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Gauged discrete symmetries and proton stabilityRabindra N. Mohapatra^{1,2,3} and Michael Ratz²¹*Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA*²*Physik Department, Technische Universität München, Garching, Germany*³*Sektion Physik, Universität München, München, Germany*

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We discuss the results of a search for anomaly-free Abelian \bar{Z}_N discrete symmetries that lead to automatic R-parity conservation and prevent dangerous higher-dimensional proton decay operators in simple extensions of minimal supersymmetric extension of the standard model based on the left-right symmetric group, the Pati-Salam group and SO(10). We require that the superpotential for the models have enough structures to be able to give correct symmetry breaking to minimal supersymmetric extension of the standard model and potentially realistic fermion masses. We find viable models in each of the extensions, and for all the cases, anomaly freedom of the discrete symmetry restricts the number of generations.

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I. INTRODUCTION

Supersymmetry (SUSY) is widely believed to be one of the key ingredients of physics beyond the standard model (SM) for various reasons:

- (i) stability of the Higgs mass and hence the weak scale;
- (ii) possibility of a supersymmetric dark matter;
- (iii) gauge-coupling unification, suggesting that there is a grand unified theory (GUT) governing the nature of all forces and matter.

The fact that the seesaw mechanism for understanding small neutrino masses also requires a new scale close to the GUT scale adds another powerful reason to believe in this general picture.

There are however many downsides to SUSY. For instance, while the standard model guarantees nucleon stability, in its supersymmetric version, there appear two new kinds of problems: (i) There are renormalizable R-parity breaking operators allowed by supersymmetry and standard model gauge invariance, e.g.

$$W_{\mathcal{R}} = \lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} Q_i L_j d_k^c + \lambda''_{ijk} u_i^c d_j^c d_k^c. \quad (1)$$

Here, Q , L , u^c , d^c and e^c denote the left-chiral quark doublet, lepton doublet, u -type, d -type and electron superfields. Combination of the last two terms leads to rapid proton decay, and present limits on nucleon stability imply (for squark masses of TeV; cf. e.g. [1–3]):

$$\lambda'_{ij} \lambda''_{ij} \lesssim 10^{-24}. \quad (2)$$

These terms also eliminate the possibility of any SUSY particle being the dark matter of the Universe.¹ Usually assumptions such as either R-parity or matter parity (cf. [4]) are invoked to forbid these couplings and rescue the proton (as well as dark matter) stability. (ii) A second, more vexing, problem is that even after imposing R-parity,

¹An exception to this statement is the case where the lightest SUSY particle is the gravitino and plays the role of the dark matter.

one may have dimension 5 R-parity conserving operators such as $\kappa_{ijkl} Q_i Q_j Q_k L_l / M_{\text{P}}$. Such operators also lead to rapid proton decay. In fact present nucleon stability limits put an upper limit on $\kappa_{1121}, \kappa_{1122} \lesssim 10^{-8}$ [1]. In what follows we will refer to these operators as $QQQL$.

To understand that these problems can be attributed to SUSY, recall that the R-parity breaking terms are forbidden in the case of SM by Lorentz invariance in the standard model, and the $QQQL$ is suppressed by two powers of Planck mass and hence not problematic.

In this study, we focus on the problem of baryon number nonconservation and require the model to satisfy the following constraints:

- (i) R-parity symmetry is exact so that it prevents catastrophic proton decay;
- (ii) R-parity conserving dimension five proton decay operators of type $QQQL$ are either forbidden or suppressed to the desired level; and
- (iii) the superpotential of the theory has enough structure for ensuring proper symmetry breaking down to the standard model and to give fermion masses.

We look for symmetries that allow all desired terms in the superpotential while forbidding the unwanted terms discussed and satisfying the anomaly constraints.

