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NONCOOPERATIVE ENTRY DETERRENCE AND THE FREE RIDER PROBLEM (?)

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1. Introduction

Beginning with the pioneering work of Bain (1956) and Sylos-Labini (1964), the entry-prevention literature has focussed on the ability of established firms to maintain positions of market dominance. This includes, for example, Dixit (1979), (1980) and Spence (1977), and the papers surveyed in Gilbert (forthcoming). Yet examples where a single firm has maintained persistent control of a market that is not a natural monopoly are quite rare. More common are situations where one or a few firms have remained dominant in an industry over significant periods and where industry concentration levels have remained higher than could be justified by technological conditions.

Central to the persistence of concentrated industries is the latitude for established firms to influence industry structure. In this paper we ask how firms entering an industry might act if they anticipate the consequences of their actions for subsequent industry competitive conditions. Included in this is the effect of "history" on the size distribution of firms and the scope for entry prevention by non-cooperating established firms.

Ours is not the first attempt to characterize noncooperative entry deterrence. Prescott and Visscher (1977) construct a location model where firms enter a market in sequence and choose locations with correct expectations about the way their decisions affect the behavior of subsequent potential entrants. Nti and Shubik (1981) consider a model with n established firms and k potential entrants where all decisions by firms are made simultaneously and independently. They show the existence of an equilibrium where potential entrants may randomize their entry decision and characterize the equilibria with respect to the cost of entry. Bernheim (1982) considers a model where established firms confront a threat of continued entry and argues that traditional public policy instruments to foster competition may have unexpected effects. Vives (1982) examines the credibility issue in noncooperative entry deterrence in the case of an established duopoly facing a potential entrant.

Our model is in the spirit of Prescott and Visscher but we distinguish between established firms, which make simultaneous and independent decisions, and potential entrants which enter in sequence. Incumbents choose outputs noncooperatively and each firm is aware of the possible reactions of future entrants. The model is a game with complete information and we restrict attention to subgame perfect equilibria where only credible threats are allowed.

For any level of the entry cost there is an associated entry preventing output Y. Given the number of established firms, m, and potential entrants, n, we find that if entry is not blockaded (i.e., if Y is larger than the m-firm Cournot output), three regions for the entry preventing output describe the possible outcomes. If Y is small entry is prevented by incumbents and typically there is a continuum of entry preventing equilibria. If Y is large, entry of all potential entrants is allowed by incumbents, which produce at Cournot levels. For Y's in an intermediate region both types of equilibria exist.

Imperfect coordination in oligopoly suggests the possibility that entry prevention may be a public good and hence competing firms may underinvest in entry-deterring capital investment. Waldman (1982) examines this problem in a model where sunk capacity is a barrier to entry, established firms may collude on price but not on capacity and where there is a pool of potential entrants with random minimum efficient scale of operation. Under certain parametrizations he finds that increasing the number of established sellers increases the probability of entry (and social welfare) and under others a limit price solution obtains.

We will show that for the model in this paper underinvestment in entry deterrence does not occur. Indeed the opposite problem arises in our oligopoly setting. Entry prevention cannot yield higher profits for incumbents in any equilibrium where entry is allowed and there are situations where incumbents' profits are higher allowing entry but the unique equilibrium calls for entry prevention. When both types of equilibria coexist the profits of each incumbent firm are higher when entry is allowed. Thus incumbents may become trapped in a Pareto dominated arrangement (in terms of profits) by preventing entry. Furthermore in all cases where an established monopoly prevents entry an oligopoly does so too and there are situations where an oligopoly prevents entry when a monopoly would allow entry to occur.

The model we develop provides an equilibrium framework for the evaluation of industry conduct, structure, and performance. This includes the consequences of collusion (coalition formation among incumbents) for market prices and the effects of changes in entry conditions. If the entry cost is positive, entry is prevented (if not blockaded) if the number of potential entrants (n) is big enough and blockaded if the number of incumbents (m) is big enough. If the entry cost is null, entry cannot be prevented and total output converges to the competitive output as n or m go to infinity but much faster with n. Total output, and hence consumer surplus, is always monotone non-decreasing in the number of potential entrants but not necessarily in the number of established firms unless we restrict attention to undominated equilibria. The number of established firms may increase and total output may fall because the type of equilibrium may switch from preventing entry to allowing entry.

Our results are robust to the introduction of (symmetric) product differentiation, keeping a linear demand structure. Relaxing linearity leads to nonconcavities and possible non-existence of equilibrium unless demand functions satisfy very strong conditions.

The plan of the paper is as follows. In Section 2 we present the model and characterize the subgame perfect equilibria. Section 3 addresses the public good problem in entry prevention. The comparative statics of entry deterrence are examined in Section 4 and concluding remarks follow.

2. The Model

Consider m established firms and n potential entrants in a homogeneous product market. All firms have constant marginal costs and there is a fixed cost of entry. Firms set output levels and the market price is the one that equates supply and demand. Our objective is to analyze how an established oligopoly, acting noncooperatively, may prevent or allow entry when new competitors must enter the industry sequentially and all firms are aware of the consequences of their actions on future entry.

We model the market as a game of complete information with $n + 1$ stages. At the initial stage, stage 0, the incumbent firms make simultaneous and independent production decisions. At stage k, the k^{th} potential entrant chooses whether or not to enter and if so what to produce, taking as fixed the outputs produced by earlier firms in the sequence and knowing that there are still $n - k$ potential entrants.

Although the model can be viewed as being atemporal, the stages of the game correspond to the order in which firms have entered and may enter the industry. Established firms move first and all firms anticipate the reactions of subsequent entrants to their output decisions. Established firms are assumed able to maintain any output level, but we restrict our attention to outputs that are equilibria of the sequential game. No distinction is made between capacity and output.

We want to exclude from our analysis any industry equilibria that are the consequence of threats which, if called, would not be enforced. For example, potential entrants cannot promise to enter at output levels that are not rational choices after they succeed in establishing themselves in the industry. Equilibria that are devoid of empty threats are "perfect". In what follows, we derive equilibrium strategies with this perfectness property by solving the sequential entry game "backwards", beginning with the decisions of the last potential entrant.

Demand is linear, $p = a - bX$, where p is the price, X total output and a and b are positive constants. Without loss of generality assume that $b = 1$. There is a finite set M of incumbent firms $(\#M = m)$ with cost function $C_i(x) = S + vx$ for $x \geq 0$, $i \in M$, where $v \geq 0$ is the constant marginal cost and S a sunk cost. There is a finite set N of potential entrants $(\#N = n)$, $N = \{1,2,\ldots,n\}$ (if $n = 0$ the set is empty), with cost function $C_j(x) = F + vx$ if $x > 0$ and equal to zero otherwise. The entry cost is $F(\, F \geq 0)$.

Given the outputs of the incumbents $(x_i)_{i \in M}$ and of the potential entrants, $(x_j)_{j \in N}$, let X be total output and X^o the total output of the incumbents. A $-i$ subscript will mean that the output of firm i is not included in the total. Let $\bar{a} = a - v$ and ignore the sunk cost S of the incumbents. Profits of incumbent i are given by

$$\pi_i(x_i, X_{-i}) = (\bar{a} - X)x_i, \; i \in M,$$

and the profits of potential entrant j by

$$\pi_j(x_j, X_{-j}) = (\bar{a} - X)x_j - F, \; j \in N.$$

Since the product is homogeneous, firms' strategies depend only on the cumulative output of already established firms. A strategy for an incumbent is simply the choice of an output while a strategy for a potential entrant is a function which assigns a nonnegative number to any possible cumulative output of incumbents and previous entrants.

Given the n-tuple of strategies of the potential entrants $(q_j)_{j \in N}$ let $Q_k(Z)$ be the total output of the last k firms when they follow the strategies $q_j(\cdot)$ and when the cumulative output of the already established firms (incumbents and n-k entrants) is Z .(To avoid unnecessary additional notation we will not script Z to denote the number of entrants in the market).

$Q_k(\cdot)$ is defined recursively:

$$Q_1(Z) = q_n(Z) \quad \text{and}$$

$$Q_k(Z) = Q_{k-1}(Z) + q_{n+1-k}(Z) \qquad k = 2,\ldots,n.$$

We restrict attention to subgame perfect Nash equilibria of the described game. In those equilibria, equilibrium strategies form a Nash equilibrium in every proper subgame (see Selten (1975)). Formally, a subgame perfect Nash equilibrium of our game is an m-tuple of nonnegative numbers, $(x_i)_{i \in M}$, and an n-tuple of functions $(q_j)_{j \in N}$ such that

(i) $\pi_i(x_i, X^o_{-i} + Q_n(X^o)) = \max\limits_{y \in R_+} \pi_i(y, X^o_{-i} + Q_n(y + X^o_{-i})), \quad i \in M$.

(ii) For all $Z \geq 0$, $\pi_j(q_j(Z), Z + Q_{n-j}(q_j(Z) + Z)) =$
$\max\limits_{y \in R_+} \pi_j(y, Z + Q_{n-j}(y + Z)), \quad j \in N$.

Condition (ii) requires the j^{th} potential entrant's strategy $q_j(\cdot)$ to yield a best response to any possible cumulated output of already established firms taking into account the reactions of the future entrants. Condition (i) requires incumbent i's output to be a best response to the outputs of the other incumbents taking into account the reactions of the entrants. Notice that for a Nash equilibrium instead of (ii) it is only required that the choices of potential entrants be optimal along the equilibrium path. That is, for firm j , the equality in (ii) only has to hold for Z equal to the cumulative equilibrium output of incumbents and the first j-1 potential entrants.

Proposition 1 below characterizes the subgame perfect equilibria (S.P.E.) of our game. If there is no entry cost we show in Lemma 1 that the Cournot reaction function is the optimal response function for any firm. If the entry cost is positive, there is room for strategic entry prevention. However a difficulty is that when total output up to firm $n - 1$ is equal to the entry preventing output, firm n, the last potential entrant, is making zero profits by either staying out or entering the market. Firm n is indifferent to entry but its decision to enter affects the profits of established firms. To get around this problem we assume that a potential entrant enters if and only if it can make positive profits.[1] Then firm n has an optimal response function and this way we can compute in Lemma 2 reaction correspondences for all firms. With these lemmas we can characterize subgame perfect equilibria in terms of these reaction correspondences. Proposition 1 describes all S.P.E. of the game according to the level of the entry cost (or, what is equivalent, according to the level of the entry preventing output).

Lemma 1. When $F = 0$ the Cournot reaction function, $r(Z) = \max(0, \frac{\overline{a} - Z}{2})$, is the optimal response function for any firm.

Proof: When $F = 0$ no entry prevention is possible and $r(\cdot)$ is the optimal response function of firm n. Now if firms $j+1, \ldots, n$ use $r(\cdot)$ and earlier firms produce $\overline{Z}$, total output of the last $n-j$ firms is (for $\overline{a} > Z$)

$$Q_{n-j}(\overline{Z}) = (1 - \frac{1}{2^{n-j}}) \quad (\overline{a} - \overline{Z}) \quad .$$

Revenue of firm j with output y when earlier firms produce Z is $(\overline{a} - (Z+y+Q_{n-j}(Z+y))y$. This expression is maximized at $r(Z)$. The argument is valid for incumbents too.

Q.E.D.

Notice that the optimal reaction of a firm when $F = 0$ is the same whether it expects entry or not. In particular, the monopoly output is equal to the

Stackelberg output of the incumbent. This is a consequence of linear demand for a homogeneous product and constant marginal costs.

