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The household activity pattern problem of analyzing/predicting the optimal path of household members through time and space as they complete a prescribed agenda of out-of-home activities is posed as a variant of the pickup and delivery problem with time windows. The most general case of the model includes provision for vehicle transfer, selective activity participation, and ridesharing options. A series of examples are solved using generic algorithms. The model is purported to remove existing barriers to the operationalization of activity-based approaches in travel behavior analysis.

INTRODUCTION

There is general consensus that the demand for travel is derived from a need or desire to participate in activities that are spatially distributed over the geographical landscape. Recognition that conventional travel demand approaches that examine each trip in isolation at best provide only limited information regarding the particular trip (since they generally ignore both the history that precedes the trip as well as the future that follows) and virtually no information on the impact of decisions regarding the particular trip on other travel decisions (both prior and subsequent), has led to a roughly decade-long quest among a cadre of transportation researchers to develop and operationalize "activity-based" travel demand analyses. A history of these developments, together with critical assessment of their limitations and potential, is provided in a special issue of Transportation (1988). In particular, Kitamura (1988) provides an extensive evaluation of the field, covering approximately 120 studies.

Goodwin (1983) capsulates the activity-based approach in simple terms as

> "the consideration of revealed travel patterns in the context of a structure of activities, of the individual or household, with a framework emphasizing the importance of time and space coordinates."

It is derived principally from the early work of Hagerstrand (1970) in time-space geography, in which travel and activity participation are recorded as passage through time and space, with the individual's location at any time represented by a continuous path in the spatial and temporal dimensions.

As noted by Stopher et al. (1993), however, "despite its conceptual appeal and clarity, the time-space geography structure has proved quite difficult to implement operationally." Commenting on challenges facing further development of activity-based approaches, Kitamura (1988) noted that, conceptually, Lancaster's utility formulation (1966) neatly applies to the problem, but that

> ". . . if a utility function can be identified at all, an array of mathematical programming methods are available. However, the problem at hand is, at the simplest, a discrete choice-continuous allocation problem with correlated multiple alternatives, combined with the traveling salesman problem, problem of collective decision-making, and household coupling constraints which is in part a logistic problem. This is an overwhelming problem. In fact no model has been constructed that determines activity patterns on the sole basis of the utility maximization principle."

In his review, Kitamura further notes that existing models are largely restricted to addressing questions of activity participation and time allocation at the level of total daily time expenditure, with a wide gap existing between these models and those modeling daily activity and travel patterns. Both Kitamura (1988) and Stopher (1993) point to the STARCHILD MODEL (Recker et al., 1986a,1986b) as the only known operationalized model that predicts a set of activity patterns from household decision-making information. Although based loosely on mathematical programming principles, the STARCHILD model is severely limited in that it: (1) provides no mechanism for household interaction, modeling the activity/travel patterns of each household member separately, (2) relies on a heuristic solution procedure based on exhaustive enumeration and evaluation of feasible solutions, (3) discretizes the temporal dimension and relies on pattern recognition algorithms to distinguish simple temporal displacements of similar solutions, and (4) has no provision for addressing either activity or vehicle allocation decisions or for consideration of complex modal choice decisions, such as carpooling.

According to Koppleman (1988), in activity-based approaches "the research need is to develop a theoretical framework within which to relate the multiple themes of human/social behavior to the generation of the need or desire to participate in activities and the derived demand for travel." This paper attempts to provide one such framework that is believed to offer potential in the operationalization of activity-based travel demand methodologies.

Specifically, the household activity pattern problem (HAPP) is posed as a variant of the pick up and delivery problem with time windows (PDPTW). In the most general case considered, the model addresses the optimization (relative to the household's utility function) of the interrelated paths through the time/space continuum of a series of household members with a prescribed activity agenda and a stable of vehicles and ridesharing options available.

In the development of the model, a deliberate attempt has been made to maintain, to the extent possible, both the notation and structure of the well-known PDPTW in the hope that this would provide a conducive environment for future development and improvement. In addition, little attention has been paid to issues of model efficiency or to the efficiency of solution algorithms; rather, reliance has been placed mainly on readily available "canned" software in an effort to demonstrate the practicality of the approach.

MODEL FORMULATION

In order to take advantage of previous work involving the PDPTW, the formulation of the general HAPP involving complex elements such as ridesharing and vehicle-switching options is developed from a progression of cases in which initial restrictions that result in an equivalence to the PDPTW are gradually removed.

CASE 1: *Each member of the household has exclusive, unrestricted use of a personal vehicle and any activity can be completed by any member of the household.*

In its most basic form, in which each member of the household has exclusive unrestricted use of a personal vehicle and any activity can be completed by any member of the household, the household activity pattern problem (HAPP) can be formulated as a variation of the well-known pickup and delivery problem with time windows (PDPTW) within the class of vehicle routing problems with time windows (VRPTW).

Following Soloman and Desrosiers (1988) we adopt the following notation

$A = \{1, 2,\ldots, i,\ldots, n\}$	the set of out-of-home activities scheduled to be completed by travelers in the household.
$V = \{1, 2,\ldots, \upsilon,..., \|V\|\}$	the set of vehicles used by travelers in the household to complete their scheduled activities.
$P^+ = \{1, 2,\ldots, i\ldots, n)$	the set designating location at which each activity is performed.
$P^- = \{n+1, n+2,\ldots, n+i,\ldots, 2n\}$	the set designating the ultimate destination of the "return to home" trip for each activity. (It is noted that the physical location of each element of P^- is "home".)
$[a_i, b_i]$	the time window of available start times for activity i. (Note: b_i must precede the closing of the availability of activity i by an amount equal to or greater than the duration of the activity.)
$[a_{n+i}, b_{n+i}]$	the time windows for the "return home" arrival from activity i.
$[a_0, b_0]$	the departure window for the beginning of the travel day.
$[a_{2n+i}, b_{2n+i}]$	the arrival window by which time all members of the household must complete their travel.
s_i	the duration of activity i.
t_{uw}	the travel time from the location of activity u to the location of activity w.
c^{υ}_{uw}	travel cost from location of activity u to the location of activity w by vehicle υ.

B_c	the household travel cost budget.
B_t^{υ}	the travel time budget for the household member using vehicle υ.
$P = P^+ \cup P^-$	the set of nodes comprising completion of the household's scheduled activities.
$N = \{0, P, 2n+1\}$	the set of all nodes, including those associated with the initial departure and final return to home.

As implied above, different elements of P^+ may correspond to the same physical location; all elements of P^- correspond to the same physical location (home) and consequently $t_{n+u,n+w} = c_{n+u,n+w} \equiv 0, \ \forall \ u,w \in P^+$.

In the analogy to the PDPTW, activities are viewed as being "picked up" by a particular household member (who, in this basic case, is uniquely associated with a particular vehicle) at the location where performed and, once completed (requiring a service time s_i) are "logged in" or "delivered" on the return trip home. Multiple "pickups" are synonymous with multiple sojourns on any given tour. The scheduling and routing protocol relative to some household objective produces the "time-space diagram" commonly referred to in travel/activity analysis.

In the PDPTW, demand functions (d_i) and a vehicle capacity (D) are introduced to ensure that the schedule of pickups and deliveries does not violate the capacity constraint of any particular vehicle. This notion is extended to the HAPP by defining as constraints

$$D = \begin{cases} D^s & = \text{ maximum number of sojourns in any tour} \\ \underline{\text{or}} & \\ D^T & = \text{ maximum time spent away from home on any tour} \end{cases}$$

with the corresponding demand

$$d_u = \begin{cases} d_u^s = 1 \\ \underline{\text{or}} \\ d_u^T = s_u + t_{w'u};\ w' = \text{stop on tour immediately preceding i.} \end{cases}$$

Decision variables directly analogous to those of the PDPTW are defined as:

X_{uw}^{υ} , $u, w \in N, \upsilon \in V, u \neq w$	binary decision variable equal to unity if vehicle υ travels from activity u to activity w, and zero otherwise.
T_u , $u \in P$	the time at which participation in activity u begins.
$T_o^{\upsilon}, T_{2n+1}^{\upsilon}$, $\upsilon \in V$	the times at which vehicle υ first departs from home and last returns to home, respectively.
Y_u , $u \in P$	the total accumulation of either sojourns or time (depending on the selection of D and d_u) on a particular tour immediately following completion of activity u.

