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ABSTRACT

A variable-length vacuum-insulated liquid hydrogen transfer line is described. The vacuum system is semipermanent, and segments of the line are assembled with only threaded vacuum fittings. Thermal stress calculations are presented for a statically indeterminate union coupling.

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INTRODUCTION

This paper describes calculations employed in the design of a variable-length liquid hydrogen transfer line. The presentation of calculations in this report is in general form; the original design calculations have been given in an unpublished paper.[1]

DESCRIPTION OF THE TRANSFER LINE

The transfer line is basically of the Johnston type, employing sets of interchangeable vacuum-insulated sections in the form of straight runs, unions, and elbows.[2] The design of the flexible ends for the receiver is similar to the type used at the National Bureau of Standards Cryogenic Engineering Laboratory at Boulder, Colorado.

The line differs from those described above in that sections can be completely assembled with wrenches, and require vacuum pumping only if used or stored for extended periods— not during assembly. The particularly undesirable requirement of soldering sections of the line at assembly in the possible presence of explosive hydrogen gas is eliminated.

Figure 1 is a photograph of the assembled line. Figure 2 is a close-up of the Teflon[3] seal of the straight section and the coupling of the flexible receiver end. The outer jacket of the flexible portion of the line is amber gun rubber, 1 in. o.d. by 1/4 in. wall.

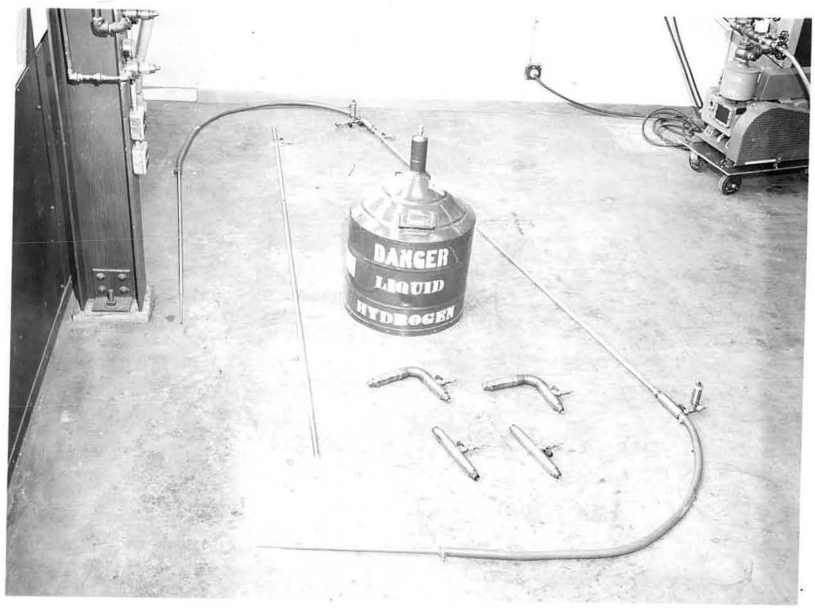


Figure 1. A portion of the assembled line with interchangable fittings.

Figure 2. Close up of Teflon seal on straight section and coupling on flexible receiver end.

