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Efficient Decentralized Economic Dispatch for Microgrids with Wind Power Integration

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***Abstract*—Decentralized energy management is of paramount importance in smart microgrids with renewables for various reasons including environmental friendliness, reduced communication overhead, and resilience to failures. In this context, the present work deals with distributed economic dispatch and demand response initiatives for grid-connected microgrids with high-penetration of wind power. To cope with the challenge of the wind's intrinsically stochastic availability, a novel energy planning approach involving the actual wind energy as well as the energy traded with the main grid, is introduced. A stochastic optimization problem is formulated to minimize the microgrid net cost, which includes conventional generation cost as well as the expected transaction cost incurred by wind uncertainty. To bypass the prohibitively high-dimensional integration involved, an efficient sample average approximation method is utilized to obtain a solver with guaranteed convergence. Leveraging the special infrastructure of the microgrid, a decentralized algorithm is further developed via the alternating direction method of multipliers. Case studies are tested to corroborate the merits of the novel approaches.**

***Index Terms*—Microgrids, economic dispatch, renewable energy, sample average approximation, ADMM**

NOMENCLATURE

A. Indices, numbers, and sets

T, t	Number of scheduling periods, and period index.
M, m	Number of conventional distributed generation (DG) units, and their index.
N, n	Number of dispatchable loads, and load index.
I, i	Number of wind farms, and their index.
$\mathcal{M}$	Set of conventional DG units.
$\mathcal{N}$	Set of dispatchable loads.

B. Variables

$P_{G_m}^t$	Power output of DG unit m over time slot t.
$P_{D_n}^t$	Power consumption of load n over slot t.
W_i^t	Power output from ith wind farm over slot t.
P_R^t	Wind power delivered to the microgrid in slot t.

C. Constants

$P_{G_m}^{\min}$, $P_{G_m}^{\max}$	Minimum and maximum power output of conventional DG unit m.
R_m^{up}, R_m^{down}	Ramp-up and ramp-down limits of conventional DG unit m.
SR^t	Spinning reserve for conventional DG.
L^t	Fixed power demand of critical loads over slot t.
$P_{D_n}^{\min}$, $P_{D_n}^{\max}$	Minimum and maximum power consumption of load n.
$P_R^{\min}$, $P_R^{\max}$	Lower and upper bounds for P_R^t.
α^t, β^t	Purchase and selling prices per slot t.

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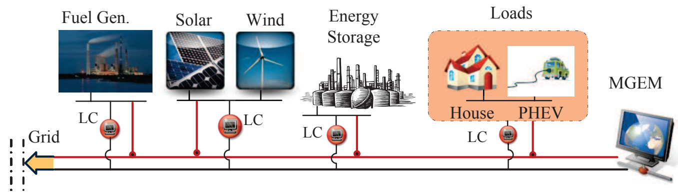

Fig. 1. Decentralized infrastructure of a microgrid with communications (black) and energy flow (red) networks.

D. Functions

$C_m^t(\cdot)$	Cost of conventional DG unit m in slot t.
$U_n^t(\cdot)$	Utility of load n in slot t.
$G(\cdot)$, $\hat{G}(\cdot)$	Expected and sample-averaged transaction cost across entire horizon.
$\mathcal{L}_\rho(\cdot)$	Partially augmented Lagrangian function.

I. INTRODUCTION

As contemporary small-scale counterparts of the bulk power grid, smart microgrids comprise distributed energy resources (DERs) and electricity end users, all deployed within a limited geographical area [1]. Depending on their origin, DERs can come either from conventional energy sources including oil, gas and coal, or, from renewable energy sources (RES), such as wind and solar energy. Bypassing limitations of a congested transmission network, microgrids can generate, distribute, and regulate power flows at the community level to efficiently meet growing consumer demands. Besides critical non-dispatchable loads, elastic controllable ones allow residential or commercial customers to participate in the electricity enterprise. A typical microgrid configuration is depicted in Fig. 1. Through the communications network, a so-termed microgrid energy manager (MGEM) coordinates the DERs and the controllable loads, each of which has a local controller (LC).

