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Robust Design, Analysis and Evaluation of Variable Speed Limit Control in a Connected Environment with Uncertainties: Performance Evaluation and Environmental Benefits

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# Robust Design, Analysis and Evaluation of Variable Speed Limit Control in a Connected Environment with Uncertainties: Performance Evaluation and Environmental Benefits

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**16. Abstract**
Connectivity between vehicles and infrastructure allows the efficient flow of information in a dynamic traffic environment. This information can be used to provide recommendations to vehicles in order to alleviate traffic congestion, improve mobility with considerable benefits to the environment. The traffic flow environment however is very complex and involves many uncertainties that include inaccurate measurements, missing data, etc. Any approach to manage or control traffic should be able to handle such uncertainties in a robust way. This project focusses on variable speed limit (VSL) control as an approach to reduce congestion at bottlenecks despite the presence of uncertainties. Numerous research efforts have been made over the years in the field of VSL control in order to resolve bottleneck congestion and improve traffic mobility. Nevertheless, few of them have looked into the issue of robustness with respect to measurement or model uncertainties. In this project, a robust VSL controller is designed based on a modified multi-section cell transmission model (CTM) to alleviate freeway traffic congestion and reject uncertainties. The proposed VSL controller computes the speed limit recommendations using measured flows and densities and communicates them to the upstream vehicles. The optimum location where the speed limit recommendation should be communicated to vehicles is another control variable addressed in the project in order to maximize performance and benefits to the environment. The proposed VSL controller is integrated with ramp metering (RM) controllers and lane change (LC) recommendations to maximize performance. The effectiveness of the integrated control scheme is demonstrated using extensive Monte Carlo microscopic simulations under several traffic demand scenarios and different types and levels of uncertainties. The microscopic simulations are carried out using the commercial traffic software VISSIM. Real data are used to validate the traffic simulator. The benefits in terms of mobility, safety and emissions are quantified.

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# Robust Design, Analysis and Evaluation of Variable Speed Limit Control in a Connected Environment with Uncertainties: Performance Evaluation and Environmental Benefits

A National Center for Sustainable Transportation Research Report

March 2022

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## EXECUTIVE SUMMARY

Connectivity between vehicles and infrastructure allows the efficient flow of information in a dynamic traffic environment. This information can be used to provide recommendations to vehicles in order to alleviate traffic congestion, improve mobility with considerable benefits to the environment. The traffic flow environment however is very complex and involves many uncertainties that include inaccurate measurements, missing data etc. Any approach to manage or control traffic should be able to handle such uncertainties in a robust way. This project focusses on variable speed limit (VSL) control as an approach to reduce congestion at bottlenecks despite the presence of uncertainties.

Numerous research efforts have been made over the years in the field of VSL control in order to resolve bottleneck congestion and improve traffic mobility. Nevertheless, few of them have looked into the issue of robustness with respect to measurement or model uncertainties. In this project, we design a robust VSL controller based on a modified multi-section cell transmission model (CTM) to alleviate freeway traffic congestion and reject uncertainties. The proposed VSL controller computes the speed limit recommendations using measured flows and densities and communicates them to the upstream vehicles. The optimum location where the speed limit recommendation should be communicated to vehicles is another control variable addressed in the project in order to maximize performance and benefits to the environment. The proposed VSL controller is integrated with ramp metering (RM) controllers and lane change (LC) recommendations to maximize performance. The effectiveness of the integrated control scheme is demonstrated using extensive Monte Carlo microscopic simulations under several traffic demand scenarios and different types and levels of uncertainties.

The proposed integrated controller is simulated using a microscopic simulation traffic flow model of the road network that includes a 15-km segment of the I-710 freeway from I-105 to the Long Beach Port in California, United States. The mainline has a fixed lane number of 5 with 5 on ramps and 5 off ramps. The traffic flow simulation model is built using the commercial software VISSIM and is validated in previous studies using actual traffic data from PeMS and involves realistic flows and traffic scenarios. The traffic simulation model is integrated with the emission model MOVES provided by the Environment Protection Agency (EPA) to calculate the average emission rates of vehicles in each microscopic simulation. The integrated controller is evaluated in both high traffic demand and moderate-traffic demand scenarios in terms of the average travel time (ATT), the average number of stops, the emission rates of $CO_2$ and the relative root mean square error (RRMSE) with respect to the desired equilibrium. Ten

![NCST logo]

independent Monte-Carlo simulations for each scenario are carried out and results are averaged in order to reduce randomness and improve reliability.

The proposed integrated controller reduces the average number of stops by 70% and the emission rates of $CO_2$ by 8% compared to the corresponding scenarios with no control.

The proposed integrated controller can tolerate 20% of uncertainties in measured mainline flows and densities without losing convergence. We found out that slowing down the traffic excessively produces worse performance than speeding it up due to the extra shockwaves created. We found that the performance of the closed-loop system is more sensitive to the uncertainties in measured mainline flows and densities, and less sensitive to the uncertainties in measured ramp flows and model parameters.

## Introduction

Freeway bottlenecks created by incidents, constructions, road merging, slow vehicles, etc., trigger severe traffic congestion when the traffic demand is higher than the bottleneck capacity [1], which occurs frequently in urban areas where the transportation activities are heavy. To prevent or alleviate bottleneck congestion, various traffic management strategies such as variable speed limit (VSL), ramp metering (RM), lane change (LC), dynamic routing, have been explored and tested in past decades [2-5].

VSL control regulates the mainstream traffic flow by adjusting the speed limit signs displayed along the freeway. It is the most widely used traffic flow control method due to its easy implementation and effectiveness in improving traffic mobility and safety [6]. Many existing VSL algorithms use an optimization approach referred to as model predictive control (MPC) to compute the VSL commands that return the optimal objective function typically consisting of total travel time (TTT), safety measurements, emission and fuel consumption [7-10]. These algorithms do not guarantee the convergence of the closed-loop system and require long-time computations, which may not suit real-time implementations. Feedback-based VSL controllers are capable of delivering close performance in terms of traffic mobility and safety compared to optimization-based controllers with much less computational effort [11-13]. However, most of the feedback-based controllers require accurate measurements of traffic states which is a limitation in practice.