With these definitions, the basic HAPP can be represented as:

Minimize Z = Household Travel Disutility (1)

subject to:

$$\sum_{\upsilon \in V} \sum_{w \in N} X_{uw}^{\upsilon} = 1 \quad , \quad u \in P^{+} \tag{2}$$

$$\sum_{w \in N} X_{uw}^{\upsilon} - \sum_{w \in N} X_{wu}^{\upsilon} = 0 \quad u \in P,\ \upsilon \in V \tag{3}$$

$$\sum_{w \in P^{+}} X_{0w}^{\upsilon} = 1 \quad , \quad \upsilon \in V \tag{4}$$

$$\sum_{u \in P^{-}} X_{u,2n+1}^{\upsilon} = 1 \quad , \quad \upsilon \in V \tag{5}$$

$$\sum_{w\in N} X^{\upsilon}_{wu} - \sum_{w\in N} X^{\upsilon}_{w,n+u} = 0 \quad u \in P^{+},\ \upsilon \in V \tag{6}$$

$$T_u + s_u + t_{u,n+u} \le T_{n+u} \quad u \in P^{+} \tag{7}$$

$$X^{\upsilon}_{uw} = 1 \Rightarrow T_u + s_u + t_{uw} \le T_w \quad , \quad u, w \in P,\ \upsilon \in V \tag{8}$$

$$X^{\upsilon}_{0w} = 1 \Rightarrow T^{\upsilon}_0 + t_{0w} \le T_w \quad , \quad w \in P^{+},\ \upsilon \in V \tag{9}$$

$$X^{\upsilon}_{u,2n+1} = 1 \Rightarrow T_u + s_u + t_{u,2n+1} \le T^{\upsilon}_{2n+1} \quad , \quad u \in P^{-},\ \upsilon \in V \tag{10}$$

$$a_u \le T_u \le b_u \quad , \quad u \in P \tag{11}$$

$$a_0 \le T^{\upsilon}_0 \le b_0 \quad , \quad \upsilon \in V \tag{12}$$

$$a_{2n+1} \le T^{\upsilon}_{2n+1} \le b_{2n+1} \ , \ \upsilon \in V \tag{13}$$

$$X^{\upsilon}_{uw} = 1 \Rightarrow Y_u + d_w = Y_w \quad u \in P,\ \ w \in P^{+}\ \upsilon \in V \tag{14}$$

$$X^{\upsilon}_{uw} = 1 \Rightarrow Y_u - d_{w-n} = Y_w \quad u \in P,\ \ w \in P^{-}\ \upsilon \in V \tag{15}$$

$$X^{\upsilon}_{0w} = 1 \Rightarrow Y_0 + d_w = Y_w \quad , \quad w \in P^{+},\ \upsilon \in V \tag{16}$$

$$Y_0 = 0 \quad , \quad 0 \le Y_u \le D \ , \ u \in P^{+} \tag{17}$$

$$X^{\upsilon}_{uw} = \begin{cases} 0 \\ 1 \end{cases} \ ; \ u, w \in N \quad , \quad \upsilon \in V \tag{18}$$

$$\sum_{\upsilon \in V} \sum_{u \in N} \sum_{w \in N} c_{uw}^{\upsilon} X_{uw}^{\upsilon} \leq B_c \tag{19}$$

$$\sum_{u \in N} \sum_{w \in N} t_{uw} X_{uw}^{\upsilon} \leq B_t^{\upsilon} \quad , \quad \upsilon \in V \tag{20}$$

$$\sum_{w \in P^-} X_{0,w}^{\upsilon} = 0 \quad , \quad \upsilon \in V \tag{21}$$

$$\sum_{u \in N} X_{u,0}^{\upsilon} = 0 \quad , \quad \upsilon \in V \tag{22}$$

$$\sum_{u \in P^+} X_{u,2n+1}^{\upsilon} = 0 \quad , \quad \upsilon \in V \tag{23}$$

$$\sum_{w \in P^-} X_{2n+1,w}^{\upsilon} = 0 \quad , \quad \upsilon \in V \tag{24}$$

Note that Equations (8), (9), and (10) may be rewritten:

$$T_u + s_u + t_{uw} - T_w \leq (1 - X_{uw}^{\upsilon})M, \quad u, w \in P, \ \upsilon \in V \tag{8′}$$

$$T_0^{\upsilon} + t_{0w} - T_w \leq (1 - X_{0w}^{\upsilon})M, \quad w \in P^+, \ \upsilon \in V \tag{9′}$$

$$T_u + s_u + t_{u,2n+1} - T_{2n+1}^{\upsilon} \leq (1 - X_{u,2n+1}^{\upsilon})M, \ \ u \in P^-, \ \upsilon \in V \tag{10′}$$

where M is a large positive number.

Equations (2) through (20) are virtually identical to those specified by Solomon and Desrosiers (*op. cit.*) for the PDPTW, with the addition of the budget constraints (i.e., Equations

(19) and (20)) and subject to the redefinition of terms, and have an analogous interpretation in the HAPP. Equations (21) through (24) explicitly state conditions implicit in the PDPTW.

Examples of potential components of the disutility function of the household that may be easily specified in the objective function of Equation (1) include:

$\sum_{\upsilon \in V} \sum_{u \in N} \sum_{w \in N} c_{uw}^{\upsilon} X_{uw}^{\upsilon}$ total household travel cost. (1a)

$\sum_{\upsilon \in V} \sum_{u \in N} \sum_{w \in N} t_{uw} X_{uw}^{\upsilon}$ total travel time. (1b)

$\sum_{u \in P^+} (T_u - b_u)$ a measure of the risk of the inability to complete activities because of stochastic variations in travel times and/or activity durations. (1c)

$\sum_{u \in P^-} (T_u - b_u)$ a measure of the risk of not returning home in time due to stochastic variations in travel time or activity participation. (1d)

$\sum_{u \in P^+} (T_{u+n} - T_u)$ a measure of the delay in returning home incurred by trip chaining. (1e)

$(T_{2n+1}^{\upsilon} - T_0^{u}) \quad , \quad \upsilon \in V$ the extent of the travel day for each household member. (1f)

Although the assumptions used in formulating the base case model described by Equations (1) - (24) (e.g., interchangeable activity participation among household members, *a priori* identification of the subset of travelers in the household, exclusive and unrestricted use of personal automobile) are too restrictive for practical application in travel activity analysis, the model nonetheless provides both a bridge to existing operations research formulations as well as a point of departure in the development of more general models.

As an example of the application of this basic HAPP formulation, we consider the case of a two-person household with three scheduled activities with durations

$$S = [s_1, s_2, s_3] = [8, 1, 2]$$

and time availability windows

$$[a_{i,} b_i] = \begin{bmatrix} 8, & 8.5 \\ 10, & 20 \\ 12, & 13 \end{bmatrix} \quad ,$$

corresponding return-home windows

$$[a_{n+i,} b_{n+i}] = \begin{bmatrix} 17, & 19 \\ 10, & 21 \\ 12, & 21 \end{bmatrix} \quad ,$$

and initial departure and end-of-travel day windows

$$[a_{0,} b_0] = [6, \quad 20]$$
$$[a_{2n+1,} b_{2n+1}] = [6, \quad 21] \quad .$$

We additionally assume the following travel time and cost matrices (assumed constant for all vehicles) associated with the locations of the three activities:

t_{uw}

FROM \ TO	0	1	2	3
0	0.00	1.00	0.25	0.50
1	1.00	0.00	1.00	0.50
2	0.25	1.00	0.00	0.50
3	0.50	0.50	0.50	0.00

c_{uw}

FROM \ TO	0	1	2	3
0	0.00	2.00	1.00	1.00
1	2.00	0.00	1.00	1.00
2	1.00	1.00	0.00	0.50
3	1.00	1.00	0.50	0.00

and budget and tour constraints

$$B_c = 8.00$$
$$B_t^1 = B_t^2 = 3.50$$
$$D_s = 4 \quad .$$

The household's objective function is assumed to be comprised of terms (1a), (1e) and (1f), i.e.,

$$\text{Min } Z = \sum_{\upsilon \in V} \sum_{u \in N} \sum_{w \in N} c_{uw} \, X_{uw}^{\upsilon} + \sum_{u \in P^+} \left(T_{u+n} - T_u \right) + \sum_{\upsilon \in V} \left(T_{2n+1}^{\upsilon} - T_0^{\upsilon} \right) \quad .$$

The HAPP mixed-integer model specified by the above parameters and Equations (1) - (24) was solved using the ZOOM algorithm (Singhal, et al., 1987) in the GAMS software package developed by the World Bank. The resulting solution for this base case (denoted as CASE 1) is summarized in Figure 1, which displays the optimal time/space paths taken by the individual household members and vehicles (in this case, synonymous) in the completion of the household's scheduled activities.

