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Authors
Nomura, Yasunori
Sanches, Fabio
Weinberg, Sean J

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The Black Hole Interior in Quantum Gravity

Yasunori Nomura, Fabio Sanches, and Sean J. Weinberg
Berkeley Center for Theoretical Physics, Department of Physics,
University of California, Berkeley, CA 94720 and
Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

We discuss the interior of a black hole in quantum gravity, in which black holes form and evaporate unitarily. The interior spacetime appears in the sense of complementarity because of special features revealed by the microscopic degrees of freedom when viewed from a semiclassical standpoint. The relation between quantum mechanics and the equivalence principle is subtle, but they are still consistent.

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INTRODUCTION

Despite much effort, the relation between quantum mechanics and the spacetime picture of general relativity has never been clear. The issue becomes particularly prominent in black hole physics [1]. Quantum mechanics suggests that the black hole formation and evaporation processes are unitary—a black hole simply appears as an intermediate resonance between the initial collapsing matter and final Hawking radiation states [2]. Meanwhile, general relativity suggests that an observer falling into a large black hole does not feel anything special at the horizon. These two assertions are surprisingly hard to reconcile. With naive applications of quantum field theory on curved spacetime, one is led to the conclusion that unitarity of quantum mechanics is violated [3] or an infalling observer finds something dramatic (a firewall) at the horizon [4–7].

In this letter, we argue that the resolution to this puzzle lies in how a semiclassical description of the system arises from the microscopic theory of quantum gravity. While a semiclassical description employs an *exact* spacetime background, the quantum uncertainty principle implies that there is no such thing—there is an intrinsic uncertainty for background spacetime for any finite energy and momentum. This implies that at the microscopic level there are many different ways to arrive at the same background for the semiclassical theory, within the precision allowed by quantum mechanics. This is the origin of the Bekenstein-Hawking entropy [8, 9]. The semiclassical picture is obtained after coarse-graining these degrees of freedom, which we call *vacuum degrees of freedom* [10].

We argue that much of the puzzle regarding unitary evolution and the interior spacetime of a black hole arises from peculiar features the vacuum degrees of freedom exhibit when viewed from the semiclassical standpoint. In particular, they show properties which we call *extreme relativeness* and *spacetime-matter duality*. The first refers to the fact that the spacetime distribution of these degrees of freedom changes when we adopt a different "reference frame." This change occurs in a way that the answers to any physical question are consistent with each other when asked in different reference frames. Together with the reference frame dependence of the semiclassical degrees of freedom discussed earlier [11, 12], this comprises basic features of how general coordinate transformations work in the full theory of quantum gravity.

The second property is related to the following fact: while the vacuum degrees of freedom are interpreted as how the semiclassical spacetime is realized at the microscopic level, their interactions with semiclassical degrees of freedom make them look like thermal radiation. In fact, these degrees of freedom are neither spacetime nor matter/radiation, as indicated by the fact that their spacetime distribution is frame dependent, and that their detailed dynamics cannot be treated in semiclassical theory. This situation reminds us of wave-particle duality—a quantum object exhibits dual properties of waves and particles while the "true" (quantum) description does not fundamentally rely on either of these classical concepts.

The two properties described above allow us to avoid the arguments in Refs. [4–6] and make the existence of the black hole interior consistent with unitary evolution, in the sense of complementarity [11] as envisioned in Refs. [13, 14]. A notion of geometry carrying information has also been considered recently in Ref. [15] in a different model of black hole evolution; see also Ref. [16] for early discussions. In our picture, we assume that a black hole evaporates through Hawking radiation [9]; for an alternative view, see Ref. [17].

In the rest of the letter, we present our picture using the example of a Schwarzschild black hole formed by collapsing matter in 4-dimensional spacetime. More detailed descriptions are given in the accompanying paper [18].