The anomaly constraints can be related to triangle graphs involving the not only discrete symmetry and the gauge symmetries but also gravity. The reason for requiring anomaly freedom is the following: there is a general belief that nonperturbative gravitational effects such as black holes and worm holes break global symmetries of nature [5]. In an effective field theory language, they are parametrized by Planck scale suppressed higher-dimensional operators. In fact the above-mentioned $QQQL$ -type operators are a manifestation of the fact that global baryon number symmetry is broken by such effects. On the other hand no hair theorem of general relativity says that any nonperturbative gravitational effect must respect the gauge symmetries. Therefore if there is a gauged discrete symmetry in the theory that prevents the undesirable

terms under discussion, they will be absent even after all nonperturbative effects are taken into account.

How does one ensure that a discrete symmetry is a gauge symmetry? This problem has been extensively studied in the literature in the context of the minimal supersymmetric extension of the SM (MSSM) [1,2,6–10], and the general procedure is to calculate the various anomaly equations involving the discrete group with gravity and the gauge group, i.e. vanishing of Dgg , DG^2 anomalies where D stands for the discrete symmetry group in question, g is the graviton and G is the (continuous) gauge symmetry on which the theory is based.

In the context of the MSSM, a discrete \mathbb{Z}_6 symmetry has been identified, dubbed “proton-hexality” in [2], that contains R-parity as \mathbb{Z}_2 subgroup and forbids $QQQL$ [7]. Remarkably, anomaly freedom of this \mathbb{Z}_6 requires the number of generations to be 3 [1]. Moreover, it has been shown that (given 3 generations) this is the only anomaly-free symmetry that allows the MSSM Yukawa couplings and neutrino masses while forbidding the dangerous operators [2]. Such symmetries can be extended so as to also forbid the μ term [11]. (For an approach to ensure proton stability by flavor symmetries see e.g. [12].) On the other hand, the charge assignment is different for different standard model representations. This raises the question whether nucleon stability can be ensured by discrete symmetries in (unified) theories where standard model representations get combined in larger multiplets, and the charge assignment is hence restricted more strongly. We therefore seek discrete symmetries ensuring sufficient proton stability in three gauge extensions of the supersymmetric standard model:

- (i) the left-right symmetric model [13],
- (ii) the Pati-Salam model [14] and
- (iii) SO(10).

All these models incorporate the $B - L$ gauge group, which is generally used in the discussion of the seesaw mechanism for neutrino masses [15–19] (for a review see e.g. [20]) and also provides one way to guarantee R-parity conservation [21–23]. Because of the higher-gauge symmetry, which must be broken down to the SM gauge symmetry, specific terms must be present in the superpotential. This poses constraints on the discrete symmetry.

One main result of the study is a connection between the order of the discrete group and the number of generations in all the cases. We give examples of viable models for all the different gauge groups. In our discussion, we follow a certain “route of unification.” We start with the left-right symmetric extension of the supersymmetric standard model, proceed via the Pati-Salam model to SO(10) GUTs and finally comment on how our results might be used in string compactifications.

This paper is organized as follows: in Sec. II, we discuss anomaly-free gauge symmetries ensuring proton stability

in left-right models; we proceed to the Pati-Salam model in Sec. III, and in Sec. IV we discuss SO(10) models. We give our conclusions in Sec. V.

II. LEFT-RIGHT MODEL AND DISCRETE SYMMETRIES

In this section, we discuss the left-right symmetric extension of MSSM, i.e. the gauge group is $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This amounts to breaking $B - L$ symmetry of the model by either a $B - L = 2$ triplet or a $B - L = 1$ doublet. In Table I (a), we show the assignments of the quarks and leptons and Higgs bosons. Tables I (b) and (c) list the Higgs sectors of the doublet and triplet models, respectively.