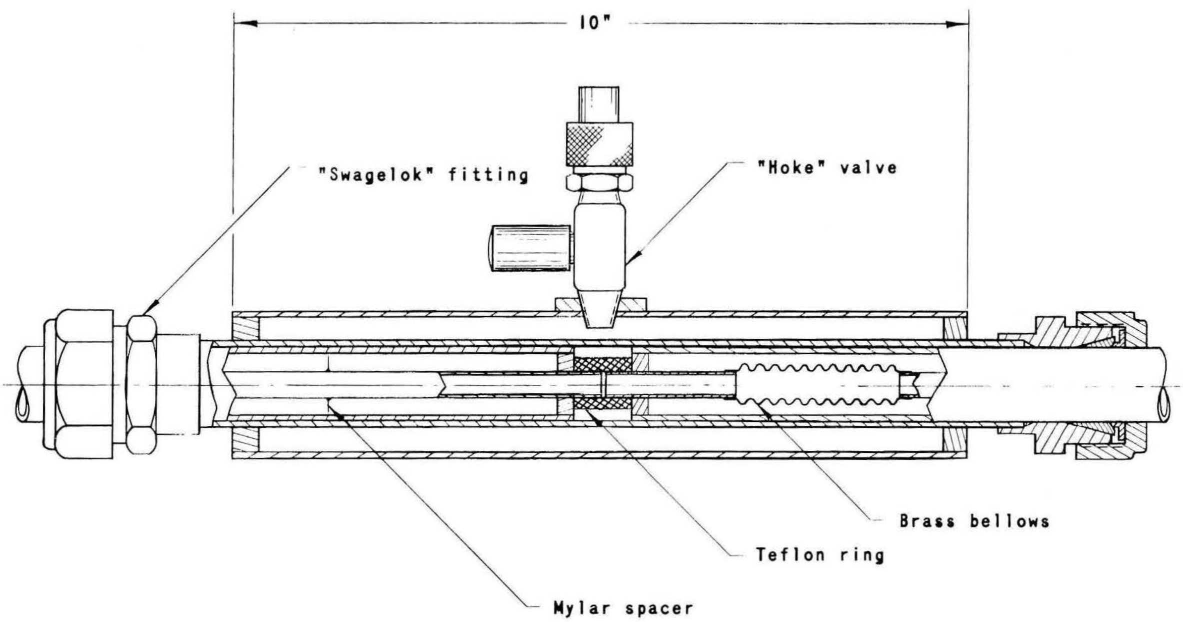


Figure 3. Section through union and 2 straight tube sections.

The thin-wall stainless steel inner tube and outer hose are kept separated by narrow Teflon rings which fit loosely on the inner tube. The spacers are retained in position on the inner tube with a small gobber of soft solder on either side. The spacers for the concentric tubes in the 10-ft straight sections are 0.01-in.-thick Mylar[4] disks cemented sparingly to the inner tube with a 50-50 mix of Epon #828[5] and Versamid #125.[6]

Figure 3 is a section of the assembled union and two straight tube sections. The Teflon ring provides the liquid seal and the Swagelok[7] fitting provides the gas seal. The union has a "straight-through" design, so that if the tubes are damaged when the joint is being made the union may be moved slightly to either side and the Swagelok joint re-made.

