.

Risk Taking and Gender in Hierarchies

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Abstract: If promotion in a hierarchy is based on a random signal of ability, rates of promotion are affected by risk-taking. Further, the statistical properties of the surviving populations of risk-takers and non-risk-takers will be different, and will be changing throughout the hierarchy. I define promotion hierarchies with and without memory, where memory means that promotion depends on the entire history of success. In both types of hierarchies, surviving risk-takers have lower average ability than surviving non risk-takers at any stage where they have a higher probability of survival. However, that will not apply in the limit. With a common set of promotion standards, risk-takers will survive with lower probability than non risk-takers, and will have higher average ability. I give several interpretations for how these theorems relate to affirmative action, in light of considerable evidence that males are more risk-taking than females.

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1 Introduction

In this paper, I study the effect of risk-taking on promotion in hierarchies, where promotion at each stage depends on a signal of ability. The motivation comes from a substantial body of evidence that males are more risk-taking than females, and from the continuing controversy about why males and females have different patterns of success in labor markets. Granting the premise that the genders differ in risk-taking, does this have explanatory power for labor markets? The answer is mixed, partly because the theorems below can be applied to labor markets in different ways.

The theorems proved below compare promotions drawn from two subpopulations, one of which generates accurate signals of ability and the other of which generates noisy signals of ability. The premise is that true abilities (which may be defined differently in different hierarchies) have the same distribution in both populations, at least initially, but that agents in one population give a noisy signal to the decision maker. This is a reduced-form hypothesis which might follow from preferences and optimizing behavior, or might reflect behavior that is hard-wired. This distinction does not matter for the theorems that I prove, although it may matter for the interpretation.

The main point of the paper is to understand how promotion plays out for the two populations in a hierarchy with a large (infinite) number of stages, under various assumptions about the promotion standards, which may or may not be gender blind.

I define two types of promotion hierarchies: those with memory and those without memory. In a hierarchy without memory, promotion at stage t depends only on the signal of ability generated in stage t. With memory, promotion can depend on the entire history of signals. Hierarchies such as sports tournaments do not have memory, since survival depends only on winning the current match. Hierarchies such as academic labor markets have memory, although promotion would typically depend more heavily on current performance than on past performance. To maximize the difference between hierarchies with memory and those without, I assume for the case of memory that all past signals are used symmetrically. There is no extra weight given to recent performance. The thrust of the discussion below is that memory does not matter very much for the main conclusions.

In section 2, I discuss some of the evidence that males are more risk taking than females, and draw out some contradictions among the promotion objectives of (a) promoting according to gender-blind standards, (b) promoting equal numbers, and (c) promoting populations with equal average abilities. There is no promotion policy that equalizes both the numbers of survivors and their average abilities.

In sections 3 and 4, respectively, I develop formal results about hierarchies with and without memory. In both cases, if the objective is to equalize abilities, then more of the non risk-takers (females) must be promoted than risk takers. Risk taking can boost the probability of survival, but a surfeit in the number of surviving risk-takers at any stage coincides with a deficit in their ability, regardless of how the standards are chosen.

Under gender-blind standards that are relatively stringent at the beginning, so that fewer than half the population survive, the surviving risk takers will be more numerous and less able than non risk takers. However, this cannot persist in the limit. The ratio of surviving risk-takers to surviving non risk-takers declines until risk-takers are eventually underrepresented. At the same time, their average ability increases, and eventually exceeds that of non risk-takers.

Standards chosen to equalize either the numbers or abilities of survivors will not be gender blind. If the goal is to equalize numbers or abilities, it may be either the males or females who need an affirmative action boost in the beginning, depending on whether the standards are relatively lenient or stringent. However, to achieve either equal numbers or equal abilities, it is the males (risk takers) who need an affirmative action boost at the end.

The approach of this paper is to take the promotion standards as exogenous, characterized by weak properties such as nondecreasing or bounded, and to compare the statistical properties of two populations of survivors. This approach can be seen as a complement to the literature that seeks to explain how promotion standards are chosen in the first place, *e.g.*, Sobel (2001), and to papers on promotion hierarchy where other aspects of behavior are being selected, *e.g.*, Harrington (1999).

2 Risk-Taking and Promotion in a Hierarchy

One of the motivations for this inquiry is the considerable evidence that males are more risk-taking than females. In particular, see Eckel and Grossman (2005a). Their own experiments (2002) show that males and females have different gambling behavior. In other experiments (2005b) they show not only that females are more risk averse, but that other agents (not just researchers on gender) perceive this to be true. Eckel and Grossman (2005a) argue that the evidence on a discrepancy in risk-taking is especially strong in "field studies" (natural experiments such as observing behavior in placing bets), but less conclusive in "contextual environmental" experiments such as experiments involving insurance choices. One of the most interesting risk-taking contexts is investment. In a study that used measures of risk tolerance reported in the Wall Street Journal, and measures of personality traits developed by psychologists, Stanford and Vellenga (2002) found that males have much higher risk tolerance than females. Jianakoplos and Bernasek (1998) came to the same conclusion by observing investment portfolios. Much of the experimental evidence comes from disciplines other than economics. For example, psychologists Ginsburg et al (2002) observed children at a zoo in contexts where the children could choose to engage in a risky activity or not. They concluded strongly that young boys were much more inclined to put themselves at risk than young girls.

Many scholars have suggested evolutionary arguments for the discrepancy in risk-taking behavior. For example, Dekel and Scotchmer (1999) postulated that males play "winner-take-all" games, and explored a precise sense in which such games do (or do not) lead to riskier behavior. The premise in that paper, which is also the easiest interpretation of the model below, is that risk taking is genetically coded. But this does not preclude that agents make choices. For example, in academic life, a risk-taker might work on new and unfashionable topics, while a non risk-taker might extend the work of others. If the risk-taker fails to find an audience, he or she will fail to get tenure. To some extent, the risky choice can follow from cold, hard calculation about the merits of risk-taking, but if males and remales have different genetic predispositions, they will make different tradeoffs. That is why I refer to the hypothesis on risk taking as "reduced form."