Aligned with the goal of high-penetration RES in future smart grids, economic dispatch (ED) with renewables has been extensively studied recently. ED penalizing over- and under-estimation of wind energy is investigated in [2]. Worst-case robust distributed ED is proposed for grid-connected microgrids with DERs in [3]. Leveraging the scenario approximation technique, risk-constrained ED with correlated wind farms have been developed recently in [4]. A multi-stage stochastic control approach is pursued for risk-limiting dispatch of wind power in [5]. A chance-constrained two-stage stochastic program is formulated in [6] for unit commitment with wind power uncertainty. Capitalizing on the hierarchical multi-agent coordination, distributed ED via heterogeneous wireless networks is studied in [7].

Notwithstanding their merits, the aforementioned works have limitations. For example, it is unlikely to have the worst-case scenario in real time operation [3]. Globally optimal solutions are generally hard to obtain for non-convex chance-constrained problems. Convex relaxation using the scenario sampling can afford efficient optimization solvers, but it turns out to be too conservative for scheduling the delivered renewables in certain scenarios [4]. Moreover, slow convergence of conventional distributed algorithms may have scalability issues facing large-scale problems; e.g., the subgradient ascent based dual decomposition approach [3].

This paper considers day-ahead ED for microgrids with high penetration of wind energy, operating in the grid-connected mode. By introducing what is termed scheduled wind power, a novel energy transaction mechanism is put forth to address the challenge of maintaining the supply-demand balance imposed by the uncertainty of wind power. A stochastic optimization program is formulated to minimize the microgrid net cost, which consists of costs for conventional generation, utility of elastic loads, as well as the expected transaction cost (Section II). A *sample average approximation (SAA)* approach with convergence guarantees is efficiently utilized to deal with the involved multidimensional integral in the expectation function. With the attractive advantages of being computationally efficient and resilient to communication outages, *decentralized* scheduling over the microgrid communications network is developed based on the *alternating direction method of multipliers (ADMM)* (Section III). Numerical tests are reported to corroborate the merits of the novel approaches (Section IV).

Notation. Boldface lower case letters represent vectors; $(\cdot)'$ indicates transpose; and $\mathbb{E}[\cdot]$ denotes the expectation operator.

II. Robust Energy Management Formulation

Consider a microgrid comprising M conventional generators, N controllable (dispatchable) loads, and I wind farms. The scheduling horizon of interest is $\mathcal{T} := \{1, 2, \ldots, T\}$ (e.g., one day ahead). Let $P_{G_m}^t$ be the power produced by the mth conventional generator, and $P_{D_n}^t$ the power consumed by the nth dispatchable load at slot t, where $m \in \mathcal{M} := \{1, \ldots, M\}$, $n \in \mathcal{N} := \{1, \ldots, N\}$, and $t \in \mathcal{T}$. Let P_R^t denote the *committed (scheduled)* wind energy delivered to the microgrid at slot t. The ensuing subsection details the transaction mechanism between the microgrid and the main grid. Subsection II-B formulates the microgrid ED problem, which boils down to optimally dispatching the powers $\{P_{G_m}^t\}_m$, $\{P_{D_n}^t\}_n$, and P_R^t for all $t \in \mathcal{T}$.

A. Expected Transaction Cost

Let W_i^t denote the *actual* wind power harvested from the ith wind farm at time slot t. Suppose that the microgrid operates in a grid-connected mode, and a transaction mechanism with the main grid is in place, where the microgrid can buy (sell) energy from (to) the spot market. Specifically, the shortfall between the actual wind power produced and the one scheduled per slot t is $\left[P_R^t - \sum_{i=1}^I W_i^t\right]^+$, while the corresponding surplus is $\left[P_R^t - \sum_{i=1}^I W_i^t\right]^-$, where $[a]^+ := \max\{a, 0\}$, and $[a]^- := -\min\{a, 0\}$. The amount of energy shortage $\left[P_R^t - \sum_{i=1}^I W_i^t\right]^+$ is bought with a known fixed purchase price α^t, while the energy surplus $\left[P_R^t - \sum_{i=1}^I W_i^t\right]^-$ is sold back to the main grid with a fixed selling price β^t. Clearly, only one of these two quantities is nonzero at each slot t. Wind power W_i^t is a function of the random wind speed v_i^t, for which different models and wind-speed-to-wind-power mappings $W_i^t(v_i^t)$ are available [8]. The *expected* transaction cost can be readily expressed as

$$G(\mathbf{p}_R) := \mathbb{E}_{\mathbf{v}}\Big[\sum_{t=1}^{T}\Big(\alpha^t[P_R^t - \sum_{i=1}^{I} W_i^t(v_i^t)]^+ \\ -\beta^t[P_R^t - \sum_{i=1}^{I} W_i^t(v_i^t)]^-\Big)\Big] \tag{1}$$

where $\mathbf{v} := [v_1^1, \ldots, v_1^T, \ldots, v_I^1, \ldots, v_I^T]'$ and $\mathbf{p}_R := [P_R^1, \ldots, P_R^T]'$.