When $F > 0$ the entry preventing output is the solution to

$$(\bar{a} - (Y + r(Y)))r(Y) = F$$

in $[0,\bar{a}]$ and equals $\max\{0,\bar{a} - 2\sqrt{F}\}$. That is, if $Z \geq Y$ then profits of the potential entrant are nonpositive. Assuming that a potential entrant enters if and only if it can make positive profits, we show in Lemma 2 below that there exist reaction correspondences $(\psi_j)_{j\in N}$ and $(\phi_i)_{i\in M}$ satisfying:

(a) $\psi_n(\cdot)$ is the optimal response function of firm n .

(b) For $j = 1,\ldots,n-1$, $y \in \psi_j(Z)$ if and only if, given that total output up to firm $j-1$ is Z , y is best for firm j when firms $j+1,\ldots,n$ use any selection of $\psi_{j+1},\ldots,\psi_n$.

(c) For $i \in M$, $y \in \phi_i(Z)$ if and only if y is best for firm i given that the other incumbents produce Z , and when potential entrants use any selection of $(\psi_j)_{j\in N}$.

A key fact is that firm j will not allow a downstream firm to enter the industry and prevent entry. Firm j makes more preventing entry itself since marginal costs are constant and total output will be at least Y anyway. Therefore if firm j allows entry it must be the case that everyone else downstream is going to allow entry too. Potential entrant j , given Z , the cumulative output of all firms up to $j-1$, and anticipating the responses of subsequent entrants will respond in the following manner. If Z is larger than or equal to the limit output Y , firm j will stay out. Let Z_0 solve $Z + r(Z) = Y$ if $Y > \frac{\bar{a}}{2}$ and be zero otherwise. $Z_0 = \max\{0,2Y - \bar{a}\}$. Note that if $Y > Z > Z_0$, then $Z + r(Z) > Y$ and firm j will blockade entry by producing $r(Z)$, as no subsequent firm would want to enter the markert. In deciding whether to allow or prevent entry firm j has to compare the profits derived from allowing entry of all the remaining firms (using $r(\cdot)$) with the profits derived from preventing entry. Allowing entry of all the remaining $n-j$ firms (according to $r(\cdot)$) firm j gets a maximum revenue of

$$\frac{1}{2^{n-j}} \left(\frac{\bar{a} - Z}{2} \right)^2$$

by producing $r(\cdot)$. Preventing entry it gets $(\bar{a} - (Z + y))y$ by producing y (if $Z + y \geq Y$). Let $f^{(n-j)}(Z)$ be the unique y larger than $r(Z)$ which equates both expressions. One gets

$$f^{(n-j)}(Z) = \frac{1 + \Delta_{n-j}}{2} (\bar{a} - Z) \text{ , where } \Delta_{n-j} = \sqrt{1 - 1/2^{n-j}} \text{ .}$$

Let Z_{n-j} solve $f^{(n-j)}(Z) + Z = Y$ if the solution is positive and be zero otherwise.

$$Z_{n-j} = \max\left\{0, \frac{2Y - (1+\Delta_{n-j})\bar{a}}{1 - \Delta_{n-j}}\right\}$$

If $Z = Z_{n-j}$ firm j is indifferent between allowing and preventing entry. For $Z_0 \geq Z \geq Z_{n-j}$ firm j will prevent entry and for $Z \leq Z_{n-j}$ it will allow entry and produce $r(Z)$. It is easily checked that $Z_{k+1} + r(Z_{k+1}) < Z_k$ for $k = 0,1,\ldots,n$ so that by producing $r(Z)$ when $Z \leq Z_{n-j}$ firm j induces the remaining firms to enter. Notice again that firm j will produce according to its Cournot reaction function $r(\cdot)$ whenever entry is not prevented. (See Figure 1, where the reaction correspondence of firm j, ψ_j, is represented). For incumbent firms the reasoning is similar except that they cannot be driven out the market and they face all n potential entrants. Lemma 2, gives the reaction correspondences of potential entrants, $\psi_j(\cdot), j \in N$, and incumbents, $\phi_i(\cdot)$, $i \in M$.

[Figure 1 about here]

Lemma 2.

$\psi_n(Z) = r(Z)$ if $Z < Y$ and zero otherwise.

$$\psi_j(Z) = \begin{cases} r(Z) & \text{if } Z_{n-j} \geq Z \geq 0 \\ Y - Z & \text{if } Z_0 \geq Z \geq Z_{n-j} \\ r(Z) & \text{if } Y > Z > Z_0 \\ 0 & \text{otherwise} \end{cases} \qquad j \in \{1,\ldots,n-1\}$$

$$\phi_i(Z) = \begin{cases} r(Z) & \text{if } Z_n \geq Z \geq 0 \\ Y - Z & \text{if } Z_0 \geq Z \geq Z_n \\ r(Z) & \text{otherwise} \end{cases} \qquad i \in M$$

where $$Z_k = \max \left\{ 0 \, , \, \frac{2Y - (1 + \sqrt{1 - 1/2^k})\,\bar{a}}{1 - \sqrt{1 - 1/2^k}} \right\} \qquad k = 0,1,\ldots,n \, .$$

Furthermore, $Z_{k+1} + r(Z_{k+1}) < Z_k$.

Proof: By backwards induction. We compute ψ_n first. We derive ψ_j assuming that firms $j+1,\ldots,n$ use any selection of $\psi_{j+1},\ldots,\psi_n$ $(j = 1,\ldots,n-1)$. For incumbents we go one step further back. Since Y is the output for which $\pi_n(r(Z),Z) \leq 0$ if $Z \geq Y$ and firm n enters only if it can make positive profits, $\psi_n(Z) = r(Z)$ if $Z < Y$ and equals zero otherwise. Consider potential entrant j $(j \leq n - 1)$ and let firms $j+1,\ldots,n$ use any selection of $\psi_{j+1},\ldots,\psi_n$, where the correspondences are given as in the statement of the Lemma. Then reasoning as before ψ_j follows. The same reasoning applies to the ϕ_i's with the exception that incumbent firms do not face an entry decision. Q.E.D.

The Cournot output of an incumbent is $\frac{\bar{a}}{m+1}$ and the total Cournot output of the m incuments is $\frac{m}{m+1}\bar{a}$. In Lemma 3 we compute two values that bound the regions of entry deterring behavior. The largest entry preventing output for which to prevent entry is profitable for all incumbents is denoted by $\bar{Y}_{m,n}$. The entry preventing output which makes incumbent i indifferent between preventing or allowing entry given that the other incumbents produce at Cournot levels is denoted by $\underline{Y}_{m,n}$.

Lemma 3.

$$\bar{Y}_{m,n} = m\left(m + \frac{1-\Delta_n}{1+\Delta_n}\right)^{-1}\bar{a}, \text{ and}$$

$$\underline{Y}_{m,n} = \frac{m+\Delta_n}{m+1}\bar{a} \qquad \text{where } \Delta_n = \sqrt{1 - 1/2^n}.$$

Furthermore $\bar{Y}_{m,n} \geq \underline{Y}_{m,n} \geq \frac{m}{m+1}\bar{a}$. The first inequality is strict if $n \geq 1$ and $m \geq 2$, the second is strict if $n \geq 1$.

Proof: Recall that if Z is the output of all the incumbents except for firm i, $f^{(n)}(Z)$ gives the maximum output incumbent i is willing to produce in order to prevent entry. If $f^{(n)}(Z) \geq Y - Z$ then it is profitable for firm i to prevent entry.

$$f^{(n)}(Z) = \frac{1+\Delta_n}{2}(\bar{a} - Z).$$

$\bar{Y}_{m,n}$ is the largest entry preventing output for which to prevent entry is profitable for all incumbents. Notice that the largest entry preventing output occurs when the incumbents share total production equally. (See Figure 2). Thus $\bar{Y}_{m,n}$ solves $mf^{(n)}\left(\frac{m-1}{m}Y\right) = Y$.

$$\bar{Y}_{m,n} = m\left(m + \frac{1-\Delta_n}{1+\Delta_n}\right)^{-1}\bar{a}.$$

The Cournot output for firm i is $\frac{\bar{a}}{m+1}$. $\underline{Y}_{m,n}$ is the entry preventing output which, given that $X^{o}_{-i} = (m-1)\frac{\bar{a}}{m+1}$, makes firm i indifferent between preventing or allowing entry. Thus

$$\underline{Y}_{m,n} = \frac{m-1}{m+1}\bar{a} + f^{(n)}\left(\frac{m-1}{m+1}\bar{a}\right), \text{ this equals } \frac{m+\Delta_n}{m+1}\bar{a}.$$

The inequalities are easily checked. Q.E.D.

We are now ready to characterize completely the subgame equilibria of the game according to the level of the entry preventing output Y. Write x for $(x_i)_{i \in M}$, q for $(q_j)_{j \in N}$ and ψ for $\prod_{j \in N} \psi_j$.

Proposition 1 Let $m > 1$ and $n \geq 0$.

(i) If $Y < \frac{m}{m+1}\bar{a}$, (x,q) is a S.P.E. if and only if $x_i = \frac{\bar{a}}{m+1}$, $i \in M$ and q is a selection of ψ. At the equilibrium path incumbents blocade entry by producing at Cournot levels.

(ii) If $\frac{m}{m+1}\bar{a} \leq Y \leq \overline{Y}_{m,n}$, (x,q) such that $x \in \{z \in R^m_+ : \sum_{i \in M} z_i = Y, \ r(Z_{-i}) \leq z_i \leq f^{(n)}(Z_{-i}), \ i \in M\}$ and q is a selection of ψ is a S.P.E. At the equilibrium path incumbents prevent entry by producing a total output of Y.

(iii) If $Y > \underline{Y}_{m,n}$, (x,q) such that $x_i = \frac{\bar{a}}{m+1}$, $i \in M$, and q is a selection of ψ is a S.P.E. At the equilibrium path incumbents allow entry by producing at Cournot levels and all potential entrants enter according to their Cournot reaction functions, $r(\cdot)$.

In summary, when the total Cournot output is larger than the entry preventing output Y, entry is blockaded. When Y is larger than (or equal to) the total Cournot output but smaller than $\underline{Y}_{m,n}$, incumbents allow entry of all potential entrants by producing at Cournot levels. In the intermediate region where $\underline{Y}_{m,n} \leq Y \leq \overline{Y}_{m,n}$, we have both types of S.P.E. When entry is prevented there is typically a continuum of entry preventing S.P.E. with the incumbents producing at any point $x \in R^m_+$ above the Cournot reaction functions $r(\cdot)$ and below the profitability surfaces $f^{(n)}(\cdot)$ on the hyperplane defined by $\sum_{i \in M} x_i = Y$. (Figure 2 illustrates the entry preventing equilibria for the case $m = 2$).

[Figure 2 about here].

Proof: Given x write $\phi(x)$ for $\prod_{i \in M} \phi_i(X^o_{-1})$. From Lemma 2 it is clear that (x,q) is a S.PE. if and only if $x \in \phi(x)$ and q is a selection of ψ . By inspection of $(\phi_i)_{i \in M}$ we see that the only candidate for an entry S.P.E. is for each incumbent to produce at the Cournot output. This is profitable for the incumbents to do if $Y > \underline{Y}_{m,n}$ since then $f^{(n)}(\frac{m-1}{m+1}\overline{a}) \leq Y - \frac{m-1}{m+1}\overline{a}$. If $\frac{m}{m+1}\overline{a} \leq Y \leq \overline{Y}_{m,n}$ then the set $\{z \in R^m_+ : \sum_{i \in M} z_i = Y , r(Z_{-i}) \leq z_i \leq f^{(n)}(Z_{-i}) , i \in M\}$ is nonempty and any point in it forms a S.P.E. with any selection of q of ψ . Case (i), when entry is blockaded, should be clear by now. Q.E.D.