Affirmative action policies have been justified and evaluated on both efficiency grounds and equity grounds. For the most part, economists have focussed on efficiency, especially productive efficiency. For example, Holzer and Neumark (2000) argue from an extensive empirical literature that "affirmative action offers significant redistribution toward females and minorities, with relatively small efficiency consequences" (page 559). Among the ingenious theoretical arguments for why affirmative action policies enhance efficiency are those of Lundberg and Startz (1983) and Lundberg (1991), who consider a model of statistical discrimination where wages depend on imperfect signals of ability. They show, among other things, that if workers with different signaling ability are pooled, there is more incentive to invest in human capital. Milgrom and Oster (1987) argue that affirmative action policies can efficiently prevent employers from underpromoting females and minorities. The incentive to underpromote derives from a fear of revealing the worth of their employees to rival firms, a threat which is higher for the more "invisible" workers, such as females and minorities.

Hierarchies introduce a new dimension to the affirmative action problem, since the relative success of two populations at any stage is importantly determined by policies at previous stages, and risk-taking matters due to randomness. I consider labor market hierarchies, in which promotion to stage t requires prior promotion to stage $t-1$. I take investments in human capital as exogenous, and assume that wages at each stage of the hierarchy are immutable. My focus is entirely on rates of promotion and whether the "right" workers are promoted, if the objective is to select on ability.

Examples of such hierarchies might be

- law, where law students are promoted to associates in law firms, associates are promoted to partner, and some partners eventually become judges;
- the executive hierarchy of corporations;
- academic life where undergraduates are promoted to graduate student, graduate students are promoted to assistant professor, and assistant professors are promoted to full professor.

The modern legal environment prohibits discrimination in labor practices. However, discrimination is hard to define. Figure 1 shows that the following three objectives are pairwise inconsistent:

- equal promotion standards
- equal numbers of promotions
- promotion of a pool of agents with equal average ability.

In Figure 1, the distribution of true ability a is shown by density g. The distribution of true ability is assumed to be the same in both populations, a risk-taking population (say, males) and a risk-averse population (say, females). The density $\tilde{g}$ represents the distribution of signals that the risk-taking population will generate, when their true ability a is confounded by noise. The signal of a random male will be $\sigma = a + u$, where a is his true ability, and u is distributed according to a cumulative distribution function Φ with mean zero.

Consider the first round of promotions. Suppose that the promotion standard for males is $\bar{c}$. That is, every male who generates a signal above $\bar{c}$ is promoted. The

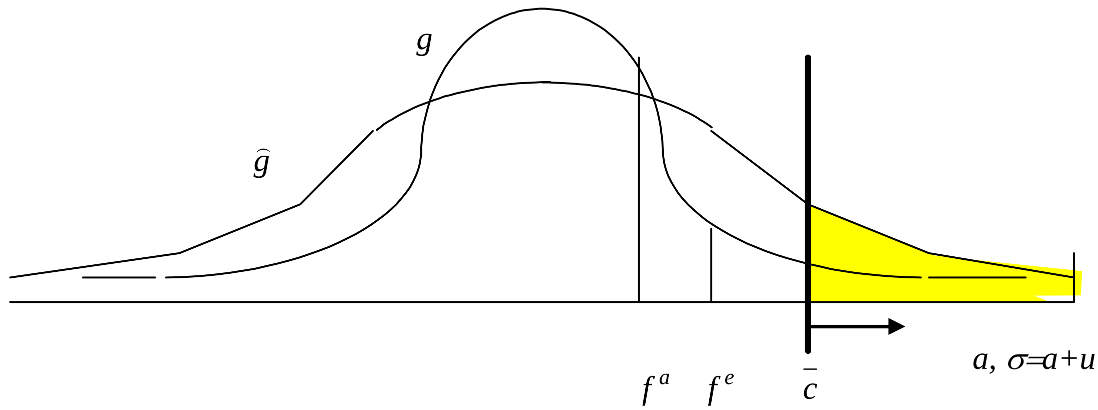

Figure 1: First Stage of a Hierarchy

other promotion standards are for females: The promotion standard f^e will ensure that females are promoted with the same probability as males, and the promotion standard f^a will ensure that the expected ability of promoted females is the same as that of promoted males. If the promotion policy is gender blind, then females are also promoted according to the standard $\bar{c}$. In the example of figure 1, where more males than females are promoted (because $\bar{c}$ is above the mean), the promoted females have higher expected ability than promoted males.

The average ability of promoted females is higher in figure 1 because there are fewer of them. To promote more males, it is necessary to reach further down into the ability distribution. In addition, some of the male promotions are mistakes. This insight is formalized below in Lemmas 1 and 6 below. At every stage of the hierarchy, surviving females have higher expected ability whenever the expected number of surviving males is at least as large, regardless of what proportion of the total pool is promoted.

A gender-blind policy that promotes more males than females, such as the one shown in figure 1, is inhospitable to women, however reasonable it may seem from a procedural point of view, due to the gender-blind promotion standard. Consider instead an "affirmative action" policy to promote equal *numbers*, as shown by the promotion standard f^e in Figure 1. Then

- if equal numbers are promoted, the promoted females have higher ability than the promoted males; and
- the promotion standard for females is lower than for males if fewer than half are promoted at stage 1, and otherwise higher.

An affirmative action policy aimed at equal numbers is still inhospitable to females in the sense that, on average, promoted females have higher ability. Their superior ability is due to the fact that, in promoting males, mistakes are made in both directions. Low-ability males are promoted, and high-ability males are excluded. Females could reasonably argue that the system should impose an even *lower* bar for females, in order to remedy the discrepancy in average (and marginal) ability.

Consider then an affirmative action policy aimed at ensuring equal *ability* of both promoted groups, instead of equal numbers. The female standard is shown as f^a in figure 1. Then

- fewer males than females will be promoted; and
- the standard for female promotion should be even lower than the one that equalizes numbers.

The much lower promotion standard for females is a bit paradoxical, especially when fewer than half of the pool are promoted, as shown in figure 1. The lower promotion standard appears to favor females of lower ability than males, but a higher standard must be applied to males in order to compensate for the mistakes.

The graphical discussion only illuminates the first stage of promotion. The question, however, is what happens in subsequent stages, as the distribution of abilities in the pool changes. At the second stage, some high-ability males have been eliminated due to randomness, and some low-ability males remain. Males who survived stage one have another opportunity to eliminate themselves at every stage. Despite their good start, fewer and fewer males are promoted as the hierarchy progresses, and the statistical properties of the surviving populations may invert, as discussed in the introduction.