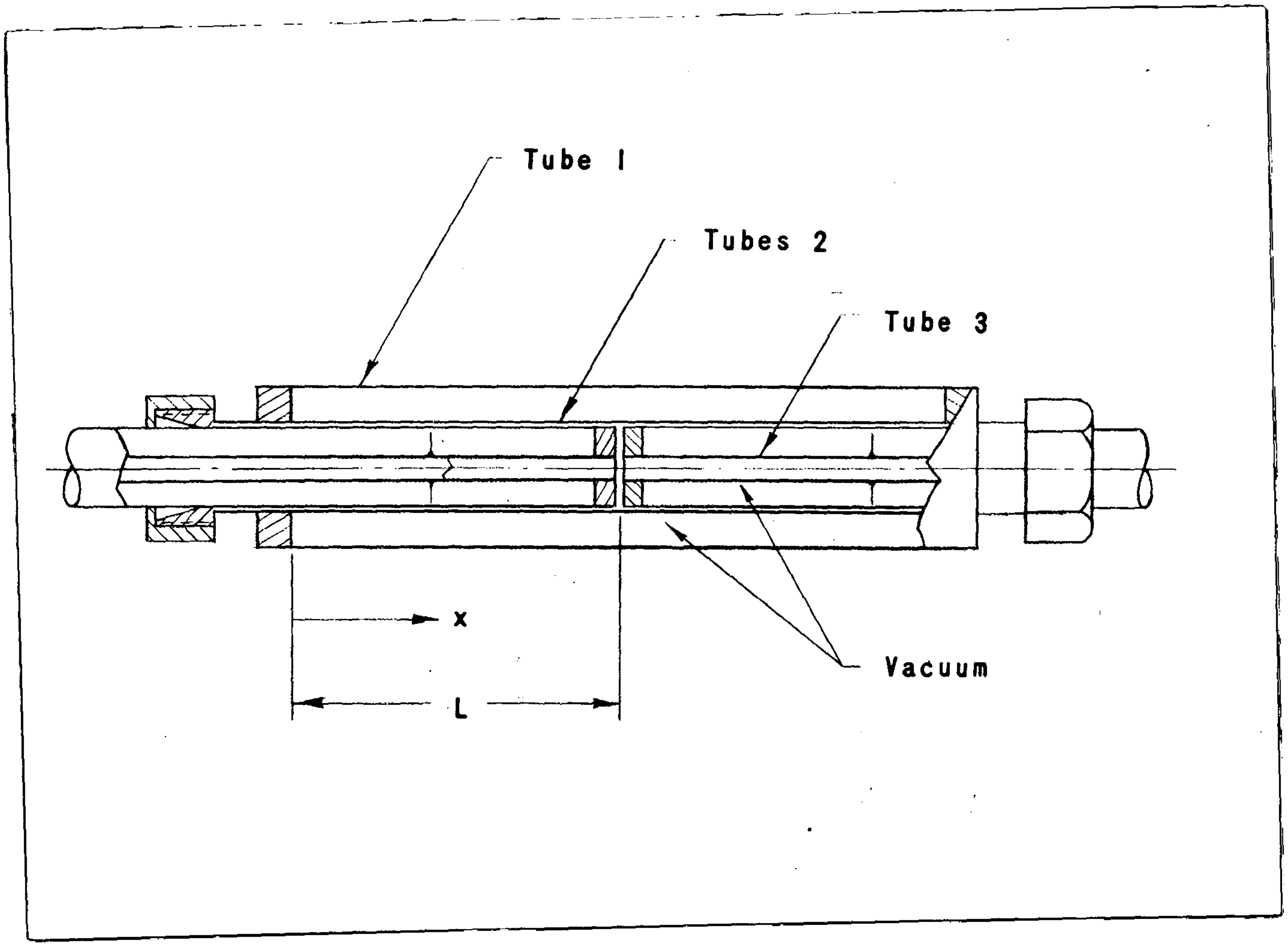
Because of the nature of the design, the union is subjected to high thermal stresses.

STATEMENT OF THE PROBLEM

The problem is to determine the steady-state axial thermal stresses in the union when the inner tube of the straight section is transferring liquid hydrogen at 36.8°R. The thermal stresses can be calculated after the temperature distribution in Tubes 2 of Fig. 4 has been determined.

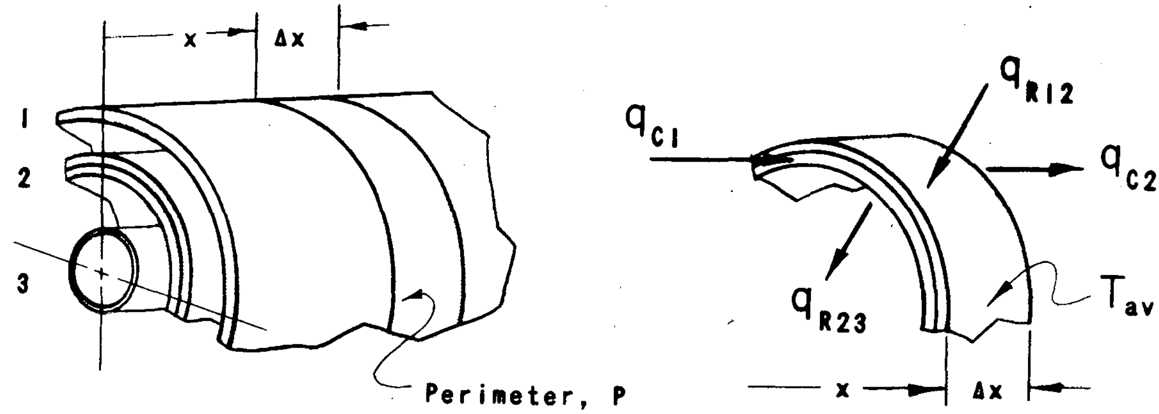
The outer tube and the ends of Tube 2 are essentially at the same temperature (a few degrees below room temperature), and the innermost tube and the center of Tube 2 are a few degrees above liquid hydrogen temperature. Radiation exchange occurs between Tubes 1 and 2, conduction along Tube 2, and radiation exchange between Tubes 2 and 3. Boundary temperature gradients are unknown, and the thermal conductivity and expansion coefficient of the tubes are not constant.

