays less aggressively if conditions are predicted to improve, while retaining the option of imposing additional delays if in fact they don't. This results in higher utilization of airport capacities. Repeated application of a static stochastic model (such as Ball et al. 2003) can also utilize updated information on airport operating conditions, but at any stage of application, the model cannot account for the possibility of revising decisions later. Therefore such heuristics cannot “plan for re-plan”, as our model does by virtue of its dynamic formulation.

Our model produces lower expected total delay costs than the Richetta-Odoni (dynamic) model, and the Ball et al (static) model. In some cases, the difference is substantial. Experiments in Section 2.5 suggest that the expected gain from using our model is positively related to the unit-cost ratio of airborne delay to ground delay, and how soon after the beginning of the planning period new information becomes available.

The decision variables in our model allow us to consider alternative objective functions that are non-linear in ground delay, while retaining a linear formulation. The alternative objectives can act as tie-breakers for choosing among multiple solutions minimizing the original linear objective function, or replace the linear objective altogether. When the alternative objective functions are optimized, the delays are distributed fairly evenly across flights, in contrast to the original solution in which a small number of short haul flights are assigned high delays, while long-haul flights are for the most part released on time.

Our model thus incorporates the efficiency objectives that motivate the FAA to exempt long-haul flights from ground delay programs, but can also mitigate the inequities that result from this strategy. Rather than dealing with distance explicitly, however, our model focuses on the availability of information at the time flights are released. Depending on the capacity scenario probabilities, as well as the objective function, a given solution may “favor” long-haul flights, short-haul flights, or neither. Moreover, in our model any such favoritism is *de facto* instead of *de jure*. This may be more acceptable to airlines and other stakeholders than policies that explicitly discriminate on the basis of flight distance.

In the dynamic stochastic SAGHP model presented in this chapter, the information on the state of the system, which is considered while deciding on individual flight release times, includes only the weather conditions that influence the capacities of various resources and not the release times of other flights. The coupling constraints and the capacity constraints together capture the effect of the individual flight release decisions on system-wide delays. By virtue of reduced state-space in representing the scenarios, the solutions of the stochastic dynamic SAGHP can be easily applied in collaborative decision making. While re-scheduling the release time individual flights, airlines need to consider only the evolving weather states, and not the release times of other flights in the system. This is particularly advantageous because while making intra-airline substitutions, an airline does not have the information on what other airline’s actions are going to be. The applicability of the solutions of DRGH model in decentralized decision making by individual airlines is discussed in next chapter.

A possible approach to solve the dynamic SAGHP is through time non-homogeneous Markov decision process. In such model, the time-varying state space must include the individual flights release times along with the weather information. This may cause dimensionality problem as commonly faced in solving large-scale MDPs. Furthermore, it is difficult to represent weather state evolution, which is the main cause of capacity reduction in the NAS, through Markov chains because there is high degree of temporal and spatial correlation in the process. However, research on comparing computational complexity of formulating the ATFM optimization problem through MDP and weather-scenario based stochastic programming, will be valuable.

CHAPTER 3: COLLABORATIVE DECISION MAKING IN A GDP UNDER UNCERTAINTY IN AIRPORT ARRIVAL CAPACITIES

3.1. INTRODUCTION

As theoretical research on ground holding problems has progressed, air traffic flow management has evolved from a completely centralized system to one in which users (i.e. airlines) have more autonomy about how to adapt their schedules when adverse conditions reduce airport capacity. In Chapter 1 of this dissertation, we described how GDPs are implemented in practice. The ATSCSS allocates arrival slots to flights based on first-scheduled-first-served (or RBS) policy. In order to increase efficiency, airlines are then allowed to re-allocate their slots so as to better realize their own internal objectives—for example by giving priority to flights that are the most time-critical. A major limitation of the current practice in GDP planning is that the time-varying airport capacity is assumed to be deterministic. The ATCSCC handles uncertainty by geographical exemptions.

In Chapter 2, we presented a dynamic stochastic optimization model for a SAGHP – the DRGH model – that addresses the issue of uncertainty in planning for a GDP. Uncertainty in airport arrival capacity is addressed by considering a set of scenarios, each representing a time-varying capacity profile. The evolution of capacity scenarios is represented through a scenario tree. Each branching point of the scenario tree reveals new information on evolving capacity as time of day progresses.

The model assigns scenario-specific ground delays (and hence arrival times) to individual flights. Instead of exempting flights solely based on their expected duration, the model generates *de facto* exemptions depending upon the probabilistic information on airport capacity. Depending on the capacity scenario probabilities, as well as the objective function, a given solution may “favor” long-haul flights, short-haul flights, or neither. The scenario-specific release time of each flight depends on its scheduled departure, and a set of coupling constraints in the model that account for incomplete information about airport arrival capacities during future time periods.

The solution to the DRGH model, although assigning slots to flights, may in practice be used as a means of allocating slots to airlines. The arrival slots are however, conditional upon which scenario (or a bundle of scenarios) is realized at the time a flight is released. In this chapter, we first present a methodology for allowing airlines to perform flight substitutions and cancellations when slot allocations are scenario-specific. We then present an optimization model that functions analogously to the CDM Compression algorithm, which can be applied after the round of airline specific substitutions and cancellations. The objective of this model is to efficiently utilize the vacant slots that become available after cancellations. Throughout this chapter we use the same notation for the DRGH model as that in Chapter 2.

3.2. SCENARIO-CONTINGENT SUBSTITUTIONS AND CANCELLATIONS

The decision variables in the DRGH model are related to the scenario-specific arrival time periods of flights. The planned arrival time of a flight $f \in \Phi$ under scenario ξ is given by the expression $\sum_{t=arr_f}^{T+1} t \times (X_{f,t}^{\xi} - X_{f,t-1}^{\xi})$. We denote this value by θ_f^{ξ} . Therefore for each flight, the model solves for an optimal portfolio of scenario contingent arrival times. The objective function in this assignment can be to minimize any of the expressions (2.4.1), (2.6.1a), (2.6.1b), and (2.6.1c), proposed in Chapter 2. However, individual airlines are likely to attach different priorities to different flights. In this section we provide a methodology to allow each airline to reassign the scenario contingent slots in light of these priorities. At first we consider the problem of re-assigning slots without the option of canceling flights. We then consider the implications of flight cancellations.

3.2.1. Substitution between Two Flights

For any given flight and a portfolio of scenario contingent arrival time periods (or slots) it is possible to perform a feasibility check of assigning the portfolio of slots to the flight. Feasibility depends on the fight's scheduled departure time and the coupling constraints given by expression (2.4.6) in Chapter 2. If a flight f is assigned a set of scenario specific arrival time periods given by $\tau^{\xi}, \xi \in \Theta$, then the decision variables $X_{f,t}^{\xi}$ attain the value 0 until $t < \tau^{\xi}$ and 1 for $t \geq \tau^{\xi}$. The auxiliary decision variables $Y_{f,t}^{\xi}$ in the DRGH model represent the scenario-specific flight departure times.

Definition 3.21. The assignment of flight f to the portfolio of scenario-contingent arrival *time periods* $\tau^{\xi}, \xi \in \Theta$ *is feasible, iff* $\tau^{\xi} \geq Arr_f, \forall \xi \in \Theta$ *, and the variables* $Y^{\xi}_{f,t}, t \geq Dep_f$ *satisfy the coupling constraints (2.4.6) in the DRGH model.*

Any substitution involving two flights requires swapping the scenario specific arrival slots between them. Therefore, for the substitution to occur, each flight must be assigned a new portfolio of slots that was owned by the other flight.

Definition 3.2.2. A substitution of scenario contingent slots between any two flights f_1 and f_2 *is allowed if* θ^{ξ}_{f1} *is a feasible assignment for* f_2*, and* θ^{ξ}_{f2} *is feasible for* f_1.

Scenario based substitution interchanges arrival time periods between two flights under different scenarios, and therefore doesn't affect the arrivals (and hence ground delays) of other flights that are not involved in the substitutions. Also, the scenario-specific total ground delays are not affected by a substitution of scenario contingent arrival times between two flights. This is proved in the following proposition.

Proposition 3.2.1. A scenario contingent slot substitution between two flights does not change the total ground delays under different scenarios.

Proof: Let f_1 and f_2 be two flights involved in substitution. Let scenario-specific ground delays of flights $f \in \Phi$ before substitution be denoted by g^{ξ}_f. After substitution, the scenario specific ground delays of flight $f \in \Phi \setminus \{f_1, f_2\}$ remains unchanged. For flights

f_1 and f_2 the ground delays after substitution are given by $g_{f_1}^{\xi} + \theta_{f_2}^{\xi} - \theta_{f_1}^{\xi}$ and $g_2^{\xi} + \theta_{f_1}^{\xi} - \theta_{f_2}^{\xi}$ respectively. Therefore the scenario-specific total ground delays of the two flights involved in the substitution remains unchanged. The proof follows.

Proposition 3.2.2. Scenario-specific planned arrival rates during any time period $t \in \Gamma$ remain unchanged after substitutions.

Proof: Let f_1 and f_2 be two flights involved in a substitution. Scenario-specific arrival time periods of flights $f \in \Phi \setminus \{f_1, f_2\}$ – i.e. $X_{f,t}^{\xi}$ -- remain unchanged after substitution. Therefore for time periods $t \in \Gamma \setminus \{\theta_{f_1}^{\xi} \cup \theta_{f_2}^{\xi}\}$ scenario-specific planned arrival numbers, given by $\sum_{f \in \Phi \setminus \{f_1, f_2\}} (X_{f,t}^{\xi} - X_{f,t-1}^{\xi})$, remains same before and after substitution.

Before substitution the expression $(X_{f_1,t}^{\xi} - X_{f_1,t-1}^{\xi})$ attains a value 1 for $t = \theta_{f_1}^{\xi}$ and 0 otherwise. Similarly before substitution $(X_{f_2,t}^{\xi} - X_{f_2,t-1}^{\xi}) = 1$ iff $t = \theta_{f_2}^{\xi}$. After substitution, $(X_{f_1,t}^{\xi} - X_{f_1,t-1}^{\xi}) = 1$ iff $t = \theta_{f_2}^{\xi}$ and $(X_{f_2,t}^{\xi} - X_{f_2,t-1}^{\xi}) = 1$ iff $t = \theta_{f_1}^{\xi}$.

Therefore during time periods $t \in \{\theta_{f_1}^{\xi} \cup \theta_{f_2}^{\xi}\}$, the scenario specific planned arrivals after substitution remains same as that before. Q.E.D.

Corollary 3.2.1. Scenario-specific airborne delays remain same after substitution.

Proof: From proposition 3.2.2, during any time period $t \in \Gamma$ the number of planned arrivals - $\sum_{f \in \Phi} (X_{f,t}^{\xi} - X_{f,t-1}^{\xi})$ - remains the same before and after substitution. As there is no change in the scenario-specific airport capacities- M_t^{ξ}, the time-varying airborne queueing delay – W_t^{ξ} -- remains unchanged after substitution.

Corollary 3.2.2. The value of the objective function minimizing total expected linear delay costs – i.e. expression (2.4.1) in the DRGH model (Chapter 2) – remains invariant to a feasible substitution.

Proof. The proof follows from Proposition 3.2.1 and Corollary 3.2.1.

3.2.2. Substitutions and Cancellations Involving Many Flights

The above discussion focused on scenario-contingent slot substitution between two flights at a time. However, an airline may want to perform substitutions involving multiple flights. For example, a flight f_1 may be substituted for f_2 under a particular scenario, and for f_3 under a different scenario. Furthermore, an airline may cancel some of its flights and utilize the vacant slots to reduce delays of some of its other flights. In this section we present a linear math-programming model that can be used by individual airlines to re-optimize their internal objective functions. Without loss of generality, we assume that each airline assigns a delay cost function to each of its flights and minimizes the total expected cost, abiding by certain constraints imposed by the ATCSCC that are explained later.

The number of flights belonging to different airlines, planned to arrive during different time periods under each scenario can be determined from the solutions of the DRGH model. These are the scenario-specific numbers of slots owned by airlines during different time periods. The airlines can cancel some of their flights and utilize the slots to reschedule some other flights. While performing any substitutions, the airlines must adhere to the coupling constraints that impose restrictions on scenario-specific arrival times of flights. Also, the total number of scenario-specific slots owned by an airline during a time period cannot be exceeded by any substitution. We now present an optimization model that can be used for re-optimizing airline-specific cost functions. We refer to this model as the Dynamic Substitution Model.

Model Parameters

As mentioned before, we use the same notations as in DRGH model wherever possible. Here we introduce some new parameters.

Let A denote the set of airlines, and $F_a, a \in A$ denote the set of flights of airline $a \in A$ that are not cancelled. Let $\nu_{a,t}^{\xi}$ denote the number of slots owned by airline $a \in A$ during time period $t \in \Gamma$ under scenario $\xi \in \Theta$. As mentioned earlier, the $v^{\xi}_{a,t}$ are determined from the solution of the DRGH model.

We focus on the flights of an airline $a \in A$, and present the optimization model whose objective is to minimize the total expected cost of this airline. An airline can assign a ground delay cost function to each of its flight. Let $c(f,\tau)$ denote the cost of ground holding a flight $f \in F_a$ for τ time periods. Even if the cost functions are non-linear, the optimization model remains a linear programming problem by virtue of the decision variables described below.

Decision Variables

The decision variables are the same as those in the DRGH model, and are related to the scenario-specific arrival time periods of flights belonging to individual airlines. Let $X^{\xi}_{f,t} \in \{0,1\}, f \in F_a, t \in \{Arr_f,..,T+1\}, \xi \in \Theta$ denote the decision variables. The variables $X^{\xi}_{f,t}$ are 0 through the time period before flight f is planned to arrive under scenario ξ, and 1 thereafter. The auxiliary variables capturing the scenario-specific departure times are given by :

$$Y^{\xi}_{f,t} = \begin{cases} X^{\xi}_{f,t+Arr_f-Dep_f} \text{ if } t + Arr_f - Dep_f \leq T \\ 1 \, \text{otherwise} \end{cases} \quad ; \; f \in F_a, t \in \{Dep_f,..,T+1\}, \xi \in \Theta$$

Objective Function

Each airline $a \in A$ minimizes their total expected ground delay cost function given by:

$$z = \sum_{f \in F_a} \sum_{\xi \in \Theta} P\{\xi\} \times \sum_{t=Arr_f}^{T+1} c(f, t - Arr_f) \times (X_{f,t}^{\xi} - X_{f,t-1}^{\xi}) \qquad (3.2.1)$$

Constraints

The set of constraints that an airline $a \in A$ must adhere to while performing substitutions are defined as follows:

$$X_{f,t}^{\xi} - X_{f,t-1}^{\xi} \geq 0; \forall f \in F_a, t \in \{Arr_f, .., T+1\}, \xi \in \Theta \qquad (3.2.2)$$

$$\sum_{f \in F_a} (X_{f,t}^{\xi} - X_{f,t-1}^{\xi}) \leq \nu_{a,t}^{\xi}; \quad \forall t \in \Gamma, \xi \in \Theta \qquad (3.2.3)$$

$$X_{f,T+1}^{\xi} = 1; \quad \forall f \in F_a, \xi \in \Theta \qquad (3.2.4)$$

$$Y_{f,t}^{S_1^i} = ... = Y_{f,t}^{S_k^i} = ... = Y_{f,t}^{S_{N_i}^i}; \quad \forall f \in F_a, t \in \Gamma, S_k^i \in \Omega_i : N_i \geq 2 \text{ and } o_i \leq t \leq \mu_i \qquad (3.2.5)$$

Constraints (3.2.2) reflect the requirement that the decision variables are non-decreasing. Constraints (3.2.3) implies that the total number of flights of the airline $a \in A$ planned to arrive during any time period $t \in \Gamma$ must be not exceed the number of slots the airline owns from the initial assignment – $\nu_{a,t}^{\xi}$. Constraint set (3.2.4) ensures that all flights of the airline, that are not cancelled, arrive within the end of the planning horizon. Finally, the constraints (3.2.5) are the coupling constraints, similar to those in DRGH model, which impose the restriction that the scenario-specific departure times (and hence arrival times) of a flight must be same for those scenarios belongs to the active branch at the flight's release time.

