### Title
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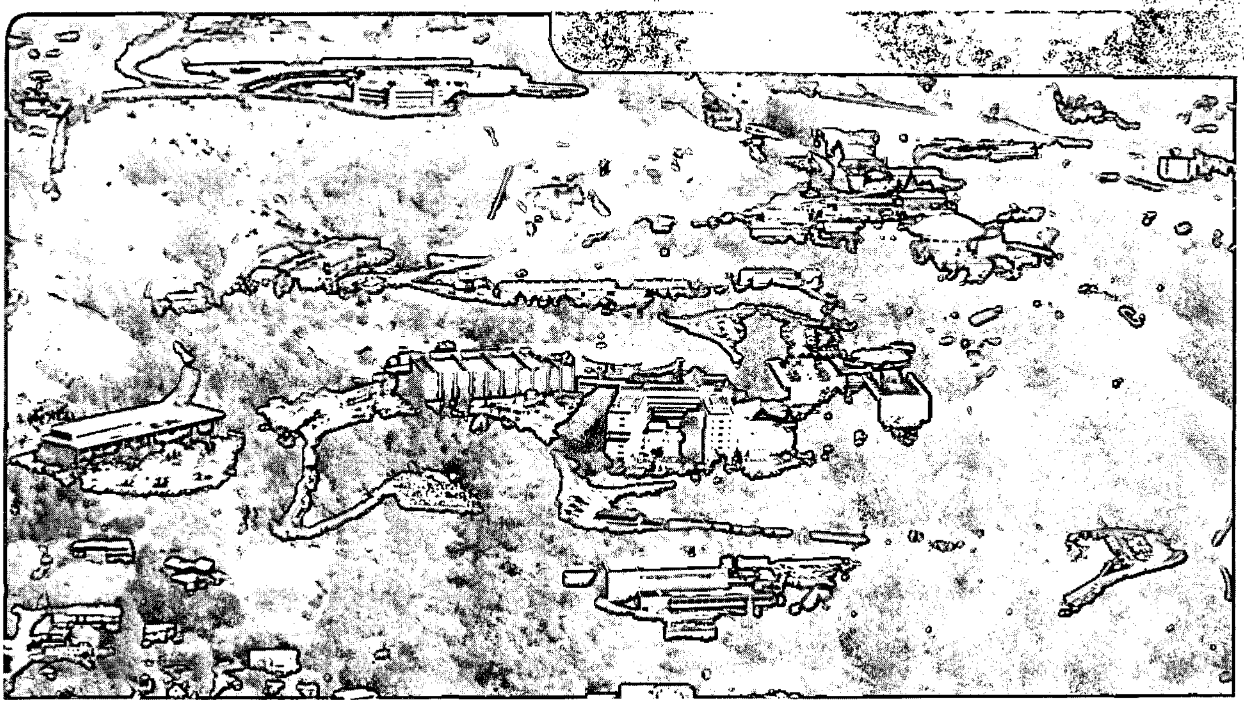



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THE HIGH SENSITIVITY OF THE MAXIMUM LIKELIHOOD ESTIMATOR METHOD OF TOMOGRAPHIC IMAGE RECONSTRUCTION

LBL-21874

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## Summary

In recent work we have shown that PET images obtained by the MLE iterative method of image reconstruction converge towards strongly deteriorated versions of the original source image. In the present work we show that the image deterioration is caused by an excessive attempt by the algorithm to match the projection data with high counts and that we can modulate this effect. We compare a source image with reconstructions by filtered backprojection and by the MLE algorithm and show that the MLE images can have similar noise to the filtered backprojection images at regions of high activity and very low noise, comparable to the source image, in regions of low activity, if the iterative procedure is stopped at an appropriate point.

## Introduction

After the Maximum Likelihood Estimator (MLE) method of image reconstruction was proposed by Shepp and Vardi[1] for emission tomography, it has often been observed that continuation of the iterative process beyond a certain point results in strong image deterioration. Starting from the original activity distribution shown in Fig. 1, we recently calculated images in a 128 x 128 pixel image plane by a random process[2]. These images are called "source images". Using a matrix of detection probabilities calculated for one 512-detector ring of the ECAT-III instrument at UCLA[3], projection data were obtained again by a random process. The data were then used as input to the MLE algorithm. By using the same matrix of detection probabilities in the reconstruction as in the source image generation we avoided questions regarding accuracy of a particular matrix to define an instrument accurately. We looked exclusively at the behavior of the MLE algorithm.

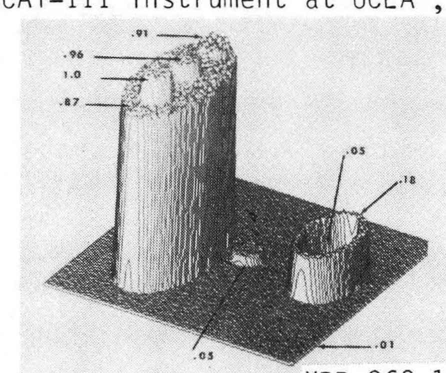


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Fig. 1 Initial source distribution for the images shown in this paper. The relative intensities are indicated.