The theorems in the next two sections, which are the main content of the paper, can be interpreted in several ways. I return to these various interpretations in section 5.

3 The Hierarchy without Memory

Each agent's ability, denoted $a \in \mathbf{R}$, is drawn independently from a distribution G with density g. An agent generates a signal of ability $\sigma_t \in \mathbf{R}$ in period t. If the agent is female with ability a, we assume that $\sigma_t = a$ (the signal is nonrandom). If the agent is a male with ability a, $\sigma_t = a + u_t$, where the random noise u_t is distributed according to a cumulative distribution function Φ with mean zero and finite variance, and the random draws of noise in different stages of the hierarchy are independent. The designations "male" and "female" refer to the riskiness of the signals that are produced. This analysis would obviously apply to any two groups that differ in the randomness of their signals. In that sense, the designations male and female are only illustrative, and can even be reversed (see below).

Promotion standards are a sequence of real numbers, $c = c_1, c_2, ...c_t...$ If the hierarchy does not have memory, a male agent with ability a *survives* to stage t if $\sigma_d \geq c_d$ for each $d \leq t$ (or $a + u_t \geq c_d$ for each $d \leq t$) and a female agent with ability a *survives* to stage t if $a \geq c_d$ for each $d \leq t$. We say that the promotion standards are *gender-blind* if males and females face the same promotion standards. When we do not assume gender-blind promotion standards, we will refer to the males' promotion standards by $m = m_1, m_2, ..., m_t, ...$ and to the females' promotion standards by $f = f_1, f_2, ..., f_t, ...$

For females, we can assume without loss of generality that the promotion standards are nondecreasing. If at any point a higher cutoff is followed by a lower cutoff, that is, $f_{t+1} < f_t$, then f_{t+1} can be replaced by f_t with no consequence. If f is nondecreasing, a female survives to stage t if $a \geq f_t$ and does not survive otherwise. Hence the probability that a random female survives to stage t is

$$\int_{f_t}^{\infty} g(a)da \tag{1}$$

A male with ability a survives to stage t if $a + u_d \geq m_d$ for all $d \leq t$. Hence the probability that a random male survives to stage t is

$$\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \Phi(m_d - a))da \tag{2}$$

The expected ability of a random female who survives to stage t is

$$\int_{f_t}^{\infty} a \, \frac{g(a)}{\int_{f_t}^{\infty} g(a)da} da = \int_{f_t}^{\infty} a \, \frac{g(a)}{1 - G(f_t)} da \tag{3}$$

and the expected ability of a random male who survives to stage t is

$$\int_{-\infty}^{\infty} a \, \frac{g(a)\Pi_{d=1}^{t}(1 - \Phi(m_d - a))}{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \Phi(m_d - a))da} da \tag{4}$$

We use the following assumptions, which are assumed throughout.

1. The distribution G is symmetric[2] and strictly increasing, has a density g that is strictly quasiconcave and continuous, has the real line as support, and has finite variance.

2. The distribution Φ is symmetric and strictly increasing, has a density ϕ, has the real line as support, has zero mean, and has finite variance.

We begin with two lemmas. The intuition for the first lemma is that the promoted males include mistakes in both directions. Lower-ability males are promoted

[2] For all x in the support, $G(x) = 1 - G(-x)$ and $g(x) = g(-x)$.

by mistake, and higher-ability males are excluded by mistake. Since no mistakes are made in promoting females, the only way to ensure that promoted males have as high ability as females is to promote fewer of them.

Lemma 1 *Let m, f be promotion standards for males (risk-takers) and females (non risk-takers) in a hierarchy without memory. The expected ability of a random surviving male is lower than the expected ability of a random surviving female at any stage t at which males have at least as high a probability of survival.*

Proof: With a change of variables, $y = a - f_t$, the females' expected ability conditional on survival to t, (3), can be written:

$$\int_0^{\infty} (f_t + y) \frac{g(f_t + y)}{\int_0^{\infty} g(f_t + y)\, dy} dy = f_t + \int_0^{\infty} y \frac{g(f_t + y)}{\int_0^{\infty} g(f_t + y)\, dy} dy \tag{5}$$

For males, with a change of variables $y = a - f_t$, the expected ability conditional on survival to t, (4), can be written:

$$\int_{-\infty}^{\infty} (f_t + y) \frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))dy} dy$$

$$= f_t + \int_{-\infty}^{\infty} y \frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))dy} dy \tag{6}$$

$$= f_t + \int_{-\infty}^{0} y \frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))dy} dy + \tag{7}$$

$$\int_{0}^{\infty} y \frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))dy} dy$$

Since the middle term of (7) is negative, to show (5) is greater than (6), it is enough to show that the following inequality holds for $y \geq 0$:

$$\frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \Phi(m_d - f_t - y))dy} < \frac{g(f_t + y)}{\int_0^{\infty} g(f_t + y)\, dy}$$

Since $g(f_t+y)\Pi_{d=1}^{t}(1-\Phi(m_d-f_t-y)) \leq g(f_t + y)$, and since by hypothesis the denominator of the lefthand side is no smaller than the denominator of the righthand side (these are the probabilities that a random male and female survive, respectively), it holds that (5) is greater than (6). □

In the next lemma, the first part reflects the fact that, regardless of the promotion standards, each male has positive probability of being eliminated at each stage. Since excluded agents cannot re-enter the pool, only few males survive in the long run.

The second part reflects the fact that, regardless of the promotion standards, only the males with very high ability are likely to survive many opportunities to be eliminated. Thus, in the "long run", it does not matter very much what the promotion standards are, as long as there is a possibility to be eliminated at each stage. Males who survive will likely have very high ability. In contrast, a female survives with probability one if her ability is above the maximum promotion standard. This means that more females survive in the long run even without extraordinary ability.

Lemma 2 *Let m, f be promotion standards that are bounded above and below in a hierarchy without memory. Then*
(1) Given $\varepsilon > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the probability that a male (risk-taker) survives to stage t is less than ε; and
(2) There exists $\hat{t}$ such that for $t > \hat{t}$, the expected ability of a random surviving male (risk-taker) is larger than the expected ability of a random surviving female (non risk-taker).