A. Left-right symmetric models—doublet Higgs case

The (wanted) superpotential is given by

$$\begin{aligned}
 W = & ihQ^T \tau_2 \Phi Q^c + ih'L^T \tau_2 \Phi L^c + if_c L^{cT} \tau_2 \bar{\chi}^c \bar{\chi}^{cT} \tau_2 L^c \\
 & + if_c L^T \tau_2 \bar{\chi} L^T \tau_2 \bar{\chi} + S(\chi^c \bar{\chi}^c + \chi \bar{\chi} - v_R^2) \\
 & + \text{Tr} \mu(\Phi^2).
 \end{aligned} \tag{3}$$

TABLE I. Assignment of the fermion and Higgs fields to the representation of the left-right symmetry group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. (a) shows the MSSM sector, (b) shows the Higgs sector of the doublet model and (c) the Higgs sector of the triplet model.

(a) MSSM part

Field	Quantum numbers
Q	$(\mathbf{3}, \mathbf{2}, \mathbf{1}, +\frac{1}{3})$
Q^c	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -\frac{1}{3})$
L	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$
L^c	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, +1)$
Φ	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})$

(b) Doublet model

Field	Quantum numbers
χ^c	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, +1)$
$\bar{\chi}^c$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, -1)$
χ	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$
$\bar{\chi}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, +1)$
S	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{0})$

(c) Triplet model

Field	Quantum numbers
Δ^c	$(\mathbf{1}, \mathbf{1}, \mathbf{3}, -2)$
$\bar{\Delta}^c$	$(\mathbf{1}, \mathbf{1}, \mathbf{3}, +2)$
Δ	$(\mathbf{1}, \mathbf{3}, \mathbf{1}, +2)$
$\bar{\Delta}$	$(\mathbf{1}, \mathbf{3}, \mathbf{1}, -2)$

On the other hand, the following couplings must be forbidden (we suppress coefficients):

$$W_{\text{unwanted}} = Q^3 L + Q^{c3} L^c + Q^3 \chi + Q^{c3} \chi^c + L \bar{\chi} + L^c \bar{\chi}^c + L Q Q^c \chi^c + Q \chi Q^c L^c + L^2 L^c \chi^c + \bar{\chi} L \Phi^2 + \bar{\chi}^c L^c \Phi^2 + L \Phi \chi^c + L^c \Phi \chi. \quad (4)$$

To study the anomaly constraints in this model for an arbitrary \mathbb{Z}_N group, we start by giving the charge assignments under \mathbb{Z}_N to the various superfields (denoted as q_F

TABLE II. A viable \mathbb{Z}_6 charge assignment in the left-right model with doublets.

q_L	q_Q	q_{L^c}	q_{Q^c}	q_Φ	q_χ	q_{χ^c}	$q_{\bar{\chi}}$	$q_{\bar{\chi}^c}$	q_S
1	1	5	5	0	0	2	0	4	0

for the field F in the equation below) and writing down the anomaly constraints. The anomaly constraints for $\mathbb{Z}_N g g$, $\mathbb{Z}_N [\text{SU}(3)_c]^2$, $\mathbb{Z}_N [\text{SU}(2)_L]^2$ and $\mathbb{Z}_N [\text{SU}(2)_R]^2$ respectively are²

$$N_g [6(q_Q + q_{Q^c}) + 2(q_L + q_{L^c})] + 4q_\Phi + 2(q_\chi + q_{\chi^c} + q_{\bar{\chi}} + q_{\bar{\chi}^c}) = 0 \pmod{N'}, \quad (5a)$$

$$N_g [2(q_Q + q_{Q^c})] = 0 \pmod{N}, \quad (5b)$$

$$N_g [3q_Q + q_L] + 2q_\Phi + q_\chi + q_{\bar{\chi}} = 0 \pmod{N}, \quad (5c)$$

$$N_g [3q_{Q^c} + q_{L^c}] + 2q_\Phi + q_{\chi^c} + q_{\bar{\chi}^c} = 0 \pmod{N}, \quad (5d)$$

where N_g denotes the number of generations. In the first equation

$$N' = \begin{cases} N, & N \text{ odd,} \\ N/2, & N \text{ even,} \end{cases} \quad (6)$$

which follows from Eq. (10) of [7].