Remark 1: Positive profits and the decision to enter.

To derive Proposition 1 we have assumed that a potential entrant enters if and only if it can make strictly positive profits. We can dispense with this assumption. In a subgame perfect equilibrium potential entrant j , $j > 2$, will enter if and only if it makes positive profits (If firm j , $j \geq 2$, were to enter making zero profits then firm $j-1$ would not have an optimal reaction to the total output of the previous firms Z in the region $Z_0 \geq Z \geq Z_{n-j+1}$ since its profits would increase as its output decreases to the limit output but at the limit output firm j would enter and firm j-1 · profits would fall discontinuously. This contradicts the definition of S.P.E. since firm j-1 must have an optimal response for <u>all</u> Z.) The first potential entrant, firm 1, may threaten to enter making zero profits in the cases (i) and (iii), i.e. when entry is blockaded or allowed.

<u>Remark 2: Perfect versus nonperfect equilibria.</u>

Proposition 1 characterizes subgame perfect Nash equilibria, which involve only credible threats. There are many more nonperfect Nash equilibria. For example, let $m = n = 1$ and suppose that $(1 + \sqrt{1/2})\,\bar{a} > Y > \bar{a}/2$. In this case the unique S.P.E is for the incumbent to set Y and the entrant to use $q(\cdot)$ where $q(Z) = r(Z)$ if $Z < Y$ and 0 otherwise. Therefore the incumbent prevents entry. However the incumbent producing zero and the entrant using $\tilde{q}$, with $\tilde{q}(0) = \bar{a}/2$ and $\tilde{q}(Z) = a$ for $Z > 0$ is a Nash equilibrium where the incumbent does not produce anything and the entrant produces the monopoly output. The threat of producing up to a if the incumbent produces a positive output is clearly not credible.

<u>Remark 3: Asymptotic properties.</u>

When $F = 0$ no entry prevention is possible. Total output is $X_{n,m}$, which equals $\left(1 - \frac{1}{2^n(m+1)}\right)\bar{a}$. This increases to the competitive output $\bar{a}$ as n or m go to infinity but much faster with n. For a given n the rate of convergence is $\frac{1}{m}$, for a given m the rate is 2^{-n}.

When $F > 0$, for n big enough entry is prevented (if not blockaded) and for m big enough entry is blockaded. The first assertion is clear since $\underline{Y}_{m,n}$ goes to $\bar{a}$ as n goes to infinity. The second follows from the convergence of the Cournot output $\frac{m}{m+1}\bar{a}$ to the competitive output $\bar{a}$.

3. Is Entry Prevention a Public Good?

Imperfect coordination in oligopoly suggests the possibility that entry prevention may be a public good and hence competing firms may underinvest in entry-deterring capital investment (see, e.g. Waldman (1982)). We will show that this intuition is false, at least for the model in this paper. Indeed the opposite problem arises in our oligopoly setting.

Entry deterrence has the characteristics of a public good since if a group of incumbents prevent entry by producing an aggregate output at least as large as Y then entry will not occur whatever action incumbents outside the group might take. In this situation all incumbents enjoy the same amount of entry prevention and one incumbent's "consumption" of entry prevention does not decrease the amount of entry prevention enjoyed by other incumbents. The public good analogy for entry prevention suggests that incumbent firms in a noncooperative oligopoly would tend to underinvest in entry deterrence. Underinvestment in entry prevention would be associated with one or more of the following.

(a) Incumbents' total profits are higher preventing than allowing entry, but the (unique) industry equilibrium allows entry.

(b) Either entry prevention or entry may be an industry equilibrium, but incumbents' profits are higher when entry is prevented.

(c) An established monopoly (or colluding incumbents) prevents entry in more situations than an established, noncooperating, oligopoly.

We show in Proposition 2 below that in none of these respects is there underinvestment in entry prevention and that "too much" entry prevention definitely can occur.

Proposition 2. Suppose $m \geq 2$ and $n \geq 1$, then the following results hold.

(i) To prevent entry cannot yield higher total profits for incumbents in any equilibrium where entry is allowed and there is an interval of limit outputs $(Y^o, \underline{Y}_{m,n})$ where incumbents total profits are higher allowing entry but the unique equilibrium calls for entry prevention.

(ii) When $\underline{Y}_{m,n} \leq Y \leq \overline{Y}_{m,n}$ either entry prevention or entry is an equilibrium but the profits of each incumbent firm are higher when entry is allowed.

(iii) In all cases where an established monopoly prevents entry an oligopoly does so too and there are situations where an oligopoly prevents entry when a monopoly would allow entry to occur.

Proof: When entry is allowed the incumbents produce $\frac{m}{m+1}\bar{a}$ and the potential entrants $(1 - \frac{1}{2^n})\frac{\bar{a}}{m+1}$. Therefore total output with entry, $X_{m,n}$, equals $(1 - \frac{1}{2^n(m+1)})\bar{a}$.

(i) Incumbents' total profits with entry, Π^E, are given by

$$\Pi^E = (\bar{a} - X_{n,m})\frac{m}{m+1}\bar{a}\text{, which equals } \frac{m}{2^n}\left(\frac{\bar{a}}{m+1}\right)^2 .$$

Incumbents total profits when entry is deterred, Π^{NE} are given by $\Pi^{NE} = (\bar{a} - Y)Y$. Now when $Y = \underline{Y}_{m,n}$, Π^{NE} is easily seen to equal

$$\left(\frac{\bar{a}}{m+1}\right)^2 \left(\frac{m}{2^n} - (m-1)\,\Delta_n(1-\Delta_n)\right) \text{ where } \Delta_n = \sqrt{1 - 1/2^n} .$$

Therefore in this case $\Pi^{NE} < \Pi^E$ (recall that $m \geq 2$) and in fact $\Pi^{NE} < \Pi^E$ for all $Y > Y^o$ where Y^o is the largest root of

$$(\bar{a} - Y)\,Y = \frac{m}{2^n}\left(\frac{\bar{a}}{m+1}\right)^2 .$$

Note that $(\bar{a} - Y)Y$ peaks at $Y = \frac{\bar{a}}{2}$ and then declines. We have then that when $Y \geq \underline{Y}_{m,n}$ to allow entry is an equilibrium and it yields higher profits than to prevent entry since $\underline{Y}_{m,n} > Y^o$. Furthermore when $Y \in (Y^o, \underline{Y}_{m,n})$ to prevent entry is the unique equilibrium but total incumbent profits are higher with entry.

(ii) Profits for incumbent i with entry are $\pi_i^E = (\bar{a} - X_{m,n}) \frac{\bar{a}}{m+1}$ and without entry, when incumbent i produces x_i, $\pi_i^{NE} = (\bar{a} - Y)\, x_i$. Let $Y = \underline{Y}_{m,n}$. When $x_i = \underline{Y}_{m,n} - \frac{m-1}{m+1}\bar{a}$, then $\pi_i^E = \pi_i^{NE}$. If x_i is less, then $\pi_i^E > \pi_i^{NE}$ (Note that $x_i \leq \underline{Y}_{m,n} - \frac{m-1}{m+1}\bar{a}$ in equilibrium). When $Y > \underline{Y}_{m,n}$ profits preventing entry are even lower because the price is lower, and therefore $\pi_i^E > \pi_i^{NE}$. We conclude that $\pi_i^E \geq \pi_i^{NE}$ for all $i \in M$ with at least one inequality strict.

(iii) Suppose entry is not blockaded either by a monopolist or by an m-firm oligopoly. That is $Y > \frac{m}{m+1}\bar{a}$. A monopolist would prevent entry if $Y \leq \underline{Y}_{1,n}$ (Note that $\underline{Y}_{1,n} = \overline{Y}_{1,n}$). An entry prevention equilibrium with m-firms exists whenever $Y \leq \overline{Y}_{m,n}$ and it is the unique equilibrium whenever $Y < \underline{Y}_{m,n}$. Note that $\underline{Y}_{m,n} > \underline{Y}_{1,n}$ (See Lemma 3). Thus whenever a monopolist would prevent entry the established oligopoly would do so also and when $Y \in (\underline{Y}_{1,n}, \underline{Y}_{m,n})$ the oligopoly would prevent entry when a monopoly would not.

Q.E.D.

It is easily seen that total ouput with entry, $X_{m,n}$, is smaller than $\underline{Y}_{m,n}$. Therefore when $Y \geq \underline{Y}_{m,n}$ total output is less and price is higher at the entry equilibrium. The intuition for this result is clear, by allowing entry when $Y \geq \underline{Y}_{m,n}$ incumbents exploit the tendency of actual entrants to hold back output, the result being a total production less than the entry preventing output. This same basic intuition is behind (i) and (ii) in Proposition 2.

NONCOOPERATIVE ENTRY DETERRENCE AND THE FREE RIDER PROBLEM (?)

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1. Introduction

Beginning with the pioneering work of Bain (1956) and Sylos-Labini (1964), the entry-prevention literature has focussed on the ability of established firms to maintain positions of market dominance. This includes, for example, Dixit (1979), (1980) and Spence (1977), and the papers surveyed in Gilbert (forthcoming). Yet examples where a single firm has maintained persistent control of a market that is not a natural monopoly are quite rare. More common are situations where one or a few firms have remained dominant in an industry over significant periods and where industry concentration levels have remained higher than could be justified by technological conditions.

Central to the persistence of concentrated industries is the latitude for established firms to influence industry structure. In this paper we ask how firms entering an industry might act if they anticipate the consequences of their actions for subsequent industry competitive conditions. Included in this is the effect of "history" on the size distribution of firms and the scope for entry prevention by non-cooperating established firms.

Ours is not the first attempt to characterize noncooperative entry deterrence. Prescott and Visscher (1977) construct a location model where firms enter a market in sequence and choose locations with correct expectations about the way their decisions affect the behavior of subsequent potential entrants. Nti and Shubik (1981) consider a model with n established firms and k potential entrants where all decisions by firms are made simultaneously and independently. They show the existence of an equilibrium where potential entrants may randomize their entry decision and characterize the equilibria with respect to the cost of entry. Bernheim (1982) considers a model where established firms confront a threat of continued entry and argues that traditional public policy instruments to foster competition may have unexpected effects. Vives (1982) examines the credibility issue in noncooperative entry deterrence in the case of an established duopoly facing a potential entrant.

Our model is in the spirit of Prescott and Visscher but we distinguish between established firms, which make simultaneous and independent decisions, and potential entrants which enter in sequence. Incumbents choose outputs noncooperatively and each firm is aware of the possible reactions of future entrants. The model is a game with complete information and we restrict attention to subgame perfect equilibria where only credible threats are allowed.

For any level of the entry cost there is an associated entry preventing output Y. Given the number of established firms, m, and potential entrants, n, we find that if entry is not blockaded (i.e., if Y is larger than the m-firm Cournot output), three regions for the entry preventing output describe the possible outcomes. If Y is small entry is prevented by incumbents and typically there is a continuum of entry preventing equilibria. If Y is large, entry of all potential entrants is allowed by incumbents, which produce at Cournot levels. For Y's in an intermediate region both types of equilibria exist.

Imperfect coordination in oligopoly suggests the possibility that entry prevention may be a public good and hence competing firms may underinvest in entry-deterring capital investment. Waldman (1982) examines this problem in a model where sunk capacity is a barrier to entry, established firms may collude on price but not on capacity and where there is a pool of potential entrants with random minimum efficient scale of operation. Under certain parametrizations he finds that increasing the number of established sellers increases the probability of entry (and social welfare) and under others a limit price solution obtains.