Proof: Let $\underline{m} \leq m_t \leq \bar{m}$, and $\underline{f} \leq f_t \leq \bar{f}$ for all $t = 1, 2, ...$

(1) Let $\varepsilon > 0$. Let $\tilde{a} > 0$ satisfy $0 < 1 - G(\tilde{a}) < \varepsilon/2$ and let $\tilde{t}$ satisfy

$\Phi(a-\underline{m})^{\tilde{t}} < \varepsilon/2$ for all $a \le \tilde{a}$. Then for $t \ge \tilde{t}$,

$$\begin{aligned}
& \int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1-\Phi(m_d-a))da \\
= & \int_{-\infty}^{\tilde{a}} g(a)\Pi_{d=1}^{t}(1-\Phi(m_d-a))da + \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}(1-\Phi(m_d-a))da \\
\le & \int_{-\infty}^{\tilde{a}} g(a)\Phi(a-\underline{m})^t da + \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}(1-\Phi(m_d-a))da \\
< & G(\tilde{a})\varepsilon/2 + (1-G(\tilde{a}) < \varepsilon
\end{aligned}$$

(2) Let $\bar{a}^f$ be an upper bound on the expected ability (3) of surviving females at each stage:

$$\bar{a}^f = \int_{\bar{f}}^{\infty} a \; \frac{g(a)}{1-G(\bar{f})} da$$

Let $1 > \delta > 0$. Let $\tilde{a} > 0$ satisfy $-\tilde{a}-\underline{m} < \tilde{a}-\bar{m}$ and $\frac{\bar{a}^f}{1-\delta} < \tilde{a}$. Let $\hat{a}$ satisfy $\tilde{a}-\underline{m} < \hat{a}-\bar{m}$ Let $\hat{t}$ be such that for $t > \hat{t}$

$$\frac{\bar{a}^f}{1-\delta}\; G(\tilde{a})\; \Phi(\tilde{a}-\underline{m})^t < (\tilde{a}-\frac{\bar{a}^f}{1-\delta})\; (1-G(\hat{a}))\; \Phi(\hat{a}-\bar{m})^t$$

$$\text{and} \qquad \left(\frac{\Phi(-\tilde{a}-\underline{m})}{\Phi(\tilde{a}-\bar{m})}\right)^t < \delta$$

To give a lower bound on the expected ability (4) of surviving males we will use the following inequality:

$$[1-\left(\frac{\Phi(-\tilde{a}-\underline{m})}{\Phi(\tilde{a}-\bar{m})}\right)^t]\;\; \tilde{a}\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}\Phi(a-m_d)da$$

$$\begin{aligned}
< & \int_{\tilde{a}}^{\infty} ag(a)\;\; \Pi_{d=1}^{t}\Phi(a-m_d)\;\; [1-\frac{\Pi_{d=1}^{t}\Phi(-a-m_d)}{\Pi_{d=1}^{t}\Phi(a-m_d)}]\;\; da \\
< & \int_{0}^{\infty} ag(a)\;\; [\Pi_{d=1}^{t}\Phi(a-m_d)-\Pi_{d=1}^{t}\Phi(-a-m_d)]\;\; da \\
= & \int_{0}^{\infty} ag(a)\; \Pi_{d=1}^{t}\Phi(a-m_d)da + \int_{0}^{\infty}(-a)g(a)\Pi_{d=1}^{t}\Phi(-a-m_d)\; da \\
= & \int_{-\infty}^{\infty} ag(a)\;\; \Pi_{d=1}^{t}\Phi(a-m_d)\;\; da
\end{aligned} \qquad (8)$$

Then

$$
\begin{aligned}
\frac{\bar{a}^f}{1-\delta}\int_{-\infty}^{\tilde{a}} g(a)\Pi_{d=1}^t \Phi(a-m_d)da &< \frac{\bar{a}^f}{1-\delta}\ G(\tilde{a})\ \Phi(\tilde{a}-\underline{m})^t < \\
(\tilde{a}-\frac{\bar{a}^f}{1-\delta})\ (1-G(\hat{a}))\Phi(\hat{a}-\bar{m})^t &\leq (\tilde{a}-\frac{\bar{a}^f}{1-\delta})\int_{\hat{a}}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da \\
&< (\tilde{a}-\frac{\bar{a}^f}{1-\delta})\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da
\end{aligned}
$$

which implies

$$
\frac{\bar{a}^f}{1-\delta}\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da < \tilde{a}\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da
$$

Hence, combining with (8):

$$
\begin{aligned}
\frac{\bar{a}^f}{1-\delta}\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da &< \tilde{a}\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da \\
&< \frac{1}{(1-\left(\frac{\Phi(-\tilde{a}-\underline{m})}{\Phi(\tilde{a}-\bar{m})}\right)^t)}\int_{-\infty}^{\infty} ag(a)\Pi_{d=1}^t \Phi(a-m_d)da
\end{aligned}
$$

Since $1<(1-\left(\frac{\Phi(-\tilde{a}-\underline{m})}{\Phi(\tilde{a}-\bar{m})}\right)^t)/(1-\delta)$, the result follows:

$$
\bar{a}^f < \frac{(1-\left(\frac{\Phi(-\tilde{a}-\underline{m})}{\Phi(\tilde{a}-\bar{m})}\right)^t)}{1-\delta}\bar{a}^f < \int_{-\infty}^{\infty} a\frac{g(a)\Pi_{d=1}^t \Phi(a-m_d)}{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t \Phi(a-m_d)da}da
$$

For $t>\hat{t}$, female ability (3) is less than male ability (4). □

I use these lemmas to characterize the consequences of gender-blind promotion standards.

Proposition 3 (Gender Blind Promotions) *Let $c=c_1,c_2,...$ be gender-blind promotion standards in a hierarchy without memory that are bounded above and below and satisfy $G(c_t)<\delta<1$ for some $\delta\in(0,1)$ and all t. Then*
(1) At the first stage, if $c_1>E_G(a)$, a random male has a higher probability of survival than a random female, and a random surviving female has higher expected ability than a random surviving male.