B. Microgrid Net Cost Minimization

The cost of the mth conventional generator is a convex increasing function $C_m^t(P_{G_m}^t)$, typically chosen either as piecewise linear or as smooth quadratic. Moreover, the utility function of the nth dispatchable load, $U_n^t(P_{D_n}^t)$, is selected to be concave increasing, and likewise either piecewise linear or smooth quadratic. Apart from dispatchable loads, there is also a fixed power demand from critical loads, denoted by L^t. For notational brevity, let $\mathbf{p}_G$ and $\mathbf{p}_D$ denote the vectors collecting $\{P_{G_m}^t\}_{m,t}$ and $\{P_{D_n}^t\}_{n,t}$, respectively.

ED aims at minimizing the microgrid-wide net cost; that is, the cost of conventional generation, minus the load utility as well as the expected transaction cost:

$$\text{(P1)}\quad \min_{\{\mathbf{p}_G, \mathbf{p}_D, \mathbf{p}_R\}} \Big\{\sum_{t=1}^{T}\Big(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t)\Big) \\ + G(\{P_R^t\})\Big\} \tag{2a}$$

subject to:

$$P_{G_m}^{\min} \le P_{G_m}^t \le P_{G_m}^{\max},\ \forall\, m \in \mathcal{M},\ \forall\, t \in \mathcal{T} \tag{2b}$$

$$P_{G_m}^t - P_{G_m}^{t-1} \le R_m^{\text{up}},\ \forall\, m \in \mathcal{M},\ \forall\, t \in \mathcal{T} \tag{2c}$$

$$P_{G_m}^{t-1} - P_{G_m}^t \le R_m^{\text{down}},\ \forall\, m \in \mathcal{M},\ \forall\, t \in \mathcal{T} \tag{2d}$$

$$\sum_{m=1}^{M}(P_{G_m}^{\max} - P_{G_m}^{t}) \geq \mathsf{SR}^t,\ \forall\, t \in \mathcal{T} \tag{2e}$$

$$P_{D_n}^{\min} \leq P_{D_n}^{t} \leq P_{D_n}^{\max},\ \forall\, n \in \mathcal{N},\ \forall\, t \in \mathcal{T} \tag{2f}$$

$$P_{R}^{\min} \leq P_{R}^{t} \leq P_{R}^{\max},\ \forall\, t \in \mathcal{T} \tag{2g}$$

$$\sum_{m=1}^{M} P_{G_m}^{t} + P_R^t = \sum_{n=1}^{N} P_{D_n}^{t} + L^t,\ \forall\, t \in \mathcal{T}. \tag{2h}$$

Constraints (2b)-(2f) stand for the minimum/maximum conventional generation, ramping up/down limits, spinning reserves, and the minimum/maximum power of the dispatchable loads, respectively. They capture the typical physical limits of the power generators and loads. Constraint (2g) places upper and lower limits on the committed wind power, which are imposed by the capacity of the transmission lines over which the energy is transacted. Finally, constraint (2h) is the supply-demand *balance equation* ensuring that the total demand is satisfied by the power generation at any time.

Note that (2b)-(2h) are linear, while $C_m^t(\cdot)$ and $-U_n^t(\cdot)$ are convex. Consequently, the convexity of (P1) depends on that of $G(\mathbf{p}_R)$, which is established in the following proposition.

