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THE ONE–LOOP EFFECTIVE ACTION IN THE CASE OF NON–CANONICAL GAUGE KINETIC ENERGY[1]

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Abstract

We calculate the one–loop effective action for a four dimensional bosonic model when the gauge kinetic energy and F–F–dual terms are coupled to the scalar fields. The simple form of the coupling we choose is particularly relavent to string inspired supergravity models.

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DISCLAIMER

1. Introduction.

In the present paper we find the one–loop effective action for a four dimensional bosonic theory with scalars, gauge vectors, and gravitons, when the gauge kinetic energy term involves couplings to scalars. The Lagrangian we consider is

$$\begin{aligned}\mathcal{L} &= \frac{1}{2\kappa}\sqrt{g}R + \frac{1}{2}\sqrt{g}g^{\mu\nu}\mathcal{D}_\mu\phi^i\mathcal{D}_\nu\phi^j Z_{ij}(\phi) - \sqrt{g}V(\phi) \\ &\quad -\frac{k}{4}x(\phi)\sqrt{g}g^{\mu\alpha}g^{\nu\beta}Tr\ F_{\mu\nu}F_{\alpha\beta} - \frac{k}{4}y(\phi)\frac{\epsilon^{\mu\nu\alpha\beta}}{2}Tr\ F_{\mu\nu}F_{\alpha\beta}. \qquad (1)\end{aligned}$$

Our notation is that of reference [1], and is outlined briefly in the appendix. Here $x(\phi)$ and $y(\phi)$ are gauge invariant real scalar functions of the scalar fields ϕ^i ($i = 1 \ldots N$) and, in the case that (1) is part of a supergravity Lagrangian, are the real and imaginary parts, respectively, of a single holomorphic function. We take the normalization $k^{-1} = (1/N_G)\mathrm{Tr}K$, where K is the Casimir operator of the gauge group $SO(N)$ in the representation on which the matrices $F_{\mu\nu}$ are valued, and N_G is the number of gauge degrees of freedom. Also, $\kappa = 1/\sqrt{8\pi G_N}$.

Although not the most general such Lagrangian, equation (1) is of the form found in effective four dimensional supergravity models from superstrings [2]. In [1] we found all the leading one–loop corrections for $x(\phi)$ and $y(\phi)$ constant.[2] Here we find additional logarithmically divergent *and* quadratically divergent corrections which will be important in understanding the physical content of string inspired supergravity.

In Section 2 we will give the background field expansion for the last two terms in (1), and in Section 3 we incorporate this with the results of [1] to give the complete divergent correction. Section 4 closes with a summary. Throughout, we follow closely the methodology developed in references [1,3,4], including the double subtraction scheme [3] to regulate divergences.

2. Background field expansion.

We expand the last two terms of (1) to second order in the quantum fields $(\hat{\phi}^i, \hat{A}_\mu, h_{\mu\nu})$ about a background configuration $(\tilde{\phi}^i, \tilde{A}_\mu, \eta_{\mu\nu})$. In this section $\eta_{\mu\nu}$ is general. The (non-abelian) field strength is given in curved space-time by

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu + e[A_\mu, A_\nu]. \qquad (2)$$

Here ∇_μ is the general coordinate transformation covariant derivative, which is just the ordinary space–time derivative ∂_μ on Lorentz scalars, and on Lorentz vectors is just

$$\nabla_\mu A_\nu = (\delta^\sigma_\nu\partial_\mu - \gamma^\sigma_{\mu\nu})A_\sigma,$$

[2] In [1] we included the noncanonical normalization of the gaugino kinetic energy terms, so that the terms calculated here and the parity–odd fermion loop corrections reported in [8] are the only modifications to [1] needed for the complete leading bosonic corrections to $N = 1$ Supergravity for flat background space–time.

$$\gamma^{\sigma}_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}). \tag{3}$$

A straightforward covariant Taylor expansion gives

$$y = \tilde{y}\left[1 + \frac{1}{\tilde{y}}\hat{\phi}^i D_{\tilde{\phi}^i}\tilde{y} + \frac{1}{2\tilde{y}}\hat{\phi}^i\hat{\phi}^j D_{\tilde{\phi}^i}D_{\tilde{\phi}^j}\tilde{y} + \cdots\right], \tag{4}$$

for the expansion of $y(\phi)$ about a background $\tilde{y}(\tilde{\phi})$, and

$$x = \tilde{x}\left[1 - 2\hat{\phi}^i D_{\tilde{\phi}^i}\ln\tilde{e} + 2\hat{\phi}^i\hat{\phi}^j\left(D_{\tilde{\phi}^i}\ln\tilde{e}D_{\tilde{\phi}^j}\ln\tilde{e} - \frac{1}{2}D_{\tilde{\phi}^i}D_{\tilde{\phi}^j}\ln\tilde{e}\right) + \cdots\right], \tag{5}$$

for the expansion of $x(\phi)$ about a background $\tilde{x}(\tilde{\phi}) \equiv e^2/\tilde{e}^2(\tilde{\phi})$. $D_{\tilde{\phi}^i}$ is the covariant derivative with respect to the background field $\tilde{\phi}^i$ [1]. Notice

$$\sqrt{\tilde{x}}\nabla_\mu A_\nu = (\nabla_\mu + [\nabla_\mu, \ln\tilde{e}])\sqrt{\tilde{x}}A_\nu, \tag{6}$$

so that we can make the gauge kinetic energy canonical by working with rescaled gauge fields:

$$\mathcal{A}_\mu \equiv \sqrt{\tilde{x}}A_\mu, \quad \mathcal{F}_{\mu\nu} \equiv \sqrt{\tilde{x}}F_{\mu\nu}, \tag{7}$$

where

$$\mathcal{F}_{\mu\nu} = (\nabla_\mu + [\nabla_\mu, \ln\tilde{e}])\mathcal{A}_\nu - (\nabla_\nu + [\nabla_\nu, \ln\tilde{e}])\mathcal{A}_\mu + \tilde{e}[\mathcal{A}_\mu, \mathcal{A}_\nu]. \tag{8}$$

That is, $\tilde{x}F_{\mu\nu}F_{\alpha\beta} = \mathcal{F}_{\mu\nu}\mathcal{F}_{\alpha\beta}$. Interestingly, the combination $\nabla_\mu + [\nabla_\mu, \ln\tilde{e}]$ functions as a covariant derivative for rescalings of $\tilde{e}$ (or $\tilde{x}$) and $\mathcal{A}_\mu$ by a function of the background scalars:

$$\tilde{e} \to \tilde{g}^{-1}\tilde{e}, \quad \mathcal{A}_\mu \to \tilde{g}\mathcal{A}_\mu. \tag{9}$$

However, the gauge kinetic energy term changes by an overall factor $\tilde{g}^2$ under these transformations, so it is not a symmetry of the Lagrangian.

Working with the rescaled fields, the gauge covariant derivative on the scalars is

$$\mathcal{D}_\mu\phi^i = (\delta^i_j\nabla_\mu + \tilde{e}\mathcal{A}^i_{\mu\,j})\phi^j, \tag{10}$$

which by virtue of $[\nabla_\mu, \tilde{e}] = \tilde{e}[\nabla_\mu, \ln\tilde{e}]$ satisfies

$$[\mathcal{D}_\mu, \mathcal{D}_\nu]\phi^i = \tilde{e}\mathcal{F}_{\mu\nu}\phi^i. \tag{11}$$

The gauge transformations are:

$$\begin{aligned} \phi &\to U\phi, \\ \mathcal{A}_\mu &\to U\mathcal{A}_\mu U^{-1} + \frac{1}{\tilde{e}}(\nabla_\mu U)U^{-1}. \end{aligned} \tag{12}$$

We now write the entire Lagrangian (1) in terms of the gauge fields $\mathcal{A}$. The last term in (1) is a total divergence when $y(\phi)$ is just a constant, in which case we could neglect it for our purposes. However, more generally, it will contribute to the one–loop Lagrangian. It is convenient to write it using the Chern–Simons term. In terms of the unscaled gauge fields

$$\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}TrF_{\mu\nu}F_{\alpha\beta} = \epsilon^{\mu\nu\alpha\beta}\nabla_\mu Tr\left[A_\nu\nabla_\alpha A_\beta + \frac{2}{3}eA_\nu A_\alpha A_\beta\right]. \tag{13}$$