(2) At later stages, $t > \tilde{t}$ for some appropriate $\tilde{t}$, the probability that a random male survives is smaller than the probability a random female survives, but the expected ability of surviving males is larger than the expected ability of surviving females.

Proof: (1) At stage 1, the probability (2) that a male survives can be written as follows with a change of variables $x = a - c_1$, and using symmetry of Φ :

$$\int_{-\infty}^{\infty} g(a)(1-\Phi(c_1-a))da = \int_{-\infty}^{\infty} g(c_1+x)\Phi(x)dx$$

$$\begin{aligned}
&= \int_{-\infty}^{0} g(c_1+x)\Phi(x)dx + \int_{0}^{\infty} g(c_1+x)\Phi(x)dx \\
&= \int_{0}^{\infty} g(c_1-x)\Phi(-x)dx + \int_{0}^{\infty} g(c_1+x)(1-\Phi(-x))dx \\
&= \int_{0}^{\infty} [g(c_1-x)-g(c_1+x)]\Phi(-x)dx + \int_{0}^{\infty} g(c_1+x) \quad dx \\
&> \int_{c_1}^{\infty} g(a) \quad da
\end{aligned}$$

The inequality holds because $[g(c_1-x)-g(c_1+x)]\Phi(-x)dx > 0$ at every x. Due to the strict quasiconcavity and symmetry of g and $c_1 > E_g(a)$, $g(c_1-x) > g(-c_1-x) = g(c_1+x)$. Hence (2) is larger than (1) at $t = 1$. Using Lemma 1, the expected ability of a surviving male is lower than the expected ability of a surviving female.

(2) follows directly from Lemma 2 by choosing $\varepsilon > 0$ such that $(1 - G(c_t)) > \varepsilon$ for all t. $\square$

We now turn to alternative policy goals. We first consider the goal of equalizing the probabilities of promotion at each stage, and then consider the goal of equalizing the average ability of the survivors at each stage.

It follows directly from Lemma 2(1) that if the promotion standards m, f are bounded, and a nontrivial fraction of females survive in the limit, the survival rates of males and females in the limit are different. Proposition 4(2) says this in a different way: If survival rates are the same, the males' promotion standards cannot be bounded below, and in particular, the promotion standards cannot be increasing. Increasing standards would be the natural interpretation of a promotion hierarchy.

Proposition 4 (Promoting Equal Numbers) *Let m, f be promotion standards in a hierarchy without memory such that males and females have the same probability of survival at each stage t.*
(1) If $f_1, m_1 > E_G(a)$, then $f_1 < m_1$.
(2) If the sequence f converges to a finite limit, then the sequence m is not bounded below.

Proof: (1) follows from Proposition 3(1), which implies that if $m_1 = f_1$, males have a higher probability of survival than females. Since the probability of survival is decreasing in m_1, the probabilities can only be equal if $m_1 > f_1$.

(2) Since the sequence f converges, the sequence of female survival rates $\{1 - G(f_t)\}_{t=1,...}$ also converges, and, by hypothesis, the sequence of male survival rates $\{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1-\Phi(m_d - a))da\}_{t=1,...}$ converges to the same limit, say L. Choose an $\varepsilon > 0$ such that $\varepsilon < L$. Suppose, contrary to the proposition, that the sequence m is bounded below by $\underline{m}$. The male survival rate at stage t satisfies

$$\begin{aligned} & \int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1-\Phi(m_d - a))da \\ \leq & \int_{-\infty}^{\infty} g(a)(1-\Phi(\underline{m} - a))^t da \end{aligned} \tag{9}$$

Choose $\tilde{a}, \hat{a}$ such that $\hat{a} < \tilde{a}$ and

$$\begin{aligned} 1 - G(\tilde{a}) &< \varepsilon/3 \\ G(\hat{a}) &< \varepsilon/3 \end{aligned}$$

Choose $\hat{t}$ such that $(1 - \Phi(\underline{m} - \tilde{a}))^{\hat{t}} < \varepsilon/3$. Then if $t > \hat{t}$, the upper bound on the male survival rate at stage t, (9), can be written

$$\int_{-\infty}^{\hat{a}} g(a)(1-\Phi(\underline{m} - a))^t da + \int_{\hat{a}}^{\tilde{a}} g(a)(1-\Phi(\underline{m} - a))^t da + \int_{\tilde{a}}^{\infty} g(a)(1-\Phi(\underline{m} - a))^t da$$

$$
\begin{aligned}
&< \int_{-\infty}^{\hat{a}} g(a)da + [G(\tilde{a}) - G(\hat{a})](1 - \Phi(\underline{m} - \tilde{a}))^t + \int_{\tilde{a}}^{\infty} g(a)da \\
&< \varepsilon/3 + (1 - \Phi(\underline{m} - \tilde{a}))^t + \varepsilon/3 < \varepsilon < L
\end{aligned}
$$

This is a contradiction. □

Proposition 5 (Promoting Equal Average Ability) *(1) Suppose that the expected abilities of surviving males and females are the same at stage $\hat{t}$ under the promotion standards m, f in a hierarchy without memory. Then the survival rate of females at stage $\hat{t}$ is greater than that of males. (2) In a hierarchy without memory, there are no bounded promotion standards m, f for which promoted males have the same average ability as promoted females at each t.*

Proof: (1) By Lemma 1, if the expected number of surviving risk-takers (males) at stage $\hat{t}$ is greater than or equal to the expected number of surviving non risk-takers (females), then the average ability of surviving risk-takers is lower. Part (1) is implied by the following equivalent statement: If the average ability of surviving males is as great or greater than surviving females, then there are fewer surviving males.

(2) follows from Lemma 2(2), which says that, for any bounded sequences, the average ability of surviving males is higher than the average ability of surviving females for late stages of the hierarchy (large t). □

4 The Hierarchy with Memory

Say that the hierarchy has *memory* if promotion depends on the performance in all periods up to the promotion date. In the hierarchy without memory defined above, promotion depends only on performance since the last promotion.