Although most aforementioned studies showed remarkable achievements in reducing traffic congestion using VSL control in macroscopic model simulations, inconsistent improvements have been observed in various traffic scenarios in microscopic model simulations and field tests [14-16]. The primary cause for the inconsistency is the capacity drop phenomenon at the bottleneck, which is not captured by macroscopic models but exerts great influence on microscopic simulations according to [17]. The authors of [17] developed a LC controller to reduce the forced lane change maneuvers and the capacity drop by providing lane change recommendations to vehicles upstream of the incident location. The shockwave generated by slowing down the traffic with VSL commands may also deteriorate the traffic mobility . The performance is even worse when the VSL commands are reduced excessively due to inaccurate measurements [18]. Hegyi et al. proposed a VSL algorithm to detect and suppress the shockwave [19], but it is difficult to extend the algorithm to resolve bottleneck congestion in multi-section road networks.

Another important factor in VSL control is the effect of VSL sign locations. Most researchers choose VSL sign locations empirically or based on the road configuration, leading to non-optimal control performance. In [20], Seraj et al. suggested that the optimal VSL sign location can be determined by the space required for the traffic to reach the bottleneck capacity. In [21], Xu et al. placed the VSL signs based on the collision risk at freeway recurrent bottlenecks. Although these studies provided some insight on the 'optimal' VSL sign location, there is no rigorous analysis that explains the impact of the VSL sign location on the performance of the VSL control. Exception is the work by Martinez and Jin [22], where the authors treated the distance of the discharging zone as a control variable and optimized it based on the bottleneck

capacity. The result indicates that higher speed limit commands lead to larger optimal discharging distance. To the best of our knowledge, the optimal distance of the upstream VSL zone and its impact on the closed-loop performance is still an open topic.

The combination of VSL control with other traffic regulation methods such as ramp metering (RM) and lane change (LC) has been proven to be more beneficial than implementing VSL alone in various scenarios [2, 17, 23, 24]. Many integrated approaches use optimal control or MPC to coordinate different controllers and optimize the cost function [2, 23, 25]. In [26], Zhang et al. demonstrated that a feedback-based scheme performs no worse than MPC with less computational efforts in terms of combining VSL with LC. In [27], Frejo and De Schutter proposed a logic-based scheme that determines the flow rates of VSL and RM by estimating the number of vehicles to be held or released in order to match the bottleneck flow with the capacity, which also revealed the fact that a well-tuned easy-to-implement integrated controller delivers similar performance compared with an optimal controller, and thus, more suitable for real-world implementations.

In this project, we propose an integrated VSL, LC and RM controller based on a modified multi-section cell transmission model (CTM) with the purpose of alleviating freeway bottleneck congestion created by lane drop and rejecting uncertainties in measured traffic states and model parameters. The VSL and RM control are coordinated in a feedback structure to ensure fast computations. The section length and lower bounds of VSL commands are modified from classical VSL control to minimize the speed difference and the consequential shockwave. We perform a comprehensive stability analysis to prove the convergence of the closed-loop system under different traffic demands, and then verify it by microscopic simulations using VISSIM. We also incorporate various types and levels of uncertainties to examine the robustness of the proposed controller.

## System Modeling

### Multi-section Cell Transmission Model with Ramps

The Cell Transmission Model (CTM) is a macroscopic first-order traffic flow model proposed by Daganzo [28]. It is a discrete approximation of the Lighthill-Whitham-Richards (LWR) model [29, 30] with high computational efficiency and reasonable accuracy in describing traffic dynamics. In the CTM framework, a freeway segment is partitioned into $N$ homogeneous sections/cells and consecutively numbered from 1 to $N$ in the traffic flow direction, as shown in Figure 1. Each section/cell is characterized by the vehicle density, mainstream inflow, mainstream outflow, on-ramp inflow, off-ramp outflow and length, denoted as $\rho_i, q_i, q_{i+1}, r_i, s_i, L_i$ respectively, where $i = 1, 2, \ldots, N$. The density is updated using a first-order ordinary differential equation based on the conservation law of traffic flow, where the mainstream inflow and outflow are determined by the supply (or receiving) and demand (or sending) functions, which define a flow-density relationship known as the fundamental diagram [31].

**Figure 1. CTM Road Network**

Although the original form of CTM can reproduce the traffic dynamics under both uncongested and congested conditions, it does not capture the capacity drop and bounded acceleration effects due to forced lane change maneuvers at the congested freeway bottleneck or ramp merging area [32, 33]. Nor does it consider the uncertainties in measured traffic states and model parameters. Therefore, the original CTM has been modified over the years in order to be consistent with the microscopic traffic dynamics [13, 17, 34, 35].

In this project, we adopt the most updated multi-section CTM that takes into account the effect of both capacity drop and bounded acceleration [1]. Moreover, the uncertainties in measurements and parameters are represented as an additional disturbance term $\mu_i$ in the conservation law of traffic flow [36]. Without loss of generality, it is assumed that the geometry of all the sections is identical. Accordingly, the evolution of the vehicle density $\rho_i$ in each section is described by the following equations:

$$\dot{\rho}_i = \frac{1}{L_i}(q_i - q_{i+1} + r_i - s_i) + \mu_i \qquad \text{for } i = 1, \dots, N \qquad (1)$$

where

$$q_1 = \min\{d, C, w(\rho^j - \rho_1)\}$$

$$q_i = \min\{v_f \rho_{i-1}, \tilde{w}(\tilde{\rho}^j - \rho_{i-1}), C, w(\rho^j - \rho_i)\} \text{ for } i = 2, \dots, N$$

$$q_{N+1} = \begin{cases} \min\{v_f \rho_N, \tilde{w}(\tilde{\rho}^j - \rho_N), (1 - \epsilon(\rho_N))C_d\}, & \text{if } C_d < C \\ \min\{v_f \rho_N, \tilde{w}(\tilde{\rho}^j - \rho_N), C_d\}, & \text{otherwise} \end{cases}$$

and                                                                                          (2)

$$\epsilon(\rho_N) = \begin{cases} 0, & \text{if } 0 \leq \rho_N \leq \frac{C_d}{v_f} \\ \epsilon_0, & \text{otherwise} \end{cases}$$

The parameters in (1) and (2) are defined in Table 1.