CASE 2: *Each member of the household has a personal vehicle; a subset of activities can be performed by any member of the household, the remainder must be performed by certain members (HAPPAA: The household activity pattern problem with assigned activities).*

As already emphasized, the Case 1 model has only very limited practical application owing to its restrictive assumptions. However, much more realistic models of the HAPP are obtained from the base case with only slight modification. For example, the restriction that activity participation is interchangeable among household members is easily addressed by the addition of a single set of constraints:

$$\sum_{w \in \Omega_v^{\upsilon}} \sum_{u \in P} X_{uw}^{\upsilon} = 0 \quad , \quad \upsilon \in V \tag{25}$$

where $\Omega_v^{\upsilon} \in A$ is the subset of activities that cannot be performed by vehicle/person υ. Figure 2 presents results for this case (labeled Case 2) in which:

$$\Omega_v^1 = \{1\}$$
$$\Omega_v^2 = \{2\} \quad ,$$

(i.e., either person in the household can perform activity 3, but person 1 must perform activity 2 and person 2 must perform activity 1) and all other parameters are as in the previous case.

CASE 3: *Each member of the household has a personal vehicle; a subset of activities can be performed by any member of the household, the remainder must be performed by certain members. Some members may not perform any activities (i.e., stay at* *home); there is some "cost" to performing out-of-home activities (or, conversely, some benefit to staying home).*

The restriction of *a priori* knowledge of the subset of household members who are travelers on any given day is removed by redefining the set A to include all household members with unrestricted exclusive access to a personal vehicle, revising Equations (4) and (5) as:

$$\sum_{w \in P^+} X_{0w}^{\upsilon} \leq 1 \quad , \quad \upsilon \in V \tag{4'}$$

$$\sum_{u \in P^-} X_{u,2n+1}^{\upsilon} \leq 1 \quad , \quad \upsilon \in V \tag{5'}$$

and adding a term to the objective function to reflect the base disutility of performing any discretionary activities outside the home on a given day, say

$$\sum_{\upsilon \in V} \sum_{w \in P^+} K X_{0w}^{\upsilon} \quad , \tag{1g}$$

where K = "cost" of performing out-of-home activities. These revisions, together with those of CASE 2, then represent the optimal solution to the HAPP in which each member of the household has exclusive use of a personal vehicle; a subset of activities can be performed by any member of the household, the remainder must be performed by certain members; some members may not perform any activities (i.e., stay at home); there is some "cost" to performing out-of-home activities (or, conversely, some benefit to staying home).

The solution to this version of the HAPP (labeled CASE 3) for an arbitrarily selected value of K = 100, and where Ω^2 = {null} and where the windows of availability for the activities have been adjusted to:

$$[a_i, b_i] = \begin{bmatrix} 8, & 8.5 \\ 6, & 20 \\ 12, & 22 \end{bmatrix} \quad ,$$

to illustrate a solution in which one household member does not travel, is shown in Figure 3.

CASE 4: *Members of the household share a stable of vehicles; a subset of vehicles may be available for use by any member of the household, the remainder may be reserved for use by certain members. A subset of activities can be performed by any member of the household, the remainder must be performed by certain members. Some members may perform no activities; some vehicles may not be used.*

The decoupling of vehicles and household members can be accomplished simply by posing "companion" vehicle and person PDPTW's (with appropriate redefinitions and coupling constraints). Specifically, we introduce the new decision variables associated with the set of household members $\eta = \{1,2,\ldots,|\eta|\}$:

$H^{\alpha}_{uw}, \quad u, w \in N, \eta, u \neq w$	binary decision variable equal to unity if household member α travels from activity u to activity w, and zero otherwise.
$\overline{T}^{\alpha}_{0}, \overline{T}^{\alpha}_{2n+1}, \quad \alpha \in \eta$	the times at which household member α first departs from home and last returns to home, respectively.

Associated with these new decision variables we add the parameters:

$\overline{a}^{\alpha}_{0}$	the earliest possible departure time for household member α.
$\overline{b}^{\alpha}_{2n+1}$	the latest return home time for household member α.

The constraints on the household member decision variables are merely a subset of the equivalent relationships on the vehicle flows:

$$\sum_{\alpha \in \eta} \sum_{w \in N} H^{\alpha}_{uw} = 1 \quad , \quad u \in P^{+} \tag{26}$$

$$\sum_{w \in N} H_{uw}^{\alpha} - \sum_{w \in N} H_{wu}^{\alpha} = 0, \quad u \in P \quad , \quad \alpha \in \eta \tag{27}$$

$$\sum_{w \in P^{+}} H_{0w}^{\alpha} \leq 1, \quad \alpha \in \eta \tag{28}$$

$$\sum_{u \in P^{-}} H_{u,2n+1}^{\alpha} \leq 1, \quad \alpha \in \eta \tag{29}$$

$$\sum_{w \in N} H_{wu}^{\alpha} - \sum_{w \in N} H_{w,n+u}^{\alpha} = 0, \quad u \in P^{+} \quad , \quad \alpha \in \eta \tag{30}$$

$$T_u + s_u + t_{uw} - T_w \leq (1 - H_{uw}^{\alpha})M, \quad u, w \in P \quad , \quad \alpha \in \eta \tag{31}$$

$$\overline{T}_0^{\alpha} + t_{0w} - T_w \leq (1 - H_{0w}^{\alpha})M, \quad w \in P^{+} \quad , \quad \alpha \in \eta \tag{32}$$

$$T_u + s_u + t_{u,2n+1} - \overline{T}_{2n+1}^{\alpha} \leq (1 - H_{u,2n+1}^{\alpha})M, \quad u \in P^{-} \quad , \quad \alpha \in \eta \tag{33}$$

$$\overline{T}_0^{\alpha} \geq \overline{a}_0^{\alpha}, \quad \alpha \in \eta \tag{34}$$

$$\overline{T}_{2n+1}^{\alpha} \leq \overline{b}_{2n+1}^{\alpha} \quad \alpha \in \eta \tag{35}$$

$$\sum_{w \in \overline{P}} H_{0,w}^{\alpha} = 0 \quad \alpha \in \eta \tag{36}$$

$$\sum_{u \in N} H_{u,0}^{\alpha} = 0 \quad \alpha \in \eta \tag{37}$$

$$\sum_{u \in P^{+}} H_{u,2n+1}^{\alpha} = 0 \quad \alpha \in \eta \tag{38}$$

$$\sum_{w \in \Omega_H^{\alpha}} \sum_{u \in P} H_{uw}^{\alpha} = 0 \quad , \quad \alpha \in \eta \tag{39}$$

where Ω_H^α is the set of activities that cannot be performed by household member α.

To these we add the coupling constraints:

$$\sum_{\alpha\in\eta} H_{uw}^{\alpha} = \sum_{\upsilon\in V} X_{uw}^{\upsilon} \quad , \quad u \in P^+, w \in P \tag{40a}$$

$$\sum_{\alpha\in\eta} H_{0w}^{\alpha} = \sum_{\upsilon\in V} X_{0w}^{\upsilon} \quad , \quad w \in P \tag{40b}$$

that ensure that only one household member may be assigned to travel between nodes u and w by vehicle υ.

Equations (1) - (40) constitute the HAPP formulation for the general case in which the only practical restriction is that of solo driving (i.e., excludes the potential to carpool). The optimal solution to this problem for

$$\Omega_1^V = \{1\}$$
$$\Omega_2^V = \{2\}$$
$$\Omega_1^H = \{1\}$$
$$\Omega_2^H = \{2\}$$

$$\bar{a}_0^1 = \bar{a}_0^2 = 6$$
$$\bar{b}_{2n+1}^1 = \bar{b}_{2n+1}^2 = 22,$$

and all other parameter values as specified in CASE 3 (the previous example), is displayed in Figure 4 (and labeled CASE 4A). When the exclusionary sets Ω are revised to

$$\Omega_1^V = \{1,3\}$$
$$\Omega_2^V = \{2\}$$
$$\Omega_1^H = \{1\}$$
$$\Omega_2^H = \{2,3\}$$

the solution shown in Figure 5 (and labeled CASE 4B) is obtained. It is noted that in this latter example, the optimal solution involves household member 1 using vehicle 1 to complete activity 2, and then using vehicle 2 to complete activity 3 after household member 2's return to home in vehicle 2.

CASE 5: Same as Case 4, but with the addition of ridesharing option, representing the general HAPP with some assigned activities and vehicles and with ridesharing and non-traveler options.