We will show that for the model in this paper underinvestment in entry deterrence does not occur. Indeed the opposite problem arises in our oligopoly setting. Entry prevention cannot yield higher profits for incumbents in any equilibrium where entry is allowed and there are situations where incumbents' profits are higher allowing entry but the unique equilibrium calls for entry prevention. When both types of equilibria coexist the profits of each incumbent firm are higher when entry is allowed. Thus incumbents may become trapped in a Pareto dominated arrangement (in terms of profits) by preventing entry. Furthermore in all cases where an established monopoly prevents entry an oligopoly does so too and there are situations where an oligopoly prevents entry when a monopoly would allow entry to occur.

The model we develop provides an equilibrium framework for the evaluation of industry conduct, structure, and performance. This includes the consequences of collusion (coalition formation among incumbents) for market prices and the effects of changes in entry conditions. If the entry cost is positive, entry is prevented (if not blockaded) if the number of potential entrants (n) is big enough and blockaded if the number of incumbents (m) is big enough. If the entry cost is null, entry cannot be prevented and total output converges to the competitive output as n or m go to infinity but much faster with n. Total output, and hence consumer surplus, is always monotone non-decreasing in the number of potential entrants but not necessarily in the number of established firms unless we restrict attention to undominated equilibria. The number of established firms may increase and total output may fall because the type of equilibrium may switch from preventing entry to allowing entry.

Our results are robust to the introduction of (symmetric) product differentiation, keeping a linear demand structure. Relaxing linearity leads to nonconcavities and possible non-existence of equilibrium unless demand functions satisfy very strong conditions.

The plan of the paper is as follows. In Section 2 we present the model and characterize the subgame perfect equilibria. Section 3 addresses the public good problem in entry prevention. The comparative statics of entry deterrence are examined in Section 4 and concluding remarks follow.

2. The Model

Consider m established firms and n potential entrants in a homogeneous product market. All firms have constant marginal costs and there is a fixed cost of entry. Firms set output levels and the market price is the one that equates supply and demand. Our objective is to analyze how an established oligopoly, acting noncooperatively, may prevent or allow entry when new competitors must enter the industry sequentially and all firms are aware of the consequences of their actions on future entry.

We model the market as a game of complete information with $n + 1$ stages. At the initial stage, stage 0, the incumbent firms make simultaneous and independent production decisions. At stage k, the k^{th} potential entrant chooses whether or not to enter and if so what to produce, taking as fixed the outputs produced by earlier firms in the sequence and knowing that there are still $n - k$ potential entrants.

Although the model can be viewed as being atemporal, the stages of the game correspond to the order in which firms have entered and may enter the industry. Established firms move first and all firms anticipate the reactions of subsequent entrants to their output decisions. Established firms are assumed able to maintain any output level, but we restrict our attention to outputs that are equilibria of the sequential game. No distinction is made between capacity and output.

We want to exclude from our analysis any industry equilibria that are the consequence of threats which, if called, would not be enforced. For example, potential entrants cannot promise to enter at output levels that are not rational choices after they succeed in establishing themselves in the industry. Equilibria that are devoid of empty threats are "perfect". In what follows, we derive equilibrium strategies with this perfectness property by solving the sequential entry game "backwards", beginning with the decisions of the last potential entrant.

Demand is linear, $p = a - bX$, where p is the price, X total output and a and b are positive constants. Without loss of generality assume that $b = 1$. There is a finite set M of incumbent firms $(\#M = m)$ with cost function $C_i(x) = S + vx$ for $x \geq 0$, $i \in M$, where $v \geq 0$ is the constant marginal cost and S a sunk cost. There is a finite set N of potential entrants $(\#N = n)$, $N = \{1,2,\ldots,n\}$ (if $n = 0$ the set is empty), with cost function $C_j(x) = F + vx$ if $x > 0$ and equal to zero otherwise. The entry cost is $F(F \geq 0)$.

Given the outputs of the incumbents $(x_i)_{i \in M}$ and of the potential entrants, $(x_j)_{j \in N}$, let X be total output and X^o the total output of the incumbents. A $-i$ subscript will mean that the output of firm i is not included in the total. Let $\bar{a} = a - v$ and ignore the sunk cost S of the incumbents. Profits of incumbent i are given by

$$\pi_i(x_i, X_{-i}) = (\bar{a} - X)x_i, \; i \in M,$$

and the profits of potential entrant j by

$$\pi_j(x_j, X_{-j}) = (\bar{a} - X)x_j - F, \; j \in N.$$

Since the product is homogeneous, firms' strategies depend only on the cumulative output of already established firms. A strategy for an incumbent is simply the choice of an output while a strategy for a potential entrant is a function which assigns a nonnegative number to any possible cumulative output of incumbents and previous entrants.

Given the n-tuple of strategies of the potential entrants $(q_j)_{j \in N}$ let $Q_k(Z)$ be the total output of the last k firms when they follow the strategies $q_j(\cdot)$ and when the cumulative output of the already established firms (incumbents and n-k entrants) is Z .(To avoid unnecessary additional notation we will not script Z to denote the number of entrants in the market).

$Q_k(\cdot)$ is defined recursively:

$$Q_1(Z) = q_n(Z) \quad \text{and}$$

$$Q_k(Z) = Q_{k-1}(Z) + q_{n+1-k}(Z) \qquad k = 2,\ldots,n.$$

We restrict attention to subgame perfect Nash equilibria of the described game. In those equilibria, equilibrium strategies form a Nash equilibrium in every proper subgame (see Selten (1975)). Formally, a subgame perfect Nash equilibrium of our game is an m-tuple of nonnegative numbers, $(x_i)_{i\in M}$, and an n-tuple of functions $(q_j)_{j\in N}$ such that

$$\text{(i)} \quad \pi_i(x_i, X^o_{-i} + Q_n(X^o)) = \max_{y\in R_+} \pi_i(y, X^o_{-i} + Q_n(y + X^o_{-i})), \quad i \in M .$$

$$\text{(ii)} \quad \text{For all} \quad Z \geq 0, \quad \pi_j(q_j(Z), Z + Q_{n-j}(q_j(Z) + Z)) = \max_{y\in R_+} \pi_j(y, Z + Q_{n-j}(y + Z)), \quad j \in N .$$

Condition (ii) requires the j^{th} potential entrant's strategy $q_j(\cdot)$ to yield a best response to any possible cumulated output of already established firms taking into account the reactions of the future entrants. Condition (i) requires incumbent i's output to be a best response to the outputs of the other incumbents taking into account the reactions of the entrants. Notice that for a Nash equilibrium instead of (ii) it is only required that the choices of potential entrants be optimal along the equilibrium path. That is, for firm j, the equality in (ii) only has to hold for Z equal to the cumulative equilibrium output of incumbents and the first $j-1$ potential entrants.

Proposition 1 below characterizes the subgame perfect equilibria (S.P.E.) of our game. If there is no entry cost we show in Lemma 1 that the Cournot reaction function is the optimal response function for any firm. If the entry cost is positive, there is room for strategic entry prevention. However a difficulty is that when total output up to firm $n - 1$ is equal to the entry preventing output, firm n, the last potential entrant, is making zero profits by either staying out or entering the market. Firm n is indifferent to entry but its decision to enter affects the profits of established firms. To get around this problem we assume that a potential entrant enters if and only if it can make positive profits.[1] Then firm n has an optimal response function and this way we can compute in Lemma 2 reaction correspondences for all firms. With these lemmas we can characterize subgame perfect equilibria in terms of these reaction correspondences. Proposition 1 describes all S.P.E. of the game according to the level of the entry cost (or, what is equivalent, according to the level of the entry preventing output).

Lemma 1. When $F = 0$ the Cournot reaction function, $r(Z) = \max(0, \frac{\bar{a} - Z}{2})$, is the optimal response function for any firm.

Proof: When $F = 0$ no entry prevention is possible and $r(\cdot)$ is the optimal response function of firm n. Now if firms $j+1,\ldots,n$ use $r(\cdot)$ and earlier firms produce $\bar{Z}$, total output of the last $n-j$ firms is (for $\bar{a} > Z$)

$$Q_{n-j}(\bar{Z}) = (1 - \frac{1}{2^{n-j}}) \quad (\bar{a} - \bar{Z}) \quad .$$

Revenue of firm j with output y when earlier firms produce Z is $(\bar{a} - (Z+y+Q_{n-j}(Z+y))y$. This expression is maximized at $r(Z)$. The argument is valid for incumbents too.

Q.E.D.

Notice that the optimal reaction of a firm when $F = 0$ is the same whether it expects entry or not. In particular, the monopoly output is equal to the

Stackelberg output of the incumbent. This is a consequence of linear demand for a homogeneous product and constant marginal costs.

When $F > 0$ the entry preventing output is the solution to

$$(\bar{a} - (Y + r(Y)))r(Y) = F$$

in $[0,\bar{a}]$ and equals $\max\{0,\bar{a} - 2\sqrt{F}\}$. That is, if $Z \geq Y$ then profits of the potential entrant are nonpositive. Assuming that a potential entrant enters if and only if it can make positive profits, we show in Lemma 2 below that there exist reaction correspondences $(\psi_j)_{j\in N}$ and $(\phi_i)_{i\in M}$ satisfying:

(a) $\psi_n(\cdot)$ is the optimal response function of firm n .

(b) For $j = 1,\ldots,n-1$, $y \in \psi_j(Z)$ if and only if, given that total output up to firm $j-1$ is Z , y is best for firm j when firms $j+1,\ldots,n$ use any selection of $\psi_{j+1},\ldots,\psi_n$.

(c) For $i \in M$, $y \in \phi_i(Z)$ if and only if y is best for firm i given that the other incumbents produce Z , and when potential entrants use any selection of $(\psi_j)_{j\in N}$.

A key fact is that firm j will not allow a downstream firm to enter the industry and prevent entry. Firm j makes more preventing entry itself since marginal costs are constant and total output will be at least Y anyway. Therefore if firm j allows entry it must be the case that everyone else downstream is going to allow entry too. Potential entrant j , given Z , the cumulative output of all firms up to $j-1$, and anticipating the responses of subsequent entrants will respond in the following manner. If Z is larger than or equal to the limit output Y , firm j will stay out. Let Z_0 solve $Z + r(Z) = Y$ if $Y > \frac{\bar{a}}{2}$ and be zero otherwise. $Z_0 = \max\{0,2Y - \bar{a}\}$. Note that if $Y > Z > Z_0$, then $Z + r(Z) > Y$ and firm j will blockade entry by producing $r(Z)$, as no subsequent firm would want to enter the markert. In deciding whether to allow or prevent entry firm j has to compare the profits derived from allowing entry of all the remaining firms (using $r(\cdot)$) with the profits derived from preventing entry. Allowing entry of all the remaining $n-j$ firms (according to $r(\cdot)$) firm j gets a maximum revenue of

$$\frac{1}{2^{n-j}} \left(\frac{\bar{a} - Z}{2} \right)^2$$

by producing $r(\cdot)$. Preventing entry it gets $(\bar{a} - (Z + y))y$ by producing y (if $Z + y \geq Y$). Let $f^{(n-j)}(Z)$ be the unique y larger than $r(Z)$ which equates both expressions. One gets

$$f^{(n-j)}(Z) = \frac{1 + \Delta_{n-j}}{2} (\bar{a} - Z) \text{ , where } \Delta_{n-j} = \sqrt{1 - 1/2^{n-j}} \text{ .}$$

Let Z_{n-j} solve $f^{(n-j)}(Z) + Z = Y$ if the solution is positive and be zero otherwise.

$$Z_{n-j} = \max\left\{0, \frac{2Y - (1+\Delta_{n-j})\bar{a}}{1 - \Delta_{n-j}}\right\}$$

If $Z = Z_{n-j}$ firm j is indifferent between allowing and preventing entry. For $Z_0 \geq Z > Z_{n-j}$ firm j will prevent entry and for $Z \leq Z_{n-j}$ it will allow entry and produce $r(Z)$. It is easily checked that $Z_{k+1} + r(Z_{k+1}) < Z_k$ for $k = 0,1,\ldots,n$ so that by producing $r(Z)$ when $Z \leq Z_{n-j}$ firm j induces the remaining firms to enter. Notice again that firm j will produce according to its Cournot reaction function $r(\cdot)$ whenever entry is not prevented. (See Figure 1, where the reaction correspondence of firm j, ψ_j, is represented). For incumbent firms the reasoning is similar except that they cannot be driven out the market and they face all n potential entrants. Lemma 2, gives the reaction correspondences of potential entrants, $\psi_j(\cdot), j \in N$, and incumbents, $\phi_i(\cdot)$, $i \in M$.