The assignments must be consistent with the superpotential (3) and have to forbid the terms in (4). We scanned over possible nonanomalous $\mathbb{Z}_{N \leq 12}$ symmetries, keeping the number of generations N_g as a free parameter. Remarkably, the smallest viable N_g we found is 3, and the smallest N that works with 3 generations is 6. An example is shown in Table II.

By giving vacuum expectation values (vevs) to the fields χ^c and $\bar{\chi}^c$, the \mathbb{Z}_6 symmetry is broken to a \mathbb{Z}_2 symmetry under which matter is odd while the MSSM Higgs are even. That is, we have obtained an effective R-parity which, although there is a gauged $B - L$ symmetry, originates from an ‘‘external’’ \mathbb{Z}_6 .

B. Left-right models—triplet case

The transformation properties of the fields under the gauge group are shown in Table I (a) and (c). In this case, right-handed neutrino masses arise from the renormalizable couplings in the theory. We have to forbid $Q^3 L$ and $(Q^c)^3 L^c$. There are many anomaly-free discrete symmetries which do the job. The interesting point is that in this case, the minimum number of generations is $N_g = 2$ with $N = 2$ for the discrete group. The smallest symmetry that works for 3 generations is \mathbb{Z}_3 .

²We ignore cubic constraints (cf. [8]).

III. PATI-SALAM MODEL

We now proceed to the Pati-Salam model, i.e. the gauge group is $G_{\text{PS}} = \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$. The new feature of this model compared to the left-right model just discussed is that the quarks and leptons belong to the same representation (see Table III).

TABLE III. Assignment of the fermion and Higgs fields to the representation of the left-right symmetry group $\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$: (a) shows the MSSM sector, (b) shows the Higgs sector of the doublet model and (c) the Higgs sector of the triplet model.

(a) MSSM part

Field	Quantum numbers
ψ	(4 , 2 , 1)
ψ^c	($\bar{4}$, 1 , 2)
Φ	(1 , 2 , 2)

(b) Doublet model

Field	Quantum numbers
χ^c	(4 , 1 , 2)
$\bar{\chi}^c$	($\bar{4}$, 1 , 2)
χ	(4 , 2 , 1)
$\bar{\chi}$	($\bar{4}$, 2 , 1)
S	(1 , 1 , 1)

(c) Triplet model

Field	Quantum numbers
Δ^c	(10 , 1 , 3)
$\bar{\Delta}^c$	($\bar{10}$, 1 , 3)
Δ	(10 , 3 , 1)
$\bar{\Delta}$	($\bar{10}$, 3 , 1)

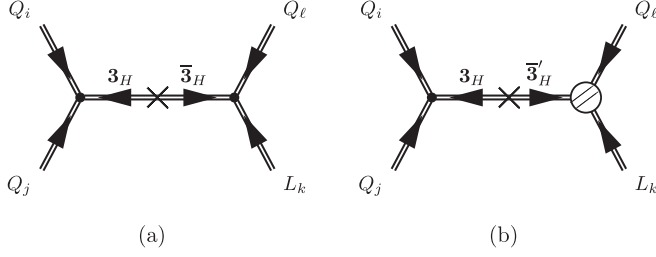


FIG. 1. Effective proton decay operator. (a) shows the usual triplet exchange diagram [24,25]. (b) illustrates that the amplitude vanishes if the mass partner of the Higgs triplet does not couple to SM particles [27].

The rest of the discussion is completely analogous to the left-right case, and in fact the solutions displayed in Table II for the discrete charges apply to the doublet case, and as in the triplet version of the left-right model there are many solutions. Note that the triplet version of Pati-Salam model does not have proton decay due to $SU(3)_c$ invariance.

IV. SO(10) GUT AND DISCRETE SYMMETRIES

We now turn our attention to the discussion of discrete symmetries in SO(10) GUTs. In SO(10) models, the dimension 5 proton decay operator $QQQL$ has two sources:

- (i) the higher-dimensional coupling $[\mathbf{16}_m]^4$ and
- (ii) effective operators emerging from integrating out Higgs triplets [24,25] (see Fig. 1).