DISTANT DESCRIPTION

Consider a quantum state representing a black hole of mass M located at some place at rest, as described in a distant reference frame. (We adopt the Schrödinger picture throughout.) Because of the uncertainty principle, such a state must involve a superposition of energy and momentum eigenstates. In particular, since a black hole of mass M will evolve after Schwarzschild time $\varDelta t \approx O(M l_{\rm P}^2)$ into a state representing a Hawking quantum and a smaller mass black hole, the state must involve a superposition with

$$\Delta E \approx \frac{1}{\Delta t} \approx O\biggl(\frac{1}{M l_{\rm P}^2}\biggr), \tag{1}$$

where E is defined in the asymptotic region, and $l_{\rm P}$ the Planck length. Requiring that the position uncertainty is comparable to the quantum stretching of the horizon $\Delta r \approx O(1/M)$, where r is the Schwarzschild radial coordinate, the momentum spread is $\Delta p \approx O(1/M l_{\rm P}^2)$. This gives an uncertainty of the kinetic energy much smaller than ΔE, so the spread of the energy comes mostly from a superposition of different rest masses: $\Delta E \approx \Delta M$.

How many different independent ways are there to superpose the energy eigenstates to arrive at the same black hole geometry within this precision? We assume that the Bekenstein-Hawking entropy, $\mathcal{A}/4l_{\rm P}^2$, gives the logarithm of this number (at the leading order in $l_{\rm P}^2/\mathcal{A}$), where $\mathcal{A} = 16\pi M^2 l_{\rm P}^4$ is the area of the horizon. The nonzero Bekenstein-Hawking entropy implies that there are exponentially many independent black hole *vacuum* states in a small energy interval of Eq. (1):

$$S_0 = \frac{\mathcal{A}}{4 l_{\rm P}^2} + O\biggl(\frac{\mathcal{A}^q}{l_{\rm P}^{2q}};\, q<1\biggr), \tag{2}$$

i.e. the states that do not have a field/string theoretic excitation on the semiclassical black hole background and in which the stretched horizon, located at $r = 2Ml_{\rm P}^2 + O(1/M) \equiv r_{\rm s}$, is not excited.

Labeling these exponentially many states by k, which we call the *vacuum index*, basis states for the general microstates of a black hole of mass M (within the uncertainty ΔM) can be given by

$$|\Psi_{\bar{a}\, a\, a_{\rm far}; k}(M)\rangle \approx |\psi_{\bar{a}a;k}(M)\rangle |\phi_{a_{\rm far}}(M)\rangle. \tag{3}$$

Here, $\bar{a}$, a, and $a_{\rm far}$ label the excitations of the stretched horizon, in the zone (i.e. the region within the gravitational potential barrier defined, e.g., as $r \leq R_{\rm Z} \equiv 3Ml_{\rm P}^2$), and outside the zone ($r > R_{\rm Z}$), respectively, and $|\psi_{\bar{a}a;k}(M)\rangle$ and $|\phi_{a_{\rm far}}(M)\rangle$ are black hole and exterior states. (Here, we have used the fact that k can be regarded as being mostly in $r \leq R_{\rm Z}$; see later.) As we have argued, the index k runs over $1, \cdots, e^{S_0}$ for the vacuum states $\bar{a} = a = a_{\rm far} = 0$. In general, the range for k depends on $\bar{a}$ and a, but its dependence is higher order in $l_{\rm P}^2/\mathcal{A}$ so we mostly ignore it. This small dependence, however, becomes relevant when we discuss negative energy excitations associated with Hawking emission.

Excitations here are defined as fluctuations with respect to a fixed background, so their energies as well as entropies can be either positive or negative, although their signs must be the same. As discussed in Refs. [19, 20], the contribution of the excitations to the total entropy is subdominant in $l_{\rm P}^2/\mathcal{A}$. The total entropy in the near black hole region, $r \leq R_{\rm Z}$, is thus given by $S = \mathcal{A}/4l_{\rm P}^2$ at the leading order.