[Figure 1 about here]

Lemma 2.

$\psi_n(Z) = r(Z)$ if $Z < Y$ and zero otherwise.

$$\psi_j(Z) = \begin{cases} r(Z) & \text{if } Z_{n-j} \geq Z \geq 0 \\ Y - Z & \text{if } Z_0 \geq Z \geq Z_{n-j} \\ r(Z) & \text{if } Y > Z > Z_0 \\ 0 & \text{otherwise} \end{cases} \qquad j \in \{1,\ldots,n-1\}$$

$$\phi_i(Z) = \begin{cases} r(Z) & \text{if } Z_n \geq Z \geq 0 \\ Y - Z & \text{if } Z_0 \geq Z \geq Z_n \\ r(Z) & \text{otherwise} \end{cases} \qquad i \in M$$

$$\text{where } \quad Z_k = \max \left\{ 0 \ , \ \frac{2Y - (1 + \sqrt{1 - 1/2^k})\ \bar{a}}{1 - \sqrt{1 - 1/2^k}} \right\} \qquad k = 0,1,\ldots,n \ .$$

Furthermore, $Z_{k+1} + r(Z_{k+1}) < Z_k$.

Proof: By backwards induction. We compute ψ_n first. We derive ψ_j assuming that firms $j+1,\ldots,n$ use any selection of $\psi_{j+1},\ldots,\psi_n$ $(j = 1,\ldots,n-1)$. For incumbents we go one step further back. Since Y is the output for which $\pi_n(r(Z),Z) \leq 0$ if $Z \geq Y$ and firm n enters only if it can make positive profits, $\psi_n(Z) = r(Z)$ if $Z < Y$ and equals zero otherwise. Consider potential entrant j $(j \leq n - 1)$ and let firms $j+1,\ldots,n$ use any selection of $\psi_{j+1},\ldots,\psi_n$, where the correspondences are given as in the statement of the Lemma. Then reasoning as before ψ_j follows. The same reasoning applies to the ϕ_i's with the exception that incumbent firms do not face an entry decision. Q.E.D.

The Cournot output of an incumbent is $\frac{\bar{a}}{m+1}$ and the total Cournot output of the m incuments is $\frac{m}{m+1}\bar{a}$. In Lemma 3 we compute two values that bound the regions of entry deterring behavior. The largest entry preventing output for which to prevent entry is profitable for all incumbents is denoted by $\bar{Y}_{m,n}$. The entry preventing output which makes incumbent i indifferent between preventing or allowing entry given that the other incumbents produce at Cournot levels is denoted by $\underline{Y}_{m,n}$.

Lemma 3.

$$\bar{Y}_{m,n} = m\left(m + \frac{1 - \Delta_n}{1 + \Delta_n}\right)^{-1} \bar{a} \quad \text{, and}$$

$$\underline{Y}_{m,n} = \frac{m + \Delta_n}{m + 1}\bar{a} \qquad \text{where } \Delta_n = \sqrt{1 - 1/2^n} .$$

Furthermore $\bar{Y}_{m,n} \geq \underline{Y}_{m,n} \geq \frac{m}{m+1}\bar{a}$. The first inequality is strict if $n \geq 1$ and $m \geq 2$, the second is strict if $n \geq 1$.

Proof: Recall that if Z is the output of all the incumbents except for firm i, $f^{(n)}(Z)$ gives the maximum output incumbent i is willing to produce in order to prevent entry. If $f^{(n)}(Z) \geq Y - Z$ then it is profitable for firm i to prevent entry.

$$f^{(n)}(Z) = \frac{1 + \Delta_n}{2}(\bar{a} - Z) .$$

$\bar{Y}_{m,n}$ is the largest entry preventing output for which to prevent entry is profitable for all incumbents. Notice that the largest entry preventing output occurs when the incumbents share total production equally. (See Figure 2). Thus $\bar{Y}_{m,n}$ solves $mf^{(n)}\left(\frac{m-1}{m}Y\right) = Y$.

$$\bar{Y}_{m,n} = m\left(m + \frac{1 - \Delta_n}{1 + \Delta_n}\right)^{-1}\bar{a} .$$

The Cournot output for firm i is $\frac{\bar{a}}{m+1}$. $\underline{Y}_{m,n}$ is the entry preventing output which, given that $X^o_{-i} = (m-1)\frac{\bar{a}}{m+1}$, makes firm i indifferent between preventing or allowing entry. Thus

$$\underline{Y}_{m,n} = \frac{m-1}{m+1}\bar{a} + f^{(n)}\left(\frac{m-1}{m+1}\bar{a}\right), \text{ this equals } \frac{m+\Delta_n}{m+1}\bar{a}.$$

The inequalities are easily checked. Q.E.D.

We are now ready to characterize completely the subgame equilibria of the game according to the level of the entry preventing output Y. Write x for $(x_i)_{i\in M}$, q for $(q_j)_{j\in N}$ and ψ for $\prod_{j\in N}\psi_j$.

Proposition 1 Let $m > 1$ and $n \geq 0$.

(i) If $Y < \frac{m}{m+1}\bar{a}$, (x,q) is a S.P.E. if and only if $x_i = \frac{\bar{a}}{m+1}$, $i \in M$ and q is a selection of ψ. At the equilibrium path incumbents blocade entry by producing at Cournot levels.

(ii) If $\frac{m}{m+1}\bar{a} \leq Y \leq \overline{Y}_{m,n}$, (x,q) such that $x \in \{z \in R^m_+ : \sum_{i\in M} z_i = Y,\ r(Z_{-i}) \leq z_i \leq f^{(n)}(Z_{-i}),\ i \in M\}$ and q is a selection of ψ is a S.P.E. At the equilibrium path incumbents prevent entry by producing a total output of Y.

(iii) If $Y > \underline{Y}_{m,n}$, (x,q) such that $x_i = \frac{\bar{a}}{m+1}$, $i \in M$, and q is a selection of ψ is a S.P.E. At the equilibrium path incumbents allow entry by producing at Cournot levels and all potential entrants enter according to their Cournot reaction functions, $r(\cdot)$.

In summary, when the total Cournot output is larger than the entry preventing output Y, entry is blockaded. When Y is larger than (or equal to) the total Cournot output but smaller than $\underline{Y}_{m,n}$, incumbents allow entry of all potential entrants by producing at Cournot levels. In the intermediate region where $\underline{Y}_{m,n} \leq Y \leq \overline{Y}_{m,n}$, we have both types of S.P.E. When entry is prevented there is typically a continuum of entry preventing S.P.E. with the incumbents producing at any point $x \in R^m_+$ above the Cournot reaction functions $r(\cdot)$ and below the profitability surfaces $f^{(n)}(\cdot)$ on the hyperplane defined by $\sum_{i \in M} x_i = Y$. (Figure 2 illustrates the entry preventing equilibria for the case $m = 2$).

[Figure 2 about here].

Proof: Given x write $\phi(x)$ for $\prod_{i \in M} \phi_i(X^o_{-i})$. From Lemma 2 it is clear that (x,q) is a S.PE. if and only if $x \in \phi(x)$ and q is a selection of ψ. By inspection of $(\phi_i)_{i \in M}$ we see that the only candidate for an entry S.P.E. is for each incumbent to produce at the Cournot output. This is profitable for the incumbents to do if $Y \geq \underline{Y}_{m,n}$ since then $f^{(n)}(\frac{m-1}{m+1}\overline{a}) \leq Y - \frac{m-1}{m+1}\overline{a}$. If $\frac{m}{m+1}\overline{a} \leq Y \leq \overline{Y}_{m,n}$ then the set $\{z \in R^m_+ \colon \sum_{i \in M} z_i = Y,\ r(Z_{-i}) \leq z_i \leq f^{(n)}(Z_{-i}),\ i \in M\}$ is nonempty and any point in it forms a S.P.E. with any selection of q of ψ. Case (i), when entry is blockaded, should be clear by now. Q.E.D.

Remark 1: Positive profits and the decision to enter.

To derive Proposition 1 we have assumed that a potential entrant enters if and only if it can make strictly positive profits. We can dispense with this assumption. In a subgame perfect equilibrium potential entrant j, $j \geq 2$, will enter if and only if it makes positive profits (If firm j, $j \geq 2$, were to enter making zero profits then firm $j-1$ would not have an optimal reaction to the total output of the previous firms Z in the region $Z_0 \geq Z \geq Z_{n-j+1}$ since its profits would increase as its output decreases to the limit output but at the limit output firm j would enter and firm j-1 · profits would fall discontinuously. This contradicts the definition of S.P.E. since firm j-1 must have an optimal response for <u>all</u> Z.) The first potential entrant, firm 1, may threaten to enter making zero profits in the cases (i) and (iii), i.e. when entry is blockaded or allowed.

Remark 2: Perfect versus nonperfect equilibria.

Proposition 1 characterizes subgame perfect Nash equilibria, which involve only credible threats. There are many more nonperfect Nash equilibria. For example, let $m = n = 1$ and suppose that $(1 + \sqrt{1/2})\, \bar{a} > Y > \bar{a}/2$. In this case the unique S.P.E is for the incumbent to set Y and the entrant to use $q(\cdot)$ where $q(Z) = r(Z)$ if $Z < Y$ and 0 otherwise. Therefore the incumbent prevents entry. However the incumbent producing zero and the entrant using $\tilde{q}$, with $\tilde{q}(0) = \bar{a}/2$ and $\tilde{q}(Z) = a$ for $Z > 0$ is a Nash equilibrium where the incumbent does not produce anything and the entrant produces the monopoly output. The threat of producing up to a if the incumbent produces a positive output is clearly not credible.

Remark 3: Asymptotic properties.

When $F = 0$ no entry prevention is possible. Total output is $X_{n,m}$, which equals $\left(1 - \frac{1}{2^n(m+1)}\right)\bar{a}$. This increases to the competitive output $\bar{a}$ as n or m go to infinity but much faster with n. For a given n the rate of convergence is $\frac{1}{m}$, for a given m the rate is 2^{-n}.

When $F > 0$, for n big enough entry is prevented (if not blockaded) and for m big enough entry is blockaded. The first assertion is clear since $\underline{Y}_{m,n}$ goes to $\bar{a}$ as n goes to infinity. The second follows from the convergence of the Cournot output $\frac{m}{m+1}\bar{a}$ to the competitive output $\bar{a}$.