**Table 1. Definition of the Model Parameters**

| Symbol | Definition | Unit |
|---|---|---|
| $d$ | the demand of the mainstream traffic | veh/h |
| $C$ | the capacity of each section/cell | veh/h |
| $C_d$ | the downstream capacity | veh/h |
| $v_f$ | the free flow speed | km/h |
| $w$ | the back propagation speed | km/h |
| $\widetilde{w}$ | the rate that the outflow $q_{i+1}$ decreases with density $\rho_i$ when $\rho_i \geq \rho_c$ | km/h |
| $\rho_c$ | the critical density of the section/cell at which $v_f \rho_c = \widetilde{w}(\tilde{\rho}^j - \rho_c) = w(\rho^j - \rho_c) = C$ | veh/km |
| $\rho^j$ | the jam density; the highest possible density at which the inflow $q_i = 0$ | veh/km |
| $\tilde{\rho}^j$ | the jam density associated with outflow $q_{i+1}$ | veh/km |
| $L_i$ | the length of each section/cell | km |
| $\epsilon_0$ | the capacity drop factor, where $\epsilon_0 \in (0,1)$ | unitless |

# Control Design

This section aims to develop an integrated VSL, LC and RM controller based on the multi-section CTM presented above so that the traffic conditions of all the mainstream sections operate within the free-flow region in the fundamental diagram, despite the activation of the downstream bottleneck. The VSL controller regulates the mainstream inflow of each section so that the density converges to the desired value and the potentially existing uncertainties are rejected. Lane change recommendations are provided for vehicles approaching the bottleneck to manage the forced lane change maneuvers and increase the bottleneck throughput. In addition, the RM control is applied to prevent mainstream traffic from being disturbed by large ramp input when the on-ramp queue is not saturated.

## Robust Variable Speed Limit Control

We propose a feedback-based VSL controller with the purpose of rejecting the disturbance $\mu_i$ in (1) and making the density of each section $\rho_i$ converge to a predefined value, denoted as $\rho^*$. In the ideal case where $\mu_i = 0$, a trivial choice is to let $\rho^* = C_d/v_f$, which corresponds to the highest possible flow-rate through the bottleneck. However, a small disturbance may drive the density towards the capacity-drop region, which introduces unwanted oscillatory behavior of the closed-loop system and negatively impacts convergence [36]. On one hand, the value of $\rho^*$ needs to be compromised for the sake of robustness, i.e., $\rho^* < C_d/v_f$ on the other hand, it should be chosen without losing potential road capacity excessively.
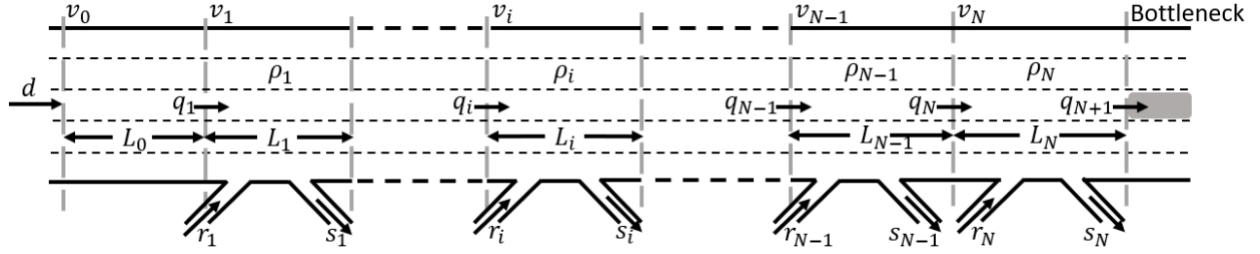
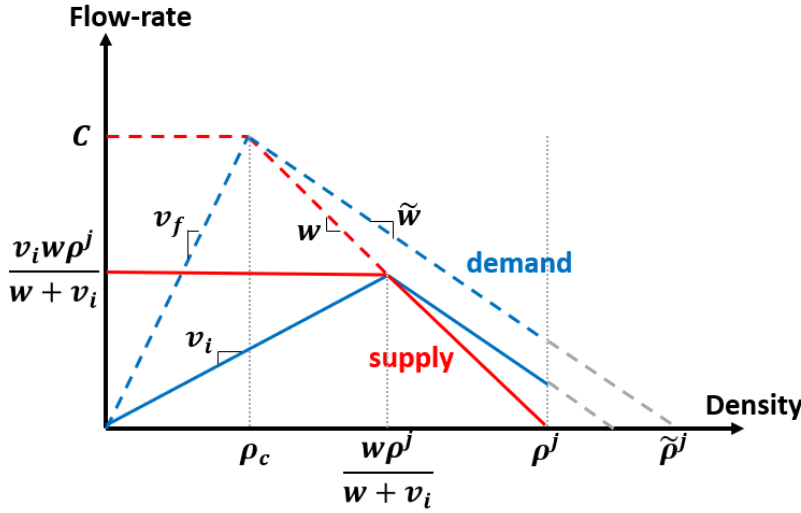**Figure 2. Road Network with VSL Control**



**Figure 3. Fundamental Diagram with VSL Control**

Figure 2 presents the road network configuration after incorporating the VSL control. Each VSL command takes effect at the beginning of the section. According to the geometry of the fundamental diagram shown in Figure 3, the maximum possible flow governed by the speed of flow $v_i$ is $\frac{v_i w \rho^j}{v_i + w}$. Therefore, the dynamics of the traffic flows when the VSL control is activated are described as follows:

$$q_1 = \min\{d, \frac{v_0 w \rho^j}{v_0 + w}, \frac{v_1 w \rho^j}{v_1 + w}, w(\rho^j - \rho_1)\}$$

$$q_i = \min\{v_{i-1}\rho_{i-1}, \frac{v_{i-1} w \rho^j}{v_{i-1} + w}, \frac{v_i w \rho^j}{v_i + w}, w(\rho^j - \rho_i)\} \text{ for } i = 2, \dots, N \tag{3}$$

$$q_{N+1} = \min\{v_N \rho_N, \widetilde{w}(\tilde{\rho}^j - \rho_N), (1 - \epsilon(\rho_N))C_d\}$$