Integrating by parts, and using (7) and (8) and the antisymmetry of $\epsilon^{\mu\nu\alpha\beta}$ to simplify the results, and dropping total divergences, we find for the last two terms in (1)

$$\begin{aligned}\mathcal{L} \ni & -\frac{k}{4}(x/\tilde{x})\sqrt{g}g^{\mu\alpha}g^{\nu\beta}Tr\ \mathcal{F}_{\mu\nu}\mathcal{F}_{\alpha\beta} \\ & +k\frac{[\nabla_\mu, y]}{2\tilde{x}}\epsilon^{\mu\nu\alpha\beta}Tr\ \left[\mathcal{A}_\nu\nabla_\alpha\mathcal{A}_\beta + \frac{2}{3}\tilde{e}\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right].\end{aligned} \tag{14}$$

The expansion of the space–time metric $g_{\mu\nu}$ about its background $\eta_{\mu\nu}$ is

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \\ g^{\mu\nu} &= \eta^{\mu\nu} - \eta^{\mu\sigma}\eta^{\nu\rho}h_{\sigma\rho} + \eta^{\mu\sigma}\eta^{\nu\rho}\eta^{\beta\gamma}h_{\sigma\beta}h_{\gamma\rho} + \cdots, \end{aligned} \tag{15}$$

where $\eta^{\mu\nu}$ is the inverse of $\eta_{\mu\nu}$. From now on, all Lorentz indices on background quantities are lowered and raised with the background space–time metric and its inverse, respectively. Also, we denote by $\tilde{\nabla}_\mu$ the background covariant derivative found from ∇_μ by replacing the space–time metric and its derivatives with the appropriate background quantities.

We now expand the gauge fields about a background $\tilde{\mathcal{A}}_\mu$. Then,

$$\begin{aligned}\mathcal{F}_{\mu\nu} = & \tilde{\mathcal{F}}_{\mu\nu} + \tilde{\mathcal{D}}_\mu\hat{\mathcal{A}}_\nu - \tilde{\mathcal{D}}_\nu\hat{\mathcal{A}}_\mu + \tilde{e}[\hat{\mathcal{A}}_\mu, \hat{\mathcal{A}}_\nu] \\ & +[\tilde{\nabla}_\mu, \ln\tilde{e}]\hat{\mathcal{A}}_\nu - [\tilde{\nabla}_\nu, \ln\tilde{e}]\hat{\mathcal{A}}_\mu,\end{aligned} \tag{16}$$

where

$$\begin{aligned}\tilde{\mathcal{D}}_\mu\hat{\mathcal{A}}_\nu &= \tilde{\nabla}_\mu\hat{\mathcal{A}}_\nu + \tilde{e}[\tilde{\mathcal{A}}_\mu, \hat{\mathcal{A}}_\nu], \\ \tilde{\mathcal{F}}_{\mu\nu} &= (\tilde{\nabla}_\mu + [\tilde{\nabla}_\mu, \ln\tilde{e}])\tilde{\mathcal{A}}_\nu - (\tilde{\nabla}_\nu + [\tilde{\nabla}_\nu, \ln\tilde{e}])\tilde{\mathcal{A}}_\mu + \tilde{e}[\tilde{\mathcal{A}}_\mu, \tilde{\mathcal{A}}_\nu].\end{aligned} \tag{17}$$

Notice that there are no terms with $h_{\mu\nu}$ dependence in the expansion of the field strength about its background. This is because $\tilde{\mathcal{F}}_{\mu\nu}$ is antisymmetric in μ and ν, whereas $\gamma^\sigma_{\mu\nu}$ is symmetric in these indices, so that such terms cancel. Terms in the Lagrangian quadratic in $\hat{\mathcal{A}}$ yield the gauge kinetic energy operator. To find this we need only work with the background space–time metric $\eta_{\mu\nu}$, and the corresponding background connection $\tilde{\gamma}^\sigma_{\mu\nu}$. Use of the relations

$$\begin{aligned}\nabla_\alpha\sqrt{g} = \nabla_\alpha g_{\mu\nu} = \nabla_\alpha g^{\mu\nu} = 0, \\ \tilde{\nabla}_\alpha\sqrt{\eta} = \tilde{\nabla}_\alpha\eta_{\mu\nu} = \tilde{\nabla}_\alpha\eta^{\mu\nu} = 0,\end{aligned} \tag{18}$$

and the gauge choice

$$F(\hat{\mathcal{A}}) = \frac{\tilde{e}}{e}(\tilde{\mathcal{D}}^\mu + [\tilde{\nabla}^\mu, \ln\tilde{e}])\hat{\mathcal{A}}_\mu = 0, \tag{19}$$

simplifies the analysis. Note that this gauge choice is just $\tilde{\mathcal{D}}^\mu \hat{A}_\mu = 0$ in terms of the unscaled quantum gauge fields.

Dropping total divergences, terms in the Lagrangian quadratic in $\hat{\mathcal{A}}$ reveal themselves to be

$$\mathcal{L}(\hat{\mathcal{A}}^2) = \frac{1}{2}\sqrt{\eta}\hat{\mathcal{A}}^a_\mu (\Delta^{-1})^{\mu\nu}_{ab} \hat{\mathcal{A}}^b_\nu, \tag{20}$$

where

$$(\Delta^{-1})^{\mu\nu}_{ab} = (\eta^{\mu\nu}\tilde{\mathcal{D}}_\alpha \tilde{\mathcal{D}}^\alpha + 2\tilde{e}\tilde{\mathcal{F}}^{\mu\nu})_{ab} + \delta_{ab}T^{\mu\nu} + W^{\mu\nu}_{ab}, \tag{21}$$

and

$$\begin{aligned} T^{\mu\nu} &= \eta^{\mu\nu}\left[(\tilde{\nabla}_\alpha \tilde{\nabla}^\alpha \ln \tilde{e}) - (\tilde{\nabla}_\alpha \ln \tilde{e})(\tilde{\nabla}^\alpha \ln \tilde{e})\right] \\ &\quad -2(\tilde{\nabla}^\mu \tilde{\nabla}^\nu \ln \tilde{e}) + 4(\tilde{\nabla}^\mu \ln \tilde{e})(\tilde{\nabla}^\nu \ln \tilde{e}) + r^{\mu\nu}, \\ W^{\mu\nu}_{ab} &= \frac{\epsilon^{\mu\alpha\nu\beta}}{\sqrt{\eta}} \frac{[\tilde{\nabla}_\beta, \tilde{y}]}{\tilde{x}} (\tilde{\mathcal{D}}_\alpha)_{ab}, \end{aligned} \tag{22}$$

and the background Ricci tensor $r^{\mu\nu}$ is found from $r_{\mu\nu}\mathcal{A}^\mu = -[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu]\mathcal{A}^\mu$ to be

$$r^{\mu\nu} = r^\beta_{\mu\beta\nu}, \quad r^\mu_{\sigma\alpha\beta} = \partial_\beta \tilde{\gamma}^\mu_{\alpha\sigma} - \partial_\alpha \tilde{\gamma}^\mu_{\beta\sigma} + \tilde{\gamma}^\mu_{\beta\rho}\tilde{\gamma}^\rho_{\alpha\sigma} - \tilde{\gamma}^\mu_{\alpha\rho}\tilde{\gamma}^\rho_{\beta\sigma}, \tag{23}$$

where the background Riemann tensor satisfies $r^\mu_{\sigma\alpha\beta}\mathcal{A}^\sigma = [\tilde{\nabla}_\beta, \tilde{\nabla}_\alpha]\mathcal{A}^\mu$.