3. Is Entry Prevention a Public Good?

Imperfect coordination in oligopoly suggests the possibility that entry prevention may be a public good and hence competing firms may underinvest in entry-deterring capital investment (see, e.g. Waldman (1982)). We will show that this intuition is false, at least for the model in this paper. Indeed the opposite problem arises in our oligopoly setting.

Entry deterrence has the characteristics of a public good since if a group of incumbents prevent entry by producing an aggregate output at least as large as Y then entry will not occur whatever action incumbents outside the group might take. In this situation all incumbents enjoy the same amount of entry prevention and one incumbent's "consumption" of entry prevention does not decrease the amount of entry prevention enjoyed by other incumbents. The public good analogy for entry prevention suggests that incumbent firms in a noncooperative oligopoly would tend to underinvest in entry deterrence. Underinvestment in entry prevention would be associated with one or more of the following.

(a) Incumbents' total profits are higher preventing than allowing entry, but the (unique) industry equilibrium allows entry.

(b) Either entry prevention or entry may be an industry equilibrium, but incumbents' profits are higher when entry is prevented.

(c) An established monopoly (or colluding incumbents) prevents entry in more situations than an established, noncooperating, oligopoly.

We show in Proposition 2 below that in none of these respects is there underinvestment in entry prevention and that "too much" entry prevention definitely can occur.

Proposition 2. Suppose $m \geq 2$ and $n \geq 1$, then the following results hold.

(i) To prevent entry cannot yield higher total profits for incumbents in any equilibrium where entry is allowed and there is an interval of limit outputs $(Y^o, \underline{Y}_{m,n})$ where incumbents total profits are higher allowing entry but the unique equilibrium calls for entry prevention.

(ii) When $\underline{Y}_{m,n} \leq Y \leq \overline{Y}_{m,n}$ either entry prevention or entry is an equilibrium but the profits of each incumbent firm are higher when entry is allowed.

(iii) In all cases where an established monopoly prevents entry an oligopoly does so too and there are situations where an oligopoly prevents entry when a monopoly would allow entry to occur.

Proof: When entry is allowed the incumbents produce $\frac{m}{m+1}\bar{a}$ and the potential entrants $(1 - \frac{1}{2^n}) \frac{\bar{a}}{m+1}$. Therefore total output with entry, $X_{m,n}$, equals $(1 - \frac{1}{2^n(m+1)})\bar{a}$.

(i) Incumbents' total profits with entry, Π^E, are given by

$$\Pi^E = (\bar{a} - X_{n,m}) \frac{m}{m+1} \bar{a}\text{, which equals } \frac{m}{2^n} (\frac{\bar{a}}{m+1})^2 .$$

Incumbents total profits when entry is deterred, Π^{NE} are given by $\Pi^{NE} = (\bar{a} - Y)Y$. Now when $Y = \underline{Y}_{m,n}$, Π^{NE} is easily seen to equal

$$(\frac{\bar{a}}{m+1})^2 (\frac{m}{2^n} - (m-1) \Delta_n(1 - \Delta_n)) \text{ where } \Delta_n = \sqrt{1 - 1/2^n} .$$

Therefore in this case $\Pi^{NE} < \Pi^E$ (recall that $m \geq 2$) and in fact $\Pi^{NE} < \Pi^E$ for all $Y > Y^o$ where Y^o is the largest root of

$$(\bar{a} - Y) Y = \frac{m}{2^n} (\frac{\bar{a}}{m+1})^2 .$$

Note that $(\bar{a} - Y)Y$ peaks at $Y = \frac{\bar{a}}{2}$ and then declines. We have then that when $Y \geq \underline{Y}_{m,n}$ to allow entry is an equilibrium and it yields higher profits than to prevent entry since $\underline{Y}_{m,n} > Y^o$. Furthermore when $Y \in (Y^o, \underline{Y}_{m,n})$ to prevent entry is the unique equilibrium but total incumbent profits are higher with entry.

(ii) Profits for incumbent i with entry are $\pi_i^E = (\bar{a} - X_{m,n}) \frac{\bar{a}}{m+1}$ and without entry, when incumbent i produces x_i, $\pi_i^{NE} = (\bar{a} - Y) x_i$. Let $Y = \underline{Y}_{m,n}$. When $x_i = \underline{Y}_{m,n} - \frac{m-1}{m+1} \bar{a}$, then $\pi_i^E = \pi_i^{NE}$. If x_i is less, then $\pi_i^E > \pi_i^{NE}$ (Note that $x_i \leq \underline{Y}_{m,n} - \frac{m-1}{m+1} \bar{a}$ in equilibrium). When $Y > \underline{Y}_{m,n}$ profits preventing entry are even lower because the price is lower, and therefore $\pi_i^E > \pi_i^{NE}$. We conclude that $\pi_i^E \geq \pi_i^{NE}$ for all $i \in M$ with at least one inequality strict.

(iii) Suppose entry is not blockaded either by a monopolist or by an m-firm oligopoly. That is $Y > \frac{m}{m+1} \bar{a}$. A monopolist would prevent entry if $Y \leq \underline{Y}_{1,n}$ (Note that $\underline{Y}_{1,n} = \overline{Y}_{1,n}$). An entry prevention equilibrium with m-firms exists whenever $Y \leq \overline{Y}_{m,n}$ and it is the unique equilibrium whenever $Y < \underline{Y}_{m,n}$. Note that $\underline{Y}_{m,n} > \underline{Y}_{1,n}$ (See Lemma 3). Thus whenever a monopolist would prevent entry the established oligopoly would do so also and when $Y \in (\underline{Y}_{1,n}, \underline{Y}_{m,n})$ the oligopoly would prevent entry when a monopoly would not. Q.E.D.

It is easily seen that total ouput with entry, $X_{m,n}$, is smaller than $\underline{Y}_{m,n}$. Therefore when $Y \geq \underline{Y}_{m,n}$ total output is less and price is higher at the entry equilibrium. The intuition for this result is clear, by allowing entry when $Y \geq \underline{Y}_{m,n}$ incumbents exploit the tendency of actual entrants to hold back output, the result being a total production less than the entry preventing output. This same basic intuition is behind (i) and (ii) in Proposition 2.

We have seen that when $\underline{Y}_{m,n} \leq Y \leq \overline{Y}_{m,n}$ there are two types of S.P.E., one where entry is prevented and another where entry is allowed and that the entry equilibrium dominates in terms of profits those equilibria where entry is prevented. This means that incumbents may get trapped in a Pareto dominated arrangement preventing entry. On the other hand if we restrict attention to undominated S.P.E. then the equilibrium path is unique in terms of total output for any given Y. If Y is less than the total Cournot output entry is blockaded. If $\frac{m}{m+1}\overline{a} \leq Y \leq \underline{Y}_{m,n}$ then entry is prevented, otherwise it is allowed.

4. Comparative Statics of Entry Deterrence

The model developed in the preceding sections provides an equilibrium framework for the evaluation of industry conduct, structure, and performance. This includes the consequences of collusion for market prices and the effects of changes in entry conditions. The fixed cost of entry, assumed sunk once incurred, is a measure of the height of entry barriers, while the number of potential entrants indicates constraints imposed by technology or resource availability.

Suppose there are m_o established firms. One measure of collusion is to take the number of "effective" incumbents m less than m_o. This may come about through explicit or implicit coalition formation among incumbents. For example, two incumbents merge and act as a single unit and therefore the number of "effective" incumbents decreases by one. We do not discuss here the profitability of such coalition formation (See the article by Salant, et al. (1983) for a discussion of the Cournot case).

A reduction in the number of "effective" incumbents does not reduce the feasible level of profits for established firms and may increase actual profits through better coordination. While collusion generally leads to higher prices and profits, we identify regions where collusion has no effect on price or profits, and where collusion even can leave established firms worse off. We will also show how a small change in the number of effective incumbents can result in a discontinuous change in the equilibrium price and in market performance as measured by total economic surplus.

Some preliminary results are useful for determining the range of possible outcomes. Since the effects of collusion depend on entry, we need to know how collusion might change entry conditions. Quantities which relate to entry incentives are the limit output, Y and the critical values $\underline{Y}_{m,n}$ and $\overline{Y}_{m,n}$

which determine the profitability of entry prevention. Recall that if entry is not blockaded, then incumbents will allow entry if $Y > \overline{Y}_{m,n}$ and will prevent entry if $Y < \underline{Y}_{m,n}$. For $\underline{Y}_{m,n} \leq Y \leq \overline{Y}_{m,n}$ either outcome is possible.

The effects of collusion will depend on how it changes the critical values $\underline{Y}_{m,n}$ and $\overline{Y}_{m,n}$, and in particular on whether collusion makes these values larger or smaller than the limit output, Y. This suggests three critical numbers m_1, m_2, m_3 (not necessarily integers) defined as follows.

(1) $$\frac{m_1}{m_1 + 1}\bar{a} = Y$$

(2) $$\underline{Y}_{m_2,n} = Y$$

(3) $$\overline{Y}_{m_3,n} = Y$$

Equation (1) gives $m_1(Y)$, the lower bound for the number of incumbents m whose Cournot outputs blockade entry. Equation (2) gives $m_2(n,Y)$, the upper bound on m for which allowing entry is an equilibrium. Equation (3) gives $m_3(n,Y)$, the lower bound on m for which preventing entry is an equilibrium. Recall that if $Y = \overline{Y}_{m,n}$ all incumbents sharing the entry preventing output is an equilibrium. It is easily checked that, with $m \geq 1$, $m_1(Y) \geq m_2(n,Y) \geq m_3(n,Y)$ with strict inequality if $n \geq 1$ and if they are larger than one. (Appendix 1 gives the expressions for the m_i's)

In summary, given the number of potential entrants, n and the limit output Y, entry conditions depend on how the number of effective incumbents compares to the critical values m_1, m_2, m_3. Proposition 3 is just a restatement of Propostition 1.

Proposition 3. For fixed n and Y ,

(i) if $m > m_1(Y)$, then entry is blockaded;

(ii) if $m_3(n,Y) \leq m \leq m_1(Y)$, then preventing entry is an equilibrium;

(iii) if $m \leq m_2(n,Y)$, then allowing entry is an equilibrium.

Note that if $m_3(n,Y) \leq m \leq m_2(n,Y)$, either preventing or allowing entry can be an equilibrium.

[Figure 3 about here]

Figure 3 shows the regions in (n,m) space corresponding to the possible equilibria for a given Y . For a sufficiently large number of incumbent firms, entry is blockaded by the Cournot output. As the number of incumbents decreases, perhaps reflecting collusion, there is a region $[m_2(n,Y) , m_1(Y)]$ where incumbents will prevent entry by setting the limit output. This region is nonempty for any $n \geq 1$. Thus there is a transition as m decreases from blockaded entry to limit pricing by the established firms. If the number of potential entrants exceeds the value n' shown in figure 3, the incumbents limit price for all $m < m_1(Y)$, so that the possible outcomes are only blockaded entry or limit pricing.

If the number of potential entrants is fewer than n' , incumbent firms may blockade, prevent, or allow entry. Entry is always blockaded if the number of incumbents is sufficiently large and allowed if the number of incumbents is sufficiently small.

For example let $Y = .98\bar{a}$ and $n = 2$. Then $m_1 \doteq 49$, $m_2 \approx 5.7$ and $m_3 \approx 3.5$ so that for $m \geq 50$ entry is blockaded, for $49 \geq m \geq 6$ entry is prevented, for $3 \geq m$ entry is allowed and if m equals 4 or 5 both types of equilibria exist.