2.2.5. Experimental Setup

We perform experiments with the dynamic substitution model presented in this section using the same setup as that in Section 5 of Chapter 2. We use the solutions from DRGH model applied to Case 2 (high cost ratio -- $\lambda = 25$) to perform our experiments here. Due to high cost ratio there is no airborne delay under any scenario. There are a total of 351 flights belonging to 24 airlines. Table 3.2.1 shows the number of their flights scheduled to arrive within the planning horizon and the expected ground delay (in flight-hours) of each airline as obtained from the solutions of DRGH model. The airlines are denoted by their three letter codes. The major airlines that operate at DFW are the American Airlines (AAL), and the Delta Airlines (DAL). The airlines CAA, EGF and SKW mostly operate commuter flights to DFW.

The cost of ground delay for one time period was assigned the value 1 for all flights in the experiments presented in Chapter 2. Here we assign, for each flight, a randomly chosen ground delay unit cost that is uniformly distributed within the interval $[0.5,1.5]$ -- i.e. with an expected value of 1. The flight-specific unit costs reflect differences in operating cost, payload, fare, class mix, downstream connectivity, and other factors that may cause an airline to attach higher or lower priority to certain flights. Just as in present-day CDM, our substitution model enables airlines to take these differences into account in re-assigning slots. In the experiments performed in this section, we illustrate the benefits of using the substitution model (and the compression model presented later), and to show that these models are suitable for use in a collaborative setting where a stochastic planning of GDP is performed using the DRGH model.

We apply the Substitution model to the following two cases:

Case I: We assume there are no cancellations. The airlines use the substitution model to minimize the total expected ground delay cost of all their flights. We apply the substitution model for 100 different realizations of the ground delay cost of each flight.

Case II: We impose cancellations on nine flights shown in Table 3.2.2. Each of these flights is a plausible candidate for cancellation because it has high expected delay assigned by the DRGH model and duplicates another flight with respect to origin and airline. We apply the substitution model for performing airline-specific substitutions, for 100 different realizations of the ground delay cost for each of the remaining flights.

Table 3.2.1 Major Airlines Operating at DFW During the Planning Horizon

Three letter carrier code	Number of flights scheduled to arrive within the planning horizon	Expected ground delay from DRGH Model (flight-hours)
AAL	149	9.1
AMT	1	0
AMW	5	1.15
AMX	1	0
AWE	3	0
BTA	3	0
CAA	43	5.65
CHQ	9	1.08
COA	3	0.8
COM	1	0
DAL	14	0.25
DLH	1	0
EGF	76	11.63
FFT	1	0
JZA	1	0
KAL	1	0
MEP	1	0
NWA	4	0
SCX	1	0
SKW	21	3.73
TRS	2	0
UAL	2	0
UPS	4	0
USA	4	0

Table 3.2.2. Cancelled Flights

Flight ID	Three Letter Airline Code	Three-letter origin airport code	Scheduled arrival time at DFW	Expected ground delay assigned by the DRGH model (minutes)	Worst scenario delay assigned by the DRGH model (minutes)
167_AAL	AAL	AUS	9:00AM	40	90
118_AAL	AAL	TUL	8:32AM	54	75
152_AAL	AAL	SAT	8:50AM	54	75
113_CAA	CAA	SHV	8:30AM	70	120
97_CAA	CAA	HOU	8:20AM	80	120
104_EGF	EGF	FSM	8:24AM	70	75
107_EGF	EGF	LAW	8:25AM	80	120
157_EGF	EGF	SHV	8:51AM	48	60
124_SKW	SKW	AUS	8:35AM	70	120

Results and Discussion

In Case I, the mean and standard deviation of percentage reduction in total expected delay cost compared to that from initial assignment by the DRGH model after applying the substitution model for intra-airline substitutions, were 11% and 2% respectively. In other words, on average the airlines reduce delay cost 11% by applying the substitution model. As there are no cancellations in this case, the substitutions did not change the total delay.

Figure 3.2.1 presents box-and-whisker plots summarizing delay cost reduction, by airline, after substitution under Case I. (Results are presented only for airlines subject to delay after the DRGH slot assignment). Airlines that operate larger numbers of flights, such as AAL, benefit more from substitutions. This is intuitive, because an airline with a larger set of flights has more substitution opportunities.

Figure 3.2.1 also shows the variation in cost reduction for the 100 different realizations of flight-specific ground delay unit cost. The upper and lower edge of each box represents the upper and lower quartiles respectively, while the diamond depicts the two standard deviation interval around the mean. There is considerable variability in cost reduction for certain airlines such as DAL and CHQ. In the former case, the explanation is that one DAL flight was assigned delay by the DRGH model. Depending on the cost reduction, this flight is substituted with another flight with lower unit delay cost in the substitution phase.

Table 3.2.3 shows some of the substitutions that occur for one realization of delay costs. The slot portfolios of flights 51_AAL and 44_AAL are exchanged because the former has higher unit delay cost. A similar substitution involving two flights happens between the Delta Airlines flights 103_DAL and 106_DAL. A more complicated case is 108_AAL, which in effect is given the slot portfolio of flight 95_AAL. The latter is not, however, given 108_AAL's portfolio. As it is a short haul flight with low unit delay cost, it is assigned a scenario-contingent portfolio with from 4 to 9 additional time periods of ground delay. The scenario-specific slots vacated by 108_AAL are filled up by another American Airlines flight – 140_AAL – that has high unit delay cost.

Table 3.2.4 compares delay cost reductions with and without cancellation of the flights in Table 3.2.2. For the airlines that had no cancellations, there is no change, because at this stage, they do not have access to unused slots. Airlines that cancelled flights benefit by substituting their higher priority flights into the vacant slots.

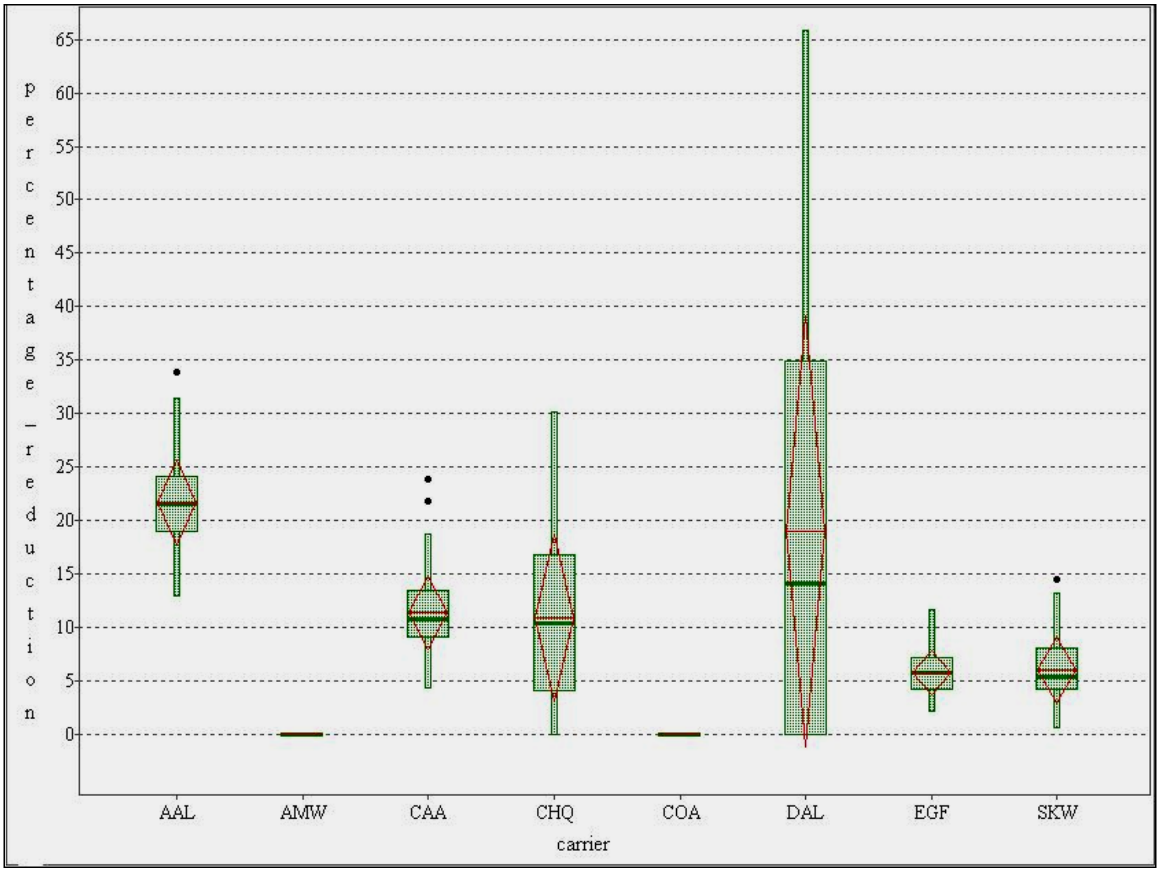

Figure 3.2.1 Box-and-Whisker Plot Showing Percentage Reductions in Expected Delay Costs of Different Airlines for Case I

Table 3.2.3 Examples of Flight Substitutions

Flight ID	Carrier	Ground Delay Cost	Scenario-Specific Arrival Time Periods Before Substitutions						Scenario-Specific Arrival Time Periods After Substitutions					
			ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
44_AAL	AAL	0.6	29	29	29	29	29	29	30	30	30	30	30	30
51_AAL	AAL	0.67	30	30	30	30	30	30	29	29	29	29	29	29
103_DAL	DAL	0.79	34	34	34	34	34	34	35	35	35	35	35	35
106_DAL	DAL	1.35	35	35	35	35	35	35	34	34	34	34	34	34
95_AAL	AAL	0.57	34	34	34	34	34	34	38	40	42	43	43	43
108_AAL	AAL	1.36	35	35	35	35	35	35	34	34	34	34	34	34
140_AAL	AAL	1.45	36	36	36	36	36	36	35	35	35	35	35	35

Table 3.2.4. Effect of Cancellations on Delay Cost Reduction

Airline	Cancellations	Percentage Reduction in Delay Cost after Substitution	
		w/o Cancellations	w/Cancellations
AAL	3	22	53
AMT	0	0	0
AMW	0	0	0
AMX	0	0	0
AWE	0	0	0
BTA	0	0	0
CAA	2	12	57
CHQ	0	11	11
COA	0	0	0
COM	0	0	0
DAL	0	19	19
DLH	0	0	0
EGF	3	5	40
FFT	0	0	0
JZA	0	0	0
KAL	0	0	0
MEP	0	0	0
NWA	0	0	0
SCX	0	0	0
SKW	1	6	39
TRS	0	0	0
UAL	0	0	0
UPS	0	0	0
USA	0	0	0

3.3. DYNAMIC COMPRESSION MODEL

If, in substitution, airlines cancel flights, a set of vacant slots is created. In present day CDM, the ATCSCC performs compression to fill the vacant slots in order to reduce delays of some flights. While making substitutions to vacant slots, the ATCSCC gives preference to the flights of airlines that released the slots. In this section, we present an optimization model that performs compression within a scenario-based framework. Our goal is to illustrate how compression, like ration-by-schedule and substitution, can be adopted to a setting featuring probabilistic scenario trees and scenario-contingent GDP planning.

3.3.1. Compression Model Parameters and Data

Wherever possible, we use the same notation and parameters as that in the DRGH and dynamic substitution models. Our additional notation is as follows. Let G be the set of flights that are not cancelled – i.e. $G = \bigcup_{a \in A} F_a$. Each flight is assigned a portfolio of scenario-specific arrival time after the substitution process. We denote this portfolio by ρ_f^{ξ}, $f \in G, \xi \in \Theta$. We denote the number of cancelled flights by an airline $a \in A$ by the parameter can_a.

The number of flights during any time period that is subject to airborne holding at the airport is obtained from the optimal values of W_t^{ξ} given by the solution of the DRGH model. We denote these values by $\hat{W}_t^{\xi}$.

Decision Variables

Decision variables are the same as those in the DRGH model, and are given by:

$$X_{f,t}^{\xi} = \begin{cases} 1 \text{ if a flights f is planned to arrive by end of time period t} \\ 0 \text{ otherwise} \end{cases} ; \begin{array}{l} f \in G, \xi \in \Theta, \\ t \in \{Arr_f, .., T+1\} \end{array}$$

The auxiliary variables related to the release times of each flight are given by:

$$Y^{\xi}_{f,t} = \begin{cases} X^{\xi}_{f,t+Arr_f-Dep_f} & \text{if } t + Arr_f - Dep_f \le T \\ 1 & \text{otherwise} \end{cases}$$

Objective Function

The objective function is to minimize the weighted expected ground delays of all flights that are not cancelled. The weights are airline specific, and are introduced to give priority in slot re-allocation to airlines that cancel more flights. To be used in this way, the weight for an airline should be positive and monotonically increasing in the number of cancellations it makes in the substitution stage. Here, we assume the weight of $(1+can_a)$. The objective function is:

$$\min z = \sum_{\xi \in \Theta} P\{\xi\} \times \left(\sum_{a \in A} (1+can_a) \times \sum_{f \in F_a} \sum_{t=Arr_f}^{T+1} (t - Arr_f)\left(X^{\xi}_{f,t} - X^{\xi}_{f,t-1}\right) \right) \qquad (3.3.1)$$

Constraints

The set of constraints are:

$$X^{\xi}_{f,t} - X^{\xi}_{f,t-1} \ge 0; \ \forall f \in G, t \in \{Arr_f,..,T+1\}, \xi \in \Theta \qquad (3.3.2)$$

$$\sum_{t=Arr_f}^{T+1} t \times (X^{\xi}_{f,t} - X^{\xi}_{f,t-1}) \le \rho^{\xi}_f; \ \forall f \in G, \xi \in \Theta \qquad (3.3.3)$$

$$\sum_{f \in G} (X^{\xi}_{f,t} - X^{\xi}_{f,t-1}) + W^{\xi}_{t-1} - W^{\xi}_{t} \leq M^{\xi}_{t}; \quad \forall t \in \Gamma, \xi \in \Theta \qquad (3.3.4)$$

$$W^{\xi}_{t} \leq \hat{W}^{\xi}_{t}; \quad \forall t \in \Gamma, \xi \in \Theta \qquad (3.3.5)$$

$$X^{\xi}_{f,T+1} = 1; \quad \forall f \in G, \xi \in \Theta \qquad (3.3.6)$$

$$Y^{S^{i}_{1}}_{f,t} = \ldots = Y^{S^{i}_{k}}_{f,t} = \ldots = Y^{S^{i}_{N_i}}_{f,t}; \quad \forall f \in G, t \in \Gamma, S^{i}_{k} \in \Omega_i : N_i \geq 2 \text{ and } o_i \leq t \leq \mu_i \qquad (3.3.7)$$

Constraints (3.3.2) capture the non-decreasing property of the decision variables. Constraints (3.3.3) prohibit assigning a later time period to a flight than what it has after substitutions, under any scenario. This ensures that compression yields a Pareto improvement. Constraints (3.3.4), similar to constraints (2.4.3) in the DRGH model (see Chapter 2), introduce arrival capacity constraints at the destination airport. Constraints (3.3.5) specify that the number of flights subject to airborne holding must not exceed the corresponding values assigned by the DRGH model during the initial assignments – i.e. solution to SAGHP. Constraints (3.3.6) and (3.3.7) are feasibility and coupling constraints similar to those in the DRGH and the substitution models.