The simplified assembly is represented as follows.



SYMBOLS AND UNITS USED

T	Absolute temperature	(^{o}R)
A	Cross-sectional area of tube	(ft^2)
$K = \frac{1}{\Delta T}\int_{T_1}^{T_3} K_t\, dt$	Average thermal conductivity between T_1 & T_3	$(Btu/hr\ ft^2\ {}^{o}F/ft)$
σ	Stefan Boltzmann constant $(0.1713\times 10^{-8}\ Btu/hr\ ft^2\ {}^{o}R^4)$	
ϵ	Emissivity (ϵ_2 evaluated at $T = \frac{T_1 + T_3}{2}$	
F	Shape factor for cylinders	
A, B, C,	constants	
P	Perimeter of tube	(ft)
L	Length of tube	(ft)



$$q_{C2} = q_{C1} + q_{R12} - q_{R23}$$

Fig. 5

SOLUTION

The following assumptions are made in the solution of the problem:

1. The outer tube is at a uniform temperature T_1 and the innermost tube is at a uniform temperature T_3.

2. The spaces between the three concentric tubes are evacuated, and gray-body conditions exist for radiation exchange.

3. The reflection is diffuse. This assumption may not be true for the long wavelengths, but will lead to a conservative result in this example.

4. The annular spaces are sufficiently long and narrow that end effects may be neglected in the shape factor.

5. The inner tube of the union and the outer tube of the straight section are one homogeneous piece at temperature T_x. Actually the annular space between is a few mils thick and contains hydrogen gas which offers some resistance to radial heat flow.

6. Conduction along the two tubes (treated as one in Assumption 5 above) is unidimensional, i.e., axial.

Figure 5 shows the isolated system and the steady-state heat balance on a cylindrical element of Tube 2.

Heat flow by conduction into the left face of the cylindrical element is given by

$$q_{c1} = - KA_2 \frac{\partial T}{\partial x} , \tag{2}$$

and out the right face by

$$q_{c1} = - \left[KA_2 \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(KA_2 \frac{\partial T}{\partial x} \right) \Delta x \right] . \tag{3}$$

The net radiation exchange between surfaces 1 and 2 is

$$q_{r12} = \sigma F_{12} A_1 (T_1^4 - T_x^4) = \sigma F_{21} P_{20} \Delta_x (T_1^4 - T_x^4). \tag{4}$$

Similarly, between surfaces 2 and 3,

$$q_{r23} = \sigma F_{23} A_2 (T_x^4 - T_3^4) = \sigma F_{23} P_{2i} \Delta x (T_x^4 - T_3^4). \tag{5}$$

The average temperature of the element of length Δx can be described by

$$T_{x(av)} = \frac{1}{2}\left[T_x + \left(T_x + \frac{\partial T}{\partial x} \Delta x\right)\right] \tag{6}$$

and

$$(T_{x(av)})^4 = T_x^4 + a T_x^3 \frac{\partial T}{\partial x} \Delta x + b T_x^2 \left(\frac{\partial T}{\partial x}\right)^2 \Delta x^2 \cdots \tag{7}$$

Substitution of Eqs. (2), (3), (4), and (5) into (1) gives

$$-\frac{\partial}{\partial x}\left(KA_2 \frac{\partial T}{\partial x}\right) \Delta x = \sigma F_{21} P_{20} \Delta x (T_1^4 - T_x^4)$$

$$- \sigma F_{23} P_{2i} \Delta x (T_x^4 - T_3^4).$$

Substituting Eq. (7) into this, dividing by Δx, and taking the limit as Δx approaches zero gives

$$\frac{d^2T}{dx^2} - \left(\frac{\sigma}{KA_2}\right)\left[F_{21}P_{20} + F_{23}P_{2i}\right] T_x^4 + \left(\frac{\sigma}{KA_2}\right)\left[F_{21}P_{20}T_1^4 + F_{23}P_{2i}T_3^4\right] = 0. \tag{8}$$