Triplet masses of the order M_{GUT} appear to be too small to be consistent with the observed proton lifetime [26]. As discussed, the coefficient of the $QQQL$ operator has to be strongly suppressed. This requires an explanation of why both contributions to the operator are simultaneously small. As we shall see, such an explanation might arise from a simple discrete symmetry.

The proton decay via Higgs triplet exchange can be forbidden by eliminating the mass term for the $\mathbf{10}$ multiplet (H). However one needs to make the color triplets in H heavy so that coupling unification is maintained. This can be done by introducing a second $\mathbf{10}$ -plet (H') such that it forms a mass term with H but the color triplet field in it does not couple to standard model matter [27]. Therefore, from now on we will consider SO(10) models with 2 $\mathbf{10}$ -plets.

We will discuss two classes of models:

- (i) where $B - L$ is broken by $\mathbf{16}$ -Higgs fields (cf. Table IV (b)) and
- (ii) where $B - L$ is broken by $\mathbf{126}$ -Higgs fields [28,29]

TABLE IV. SO(10) model. (a) shows the MSSM sector, (b) the Higgs content of the $\mathbf{16}$ -Higgs model and (c) the Higgs content of the $\mathbf{126}$ -Higgs model. \mathbb{Z}_6 charges (see text) appear as subscript.

(a) MSSM part

Field	Quantum numbers
ψ_m	$\mathbf{16}_1$
H	$\mathbf{10}_{-2}$
H'	$\mathbf{10}_2$

(b) 16-Higgs model

Field	Quantum numbers
ψ_H	$\mathbf{16}_{-2}$
$\bar{\psi}_H$	$\mathbf{16}_2$
A	$\mathbf{45}_0$
S	$\mathbf{54}_0$

(c) 126-Higgs model

Field	Quantum numbers
Δ	$\mathbf{126}_2$
$\bar{\Delta}$	$\mathbf{126}_{-2}$
Φ	$\mathbf{210}_0$

(cf. Table IV (c)).

We consider Abelian discrete (\mathbb{Z}_N) symmetries and require that higher-dimensional R-parity conserving leading order $\Delta B \neq 0$ operators which in this case are of type $\mathbf{16}_m^4$ (where the subscript m stands for matter) are forbidden as are all R-parity breaking terms while allowing all terms in the superpotential that are needed to break the GUT symmetry down to the MSSM. Our focus is on proton stability, and we leave other issues such as fermion masses and doublet-triplet splitting for future studies.

A. 16-Higgs models

In this class of models, one has an independent motivation for introducing a second $\mathbf{10}$ -plet coming from doublet-triplet splitting. Further, apart from a pair of $\mathbf{16} \oplus \mathbf{16}$ -Higgses, $\mathbf{45}$ - and $\mathbf{54}$ -plets are required to ensure proper GUT symmetry breaking down to MSSM (cf. [30–33]). The superpotential terms that must be allowed are $\psi_m \psi_m H$, $(\psi_m \bar{\psi}_H)^2 / M_P$, $\psi_H \bar{\psi}_H$, A^2 , $S^{2,3}$ and SA^2 , where ψ_m, H are matter and Higgs $\mathbf{16}$ -plets; H , A , S are $\mathbf{10}$ -, $\mathbf{45}$ -, $\mathbf{54}$ -plets, respectively, (see Tables IV (a) and (b)). This leads to the following constraints on the \mathbb{Z}_N charges:

$$2q_{\psi_m} + q_H = 0 \pmod{N}, \quad 2q_{\psi_m} + 2q_{\bar{\psi}_H} = 0 \pmod{N}, \quad (7a)$$

$$q_{\psi_H} + q_{\bar{\psi}_H} = 0 \pmod{N}, \quad q_H + q_{H'} = 0 \pmod{N}, \quad (7b)$$