I will study the special case in which promotion depends symmetrically on the signals generated in the entire history to date. Non risk-taking (female) agents generate the same signal in each period, so the survival condition is the same as without memory. If the promotion standards are c, a female agent with ability a *survives* to stage t if $a \geq c_d$ for each $d \leq t$. A risk-taking (male) agent *survives* to stage t in a hierarchy with memory if $(1/d)\sum_{k=1}^{d}\sigma_k \geq c_d$ for each $d \leq t$, which means that $a + (1/d)\sum_{k=1}^{d} u_k \geq c_d$ for all $d \leq t$ when the agent's ability is a. Hence the probability that a male with ability a survives to stage t is given by $S_t(a, c)$ below. $S_t(\cdot, c)$ is a continuous and increasing function.

$$S_t(a,c) = \int_{c_1-a}^{\infty} \phi(u_1) \int_{2(c_2-a)-u_1}^{\infty} \phi(u_2) \ldots \int_{t(c_t-a)-\sum_{i=1}^{t-1} u_t}^{\infty} \phi(u_t)\, du_t \ldots du_2\, du_1$$

At each a, the probability of survival $S_t(a, c)$ is decreasing with t, and bounded below by zero. Hence the sequence converges at each a. Let

$$S(a,c) = \lim_{t\to\infty} S_t(a,c) \text{ for each } a \in \mathbf{R}$$

The limiting expected ability of surviving risk takers is the following, provided the probability of survival in the limit (the denominator) is positive.

$$\int_{-\infty}^{\infty} a \frac{S(a;m)\, g(a)}{\int_{-\infty}^{\infty} S(a,c)\, g(a)\, da} da \tag{10}$$

For hierarchies with memory, Lemma 6 is the analog of Lemma 1.

Lemma 6 *Let m, f be sequences of promotion standards for males and females, respectively, in a hierarchy with memory. The expected ability of a random surviving male is lower than the expected ability of a random surviving female at any stage t at which males have at least as high a probability of survival.*

Proof: With a change of variables, $y = a - f_t$, the females' expected ability conditional on survival to t, (3), can be written as (5) above.

For males, with a change of variables $y = a - f_t$, the expected ability conditional on survival to t, (10), can be written:

$$\int_{-\infty}^{\infty} (f_t + y) \frac{g(f_t + y) S_t(f_t + y, m)}{\int_{-\infty}^{\infty} g(f_t + y) S(f_t + y, m)\, dy} dy$$

$$= f_t + \int_{-\infty}^{\infty} y \frac{g(f_t+y)S_t(f_t+y,m)}{\int_{-\infty}^{\infty} g(f_t+y)S_t(f_t+y,m)\,dy} dy \tag{11}$$

$$= f_t + \int_{-\infty}^{0} y \frac{g(f_t+y)S_t(f_t+y,m)}{\int_{-\infty}^{\infty} g(f_t+y)S_t(f_t+y,m)\,dy} dy \tag{12}$$

$$+ \int_{0}^{\infty} y \frac{g(f_t+y)S_t(f_t+y,m)}{\int_{-\infty}^{\infty} g(f_t+y)S_t(f_t+y,m)\,dy} dy$$

Since the middle term of (12) is negative, to show that (5) is greater than (11), it is enough to show that the following inequality holds for each $y \geq 0$:

$$\frac{g(f_t+y)S_t(f_t+y,m)}{\int_{-\infty}^{\infty} g(f_t+y)S_t(f_t+y,m)\,dy} < \frac{g(f_t+y)}{\int_{0}^{\infty} g(f_t+y)\,dy}$$

Since $g(f_t+y)S_t(f_t+y,m) \leq g(f_t+y)$, and since by hypothesis the denominator of the lefthand side is no smaller than the denominator of the righthand side (these are the probabilities that a random male and female survive, respectively), it holds that (5) is greater than (11). □

For hierarchies without memory, we showed in Lemma 2 and Proposition 4 that most risk-takers will eventually be eliminated, provided the standards are bounded below. Each risk-taker has infinitely many opportunities to throw himself out of the pool, and if any risk-takers survive, it is only those with exceptional ability. The expected ability of risk-taking survivors is therefore higher than that of non risk-taking survivors in the limit.

I now show that, with memory, risk-takers survive in the limit. Nevertheless, it is still true, as in hierarchies without memory, that surviving risk-takers (males) will be less numerous than surviving non risk-takers (females), and will have higher average ability. This must be proved in a different manner than Proposition 4, since the analog to Lemma 2 does not hold. Further, Proposition 8 only holds for promotion standards that are nondecreasing. With decreasing promotion standards, all non-risk-takers with ability above c_1 would survive, and none would be eliminated after stage one. This is not true of risk-takers. If the performance standards decrease rapidly, risk-takers with abilities lower than c_1 might survive in large numbers, and the limiting

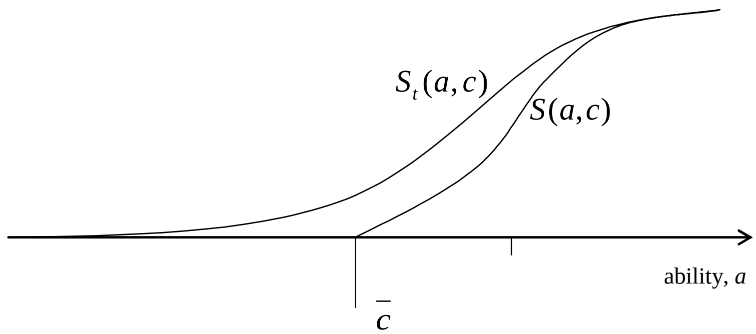

Figure 2: Survival of Risk Takers in a Hierarchy with Memory

expected ability of risk-takers could be lower than that of non risk-takers.

In any case, increasing promotion standards are the more natural case.

Proposition 8 follows from the shape of the limiting survival function S, described in Lemma 7 and shown in Figure 2, where the promotion standards converge to $\bar{c}$.

Lemma 7 *Suppose that* $c_1, c_2 ... c_t ..$ *converges to* $\bar{c}$. *For each* $a \in \mathbf{R}$ *such that* $\bar{c} - a \geq 0$, $S(a, c) = 0$. *For each* $a \in \mathbf{R}$ *such that* $\bar{c} - a < 0$, $S(a, c) > 0$. *The limit function* S *is nondecreasing, and there exists* $a > \bar{c}$ *such that* $S(a, c) > S(\bar{c}, c) = 0$.