3.3.2. Experimental Results

We use the same experimental setup as described in Section 2.2.5 of this chapter. We apply the dynamic compression model after the dynamic substitution model has been applied to Case II. The results shown here correspond to one realization of ground delay cost for each flight. As before, the unit ground delay cost of a flight is assumed to be uniformly distributed in [0.5,1.5].

Figure 3.3.1a shows the expected ground delay (in flight-hours) of each airline as obtained from the initial delay assignment by DRGH model, after substitutions, and finally after applying the dynamic compression model. Those airlines that face no delays after the initial DRGH assignment are not affected by substitutions and cancellations. Some of the other airlines – DAL, AMW, CHQ, and COA – perform no cancellations. Hence their delays (though not necessarily delay costs) remain same after substitutions. In Figure 3.3.1a, these airlines do not realize any gain from compression, either.

Those airlines who cancel flights – AAL, CAA, EGF, and SKW – achieve reduction in expected delays (as well as delay costs) through substitutions. This is because they can substitute their own flights into the slots vacated by cancellations. In this example, however, only AAL and EGF benefit from the compression step. These airlines operate most of the flights to DFW during the planning horizon, and hence have greater flexibility to utilize empty slots. Moreover, these airlines cancelled the highest numbers of flights – 3 each – and therefore the associated weights in the objective function for them are greater.

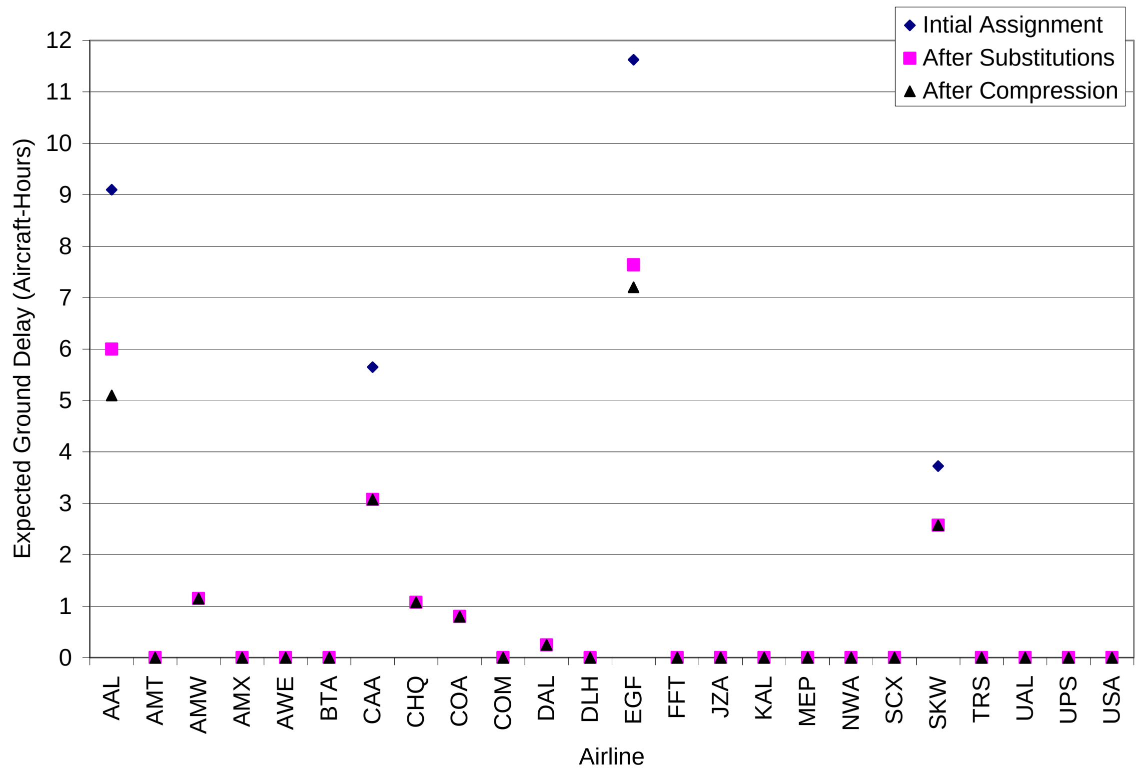

Figure 3.3.1a Changes in Expected Ground Delays of Airlines as a Result of Dynamic Substitution and Compression models Applied to Case II

While in the above example, airlines that do not cancel flights do not achieve delay reductions from compression, this is not generally true. To illustrate, we performed another experiment in which American Airlines cancels 20% of its flights, chosen at random, without any other cancellations. The results are shown in Figure 3.3.1b. The total expected ground delay of the American Airlines has been virtually eliminated after cancellations and intra-airline substitutions, while expected delays for the other carriers are unaffected. The Compression model, however, yields substantial reductions in expected ground delays for CAA and SKW, and a slight reduction in delays of the airlines COA, CHQ, and DAL (whose ground delay is eliminated).

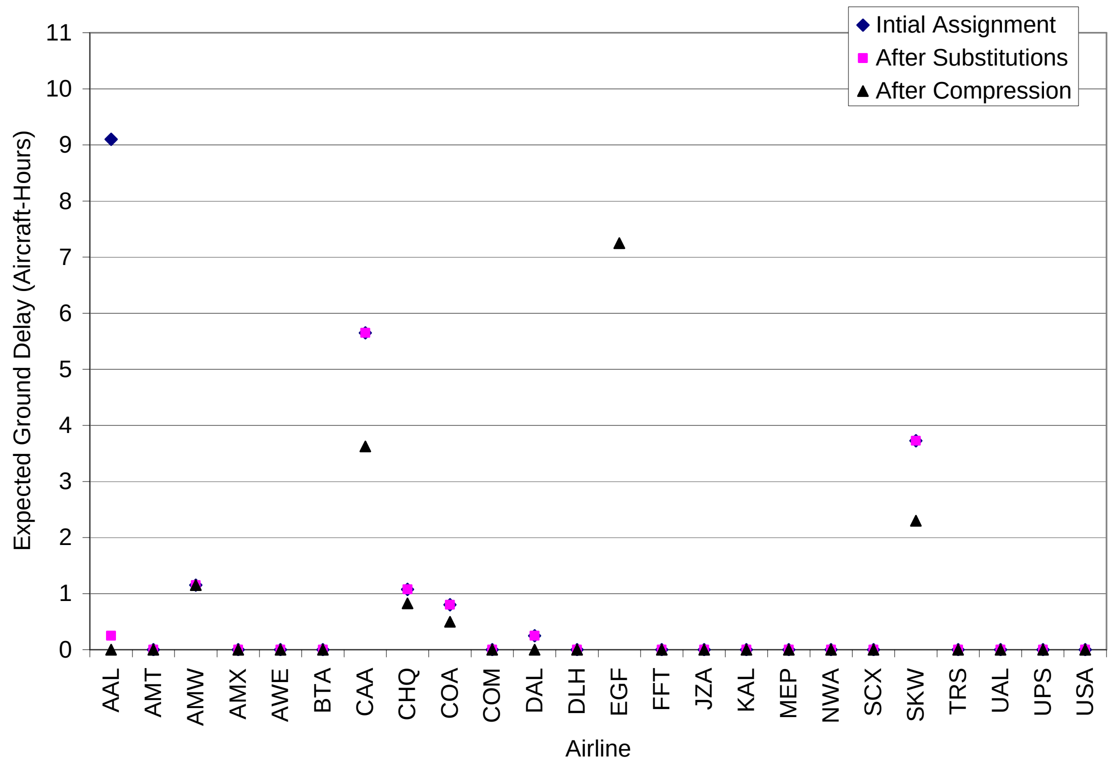

Figure 3.3.1b Changes in Expected Ground Delays of Airlines as a Result of Substitutions and Compressions When Only AAL Flights Are Cancelled

3.4. CONCLUDING REMARKS

In this chapter we have presented a methodology for allowing cancellations and substitutions to be performed by airlines, and compression to be performed by the ATCSCC, after scenario-contingent ground delays have been assigned to flights by the DRGH model. The resulting set of models work analogously to present-day CDM, but with the added ability to dynamically revise decisions in the face of updated information.

In Section 3.2 we presented an optimization model that can be used by individual airlines to re-optimize their own internal delay cost functions. Even if the cost functions are non-linear, the model remains a linear integer-programming problem by virtue of the binary and non-decreasing decision variables. We performed experiments to show that the delay cost reduction from substitutions is greater for airlines that operate many flights. Such airlines enjoy greater flexibility to make substitutions in order to reduce delays of their high-priority flights. Airlines can also use the dynamic substitution model for re-utilizing the vacant slots created by canceling some of their own flights, in order to minimize delay costs. The dynamic substitution model prevents inter-airline substitutions from happening.

After a round of cancellations and substitutions, airlines apprise the ATCSCC about their updated schedule. The dynamic compression model can then be applied to fill up the released slots while giving priorities to the flights of the airlines that cancelled flights and released those slots. The compression process yields benefits (delay reduction) for airlines that do not cancel flights, since their flights can be assigned to slots freed by cancellations made by others. One important constraint in the dynamic compression model is that no flight can be assigned a later slot under any scenario, than what it has after the substitution step. Therefore, after the compression model is applied, any airline is at least as well off, if not better-off, than before.

Further study related to the present work will involve applying the dynamic substitution and compression models to a more realistic setting. Some of the limitations of the experiments performed in this chapter are the simplistic delay cost function, and the lack of a methodology for selecting flights for cancellations. Such a methodology must consider the marginal delay cost of a flight as well as the cost of cancellation. Scenario-based cancellations decisions may also be possible. With such a cancellation model, our dynamic, scenario based reinvention of CDM will be complete.

CHAPTER 4: STOCHASTIC OPTIMIZATION MODELS FOR AIR TRAFFIC FLOW MANAGEMENT

4.1. INTRODUCTION

In this chapter, we focus on ATFM problems with enroute airspace and airport congestion, and propose solutions that recognize uncertainty in the future evolution of capacity of these multiple NAS resources. In contrast to chapter 2, here we consider multiple congestible NAS resources – enroute fixes, airports – which are used by aircraft to fly within the system. We proceed as follows. In Section 4.2, we provide a discussion on the optimization models that address both airport and enroute airspace capacities, and are relevant to our formulations that we present later. In Section 4.3 we summarize the research contributions made in this chapter. Section 4.4 presents three stochastic optimization models for ATFM, followed by experimental results in Section 4.5 and concluding remarks in Section 4.6.

4.2. BACKGROUND

Bertsimas-Stock (1998) addressed the ATFM problem under deterministic time-varying enroute airspace and airport capacities. They proposed an Integer Programming (IP) formulation. They showed the problem is equivalent to the “Job Shop Scheduling” problem (Garey and Johnson, 1979), which is *NP-hard*. Nevertheless, many constraints in their formulation are *facets* of the polyhedron defined by the set of all constraints, and therefore the formulation is particularly *strong* (Nemhauser and Wolsey, 1998). In many instances, even ones with large numbers of variables and constraints, integer solutions are obtained directly from the LP relaxation of their model.

The Bertsimas-Stock (1998) model is "disaggregate" in formulation because the decision variables are related to each individual flight's release (or departure) time, amount of airborne holding, and route. This makes the problem very large for realistic situations involving thousands of flights and congestible NAS resources. The computation time can become exorbitant if LP relaxation doesn't yield integer optimal solutions. But disaggregate formulation also has advantages. It allows flight-specific set of route(s) and cost functions to be used as inputs. There are several variations of the original formulation that include route choice, allow dynamic rerouting, consider delay propagation, and capture dependence between arrival and departure capacities of airports. A major limitation of the model is that it is deterministic, and thus ignores uncertainty in resource capacities.

Uncertainty in resource capacities in ATFM has been addressed mainly in context of the GHP (Richetta and Odoni, 1993; Richetta and Odoni, 1994; and Ball et al, 2002.) In all of these models, uncertainty has been addressed by considering a set of scenarios, each corresponding to a time-varying airport capacity profile. Richetta and Odoni (1994) also used the concept of a scenario tree that represents evolving information about which scenario will be realized. In chapter 2 of this dissertation, we formulated a multi-stage stochastic integer program with recourse that allows dynamic revision of ground delays imposed on non-departed flights, based on updated information on the capacity conditions at the destination airport. Updated information is achieved through branching of the scenario tree. We specified a set of coupling constraints that impose the condition that unless a particular scenario is realized completely, the ground holding decisions cannot be made exclusively based on that scenario. In other words, the decision to release a flight must consider all scenarios that have not been eliminated from possibility at the time of release. The decision variables specify departure times of individual flights. The disaggregate formulation also allows us to easily incorporate non-linear measures of delay, without having to reformulate the problem as a non-linear optimization model. Such measures of delay are useful to address equity issues in GHP (Vossen 2002).

Nilim et al. (2002) addressed weather related uncertainty in airspace condition in routing individual flights. They formulated a robust Markov Decision Process problem for dynamically rerouting an aircraft across convective weather impacted region. Decisions in their problem are aircraft speed and heading. This research is among the first to address uncertainty in enroute airspace conditions. However, the methodology is not directly relevant to ATFM because it considers individual flights and assumes that weather evolution is Markovian.

In summary, while there is considerable literature on ATFM optimization, a significant gap remains. While some models consider multiple constraints in both airport and enroute capacities, and others address uncertainty in airport capacity scenarios, the literature does not include models that consider both airport and enroute capacity constraints while addressing uncertainty. This void is addressed in this chapter.

4.3. RESEARCH CONTRIBUTIONS

In this chapter, we present three linear optimization models for ATFM that account for uncertainty in both airport and enroute airspace capacity. Rather than analyzing this problem in its full generality, we focus on the case in which there is a single destination airport and a small number of arrival fixes subject to blockage or reduced capacity as a result of weather. This would typically occur when there is weather in the vicinity of the airport, and this impacts throughput of some of the standard arrival fixes of the airport, along with the acceptance rate (or arrival capacity) of the airport itself. The main decisions are therefore pre-departure delay (or ground holding) and local rerouting of inbound flights.

We apply the principles of stochastic programming (Birge and Louveaux, 1997) to formulate the models. The first model is a static stochastic integer programming formulation, in which ground delay decisions are made once at the beginning of day and not revised later. Airborne delays are incurred if, as a result of adverse weather conditions, demand created by planned arrivals exceeds the capacity. The second model is two-stage stochastic, with decisions for rerouting flights taken at certain points (or fixes) in the airspace. Rerouting flights may be necessary if there is congestion downstream of the primary path. In the third model, which is multi-stage with recourse, ground delay decisions of flights maybe revised upto their departure, and while flights are enroute, decisions on rerouting and airborne holding are made. In the first two models, decision variables concern the aggregate number of flights, while in the third model they specify the departure time, and route of each individual flight.

4.4. STOCHASTIC OPTIMIZATION MODELS FOR ATFM

4.4.1. Scenario Tree

As mentioned earlier, in the literature on stochastic optimization models for GHP, uncertainty in the airport arrival capacity is addressed by considering multiple scenarios. Each scenario represents a particular profile of time-varying capacity (or, in the jargon of the industry, acceptance rates). A scenario tree consists of branches with branching points corresponding to particular times of day. Each branch corresponds to a particular scenario or group of scenarios. A branching point, or node, in the scenario tree represents a time when certain scenarios are eliminated as possibilities for a particular day. Thus, as the day progresses, more specific information about the capacity scenario that will be realized on that day becomes available. Branching points may coincide with times when capacity profiles of different scenarios diverge, or they may precede those times as a result of forecast information.

In this chapter, we specify a scenario tree with notation similar to that used in the DRGH model presented in Chapter 2. Let Θ denote the set of capacity profile scenarios, and the unconditional probability of occurrence of a scenario $\xi \in \Theta$ be given by $P\{\xi\}$. In the present context, a scenario represents the joint evolution of capacity profiles of multiple NAS resources – arrivals fixes for an airport and the airport itself.