$$2q_A = 0 \pmod{N}, \quad 2q_S = 0 \pmod{N}, \quad (7c)$$

$$3q_S = 0 \pmod{N}, \quad 2q_A + q_S = 0 \pmod{N}. \quad (7d)$$

Here, we denote the \mathbb{Z}_N charge for a field F by q_F . Next, we list the anomaly constraints,

$$16(N_g q_{\psi_m} + q_{\psi_H} + q_{\bar{\psi}_H}) + 10(q_H + q_{H'}) + 45q_A + 54q_S = 0 \pmod{N'} \quad (8a)$$

$$2N_g q_{\psi_m} + 2q_{\psi_H} + 2q_{\bar{\psi}_H} + q_H + q_{H'} + 8q_A + 12q_S = 0 \pmod{N}. \quad (8b)$$

To forbid the dangerous couplings $\psi_m \psi_m H'$, ψ_m^A , $\psi_m \bar{\psi}_H$ and $\psi_m^3 \psi_H$, $\psi_m \psi_H H$, $\psi_m \psi_H H'$ and $\psi_m \bar{\psi}_H A$, the values of the \mathbb{Z}_N charges have to be chosen such that they satisfy the inequalities

$$2q_{\psi_m} + q_{H'} \neq 0 \pmod{N}, \quad 4q_{\psi_m} \neq 0 \pmod{N}, \quad (9a)$$

$$q_{\psi_m} + q_{\bar{\psi}_H} \neq 0 \pmod{N}, \quad 3q_{\psi_m} + q_{\psi_H} \neq 0 \pmod{N}, \quad (9b)$$

$$q_{\psi_m} + q_{H',H} + q_{\psi_H} \neq 0 \pmod{N}, \quad q_{\psi_m} + q_{\bar{\psi}_H} + q_A \neq 0 \pmod{N}. \quad (9c)$$

The smallest symmetry allowing to fulfill all criteria is \mathbb{Z}_6 and requires $N_g = 3$. A possible charge assignment is $q_{\psi_m} = 1$, $q_{\psi_H} = -2$, $q_{\bar{\psi}_H} = +2$, $q_H = -2$, $q_{H'} = +2$, $q_{45} = 0$ (cf. Tables IV (a) and (c)). This charge assignment allows for seesaw couplings and the possibility of fermion masses from couplings of type $\psi_m \psi_m H$. The allowed operator $\psi_m \psi_m \bar{\psi}_H^2$ contributes to both the fermion masses as well as to the seesaw. We note that the charge assignment is such that we have

$$3 \times \text{generation} + \text{vector-like} \quad (10)$$

under $\text{SO}(10) \times \mathbb{Z}_6$. The model also eliminates the dangerous proton decay operator $QQQL$ or operator of type $(\psi_m)^4/M_p$. We note that, although there are higher-dimensional gauge (and \mathbb{Z}_6) invariant operators, they do not give rise to proton decay operators for the following reasons:

- (i) any possible combination of ψ_H and $\bar{\psi}_H$ that breaks $B - L$ cannot multiply the $B - L$ neutral proton decay operators $QQQL$ (or $u^c u^c d^c e^c$)
- (ii) any $B - L$ neutral combination is also \mathbb{Z}_6 invariant because it has to involve as many ψ_H as $\bar{\psi}_H$ fields, as only the SM singlets with $B - L$ charge ± 1 attain a vev. Therefore the product of the \mathbb{Z}_6 noninvariant combination $QQQL$ with the $B - L$ neutral combination of ψ_H and $\bar{\psi}_H$ fields cannot be \mathbb{Z}_6 invariant. Consider, for instance, the operators $\frac{1}{M_p^3} [\psi_m]^4 [\bar{\psi}_H]^4$ and $\frac{1}{M_p^3} [\psi_m]^4 [\psi_H \bar{\psi}_H]$. In the first operator, the expectation value of $[\psi_H]^4$ vanishes because of the first argument while the second operator is not \mathbb{Z}_6 invariant.