Let

$$\frac{\sigma}{KA_2}\left[F_{21}P_{20} + F_{23}P_{2i}\right] = A$$

and

$$\frac{\sigma}{KA_2}\left[F_{21}P_{20}T_1^4 + F_{23}P_{2i}T_3^4\right] = B;$$

therefore,

$$\frac{d^2T}{dx^2} - AT_x^4 + B = 0 \tag{9}$$

To solve this equation

$$\text{let} \quad V = \frac{dT}{dx}\ ; \ \text{then} \ \frac{dV}{dx} = \frac{d^2T}{dx^2} = \frac{dT}{dx} \cdot \frac{dV}{dT} = V\ \frac{dV}{dT}$$

or

$$V\ \frac{dV}{dT} = AT_x^{\ 4} - B. \qquad (10)$$

Integration of (10) gives

$$\frac{V^2}{2} = \frac{A}{5}\ T_x^{\ 5} - BT_x + C_1\ ,$$

$$V = \frac{dT}{dx} = \left[\frac{2}{5}\ AT_x^{\ 5} - 2BT_x + 2C_1\right]^{1/2} . \qquad (11)$$

Separation of variables in (11) leaves

$$X = \int_{T_1}^{T_x} \frac{dT}{\left[\frac{2}{5}\ AT_x^{\ 5} - 2BT_x + 2C_1\right]^{1/2}} . \qquad (12)$$

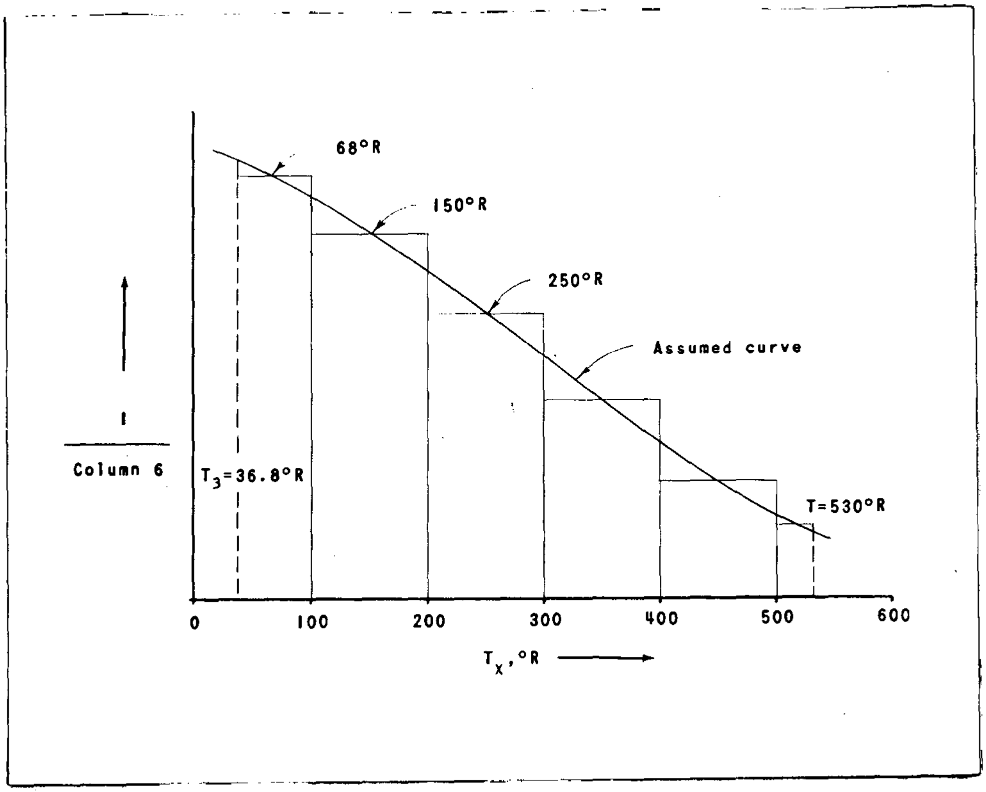
The constant of integration could not be determined from Eq. (11), because temperature gradients were not known between T_1 and T_3. In Eq. (12), however, the distance L is known. If the integrand of Eq. (12) is plotted against assumed values of T_x and graphically integrated between the limits of T_1 and T_3, the area under the curve will be numerically equal to L.