In a static stochastic optimization model, the updated information that is available from the branching of the scenario tree is not utilized. Decisions on ground delays of flights are made once at the beginning of planning horizon, and not revised later. In contrast, dynamic models adapt the new information that becomes available at branching points. For the dynamic models, the scenario tree is input through the following data and parameters. Let B be the total number of branches of the scenario tree; $B \geq |\Theta|$. Each branch corresponds to a set of scenarios. Let η_b scenarios corresponding to branch $b \in \{1,..,B\}$ be given by the set $\Omega_b = \{\psi_1^b,..,\psi_{\eta_b}^b\} \subset \Theta$. The time periods corresponding to start and end nodes of a branch are denoted by o_b and e_b.

4.2.1. Static Stochastic Model

In this model, ground delay decisions are static, and there is no option for rerouting. Each flight arrives at the destination airport via a pre-specified arrival fix. Depending on ground delays, flights are planned to arrive at their respective arrival fixes either at their schedule or later. If the capacity of an arrival fix fall below demand (created by planned arrivals of flights) at any time period, then airborne holding occurs. Airborne holding can also occur within the airport TRACON if arrival demand exceeds airport landing capacity.

Model Parameters and Data

Let P be the set of arrival fixes, and k denote the destination airport. The time of day is discretized into a set of T time periods of equal duration, and is denoted by the set $\Gamma = \{0,1,2,...T\}$. For example, Γ might be a set of 48 time periods of 15 minutes each, amounting to a planning horizon of 12 hours. Any inbound flight must fly over one of the arrival fixes before arriving at the airport TRACON. We ignore the possibility of multiple arrival fixes along a flight path. Thus each flight faces a potential capacity constraint (and hence airborne delay) either at its designated arrival fix, or at the airport itself. The time-varying cumulative count of flights scheduled to arrive at arrival fix p is denoted by $SD_p(t)$, $p \in P$, and $t \in \Gamma$. Figure 4.4.1a shows a hypothetical airspace that encompasses three arrival fixes of an airport. Key parameters and decision variables are also explained in the figure.

As mentioned earlier, the static model ignores updated information from branching of scenarios as time of day progresses; it uses information about the scenarios and scenario probabilities, but not the scenario tree. Let $C_j^{\xi}(t), j \in P \bigcup \{k\}$, denote the scenario-specific time-varying capacity profiles of various resources under each scenario $\xi \in \Theta$. A resource can be an arrival fix $p \in P$ or the destination airport k. Let $\lambda > 1$ denote the cost ratio between one unit of airborne and ground delay $\lambda > 1$.

Decision Variables

The key decision variables in this model, which are all integers, are the time-varying cumulative number of planned arrivals at various fixes - $X_p(t)$ (see Figure 4.4.1a). Any difference in $SD_p(t)$ and $X_p(t)$ amounts to delayed arrivals, and due to the cost ratio λ being greater than 1, we assume that the delay is absorbed through ground holding of flights. As shown in Figure 4.4.1b, the area between $SD_p(t)$ and $X_p(t)$ amounts to ground delay imposed on flights scheduled to arrive at the destination airport via arrival fix p.

Let $D_p^\xi(t)$ denote the cumulative number of flights that have crossed the arrival fix p by the end of time period t, under scenario $\xi \in \Theta$. At any instant, the difference between $X_p(t)$ and $D_p^\xi(t)$ is the number of flights subject to airborne queuing delay for arrival fix p under scenario ξ. Such delay will exist if the number of planned arrivals at the arrival fix exceeds its capacity. As shown in Figure 4.4.1b the area between $X_p(t)$ and $D_p^\xi(t)$ amounts to the airborne queuing delay of flights before passing through fix p under scenario ξ. The inflow of air traffic from various arrival fixes to the destination airport k, under scenario ξ, are denoted by the auxiliary variables $A_p^\xi(t)$. $A_p^\xi(t)$ are related to $D_p^\xi(t)$ by the following expression:

$$A_p^\xi(t) = D_p^\xi(\max(0, t - l_p)) \qquad (4.4.1)$$

where, l_p is the minimum flight time between arrival fix p and the airport k.

The time-varying arrival demand at the airport k , under each scenario ξ is derived from the sum of inflow of air traffic from various arrival fixes, and is given by $\sum_{p \in P} A_p^{\xi}(t)$. Due to possible capacity constraint at airport, it may not be possible to serve all of the demand, resulting in airborne queuing delays. Let $D_k^{\xi}(t)$ denote the cumulative count of landings at the airport. Any difference between $\sum_{p \in P} A_p^{\xi}(t)$ and $D_k^{\xi}(t)$ at any time period amounts to airborne holding.

Objective Function

Objective function is to minimize the total expected cost of delay, and is given by the following expression:

$$
\begin{aligned}
z = & \sum_{p \in P} \sum_{t=0}^{T-1} \left(SD_p(t) - X_p(t) \right) + \\
& \lambda \times \sum_{\xi \in \Theta} P\{\xi\} \times \left(\sum_{p \in P} \sum_{t=0}^{T-1} \left(X_p(t) - D_p^{\xi}(t) \right) + \sum_{t=0}^{T-1} \left(\sum_{p \in P} A_p^{\xi}(t) - D_k^{\xi}(t) \right) \right)
\end{aligned}
\tag{4.4.2}
$$

As mentioned before, each flight utilizes two NAS resources – the designated arrival fix of the flight and the destination airport – whose capacities are possibly reduced due to weather. The first term in the objective function computes the total ground delay due to any difference between scheduled and planned arrivals at the arrival fixes. The second term computes the total expected airborne delays at various arrival fixes, and at the destination airport, weighted by the cost ratio λ.

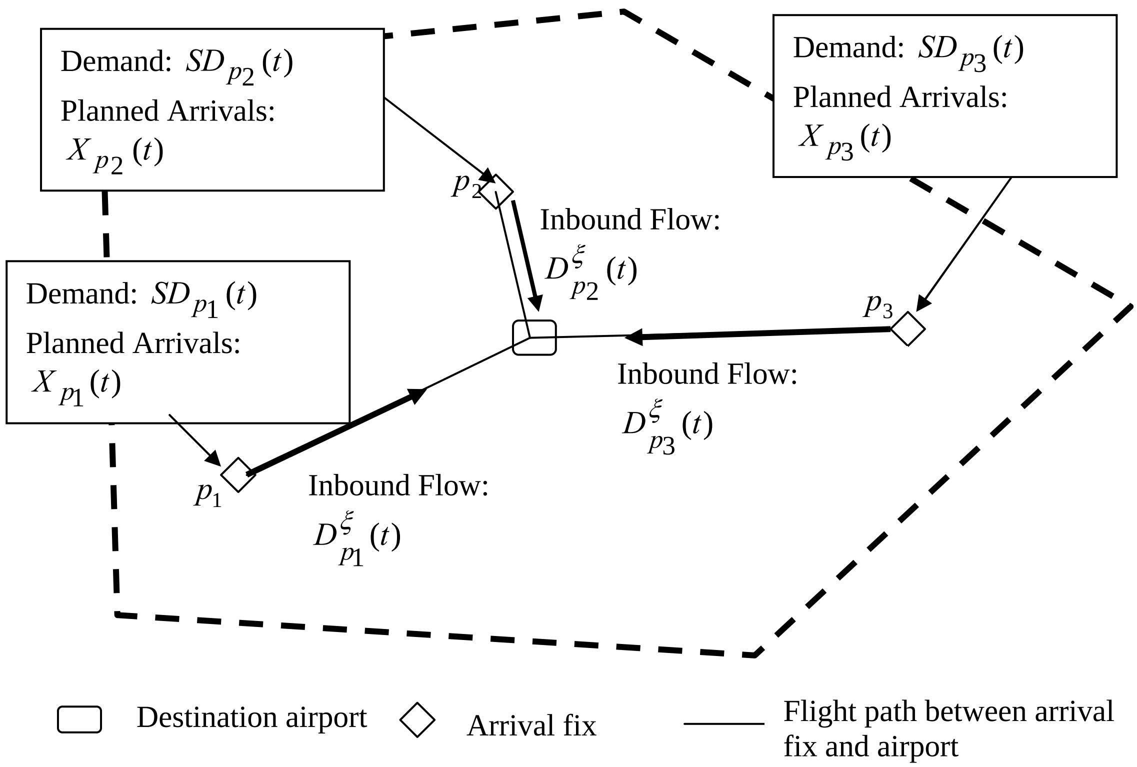

Figure 4.4.1a Hypothetical Airspace Describing Parameters and Decision Variables in Static Model

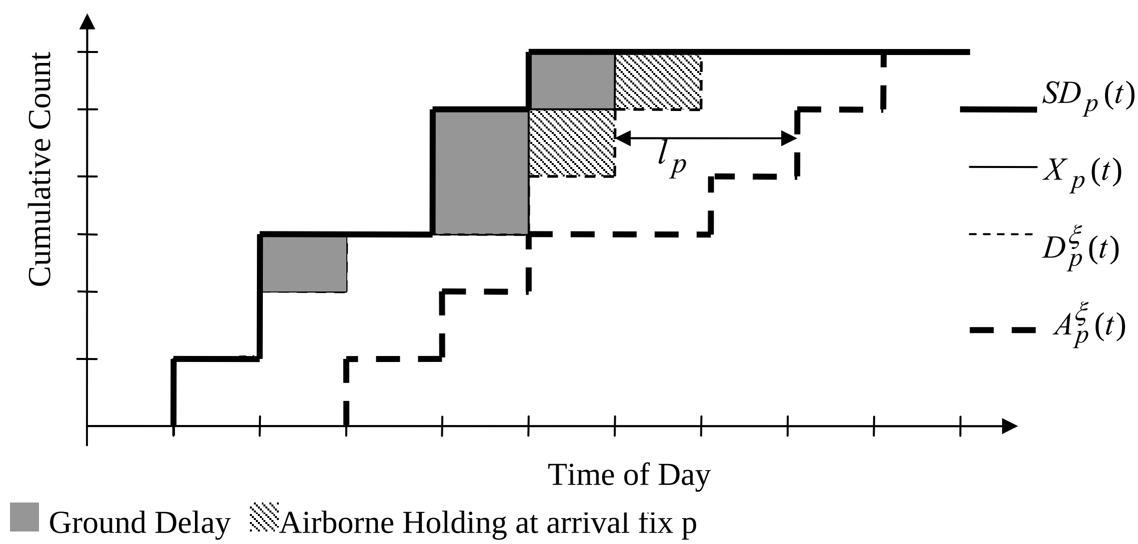

Figure 4.4.1b Queuing Diagram of Air Traffic Flow between an Arrival Fix and Destination Airport

Constraints

The set of constraints are given by the following equations:

$$SD_p(t) - X_p(t) \ge 0; \quad \forall\, p \in P, t \in \Gamma \tag{4.4.3}$$

$$X_p(t) - D_p^{\xi}(t) \ge 0; \quad \forall\, p \in P,\ \xi \in \Theta, t \in \Gamma \tag{4.4.4}$$

$$\sum_{p \in P} A_p^{\xi}(t) - D_k^{\xi}(t) \ge 0; \quad \forall\, \xi \in \Theta, t \in \Gamma \tag{4.4.5}$$

$$X_p(t) - X_p(t-1) \ge 0; \quad \forall\, p \in P, t \in \{1,2,...,T\} \tag{4.4.6}$$

$$D_p^{\xi}(t) - D_p^{\xi}(t-1) \ge 0; \quad \forall\, p \in P, \xi \in \Theta, t \in \{1,2,...,T\} \tag{4.4.7}$$

$$D_k^{\xi}(t) - D_k^{\xi}(t-1) \ge 0; \quad \forall\, \xi \in \Theta, t \in \{1,2,...,T\} \tag{4.4.8}$$

$$D_p^{\xi}(t) - D_p^{\xi}(t-1) \le C_p^{\xi}(t); \quad \forall\, p \in P, \xi \in \Theta, t \in \{1,2,...T\} \tag{4.4.9}$$

$$D_k^{\xi}(t) - D_k^{\xi}(t-1) \le C_k^{\xi}(t); \quad \forall\, \xi \in \Theta, t \in \{1,2,...T\} \tag{4.4.10}$$

$$D_k^{\xi}(T) - \sum_{p \in P} SD_p(T) = 0; \quad \forall\, \xi \in \Theta \tag{4.4.11}$$

$$X_p(t) \ge 0; \text{integer} \quad \forall p \in P, t \in \Gamma \tag{4.4.12}$$

$$D_p^{\xi}(t) \ge 0; \text{integer} \quad \forall p \in P, \xi \in \Theta, t \in \Gamma \tag{4.4.13}$$

$$D_k^{\xi}(t) \ge 0;\ \text{integer} \quad \forall\, \xi \in \Theta, t \in \Gamma \tag{4.4.14}$$

Constraints 4.4.3 specifies that the cumulative planned arrivals at a fix cannot exceed the cumulative scheduled arrivals by end of any time period. Constraint 4.4.4 imposes the restriction that the cumulative number of flights that cross any arrival fix cannot exceed the planned arrival numbers. Constraint 4.4.5 imposes a similar restriction at the destination airport k.

Constraints 4.4.6 – 4.4.8 reflect the requirement that the decision variables, defined as cumulative counts of either arrivals to or departures from various NAS resources, are non-decreasing. Constraints 4.4.12 – 4.4.14 specifies that the decision variables are non-negative.

Constraints 4.4.9 imposes the condition that the number of flights that cross an arrival fix at any time period t under a scenario ξ, must be bounded by the capacity of that fix during that time period under scenario ξ. Constraints 4.4.10 imposes a similar constraint for the number of arrivals (or landings) at the airport k. Constraints 4.4.11 ensures that all flights that are scheduled to arrive, land by the end of the planning horizon – i.e. there are no cancellations.

4.2.2. Two Stage Stochastic Model with Recourse

In this model, ground holding decisions are static – i.e. not revised as time of day progresses and updated information becomes available; but there are some enroute fixes near the destination airport, upstream of the arrival fixes, where dynamic rerouting is allowed to alleviate congestion downstream.

Model Parameters and Data

Consider a set of enroute fixes M , each of which lies on the primary (or shortest) route from several origin airports to the destination airport; and where dynamic rerouting of flights can occur. Let $SD_m(t)$ denote the time-varying cumulative demand of air traffic from different origin airports, bound towards the destination airport k , that are scheduled to pass the fix $m \in M$ by time $t \in \Gamma$. $\Gamma = \{0,1,...,T\}$ denotes the planning horizon in T discrete time intervals of equal duration.

From each enroute fix there is a primary route that delivers inbound air traffic flow to the airport via an arrival fix. Also there are R_m alternative routes, with travel times greater than the primary one, from fix $m \in M$ to the destination airport via other arrival fixes. Note that if $R_m = 0$ then there are no rerouting options for air traffic passing through node m . Let P denote the set of arrival fixes, some of which faces capacity constraints due to adverse weather. Let $p_{m,r} \in P$ denote the arrival fix that serves the air traffic flow between node m and the destination airport via route $r \in \{1,..,R_m + 1\}$.

Figure 4.4.2 shows a hypothetical airspace in which the key model parameters and decision variables of the two-stage model are explained. Rerouting can occur at an enroute fix denoted by m in the figure. There are two alternative routes available. The primary route connects m with the destination airport via arrival fix p_1 , and the alternative route connects via p_2 .