We also observe that operators like $\psi_H^4 H^2$, which would lead to small diagonal entries in the $H - H'$ mass matrix, do not exist for the same reason. That is, in this model, proton decay is only due to dimension 6 operators.

We refrain from spelling out the detailed phenomenological analysis of this model. Our preliminary studies indicate that, by extending the Higgs content, one can achieve doublet-triplet splitting and realistic fermion masses while avoiding the proton stability problem. We defer the detailed discussion to a future publication.

Let us also comment that, like in the left-right model with doublets, the \mathbb{Z}_6 symmetry gets broken by the ψ_H and $\bar{\psi}_H$ vevs down to a \mathbb{Z}_2 which forbids W_R , i.e. it acts as an R-parity. This means that R-parity in this $\text{SO}(10)$ model does not originate from $B - L$.

B. 126-Higgs models

We now discuss models where the $\mathbf{16}_H \oplus \overline{\mathbf{16}}_H$ get replaced by $\mathbf{126} \oplus \overline{\mathbf{126}}$ —the motivation being that R-parity becomes an automatic symmetry. Such models have been extensively discussed in the literature [34–37]. In our context it means that the last two of the four inequalities in Eq. (9) do not exist (see Eq. (13) below). Instead we have the following set of constraints on the charges from anomaly freedom:

$$16N_g q_{\psi_m} + 10(q_H + q_{H'}) + 126(q_\Delta + q_{\bar{\Delta}}) = 0 \pmod{N'}, \quad (11a)$$

$$2N_g q_{\psi_m} + 35(q_\Delta + q_{\bar{\Delta}}) + q_H + q_{H'} = 0 \pmod{N}. \quad (11b)$$

The superpotential constraints can be decomposed in analogs of Eq. (7)

$$2q_{\psi_m} + q_H = 0 \pmod{N}, \quad (12a)$$

$$2q_{\psi_m} + q_{\bar{\Delta}} = 0 \pmod{N}, \quad (12b)$$

$$q_\Delta + q_{\bar{\Delta}} = 0 \pmod{N}, \quad (12c)$$

and analogs of Eq. (9)

$$2q_{\psi_m} + q_{H'} \neq 0 \pmod{N}, \quad (13a)$$

$$4q_{\psi_m} \neq 0 \pmod{N}. \quad (13b)$$

Typically in a class of these models, there are only $\mathbf{210}$ dimensional representations that need to couple to Δ and $\bar{\Delta}$ fields among themselves as well as with $\mathbf{10}$ -Higgs [38,39]. These imply that the discrete charge of $\mathbf{210}$ vanishes and also that of Δ and $\bar{\Delta}$ are opposite. Substituting these conditions into Eq. (11), it becomes clear that if there is only a single $\mathbf{10}$ Higgs in the model (or, if $q_{H'} = 0$), the

TABLE V. \mathbb{Z}_3 charge assignment.

Field	q_{ψ_m}	H	H'	Δ	$\bar{\Delta}$
\mathbb{Z}_3	1	1	-1	-1	1

requirement of anomaly freedom becomes rather constraining. We could only find a \mathbb{Z}_8 symmetry with one generation. However, once we allow for nontrivial $q_{H'}$, it is possible to have simple anomaly-free \mathbb{Z}_N symmetries which satisfy all constraints, the simplest example being a \mathbb{Z}_3 symmetry with the charge assignment listed in Table V. The $\mathbb{Z}_{N \leq 12}$ symmetries with nontrivial $q_{H'}$ require $N_g = 3$ or larger.

Another possible symmetry is \mathbb{Z}_6 with the charge assignment listed in Table IV (c).

This model has the same effective structure as the **126**-model of Refs. [34–37] as far as the discussion of fermion masses go (even though it has 2 **10**-plets, one of the **10**'s does not couple to fermions due to \mathbb{Z}_6 charge assignments).

For the same reasons as in the **16**-Higgs models, the $QQQL$ -type induced proton decay operators are forbidden.