The inclusion of a ridesharing option significantly alters the basic formulation of the previous cases. While maintaining a similar structure to previous cases, the set of nodes is expanded to include "drop-off passenger" and "pick-up passenger" activities at the locations of the prescribed household activities; the former being discretionary, however, while the latter remain compulsory. The elements of the set defining the vehicles available to the household is also expanded by designating "driver seat" and "passenger seat(s)" for each vehicle in the stable. Defining these new sets as:

P_{DO}^{+}	set of serve passenger "drop off" activity locations.
P_{PU}^{+}	set of serve passenger "pick-up" activity locations.
$\overline{P}^{+}$	$P^{+} + P_{DO}^{+} + P_{PU}^{+}$
$P_{DO}^{-}, P_{PU}^{-}, \overline{P}^{-}$	respective eventual home trips to "unload".
$\hat{V}$	passenger "seats".
$\overline{V}$	$V + \hat{V}$,

with the corresponding elements:

$A = \{1, 2, \ldots, i, \ldots, n\}$

$V = \{1, 2, \ldots, |V|\}$

$\hat{V} = \{|V|+1, |V|+2, \ldots, 2|V|\}$

$\overline{V} = \{1, \ldots, |V|, |V|+1, \ldots, 2|V|\}$

$P^{+} = \{1, 2, \ldots, i, \ldots, n\}$

$P_{DO}^{+} = \{n+1, n+2, \ldots, n+i, \ldots, 2n\}$

$P_{PU}^{+} = \{2n+1, 2n+2, \ldots, 2n+i, \ldots, \overline{n}\}, \ \overline{n} = 3n$

$P^{-} = \{\overline{n}+1, \overline{n}+2, \ldots, \overline{n}+i, \ldots, \overline{n}+n\}$

$P_{DO}^{-} = \{\overline{n}+n+1, \overline{n}+n+2, \ldots, \overline{n}+n+i, \ldots, \overline{n}+2n\}$

$P_{PU}^{-} = \{\overline{n}+2n+1, \overline{n}+2n+2, \ldots, \overline{n}+2n+i, \ldots, 2\overline{n}\}$

$\overline{P}^{+} = P^{+} \cup P_{DO}^{+} \cup P_{PU}^{+} = \{1, 2, \ldots, \overline{n}\}$

$\overline{P}^{-} = P^{-} \cup P_{DO}^{-} \cup P_{PU}^{-} = \{\overline{n}+1, \overline{n}+2, \ldots, 2\overline{n}\}$

$\overline{P} = \overline{P}^{+} \cup \overline{P}^{-} = \{1, 2, \ldots, 2\overline{n}\}$

$\overline{N} = \{0, \overline{P}, 2\overline{n}+1\}$,

the constraints defining the HAPP with ridesharing options can be grouped into six broad categories: (1) temporal constraints on the vehicles, (2) temporal constraints on the household members performing the activities, (3) spatial connectivity constraints on the vehicles, (4) spatial connectivity constraints on the household members, (5) capacity, budget and participation constraints, and (6) vehicle and household member coupling constraints. These constraints are presented in detail below:

(1) Vehicle Temporal Constraints:

$$T_u + s_u + t_{u,\overline{n}+u} - T_{\overline{n}+u} \le \left(1 - \sum_{w \in \overline{P}} \sum_{\upsilon \in \overline{V}} X_{uw}^{\upsilon}\right) M, \quad u \in \overline{P}^{+}, \ \upsilon \in \overline{V} \tag{41}$$

$$T_u + s_u + t_{uw} - T_w \le (1 - X_{uw}^{\upsilon}) M, \quad u, w \in \overline{P}^{+}, \ \upsilon \in \overline{V} \tag{42}$$

$$T_0^{\upsilon} + t_{0w} - T_w \le (1 - X_{0w}^{\upsilon})M, \quad w \in \overline{P}^{+}, \ \upsilon \in \overline{V} \tag{43}$$

$$T_u + s_u + T_{u,2\overline{n}+1} - T_{2\overline{n}+1}^{\upsilon} \le (1 - X_{u,2\overline{n}+1}^{\upsilon})M, \quad u \in \overline{P}^{-}, \ \upsilon \in \overline{V} \tag{44}$$

$$T_{u+n} - T_u - s_{u+n} \le (1 - X_{w,u+n}^{\upsilon})M, \quad u \in P^{+}, \ w \in 0,\overline{P}, \ \upsilon \in V \tag{45}$$

$$T_u + s_u - T_{u+2n} \le (1 - X_{w,u+2n}^{\upsilon})M, \quad u \in P^{+}, \ w \in 0,\overline{P}, \ \upsilon \in V \tag{46}$$

$$T_u - b_u \le \left(1 - \sum_{w \in \overline{P}} \sum_{\upsilon \in \overline{V}} X_{wu}^{\upsilon}\right)M \ \ge -T_u + a_u, \quad u \in \overline{P} \tag{47}$$

$$a_0 \le T_0^{\upsilon} \le b_0, \quad \upsilon \in V \tag{48}$$

$$a_{2\overline{n}+1} \le T_{2\overline{n}+1}^{\upsilon} \le b_{2\overline{n}+1}, \quad \upsilon \in V \tag{49}$$

$$T_0^{\upsilon} - T_0^{\upsilon+|V|} = 0, \quad \upsilon \in V \tag{50}$$

$$T_{2\overline{n}+1}^{\upsilon} - T_{2\overline{n}+1}^{\upsilon+|V|} = 0, \quad \upsilon \in V \tag{51}$$

The constraints embodied in Equations (41) - (47) are roughly equivalent to the corresponding constraints for Case 4 of the HAPP and the associated PDPTW, the principal exceptions being the expansion of the activity and vehicle sets, and the introduction of discretionary "serve passenger" activities. For example, Equation (41) ensures that the constraint that the "return home" be subsequent to activity participation is enforced on only those "serve passenger" trips that are actually made; for $u \in P^{+}$ the right side of Equation (41) is identically zero. Similarly for Equation (45), which ensures that activities take place within their allotted time windows. Equations (42)- (44) ensure that travel between any two activity locations can occur if and only if there is sufficient time to reach the destination prior to commencing the associated activity. Equations (45) and (46) constrain activities that are accessed as a passenger to occur after the passenger is dropped off at the destination and be completed prior to being picked up for the return home. Equations (47) and (48) ensure that the initial vehicle departure times and final return home times fall within the allotted time windows. Equations (49) and (50) require that these times be identical for the vehicle and its passenger seat.

(2) Household Member Temporal Constraints:

$$T_u + s_u + t_{uw} - T_w \leq (1 - H^{\alpha}_{uw})M, \quad u, w \in \overline{P}\ ,\ \alpha \in \eta \tag{52}$$

$$\overline{T}^{\alpha}_0 + t_{0w} - T_w \leq (1 - H^{\alpha}_{0w})M, \quad w \in \overline{P}^{+}\ ,\ \alpha \in \eta \tag{53}$$

$$-(1 - H^{\alpha}_{u,2\overline{n}+1})M \leq T_u - \overline{T}^{\alpha}_{2n+1} \leq (1 - H^{\alpha}_{u,2\overline{n}+1})M, \quad u \in \overline{P}^{-}\ ,\ \alpha \in \eta \tag{54}$$

$$\overline{a}^{\alpha}_0 \leq \overline{T}^{\alpha}_0 \leq \overline{b}^{\alpha}_0\ \ ,\ \alpha \in \eta \tag{55}$$

$$\overline{a}^{\alpha}_{2\overline{n}+1} \leq \overline{T}^{\alpha}_{2\overline{n}+1} \leq \overline{b}^{\alpha}_{2\overline{n}+1}\ \ ,\ \alpha \in \eta \tag{56}$$

With the exception of the expansion of the activity and vehicle sets, Equations (52) - (56) are equivalent to Equations (31) - (35).