Since the mainstream demand $d$ is the major input of the road network in normal circumstances, most control efforts should be distributed into the upstream VSL section to ensure that $q_1$ is within the bottleneck capacity, and the task for the remaining downstream sections is to maintain a steady traffic flow with the assistance from the RM control. The above strategy minimizes the speed variations between consecutive sections and diminishes the stop-

and-go traffic behavior [37]. Driven by this idea, the VSL commands for each section can be computed as follows:

$$v_o = \frac{wq_{1v}}{w\rho^j - q_{1v}}$$

$$v_{i-1} = \frac{q_{iv}}{\rho_i} \qquad \text{for } i = 2, \dots, N \tag{4}$$

$$v_N = v_f$$

where $q_{iv}$ is the desired mainstream inflow of section $i$. Assume that the disturbance $\mu_i$ in (1) is bounded by a constant $\mu_m$ and satisfies $|\mu_i| \le \mu_m \ll C_d$. In order to reject $\mu_i$ and guarantee the convergence of the closed-loop system, we compute $q_{iv}$ using the following proportional-integral (PI) controller equation [13, 36]:

$$q_{iv} = q_{i+1} + s_i - r_i - \lambda_1(\rho_i - \rho^*) - \lambda_2(\int_{t_0}^{t}(\rho_i - \rho^*)d\tau - \frac{\lambda_1(\rho_i(t_0) - \rho^*) - \mu_m}{\lambda_2}) \tag{5}$$

where $q_{i+1}, \rho_i, s_i, r_i$ are measured traffic states subject to uncertainties, $\lambda_1 > 0$ and $\lambda_2 > 0$ are the proportional and integral gains, respectively. These parameters are initialized with the empirical values from [13] and tuned based on simulation results. $t_0$ denotes the time when the controller is activated.

To ensure safety and feasibility in real world, we also incorporate the following constraints on the speed limit computations:

- $v_i$ is rounded to be a multiple of 10 km/h.
- The bounds of $v_0$: 20 km/h $\le v_0 \le$ 100 km/h.
- The bounds of $v_i$ for $i = 1, \dots, N$: 70 km/h $\le v_i \le$ 100 km/h.
- $v_i$ can be increased or decreased by at most 10 km/h in each control cycle.

Note that we raise the minimum value for all downstream VSL commands ($v_i$ where $i = 1, \dots, N$) to 70 km/h to avoid excessive speed reduction caused by the disturbance $\mu_i$, which creates extra shockwave and deteriorates the control performance. Since the mainstream flow $q_1$ is regulated by $v_0$ and all ramp inputs are controlled by the RM, it is unnecessary to regulate the mainstream flow again with low speed limits in the downstream sections.

### Length of The Upstream VSL Zone

The length of the most upstream VSL zone, denoted as $L_0$, is an important control variable that strongly affects the VSL performance and needs to be carefully determined based on initial traffic states and model parameters. Since we concentrate the control efforts in section 0, $v_0 < v_i$ for $i = 1, \dots, N$ is true in general. This speed difference produces a low-density area within the road network that moves along with the traffic, which can be verified in Figure 4. Note that the vehicle input never drops down, but the inflow drops in all downstream sections as a result of the VSL control.
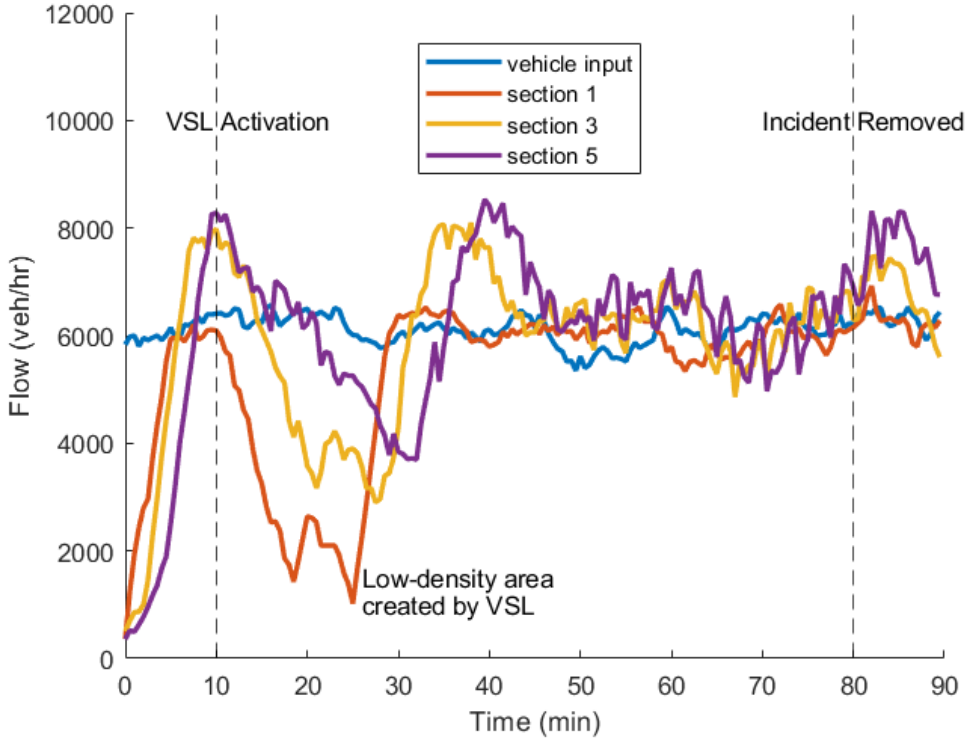
**Figure 4. Flow Curves with VSL Control**

The low-density area must be long enough so that the bottleneck congestion created by the initial traffic within the road network is completely resolved before the newly-entered traffic reaches the bottleneck, which is necessary for the closed-loop system to converge. Otherwise, the bottleneck congestion will be extended by the newly-entered traffic and the capacity drop continues to exist. Since the length of the low-density area is positively correlated with $L_0$, $L_0$ needs to be long enough to guarantee the convergence, but overextending $L_0$ may lead to the underutilization of the road capacity and may increase the average travel time. In this project, we select $L_0$ based on the following theorem:

*Theorem 1*. Consider the freeway bottleneck control problem with VSL commands given by (4)> The propagation of traffic congestion at the bottleneck can be completely absorbed by the low-density area created by the VSL control if the upstream VSL zone distance $L_0$ satisfies

$$L_0 > \frac{(Q_r + v_f \rho_d(t_0) - (1-\epsilon_0)C_d)v_{0,min}L_d}{((1-\epsilon_0)C_d - Q_r - v_{0,min}\rho_0(t_0))v_f} \tag{6}$$

where $\rho_i(t_0)$ ($i = 0, \dots, N$) is the initial density of each section; $L_i$ ($i = 0, \dots, N$) is the length of each section; $L_d = \sum_{i=1}^{N} L_i$ is the total length from section 1 to $N$; $\rho_d(t_0) = \sum_{i=1}^{N} L_i \rho_i(t_0) / L_d$ is the initial average density from section 1 to $N$; $C_d$ is the downstream capacity; $v_f$ is the free flow speed; $v_{0,min}$ is the minimum value of the upstream VSL command; $\epsilon_0$ is the capacity drop factor; $Q_r$ is the average net inflow from all ramps.

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Note that (6) is derived from a chasing problem in which the time it takes to evacuate the initial traffic plus on-ramp inputs through the bottleneck and off-ramps is strictly less than the time spent for the newly-entered traffic to reach the bottleneck. A detailed proof of no-ramp scenarios is provided in [37]. Theorem 1 extends the result by including the ramp flows.

## Lane Change Control

In order to relieve the capacity drop caused by forced lane change maneuvers and increase the throughput at the bottleneck, a Lane Change (LC) control is implemented in the discharging section [17] as shown in Figure 5.
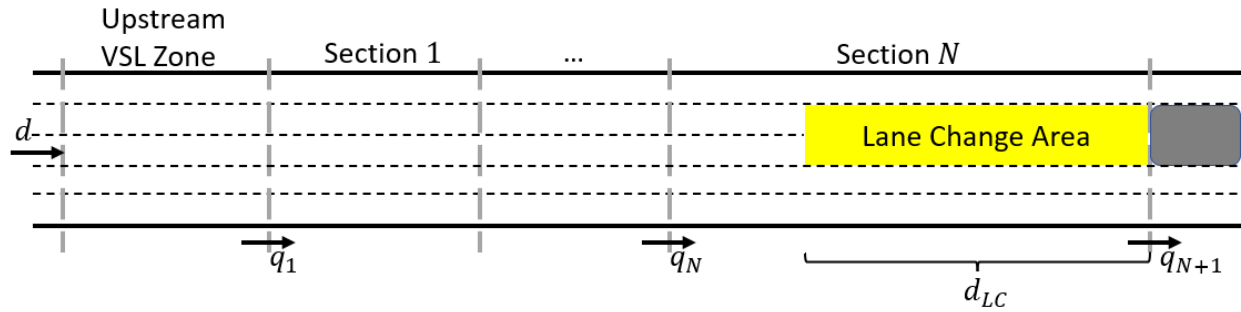


**Figure 5. Lane Change Control**

The controller provides LC recommendations to vehicles moving in the closed lane(s) before approaching the bottleneck. The distance from the bottleneck to activate the LC control, denoted as $d_{LC}$, is a key variable that needs to be determined properly. $d_{LC}$ must be long enough so that the vehicles can safely complete the lane change maneuvers, but overextending it may lead to the underutilization of the road capacity. In this project, $d_{LC}$ is determined by the empirical formula proposed in [17]:

$$d_{LC} = \xi \cdot n \tag{7}$$

where $n$ is the number of lanes closed at the bottleneck, $\xi$ is a design parameter that depends on the traffic demand and is estimated using microscopic simulations. For the specific road network presented in this report, the relationship between $\xi$ and traffic demands is shown in Figure 6.
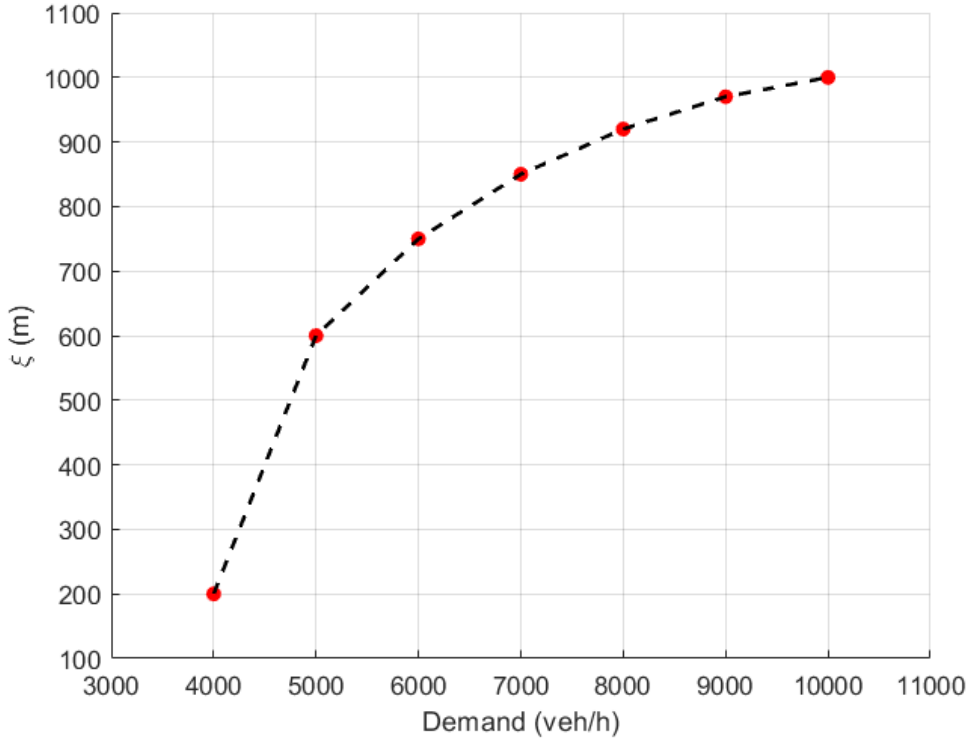
**Figure 6. Relationship between ξ and Traffic Demands**