This is best accomplished by assuming values of T_x and constructing a table as shown below.

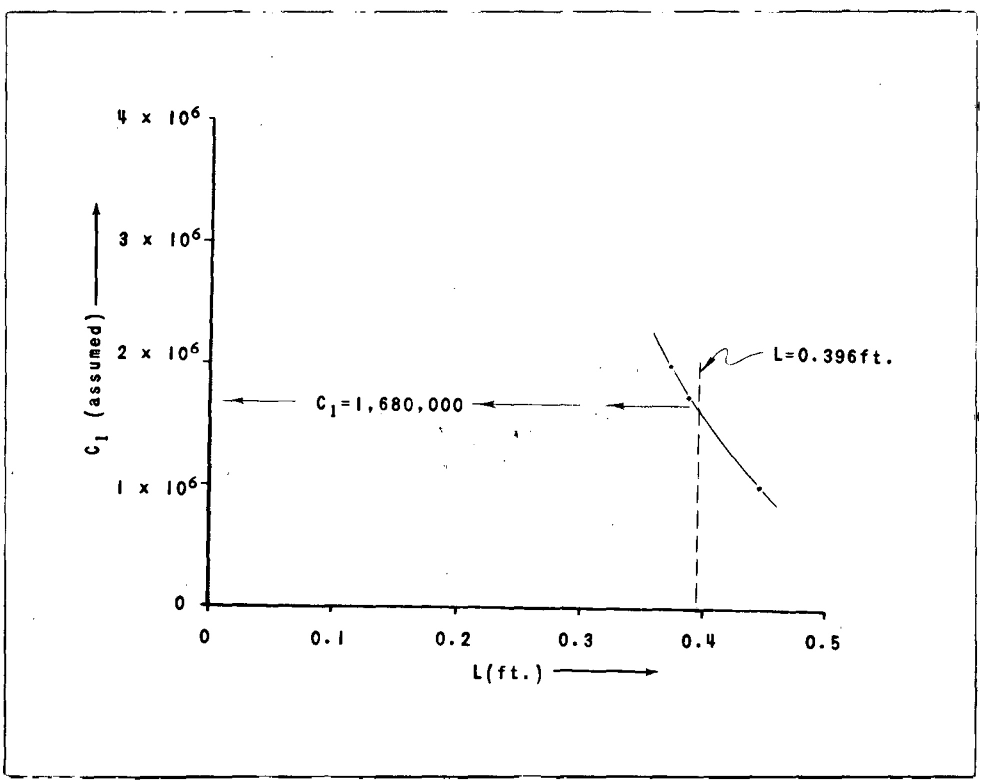
(1)	(2)	(3)	(4)	(5)	(6)
T_x (°R)	$(T_x)^5$	$2/5AT_x^5$	$-2BT_x$	(3)+(4)	$[(5)\ 2C_1]^{1/2}$
515					
450					
350					
250					
150					
68					

ΣColumn 6 = L

For further clarity, the following sketch may be assumed, showing the last column of the table plotted against T_x. Any desired accuracy may be obtained, but the smaller the temperature increments the more difficult the final trial solution for the constant C_1.