Figure 4 shows for a given (n,Y) how the industry price may change with the number of incumbent firms. The line P^C is the price corresponding to the Cournot outputs of the incumbents, $P^C = \frac{\bar{a}}{m+1} + v$. P^E is the price with entry, $P^E = \frac{\bar{a}}{2^n(m+1)} + v$, and $\bar{P}$ is the price at the limit output Y , $\bar{P} = a - Y$. For $m > m_1$, entry is blockaded and the equilibrium price is given by P^C . For $m_2 < m \leq m_1$, the unique equilibrium is entry prevention with price $\bar{P}$. Note that for $m_2 < m < \hat{m}$ the price is $\bar{P}$ although if entry occurred, the price would be given by P^E with $P^E > \bar{P}$. For $m_3 < m \leq m_2$ the equilibrium is not unique and price could be either $\bar{P}$ with entry deterred or P^E with entry. Incumbents however are strictly better off with entry and price P^E . For $m < m_3$ entry is allowed and the price is given by P^E .

[Figure 4 about here]

If we think of greater collusion continuously lowering the number of "effective" incumbents m , price increases continuously for $m > m_1$ and then stays constant for $m_2 < m \leq m_1$. For some $m \leq m_2$, the price must increase discontinuously as the equilibrium changes from entry prevention to entry. In the interval $[m_3, m_2]$ welfare, as measured by total surplus, falls abruptly when the industry equilibrium changes from entry prevention to entry. Not only does price increase abruptly, but costs increase as new firms pay the fixed cost of entry.

Next we ask how a given number of incumbents would react to a change in the number of potential entrants, holding Y fixed. Clearly for $m > m_1(Y)$, the number of potential entrants has no effect as entry is blockaded. For $m < m_1(Y)$, the unique equilibrium calls for entry prevention when n is sufficiently large (see figure 3). For n sufficiently small (possibly zero, given the integer constraint), the unique equilibrium allows entry. For n in these regions, a reduction in the number of potential entrants can increase and would never decrease price. The only region where the effect of the number of potential entrants on price is ambiguous is where $m \in [m_3(n,Y) , m_2(n,Y)]$, since in this region there are mulitple equilibria with different prices.

Nonetheless we can show that an <u>integer</u> change in the number of potential entrants has an unambiguous effect on the direction of price movements. Specifically, if $m \in [m_3(n,Y) , m_2(n,Y)]$, then $m \notin [m_3(n \mp k,Y) , m_2(n \mp k,Y]$ for any positive integer k (See Appendix 2 for a proof). If n is such that there can be multiple equilibria, with one more potential entrant we would have $m > m_2(n+1, Y)$ and the price would be $\overline{P}$. With one less potential entrant we have $m < m_3(n-1, Y)$ and this would be accompanied by a price rise. Thus price is monotone nonincreasing in the (integer) number of potential entrants.

Proposition 4 summarizes our results on the relationship between market price, m and n for a given entry preventing output Y .

<u>**Proposition 4.**</u> Fix Y . In equilibrium, market price is (for any given m) monotone nonincreasing in the number of potential entrants, n but not necessarily (for any given n) in the number of established firms m , unless we restrict attention to undominated equilibria (in terms of profits).

Finally we consider the effect of a change in the limit output, Y holding the number of incumbents and potential entrants fixed. A change in the limit output could result, for example, from a technological development that lowers the cost of entry. An increase in the limit output from Y to Y' would have no effect on price or performance if entry is either blockaded or allowed at both values. Not surprisingly, if entry is prevented at Y

and Y' , the increase lowers the equilibrium price.

The consequences of an increase in the limit output are more interesting when the change causes a switch from an equilibrium where entry is prevented to an equilibrium where entry is allowed. Suppose $Y \in [\underline{Y}_{m,n}, \overline{Y}_{m,n}]$ and entry is prevented. A sufficiently large increase in the limit output will result in $Y' > \overline{Y}_{m,n}$ and then allowing entry will be the unique equilibrium. If entry is allowed, the resulting output $X_{m,n}$ depends only on the number of incumbents and entrants, and it is independent of the limit output. (Recall furthermore that $X_{m,n} < \underline{Y}_{m,n}$) . (See figure 5 where total equilibrium output is shown for every possible Y) Thus total output will be $X_{m,n}$ and price will be higher than when entry was prevented at the limit output. In addition, costs are higher with entry, so that total surplus is unambiguously lower.

[Figure 5 about here]

Of course the increase in the limit output from Y to Y' would lower price and profits if incumbents continued to prevent entry at both Y and Y' . When incumbents switch to allowing entry at Y' , the lower entry cost has the perverse effect of reducing economic performance. Incumbents switch to an equilibrium where price, profits, and costs are greater than when entry was prevented at the original limit output. Lowering the cost of entry, and thereby making potential competitors a more credible threat, leads to an unambiguous deterioration in market performance. This situation can occur with any number of incumbents, including the case of a single dominant firm.

5. Concluding remarks

We have explored a model where m incumbents face noncooperatively n potential entrants that must enter sequentially (paying an entry cost F). Given n , m and F Proposition 1 specifies whether entry will be blockaded, prevented or allowed, except in a region where there are two types of equilibrium paths, one where entry is prevented, another where entry is allowed.

Despite noncooperative behavior among incumbent firms, we found no evidence of underinvestment in entry prevention. Indeed, the opposite result occurs in some situations: incumbents prevent entry even though their profits would be higher if entry were allowed. (See Proposition 2). A main reason why this result obtains is that entry-preventing investment earns revenues and therefore confers direct benefits on any firm that invests to exclude rivals. If entry is prevented, profits of incumbent i increase with investment up to the limit output. Thus each incumbent firm is better off if it carries the "burden" of entry prevention. This incentive to be the entry-preventer can lead to excessive investment in entry prevention. Furthermore if entry is allowed, incumbents' equilibrium outputs would fall to Cournot levels and entrants would hold back output resulting in a market price higher than the limit price.

These results suggest that limit-pricing can confer benefits for consumers. There is at least a tendency for firms to limit price and keep prices below what they would be if entry occurred, and this increases total economic surplus. Of course there are situations (low limit outputs in region P of figure 5) where firms limit price when total output would be higher (and price would be lower) with entry. This occurs at relatively small limit outputs, which reflects high costs associated with entry (and at which preventing entry is the unique equilibrium).

In the extreme these results suggest that policies which raise the cost of entry actually can have desirable welfare consequences (at least within the narrow focus of our model). If entry is easy so that $Y \geq \overline{Y}_{m,n}$ in figure 5, incumbents would allow entry and total output would be $X_{m,n}$. A tax on entry (or an entry fee) could lower the limit output to a level Y somewhat below $\underline{Y}_{m,m}$. Incumbents would then prevent entry, and total output Y would be greater (and price lower) than with $X_{m,n}$. While we remain skeptical of policies which interfere with competition - potential or actual - we merely call attention to the fact that in oligopolistic markets easier entry need not be beneficial.

Clearly the results in this paper hinge on particular assumptions (although assumptions that are quite common in the industrial organization literature), and we would like to know whether they extend to more general situations. Allowing for symmetric product differentiation while maintaining a linear demand structure suggests similar conclusions. Although optimal reactions are more complex, similar behavior with respect to entry occurs.

Nonlinear demand, even with a homogeneous product, causes difficulties. The problem is the possible absence of global concavity in firms' profits (even ignoring the entry cost F), with resulting multi-valued reactions which can lead to non-existence of equilibria. Since each firm's profits depend on the reactions of all future entrants, the conditions needed to guarantee concavity turn out to be very restrictive. To have strict concavity of profits for firm j (with respect to its own output) a sufficient condition is that the optimal reactions of the remaining $n-j$ firms be convex. This requires conditions on the $n-j+3$ derivatives of the demand function!

We have supposed from the start that outputs, once set, were maintained by the firms and that market price was the one which cleared the market. We could refine the analysis and distinguish between capacity and output (as in Dixit (1980)). Capacity would have a constant unit cost: output would have a constant marginal cost up to capacity and infinite otherwise. The $n + 1$ stage game would be as before, substituting output by capacity except that once all capacities were set the profits accruing to each firm would be the ones of the Cournot equilibrium resulting from the cost functions (capacities) chosen by the firms. One may conjecture that (as in Dixit (1980) where he analyzes the case $m = 1$, $n = 1$) there will not be excess capacity in equilibrium (S.P.E.) since capacity is costly and excess capacity does not deter entry; potential entrants figure out capacities which are not going to be used fully (i.e., capacities which are not credible). Although there will be no excess capacity in equilibrium, our reformulated game does not reduce to the original game since now firms have to worry not only about the profitability of deterring entry but also about the feasibility of doing so. There are going to be situations where it would be profitable to keep an entrant out but it is not possible to do so because the output needed to prevent entry cannot be induced in Cournot equilibrium by any capacity choice of the incumbents. The characterization of equilibria turns out to be complex, due to the feasibility (or credibility) constraint, but one thing is certain: incumbents will have a harder time preventing entry than suggested by our model.[2] Therefore one should interpret with some caution the strong entry preventing results suggested here.

Appendix 1

Let $\lambda = \frac{Y}{\bar{a}}$ and $\Delta_n = \sqrt{1 - 1/2^n}$ then $m_1(Y) = \frac{\lambda}{1 - \lambda}$, $m_2(Y,n) = \frac{\lambda - \Delta_n}{1 - \lambda}$ and $m_3(Y,n) = \frac{\lambda}{1 - \lambda} \cdot \frac{1 - \Delta_n}{1 + \Delta_n}$.

Appendix 2

We show that if $m \in \lfloor m_3(n,Y), m_2(n,Y) \rfloor$ then $m \notin \lfloor m_3(n \mp k,Y), m_2(n \mp k,Y) \rfloor$ where k is positive integer. It is sufficient to show that $\overline{Y}_{m,n} < \underline{Y}_{m,n+1}$ since if this is the case then $\lfloor \underline{Y}_{m,n}, \overline{Y}_{m,n} \rfloor \cap \lfloor \underline{Y}_{m,n+1}, \overline{Y}_{m,n+1} \rfloor = \emptyset$ for all $m \geq 1$ and n. Therefore if $Y \in \lfloor \underline{Y}_{m,n}, \overline{Y}_{m,n} \rfloor$ then for $\hat{n} = n \mp k$, $Y \notin \lfloor \underline{Y}_{m,\hat{n}}, \overline{Y}_{m,\hat{n}} \rfloor$.

Lemma: $\overline{Y}_{m,n} < \underline{Y}_{m,n+1}$ for all $m \geq 1$ and n.

Proof: After some computations we get that (let $\Delta_n = \sqrt{1 - 1/2^n}$)

$$\text{sign}\{\overline{Y}_{m,n} - \underline{Y}_{m,n+1}\} = \text{sign}\{m(2\Delta_n - \Delta_{n+1}(1 + \Delta_n)) - (1 - \Delta_n)\Delta_{n+1}\}.$$

Noticing that $\Delta_n < \Delta_{n+1}$ and $\Delta_n < 1$ for all n one can check that this is negative for all m and n. Q.E.D.

Footnotes

1. This assumption serves only to simplify the exposition. See Remark 1.

2. See Vives (1982) for an attempt to deal with the case $n = 2$ and $m = 1$, that is, an established duopoly facing a potential entrant.

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Figure 1. The reaction correspondence of firm j , $\psi_j(\cdot)$, is represented by the thick broken line.

Figure 2. Two incumbents. C is the Cournot point. The thick segment is the set of entry preventing equilibria.

Figure 3. Types of equilibria in (m,n) space.

Figure 4. P^C: Cournot price, P^E: entry price, $\overline{P}$: limit price.