The fact that all the independent microstates with different k lead to the same geometry suggests that the semiclassical picture is obtained after coarse-graining the degrees of freedom represented by this index, the vacuum degrees of freedom [10]. According to this picture, the black hole vacuum state in the semiclassical description is given by the density matrix

$$\rho_0(M) = \frac{1}{e^{S_0}} \sum_{k=1}^{e^{S_0}} |\Psi_{\bar{a}=a=a_{\rm far}=0;k}(M)\rangle \langle \Psi_{\bar{a}=a=a_{\rm far}=0;k}(M)|. \tag{4}$$

To obtain the response of this state to the operators in the semiclassical theory, we may trace out the subsystem on which they do not act. Denoting this subsystem by $\bar{C}$, the relevant reduced density matrix is

$$\tilde{\rho}_0(M) = {\rm Tr}_{\bar{C}}\, \rho_0(M). \tag{5}$$

Consistently with our identification of the origin of the Bekenstein-Hawking entropy, we assume that this represents the thermal density matrix

$$\tilde{\rho}_0(M) \approx \frac{e^{-\beta H_{\rm sc}(M)}}{{\rm Tr}\, e^{-\beta H_{\rm sc}(M)}}; \quad \beta = \begin{cases} \frac{1}{T_{\rm H}} & \text{for } r \leq R_{\rm Z}, \\ +\infty & \text{for } r > R_{\rm Z}, \end{cases} \tag{6}$$

where $T_{\rm H} = 1/8\pi M l_{\rm P}^2$, and $H_{\rm sc}(M)$ is the Hamiltonian of the semiclassical theory. Here, the expression $\beta H_{\rm sc}(M)$ should really be interpreted as β times the Hamiltonian density integrated over space.

In standard semiclassical field theory, the density matrix of Eq. (6) is obtained as a reduced density matrix by tracing out the region within the horizon in the *unique* global black hole vacuum state. Our view is that this density matrix is obtained from a mixed state of exponentially many pure states, arising from the coarse-graining in Eq. (4). We stress that the information in the vacuum index k is invisible in the semiclassical theory as it is already coarse-grained *to obtain* the theory; in particular, the dynamics of the vacuum degrees of freedom cannot be described in terms of $H_{\rm sc}(M)$.

The expression in Eq. (6) suggests that the spatial distribution of the information about k follows the thermal entropy calculated using the local temperature:

$$T(r) \simeq \begin{cases} \frac{T_{\rm H}}{\sqrt{1-\frac{2Ml_{\rm P}^2}{r}}} & \text{for } r \leq R_{\rm Z}, \\ 0 & \text{for } r > R_{\rm Z}. \end{cases} \tag{7}$$

In particular, the region around the edge of the zone, $r \leq R_{\rm Z}$ and $r - 2Ml_{\rm P}^2 \not\ll M l_{\rm P}^2$, contains $O(1)$ bits of information about k.

Semiclassical operators in the zone act nontrivially on both a and k indices; otherwise the maximal mixture in Eq. (4) is not compatible with the thermality in Eq. (6).

Since the thermal nature is prominent only for modes whose energies measured in the asymptotic region are

$$\omega \lesssim T_{\rm H}, \tag{8}$$

this feature is significant only for such infrared modes. For operators with Eq. (8), their actions on microstates can be complicated, although they act on the coarse-grained vacuum state of Eq. (4) as if it is the thermal state in Eq. (6).

There is a simple physical picture behind this phenomenon of "non-decoupling" of the a and k indices for the infrared modes. As viewed from a distance, these modes are "too soft" to be resolved clearly above the background. Since the derivation of the semiclassical theory involves coarse-graining over microstates in which the energy stored in the region $r \lesssim R_{\rm Z}$ has spreads of order $\varDelta E \approx 1/Ml_{\rm P}^2$, infrared modes with $\omega \lesssim T_{\rm H} \approx O(1/Ml_{\rm P}^2)$ are not necessarily distinguished from "spacetime fluctuations" of order $\varDelta E$.