In $(\Delta^{-1})^{\mu\nu}_{ab}$ the covariant derivative $\tilde{\mathcal{D}}_\mu$ and the field strength $\tilde{\mathcal{F}}_{\mu\nu}$ are represented as matrices in the adjoint representation of the gauged group. That is,

$$\begin{aligned} \hat{\mathcal{A}}_\mu &\equiv \hat{\mathcal{A}}^a_\mu T_a, \\ \tilde{\mathcal{D}}_\nu \hat{\mathcal{A}}_\mu &\equiv \left((\tilde{\mathcal{D}}_\nu)_{ba}\hat{\mathcal{A}}^a_\mu\right) T^b, \\ \left[\tilde{\mathcal{F}}_{\mu\nu}, \hat{\mathcal{A}}_\rho\right] &\equiv \left((\tilde{\mathcal{F}}_{\mu\nu})_{ba}\hat{\mathcal{A}}^a_\rho\right) T^b, \end{aligned} \tag{24}$$

where the T^a are the generators of the gauged group satisfying $\mathrm{Tr}T_aT_b = \delta_{ab}/k$.

Before determining the terms involving the other quantum fields $(\hat{\phi}^i, h_{\mu\nu})$ we use the Faddeev–Popov procedure to determine the correct gauge field measure for our path integral. The procedure remains unchanged from the canonical case. To gauge fix with (20) we add the ghost term

$$\mathcal{L}_\theta = \bar{\theta}^a(\tilde{\mathcal{D}}^\mu \tilde{\mathcal{D}}_\mu)_{ab}\theta^b, \tag{25}$$

to the Lagrangian and integrate over the Faddeev–Popov complex ghost fields θ in the path integral. Furthermore, writing $\delta(F(\hat{\mathcal{A}})) = \int d\alpha' \exp(i\alpha' F(\hat{\mathcal{A}}))$ we can implement this δ–function in the path integral by adding to the Lagrangian the term $+i(\tilde{e}/e)\alpha'(\tilde{\mathcal{D}}^\mu + [\tilde{\nabla}^\mu, ln\tilde{e}])\hat{\mathcal{A}}_\mu$ involving the auxiliary fields α'. However, it is more convenient to work with the rescaled field $\alpha = (\tilde{e}/e)\alpha' = \alpha'/\sqrt{\tilde{x}}$, for which the appropriate auxiliary field term is (in component form)

$$\mathcal{L}_\alpha = +i\alpha^b\left(\tilde{\mathcal{D}}^\mu_{ab} + \delta_{ab}[\tilde{\nabla}^\mu, \ln \tilde{e}]\right)\hat{\mathcal{A}}^a. \tag{26}$$

Returning to the Lagrangian (14), we see that only the first term contributes to terms that contain $h_{\mu\nu}$, whereas both terms contribute to terms containing $\hat{\phi}$ and/or $\hat{\mathcal{A}}$. Some algebra reveals

$$\begin{aligned}
\mathcal{L}(h^2) &= -\frac{\sqrt{\eta}}{2} h_{\alpha\beta}\left(2\kappa k F^{\alpha\beta,\mu\nu}\right) h_{\mu\nu}, \\
\mathcal{L}(\hat{\phi}^2) &= -\frac{\sqrt{\eta}}{2}\hat{\phi}^i v_{ij}\hat{\phi}^j, \\
\mathcal{L}(h,\hat{\phi}) &= -\sqrt{\eta}\hat{\phi}^i y_i^{\mu\nu} h_{\mu\nu}, \\
\mathcal{L}(h,\hat{\mathcal{A}}) &= +\sqrt{\eta} h_{\alpha\beta} K_a^{\alpha\beta,\epsilon}\hat{\mathcal{A}}_\epsilon^a, \\
\mathcal{L}(\hat{\phi},\hat{\mathcal{A}}) &= +\sqrt{\eta}\hat{\phi}^i\left[(s^\nu)_{ai} + (z^\nu)_{ai}\right]\hat{\mathcal{A}}_\nu^a, \qquad (27)
\end{aligned}$$

where

$$F^{\alpha\beta,\mu\nu} = -\frac{1}{4}P^{\alpha\beta,\mu\nu} Tr\, \tilde{\mathcal{F}}_{\gamma\delta}\tilde{\mathcal{F}}^{\gamma\delta} + 2P^{\alpha\beta,\gamma\mu} Tr\, \tilde{\mathcal{F}}_{\gamma\delta}\tilde{\mathcal{F}}^{\nu\delta} + \frac{1}{4\kappa} Tr\, \tilde{\mathcal{F}}^{\alpha\mu}\tilde{\mathcal{F}}^{\beta\nu}, \qquad (28)$$

$$\begin{aligned}
v_{ij} &= k\left(D_{\tilde{\phi}^i}\ln\tilde{e} D_{\tilde{\phi}^j}\ln\tilde{e} - \frac{1}{2}D_{\tilde{\phi}^i}D_{\tilde{\phi}^j}\ln\tilde{e}\right) Tr\, \tilde{\mathcal{F}}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu} \\
&\quad + k\frac{D_{\tilde{\phi}^i}D_{\tilde{\phi}^j}\tilde{y}}{4\tilde{x}}\frac{\epsilon^{\mu\nu\alpha\beta}}{2\sqrt{\eta}} Tr\, \tilde{\mathcal{F}}_{\mu\nu}\tilde{\mathcal{F}}_{\alpha\beta}, \qquad (29)
\end{aligned}$$

$$y_i^{\mu\nu} = 2k\kappa(D_{\tilde{\phi}^i}\ln\tilde{e})\left[P^{\alpha\beta,\delta\sigma}\eta^{\delta\epsilon} + \frac{1}{4\kappa}\eta^{\gamma\alpha}\eta^{\epsilon\beta}\eta^{\gamma\sigma}\right] Tr\, \tilde{\mathcal{F}}_{\epsilon\sigma}\tilde{\mathcal{F}}_{\gamma\delta}, \qquad (30)$$

$$K_a^{\alpha\beta,\epsilon} = -4\kappa\left[P^{\alpha\beta,\delta\sigma}\eta^{\gamma\epsilon} + \frac{1}{4\kappa}\eta^{\gamma\alpha}\eta^{\epsilon\beta}\eta^{\delta\sigma}\right]\left[(\tilde{\mathcal{D}}_\sigma\tilde{\mathcal{F}}_{\gamma\epsilon})_a - [\tilde{\nabla}_\sigma,\ln\tilde{e}](\tilde{\mathcal{F}}_{\gamma\epsilon})_a\right], \qquad (31)$$

$$(s^\nu)_{ia} = 2(D_{\tilde{\phi}^i}\ln\tilde{e})[\tilde{\nabla}_\mu,\ln\tilde{e}](\tilde{\mathcal{F}}^{\mu\nu})_a, \qquad (32)$$

$$(z^\nu)_{ia} = 2(D_{\tilde{\phi}^i}\ln\tilde{e})(\tilde{\mathcal{F}}^{\mu\nu})^b(\tilde{\mathcal{D}}_\mu)_{ab} - \frac{D_{\tilde{\phi}^i}\tilde{y}}{\tilde{x}}\frac{\epsilon^{\mu\nu\alpha\beta}}{2\sqrt{\eta}}(\tilde{\mathcal{F}}_{\alpha\beta})^b(\tilde{\mathcal{D}}_\mu)_{ab}, \qquad (33)$$

and finally

$$P^{\alpha\beta,\mu\nu} = \frac{1}{2\kappa}\left(\frac{1}{2}\eta^{\alpha\mu}\eta^{\beta\nu} - \frac{1}{4}\eta^{\alpha\beta}\eta^{\mu\nu}\right). \qquad (34)$$

Here $\tilde{\mathcal{F}}_{\mu\nu} = (\tilde{\mathcal{F}}_{\mu\nu})_a T^a$, and so on.

The above results may be combined with previously developed techniques [5,6,7] to determine the one–loop corrections in a curved space–time background. Here we will give the results only for a flat background metric.

The full expansion of (1), in the case of a flat space–time background, to second order in the quantum fields and with all the quantum gauges fixed, can be deduced from reference [1]. The new pieces due to nonconstant x and y, described by eqs. (29), (30), (32), (33), and the $\tilde{e}$–dependent term in (31), are terms that mix the different quantum fields so we generally expect them to yield at most log–divergent corrections (in four dimensions). With the the exception of (33), this is true. The z^ν term of (33) yields a quadratically divergent correction due to the appearance of the derivative. There will also be additional quadratically divergent corrections from the mass like terms of (22). We find all these in the section below.