(3) Spatial Connectivity Constraints on the Vehicles:

$$\sum_{\upsilon \in \overline{V}} \sum_{w \in \overline{N}} X^{\upsilon}_{uw} = 1 \quad , \quad u \in P^{+} \tag{57}$$

$$\sum_{\upsilon \in V} \sum_{w \in \overline{N}} X_{uw}^{\upsilon} \leq 1 \quad , \quad u \in P_{DO}^{+} \cup P_{PU}^{+} \tag{58}$$

$$\sum_{w \in \overline{P}} X_{uw}^{\upsilon+|V|} \leq \sum_{w \in \overline{P}} X_{u+jn,w}^{\upsilon} \quad , \quad u \in P^{+}, \; \upsilon \in V, \; j = 1,2 \tag{59}$$

$$X_{0u}^{\upsilon+|V|} \leq X_{0,u+n}^{\upsilon} + \sum_{w \in \overline{P}^{-}} X_{w,u+n}^{\upsilon} \quad , \quad u \in P^{+}, \; \upsilon \in V \tag{60}$$

$$X_{uw}^{\upsilon} = 0 \quad , \; u \in \overline{N}, \; w \in P_{DO}^{+} \cup P_{PU}^{+} \; , \; \upsilon \in \hat{V} \tag{61}$$

$$\sum_{w \in \overline{N}} X_{uw}^{\upsilon} - \sum_{w \in \overline{N}} X_{wu}^{\upsilon} = 0 \; , \quad u \in \overline{P}, \; \upsilon \in \overline{V} \tag{62}$$

$$\sum_{w \in \overline{P}^{+}} X_{0w}^{\upsilon} \leq 1 \quad , \; \upsilon \in \overline{V} \tag{63}$$

$$X_{uw}^{\upsilon} \leq \sum_{r \in \overline{P}^{+}} X_{0r}^{\upsilon} \quad , \; \upsilon \in V \; , \; u,w \in \overline{P} \tag{64}$$

$$\sum_{u \in \overline{P}^{-}} X_{u,2\overline{n}+1}^{\upsilon} \leq 1 \quad , \; \upsilon \in \overline{V} \tag{65}$$

$$\sum_{w \in \overline{N}} X_{wu}^{\upsilon} - \sum_{w \in \overline{N}} X_{w,\overline{n}+u}^{\upsilon} = 0 \; , \quad u \in \overline{P}^{+}, \; \upsilon \in \overline{V} \tag{66}$$

$$\sum_{w \in \overline{P}^{-}} X_{0,w}^{\upsilon} = 0 \quad , \; \upsilon \in \overline{V} \tag{67}$$

$$\sum_{u \in \overline{N}} X_{u0}^{\upsilon} = 0 \quad , \; \upsilon \in \overline{V} \tag{68}$$

$$\sum_{u \in \overline{P}^{+}} X_{u,2\overline{n}+1}^{\upsilon} = 0 \quad , \; \upsilon \in \overline{V} \tag{69}$$

$$\sum_{w \in \overline{P}} X^{\upsilon}_{2\overline{n}+1,w} = 0 \quad , \quad \upsilon \in \overline{V} \tag{70}$$

Equation (57) requires that all compulsory activities must be accessed either by a vehicle driver or as a carpool passenger; Equation (58) is the stipulation that "serve passenger" activities, if performed, must be by one and only one vehicle driver. Equations (59) and (60) ensure that activities accesses as a passenger are coupled to a corresponding "serve passenger" trip. Equation (61) precludes passengers from "serve passenger" activities. Equation (62) ensures that there is a connected path for each vehicle and no activity location is revisited. Equations (63) - (65) state that not all vehicles may be used in completing the household activity agenda, but if one is, its initial tour must begin at home. Equation (66) requires the "eventual return to home" from an activity be assigned to the vehicle that was used to accessed the activity. Equations (67) - (70) prohibit linkages among illogical activities, regardless of the specification of the objective function.

(4) Spatial Connectivity Constraints on the Household Members:

$$\sum_{\alpha \in \eta} \sum_{w \in \overline{N}} H^{\alpha}_{uw} = 1 \quad , \quad u \in P^{+} \tag{71}$$

$$\sum_{w \in \overline{N}} H^{\alpha}_{uw} - \sum_{w \in \overline{N}} H^{\alpha}_{wu} = 0 \quad , \quad u \in \overline{P} \quad , \quad \alpha \in \eta \tag{72}$$

$$\sum_{w \in \overline{P}^{+}} H^{\alpha}_{0w} \leq 1 \quad , \quad \alpha \in \eta \tag{73}$$

$$\sum_{u \in \overline{P}^{-}} H^{\alpha}_{u,2\overline{n}+1} \leq 1 \quad , \quad \alpha \in \eta \tag{74}$$

$$\sum_{w \in \overline{N}} H^{\alpha}_{wu} - \sum_{w \in \overline{N}} H^{\alpha}_{w,\overline{n}+u} = 0 \quad , \quad u \in \overline{P}^{+} \quad , \quad \alpha \in \eta \tag{75}$$

$$\sum_{w \in \overline{P}^{-}} H^{\alpha}_{0w} = 0 \quad , \quad \alpha \in \eta \tag{76}$$

$$\sum_{u \in \overline{N}} H^{\alpha}_{u0} = 0 \quad , \quad \alpha \in \eta \tag{77}$$

$$\sum_{u \in \overline{P}^{+}} H^{\alpha}_{u,2\overline{n}+1} = 0 \quad , \quad \alpha \in \eta \tag{78}$$

Equations (71) and (72) require that all compulsory activities must be completed by a member of the household, and that the household members have a connected path, respectively. Equations (73) and (74) state that some members of the household may not travel. Equations (75) - (77) are similar in interpretation to Equations (67) - (70).

(5) Capacity Budget and Participation Constraints:

$$-(1 - X^{\upsilon}_{uw})M \le Y_u + d_w - Y_w \le (1 - X^{\upsilon}_{uw})M, \;\; u \in \overline{P}, \; w \in \overline{P}^{+}, \; \upsilon \in \overline{V} \tag{79}$$

$$-(1 - X^{\upsilon}_{uw})M \le Y_u + d_{w-\overline{n}} - Y_w \le (1 - X^{\upsilon}_{uw})M, \;\; u \in \overline{P}, \; w \in \overline{P}^{-}, \; \upsilon \in \overline{V} \tag{80}$$

$$-(1 - X^{\upsilon}_{0w})M \le Y_0 + d_w - Y_w \le (1 - X^{\upsilon}_{0w})M, \;\; w \in \overline{P}^{+}, \; \upsilon \in V \tag{81}$$

$$Y_0 = 0 \quad , \quad 0 \le Y_u \le D \quad , \quad u \in \overline{P}^{+} \tag{82}$$

$$\sum_{\upsilon \in V} \sum_{u \in \overline{N}} \sum_{w \in \overline{N}} c^{\upsilon}_{uw} X^{\upsilon}_{uw} \leq B_c \tag{83}$$

$$\sum_{u \in \overline{N}} \sum_{w \in \overline{N}} t_{uw} H^{\alpha}_{uw} \leq B^{\alpha}_{t} \quad , \quad \alpha \in \eta \tag{84}$$

$$\sum_{w \in \Omega^{\upsilon}} \sum_{u \in \overline{N}} X^{\upsilon}_{uw} = 0 \quad , \quad \upsilon \in V \tag{85}$$

$$\sum_{w \in \Omega^{\alpha}_{H}} \sum_{u \in \overline{N}} H^{\alpha}_{uw} = 0 \quad , \quad \alpha \in \eta \tag{86}$$

Equations (79) - (81) specify the demand continuity relationships at each stop, while Equation (82) is the corresponding capacity constraint. Equation (83) is the household travel cost budget constraint; Equations (84) are the household member's travel time constraints. Equations (85) and (86) represent the vehicle and member activity participation exclusions.

(6) Vehicle and Household Member Coupling Constraints:

$$\sum_{\alpha \in \eta} H^{\alpha}_{uw} - \sum_{\upsilon \in \overline{V}} X^{\upsilon}_{uw} = 0 \quad , \quad u \in \overline{P}^{+},\ w \in \overline{P} \tag{87}$$

$$\sum_{\alpha \in \eta} H^{\alpha}_{0w} + \sum_{\alpha \in \eta} \sum_{u \in \overline{P}^{-}} H^{\alpha}_{uw} - \sum_{\upsilon \in \overline{V}} X^{\upsilon}_{0w} - \sum_{\upsilon \in \overline{V}} \sum_{u \in \overline{P}^{-}} X^{\upsilon}_{uw} = 0 \quad , \quad w \in \overline{P} \tag{88}$$

$$-(1 - H^{\alpha}_{0w})M - (1 - X^{\upsilon}_{uw})M \leq \overline{T}^{\alpha}_{0} - T_{u} \leq (1 - H^{\alpha}_{0w})M + \left(1 - X^{\upsilon}_{uw}\right)M, \quad w \in \overline{P}^{+},\ u \in \overline{N},\ \upsilon \in \overline{V},\ \alpha \in \eta \tag{89}$$

Equation (87) ensures that only one household member is assigned to travel between any activity location and any other location by any particular vehicle "seat". Equation (88) allows for transference of connectivity between vehicles and household members at the home location. Equation (89) requires that the time of the initial departure from home by any household member coincide with the departure time of the vehicle (initial or otherwise) that transports the individual to the activity.