The structure described above leads to the following picture for black hole evaporation.[1] Suppose a black hole of mass M is in microstate k:

$$|\Psi_k(M)\rangle = |\psi_k(M)\rangle |\phi_I\rangle, \tag{9}$$

where $|\psi_k(M)\rangle$ is the black hole state, with suppressed excitation indices, and $|\phi_I\rangle$ the exterior state. After a timescale of $t \approx O(Ml_{\rm P}^2)$, this state evolves due to Hawking emission as

$$|\psi_k(M)\rangle |\phi_I\rangle \rightarrow \sum_{i,a,k'} c^k_{iak'} |\psi_{a;k'}(M)\rangle |\phi_{I+i}\rangle, \tag{10}$$

where $|\phi_{I+i}\rangle$ is the state in which newly emitted Hawking quanta, labeled by i and having energy E_i, are added to the appropriately time evolved $|\phi_I\rangle$. The index a represents the fact that the black hole state has negative energy excitations of energy $-E_a$ around the edge of the zone, created in connection with the Hawking emission; the coefficients $c^k_{iak'}$ are nonzero only if $E_i \approx E_a$ (within the uncertainty). The negative energy excitations then propagate inward, and after a time of order $Ml_{\rm P}^2 \ln(Ml_{\rm P})$ collide with the stretched horizon, making the black hole states relax as

$$|\psi_{a;k'}(M)\rangle \rightarrow \sum_{k_a} d^{ak'}_{k_a} |\psi_{k_a}(M - E_a)\rangle. \tag{11}$$

The combination of Eqs. (10, 11) yields

$$|\psi_k(M)\rangle |\phi_I\rangle \rightarrow \sum_{i,k_i} \alpha^k_{ik_i} |\psi_{k_i}(M - E_i)\rangle |\phi_{I+i}\rangle, \tag{12}$$

where $\alpha^k_{ik_i} = \sum_{a,k'} c^k_{iak'} d^{ak'}_{k_i}$, and we have used $E_i = E_a$. This expression shows that information in the black hole can be transferred to the radiation state i.

It is important that the negative energy excitations in Eq. (10) come with *negative entropies*, so that each of the processes in Eqs. (10, 11) is separately unitary. Specifically, as k and i run over all the possible values with a being fixed, the index k' runs only over $1, \cdots, e^{S_0(M-E_a)}$, the dimension of the space spanned by k_a. Here, $S_0(M) \equiv 4\pi M^2 l_{\rm P}^2$. This is an example of the non-factorizable nature of the k and a indices discussed after Eq. (3). This structure avoids the firewall argument in Ref. [5]—unlike what is imagined there, the physical Hilbert space is smaller than the naive Fock space built on each k.

From the semiclassical standpoint, the emission of Eq. (10) is viewed as occurring locally around the edge of the zone, which is possible because the information about the black hole microstate extends into the whole zone region. In this region, information stored in the vacuum state, k, is transferred into that in modes $a_{\rm far} \neq 0$, which have clear identities over the background spacetime. Because of energy conservation, this process is accompanied by the creation of ingoing negative energy excitations. Unlike standard pair creation, however, these excitations are *not* (maximally) entangled with the emitted Hawking quanta.

The discussion here indicates that the purifiers of the emitted Hawking quanta are microstates which semiclassical theory describes as a vacuum. Unlike what was considered in Ref. [4], Hawking quanta are not modes associated solely with one of the Rindler wedges in the near horizon approximation (b modes in the notation of Ref. [4]) nor outgoing Minkowski modes (a modes), which would appear to have high energies for infalling observers. This allows for avoiding the entropy [4] and typicality [6] arguments for firewalls. Note that physics described here need not introduce nonlocality in low energy field theory; it can still respect causality in $r > r_{\rm s}$.