In the work of Ref. 2 we were able to confirm the process of image deterioration and study the convergence characteristics of the MLE algorithm. Figure 2 shows the log likelihood for the images obtained at different numbers of iterations for a source image with 2 million counts (2M). The horizontal line indicates the likelihood for the true source image. The conclusions from the previous work can be summarized as: 1) the MLE algorithm actually converges towards an image that maximizes the likelihood that the initial projection data would have come from a source distribution corresponding to the obtained image, 2) the asymptotic maximum likelihood image can be a very deteriorated version of the original source distribution, although the quality of the reconstruction increases as the number of counts in the projection data increases, and 3) the original source image is not a maximum likelihood image for the projection data. Further data, images and discussion are given in Ref. 2.

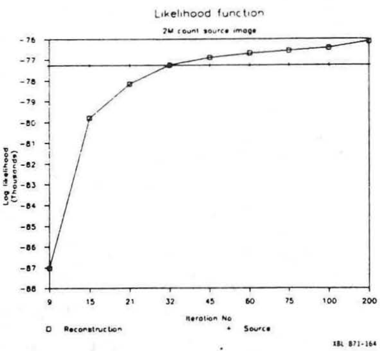


Fig. 2 Log likelihood that the image obtained at a certain number of iterations has yielded that certain set of projection data used as input for the reconstruction. The horizontal line indicates the likelihood for the 2M count source that truly generated the input data.

From the above findings one could conclude that the MLE algorithm does not have much future in medical tomography, since reconstructions converge towards images that could be sufficiently different from the source image to lead to false diagnosis. It is clear, however, that images obtained with a moderate number of iterations appear to be good representations of the source image. We have now quantified the noise in different parts of the reconstructed images and compared them with the Filtered Backprojection (FBP) method. We have understood the causes of image deterioration and controlled it within a certain range. The analysis leads us to the conclusion that the MLE algorithm can yield excellent images with very low noise in regions of low counts if used properly.

## Reconstruction Procedure

Based on our understanding of the properties of the MLE algorithm, we felt it would be important to incorporate a method of changing the weight given to tubes d in the process of maximization. The notation of Shepp and Vardi[1] is used throughout this paper. We call this modified method the Weighted Likelihood Estimator (WLE). We seek to maximize the function:

$$WL(\lambda) = WP(n^*|\lambda) = \prod_{d=1}^{D} \left[ e^{-\lambda^*(d)} \lambda^*(d)^{n^*(d)} / n^*(d)! \right]^{s \cdot n^*(d)+t} \tag{1}$$

where $n^*(d)$ is the number of counts detected in a tube d, and $\lambda^*(d)$ is the projection into tube d of the reconstructed image. With $s = 0$ and $t = 1$, the function WL is identical to the likelihood function L of Ref. 1. Keeping $t = 1$, $s > 0$ will give higher weight to those tubes that have higher number of counts, while making $s < 0$ will decrease their weight. Unlike L of Ref. 1, $WL(\lambda)$ does not have the meaning of the probability to obtain the projection data $n^*$ from the image $\lambda^*$. However, if s is small, both L and WL increase monotonically by iteration. The iterative formula for the maximization of Eq. 1, obtained by a method similar to that of Ref. 1 is the following:

$$\lambda^{new}(b) = \lambda^{old}(b) \left[ 1 + \sum_{d=1}^{D} [s \cdot n^*(d)+t]\, p(b,d) \frac{n^*(d) - 1}{\sum_{b'=1}^{B} \lambda^{old}(b')p(b',d)} \right] \tag{2}$$

## Results of Reconstructions

We have used a source image with 2M counts based on the activity distribution shown in Fig. 1. Figure 3a shows a cut through the source image. Figure 3b shows the FBP results with the Shepp-Logan filter and Figs. 3c, d and e show the results from the unmodified MLE at 9, 32 and 200 iterations, respectively. We have also carried out reconstructions with the WLE for values of $s = 0.0025$ and $-0.0015$. It is observed that the onset of image deterioration in regions of high activity comes early in the first case and is delayed in the second case. No substantial differences are observed in regions of low activity.

## Evaluation of Results

We have defined two regions, 1 and 2, in the source image representing high (1.0) and low (0.05) activity regions, respectively, as shown in Fig. 4. The mean values of the reconstructions and the standard error from the mean have been calculated in each zone. Figures 5a and b show error plots for the two regions. In region 1 we observe that the error for the FPB is a factor of 2 higher than the source image error, normal for that method of reconstruction. For the WLE method, we see a substantial influence of the parameter s on the iteration number at which the std. error is equal to that of the FBP method. In region 2, the error of the FBP is ~ 0.05, of the same magnitude as the signal, while the WLE results remain under 0.01 (near the source noise) up to iterations 40 to 60, depending weakly on parameter s. Even at iteration 200, with s = 0 or -0.0015 the std. error remains under 0.02, with a marked superiority over the FBP method.