Proof: That $S(a, c) \to 0$ for $a < \bar{c}$ follows because $(1/t) \sum_{i=1}^{t} u_t + a$ converges in probability to a. Since $a < \bar{c}$, it also holds that $(1/t) \sum_{i=1}^{t} u_t + a < \bar{c}$ with probability arbitrarily close to one for large t. That $S(\bar{c}, c) \to 0$ follows because the limit distribution of $\left(1/\sqrt{t}\right) \sum_{i=1}^{t} (u_t/v)$ is normal, centered at 0, where v^2 is the variance of Φ. If $a = \bar{c}$, then for large t, $c_t - a = c_t - \bar{c}$ is close to 0. With positive probability it holds that $\left(1/v\sqrt{t}\right) \sum_{i=1}^{t} u_t < c_t - \bar{c} < 0$, or that $(1/vt) \sum_{i=1}^{t} u_t < (c_t - \bar{c}) / \sqrt{t} < 0$.

But since survival at t requires that $(1/t)\sum_{i=1}^{t} u_t \geq 0$, this implies that the agent survives at each t with probability strictly less than one, so that the joint probability of survival at all t is zero.

To show that $S(a,c) > 0$ for $a > \bar{c}$, we argue instead that $S(a,\{\bar{c},\bar{c},...\}) > 0$, since $S(a,c) > S(a,\{\bar{c},\bar{c},...\})$. An agent with ability a fails to survive if $(1/t)\sum_{1}^{t} u_i < \bar{c} - a$ for some t. Using Lemma (6) of Dubins and Freedman (1965, p. 801), if $b_1, b_2 > 0$,

$$\Pr\left[\frac{1}{t}\sum_{i=1}^{t} u_i \leq -b_1 v^2 - \frac{b_2}{t} \text{ for some } t = 1,2,...\right] \leq \frac{1}{1+b_1 b_2}$$

Thus,

$$\Pr\left[\frac{1}{t}\sum_{i=1}^{t} u_i > -b_1 v^2 - \frac{b_2}{t} \text{ for all } t = 1,2,...\right] \geq 1 - \frac{1}{1+b_1 b_2} > 0$$

Choose $b_1, b_2 > 0$ so that $-b_1 v^2 - b_2 = \bar{c} - a$. Then

$$S(a,c) > S(a,\{\bar{c},\bar{c},...\}) = \Pr\left[\frac{1}{t}\sum_{i=1}^{t} u_i \geq -b_1 v^2 - b_2 = \bar{c} - a \text{ for all } t = 1,2,...\right] >$$

$$\Pr\left[\frac{1}{t}\sum_{i=1}^{t} u_i \geq -b_1 v^2 - b_2/t \text{ for all } t = 1,2,...\right] \geq 1 - \frac{1}{1+b_1 b_2} > 0$$

□

Proposition 8 (Gender Blind Promotions with Memory) *Let $c = c_1, c_2.,,, c_t, ...$ be a nondecreasing sequence of gender blind promotion standards that converge to $\bar{c}$ in a hierarchy with memory. Then*

(1) At the first stage, if $c_1 > E_G(a)$, a random risk-taker (male) has a higher probability of survival than a random non risk-taker (female), but lower expected ability.

(2) At later stages, $t > \hat{t}$ for some appropriate $\hat{t}$, the probability that a random risk-taker (male) survives is smaller than the probability that a random non risk-taker (female) survives, but the surviving risk-takers (males) have higher expected ability.

Proof: (1) That risk-takers have a higher probability of survival at stage 1 is proved in Proposition 3, since $S_1(a,c) = [1 - \Phi(c_1 - a)]$. The risk-takers' lower expected ability follows from Lemma 6.

(2) First, fewer risk-takers (males) than non risk-takers (females) survive in the limit. For $a < \bar{c}$, neither risk-takers nor non risk-takers survive. For $a > \bar{c}$, the probability that a non risk-taker survives is one, while, for risk-takers, the survival probability is less than one: $S(a, c) < 1 - \Pr[u_1 < c_1 - a] < 1$.

Because the risk-takers' limit probabilities of survival are nondecreasing with a, the limit distribution of their abilities first-order dominates the limit distribution of non risk-takers' abilities. Thus, the expected ability of surviving risk-takers is no smaller than that of surviving non risk-takers. But since $S(a, c) > S(\bar{c}, c)$ for some $a > c$, the limiting expected ability of surviving risk takers is strictly greater than that of surviving non risk-takers. $\square$

For hierarchies with memory, there is no analog to Proposition 4, but the following is the analog to Proposition 5.

Proposition 9 (Promoting Equal Average Ability with Memory) *Suppose that the expected abilities of surviving males and females are the same at stage $\hat{t}$ under the promotion standards m, f in a hierarchy with memory. Then the survival rate of females at stage $\hat{t}$ is greater than that of males.*

Proof: By Lemma 6, if the expected number of surviving risk-takers (males) at stage t is greater than or equal to the surviving non risk-takers (females), then the average ability of surviving risk-takers (males) is lower than that of surviving females. The proposition follows from an equivalent statement: If the average ability of surviving males is as great or greater than the average ability of surviving females, then the expected number of surviving males is lower. $\square$

For completeness, the following proposition gives some insight into how the promotion standards must differ with and without memory, in order to equalize the number of survivors.

Proposition 10 *Let $\hat{c}^a$ and c be promotion standards in hierarchies with and without* *memory, respectively, which yield the same probabilities of survival at each t for a risk-taking agent with ability a. Then it holds that $\hat{c}_1^a = c_1$ and $\hat{c}_t^a > (1/d)\sum_{d=1}^{t} c_d$ for each $t > 1$.*

Proof: A risk-taking agent with ability a will have a sequence of random errors in his signal, $\{u_t\}$. With and without memory, respectively, the agent survives the first stage in the events

$$\{u_1 \geq c_1 - a\}, \ \{u_1 \geq \hat{c}_1^a - a\}$$

so $c_1 = \hat{c}_1^a$. Without memory, the agent survives two stages in the event

$$\{u_1 \geq c_1 - a \text{ and } u_2 \geq c_2 - a\} \tag{13}$$

With memory the agent survives two stages in the event

$$\{u_1 \geq c_1 - a \text{ and } u_2 \geq c_2 - a + (c_1 - a - u_1)\} \tag{14}$$