Proposition 1. *If the selling price β^t does not exceed the purchase price α^t for any $t \in \mathcal{T}$, then the expected transaction cost $G(\mathbf{p}_R)$ is convex in $\mathbf{p}_R$.*

Proof: Using the identities $[a]^+ + [a]^- = |a|$ and $[a]^+ - [a]^- = a$, $G(\mathbf{p}_R)$ can be equivalently re-written as

$$G(\mathbf{p}_R) = \mathbb{E}_{\mathbf{v}}\Big[\sum_{t=1}^{T}\Big(\delta^t\Big|P_R^t - \sum_{i=1}^{I} W_i^t(v_i^t)\Big| + \gamma^t[P_R^t - \sum_{i=1}^{I} W_i^t(v_i^t)]\Big)\Big] \tag{3}$$

with $\delta^t := (\alpha^t - \beta^t)/2$, and $\gamma^t := (\alpha^t + \beta^t)/2$. Since the absolute value function is convex, and the operations of nonnegative weighted summation and integration preserve convexity [9, Sec. 3.2.1], the claim follows readily. ■

An immediate corollary here is that the ED problem $(P1)$ is convex if $\beta^t \leq \alpha^t$ for all t. The next section begins with a special case when $\alpha^t \equiv \beta^t$, before developing an approximation method together with an efficient decentralized solver for general transaction prices satisfying the condition of Proposition 1.

III. Sample Average Approximation and Distributed Algorithm

A. A Special Case

If the *locational marginal pricing (LMP)* mechanism is utilized to price energy purchases and sales for the microgrid, then $\alpha^t = \beta^t = \ell^t$ for all $t \in \mathcal{T}$, where $\{\ell^t\}$ are the locational marginal prices at the bus where the transaction takes place. In this case, we have $\delta^t = 0$ and $\gamma^t = \alpha^t$ for all t. It thus follows that

$$G(\mathbf{p}_R) = \mathbb{E}_{\mathbf{v}}\Big[\sum_{t=1}^{T}\alpha^t\Big(P_R^t - \sum_{i=1}^{I} W_i^t(v_i^t)\Big)\Big] \doteq \sum_{t=1}^{T}\alpha^t\Big(P_R^t - \sum_{i=1}^{I}\bar{W}_i^t\Big)$$

where $\{\bar{W}_i^t\}_{i,t}$ are sample average wind power estimates assumed to be available via statistical inference based on historical data, or, through numerical weather prediction.

In this special case, (P1) boils down to a smooth convex minimization problem. If $\{C_m^t(\cdot)\}_{m,t}$ and $\{U_n^t(\cdot)\}_{n,t}$ are convex quadratic or piece-wise linear, then (P1) is either a convex quadratic program (QP) or a linear program (LP); hence, (P1) is efficiently solvable with off-the-shelf QP/LP solvers. Next, the general case of transaction prices is investigated with the resulting optimization problem formulated using the aforementioned sample approximation method, and solved in a decentralized fashion.

B. Sample Average Approximation

Consider now the general case under the price condition of Proposition 1, which typically holds for microgrid power systems [10]. If the selling and buying prices are not always the same, then the absolute value terms in $G(\mathbf{p}_R)$ do not disappear (cf. (3)). Due to the nonlinearity of the absolute value operator, it cannot be interchanged with the expectation. In addition, although entries of $\mathbf{v}$ are Weibull distributed, their correlation prevents analytical expression of $G(\mathbf{p}_R)$. Moreover, the multidimensional integration needed to carry out the expectation cannot be computed with high accuracy numerically.

To bypass this challenge, the empirical estimate of $G(\mathbf{p}_R)$ will be adopted based on N_s Monte Carlo samples $\{W_i^t(s)\}_{s=1}^{N_s}$ for each W_i^t. In this case, $G(\mathbf{p}_R)$ is replaced by

$$\hat{G}(\mathbf{p}_R) := \frac{1}{N_s}\sum_{s=1}^{N_s}\sum_{t=1}^{T}\delta^t\Big|P_R^t - \sum_{i=1}^{I} W_i^t(s)\Big| + \sum_{t=1}^{T}\gamma^t\Big(P_R^t - \sum_{i=1}^{I}\bar{W}_i^t\Big). \tag{4}$$