3. Leading one–loop corrections.

For the remainder of this paper we take the background space–time to be flat ($\eta_{\mu\nu}$ is the Minkowski metric). In ref. [1] the fields $(\hat{\phi}, \hat{\mathcal{A}}, h)$ were considered as part of one "multiplet" of bosonic fields, Φ. For a Lagrangian given by

$$\mathcal{L} = -\frac{1}{2}\Phi^T Z(d^2 + M^2)\Phi = -\frac{1}{2}\Phi^T \Delta_\Phi^{-1}\Phi, \tag{35}$$

where Z is the metric for Φ represented by a block diagonal matrix (when Φ is thought of as a column), d_μ is the derivative, M is the mass matrix, the leading regulated one–loop corrections are [3]

$$\mathcal{L}_{reg} = -\frac{1}{64\pi^2}Tr\left[(M^4 + \frac{1}{3}d^2M^2 + \frac{1}{6}G^{\mu\nu}G_{\mu\nu})\ln(2\mu_0^2/\mu^2) + 4M^2\mu^2\ln 2\right]. \tag{36}$$

Here $G_{\mu\nu} = [d_\mu, d_\nu]$, and a specific [3] double subtraction scheme was used to obtain the results: μ is the regulating scale and μ_0 is a characteristic low energy scale. The presence of the term (33) makes the use of this relation difficult, since this term cannot be interpreted as an off–diagonal mass term. Additionally, we must worry about the derivative term in (22). Nevertheless, we may still use the general result due to the following observation.

For the derivative term in (22) we employ the following trick. Eq. (21) can be written as

$$(\Delta^{-1})^{\mu\nu}_{ab} = (\hat{\mathcal{D}}_\alpha\hat{\mathcal{D}}^\alpha + 2\tilde{e}\tilde{\mathcal{F}}^{\mu\nu})_{ab} + \delta_{ab}T^{\mu\nu} + \Omega^{\mu\nu}_{ab}, \tag{37}$$

where the hatted "covariant" derivative is

$$\begin{aligned} (\hat{\mathcal{D}}_\mu)^{\alpha b}_{\beta a} &= (\mathcal{D}_\mu)^b_a\delta^\alpha_\beta + (\Upsilon_\mu)^{\alpha b}_{\beta a}, \\ (\Upsilon_\mu)^{\alpha b}_{\beta a} &= \frac{1}{2}\epsilon^\alpha{}_{\mu\beta\rho}\delta^b_a(\partial^\rho\tilde{y})/\tilde{x}, \end{aligned} \tag{38}$$

and

$$\Omega^{\mu\nu}_{ab} = -(\Upsilon_\alpha\Upsilon^\alpha)^{\mu\nu}_{ab} = -\frac{1}{2\tilde{x}^2}\delta_{ab}\left(\eta^{\mu\nu}\partial^\rho\tilde{y}\partial_\rho\tilde{y} - \partial^\mu\tilde{y}\partial^\nu\tilde{y}\right), \tag{39}$$

where we dropped a $\mathcal{D}_\alpha\Upsilon^\alpha$ term because of antisymmetry (eq. (37) is sandwiched between quantum gauge fields).

We employ a similar trick for the term (33). Consider the simple multiplet consisting of two bosonic fields A and ϕ:

$$\Phi = \begin{pmatrix} A \\ \phi \end{pmatrix} \tag{40}$$

We define a "covariant" derivative which on Φ has the matrix form

$$D_\mu = \begin{pmatrix} \partial_\mu & -C_\mu \\ C_\mu & \partial_\mu \end{pmatrix}, \tag{41}$$

so that $D_\mu \Phi^T \Phi = \partial_\mu \Phi^T \Phi$. From this it is straightforward to verify

$$-\frac{1}{2}A\partial_\mu\partial^\mu A - \frac{1}{2}\phi\partial_\mu\partial^\mu\phi - 2\phi C^\mu(\partial_\mu A) = -\frac{1}{2}\Phi^T[D_\mu D^\mu + m^2]\Phi, \tag{42}$$

up to total divergences, and where

$$m^2 = \begin{pmatrix} C_\mu C^\mu & -\partial_\mu C^\mu \\ -\partial_\mu C^\mu & C_\mu C^\mu \end{pmatrix}. \tag{43}$$

Since we drop total divergences, we can calculate the one loop corrections from (39) in a similar vein to (35). More generally the partial derivative is replaced by a reparameterization and gauge covariant derivative, we have many fields $\hat{\mathcal{A}}^\alpha_a$ and $\hat{\phi}^i$, and the gauge kinetic term (20) is different from that of scalars. This means that the derivative, as well as the "connection" C_μ, carry indices themselves.

To include the mixing terms due to both (22) and (33) and use the compact result of (36) we define a derivative d_μ which includes the C_μ and the Υ_μ terms. This derivative is block diagonal except for the terms induced by C_μ. The block diagonal parts without the Υ_μ term, as outlined in [1], are the appropriate background gauge and reparameterization covariant derivatives on the scalars, vectors, and spin two fields. By inspection we have $(d_\mu)^{\alpha b}_{\beta a} = (\hat{\mathcal{D}}_\mu)^{\alpha b}_{\beta a}$, $(d_\mu)^\alpha_{ja} = -(C_\mu)^\alpha_{ja}$, $(d_\mu)^{ib}_\beta = -\eta_{\beta\theta}(C_\mu)^{\theta ib}$, and $(d_\mu)^i_j$ is the fully background covariant derivative on the scalars: $(d_\mu)^i_j = \partial_\mu\delta^i_j + \Gamma^i_{jk}\partial_\mu\tilde{\phi}^k$, where $\Gamma^i_{jk} = Z^{i\ell}\Gamma_{\ell jk}$ is the scalar connection defined in Eq.(56) of the Appendix. Also, $-2\hat{\phi}^i\eta_{\alpha\nu}(C_\mu)^\alpha_{bi}(\mathcal{D}^\mu)^{ab}\hat{\mathcal{A}}^a_\nu = \hat{\phi}^i(z^\nu)_{ai}\hat{\mathcal{A}}^a_\nu$. We immediately obtain from (33)

$$(C_\mu)^{\nu b}_i = +\frac{D_{\tilde{\phi}^i}\tilde{y}}{\tilde{x}}\frac{\epsilon_\mu{}^{\nu\alpha\beta}}{4}(\tilde{\mathcal{F}}_{\alpha\beta})^b - (D_{\tilde{\phi}^i}\ln\tilde{e})(\tilde{\mathcal{F}}_\mu{}^\nu)^b. \tag{44}$$

We may now write down the full expansion of (1) in the form of (35), neglecting auxiliary and ghost terms for the moment. From the results of [1], after all gauge fixing, we have for the metric

$$Z^{\lambda\sigma,\mu\nu} = P^{\lambda\sigma,\mu\nu}, \quad Z^{ab}_{\mu\nu} = -\eta_{\mu\nu}\delta^{ab}, \quad Z_{ij} = Z_{ij}(\tilde{\phi}). \tag{45}$$

The mass terms, including the modifications from C_μ and Υ_μ dependent terms, are

$$\begin{aligned}
(ZM^2)^{\lambda\sigma,\mu\nu} &= X^{\lambda\sigma,\mu\nu}, \\
(ZM^2)^{ab}_{\mu\nu} &= -\eta_{\mu\nu}(\tilde{M}^2)^{ab} - 2\tilde{e}\tilde{\mathcal{F}}^{ab}_{\mu\nu} - \delta^{ab}T_{\mu\nu} - \Omega^{ab}_{\mu\nu} - (C^\alpha)^a_{\mu i}(C_\alpha)^b_{\nu j}Z^{ij} \equiv -N^{ab}_{\mu\nu}, \\
(ZM^2)_{ij} &= U_{ij} - V_{ij} + R_{ij} + v_{ij} + (C^\alpha)^a_{\mu i}(C_\alpha)^\mu_{aj} \equiv H_{ij}, \\
(ZM^2)^{\mu\nu}_i &= Y^{\mu\nu}_i + y^{\mu\nu}_i \equiv \mathcal{Y}^{\mu\nu}_i, \\
(ZM^2)^a_{\mu i} &= -\tilde{e}S^a_{\mu i} - (D_\nu C^\nu)^a_{\mu i} - s^a_{\mu i} \equiv \mathcal{S}^a_{\mu i}, \\
(ZM^2)^{a,\lambda\sigma}_\mu &= -K^{a,\lambda\sigma}_\mu - Q^{a,\lambda\sigma}_\mu.
\end{aligned} \tag{46}$$

We have collected the undefined expressions X, U, V, R, Y, S, $\tilde{M}$, and Q from [1] in the appendix. The derivative on C_μ is given just below.