## Ramp Metering

As mentioned earlier, the success of VSL control in regulating the traffic flow relies on a steady ramp input within the mainstream receiving ability, which can be achieved by the Ramp Metering (RM) control. While controlling the ramp inflow, we want to ensure that the ramp queue does not exceed the length of the ramp. Therefore, the RM controller needs to maintain a good balance between the mainstream traffic and the ramp queue. Driven by this idea, we adopt the ALINEA/Q algorithm proposed in [38], which is modified from the classic ALINEA algorithm by considering the ramp queue capacity. Note that the original ALINEA/Q contains both the occupancy and the queue length in the feedback loop. In this project, we use the density of the mainstream section instead of the occupancy in order to be consistent with the mechanism of the VSL control [24].

Two ramp flow rates, $r_i^d(k)$ and $r_i^q(k)$, are computed respectively based on the mainstream density $\rho_i(k)$ and the ramp queue length $w_i(k)$ at each time step $k$ for the on-ramp $i$. The final ramp flow rate $r_i(k)$ is the maximum of the two, i.e.,

$$r_i^d(k) = r_i(k-1) + \beta_d(\rho^* - \rho_i(k))$$

$$r_i^q(k) = \beta_q\big(w_i^r - w_i(k)\big) + d_i(k-1) \tag{8}$$

$$r_i(k) = \max\{r_i^d(k), r_i^q(k)\}$$

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where $d_i(k-1)$ is the demand from ramp $i$ within the previous time step, $w_i^r$ is the reference queue capacity of ramp $i$, $\rho^*$ is the desired density, $\beta_d$ and $\beta_q$ are the feedback gains of the density and queue length respectively.

## Numerical Simulations

This section presents the results of microscopic simulations under different traffic demands, control schemes and uncertainties using the commercial software PTV VISSIM 10. We repeat each scenario 10 times with 10 distinct random seeds respectively and then take the average results to increase the reliability.

## Road Network and Parameter Selection

The road network shown in Figure 7 is a 16-km segment of the I-710 freeway (between I-105 junction and Long Beach Port) in California, United States. The space reserved for the most upstream VSL zone is 4 km, which means that $L_0$ is adjustable with a maximum value of 4 km. The remaining 6 downstream sections have a length of 2 km, and each of them is connected with one on-ramp or one off-ramp or both. The number of lanes is fixed to be 5 for all sections including the upstream. The simulation lasts for 90 min. An incident occurs and leads to the closure of either 1 or 2 lanes at the downstream exit of the road network at time equal to 10 min. The incident is cleared at time equal to 80 min.
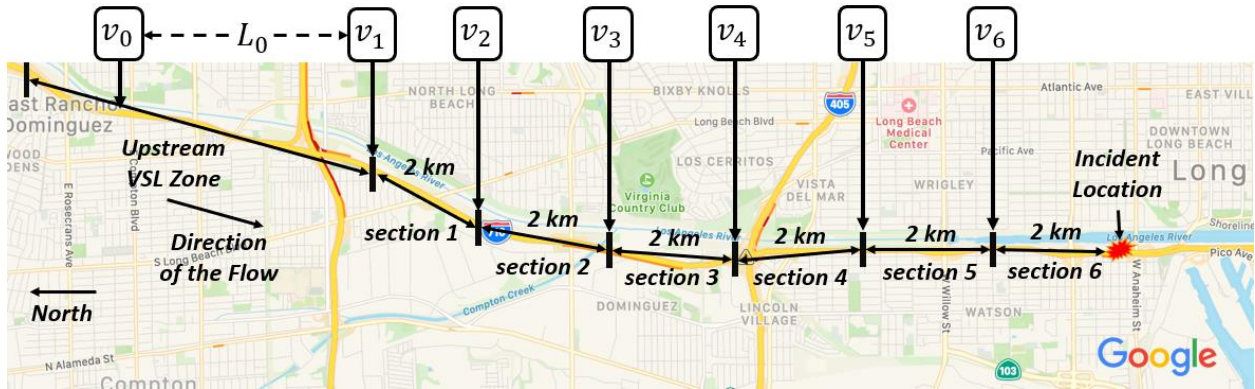


**Figure 7. I-710 Simulation Network**

To determine model parameters of the given road network, we run multiple open-loop scenarios with both one-lane-closure and two-lane-closure under a demand that gradually increases. With the collected flow and density measurements for all sections, the following parameters are obtained: the road capacity $C = 12000$ veh/h, the bottleneck capacity $C_d = \frac{5-n}{5} C$ where $n \in \{1, 2\}$ is the number of closed lane(s). The capacity drop factor $\epsilon_0 = 0.1$. The free-flow speed $v_f$ is set to be 100 km/h. The backpropagation speeds are chosen from the empirical values proposed in [36]: $w = 30$ km/h and $\tilde{w} = 15$ km/h. Using the geometry in Figure 3, we have $\rho^j = C/v_f + C/w = 520$ veh/km and $\tilde{\rho}^j = C/v_f + C/\tilde{w} = 920$ veh/km.

## Performance Measurements

The following criteria are used to evaluate the performance of the proposed controller [17]:

- Average Travel Time (ATT): the average time spent for each vehicle to travel through the whole network.

$$ATT = \frac{1}{N_v}\sum_{i=1}^{N_v}(t_{i,out} - t_{i,in}) \tag{9}$$

where $N_v$ is the number of vehicles passing through the network, $t_{i,in}$ and $t_{i,out}$ is the time vehicle $i$ enters and exits the network respectively.

- Average number of stops: the average number of stops performed by each vehicle when traveling in the network.

$$\bar{s} = \frac{1}{N_v}\sum_{i=1}^{N_v} s_i \tag{10}$$

where $s_i$ is the number of stops performed by vehicle $i$.