Figure 5. The thick line represents total equilibrium output.
B: Blockaded, P: Prevent, A: Allow, entry.

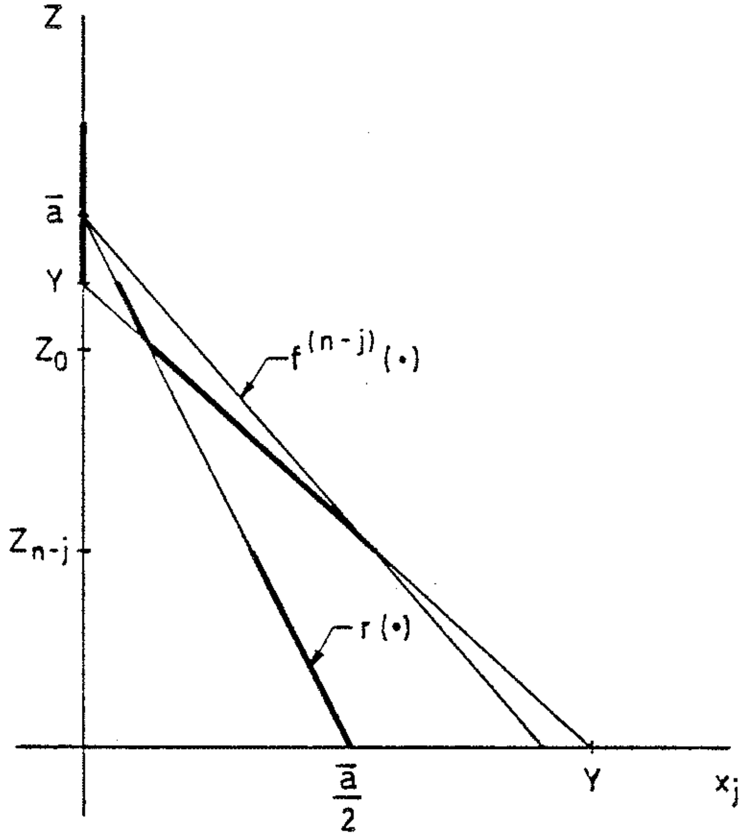


Figure 1.

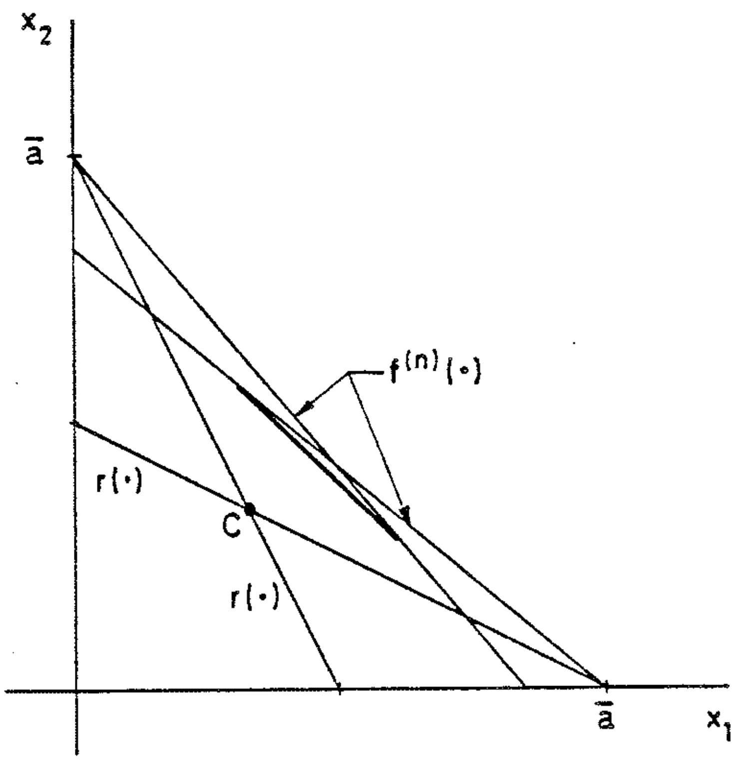
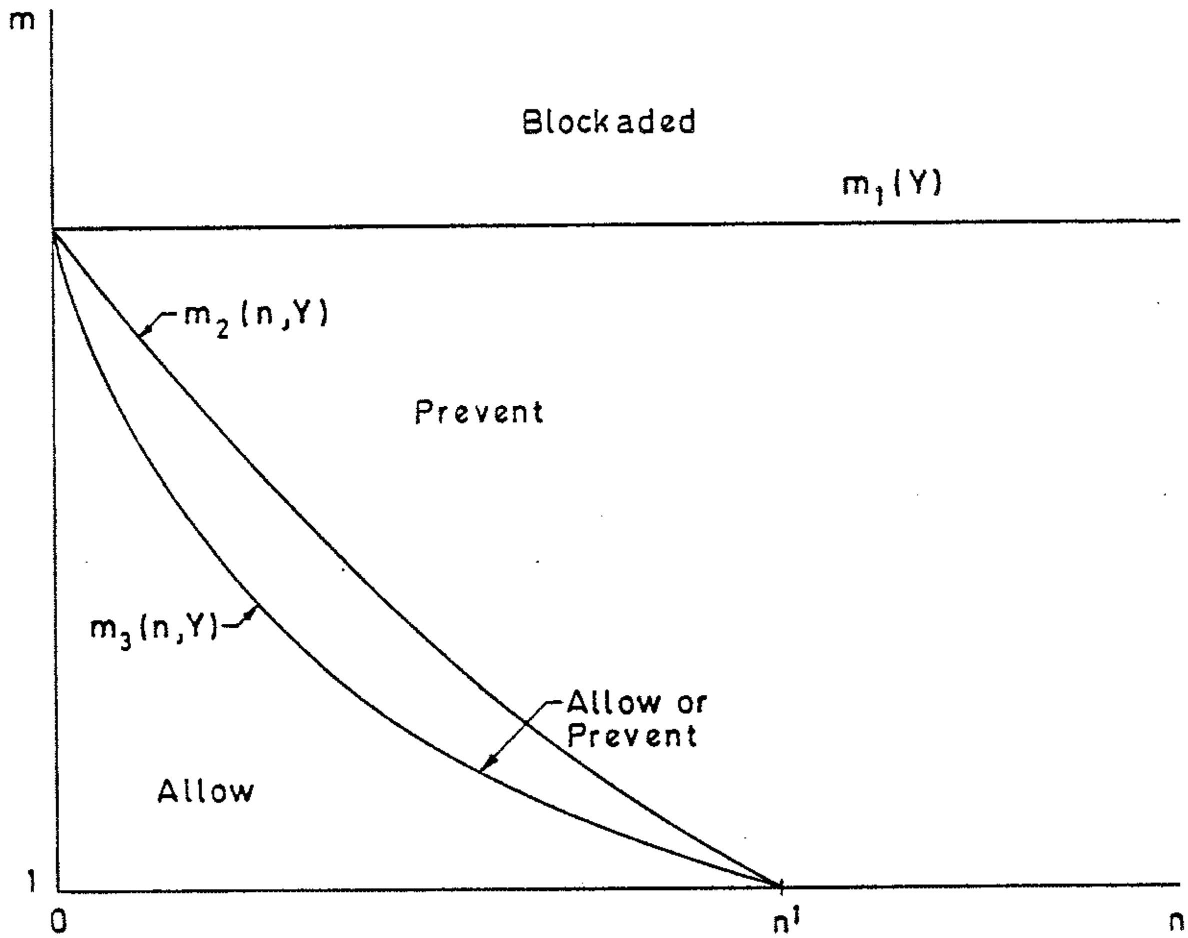


Figure 2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Blockaded
m1(Y)
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Figure 3

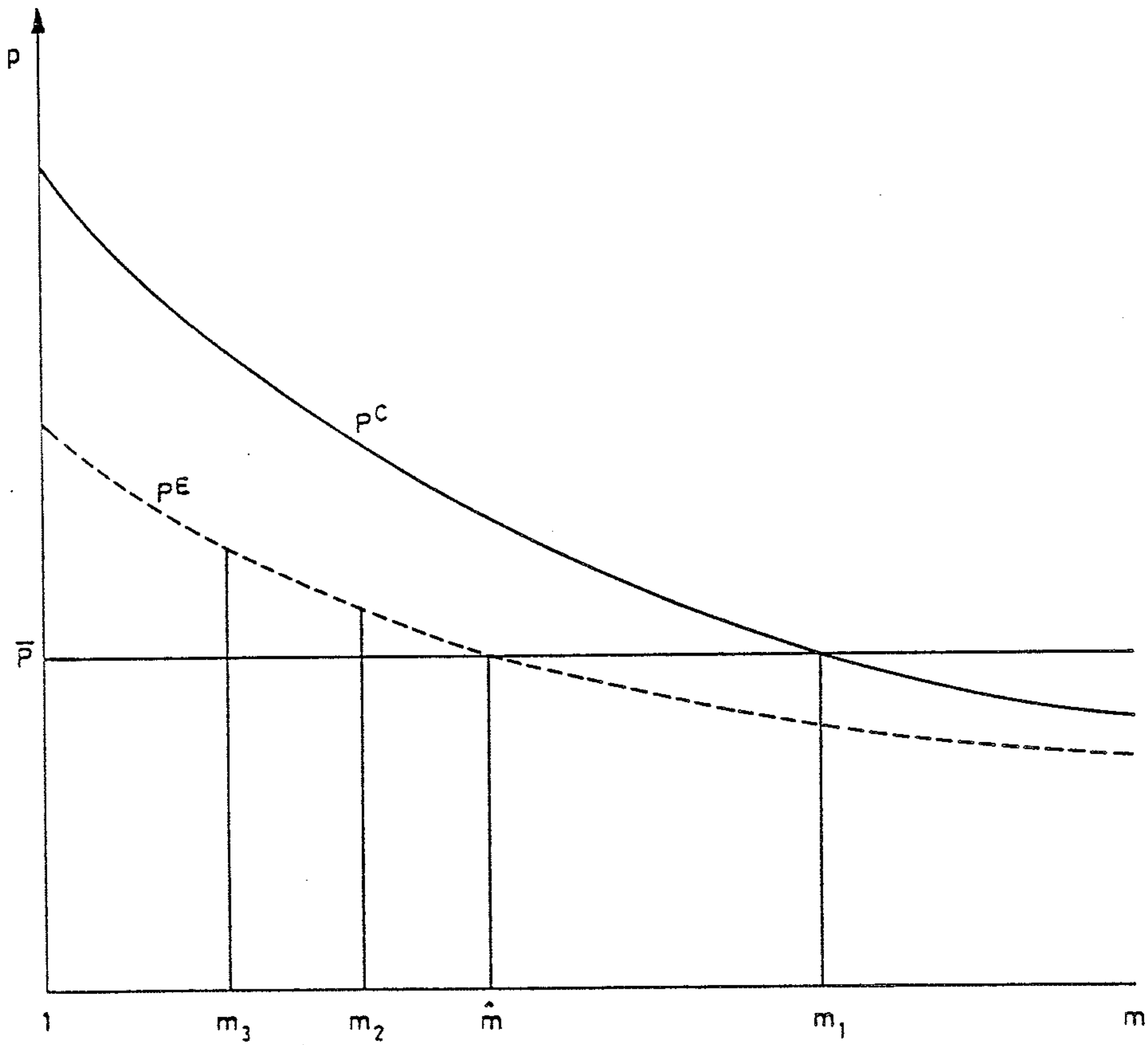

Figure 4