The only contributions to $G_{\mu\nu}$ of (36) are:

$$\begin{aligned}\begin{pmatrix} (\hat{G}_{\mu\nu})^{\alpha c}_{\gamma a} & (G_{\mu\nu})^{\alpha}_{ka} \\ (G_{\mu\nu})^{ic}_{\gamma} & (G_{\mu\nu})^{i}_{k} \end{pmatrix} &= \left[\begin{pmatrix} (\hat{\mathcal{D}}_\mu)^{\alpha b}_{\beta a} & -(C_\mu)^{\alpha}_{ja} \\ -\eta_{\beta\theta}(C_\mu)^{\theta i b} & (d_\mu)^{i}_{j} \end{pmatrix}, \begin{pmatrix} (\hat{\mathcal{D}}_\nu)^{\beta c}_{\gamma b} & -(C_\nu)^{\beta}_{kb} \\ -\eta_{\gamma\rho}(C_\nu)^{\rho j c} & (d_\nu)^{j}_{k} \end{pmatrix}\right] \\ &= \begin{pmatrix} (G^A_{\mu\nu})^c_a\delta^\alpha_\gamma + [C_\mu, C_\nu]^{\alpha c}_{\gamma a} & (D_\nu C_\mu)^{\alpha}_{ak} - (D_\mu C_\nu)^{\alpha}_{ak} \\ (D_\nu C_\mu)^{ic}_{\gamma} - (D_\mu C_\nu)^{ic}_{\gamma} & (G^\phi_{\mu\nu})^i_k + [C_\mu, C_\nu]^i_k \end{pmatrix} \end{aligned} \quad (47)$$

where $[C_\mu, C_\nu]^{\alpha c}_{\gamma a} = (C_\mu)^{\alpha}_{ja}(C_\nu)^{\rho j c}\eta_{\gamma\rho} - \mu \leftrightarrow \nu$, and the action of the fully covariant derivative D_μ is defined by $(D_\nu C_\mu)^{\alpha}_{ak} = [d_\nu, C_\mu]^{\alpha}_{ak} = (\hat{\mathcal{D}}_\nu)^{\alpha b}_{\beta a}(C_\mu)^{\beta}_{kb} - (C_\mu)^{\alpha}_{ja}(d_\nu)^{j}_{k}$ and so on. $\hat{G}^A_{\mu\nu}$ and $G^\phi_{\mu\nu}$ are seen to be

$$\begin{aligned} (\hat{G}^A_{\mu\nu})^{\alpha b}_{\beta a} &= (G^A_{\mu\nu})^b_a\delta^\alpha_\beta + [\mathcal{D}_\mu, \Upsilon_\nu]^{\alpha b}_{\beta a} - [\mathcal{D}_\nu, \Upsilon_\mu]^{\alpha b}_{\beta a} + [\Upsilon_\mu, \Upsilon_\nu]^{\alpha b}_{\beta a}, \\ (G^\phi_{\mu\nu})^i_j &= \mathcal{D}_\mu\tilde{\phi}^k\mathcal{D}_\nu\tilde{\phi}^l R^i{}_{jlk} + \tilde{e}(\tilde{\mathcal{F}}_{\mu\nu})^i_j + \tilde{e}\Gamma^i{}_{kj}(\tilde{\mathcal{F}}_{\mu\nu}\tilde{\phi})^k, \\ (G^A_{\mu\nu})^a_b &= \tilde{e}(\tilde{\mathcal{F}}_{\mu\nu})^a_b, \end{aligned} \quad (48)$$

where R and Γ are given in the appendix, eq. (60).

The ghost contributions are given in [1]. Combined with those from (36) they yield the quadratic corrections

$$\mathcal{L}^{(1)}_{quad} = -\frac{\mu^2\ln 2}{16\pi^2}Tr\ [H - 2U_{gh} + X + N], \quad (49)$$

and the logarithmic corrections

$$\begin{aligned}\mathcal{L}^{(1)}_{log} = &-\frac{\ln(2\mu_0^2/\mu^2)}{64\pi^2}Tr\ \Big[H^2 - 2U_{gh}^2 + X^2 + N^2 + 2\mathcal{Y}^2 - 2(K+Q)^2 \\ &-2S^2 + \frac{1}{3}(\partial^2 X - 2\partial^2 U_{gh} + d^2 H + d^2 N) \\ &+\frac{1}{6}(G^\phi_{\mu\nu}G^{\mu\nu}_\phi + 2G^A_{\mu\nu}G^{\mu\nu}_A + 2[C_\mu, C_\nu]^2 + 4(D_\mu C_\nu)(D^\mu C^\nu) \\ &-4(D_\mu C_\nu)(D^\nu C^\mu) + 2G^A_{\mu\nu}[C^\mu, C^\nu] + 2G^\phi_{\mu\nu}[C^\mu, C^\nu] \\ &+2(\mathcal{D}_\mu\Upsilon_\nu)(\mathcal{D}^\mu\Upsilon^\nu) - 2(\mathcal{D}_\mu\Upsilon_\nu)(\mathcal{D}^\nu\Upsilon^\mu) + [\Upsilon_\mu, \Upsilon_\nu]^2)\Big]. \end{aligned} \quad (50)$$

A few words of caution. The Tr stands for a full contraction of all indices on the matrices using when necessary the metrics $Z_{ij}, P_{\alpha\beta,\mu\nu}, \delta_{ab}, \eta_{\mu\nu}$ and their inverses. Thus, for example, $Tr\ N = +N^{ab}_{\mu\nu}\delta_{ab}\eta^{\mu\nu}$, whereas $Tr\ G^A_{\mu\nu}G^{\mu\nu}_A = (G^A_{\mu\nu})_{ab}(G^{\mu\nu}_A)_{cd}\delta^{ac}\delta^{bd}$. The traces of terms involving only C_μ (and derivatives) is over a matrix with scalar indices only (see for example the diagonal terms in (47)), e.g.

$$Tr(D_\mu C_\nu)(D^\mu C^\nu) = (D_\mu C_\nu)^{\alpha}_{ak}(D^\mu C^\nu)^{\beta}_{bj}\delta^{ab}\eta_{\alpha\beta}Z^{jk}. \quad (51)$$

As in [1], a similar convention is used for the traces over the sqares of other matrices $(\mathcal{S}, K+Q, \mathcal{Y})$ that are off-diagonal with respect to spin. The trace over terms involving only Υ_μ terms (and derivatives) is over the indices given in the first line of (48). The total derivative terms involving X, H, and N arise from the second term in (36). The non–diagonal contributions from d_μ cancel under the trace. In addition, we have not kept terms that vanish because $TrG^A_{\mu\nu} = 0$ (over gauge indices). U_{gh} is defined in (59).