Since $0 \geq (c_1 - a - u_1)$, the event (13) implies the event (14), but not vice versa. Thus, the probability of the event (13) is lower than the probability of the event (14). There exists $\tilde{c}_2 > c_2$ such that the probabilities of survival are equalized at the first two stages, when $\hat{c}_1^a = c_1$ and $\hat{c}_2^a = (1/2)(c_1 + \tilde{c}_2) > (1/2)(c_1 + c_2)$:

$$\Pr\{u_1 \geq c_1 - a \text{ and } u_2 \geq c_2 - a\} = \Pr\{u_1 \geq c_1 \text{ and } u_2 \geq \tilde{c}_2 - a + (c_1 - a - u_1)\}$$

Similarly, at stage $t = 3$,

$$\begin{aligned} & \Pr\{u_1 \geq c_1 - a \text{ and } u_2 \geq c_2 - a \text{ and } u_3 \geq c_3 - a\} \\ < \ & \Pr\left\{ \begin{array}{c} u_1 \geq c_1 - a \text{ and } u_2 \geq \tilde{c}_2 - a + (c_1 - a - u_1) \\ \text{and } u_3 \geq c_3 - a + (c_1 + \tilde{c}_2 - 2a - u_2 - u_1) \end{array} \right\} \end{aligned}$$

since $0 \geq (c_1 + \tilde{c}_2 - 2a - u_2 - u_1)$. Thus, there exists $\tilde{c}_3 > c_3$ such that

$$\begin{aligned} & \Pr\{u_1 \geq c_1 - a \text{ and } u_2 \geq c_2 - a \text{ and } u_3 \geq c_3 - a\} \\ = \ & \Pr\left\{ \begin{array}{c} u_1 \geq c_1 - a \text{ and } u_2 \geq \tilde{c}_2 - a + (c_1 - a - u_1) \\ \text{and } u_3 \geq \tilde{c}_3 - a + (c_1 + \tilde{c}_2 - 2a - u_2 - u_1) \end{array} \right\} \end{aligned}$$

Thus, there exists a sequence $\tilde{c}_1, \tilde{c}_2, ...$ such that $\tilde{c}_1 = c_1$, $\tilde{c}_t > c_t$ for $t > 1$, and for each t,

$$\Pr\{u_d \geq (c_d - a) \text{ for all } d \leq t\} = \Pr\left\{\sum_{i=1}^{d} u_i \geq \sum_{i=1}^{d} (\tilde{c}_i - a) \text{ for all } d \leq t\right\}$$

Thus, the promotion standards with memory $\hat{c}^a$ defined by $\hat{c}_1^a = c_1$, $\hat{c}_t^a = \frac{1}{t}\sum_{i=1}^{t} \tilde{c}_i$ for $t > 1$, yield the same probabilities of survival at each t as the promotion standards without memory c, and $\hat{c}_t^a > (1/d)\sum_{d=1}^{t} c_d$ for all $t > 1$. $\square$

Proposition 10 does not assert that the promotion standards $\hat{c}^a$ are the same for agents with different abilities. However it does imply that to maintain the same overall promotion rate with and without memory, the standards must satisfy $\hat{c}_t > (1/d)\sum_{d=1}^{t} c_d$ for each $t > 1$, since otherwise the promotion rate would be higher at some t for *every* a.

5 Interpretations and Open Questions

Even if we accept the hypothesis that males generate riskier signals than females, these conclusions are difficult to verify empirically. At most we can observe promotion rules, signals, and proportions promoted, but we cannot in general observe true abilities. In addition, it is hard to identify hierarchies where the same proportions of females (non risk takers) and males (risk takers) have wanted to stay in the pool. Instead, women and men drop out at different rates for self-motivated reasons such as child bearing.

Nevertheless, if we assume that the initial numbers and abilities of males and females are the same, and that they drop out for exogenous reasons at the same rate and in a way that is uncorrelated with ability, then the following implications would be consistent with this model:

1. If the promotion policy is gender blind and has stringent standards at the beginning (fewer than half are promoted), the ratio of surviving females to males is lower at the beginning than at the end.

2. Under an equal-abilities promotion policy, more females survive at every stage than males.

3. If fewer than half the females are promoted at the beginning of the hierarchy, and if the surviving males and females are equally numerous or have the same average ability at each stage, then the standards for males are more stringent at the beginning but more lenient at the end.

4. If more than half the females are promoted at the beginning of the hierarchy, and if surviving males and females have equal average abilities at each stage, then the standards for males are more stringent at the beginning but more lenient at the end. If the surviving males and females are equally numerous at each stage, the standards for males are more lenient at both the beginning and the end.

The hypothesis that males generate riskier signals than females might be inverted. Suppose, for example, that males and females are equally risk-taking, but males generate more evidence in each period about their true ability than females, or are observed more closely. Females are relatively "invisible" in a manner similar to the phenomenon discussed by Milgrom and Oster (1987). This would reverse the hypothesis that males generate riskier signals than females.

If the hypothesis on riskiness of signals is reversed, then the interpretation of the above propositions is reversed. Instead of being disfavored in numbers at the early stages of the gender-blind hierarchy and favored in later stages, females may be favored in early stages and disfavored in later stages. That would give credence to the 1970's slogan, that women have to be "twice as good to get half as far."

The analysis above is positive rather than normative. The motive behind affirmative action is a normative one, namely, to redress the apparent inequity of promoting more males than females. This leads to the question of whether there is an "efficiency/equity" tradeoff.

Efficiency is hard to define in a partial model of a labor market such as this. In fact, since affirmative action has many faces, its efficiency effects are hard to identify in general, as discussed by Holzer and Neumark (2000). I will think of efficiency as being served by the promotion of the most able agents.

If the males' signals are so random that the truth is mostly obscured, it is probably better to promote only females, for whom the ability is more observable. This wisdom is particularly compelling if the number of agents required at the next level of promotion is small relative to the pool, so that ability is not compromised by promoting enough females to fill the slots. The main prescription in this regard is given by Propositions 5 and 9, which point out that, if equal abilities are desired in the promoted pool, more females (non risk-takers) than males (risk-takers) must survive at every stage.

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