We emphasize that the vacuum degrees of freedom play *dual* roles. While they represent how the semiclassical spacetime is composed at the microscopic level, they also appear as thermal radiation when probed in the semiclassical theory. In fact, these degrees of freedom are neither spacetime nor matter/radiation. In particular, their detailed dynamics cannot be treated in semiclassical theory.

The above understanding of Hawking emission clarifies why the semiclassical calculation of Ref. [3] finds an apparent violation of unitarity. At the microscopic level, formation and evaporation of a black hole involve the vacuum degrees of freedom. Since semiclassical theory is incapable of describing their microscopic dynamics, the description of black hole evolution in semiclassical theory is necessarily non-unitary.

A similar analysis can also be performed for black hole mining [21, 22]. See Ref. [18] for details.

[1] We focus on a single Hawking emission and ignore excitations beyond those directly associated with the emission. For a more complete discussion, see Ref. [18].

INFALLING DESCRIPTION

Suppose we drop an object into a black hole. In a distant reference frame, the semiclassical description of the object (in terms of a and $a_{\rm far}$) is applicable only until it hits the stretched horizon, after which it is represented as excitations of the stretched horizon (in terms of $\bar{a}$). The information about the fallen object will then stay there, at least, for the scrambling time of order $Ml_{\rm P}^2 \ln(Ml_{\rm P})$ [23] before being transferred to k. On the other hand, the equivalence principle says that the falling object does not feel anything special when it crosses the horizon. How can these two pictures be consistent?

The idea of complementarity is that the infalling object is still described using low energy language after it crosses the Schwarzschild horizon by making an appropriate reference frame change. Here we consider a class of reference frames which reveal the spacetime structure near the Schwarzschild horizon in the clearest form. We call them *infalling reference frames*.

Let the spatial origin p_0 of a reference frame follow a timelike geodesic released from rest at $r = r_0$, with $r_0 - 2Ml_{\rm P}^2 \gtrsim Ml_{\rm P}^2$. According to complementarity, the system described in this reference frame does not have a (hot) stretched horizon at the location of the Schwarzschild horizon when p_0 crosses it; the region around p_0 appears approximately flat up to small curvature effects.

In this description, a "horizon" signaling the breakdown of the semiclassical description is expected to appear in the past-directed and inward directions from p_0. In analogy with the case of a distant frame description, we denote basis states for the general microstates as

$$|\Psi_{\bar{\alpha}\,\alpha\,\alpha_{\rm far};\kappa}(M)\rangle, \tag{13}$$

where $\bar{\alpha}$ labels the excitations of the "horizon," and α, and $\alpha_{\rm far}$ the semiclassical excitations near and far from the black hole, respectively; κ is the vacuum index.

The complementarity transformation provides a map between the states in Eq. (3) and those in Eq. (13). While the general form of this transformation can be complicated, we may consider, based on the analysis of an infalling object, that a portion of the α index representing interior excitations is transformed into the $\bar{a}$ index (and vice versa). Note that the amount of information needed to reconstruct the interior (in the semiclassical sense) is much smaller than the Bekenstein-Hawking entropy—the logarithm of the dimension of the relevant Hilbert space is of order $(\mathcal{A}/l_{\rm P}^2)^q$ with $q < 1$.

Where are the κ degrees of freedom located? We expect that most are in the region close to the "horizon"; in particular, the number of κ degrees of freedom within a distance sufficiently smaller than $Ml_{\rm P}^2$ from p_0 is of $O(1)$, since the time and length scales characterizing local deviations from Minkowski space are of order $Ml_{\rm P}^2$ there. This invites a question: how can this picture be consistent with that in the distant reference frame, which has a very different spacetime distribution of the vacuum degrees of freedom?

To see a nontrivial consistency between the two pictures, consider detectors hovering at a constant r with $r - 2Ml_{\rm P}^2 \ll Ml_{\rm P}^2$. In a distant description, the spatial density of the microscopic information in k is large there, so that these detectors can be used for black hole mining. The rate of extracting information, however, is still of order one qubit per Schwarzschild time $t \approx O(Ml_{\rm P}^2)$ *per channel* [22]—the acceleration of information extraction occurs not because of a higher rate in each channel but because of an increased number of available channels. This implies that each single detector, which we define to act on a single channel, "clicks" once per $t \approx O(Ml_{\rm P}^2)$.