Finally, we come to the auxiliary field contribution. As in [1], by a field redefinition, (26) can be rewritten in a form purely quadratic in the auxiliary fields,

$$\mathcal{L}(\alpha^2) = -\frac{1}{2}\alpha^a(\mathcal{D}^\mu_{ab} + \delta_{ab}[\partial^\mu, \ln\tilde{e}])(\Delta_\Phi)^{bc}_{\mu\nu}(\mathcal{D}^\nu_{cd} - \delta_{cd}[\partial^\nu, \ln\tilde{e}])\alpha^d. \tag{52}$$

This contains derivatives in the denominator, so that a formula like (36) is not applicable. To find the divergent corrections we may expand (52) in decreasing powers of the (covariant) derivatives, keeping only order ∂_x^{-4} terms. We find:

$$\begin{aligned}(\mathcal{D}^\mu_{ab} &+ \delta_{ab}[\partial^\mu, \ln\tilde{e}])(\Delta_\Phi)^{bc}_{\mu\nu}(\mathcal{D}^\nu_{cd} - \delta_{cd}[\partial^\nu, \ln\tilde{e}]) = \\ &\mathcal{D}^\mu_{ab}(\Delta_A)^{bc}_{\mu\nu}\mathcal{D}^\nu_{cd} + [\partial^\mu, \ln\tilde{e}]\delta_{ab}(\Delta_A)^{bc}_{\mu\nu}\mathcal{D}^\nu_{cd} - \mathcal{D}^\mu_{ab}(\Delta_A)^{bc}_{\mu\nu}\delta_{cd}[\partial^\nu, \ln\tilde{e}] \\ &+\mathcal{D}^\mu_{ab}(\Delta_A)^{ab'}_{\mu\nu'}(ZM^2)^{\nu'}_{b'i}(\Delta_\phi)^{ij}(ZM^2)^{\mu'}_{a',j}(\Delta_A)^{a'b}_{\mu'\nu}\mathcal{D}^\nu_{cd} \\ &+\mathcal{D}^\mu_{ab}(\Delta_A)^{ab'}_{\mu\nu'}(ZM^2)^{\nu'}_{b'\lambda\sigma}(\Delta_h)^{\lambda\sigma,\rho\tau}(ZM^2)^{\mu'}_{a',\rho\tau}(\Delta_A)^{a'b}_{\mu'\nu}\mathcal{D}^\mu_{cd} + \cdots,\end{aligned} \tag{53}$$

where $\Delta_\phi^{-1}, \Delta_A^{-1}, \Delta_h^{-1}$ are the appropriate block diagonal parts of Δ_Φ^{-1}.

To calculate the leading corrections we follow refs. [1,3,4]. In this section, for technical reasons, we do not rewrite (21) by introducing a new derivative as in (37), i.e. the gauge covariant derivative is $\mathcal{D}_\mu$ and not $\hat{\mathcal{D}}_\mu$. The gauge field "mass" term is then given by a matrix $N' + W$, where N' is N without the Ω terms (see eq. (46)) and W is defined in (22).

We use the following substitution rule [1,3]:

$$\begin{aligned}F &\rightarrow F - i(d_{\mu_1}F)\partial_p^{\mu_1} - \frac{1}{2}(d_{\mu_1}d_{\mu_2}F)\partial_p^{\mu_1}\partial_p^{\mu_2} + \ldots, \\ d_\mu &\rightarrow i(p_\mu + \tilde{G}_{\nu\mu}\partial_p^\nu), \\ \tilde{G}_{\mu\nu} &= \frac{1}{2}G_{\mu\nu} - \frac{i}{3}(d_{\mu_1}G_{\mu\nu})\partial_p^{\mu_1} - \frac{1}{8}(d_{\mu_1}d_{\mu_2}G_{\mu\nu})\partial_p^{\mu_1}\partial_p^{\mu_2} + \ldots,\end{aligned} \tag{54}$$

for a matrix valued function F and a covariant derivative d. In the expansion of the logarithm of (53), *viz.*, $\ln(1+\epsilon) = \epsilon - \frac{1}{2}\epsilon^2 + \cdots$, derivatives with respect to p that appear to the far right may be dropped, as may, after integration by parts derivative to the far left. We retain terms of $O(p^0)$, $O(p^{-2})$, and $O(p^{-4})$ in the expansion of the ln, and since the sum of $O(p^{-1})$ terms cancel we dropped all $O(p^{-3})$ terms in the intermediate equation that follows. Using $[\tilde{G}_{\mu\nu}p^{-2}p^\nu, \partial_p^\mu] = \delta_{ab}G^{ab}_{\mu\nu} = 0$ and the antisymmetry of $(\Upsilon^\alpha)^\mu_\nu$ in its three indices, the components of eq. (53) reduce to

$$
\begin{aligned}
[\partial^\mu, \ln\tilde{e}](\Delta_A)^{ac}_{\mu\nu}\mathcal{D}^\nu_{cb} \;\rightarrow\; & -i(\partial^\mu \ln\tilde{e})\delta^a_b p_\mu p^{-2} + i(\partial^\mu \ln\tilde{e})p^{-2}\left[-N'_{\mu\nu} + \eta_{\mu\nu}x_2\right]^a_b p^{-2}p^\nu \\
& -(\partial^\mu \ln\tilde{e})p^{-2}(d_\alpha N'_{\mu\nu})^a_b \partial^\alpha_p p^{-2}p^\nu + \frac{i}{2}(\partial^\alpha\partial^\beta\partial^\mu \ln\tilde{e})\delta^a_b \partial^\alpha_p \partial^\beta_p p_\mu p^{-2} \\
& -(\partial^\alpha\partial^\mu \ln\tilde{e})\delta^a_b \partial^\alpha_p p_\mu p^{-2} \\
& -2(\partial_\mu \ln\tilde{e})p^{-2}\left[(\Upsilon^\alpha)p_\alpha p^{-2}N'\right]^{\mu a}_{\nu b} p^{-2}p^\nu \\
\mathcal{D}^\mu_{bc}(\Delta_A)^{ca}_{\mu\nu}[\partial^\nu, \ln\tilde{e}] \;\rightarrow\; & -ip_\mu p^{-2}(\partial^\mu \ln\tilde{e})\delta^a_b + ip^\mu p^{-2}\left[\eta_{\mu\nu}x_2 - N'_{\mu\nu}\right]^a_b p^{-2}(\partial^\nu \ln\tilde{e}) \\
& -p^\mu p^{-2}(d_\alpha N'_{\mu\nu})^a_b \partial^\alpha_p p^{-2}(\partial^\nu \ln\tilde{e}) \\
& +\frac{i}{2}p_\mu p^{-2}(\partial^\alpha\partial^\beta\partial^\mu \ln\tilde{e})\partial^\alpha_p\partial^\beta_p - p_\mu p^{-2}(\partial^\alpha\partial^\mu \ln\tilde{e})\partial^\alpha_p, \\
& +2p_\mu p^{-2}\left[-\frac{1}{2}(d_\rho d_\sigma(\Upsilon^\alpha)^\mu_\nu)\partial^\rho_p\partial^\sigma_p p_\alpha\right]^a_b p^{-2}(\partial^\nu \ln\tilde{e}) \\
& -4p_\mu p^{-2}\left[(d_\rho\Upsilon^\beta)\partial^\rho_p p_\beta p^{-2}(\Upsilon^\alpha)p_\alpha\right]^{\mu a}_{\nu b} p^{-2}(\partial^\nu \ln\tilde{e}) \\
& -2p_\mu p^{-2}\left[N' p^{-2}(\Upsilon^\alpha)p_\alpha\right]^{\mu a}_{\nu b} p^{-2}(\partial^\nu \ln\tilde{e}), \\
\mathcal{D}^\mu_{ab}(\Delta_A)^{bc}_{\mu\nu}\mathcal{D}^\nu_{cd} \;\rightarrow\; & 1 - p_\mu p^{-2}x_2 p^{-2}p^\mu \\
& +4p_\mu p^{-2}x_4 p^{-2}x_4 p^{-2}p^\mu + p_\mu p^{-2}N'^{\mu\nu}_{ad}p^{-2}p_\nu \\
& -p_\mu p^{-2}N'^{\mu\nu}p^{-2}x_2 p^{-2}p_\nu - p_\mu p^{-2}x_2 p^{-2}N'^{\mu\nu}p^{-2}p_\nu \\
& -\frac{1}{2}p_\mu p^{-2}(d_{\mu_1}d_{\mu_2}N'^{\mu\nu})\partial^{\mu_1}_p\partial^{\mu_2}_p p^{-2}p_\nu + p^\mu p_\nu p^{-6}N'_{\mu\sigma}N'^{\sigma\nu} \\
& +2p_\mu p^{-2}[d_\sigma\Upsilon^\alpha\partial^\sigma_p p_\alpha]^\mu_\theta p^{-2}N'^\theta_\nu p^{-2}p^\nu
\end{aligned}
\tag{55}
$$

where the RHS of the last expression is a matrix with labels ad. For the last two terms of (53) we may simply use $\mathcal{D}^\mu \rightarrow ip^\mu$ both in the numerator and in the denominator to this order. In these expressions $x_2 = 2p_\mu \tilde{G}^{\nu\mu}\partial^p_\nu + \tilde{G}_{\nu\mu}\tilde{G}^{\nu'\mu}\partial^\nu_p\partial^p_{\nu'}$, and $x_4 = p_\mu \tilde{G}^{\nu\mu}\partial^p_\nu$.