Equations (41) - (89), together with an objective function comprised of a linear combination of activity/travel disutility components (e.g., drawn from Equations (1)), constitute the general case of the HAPP model with the provision of ridesharing options. An example of this formulation applied to the data and parameters used in CASE 4B, with the exception that the duration of Activity 2 is increased to seven hours to permit a viable ridesharing alternative, is shown in Figure 6. The optimal solution involves household member 2 driving household member 1 to the location of Activity 2 using vehicle 2 (vehicle 1 is not used in this solution), and then continuing on to Activity 1. Upon completion of Activity 1, household member 2 picks up household member 1 on the return to home. Household member 1 then drives to the location of Activity 3, while household member 2 remains home; upon completion of Activity 3, household member 1 returns home.

Because of the size of the model for this case with ridesharing options, it was not feasible to solve the model simply using the GAMS ZOOM module. Rather, a decomposition procedure was devised in which the ZOOM solver first was employed to obtain a solution to the non-ridesharing version of the problem. Then, using this as an initial feasible solution to the general problem with ridesharing, Equations (41) - (89) were decomposed into their integer (largely spatial) and non-integer (largely temporal) components. A heuristic was used to generate feasible ridesharing perturbations (branches) of the non-ridesharing solution while satisfying the integer spatial constraints and the absolute temporal constraints embodied in the input data (e.g., travel time and cost matrices, activity durations, and various time windows); the temporal portion of each branch was optimized using the GAMS LP solver and the overall optimal solution selected. For the example discussed, the solution displayed in Figure 6 required approximately 3.5 minutes on a 50 Mhz 486 PC.

CONCLUDING REMARKS

Despite their conceptual clarity, theoretical consistency, and purported unmatched potential for policy application, activity-based approaches to understanding and predicting travel behavior have not progressed much beyond the initial forays into the field over a decade ago. Principal among the contributing factors to this lack of progress has been the absence of an analytical framework that unified the complex interactions among the resource allocation decisions made by households in conducting their daily affairs outside the home, while preserving the utility-maximizing principles presumed to guide such decisions. It is believed that the formulation presented in this paper provides a promising approach toward removing this major obstacle to operationalizing activity-based behavioral travel analysis.

As indicated in the development of this particular framework, the focus has been on the demonstration that some rather well-known network-based formulations in operations research that have heretofore largely gone unnoticed in activity-based travel research offer a potentially powerful technique for advancing the general development of this approach. Reliance on generic solvers for solution of a set of examples that in the realm of activity-based research have been perceived to be at least practically intractable, demonstrates that such frameworks are not prohibitively computationally intensive; and, undoubtedly, the application of algorithms specifically tailored to the model formulation would be substantially more efficient than those employed here.

In the PDPTW, as well as in the examples considered in this paper to demonstrate the application of the mathematical framework, the specification of the objective function is known to both the decision maker and the analyst. The typical problem in demand modeling (of which the HAPP is a subset) is focused on inferring the relative weights associated with potential components, such as those contained in Equations (1), that are determinants to a population's revealed selection of the decision variables (in the model estimation phase) with subsequent forecasts made using these weights in conventional application of the model. In that sense, the modeling framework developed offers the first real analytical option for estimating the relative importances of factors associated with the spatial and temporal interrelationships among the out-of-home activities that motivate a household's need or desire to travel. Such estimation could proceed in a manner similar to utility-maximizing estimation techniques used in conventional demand analysis (e.g., regression, logit and probit analyses) in which the choice situation is presumed to be unconstrained; the proposed framework provides both the necessary constraint considerations on the household's decision alternatives within a utility-maximizing structure as well as a convenient mechanism for generating the set of feasible alternatives that are likely to be considered

Finally, it is cautioned that initial mathematical programming formulations of this complexity notoriously are prone to contain redundancies as well as "hidden" inconsistencies that may surface with their application to scenarios other than those tested in their development. The work presented here should be viewed as an initial attempt to provide direction to researchers with much more talent in operations research than the manifestly limited skills of the author.

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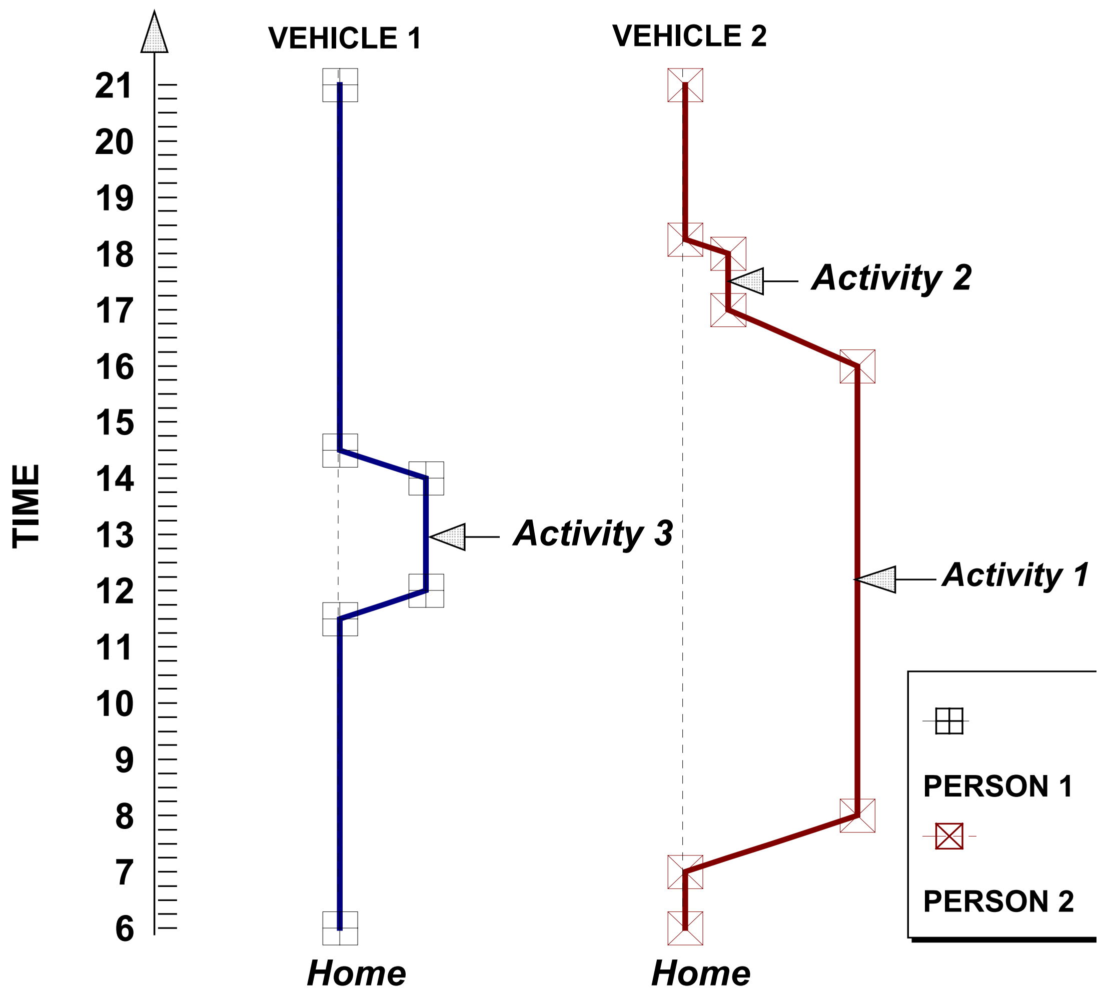