This sample average approximation (SAA) of (P1) is distribution free, and the law of large numbers (LLN) guarantees that $\hat{G}(\mathbf{p}_R)$ is a good approximation of $G(\mathbf{p}_R)$ for N_s large enough. Based on the latter, the ED problem of interest can be approximated as

$$\text{(AP1)}\ \min_{\{\mathbf{p}_G,\mathbf{p}_D,\mathbf{p}_R\}}\Big\{\sum_{t=1}^{T}\Big(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t)\Big) + \hat{G}(\mathbf{p}_R)\Big\} \tag{5a}$$

$$\text{s.t.}\quad (2b)-(2h).$$

Clearly, convexity is preserved in the SAA formulation (AP1), and this renders it efficiently solvable. The following conditions are sufficient to establish the convergence of SAA applied to (P1): A1) The optimal solution set of (P1) is nonempty; A2) The LLN holds pointwise; that is, $\hat{G}(\mathbf{p}_R) \to G(\mathbf{p}_R)$ with probability (w.p.) 1 as $N_s \to \infty$.

Let ϑ^* and $\mathcal{S}^*$ denote the optimal value and the optimal solution set of (P1), respectively. Similarly, $\hat{\vartheta}_{N_s}$ and $\hat{\mathcal{S}}_{N_s}$ for (AP1). Define further the deviation of the set $\mathcal{A}$ from the set $\mathcal{B}$ by $\mathbb{D}(\mathcal{A},\mathcal{B}) := \sup_{\mathbf{x}\in\mathcal{A}} \inf_{\mathbf{y}\in\mathcal{B}} \|\mathbf{x}-\mathbf{y}\|$. With these notational conventions, the following convergence result can be established.

Proposition 2. *If conditions A1) and A2) hold, then $\hat{\vartheta}_{N_s} \to \vartheta^*$, and $\mathbb{D}(\hat{\mathcal{S}}_{N_s}, \mathcal{S}^*) \to 0$ w.p. 1 as $N_s \to \infty$.*

Proof: It can be shown that A1)-A2) as well as the special structure of (P1) satisfy the conditions in [11, Thm. 5.4], where a convergence claim for a general problem is established. Due to space limitations, the detailed proof is omitted. ■

Note that (AP1) entails a separable convex objective (5a) with a linear equality constraint (2h), as well as the compact polyhedral feasible sets (2b)-(2g), which are in the form of a Cartesian product. This separable structure motivates solving (AP1) in a distributed fashion by resorting to the alternating direction method of multipliers (ADMM) [12], which has drawn growing interest recently, because it exhibits good performance in many large-scale distributed optimization problems in e.g., machine learning and signal processing.

By exploiting the microgrid infrastructure, an ADMM-based distributed solver is developed in the ensuing section.

C. Decentralized ED via ADMM

With reference to the microgrid depicted in Fig. 1, it is natural to implement ED across the local controllers (LCs) of conventional generators, dispatchable loads, and renewable facilities. To this end, introduce a Lagrange multiplier vector $\boldsymbol{\lambda} := [\lambda^1, \ldots, \lambda^T]'$ associated with the coupling equality constraints (2h), along with a quadratic penalty. The partially augmented Lagrangian of (AP1) is

$$\begin{aligned}\mathcal{L}_\rho(\mathbf{p}_G,\mathbf{p}_D,\mathbf{p}_R,\boldsymbol{\lambda}) &= \sum_{t=1}^{T}\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{t=1}^{T}\sum_{n=1}^{N} U_n^t(P_{D_n}^t)\\ &+ \hat{G}(\mathbf{p}_R) + \sum_{t=1}^{T}\lambda^t\left(\sum_{m=1}^{M}P_{G_m}^t + P_R^t - \sum_{n=1}^{N}P_{D_n}^t - L^t\right)\\ &+ \frac{\rho}{2}\sum_{t=1}^{T}\left(\sum_{m=1}^{M}P_{G_m}^t + P_R^t - \sum_{n=1}^{N}P_{D_n}^t - L^t\right)^2 \qquad (6)\end{aligned}$$

where $\rho > 0$ is a constant.