If $\mathcal{A}$ is (53) after the substitution rule, then the one–loop corrections are $\mathcal{L}^{(1)} = \frac{i}{2}\int \frac{d^4p}{(2\pi)^4} Tr\ \ln\mathcal{A}$, and the divergent integrals can be regulated as shown in [5]. The result of these computations is:

$$
\begin{aligned}
\mathcal{L}^{(1)}_{aux} = & \frac{\mu^2\ln 2}{64\pi^2} Tr\ \eta_{\mu\nu}N'^{\mu\nu} + \frac{\ln(\mu^2/2\mu_0^2)}{32\pi^2} Tr\left[-\frac{1}{4}N'_{\mu\nu}N'^{\nu\mu} + \frac{1}{48}\rho_{\mu\nu\alpha\beta}N'^{\mu\nu}N'^{\alpha\beta}\right. \\
& +\frac{1}{4}\mathcal{S}^2 + \frac{1}{4}(K+Q)^2 - \frac{1}{12}\rho_{\mu\nu\alpha\beta}(d^\alpha d^\beta N'^{\mu\nu}) - \frac{1}{2}N'^{\mu\nu}G^A_{\mu\nu} \\
& +\frac{1}{2}(\partial^\mu \ln\tilde{e})(d^\nu N'_{\mu\nu} + d^\nu N'_{\nu\mu}) + (\frac{1}{12}\rho_{\mu\nu\alpha\beta} - \frac{1}{4}\eta_{\mu\nu}\eta_{\alpha\beta})(\partial^\mu\partial^\nu \ln\tilde{e})N'^{\alpha\beta} \\
& \left.+\frac{1}{2}\eta^\sigma_\mu\eta^\nu_\alpha[(d_\sigma\Upsilon^\alpha)N']^\mu_\nu\right] \\
& +N_G\frac{\ln(\mu^2/2\mu_0^2)}{32\pi^2}\left[\frac{1}{12}\rho_{\mu\nu\alpha\beta}(\partial^\mu\partial^\nu \ln\tilde{e})(\partial^\alpha\partial^\beta \ln\tilde{e}) + d_\rho(\Upsilon^\beta)^\mu_\theta(\Upsilon^\alpha)^\theta_\nu\eta^\rho_\mu\eta_{\alpha\beta}\partial^\nu \ln\tilde{e}\right]
\end{aligned}
\tag{56}
$$

where

$$\rho_{\mu\nu\alpha\beta} = \eta_{\mu\nu}\eta_{\alpha\beta} + \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}. \tag{57}$$

The Tr is over gauge indices and is made explicit below. This result agrees with a previous computation [1] when the functions x and y defined in the first section are constant. We note that the precise coefficients of the quadratically divergent terms are renormalization prescription dependent. In (56) we chose the coefficient so as to reproduce the standard Coleman-Weinberg result in the canonical case $x = y =$ constant. However, the relative coefficients for the different terms in the expression for N^{μ}_{μ} can be reliably determined only when a manifestly supersymmetric and gauge invariant regularization procedure is used. Finally, the third to last term in (56) vanishes because $\delta_{ab}N'^{ab}_{\mu\nu}$ is symmetric in its Lorentz indices, whereas Υ is totally antisymmetric in all its Lorentz indices.

4. Conclusion.

The total leading correction, given by the sum of (49), (50), and (56) can be written in a more explicit form:

$$\begin{aligned}
\mathcal{L}^{(1)} \;=\; & -\frac{\mu^2\ln 2}{16\pi^2}\left[H_{ij}Z^{ij} - 2(U_{gh})_{\mu\nu}\eta^{\mu\nu} + X_{\mu\nu,\alpha\beta}P^{\mu\nu,\alpha\beta} + \frac{3}{4}N'^{ab}_{\mu\nu}\delta_{ab}\eta^{\mu\nu}\right] \\
& -\frac{\ln(2\mu_0^2/\mu^2)}{64\pi^2}\Big[H_{ij}H^{ij} - 2(U_{gh})_{\mu\nu}(U_{gh})^{\mu\nu} + X^{\alpha\beta}_{\mu\nu}X^{\mu\nu}_{\alpha\beta} + 2\mathcal{Y}^{\mu\nu}_i\mathcal{Y}^{\alpha\beta}_j Z^{ij}P_{\mu\nu,\alpha\beta} \\
& -\frac{3}{2}(K+Q)^{\mu\nu,\epsilon}_a(K+Q)^{\alpha\beta,\eta}_b P_{\mu\nu,\alpha\beta}\eta_{\epsilon\eta}\delta_{ab} - \frac{3}{2}\mathcal{S}^a_{\mu i}\mathcal{S}^b_{\nu j}\delta_{ab}Z^{ij}\eta^{\mu\nu} \\
& +\frac{1}{2}(N'_{\mu\nu}N'^{\nu\mu})^{ab}\delta_{ab} + \frac{1}{24}\rho_{\mu\nu\alpha\beta}(N'^{\mu\nu}N'^{\alpha\beta})^{ab}\delta_{ab} + \frac{1}{6}(G^{\phi}_{\mu\nu})^i_j(G^{\mu\nu}_{\phi})^j_i + \frac{1}{3}(G^A_{\mu\nu})^a_b(G^{\mu\nu}_A)^b_a \\
& -(N'^{\mu\nu})^a_b(G^A_{\mu\nu})^b_a - \frac{5}{3}(\partial^\mu\partial^\nu\ln\tilde{e})N'^{ab}_{\mu\nu}\delta_{ab} - \frac{1}{3}(\partial^2\ln\tilde{e})N'^{ab}_{\mu\nu}\eta^{\mu\nu}\delta_{ab} \\
& +\frac{N_G}{6}\rho_{\mu\nu\alpha\beta}(\partial^\mu\partial^\nu\ln\tilde{e})(\partial^\alpha\partial^\beta\ln\tilde{e}) + \frac{2}{3}(C_\mu)^\alpha_{ja}(C_\nu)^{\rho jc}(C^\mu)^\gamma_{ic}(C^\nu)^{\sigma ia}\eta_{\gamma\rho}\eta_{\alpha\sigma} \\
& -\frac{2}{3}(C_\mu)^\alpha_{ja}(C_\nu)^{\rho jc}(C^\nu)^\gamma_{ic}(C^\mu)^{\sigma ia}\eta_{\gamma\rho}\eta_{\alpha\sigma} + \frac{2}{3}(D_\mu C_\nu)^\alpha_{ak}(D^\mu C^\nu)^{kb}_\alpha \\
& -\frac{2}{3}(D_\mu C_\nu)^\alpha_{ak}(D^\nu C^\mu)^{kb}_\alpha + \frac{1}{3}(G^A_{\mu\nu})^c_a[C^\mu, C^\nu]^{\alpha a}_{\alpha c} + \frac{1}{3}(G^\phi_{\mu\nu})^i_k[C^\mu, C^\nu]^k_i + 2N'_{\mu\nu}\Omega^{\nu\mu}\Big] \\
& +\frac{3\mu^2\ln 2}{32\pi^2}N_G(\partial^\mu\tilde{y}\partial_\mu\tilde{y})/\tilde{x}^2 \\
& -\frac{1}{2\tilde{x}^4}N_G\frac{\ln(2\mu_0^2/\mu^2)}{64\pi^2}[5\partial^\mu\tilde{y}\partial_\mu\tilde{y}\partial^\nu\tilde{x}\partial_\nu\tilde{x} + (\partial^\mu\tilde{y}\partial_\mu\tilde{x})^2 - 6x^2\partial^\mu\tilde{y}\partial^2\partial_\mu\tilde{y}] \\
& +\text{total divergence.}
\end{aligned} \tag{58}$$

We have dropped all total divergences since we did not consistently keep them. The matrix N' is given by N in (46) without the Ω term, and we have explicitly evaluated the purely Υ-dependent terms from (50) and (56).