Based on the distance between m and $p_{m,r}$, there is a minimum flight time to arrive at the respective arrival fix via different routes, which is denoted by $l_{m,r}$. Let l_p denote the flight time between an arrival fix $p \in P$ and the destination airport. Based on the given data, we can derive the additional flight time – $\tau_{m,r}$, between the enroute fix m and the airport k via route $r \in \{1,..,R_m + 1\}$. For simplicity, we ignore the capacity constraints at any enroute fix other than the arrival fixes of the airport. Such constraints can be easily accounted by defining a set of nodes that lie on different routes of each $m - k$ pair, and allowing maximum flow constraints at those nodes. As in the static model, scenario-specific capacities of the arrival fixes and the airport are required as input data – $C_j^{\xi}(t), j \in P \cup \{k\}$.

Decision Variables

Let $X_m(t)$ be the time-varying cumulative number of flights planned to arrive at the enroute fix $m \in M$. As in the static model, any difference between $SD_m(t)$ and $X_m(t)$ implies ground delay. Let $D_{m,r}^{\xi}(j,t)$ represent the time-varying cumulative number of flights that have crossed various resources $j \in \{m, p_{m,r}, k\}$ along route $r \in \{1,..,R_m + 1\}$ between nodes m and k, under scenario $\xi \in \Theta$. For $j = k$, the variables $D_{m,r}^{\xi}(k,t)$ represent the number of flights arriving via enroute fix m that have landed at the destination airport, through route r under scenario ξ. Any difference between $X_m(t)$

and $\sum_{r=1}^{R_m+1} D_{m,r}^{\xi}(m,t)$ amounts to airborne queuing delay for node m under scenario ξ. Moreover, the extra airborne time of flights due to traveling via longer route, under scenario ξ, is given by $\sum_{r=2}^{R_m+1} \tau_{m,r} \times D_{m,r}^{\xi}(m,T)$.

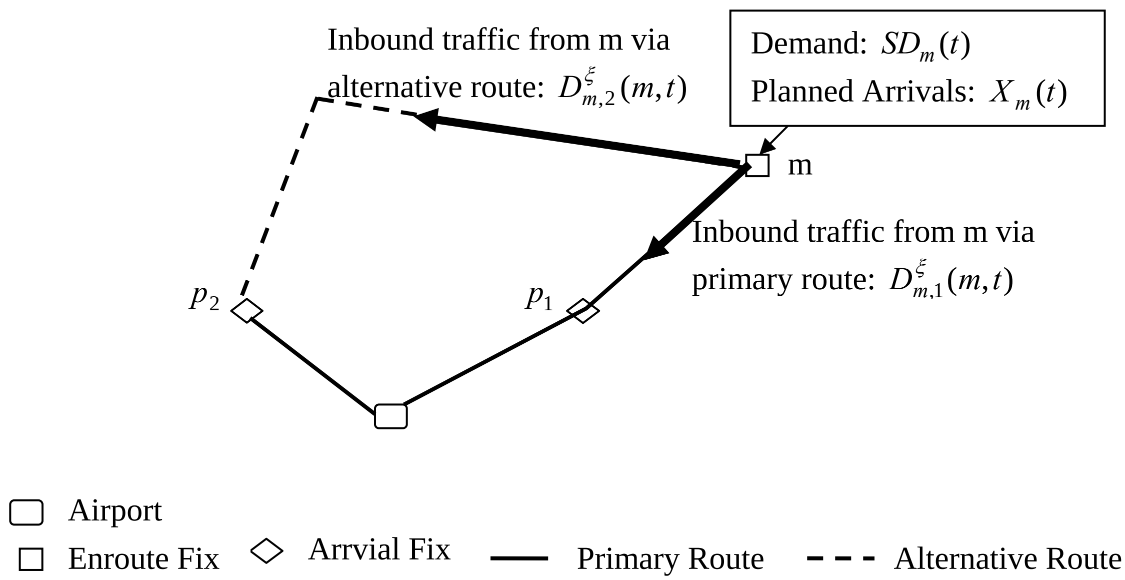

Figure 4.4.2 Hypothetical Airspace Describing Parameters and Decision Variables in the Two-Stage Model

Based on the minimum travel time on different routes, the cumulative count of flights that arrive at different arrival fixes and the destination airport, $A_{m,r}^{\xi}(j,t); r \in \{1,..,R_m+1\}, j \in \{p_{m,r}, k\}$, can be derived from the decision variables $D_{m,r}^{\xi}(j,t)$, through the relationships:

$$A_{m,r}^{\xi}(j,t) = \begin{cases} D_{m,r}^{\xi}(m, t - l_{m,r}) \text{ if } j = p_{m,r} \\ D_{m,r}^{\xi}(p_{m,r}, t - l_{p_{m,r}}) \text{ if } j = k \end{cases} \qquad (4.4.15)$$

As mentioned before, difference between $A_{m,r}^{\xi}(j,t)$ and $D_{m,r}^{\xi}(j,t)$ amounts to airborne holding.

Objective Function

The objective function is to minimize total expected delay cost, which is given by the following expression:

$$z = \sum_{m \in M} \sum_{t=0}^{T-1} \left(SD_m(t) - X_m(t) \right) + \lambda \times \sum_{\xi \in \Theta} P\{\xi\} \times \left(\begin{array}{l} \sum_{m \in M} \sum_{r=2}^{R_m+1} \tau_{m,r} \times D_{m,r}^{\xi}(m,T) \\ + \sum_{m \in M} \sum_{t=0}^{T-1} \left(X_m(t) - \sum_{r=1}^{R_m+1} D_{m,r}^{\xi}(m,t) \right) \\ + \sum_{m \in M} \sum_{r=1}^{R_m+1} \sum_{j \in \{p_{m,r},k\}} \sum_{t=0}^{T-1} \left(A_{m,r}^{\xi}(j,t) - D_{m,r}^{\xi}(j,t) \right) \end{array} \right) \qquad (4.4.16)$$

The first term computes the total ground delay, and the second term computes expected airborne delay, which is weighted by the cost ratio λ. Airborne delay has two components: extra airborne time due to longer routes flown, and airborne holding due to capacity constraints at fixes and the destination airport.

Constraints

The set of constraints are defined as follows:

$$SD_m(t) - X_m(t) \geq 0; \quad \forall m \in M, t \in \Gamma \qquad (4.4.17)$$

$$X_m(t) - \sum_{r=1}^{R_m+1} D_{m,r}^{\xi}(m,t) \geq 0; \quad \forall\, m \in M, \xi \in \Theta, t \in \Gamma \qquad (4.4.18)$$

$$A_{m,r}^{\xi}(j,t) - D_{m,r}^{\xi}(j,t) \geq 0; \quad \forall\, m \in M, j \in \{p_{m,r}, k\}, \xi \in \Theta, t \in \Gamma \qquad (4.4.19)$$

$$X_m(t) - X_m(t-1) \geq 0; \quad \forall\, m \in M, t \in \{1,..,T\} \qquad (4.4.20)$$

$$D_{m,r}^{\xi}(j,t) - D_{m,r}^{\xi}(j,t-1) \geq 0; \quad \forall\, m \in M, r \in \{1,..R_m\}, j \in \{m, p_{m,r}, k\}, \xi \in \Theta, t \in \{1,..,T\} \qquad (4.4.21)$$

$$\sum_{\substack{m \in M, \\ r: p_{m,r} = p}} \left(D_{m,r}^{\xi}(p,t) - D_{m,r}^{\xi}(p,t-1) \right) \leq C_p^{\xi}(t); \quad \forall p \in P, \xi \in \Theta, t \in \Gamma \qquad (4.4.22)$$

$$\sum_{m \in M} \sum_{r=1}^{R_m+1} \left(D_{m,r}^{\xi}(k,t) - D_{m,r}^{\xi}(k,t-1) \right) \leq C_k^{\xi}(t); \quad \forall \xi \in \Theta, t \in \Gamma \qquad (4.4.23)$$

$$\sum_{m \in M} \sum_{r=1}^{R_m+1} D_{m,r}^{\xi}(k,T) - \sum_{m \in M} SD_m(T) = 0; \quad \forall \xi \in \Theta \qquad (4.4.24)$$

$$D_{m,r}^{\psi_1^b}(m,t) = \ldots = D_{m,r}^{\psi_q^b}(m,t) = \ldots = D_{m,r}^{\psi_{\eta_b}^b}(m,t);$$
$$m \in M, r \in \{1,.., R_m + 1\}, t \in \Gamma, \psi_q^b \in \Omega_b : \eta_b \geq 2 \text{ and } o_b \leq t \leq e_b \qquad (4.4.25)$$

$$X_m(t) \geq 0; \text{integer} \quad \forall m \in M, t \in \Gamma \qquad (4.4.26)$$

$$D_{m,r}^{\xi}(j,t) \geq 0; \text{integer} \quad \forall m \in M, r \in \{1,..., R_m + 1\}, j \in \{m, p_{m,r}, k\}, t \in \Gamma \qquad (4.4.27)$$

Constraints 4.4.17 specify that the time-varying cumulative planned arrivals at any enroute fix $m \in M$ cannot exceed the cumulative scheduled arrivals. Constraints 4.4.18 specify that the cumulative number of flights that have crossed an enroute fix via all available routes by the end of time period t cannot exceed the cumulative planned arrivals. Constraints 4.4.19 impose similar restrictions on the variables $D^{\xi}_{m,r}(j,t)$, stating that these cannot exceed the corresponding cumulative arrivals at the respective resources $j \in \{p_{m,r}, k\}$ under any scenario ξ. Constraints 4.4.20 – 4.4.21 impose the condition that the decision variables, which are cumulative counts, are non-decreasing.

Constraints 4.4.22 state that the total number of flights that cross an arrival fix $p \in P$ during any time period t , under scenario ξ, must be less than or equal to the scenario-specific maximum throughput (or capacity) of the fix at that time period. Flights arrive at the fix from various fixes $m \in M$ via available routes $r \in \{1,..,R_m + 1\}$, such that the arrival fix p belongs to the route between m and k. Constraints 4.4.23 state that the total number of flights that land at the destination airport during different time periods, under various capacity scenarios, must not exceed the scenario-specific arrival capacities of the airport during those time intervals.

Constraints 4.4.24 indicate there are no cancellations - i.e. all scheduled arrivals are served by the end of planning horizon T. 4.4.26 – 4.4.27 impose non-negativity constraints on the decision variables.

Constraints 4.4.25 impose coupling constraints on the rerouting decisions at various enroute fixes $m \in M$. It states that the total number of flights that have been dispatched from enroute fix $m \in M$ along a route $r \in \{1,..,R_m + 1\}$ by end of time period t, must be same under all scenarios that belongs to the active branch of the scenario tree during that time. Before each branching point in the scenario tree, there is a set of scenarios that remain possible, some of which are eliminated when the tree branches. Thus the coupling constraints impose the condition that rerouting decisions cannot be based on a particular set of scenarios unless, at the time the decision is made, one can tell that the scenario that will be realized belongs to that set.

4.2.3. Multi-Stage (Dynamic) Stochastic Model

In this model, ground delays of non-departed flights can be revised after updated information on evolving capacity at the airport and arrival fixes becomes available. While enroute, flights can be rerouted at any enroute fix along their path where alternative routes are available

Network Representation of the Airspace

The airspace is represented via network $G = (N, A)$ formed by a set of nodes N and a set of directed links A interconnecting the nodes. A node can be an airport or an enroute fix; and a link is a pre-specified flight trajectory between two nodes. As defined in previous models, let $P \subset N$ denote the set of arrival fixes of the destination airport.

Figure 4.4.3 illustrates the network representation of the airspace. Between each origin airport and the destination, there are a set of links that forms the primary (or shortest distance) route. Also, as shown in the figure, rerouting of flights can occur at some pre-specified set of enroute fixes using alternative links. Moreover, there can be multiple fixes along the primary path from an origin, where rerouting decisions can be made. The dotted bold line in the figure represents the boundary of a hypothetical airspace in the vicinity of an airport. The standard arrival fixes for the airport lie within this region. The capacity of some of the arrival fixes as well as the airport may be affected by adverse weather. Several scenarios, as discussed earlier, depict time-varying capacity profiles of NAS resources within the impacted region.

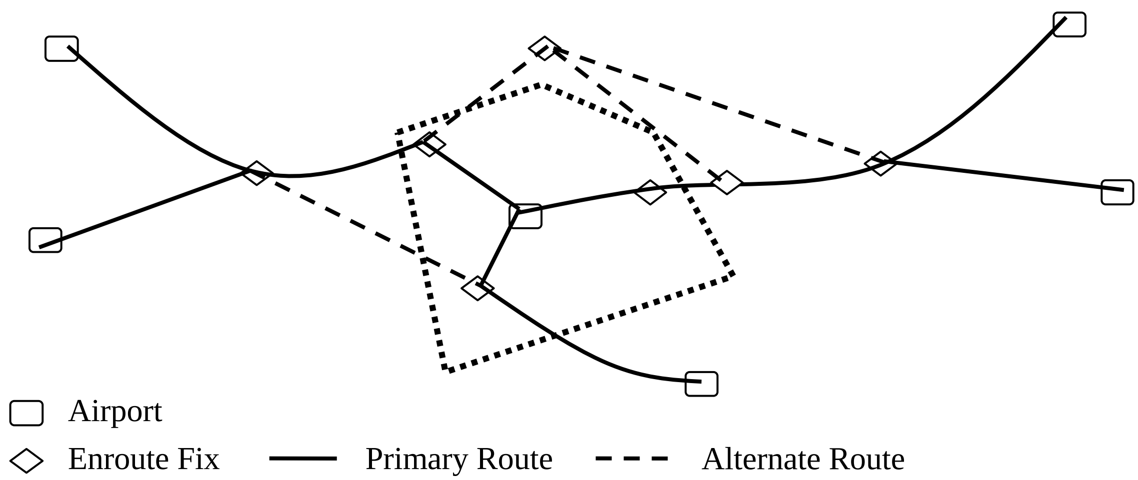

Figure 4.4.3 Network Representation of the Airspace

Model Parameters and Data

Let Φ denote the set of flights, each of which originates from an airport denoted by org_f, $f \in \Phi$. As in the previous models, the destination airport is denoted by a node k. The planning horizon is divided into time periods of equal intervals, and is denoted by a set $\Gamma = \{1,...,T\}$. Let dep_f and arr_f denote the scheduled departure and arrival time periods of a flight $f \in \Phi$. Let $\Lambda_f \subset A$ denote the set of links that forms the available routes for flight f. The minimum travel time of a flight on various links on its flight path is given by the parameters $\tau_{i,j}^{f}; (i,j) \in \Lambda_f; i,j \in N$. To reduce the problem size, we define a time window (a set of time periods) for each flight, within which it must have crossed a NAS resource, if the resource lies on the route chosen by the flight. The time window is denoted by $L_i^f; i,j \in N : (i,j) \in \Lambda_f$. Defining such time window also imposes an upper limit to the amount of airborne holding a flight can be subject to at various locations on its flight path. Finally the multi-stage dynamic model requires scenario-specific time-varying capacity profiles of various resources -- $C_j^{\xi}(t), j \in P \bigcup \{k\}$ -- and the cost ratio of airborne and ground delay – λ – defined for the previous models.

Decision Variables

The decision variables, which are binary, are defined as follows:

$$Y_{f,i,j}^{\xi}(t) = \begin{cases} 1 \text{ if flight } f \text{ has crossed node } i \text{ on its flight path} \\ \text{along link } (i,j) \quad \text{by the end of time period } t \text{ under scenario } \xi \\ \\ 0 \text{ otherwise} \end{cases}$$

$$f \in \Phi, (i,j) \in \Lambda_f, t \in L_i^f, \xi \in \Theta$$

At the origin airport ($i = org_f$), the decision variables represent the time period by when a flight has departed under different scenarios. Any difference from the scheduled departure time implies a ground delay. Ground delays can differ across scenarios. This means that the ground delay of a flight that has not yet departed may be revised based on updated information on evolving conditions at the destination airport and arrival fixes.

Dynamic rerouting occurs at fixes along the flight path that are divergence points for alternative routes to the destination airport.