ADMM is tantamount to updating first the primal variables in the Gauss-Seidel fashion (a.k.a. block coordinate descent), and then updating the dual variables in a gradient ascent manner. Specifically, with $\mathcal{P}_G := \{\mathbf{p}_G | \text{ (2b)} - \text{(2e)}\}$, $\mathcal{P}_D := \{\mathbf{p}_D | \text{ (2f)}\}$, and $\mathcal{P}_R := \{\mathbf{p}_R | \text{ (2g)}\}$, let k denote the iteration index, and $\nu > 0$ a constant stepsize. The resulting distributed ED solver is tabulated as Algorithm 1, where the last step is a reasonable termination criterion using the primal residual (see also [12, Sec. 3.3.1])

Algorithm 1 Distributed Economic Dispatch using ADMM

Initialize $\boldsymbol{\lambda}(0) = \mathbf{0}$

repeat $(k = 1, 2, \ldots)$

Update primal variables:

$$\mathbf{p}_G(k+1) = \arg\min_{\mathbf{p}_G\in\mathcal{P}_G} \mathcal{L}_\rho(\mathbf{p}_G, \mathbf{p}_D(k), \mathbf{p}_R(k), \boldsymbol{\lambda}(k)) \qquad (8)$$

$$\mathbf{p}_D(k+1) = \arg\min_{\mathbf{p}_D\in\mathcal{P}_D} \mathcal{L}_\rho(\mathbf{p}_G(k+1), \mathbf{p}_D, \mathbf{p}_R(k), \boldsymbol{\lambda}(k)) \qquad (9)$$

$$\mathbf{p}_R(k+1) = \arg\min_{\mathbf{p}_R\in\mathcal{P}_R} \mathcal{L}_\rho(\mathbf{p}_G(k+1), \mathbf{p}_D(k+1), \mathbf{p}_R, \boldsymbol{\lambda}(k)) \qquad (10)$$

Update dual variables: for all $t \in \mathcal{T}$

$$\begin{aligned}\lambda^t(k+1) = \lambda^t(k) + \nu\Big(&\sum_{m=1}^{M} P_{G_m}^t(k+1) + P_R^t(k+1)\\ &- \sum_{n=1}^{N} P_{D_n}^t(k+1) - L^t\Big) \qquad (11)\end{aligned}$$

until $\xi \le \epsilon_{\text{res}}$

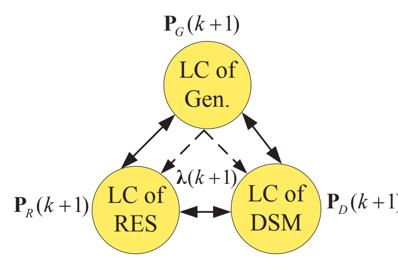

Fig. 2. ADMM message passing.

$$\xi := \left[\sum_{t=1}^{T}\left(\sum_{m=1}^{M}P_{G_m}^t + P_R^t - \sum_{n=1}^{N}P_{D_n}^t - L^t\right)^2\right]^{1/2}. \qquad (7)$$

Remark 1. *(Convergence of ADMM).* Sufficient conditions for convergence of the K-block ($K \ge 3$) ADMM have been established recently in [13] and [14]. One of these conditions requires that all subproblems of updating the primal variables are strongly convex. It is worth pointing out that although subproblem (10) is not strongly convex, the algorithm always converged in the extensive numerical tests that we performed (see Section IV). Furthermore, the proximal ADMM of [14] can be applied here with guaranteed linear convergence. Interested readers are referred to [14] for the detailed algorithm and convergence claims.

ADMM iterations easily lend themselves to a distributed implementation utilizing the microgrid communication network (cf. Fig. 1). Specifically, the LCs of conventional generation, dispatchable loads, and RES solve subproblems (8),

TABLE I

GENERATING CAPACITIES, RAMPING LIMITS, AND COST COEFFICIENTS. THE UNITS OF a_m AND b_m ARE ¢/(KWH)^2 AND ¢/KWH, RESPECTIVELY.

Unit	$P_{G_m}^{\min}$	$P_{G_m}^{\max}$	$R_{m,\text{up(down)}}$	a_m	b_m
1	5	70	30	0.006	14
2	5	80	35	0.003	20
3	10	85	50	0.004	50

TABLE II

LOAD DEMAND LIMITS, AND UTILITY COEFFICIENTS. THE UNITS OF c_n AND d_n ARE ¢/(KWH)^2 AND ¢/KWH, RESPECTIVELY.