The result (58), which differs from the canonical calculation of [1] by quite a few quadratically and logarithmically divergent terms, is easily modified for a different gauge group and complex scalars. For the case of effective four dimensional supergravity models from strings, where $x(\phi)$ and $y(\phi)$ are respectively the real and imaginary part of a single dilaton chiral multiplet, the complete results are given in ref. [8].

As mentioned above, while the coefficients of the logarithmically divergent terms are prescription independent, those of the quadratically divergent terms are not. A reliable evaluation of these coefficients requires the introduction of an explicit regularization scheme which is consistent with the symmetries of the theory and which can modify [5,9] the coefficients of some terms that grow quadratically with the regulator mass μ. In this sense, combining our results with those of [1] and [8], we have identified all the ultraviolet divergent terms at one loop in effective bosonic lagrangian of supergravity theories, and determined the coefficients of the logarithmically divergent terms, for a flat space-time background metric. The generalization to a curved background metric is implicit in the combined results of [5], [7], [10] and (for noncanonical gauge kinetic energy) the curvature dependent parts of the operators (27)-(34) that we did not explicitly evaluate here.

Appendix.

Here we collect some notation and results from ref. [1]. The space–time metric $g_{\mu\nu}$ has the flat limit $\eta_{\mu\nu}$ =diag$(-1,1,1,1)$. Z_{ij} is the metric on the scalar manifold; the canonical case corresponds for our real scalars to $Z_{ij}=\delta_{ij}$. We also take $\epsilon^{0123}=1$.

The undefined terms in (43) and (47) are

$$\begin{aligned}
X^{\lambda\sigma,\mu\nu} &= \frac{1}{2}\eta^{\lambda\sigma}\mathcal{D}^{\mu}\tilde{\phi}^{i}\mathcal{D}^{\nu}\tilde{\phi}^{j}Z_{ij}-\eta^{\sigma\nu}\mathcal{D}^{\lambda}\tilde{\phi}^{i}\mathcal{D}^{\mu}\tilde{\phi}^{j}Z_{ij}\\
&\quad+\frac{1}{4}\eta^{\lambda\mu}\eta^{\sigma\nu}\mathcal{D}_{\rho}\tilde{\phi}^{i}\mathcal{D}^{\rho}\tilde{\phi}^{j}Z_{ij}-\frac{1}{8}\eta^{\lambda\sigma}\eta^{\mu\nu}\mathcal{D}_{\rho}\tilde{\phi}^{i}\mathcal{D}^{\rho}\tilde{\phi}^{j}Z_{ij}\\
&\quad-2\kappa V(\tilde{\phi})P^{\lambda\sigma,\mu\nu}+2k\kappa F^{\lambda\sigma,\mu\nu},\\
Y_{k}^{\mu\nu} &= -Z_{km}(d^{\mu})^{m}_{j}\mathcal{D}^{\nu}\tilde{\phi}^{j}+\frac{1}{2}Z_{km}\eta^{\mu\nu}(d^{\lambda})^{m}_{j}\mathcal{D}_{\lambda}\tilde{\phi}^{j}\\
&\quad+\frac{1}{2}\eta^{\mu\nu}\partial_{\tilde{\phi}^{i}}V(\tilde{\phi}),\\
U_{ij} &= -2\kappa\mathcal{D}_{\mu}\tilde{\phi}^{m}Z_{im}\mathcal{D}^{\mu}\tilde{\phi}^{n}Z_{jn},\\
Q^{\lambda\sigma,a}_{\rho} &= -4e\kappa P^{\lambda\sigma,\mu\nu}\eta_{\mu\rho}(T^{a}\tilde{\phi})^{i}(\mathcal{D}_{\nu}\tilde{\phi})^{j}Z_{ij},\\
\tilde{M}^{2}_{ab} &= \tilde{e}^{2}(T_{a}\tilde{\phi})^{j}(T_{b}\tilde{\phi})^{i}Z_{ij},\\
(S_{\mu})^{a}_{i} &= -2[\Gamma_{ijk}(\mathcal{D}_{\mu}\tilde{\phi})^{k}(T^{a}\tilde{\phi})^{j}+(T^{a}\mathcal{D}_{\mu}\tilde{\phi})^{j}Z_{ij}],\\
R_{ij} &= \mathcal{D}^{\mu}\tilde{\phi}^{p}\mathcal{D}_{\mu}\tilde{\phi}^{q}R_{ipqj},\\
V_{ij} &= D_{\tilde{\phi}^{i}}D_{\tilde{\phi}^{j}}V(\tilde{\phi}),\\
(U_{gh})_{\mu\nu} &= -2\kappa\mathcal{D}_{\mu}\tilde{\phi}^{i}\mathcal{D}_{\nu}\tilde{\phi}^{j}Z_{ij},
\end{aligned}\tag{59}$$

where the background scalar curvature and the scalar connection are

$$\Gamma_{mij} = \frac{1}{2}\left[\partial_{\tilde{\phi}^{i}}Z_{mj}+\partial_{\tilde{\phi}^{j}}Z_{mi}-\partial_{\tilde{\phi}^{m}}Z_{ij}\right],$$

$$R^i{}_{pqj} = \partial_{\bar{\phi}^j}\Gamma^i{}_{pq} - \partial_{\bar{\phi}^q}\Gamma^i{}_{jp} + \Gamma^i{}_{lj}\Gamma^l{}_{pq} - \Gamma^i{}_{lq}\Gamma^l{}_{jp}. \tag{60}$$

In all of these, Z_{ij} is taken at its background.

References.

[1] J. W. Burton, M. K. Gaillard, and V. Jain, *Phys. Rev.* **D41**:3118, (1990).

[2] E. Witten, *Phys. Lett.* **155B**: 151 (1985); S. Ferrara, C. Kounnas and M. Porrati, *Phys. Lett.* **181B**: 263 (1986); L.J. Dixon, V.S. Kaplunovsky and J. Louis, *Nucl. Phys.* **B329**: 27 (1990); S. Ferrara, D. Lüst and S. Theisen, *Phys. Lett.* **233B**: 147 (1989).

[3] M. K. Gaillard, *Nucl. Phys.* **B268**:669 (1986).

[4] M. K. Gaillard and V. Jain, *Phys. Rev.* **D39**: 3755 (1989); V. Jain, *Phys. Rev.* **D40**: 1213 (1989).

[5] P. Binétruy and M. K. Gaillard, *Nucl. Phys.* **B312**:341 (1989).

[6] G. K. 't Hooft and M. Veltman, *Ann. Inst. Henri Poincaré* **20**: 69 (1974).

[7] C. Chiou–Lahanas, A. Kapella–Economu, A.B. Lahanas, and X.N. Maintas, *Phys. Rev.* **D42**: 469 (1990).

[8] J. W. Burton, M. K. Gaillard, and V. Jain, "The Effective One–Loop Scalar Lagrangian in No–Scale Supergravity Models II: Axial Couplings, Addenda, and Errata," LBL–31463, UCB–PTH–91/64, MPI–Ph/91–99.

[9] P. Binétruy and M.K. Gaillard, *Phys. Lett.* **220B**: 68 (1989).

[10] C. Chiou-Lahanas, A. Kapella-Economu, A.B. Lahanas and X.N. Maintas, "Gravitational Radiative Corrections in N=1 Supergravity", University of Athens preprint (1991).