The scenario-specific planned arrival time of individual flights at the destination airport (denoted by node k) can be determined from the above decision variables, and is represented by the auxiliary variable:

$$X_f^{\xi}(t) = \begin{cases} 1 \text{ if flight } f \text{ is planned to arrive at the airport k} \\ \quad \text{by end of time period } t \text{ under scenario } \xi \\ \\ 0 \text{ otherwise} \end{cases} \quad ; \quad f \in \Phi, t \in \{arrf,..,T\}, \xi \in \Theta$$

The variables $X_f^{\xi}(t)$ are related to the decision variables $Y_{f,i,j}^{\xi}(t)$ by the following expression:

$$X_f^{\xi}(t) = \sum_{p \in P:(p,k) \in \Lambda_f} Y_{f,p,k}^{\xi}(t - \tau_{p,k}^f) \qquad (4.4.28)$$

Note that the planned arrivals may be different from actual arrivals (or landings) at the airport due to capacity constraints. When arrival capacity is inadequate, certain flight will face airborne holding inside the airport TRACON. Let $W^{\xi}(t)$ denote the number of flights that are subject to airborne queuing delay in the airport TRACON area at time period $t \in \Gamma$, under scenario $\xi \in \Theta$. The variables $W^{\xi}(t)$ will be greater than zero if the airport acceptance rate at time period t is below the total arrival demand during that time period.

The scenario-specific ground delay of a flight can be derived from the decision variables as follows:

$$Ground\_Delay_f^{\xi} = \sum_{t \in L_{org_f}^f} \sum_{j:(org_f, j) \in \Lambda_f} \left(Y_{f,org_f,j}^{\xi}(t) - Y_{f,org_f,j}^{\xi}(t-1) \right) \times (t - dep_f)$$

(4.4.29)

The scenario-specific enroute delay of a flight before its planned arrival at the airport is given by the following expression:

$$Enroute\_Delay_f^{\xi} = \sum_{t=arr_f+1}^{T} \left(X_f^{\xi}(t) - X_f^{\xi}(t-1) \right) \times (t - arr_f) - Ground\_Delay_f^{\xi}$$

(4.4.30)

We can also compute the extra airborne time of a flight due to longer route taken using the following expression (although we do not use this value directly in formulating the model, we need it later to analyze the results) :

$$Extra\_flight\_time_f^{\xi} = \sum_{(i,j) \in \Lambda_f} \sum_{t \in L_i^f} \left(Y_{f,i,j}^{\xi}(t) - Y_{f,i,j}^{\xi}(t-1) \right) \times \tau_{i,j}^f - \left(arr_f - dep_f \right)$$

(4.4.31)

In expression 4.4.29, the term $\left(Y_{f,org_f,j}^{\xi}(t) - Y_{f,org_f,j}^{\xi}(t-1) \right)$ is equal to 1 for that time period when the flight f departs. Therefore, its product with the second term $\left(t - dep_f \right)$ computes the scenario-specific ground delay. In expression 4.4.30, the total scenario planned arrival delay of a flight is computed by the first term. Subtracting the ground delay from this expression gives the arrival delay due to airborne holding and/or extra flight time. In 4.4.31 the first term $\sum_{(i,j)\in\Lambda_f}\sum_{t\in L_i^f}\left(Y_{f,i,j}^{\xi}(t)-Y_{f,i,j}^{\xi}(t-1)\times\tau_{i,j}^{f}\right)$ computes the total minimum flight time based on various links chosen (accounting for dynamic rerouting), and the second term provides the scheduled flight time; thus the difference between the two yields extra airborne time due to taking a route other than the shortest one.

Objective Function

As in previous models, the objective function minimizes the expected total cost of delay, where airborne delay is weighted by the cost ratio λ. The following expression gives the objective function used in this model:

$$\min z=\sum_{\xi\in\Theta}P\{\xi\}\times\left\{\sum_{f\in\Phi}Ground\_Delay_f^{\xi}+\lambda\times\left(\sum_{f\in\Phi}Enroute\_Delay_f^{\xi}+\sum_{t=1}^{T}W^{\xi}(t)\right)\right\}$$

(4.4.32)

The last term in the above expression computes the airborne queuing delay at the destination airport. The other terms have been explained earlier.

Constraints

The set of constraints in the model are given as follows:

$$Y_{f,i,j}^{\xi}(t)-Y_{f,i,j}^{\xi}(t-1)\geq 0;\quad \forall f\in\Phi,(i,j)\in\Lambda_f,t\in L_i^f,\xi\in\Theta \tag{4.4.33}$$

$$Y_{f,i,j}^{\xi}(t)-\sum_{(i',i)\in\Lambda_f}Y_{f,i',i}^{\xi}(t-\tau_{i',i}^{f})\leq 0;\quad \forall f\in\Phi,(i,j)\in\Lambda_f,t\in L_i^f,\xi\in\Theta \tag{4.4.34}$$

$$\sum_{j:(i,j)\in\Lambda_f} Y^{\xi}_{f,i,j}(t) \le 1; \quad \forall f \in \Phi, \xi \in \Theta \tag{4.4.35}$$

$$W^{\xi}(t-1) - W^{\xi}(t) + \sum_{f\in\Phi}\left(X^{\xi}_{f}(t) - X^{\xi}_{f}(t-1)\right) \le C^{\xi}_{k}(t); \quad \forall t \in \Gamma, \xi \in \Theta \tag{4.4.36}$$

$$\sum_{f\in\Phi:(p,k)\in\Lambda_f}\left(Y^{\xi}_{f,p,k}(t) - Y^{\xi}_{f,p,k}(t-1)\right) \le C^{\xi}_{p}(t); \quad \forall t \in \Gamma, p \in P, \xi \in \Theta \tag{4.4.37}$$

$$W^{\xi}(0) = 0; \quad \forall \xi \in \Theta \tag{4.4.38}$$

$$W^{\xi}(T) = 0; \quad \forall \xi \in \Theta \tag{4.4.39}$$

$$X^{\xi}_{f}(T) = 1; \quad \forall f \in \Phi, \xi \in \Theta \tag{4.4.40}$$

$$\begin{gathered} Y^{\psi^b_1}_{f,i,j}(t) = \ldots = Y^{\psi^b_q}_{f,i,j}(t) = \ldots = Y^{\psi^b_{\eta_b}}_{f,i,j}(t); \\ \forall f \in \Phi, (i,j) \in \Lambda_f, t \in L^f_i, \psi^b_q \in \Omega_b : \eta_b \ge 2 \text{ and } o_b \le t \le e_b \end{gathered} \tag{4.4.41}$$

$$Y^{\xi}_{f,i,j}(t) \in \{0,1\}; \quad \forall f \in \Phi, (i,j) \in \Lambda_f, \xi \in \Theta, t \in L^f_i \tag{4.4.42}$$

$$W^{\xi}(t) \ge 0; \quad \forall \xi \in \Theta, t \in \Gamma \tag{4.4.43}$$

Constraints 4.4.33 reflect the non-decreasing property of decision variables $Y^{\xi}_{f,i,j}(t)$. Thus if a flight f is has crossed a node i and entered the link $(i,j) \in \Lambda_f$ by end of time period t_0, under scenario ξ, then the variables $Y^{\xi}_{f,i,j}(t)$ remain 1 for all time periods $t \ge t_0$. Constraints 4.4.34 reflect the requirement that a flight can enter a link $(i,j) \in \Lambda_f$ by time period t under any scenario $\xi \in \Theta$, only if it has flown a minimum distance corresponding to an incoming link to node i - $\tau^f_{i',i};(i',i) \in \Lambda_f$. Constraints 4.4.35 state the condition that only one among several available links can be chosen at a node i, for rerouting. Constraints 4.4.34 and 4.4.35 together ensure that each flight selects only one route (among several available) under a specific scenario. The second term in constraints 4.4.35 $\sum_{(i',i)\in\Lambda_f} Y^{\xi}_{f,i',i}(t-\tau^{f}_{i',i})$ is 1 only after a flight f has entered the link (i',i), which belongs to the route chosen by the flight, and flown a minimum flight time of $\tau^{f}_{i',i}$. Thus constraint 4.4.35 ensures that each flight travels the minimum distance on the selected route.

Constraints 4.4.36 impose an upper bound, which is the airport scenario-specific airport capacity, to the number of flights that can land at the destination airport during any time period $t \in \Gamma$, under each scenario $\xi \in \Theta$. Variables $W^{\xi}(t)$ reflect the number of flights that are planned to arrive during or before the time period t, that are subject to airborne queuing, due to deficit in the airport capacity. Constraints 4.4.37 impose the scenario-specific capacity of various arrival fixes to be the upper bound to the corresponding throughput at each of those fixes.

Constraints 4.4.38 state that the system is empty, i.e. there are no flights in airborne queuing delay at the airport, in the beginning of the planning horizon. Constraints 4.4.39 – 4.4.40 ensure all flights land by the end of planning horizon.

Constraints 4.4.41 impose the condition that the decision variables $Y_{f,i,j}^{\xi}(t)$ are the same for all scenarios ξ that are represented by the evolving branch of the scenario tree at time t - i.e. $\xi \in \Omega_b : o_b \le t \le e_b$. Note that if the number of scenarios represented by the evolving branch is one, then the coupling constraints are no longer imposed. In other words, scenario-specific decisions can be based on a specific capacity scenario only after it has been realized. As long as the active branch of the scenario tree represents more than one scenario, the decisions must be same under all those scenarios.

4.5. EXPERIMENTAL RESULTS

4.5.1. Problem Setup

In this section we present and compare the results obtained by applying the stochastic models discussed in Section 4.4, to a real world problem. Flights arriving at Dallas Fort Worth Intl. Airport (DFW) usually fly over one of the four standard arrival fixes – BYP, CQY, JEN, and UKW. Under fair weather conditions, each of the arrival fixes serve air traffic originating from a set of airports, whose primary route to DFW passes over it. In our study, we consider a hypothetical convective weather event that affects the two eastern arrival fixes – BYP and CQY – along with the acceptance rates at DFW itself. The location of the convective weather is shown by a shaded region in Figure 4.5.1. The impact of the weather activity on capacity of arrival fixes and the airport will be discussed later.

Dynamic rerouting is allowed (in the Two-Stage Stochastic Model, and the Dynamic Model) via alternative routes, as shown in Figure 4.5.1. The decision on rerouting is taken at certain enroute fixes that serve the inbound air traffic from various airports to the arrival fixes. Flights arrive at those fixes through the primary route that connects their origin airport with DFW. Three letter airport codes of some major airports from which traffic is at the fixes originates are also included in the figure. For example, air traffic from the north eastern cities such as Boston, New York, Washington D.C., and Philadelphia, normally flies into DFW via arrival fix BYP. The primary route between these airports and DFW passes over an enroute fix LIT, where rerouting decisions can be made, with some flights can be diverted to the alternative route into DFW over north-west arrival fix UKW, via IRW.

We consider 351 flights that are scheduled to arrive at DFW by 12:15 PM on July 14, 2003. The planning horizon of 12 hours and 15 minutes is divided into 49 quarter-hour time periods, starting from midnight – 12:15 AM, and ending at noon – 12:15 PM. Thus $\Gamma = \{1,..,49\}$ is the set of time periods.

The arrival demand during each time period at arrival and other enroute fixes is derived from flight schedules and minimum flight times on primary routes. The minimum flight time is based on the Great Circle Distance between two points in airspace, and an assumed average speed of an aircraft. The flight time is expressed in 15-minute units and rounded to the nearest integer. The set of arrival fixes and fixes where rerouting can occur are $P = \{BYP, CQY, JEN, UKW\}$ and $M = \{LIT, RZC, EIC, AEX\}$ respectively.

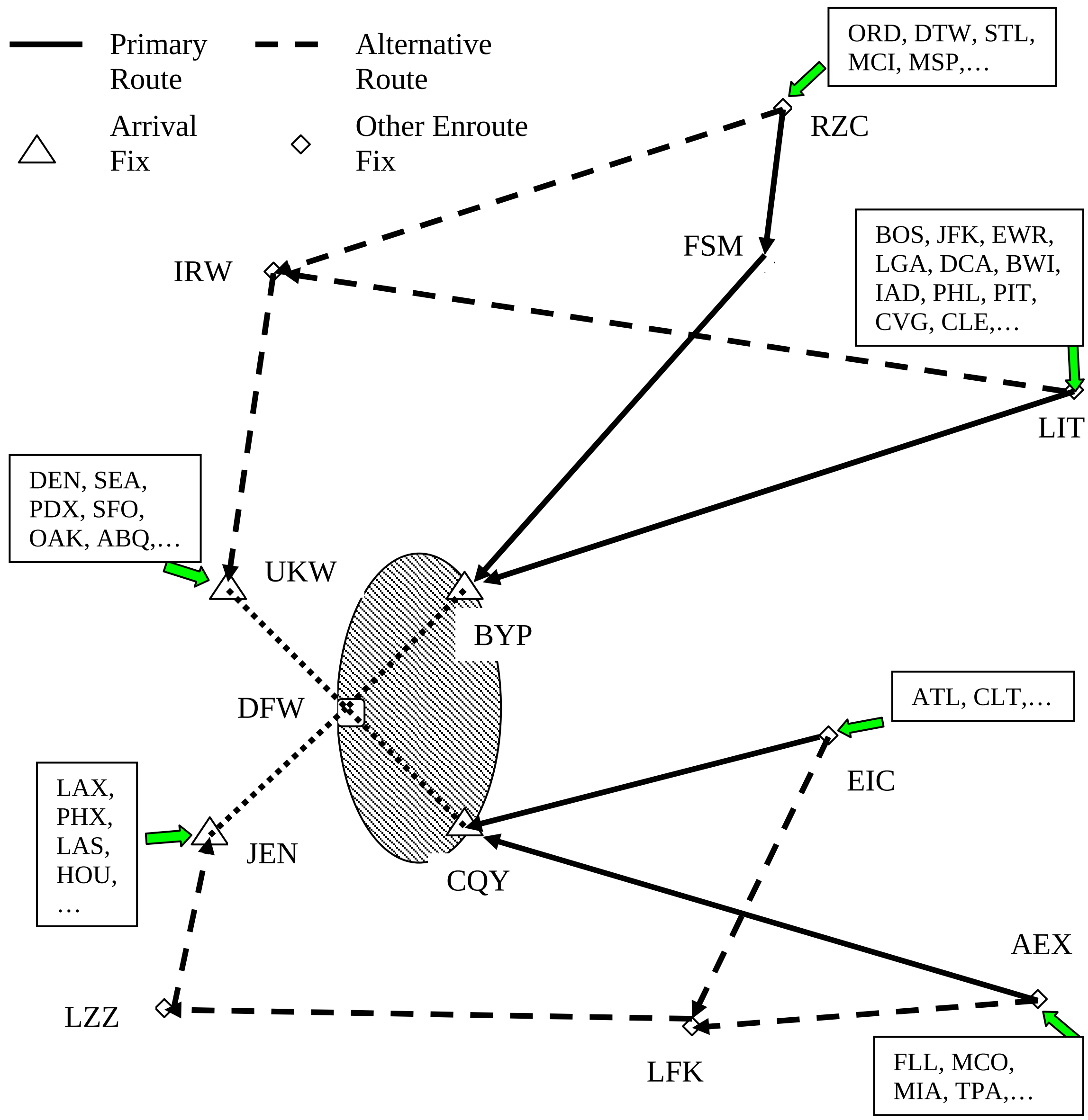

Figure 4.5.1 Standard Arrival Routes and Rerouting Options at DFW

The time-varying cumulative scheduled demands (parameters $SD_p(t)$ in the Static Model and $SD_m(t)$ in the Two-Stage Stochastic Model) can be derived from the scheduled arrival demand of individual flights at respective fixes.