In an infalling reference frame, the density of the microscopic information in κ is small at the detector location, at least when p_0 passes nearby. The rate of extracting information thus cannot be much faster than $1/Ml_{\rm P}^2$ around p_0, reflecting the fact that the spacetime appears approximately flat there. This, however, is still consistent with the distant description. By adopting the near-horizon Rindler approximation, one can show that when viewed from the infalling reference frame, the detector clicks only once in each time/space interval of

$$\varDelta T \approx \varDelta Z \approx O(Ml_{\rm P}^2), \tag{14}$$

around p_0 [18]. This is what we expect from the equivalence principle: the spacetime appears flat up to curvature effects with lengthscale $Ml_{\rm P}^2$. While the detector clicks of order $\ln(Ml_{\rm P})$ times within the causal patch of the infalling frame, these clicks occur at distances of order $Ml_{\rm P}^2$ away from p_0, where we expect a higher density of κ degrees of freedom.

The two descriptions are thus consistent. It is striking that the microscopic information about a black hole exhibits this level of reference frame dependence, a phenomenon we refer to as *extreme relativeness*.

OTHER REFERENCE FRAMES

We now discuss a reference frame whose origin follows a timelike geodesic released from rest at $r = r_0$, where r_0 is *close to* the Schwarzschild horizon, $r_0 - 2Ml_{\rm P}^2 \ll Ml_{\rm P}^2$. In the case of $r_0 - 2Ml_{\rm P}^2 \gtrsim Ml_{\rm P}^2$, we found that the detector-click time/length scales are given by Eq. (14), despite the fact that the detector clicks at a much higher rate in its own frame. Technically, this was due to a huge relative boost between p_0 and the detector when they approach. Here, however, the relevant boost is not as large, and the detector-click time/length scales appear as

$$\varDelta T \approx \varDelta Z \ll Ml_{\rm P}^2. \tag{15}$$

Since each detector click extracts an $O(1)$ amount of information from spacetime, which we expect not to occur in Minkowski space, this implies that the spacetime as viewed from this reference frame is not approximately Minkowski over the lengthscale $Ml_{\rm P}^2$ when p_0 crosses the Schwarzschild horizon. We interpret this to mean that in this reference frame, the "horizon" is at a distance of order ΔZ away from p_0, so that detector clicks occur near or "on" this surface. Since we expect that the microscopic information is located near and on the "horizon," there is no inconsistency for the clicks to extract information from the black hole.

One might worry that in this reference frame, spacetime near the Schwarzschild horizon does not appear large, $\approx O(Ml_{\rm P}^2)$, nearly flat space. However, the *existence* of an infalling reference frame discussed before ensures that an infalling physical observer sees a large black hole interior. The analysis here simply says that the spacetime around the Schwarzschild horizon is not always *described* as a large nearly flat region, even in reference frames falling freely into the black hole.

We finally discuss (non-)relations of black hole mining and the Unruh effect [24] in Minkowski space. It is often thought that these two reveal the same physics, which would mean the existence of a "horizon" in an *inertial* frame description of Minkowski space. This is, however, not true. Since the equivalence principle can make a statement only about a point at a given moment in a given reference frame, while a system in quantum mechanics is specified by a state which encodes global information on the equal-time hypersurface, there is no reason that physics of the two systems must be similar beyond a point in space. In particular, the inertial frame description of Minkowski space does not have a "horizon," so a detector reacts very differently to blueshifted Hawking radiation and Unruh radiation in Minkowski space—it extracts microscopic information about spacetime in the former case, while it does not in the latter. The relation between quantum mechanics and the equivalence principle seems subtle, but they are consistent.

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