Unit	$P_{D_n}^{\min}$	$P_{D_n}^{\max}$	c_n	d_n
1	5	30	-0.20	20
2	8	50	-0.30	30
3	3	45	-0.17	17

(9), and (10) sequentially, via efficient QP solvers. Note that after each LC solves its own subproblem, the correspondingly updated primal variables should be broadcast to all other LCs. The dual updating step (11) can be readily implemented by any one of the three LCs. The detailed message passing process is depicted in Fig. 2.

IV. NUMERICAL TESTS

In this section, case studies are presented to verify the performance of ADMM-based distributed ED for a microgrid consisting of $M = 3$ conventional generators, $N = 3$ dispatchable loads, and $I = 4$ wind farms scheduled over $T = 8$ hours. The generation costs $C_m(P_{G_m}) = a_m P_{G_m}^2 + b_m P_{G_m}$, and the utilities of elastic loads $U_n(P_{D_n}) = c_n P_{D_n}^2 + d_n P_{D_n}$ are selected time-invariant and quadratic. The corresponding parameters of generators, loads and transaction prices are listed in Table I – III, while spinning reserves are set to $\mathsf{SR}^t = 6.66$ kWh for all $t \in \mathcal{T}$. The resulting optimization problems are specified and solved via the Matlab-based modeling language `CVX` [15] along with the solver `Gurobi` [16].

To obtain the wind power samples $\{W_i^t(s)\}_{s=1}^{N_s}$ required as input to (AP1) (cf. (5a)), a simple but effective sampling approach leveraging autoregressive models with the wind-speed-to-wind-power mapping is utilized; see [4] and [17] for details. In the numerical tests, the sample size is $N_s = 1,000$, and the averaged wind power outputs $\{\bar{W}_i^t\}_{i,t}$ are obtained using $20,000$ samples of the wind speed.

Figure 3 demonstrates the convergence of the net cost (5a), and the evolution of the primal residual ξ. It is clear that the algorithm converges fast within 50 iterations. In all numerical tests, the relevant parameters are $\rho = 1$, $\nu = 0.5$, and $\epsilon_{\text{res}} = 10^{-2}$. Furthermore, as with other distributed solvers (e.g., dual decomposition using subgradient ascent), ADMM is not an iterative algorithm guaranteeing a monotonically decreasing objective. Figure 3 shows that some objective values of the iterates can be even smaller than the optimal value due to the constraint violation. However, for the day-ahead energy planning problem, ADMM outperforms alternative distributed solvers thanks to its fast convergence.

Convergence of the primal and dual variables is verified in Fig. 4, where $P_G^S := \sum_m P_{G_m}^t$ denotes the total conventional

TABLE III

FIXED LOAD DEMAND AND TRANSACTION PRICES. THE UNIT OF α^t AND β^t IS ¢/KWH

Time slot	1	2	3	4	5	6	7	8
L^t	30	34	47	60	75	67	55	43
α^t	1.40	2.20	4.70	6.30	8.50	7.80	5.60	4.50
β^t	1.12	1.76	3.76	5.04	6.80	6.24	4.48	3.60

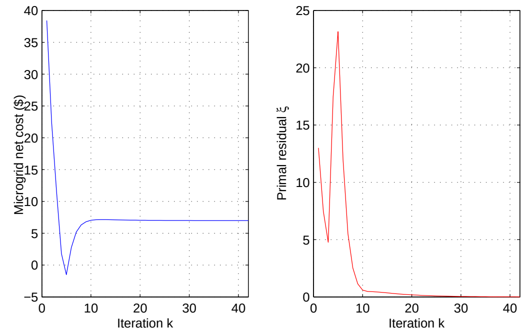

Fig. 3. Convergence of the net cost and evolution of the primal residual.

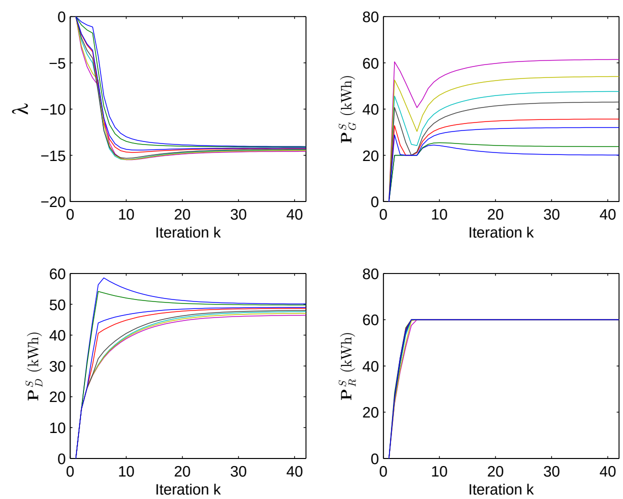

Fig. 4. Convergence of the primal and dual variables.

power generation, and likewise for P_D^S. Clearly, the iterates converge fast as shown by the 8 trajectories per subplot, each corresponding to a different time slot.