In this experiment, we assume that the presence of convective weather essentially blocks the affected arrival fixes – i.e. the capacity goes to zero. The maximum throughput of the arrival fixes in fair weather conditions is assumed to be 15 aircraft/qhr (based on 5 miles-in-trail horizontal separation between aircraft and an assumed speed of 300 nautical miles per hour). For DFW itself, we assume that the 15-minute arrival capacity reduces to 15 arrivals/qhr under convective weather, and in fair weather conditions it goes up to 35 arrivals/qhr.

Capacity scenarios and the scenario tree are derived as follows. Convective weather activity at DFW is assumed to be present from the beginning of the day (beginning of planning horizon) until the end of a time period t_{dfw}, after which the weather clears off. Similar weather activity occurs at the arrival fixes – BYP and CQY – where the clearance time period is t_{fix}. Both t_{dfw} and t_{fix} can take any one of five possible clearance times – 8:30AM, 9:00AM, 9:30AM, 10:00AM, and 10:30AM. Note that t_{dfw} and t_{fix} can be different. The scenario tree representing capacity changes at DFW is shown in Figure 4.5.2. The set of scenarios $s_i^{dfw}, i = 1,..,5$, represent the time-varying airport arrival capacity, under different weather clearance times. A similar scenario tree can be constructed for the capacity changes at the two arrival fixes; and the set of scenarios is given by $s_i^{fix}, i = 1,..,5$.

There are a maximum of 25 total number of scenarios denoted by the set $\Theta = \{\xi_{11}, \xi_{12}, ..\xi_{ij}, ..\xi_{55}\}$, each of which represents the joint evolution of capacity changes at DFW and the two arrival fixes (BYP and CQY) corresponding to the weather clearance times. Scenario ξ_{ij} denotes the capacity scenario in which t_{dfw} takes the i^{th} clearance time among the five possible (ordered in increasing time of day), and t_{fix} takes the j^{th}; i.e. $\xi_{ij} = (s_i^{dfw}, s_j^{fix}), i, j \in \{1,..,5\}$. Table 4.5.1 relates each scenario with the weather clearance times. For example, scenario ξ_{14} represents the case when weather at DFW clears at 8:30 AM and the arrival fixes clear at 10:00 AM; similarly, scenario ξ_{22} corresponds to the clearance time of 9:00AM at both airport and the arrival fixes.

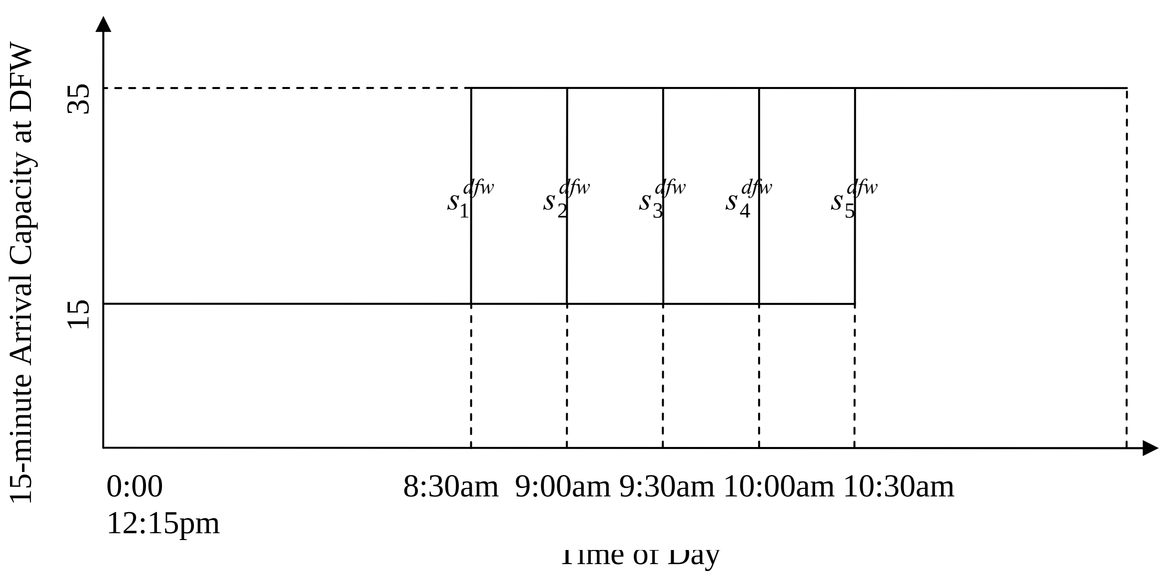

Figure 4.5.2 Capacity Scenarios of Airport Acceptance Rates at DFW

Table 4.5.1. Scenarios Corresponding to Different Combination of Weather Clearance Times					
Weather Clearance Time at DFW	Weather Clearance Time at the Arrival Fixes (BYP and CQY)				
	8:30 AM	9:00 AM	9:30 AM	10:00 AM	10:30 AM
8:30 AM	ξ_{11}	ξ_{12}	ξ_{13}	ξ_{14}	ξ_{15}
9:00 AM	ξ_{21}	ξ_{22}	ξ_{23}	ξ_{24}	ξ_{25}
9:30 AM	ξ_{31}	ξ_{32}	ξ_{33}	ξ_{34}	ξ_{35}
10:00 AM	ξ_{41}	ξ_{42}	ξ_{43}	ξ_{44}	ξ_{45}
10:30 AM	ξ_{51}	ξ_{52}	ξ_{53}	ξ_{54}	ξ_{55}

We apply the stochastic optimization models proposed in Section 4.4, to the problem described above. To analyze the sensitivity of the results, we consider a set of alternative cases in which various model inputs are varied, and compare the results with the baseline case. We define the baseline case and the set of alternative cases as follows.

Baseline Definition (Case 1):

We assume that all capacity scenarios are equally likely, i.e. $P\{\xi_{ij}\} = 0.04,$ where $\xi_{ij} \in \Theta$. The cost ratio λ is set to 3, which is fairly low. Therefore in this case, we expect to see airborne holding and extra airborne time due to longer flight paths.

Case 2: High cost ratio.

In this case we set the cost ratio λ to 25. The unconditional probabilities of capacity scenarios remain unchanged. Intuitively, we expect to see higher ground delays than in the baseline case, and little or no expected airborne delays..

Case 3: High cost ratio along with high probability of early weather clearance.

The cost ratio λ is set to 25, and the probability mass function of capacity scenarios is give as follows: $P\{\xi_{11}\} = 0.4, P\{\xi_{ij}\} = 0.025$ for $\xi_{ij} \in \Theta \setminus \xi_{11}$. We expect to see higher benefits from dynamic revision of ground delays in this case.

Case 4: Value of early information.

In this case our goal is to analyze the delay cost reduction if the information on capacity changes are obtained 30 minutes earlier than the actual changes in weather conditions. Therefore in effect, branches of the scenario tree unveil two time periods earlier than that in baseline scenario tree. All other parameters and input data are kept the same as in the baseline case.

Case 5: Convective weather affecting DFW only.

In this case we assume that the convective weather impacts arrival capacity of the airport (DFW) only, and not the arrival fixes. The weather clearance time at DFW -- t_{dfw} -- can take five possible values as in the baseline case. Assuming all scenarios to be equally likely, the probability of occurrence of each scenario becomes 0.2.

4.5.2. Results

The computing time of the aggregate models (Static and Two-Stage) in all experimental cases was approximately 5 seconds; using CPLEX version 7.1 run on a personal computer with 2.2 GHz processor speed. In the Dynamic model, the computing time was higher – about 10-12 minutes – due to the larger number of variables and constraints. In all instances, integer solutions were obtained from LP relaxation of all the three models.

Further research on analyzing the computational complexity of the models would provide insight to the structure of the integer programming formulations.

Table 4.5.2 and Figure 4.5.3 summarize the results from the three models described, along with a benchmark model that assumes perfect information about all capacity profiles is available at the beginning of the planning period. For each model and case, Table 4.5.2 presents expected costs, measured in units of aircraft hours of ground delay, airborne holding, extra flight time from taking longer routes, and the total expected cost. Figure 4.5.3 depicts expected total cost results graphically, as a percentage of the value that could be achieved with perfect information. In Table 4.5.2, the extra flight time component is excluded from that Static Model, since this model does not allow rerouting. Similarly, the Perfect Information Model does not include airborne holding since with perfect information all holding will be transferred to the ground. The Perfect Information Model does allow extra flight time, since in some cases a large amount of ground delay can be saved by choosing a longer route.

As expected, the Perfect Information Model outperforms all the models that rely on imperfect information. The relative performance of the other models is also as expected: the Dynamic Model yields the best results in each case because it allows ground delays to be revised based on updated information along with rerouting decisions, while the Two-Stage Model, which allows rerouting, outperforms the Static Model, which doesn't.

Table 4.5.2. Expected Delay Costs (Aircraft-Hours)*															
Case	Static Model			Two-Stage Stochastic Model				Dynamic Model				Perfect Information		
	Ground Delay	Air-borne Delay	Total	Ground Delay	Airborne Holding	Extra Flight Time	Total	Ground Delay	Air-borne Holding	Extra Flight Time	Total	Ground Delay	Extra Flight Time	Total
1	158.3	51.5	209.8	38.5	6.4	67.5	112.4	45.7	6.8	57	109.5	35.2	41.7	76.9
2	231.8	0	231.8	231.8	0	0	231.8	210	0	0	210	130.3	0	130.3
3	231.8	0	231.8	231.8	0	0	231.8	189.5	0	0	189.5	88.7	0	88.7
4	158.3	51.5	209.8	39.8	3.4	62	105.2	44.9	3.5	50.5	98.9	35.2	41.7	76.9
5	29.3	5.6	34.9	29.3	5.6	0	34.9	30.4	3	0	33.4	18.5	0	18.5

*Costs measured is units of ground delay, with airborne delay and extra flight time multiplied by the delay cost ratio λ.

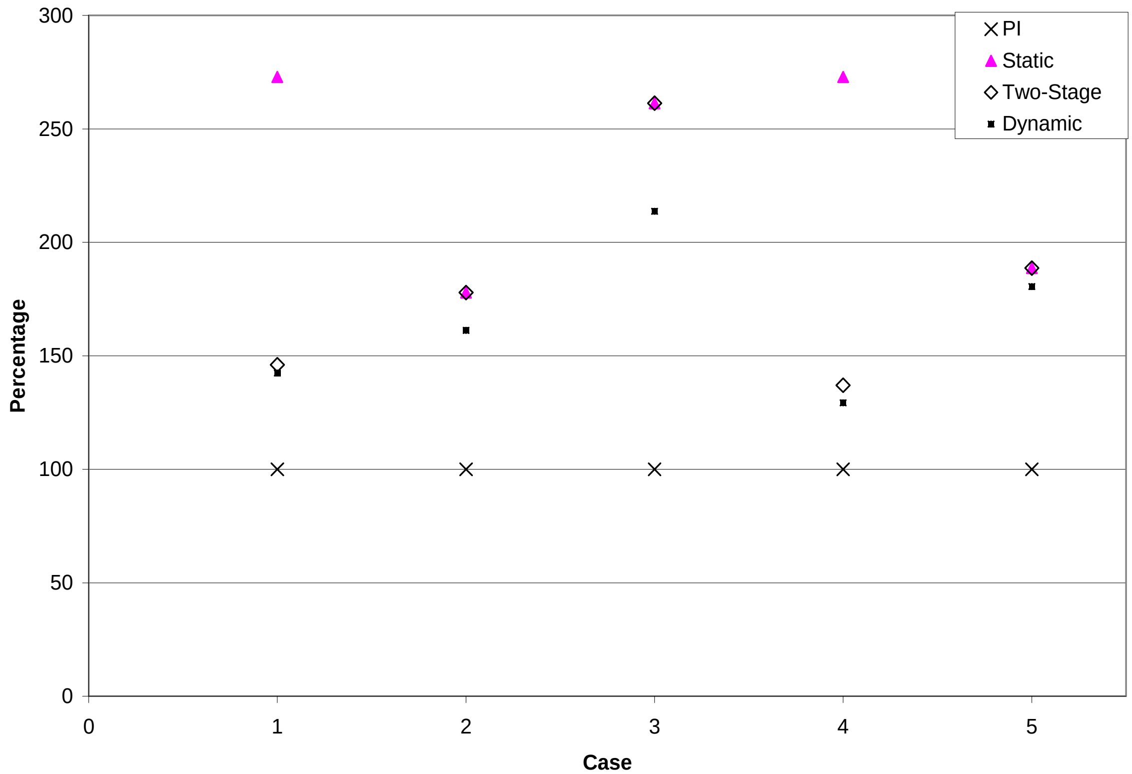

Figure 4.5.3 Expected Total Delay Cost from Stochastic Models Compared to Those under Perfect Information

Examining the Baseline Case in Table 4.5.2, we see that the Static Model results in much more ground delay and airborne delay than the Two-stage and Dynamic Models. Without the ability to either re-route aircraft or modify ground delay decisions, and faced with a wide range of equally likely capacity scenarios, all that the Static Model can do is add ground delay until the cost of doing so exceeds that of the expected airborne delay from releasing flights when capacity at the destination may be insufficient. The differences between the Two-Stage Model and the Dynamic Model are smaller but nonetheless revealing. The latter model imposes somewhat more ground delay so that better information is available when decisions are made; this improved information results in reduced airborne delay. This represents flights that are released and then rerouted under the Two-stage Model, and ground held until the primary route becomes available under the Dynamic Model.

Tables 4.5.3a and 4.5.3b provide further insight about how the models differ in the baseline case. Table 4.5.3a provides scenario-specific ground delay ratios, expressed in percentage, between the Dynamic and Two-Stage models (D/2S) and Dynamic and Perfect Information (D/PI) models. Table 4.5.3b provides similar ratios for airborne delays. Table 4.5.3a shows that under worse conditions – scenarios corresponding to late weather clearance times – the scenario-specific ground delay ratio between Dynamic and Two-Stage model exceeds 120%. But this is more than offset from 10-30% reductions in airborne delays that the dynamic model attains through adjusting ground delay times in response to updated scenario information. For any given weather clearance time at DFW, the greatest difference in airborne delay between the dynamic and two-stage models occurs for arrival fix clearance times in the 9:00-10:00 AM range—neither the worst en route scenario nor the best. When the arrival fixes clear earlier than this, rerouting is rarely necessary, while when they clear later it is rarely avoidable.

Table 4.5.3a. Scenario[1] Specific Ground Delay Ratio, Expressed in Percentage, between Dynamic Model (D) and 2-Stage Model (2S), and between Dynamic Model (D) and Perfect Information (PI), in Baseline Case

Weather Clearance Time at DFW	Weather Clearance Time at the Arrival Fixes (BYP and CQY)									
	8:30 AM		9:00 AM		9:30 AM		10:00 AM		10:30 AM	
	D/2S	D/PI	D/2S	D/PI	D/2S	D/PI	D/2S	D/PI	D/2S	D/PI
8:30 AM	100	700	109	178	114	135	118	167	119	526
9:00 AM	103	265	113	197	121	150	124	211	125	346
9:30 AM	111	128	120	150	120	128	123	125	124	185
10:00 AM	117	95	122	107	121	118	123	94	124	101
10:30 AM	117	72	122	80	121	85	123	89	124	91

Table 4.5.3b. Scenario[1] Specific Airborne Delay Ratio, Expressed in Percentage, between Dynamic Model (D) and 2-Stage Model (2S), and between Dynamic Model (D) and Perfect Information (PI), in Baseline Case

Weather Clearance Time at DFW	Weather Clearance Time at the Arrival Fixes (BYP and CQY)									
	8:30 AM		9:00 AM		9:30 AM		10:00 AM		10:30 AM	
	D/2S	D/PI	D/2S	D/PI	D/2S	D/PI	D/2S	D/PI	D/2S	D/PI
8:30 AM	94	336	89	288	87	160	83	115	98	98
9:00 AM	94	308	89	265	86	151	83	108	91	97
9:30 AM	94	400	89	246	86	168	83	125	93	104
10:00 AM	84	501	86	243	86	157	83	155	93	121
10:30 AM	75	607	69	260	76	185	81	134	91	129

[1] See Table 5.1.