Again we refrain from a detailed phenomenological analysis of the way this model leads to MSSM at low energies. We close the discussion with a comment on how easily the MSSM Higgs fields emerge from the superpotential: the relevant part of the superpotential has the form

$$W = M_H HH' + M_0 \Delta (\bar{\Delta} + \lambda \Phi H) + M_\Phi \Phi^2 + \lambda' \Phi^3 + \lambda'' \Delta \Phi H' + \lambda''' \Delta \Phi \Delta, \quad (14)$$

which has the right linear combination of MSSM doublets to maintain all the simple form for the fermion mass results of Refs. [34–37]. A detailed analysis of these issues will be given elsewhere.

C. SO(10) GUTs in higher dimensions

Let us now comment on implications of our findings for higher-dimensional models of grand unification, such as “orbifold GUTs” [40–46], which provide a simple solution to the doublet-triplet splitting problem. In such models the dimension 5 proton decay can be naturally suppressed [42,43] (while dimension 6 proton decay is slightly enhanced [47]) since here the mass partner of the Higgs triplet has vanishing couplings to matter, as in the discussion above. However, (brane) couplings like ψ_m^4 , also leading to proton decay, have not been discussed in this scheme. A reliable discussion of such operators seems hardly possible in the effective higher-dimensional field theory framework.

One possible way to address this question is thus to embed the model into string theory or, in other words, to derive orbifold GUT models from string theory. The first

steps for doing so have been performed in Refs. [48–50]. This has further led to the scheme of “local grand unification” [51–54], which facilitates the construction of supersymmetric standard models from the heterotic string [52,53,55,56]. Here, the two light MSSM matter generations originate from **16**-plets localized at points with SO(10) gauge symmetry. Some of these models can have an R-parity arising as a \mathbb{Z}_2 subgroup of a gauged, non-anomalous $B - L$ symmetry [55–57], like in ordinary GUTs (however, without the need for $\overline{\mathbf{126}}$ -plets). On the other hand, $QQQL$ operators remain a challenge [53,56]. The fact that these operators could be eliminated so easily in conventional GUTs by simple symmetries leads to the expectation that similar symmetries will be helpful in the string-derived supersymmetric standard models with (local) GUT structures. One lesson which one might learn from our analysis is that one may derive an effective R-parity and suppress $QQQL$ by a discrete (possibly \mathbb{Z}_6) symmetry under which matter **16**-plets have a universal charge. One might further hope to get insights about the origin of the discrete symmetries (which remains somewhat obscure in the 4D field-theoretic approach) in string models. These issues will be studied elsewhere.

V. CONCLUSION AND COMMENTS

Motivated by the beauty of the ideas of supersymmetry and unification, we have started a search for discrete symmetries that forbid proton decay operators in gauge extensions of the supersymmetric standard model. We required the symmetries to allow the standard interactions and to be anomaly-free. Considering the left-right symmetric, Pati-Salam and SO(10) GUT models with various Higgs contents, we could identify (surprisingly simple) symmetries that satisfy all our criteria. In many cases, there is a connection between the anomaly freedom and the number of generations. Often, simple symmetries exist only for 3 generations (or multiples thereof), as in [1]. In the SO(10) models, our symmetries forbid dimension 5 proton decay operators.

Our findings can be interpreted in the following way. Supersymmetric models with an extended or GUT symmetry are often challenged by proton decay. To rectify this, one might be forced to introduce additional (discrete) symmetries. Our examples then show that R-parity can be a consequence of these additional symmetries rather than being related to $B - L$. From this one might conclude that the appearance of fields with even $B - L$ charges is not a necessity, and, for instance, **16**-Higgs and **126**-Higgs SO(10) models can be on the same footing.

It is also interesting that the minimal **126**-based SO(10) models become free of all dangerous proton decay operators without losing their ability to be predictive in the fermion sector once we add a simple anomaly-free discrete symmetry.

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