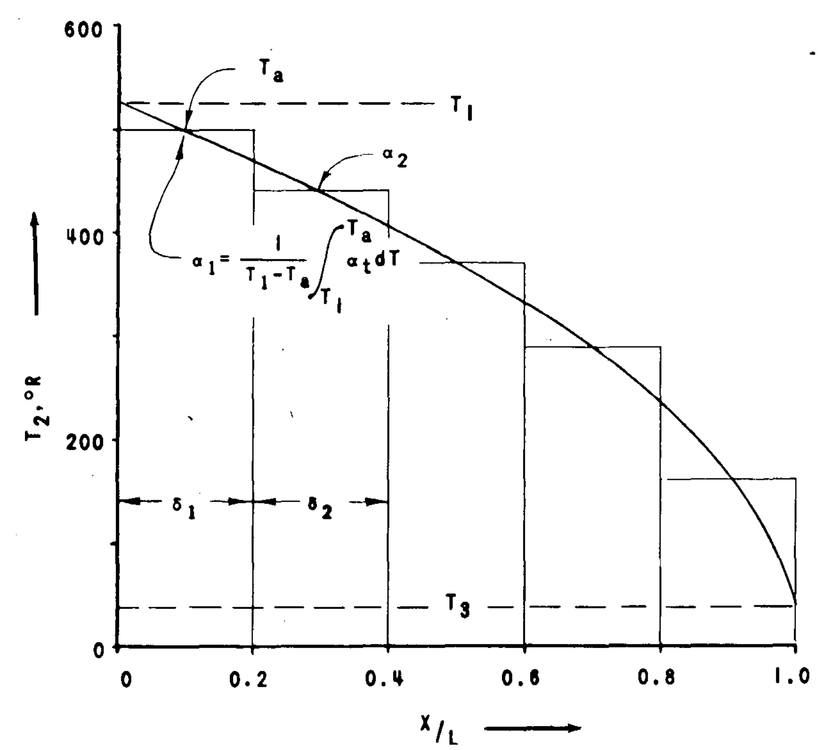


To simplify the trial solution for C_1, assumed values are plotted against L in the proper units. The sketch below shows how this was done for the particular hydrogen transfer line discussed in Ref. 1