Figure 1. Optimal Time-Space Path for CASE 1 Example

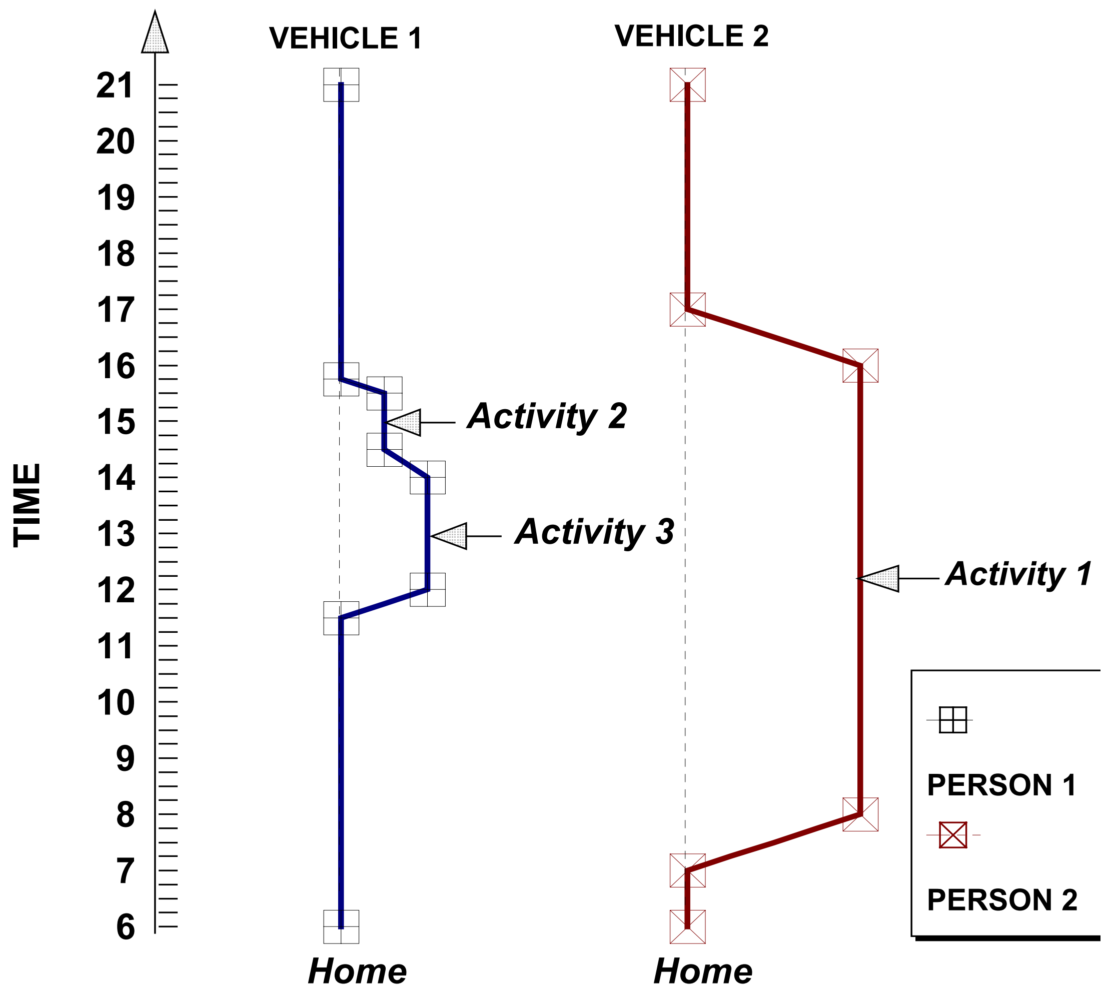

Figure 2. Optimal Time-Space Path for CASE 2 Example

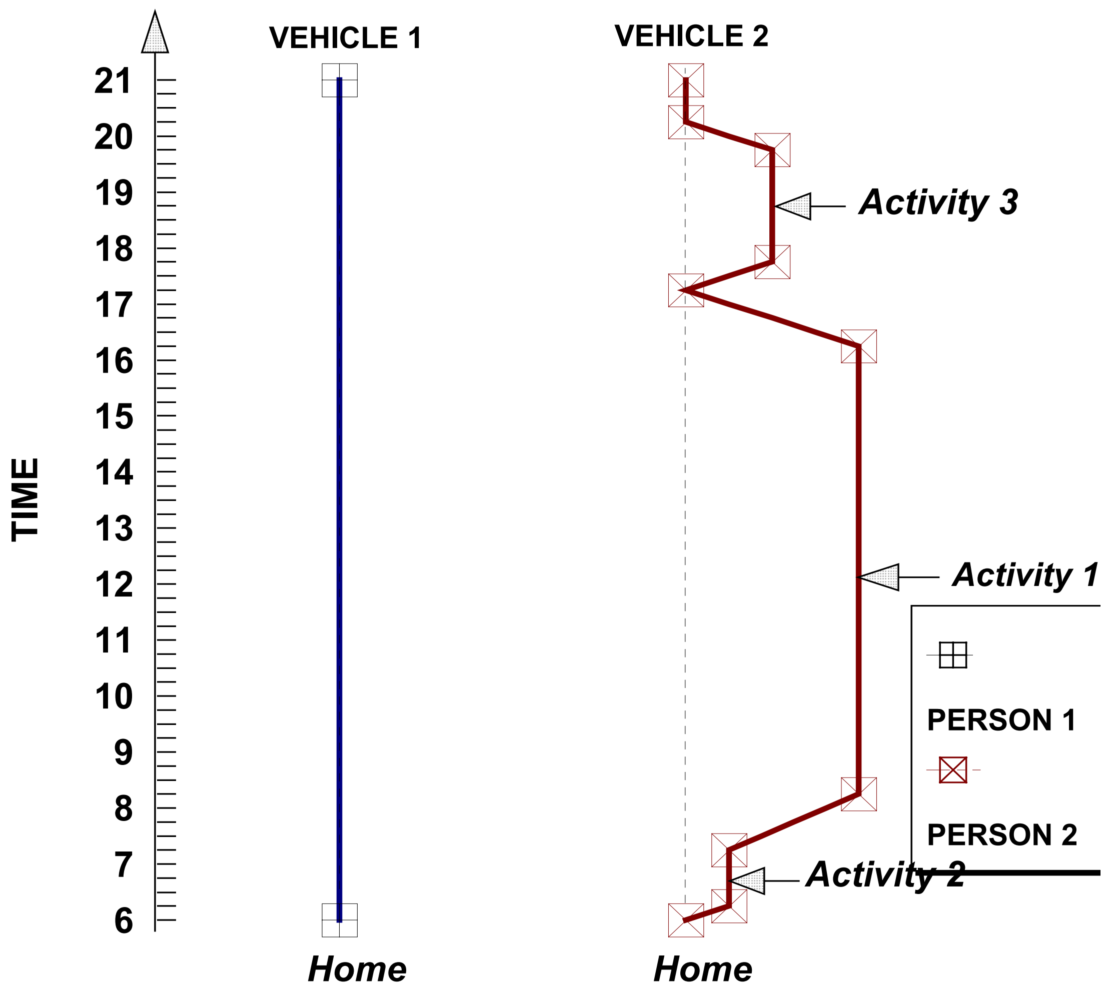

Figure 3. Optimal Time-Space Path and Activity Allocation for CASE 3 Example

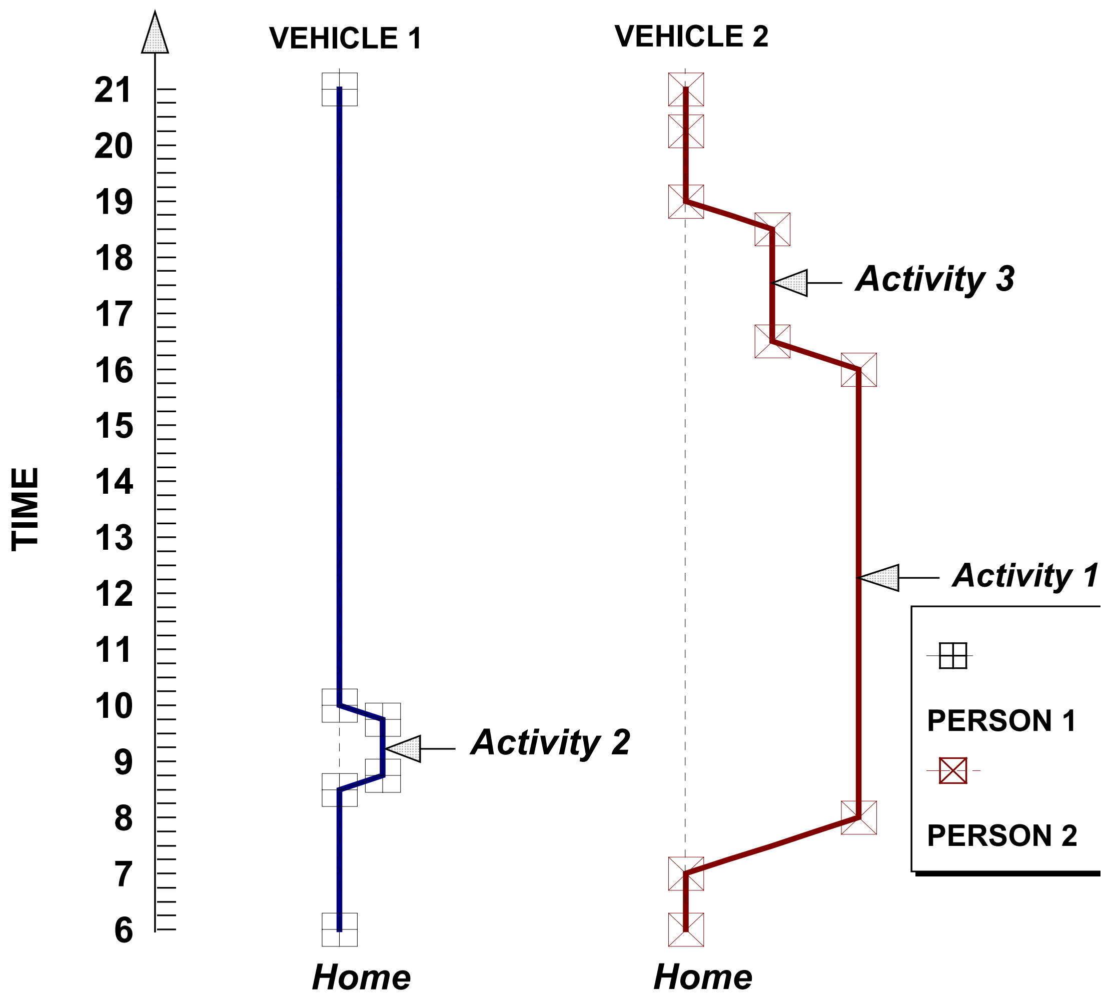

Figure 4. Optimal Time-Space Path and Vehicle and Activity Allocation for CASE 4A Example