The optimal power schedules are depicted in Fig. 5. As expected, the total conventional power generation P_G^S varies across t with the same trend as the fixed load demand L. Moreover, the elastic demand P_D^S exhibits opposite trend with respect to L. This is because when L^t is low, P_D^t can increase to gain in utility, as long as the total load demand can be satisfied. As shown in the slots from 4 to 7, this behavior illustrates the peak-load shifting ability of the proposed design. It is also interesting to see that the optimal scheduled wind power P_R is set equal to $P_R^{\max} = 60$ kWh across time. This is because with the energy purchase price α^t being much smaller than the generation costs $\{a_m, b_m\}_m$ (cf. Tables I and III), the

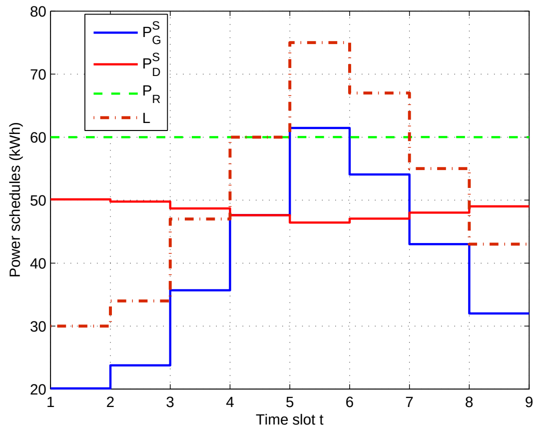

Fig. 5. Optimal power schedules.

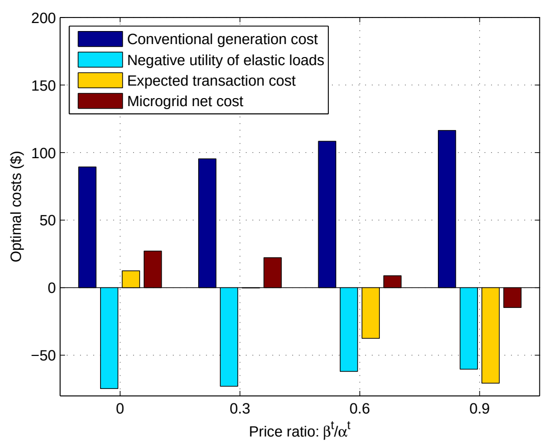

Fig. 6. Optimal costs with different price ratios.

economic scheduling decision is to reduce the conventional generation while purchasing as much energy as possible to keep the supply-demand balance.

Finally, Fig. 6 shows the effect of different transaction prices on the optimal costs, where five times of α^t in Table III is used. Clearly, the net cost decreases as the selling-to-purchase-price ratio β^t/α^t increases. When this ratio increases, the microgrid can afford higher margin for revenue by selling renewable energy back to the main grid. Thus, if more energy is sold instead of being used within the microgrid, the cost of conventional generation will increase to supply the loads. Therefore, as depicted in Fig. 6, the microgrid net cost can be reduced so long as the obtained transaction profit exceeds the extra generation cost.

V. Conclusions and Future Work

A distributed energy planning approach was developed in this paper tailored for microgrids with high penetration of wind power. By introducing the quantity of scheduled wind power, a transaction model was proposed to maintain the supply-demand balance challenged by the intermittent nature of RES. A stochastic optimization problem was formulated with the objective of minimizing the microgrid net cost. The SAA method was efficiently utilized to overcome the high-dimensional integration involved. Finally, the robust ED problem was solved in a distributed fashion using an ADMM-based solver whose fast convergence was corroborated by extensive numerical tests.

A number of appealing future directions open up, including real-time dispatch and the incorporation of uncertainty stemming from critical loads and the transaction prices.

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