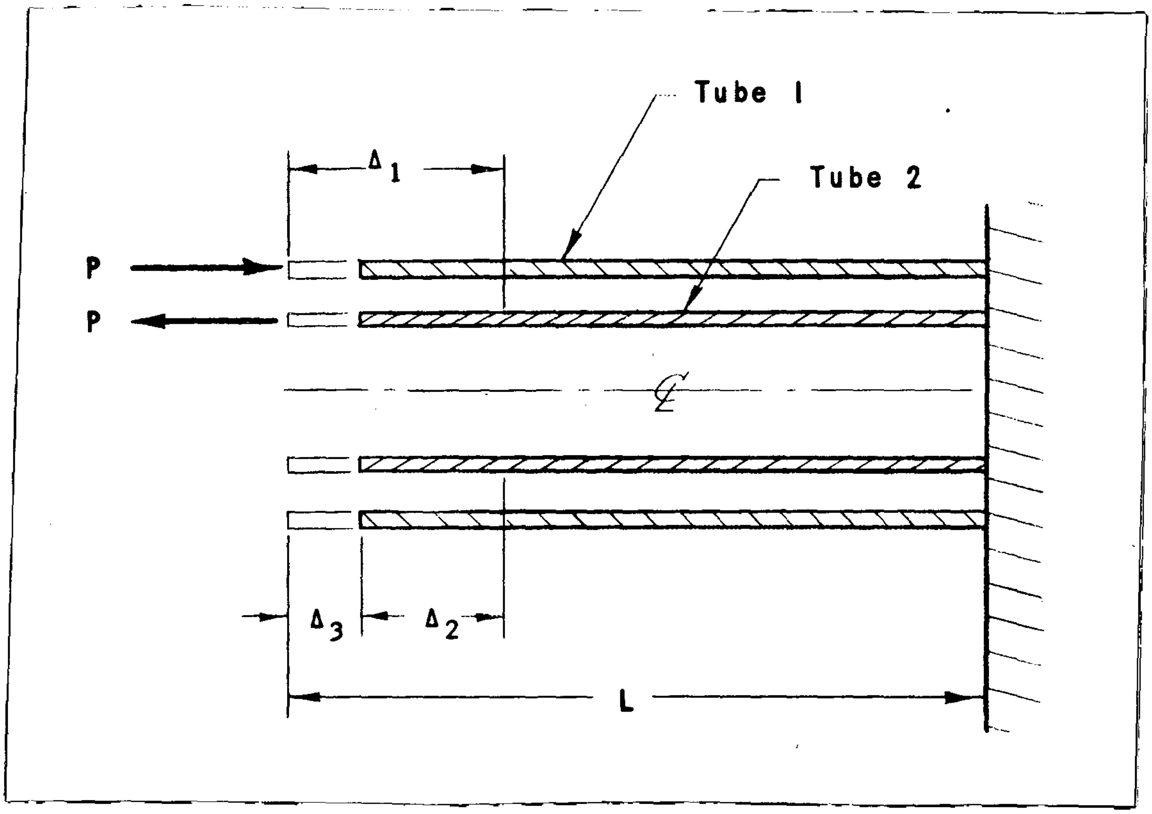


Once the constant of integration is found, the temperature distribution is known and thermal stresses can be calculated. Because of the drastic changes in the coefficient of thermal expansion of most metals between room temperature and liquid hydrogen temperature, it is desirable to solve this part of the problem numerically also.

As shown below, the temperature distribution is plotted against L between the limits of T_1 and T_3. The curve is then divided into a convenient number of length increments. The thermal coefficient of expansion corresponding to the average temperature drop of the element below T_1 is taken from the literature. Curves of the expansion coefficient α_t as a function of temperature are integrated between T_1 and T_x for each increment of length taken.



The coaxial tube assembly of the union is statically indeterminate, and geometric considerations must be used to determine the stresses. The "fixed-end" representation of the left half of the union is shown in the sketch below.



We identify

Δ_1 = thermal contraction of Tube 2,

Δ_2 = elastic elongation of Tube 2,

Δ_3 = elastic contraction of Tube 1.

Therefore, from the sketch, and neglecting the small thermal contraction of the outer tube which operates at a few degrees below room temperature, we have $\Delta_1 - \Delta_2 = \Delta_3$. (13)

Substitution of the appropriate expression into (13) gives

$$L_2 \int a_t \, dT - \frac{PL_2}{A_2 E} = \frac{PL_1}{A_1 E}$$

or

$$\int a_t \, dT = \frac{P}{E} \left(\frac{1}{A_1} + \frac{1}{A_2} \right) , \quad (14)$$

where a_t is the temperature-dependent, linear coefficient of thermal expansion of Tube 2,

and

$$\int a_t \, dT = \frac{1}{L_0} \int L_0 \, a_t \, dT = \frac{1}{L_0} \left\{ \delta_1 a_1 \Delta T_1 + \delta_2 a_2 \Delta T_2 + \cdots \right\} .$$

Therefore, we obtain

$$\sigma_2 = \frac{P}{A_2} = \frac{E A_1}{A_1 + A_2} \left(\frac{1}{L_0} \right) \left\{ \delta_1 a_1 \Delta T_1 + \delta_2 a_2 \Delta T_2 + \cdots \right\} , \quad (15)$$

where δ is the length of the increments in Fig. 8.

DISCUSSION AND CONCLUSIONS

If two-dimensional conduction had been included in the calculation, and an appropriate thermal resistance applied for the hydrogen gas, a measurable radial temperature difference would have existed across the two tubes treated as one in the previous solution. This would have increased the average temperature of the inner tube of the union near the center ($x \cong L$) where the gas was coldest and the thermal conductivity low. Even though the expansion (contraction) coefficients are smallest at low temperatures, the stress is lower, because the rate of change of a with T is very small (approx. 2×10^{-7} for stainless steel at 100°R.[8]) and the ΔT terms in the final equation predominate.

The thermal contraction of the outer tube was also neglected in deriving the final expression, because this tube operates at only a few degrees below room temperature. The expansion coefficient is generally large in this region, however, and inclusion of this contraction term would also lead to a lower calculated stress.

The annular ring that separates the inner and outer tubes of the union was assumed to be ideally rigid in the analysis. Any deflections of the supports in the particular case reduces the axial stress.

ACKNOWLEDGMENTS

The transfer line described here has evolved gradually from previous designs at the Lawrence Radiation Laboratory and those described in Ref. 2. Mr. R. Scott Hickman and Mr. Edwin F. McLaughlin offered many valuable suggestions which have been incorporated into the present design.

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FOOTNOTES AND REFERENCES

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