When the destination airport (DFW) is the main bottleneck, such as under scenario ξ_{51}, the perfect information solution absorbs virtually all delay through ground holding flights, since rerouting confers little benefit. The ground delay assigned by the Dynamic model under this scenario is 17% more than the Two-Stage model, but still 28% less than that necessary under perfect information (see Table 4.5.3a). The airborne delay is considerably less -- 25% -- in the Dynamic model compared to the Two-Stage model, but the increase in airborne delay due to imperfect information is still high – 507% (see Table 4.5.3b). The difference is reduced as en route congestion becomes increasingly severe, resulting in a perfect information solution with more rerouting. For example under the scenario ξ_{55}, the Dynamic model faces 29% excess airborne delay compared to perfect information.

Another revealing scenario in the baseline case is ξ_{15}, in which conditions at the airport improve early while adverse weather at the arrival fixed persists. Perfect information in this scenario necessitates rerouting many flights via alternative routes, in order to avoid excessive ground delays. Therefore the total ground delay with perfect information on scenario ξ_{15} has a lower value compared other scenarios, while there are higher airborne delays due to extra flight time on longer routes. The "wait-and-see" policy for assigning ground delays in the Dynamic model backfires if scenario ξ_{15} occurs. In this case, the excessive ground delays assigned by the Dynamic model in anticipation of scenarios ξ_{12} and ξ_{13} cannot be recovered if ξ_{15} occurs. Thus we see a 426% increase in total ground delay from using the Dynamic model compared to what would happen if we knew that at the beginning of the day that scenario ξ_{15} would be realized. Indeed, the Two-stage model, which cannot use the "wait-and-see" approach to assign ground delays, yields a better outcome under ξ_{15}, featuring a considerably lower ground delay and virtually the same airborne delay.

Figure 4.5.3 reveals that, when compared to delays under perfect information, the Dynamic model yields 42% higher total expected delay cost, while the corresponding values from the Two-Stage and Static models are 46 and 173% respectively. The substantially higher total delay cost in the Static model as compared to the others results from excessive ground delays due to unavailability of alternative routes to reroute flights.

Table 4.5.2 shows that in Case 2, no airborne delay is faced by any model due to high cost ratio ($\lambda = 25$). Both the Static and Two-Stage models assign ground delays corresponding to the worst scenario, which is ξ_{55}. The flexibility to reroute flights in the Two-Stage or Dynamic model is not utilized in order to prevent any airborne delay. The Dynamic model releases flights with lower ground delays if favorable conditions (scenarios corresponding to early clearance time of convective weather) are realized. Therefore, the total expected ground delay assigned by the Dynamic model is about 10% less than the Two-Stage and Static model ground delay.

The effect of dynamic revision of ground delays in Dynamic model is greater when the probabilities of favorable conditions are higher, as in Case 3. In this case, the Static and the Two- Stage models assign the same total ground delays (corresponding to worst scenario ξ_{55}) as in Case 2, whereas the Dynamic model produces about 18% less expected ground delay. The loss due to imperfect information is 50% less in the Dynamic model as compared to the Two-Stage and Static models.

The effect of earlier information is revealed from the results obtained in Case 4. The Static model produces the same delays as in the Baseline case. This is because updated information on weather conditions is not utilized by the model. Expected airborne delays are reduced by 12% and 16% in the Two-Stage and Dynamic model respectively. Percentage loss due to imperfect information is reduceed by approximately 20% in the dynamic models.

In Case 5, when there is no congestion in the enroute arrival fixes, rerouting is no longer necessary to avoid enroute weather. Therefore, there is no extra flight time in the Two-Stage and Dynamic models. The Two-Stage model faces the same expected delays as the Static model in this case, because there is no benefit from rerouting, and ground delay decisions are static in both cases. The Dynamic model assigns slightly higher ground delays (3% more) than the Two-Stage and Static models, but yields 46% less expected airborne holding. The “wait-and-see” policy of the Dynamic model results in higher ground delay but less airborne holding.

4.6. CONCLUDING REMARKS

In this chapter, we present three stochastic integer programming models for managing inbound air traffic flow of an airport. These are the first such models to consider uncertainty of future capacities of multiple NAS resources. We assume that convective weather, which is present in the vicinity of the airport, affects airport acceptance rates (or arrival capacity) and maximum throughput of some of the standard arrival fixes of the airport. In the first two models, in which ground delay decisions are static, the decision variables are the number of flights planned to arrive either at an arrival fix (in the Static model), or at an enroute fix where rerouting decisions can be made (in the Two-Stage model). The third model, which is dynamic, and allows decisions on rerouting and ground delay of flights to be revised based on updated information.

A set of capacity scenarios, each representing the joint evolution of time-varying capacities of multiple NAS resources (arrival fixes, and airport), are considered. A scenario tree, in which branching occurs when new information eliminates certain scenarios, provides the information on evolution of the system with time. The probability of occurrence of each scenario is assumed to be known at the beginning of planning horizon, and is provided as input to the optimization models.

The value of rerouting is evident in the improved performance of the Two-Stage and Dynamic models over the Static one. As expected, this value is highest when the airborne delay cost is not so high as to foreclose trades between airborne delay and ground delay. An interesting question in this regard is how to assess the relative unit costs of airborne holding time and extra flight time from taking longer routes. While these were assumed equal in our experiments, there is reason to believe that the latter is actually less expensive, which would further enhance the value of rerouting.

The value of dynamically adjusting ground delay decisions is evident from comparing the Dynamic and Two-Stage model results. On the whole, dynamic adjustment generates lower gains than rerouting in our experiments; its value is greatest when the probability of adverse conditions is large. None of the models with imperfect information approaches yield results that come close to what can be obtained if the capacity scenario is known with certainty at the beginning of the planning period. While there is benefit from planning cleverly with poor information, there remains great value from making the information better.

The Dynamic model is more complex and computationally burdensome than the static and Two-Stage ones. It may also be more challenging to implement in practice, since it must be incorporated into the CDM process which is designed to work with aggregate rates rather than the routing and scheduling of individual flights. One the other hand, the Dynamic model generates substantial cost reductions in certain circumstances. Moreover it can handle a much wider range of objective functions taking into account equity issues and non-linearity in the delay cost function. Even though the runtime of the Dynamic model is considerably longer, it is still on the order of seconds for the realistic-scale problems run in our experiments. One reason for this is that the LP relaxations of the IP models yielded integer solutions in all cases. There is a need for further investigation of the general conditions under which this will occur.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this dissertation, we have developed dynamic stochastic linear optimization models for decision making in ATFM. The models account for uncertainty in the future capacities of NAS resources, and are adaptive to updated information on capacity evolution that becomes available as the time of day progresses.

The DRGH model presented in Chapter 2 optimally solves the SAGHP when the evolution of arrival capacities of an airport is uncertain. Uncertainty is represented by a set of capacity scenarios, each depicting a particular time-varying profile of arrival capacity of the airport. The probability of occurrence of each scenario is assumed to be known at the beginning of the planning horizon, and is provided as input to the optimization model. A scenario tree is also specified as input. The scenario tree includes branching points when new information eliminates the possibility of certain scenarios. The model is designed to optimally exploit this new information.

The DRGH model overcomes some of the major limitations of the existing models in the literature for SAHGP. Ground delays of non-departed flights are revised, based on updated information from scenario tree branching. By virtue of the decision variables, a wide range of objective functions, which may be non-linear in ground delays, can be incorporated in the model. In contrast to the existing practice of exempting flights solely based on the distance between their origin airports and the destination, the model generates *de facto* exemptions depending upon the probabilistic information on airport capacity as well as the objective function. Thus the model incorporates the efficiency objectives that motivate the FAA to exempt long-haul flights from GDPs, but can also mitigate the inequities associated with this strategy.

We presented a methodology and two optimization models – the dynamic substitution and compression models – that can be used to implement the DRGH model for GDP planning in practice under the CDM paradigm. We proposed a rule for allowing airlines to make substitutions among their flights. The substitution model can be used by airlines to minimize the expected delay cost of their flights by re-assigning slots. The dynamic compression model allows the FAA to fill the vacant slots that are created after a round of substitutions and cancellations by the airlines.

Finally, we presented three stochastic integer programming models – the Static, Two-Stage, and Dynamic – for managing inbound air traffic flow of an airport, when there is adverse weather impacting both the arrival capacity of the airport and its arrival fixes. These are the first ATFM optimization models to consider uncertainty of future capacities of multiple NAS resources. As in the DRGH model, the uncertainty in resource capacities is represented by a set of capacity scenarios, and a scenario tree that represents evolving information about which scenarios will be realized.

In the two-stage and dynamic models, dynamic rerouting of flights is allowed at certain enroute fixes, in order to avoid weather affected airspace. The benefit from dynamic rerouting is most profound under adverse conditions of enroute airspace and low ratio between airborne and ground delay. The dynamic model also allows revision of ground delays of flights, based on updated information from scenario tree branching, and can also accommodate cost functions that are non-linear measures of ground delay without changing the linearity of the math-programming model.

By re-assessing ATFM decisions, the dynamic models can adapt to early information on evolving weather, which may be available from forecasts. The reduced value of the objective function due to adaptive decisions reveals the value of early information. This makes it possible to derive the value of more timely weather forecasts. Also the predictability of individual flight release times can increase by advanced forecasts.

The dynamic stochastic models proposed in this dissertation require as inputs a pre-specified set of capacity scenarios, scenario probabilities, and a scenario tree. A major challenge to implementing these models is to construct tractable, realistic decision trees from real-world data. Scenario trees for multiple NAS resources can be constructed from those of individual resources, by considering their spatial and temporal correlations.

Further research is required to address problems in which the scenario probabilities themselves are uncertain. The scenario tree branching times may also be uncertain. Such problem can however be solved using a similar approach as in the models presented in this dissertation, by constructing scenarios and scenario tree from a distribution of probabilities of occurrence and scenario branching times. However, if the weather forecasts provide completely new information, such as a new set of capacity scenarios and probabilities that were not considered at the beginning, the problem may not be solvable by purely dynamic programming techniques. Such problem will require approximations and heuristic methods to achieve solutions.

The expected airborne delays can be controlled by varying the cost ratio between airborne and ground holding in the objective functions of the stochastic models for optimizing ATFM. However, if the ATC facilities put upper-limits on the numbers of flights that can be handled during airborne holding in the enroute sectors or at airport TRACONs, additional constraints in the models are needed to impose such restrictions. Such constraints must account for the variance in the number of aircraft subject to airborne holding at various regions in the airspace.

In the dynamic substitution and compression models presented in Chapter 3, airlines are provided with the flexibility to perform cancellations at their own discretion. However, an important research issue is to include a set of criteria for canceling flights during the substitutions, in the substitution model. Such criteria will be based on the marginal delays caused by various flights to the system. Furthermore, allowing airlines to perform scenario-contingent cancellations, and to adapt such decisions in the substitutions and compression, is an important extension that requires further research.

Scalability of the models that address enroute capacities is a further issue. Computational complexity may become a major burden in extending them to deal with a region that encompasses multiple airports and enroute fixes. Sector capacity constraints, along with airport and enroute fixes, must be added. In such cases, aggregate models may have much lower computing times than the disaggregate ones. Analysis of computational complexity of the stochastic optimization models will determine if heuristics are required to handle these problems. Furthermore, applying the solutions obtained from the models in a CDM environment needs further analysis, such as that we presented for the SAGHP solution in this dissertation.

Finally, developing resource allocation methods for constrained enroute airspace is an important research issue. Unlike in GDPs, where RBS and Compression algorithms provide accepted resource allocation mechanisms, there is a lack of resource allocation models for enroute airspace. This is a very complex problem because of the presence of multiple resources and multiple airport pairs.

Applying optimization methods in ATFM is the key to increase NAS efficiency. Exorbitant amount of delays and risks are inevitable if within next few years, due to predicted increase in the air traffic, if decision making in ATFM is not optimized. The models developed in this dissertation address uncertainty in resource capacities, which is absolutely necessary to implement such decision-making tools in practice. Equally important, these tools are suited to the real-world environment in which ATFM decisions are made – one featuring unfolding events and accumulating information where managers must “plan-to-replan”. In the future years, along with research on optimizing ATFM, efforts must be put to implement those tools in real world decision making. By adopting optimization to this environment, we have made the prospects of this to occur.

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APPENDIX 1

We demonstrate that the DRGH model always produces solutions with expected delay costs less than or equal to those of Ball et al. (2003) and Richetta-Odoni (1994). To do this, we demonstrate that the latter models are equivalent to our model except that they impose additional constraints, the removal of which can only improve the value of the objective function at optimum. When constraints (2.4.7) are modified as in (7a), given below, the model produces same expected delays as that obtained by applying the Ball et al. static model. As each of the sets Ω_i (in constraint set 2.4.7) are subset of Θ (set of all possible scenarios), the modified set of constraints (7a) implies that a larger number of constraints are imposed to the model. Furthermore, it means that the ground delay decision of individual flights do not depend on scenarios realized as the time of day progresses. In other words, constraint (7a) implies no utilization of information on evolving conditions, i.e. the decision making process is no longer dynamic. Thus, the results (expected delays and planned arrival numbers) match that from the static stochastic model.

$$Y_{f,t}^{q_1} = Y_{f,t}^{q_2} = ... = Y_{f,t}^{q_i} = ...Y_{f,t}^{q_{|\Theta|}} ; \ \forall f \in \Phi, t \in \Gamma, q_i \in \Theta \quad (7a)$$

When the constraint set (2.4.7) in the DRGH model is replaced by constraints (7b), given below, the expected delays and planned arrival numbers obtained matches that from Richetta-Odoni Model. Constraints (7b) implies that the scenario-specific ground delays of a flight has to be same under all scenarios that are associated with the branch of the scenario tree that occurs during the scheduled departure time of the flight. In other words, the ground delay of a flight cannot be revised if a specific scenario is realized later. Therefore constraints (7b) is more restrictive than (7), because they prohibit ground delays from being revised once they are assigned.

$$Y_{f,t}^{S_1^i} = ... = Y_{f,t}^{S_k^i} = ... = Y_{f,t}^{S^i_{N_i}}; \qquad f \in \Phi, S_k^i \in \Omega_i : \mathrm{N}_i \geq 2 \;\; and \;\; o_i \leq dep_f \leq \mu_i$$

(7b)

APPENDIX 2: LIST OF ABBREVIATIONS

ARTCC	Air Route Traffic Control Center
ASPM	Aviation Systems Performance Metrics
ATC	Air Traffic Control
ATCSCC	Air Traffic Control System Command Center
ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
CDM	Collaborative Decision Making
CDR	Coded Departure Routes
CR	Collaborative Routing
CRCT	Collaborative Routing Coordination Tool
CCFP	Collaborative Convective Forecast Product
DRGH	Dynamic Revisable Ground Holding
ETMS	Enhanced Traffic Management System
FAA	Federal Aviation Administration
FSM	Flight Schedule Monitor
GDP	Ground Delay Program
GHP	Ground Holding Problem
IP	Integer Programming
LAADR	Low Altitude Arrival and Departure Routes
LP	Linear Programming
MAGHP	Multi-Airport Ground Holding Problem
MDP	Markov Decision Process

MIT	Miles-in-Trail
NAS	National Airspace System
OAG	Official Airlines Guide
RBS	Ration-by-Schedule
SAGHP	Single Airport Ground Holding Problem
TRACON	Terminal Area Radar Approach Control