## Discussions and Conclusions

The behavior of the WLE reconstructions when the parameter s is changed indicates that the progressive deterioration of the images is due to an attempt by the algorithm to match excessively well projection data with high number of counts. We have made the observation earlier[2] that the MLE algorithm gains more likelihood by matching projection data with low number of counts than with high counts. We are now finding that, due to the imperfect nature of the count limited projection data, the MLE method still tries too hard to match regions of high activity and yields unacceptable images if allowed to iterate without limits. Considering that it is possible to obtain images

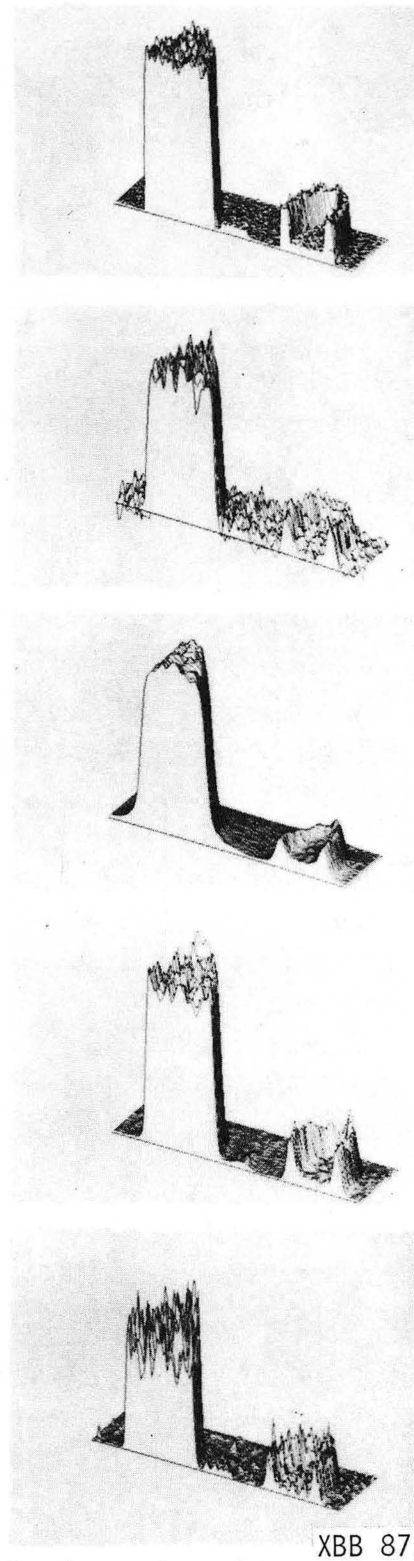

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Fig. 3 Cuts through source and reconstructed images. a) source image with 2 million counts. b) reconstruction by filtered backprojection, Shepp-Logan filter. c) reconstruction by maximum likelihood estimate, 9 iterations. d) ditto, 32 iterations. e) ditto, 200 iterations.

with the MLE that have similar noise as the FBP in regions of high activity, and much lower noise in regions of low activity when the iterations are stopped at an appropriate point, it appears that it would be fruitful to define a criterion for iteration stopping based on statistical considerations. We are continuing work in that direction.

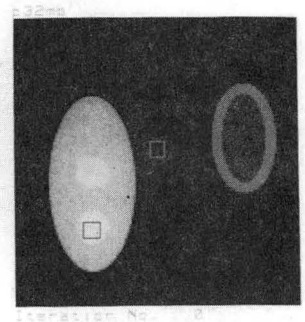

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Fig. 4 Source distribution showing Regions 1 and 2 for noise evaluation.

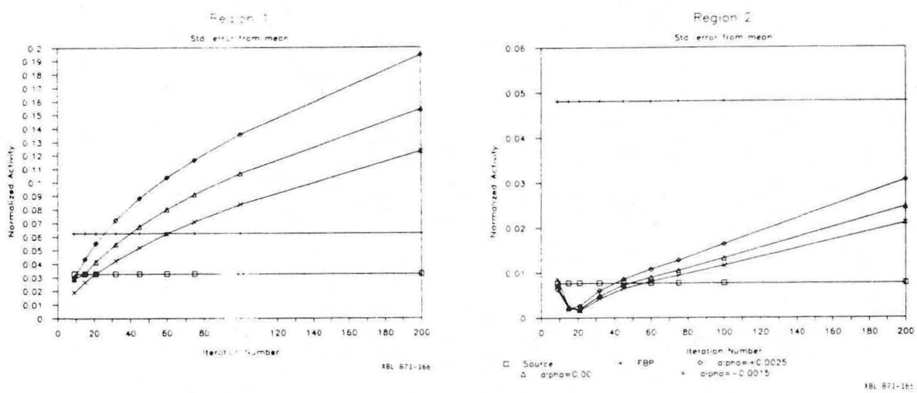


Fig. 5 Plots of standard error from the mean as a function of iteration number for different values of parameter s in Eqs. 1 and 2. a) for region 1, with high counts (1.0). b) for region 2, with low counts (0.05).

## Acknowledgment

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