- Average emission rates of $CO_2$: the calculation of emission rates is based on the MOVES model provided by the Environment Protection Agency (EPA) [39].

$$\bar{R} = \sum_{i=1}^{N_v} E_i \,/\, \sum_{i=1}^{N_v} d_i \tag{11}$$

where $E_i$ is the emission produced by vehicle $i$ and $d_i$ is the travelled distance of vehicle $i$.

- The relative root mean square error (RRMSE): we compare the average density measurements of each downstream sections with the desired equilibrium and compute the RRMSE to indicate whether the convergence is achieved for the closed-loop system.

$$e_\rho = \frac{1}{\rho^*}\sqrt{\frac{1}{t_e-t_s}\int_{t_s}^{t_e}(\bar{\rho}(\tau) - \rho^*)^2} \tag{12}$$

where $\rho^*$ is the desired density, $t_s$ is the time when bottleneck congestion created by the initial traffic is cleared, $t_e$ is the time when the incident is removed, $\bar{\rho}$ is the average density measurement.

## Two-lane Closure with Moderate Demand

In this subsection, we present the simulation results under a two-lane closure scenario with moderate traffic demands, i.e., the mainstream demand $d = 6000$ veh/h and the ramp demands $= [800, 800, 800, 300, 300]$ veh/h from upstream to downstream. Since the overall demand exceeds the downstream capacity $C_d = 7200$ veh/h, $\rho^* < C_d/v_f = 72$ veh/km, according to (6), $L_0 > 3.2$ km. Therefore, we select $\rho^* = 68$ veh/km and $L_0 = 4$ km to accommodate for potentially existing uncertainties. We first compare different control schemes including the open-loop control to demonstrate the effectiveness of the proposed integrated controller, and then incorporate various types of uncertainties in different levels to test the robustness of the proposed controller. All the evaluation results are shown in Table 2. Note that $\sigma$ denotes the types and levels of uncertainties. For example, $\sigma_q = -0.1$ indicates that the flow

measurement is corrupted by a factor of $1 + \sigma_q = 0.9$. $\sigma_{qr} = 0.2$ indicates that the ramp flow measurement is corrupted by a factor of $1 + \sigma_{qr} = 1.2$.

**Table 2. Evaluations of Two-lane Closure with Moderate Demand**

| Scenarios | ATT (min) | $\bar{s}$ | $CO_2$ (g/veh/km) | $e_\rho$ |
|---|---|---|---|---|
| No Control | 15.8 | 40.2 | 271.6 | 213.1% |
| VSL+LC (No RM) | 16.3 | 15.4 | 254.0 | 136.8% |
| Proposed | 17.3 | 12.4 | 249.0 | 36.8% |
| $\sigma_q = -0.1$ | 17.4 | 11.5 | 244.8 | 37.7% |
| $\sigma_q = -0.2$ | 17.8 | 12.7 | 240.6 | 40.9% |
| $\sigma_q = -0.3$ | 17.8 | 11.9 | 238.8 | 40.2% |
| $\sigma_q = 0.1$ | 16.9 | 13.4 | 250.8 | 34.8% |
| $\sigma_q = 0.2$ | 16.6 | 16.2 | 256.7 | 37.7% |
| $\sigma_q = 0.3$ | 16.2 | 17.7 | 259.1 | 53.7% |
| $\sigma_\rho = -0.1$ | 16.9 | 14.6 | 253.2 | 34.4% |
| $\sigma_\rho = -0.2$ | 16.6 | 15.5 | 256.1 | 43.1% |
| $\sigma_\rho = -0.3$ | 16.3 | 17.8 | 258.8 | 58.7% |
| $\sigma_\rho = 0.1$ | 17.7 | 12.4 | 243.7 | 39.2% |
| $\sigma_\rho = 0.2$ | 17.7 | 12.4 | 240.4 | 40.6% |
| $\sigma_\rho = 0.3$ | 17.8 | 12.2 | 238.5 | 39.2% |
| $\sigma_{qr} = -0.1$ | 17.3 | 12.5 | 248.4 | 37.5% |
| $\sigma_{qr} = -0.2$ | 17.1 | 12.0 | 248.0 | 35.4% |
| $\sigma_{qr} = -0.3$ | 17.1 | 11.5 | 247.3 | 33.8% |
| $\sigma_{qr} = 0.1$ | 17.3 | 12.7 | 248.1 | 35.3% |
| $\sigma_{qr} = 0.2$ | 17.1 | 12.1 | 248.9 | 33.7% |
| $\sigma_{qr} = 0.3$ | 17.2 | 11.6 | 246.8 | 37.1% |
| $\sigma_w = -0.1$ | 17.3 | 13.1 | 248.7 | 34.9% |
| $\sigma_w = -0.2$ | 17.2 | 11.8 | 247.4 | 33.6% |
| $\sigma_w = -0.3$ | 17.2 | 11.5 | 246.2 | 34.4% |
| $\sigma_w = 0.1$ | 17.1 | 11.5 | 247.7 | 36.1% |
| $\sigma_w = 0.2$ | 17.1 | 11.8 | 247.8 | 36.6% |
| $\sigma_w = 0.3$ | 17.0 | 12.1 | 248.1 | 35.4% |

From Table 2, we observe that the proposed integrated controller improves the average number of stops and the emission of $CO_2$ significantly compared with the open-loop case. The performance of combined VSL and LC controller cannot catch up with the proposed controller without the assistance of RM. The robustness of the closed-loop system is satisfactory. It is capable of tolerating up to 20% of uncertainties in sensitive measurements such as flows and densities without losing convergence. The tolerance is even higher for less sensitive quantities such as the ramp flow measurement and the model parameter $w$. Note that the convergence deteriorates as we increase $\sigma_q$ from 0.2 to 0.3 or decrease $\sigma_\rho$ from -0.2 to -0.3. In such situation

the corrupted measurements are way off for feedback to be effective. The significant of this result is that it gives an indication how accurate traffic measurement sensors should be.