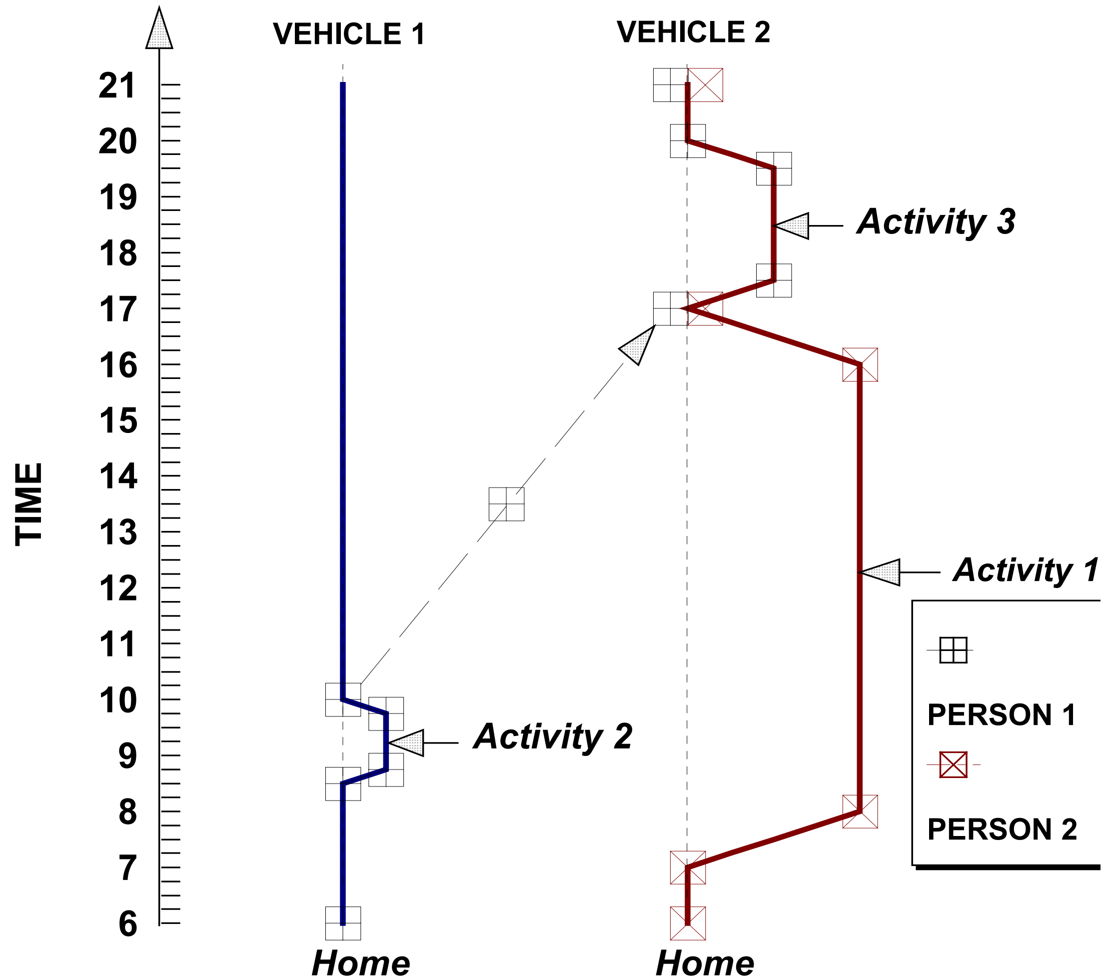

Figure 5. Optimal Time-Space Path and Vehicle and Activity Allocation for CASE 4B Example

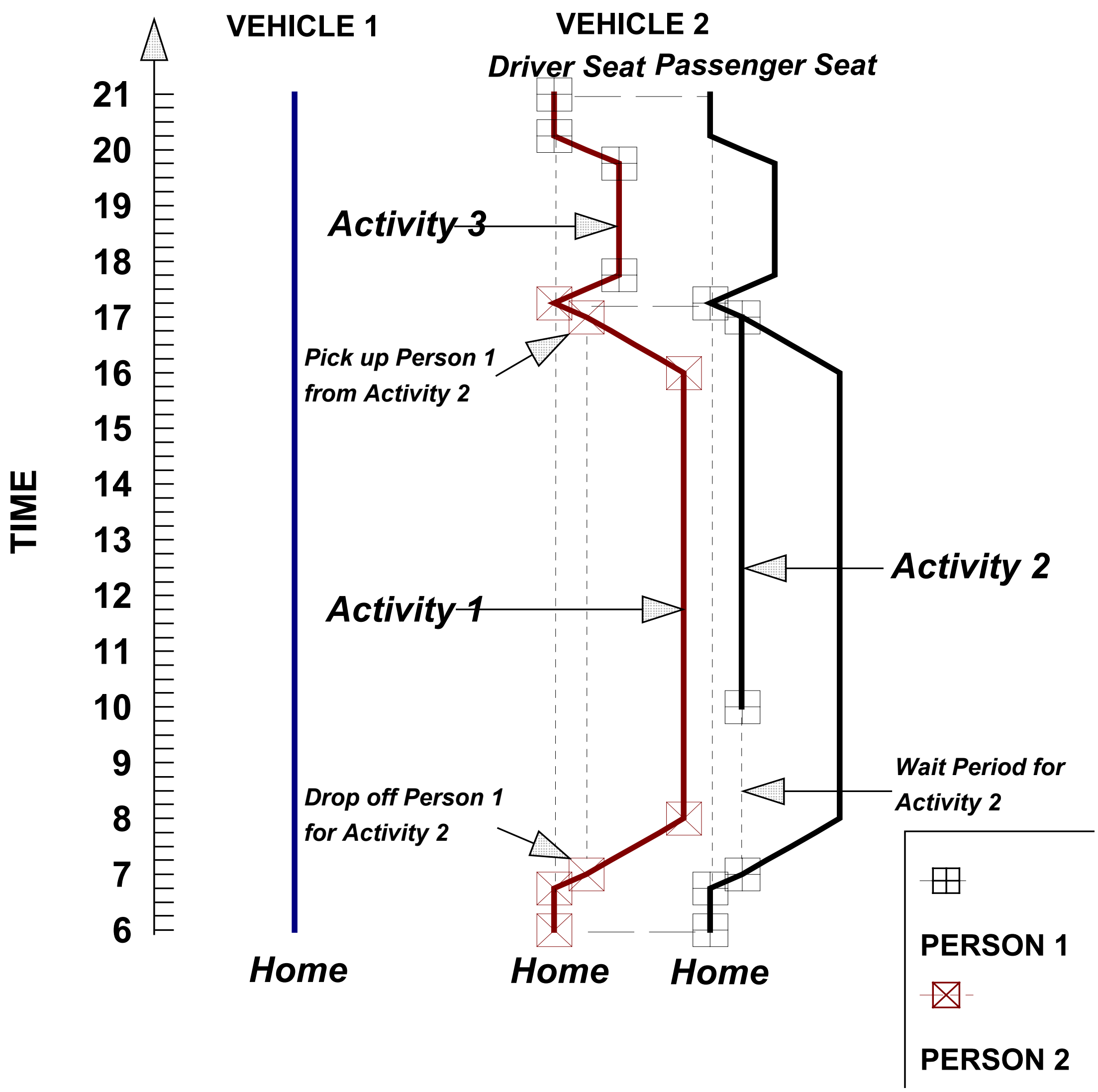

Figure 6. Optimal Time-Space Path, Vehicle, Activity Allocation, and Model Choice for CASE 5 Example