## One-lane Closure with High Demand

In this subsection, we repeat the microscopic simulations under a one-lane closure scenario with high traffic demand, i.e., the mainstream demand $d = 7500$ veh/h and ramp demands $= [900, 900, 900, 400, 400]$ veh/h from upstream to downstream. Since the overall demand exceeds the downstream capacity $C_d = 9600$ veh/h, $\rho^* < C_d/v_f = 96$ veh/km according to (6), $L_0 > 3.0$ km. Therefore, we select $\rho^* = 92$ veh/km and $L_0 = 4$ km to accommodate for potentially existing uncertainties. We present all the evaluation results in Table 3 in the same manner as Table 2.

From Table 3, we can still observe significant benefits in the average number of stops and the emission of $CO_2$ by applying the proposed controller, while by removing RM lowers the overall performance. The closed-loop system also achieves the same level of robustness as the previous scenario.

**Table 3. Evaluations of One-lane Closure with High Demand**

| Scenarios | ATT (min) | $\bar{s}$ | $CO_2$ (g/veh/km) | $e_\rho$ |
|---|---|---|---|---|
| No Control | 13.6 | 12.9 | 268.4 | 130.6% |
| VSL+LC (No RM) | 15.8 | 3.4 | 254.4 | 69.4% |
| Proposed | 16.3 | 4.3 | 248.3 | 7.1% |
| $\sigma_q = -0.1$ | 16.7 | 4.2 | 240.4 | 6.1% |
| $\sigma_q = -0.2$ | 16.8 | 4.1 | 232.6 | 7.0% |
| $\sigma_q = -0.3$ | 17.0 | 4.2 | 231.5 | 7.4% |
| $\sigma_q = 0.1$ | 15.8 | 6.4 | 254.3 | 9.2% |
| $\sigma_q = 0.2$ | 15.5 | 9.8 | 257.9 | 17.8% |
| $\sigma_q = 0.3$ | 15.1 | 9.0 | 265.4 | 42.2% |
| $\sigma_\rho = -0.1$ | 15.9 | 5.0 | 254.1 | 9.0% |
| $\sigma_\rho = -0.2$ | 15.4 | 6.8 | 252.6 | 13.9% |
| $\sigma_\rho = -0.3$ | 15.2 | 8.4 | 256.2 | 11.8% |
| $\sigma_\rho = 0.1$ | 16.6 | 4.3 | 240.1 | 6.2% |
| $\sigma_\rho = 0.2$ | 16.9 | 4.3 | 234.2 | 6.5% |
| $\sigma_\rho = 0.3$ | 16.9 | 4.6 | 233.9 | 6.5% |
| $\sigma_{qr} = -0.1$ | 16.2 | 4.4 | 250.2 | 6.9% |
| $\sigma_{qr} = -0.2$ | 16.2 | 4.5 | 249.5 | 7.2% |
| $\sigma_{qr} = -0.3$ | 16.2 | 4.7 | 250.2 | 6.8% |
| $\sigma_{qr} = 0.1$ | 16.2 | 4.4 | 248.1 | 7.5% |
| $\sigma_{qr} = 0.2$ | 16.4 | 4.4 | 247.1 | 6.7% |
| $\sigma_{qr} = 0.3$ | 16.3 | 4.4 | 246.8 | 7.4% |
| $\sigma_w = -0.1$ | 16.1 | 4.1 | 248.3 | 7.2% |
| $\sigma_w = -0.2$ | 16.2 | 4.2 | 246.9 | 7.1% |
| $\sigma_w = -0.3$ | 16.2 | 4.2 | 247.8 | 6.7% |
| $\sigma_w = 0.1$ | 16.2 | 4.5 | 247.2 | 6.6% |
| $\sigma_w = 0.2$ | 16.2 | 5.2 | 249,3 | 7.0% |
| $\sigma_w = 0.3$ | 16.2 | 5.8 | 251.7 | 8.7% |

## Conclusion

A robust integrated VSL, RM and LC controller to alleviate freeway bottleneck congestion caused by lane drop is proposed. A modified multi-section CTM is used to describe the traffic behavior and capture more complex traffic flow phenomena, such as the capacity drop and bounded acceleration. The VSL commands are computed based on flow and density measurements in a feedback manner. Most control efforts are concentrated on the most upstream VSL section to match the inflow of the road network with the bottleneck capacity in order to minimize downstream speed variations and suppress shockwave. The length of the most upstream VSL section is treated as a control variable and its lower bound is derived as a necessary condition to achieve convergence. Feasibility constraints are applied to ensure safety and enhance robustness. The LC controller is implemented in order to reduce the capacity drop.

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The RM is used for each on-ramp to restrict the ramp input when the ramp queue capacity is available. The integrated controller shows significant improvements in traffic mobility, safety and environmental impact, as demonstrated by microscopic simulations of an actual freeway network in Southern California. The closed-loop system is able to tolerate up to 20% uncertainties in sensitive measurements such as mainstream flows and densities.

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## Data Summary

### Products of Research

We collected all the vehicle records, densities, flows, VSL commands of each microscopic simulation. Then we evaluated the ATT, the average number of stops and the average emission rates of $CO_2$ using the vehicle records. We also computed the RRMSE with respect to the desired density using the density measurements.

### Data Format and Content

We compressed all the data into zip files with a common name "RVSL_Project_Data". Each vehicle record is a fzp file generated by VISSIM that contains all vehicle information in the specific simulation run. Since we ran each scenario 10 times, there are 10 fzp files for each scenario. The evaluation results of all the vehicle records plus the average results for each scenario are stored in an xlsx file named "VehRecords_Evals.xlsx". The densities, flows and VSL commands of all simulation runs are stored in two xlsx files named "FlowDenVSL_D750099944_1Lane.xlsx" and "FlowDenVSL_D600088833_2Lanes.xlsx", corresponding to one-lane closure with high demand and two-lane closure with moderate demand respectively.

In addition, we prepared another zip file containing the VISSIM model and Python3 scripts for running VISSIM simulations and evaluations.

### Data Access and Sharing

All of the above-mentioned data and files are available via Harvard Dataverse under the name "Replication Data for: Robust Design, Analysis and Evaluation of Variable Speed Limit Control in a Connected Environment with Uncertainties". They can be accessed with the following DOI: https://doi.org/10.7910/DVN/7LKDFV.