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Corrigendum

Corrigendum to “Extended theory of harmonic maps connects general relativity to chaos and quantum mechanism” [Chaos, Solitons and Fractals 103 (2017) 567–570]

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Abstract

In this corrigendum, we corrected the potential function $U(\Phi, \sigma)$ that used to derive the Schrödinger equation for a one-dimensional harmonic oscillator in [Chaos, Solitons and Fractals 103 (2017) 567–570]. We also corrected several typos in the equations in the paper and the supplementary information. These corrections do not alter the conclusion of the article.

Keywords

Extended harmonic maps; Chaos equation; General relativity; Quantum physics.

1. The definition of the potential function U in Eq. (31)

The definition of the potential function U that used to derive the Schrödinger equation for a one-dimensional harmonic should be

$$U(\Phi, \sigma) = e^{\Phi+\sigma} \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) \right] \quad (1)$$

instead of

$$U(\Phi, \sigma) = e^{\Phi+\sigma} \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 - \frac{m}{4\hbar^2} K\sigma^2 - \frac{m}{2\hbar^2} K\Phi\sigma + \frac{2m}{\hbar^2} (E - K)\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{\hbar^2} \left(\frac{3}{4}K - E \right) \right] \quad (2)$$

in Eq. (31) in [1].

Moreover, the Eq. (2) that was also showed in the 2nd equation in the 15th page of supplementary information should be removed.

2. Missed a “K” in the definition of the potential function U of the supplementary information (line #4 in page #15)

The equation in line #4 of page #15 of the supplementary information [1] should be

$$U(\Phi, \sigma) = e^{\Phi+\sigma} \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) \right] \quad (3)$$

instead of

$$U(\Phi, \sigma) = e^{\Phi+\sigma} \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} \sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) \right] \quad (4)$$

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3. The typos in Eq. (5)

The Eq. (5) should be

$$\frac{\partial L}{\partial \Phi^A} - \partial_\mu \frac{\partial L}{\partial \partial_\mu \Phi^A} = 0; A = 1, 2, \dots, n \tag{5}$$

instead of

$$\frac{\partial L}{\partial \Phi^A} - \partial_\mu \frac{\partial L}{\partial \partial_{\mu X} \Phi^A} = 0; A = 1, 2, \dots, n \tag{6}$$

4. The typos in the equation of supplementary information (line #3 in page #2)

In the line #3 of page #2 of supplementary information [1], for the Christoffels symbol Γ_{12}^2 should be

$$\begin{aligned} \Gamma_{12}^2 &= \frac{1}{2} G^{2D} \left[\frac{\partial G_{1D}}{\partial \Phi^2} + \frac{\partial G_{2D}}{\partial \Phi^1} - \frac{\partial G_{12}}{\partial \Phi^D} \right] \\ &= \frac{1}{2} G^{21} \left[\frac{\partial G_{11}}{\partial \sigma} + \frac{\partial G_{21}}{\partial \Phi} - \frac{\partial G_{12}}{\partial \Phi} \right] + \frac{1}{2} G^{22} \left[\frac{\partial G_{12}}{\partial \sigma} + \frac{\partial G_{22}}{\partial \Phi} - \frac{\partial G_{12}}{\partial \sigma} \right] \\ &= \frac{1}{2} G^{22} \left[\frac{\partial G_{22}}{\partial \Phi} \right] = \frac{1}{2} \left(\frac{1}{k-1} e^{-\Phi-\sigma} \right) \left[\frac{\partial [(k-1)e^{\Phi+\sigma}]}{\partial \Phi} \right] = \frac{1}{2} \end{aligned} \tag{7}$$

instead of

$$\begin{aligned} \Gamma_{12}^2 &= \frac{1}{2} G^{2D} \left[\frac{\partial G_{1D}}{\partial \Phi^2} + \frac{\partial G_{2D}}{\partial \Phi^1} - \frac{\partial G_{12}}{\partial \Phi^D} \right] \\ &= \frac{1}{2} G^{21} \left[\frac{\partial G_{11}}{\partial \Phi} + \frac{\partial G_{21}}{\partial \Phi} - \frac{\partial G_{12}}{\partial \Phi} \right] + \frac{1}{2} G^{22} \left[\frac{\partial G_{12}}{\partial \Phi} + \frac{\partial G_{22}}{\partial \Phi} - \frac{\partial G_{12}}{\partial \sigma} \right] \\ &= \frac{1}{2} G^{22} \left[\frac{\partial G_{22}}{\partial \Phi} \right] = \frac{1}{2} \left(\frac{1}{k-1} e^{-\Phi-\sigma} \right) \left[\frac{\partial [(k-1)e^{\Phi+\sigma}]}{\partial \Phi} \right] = \frac{1}{2} \end{aligned} \tag{8}$$

5. The differential symbol of the equation of supplementary information (line #10 in page #6)

The equation in line #10 of page #6 of the supplementary information [1] should be

$$\frac{d^2 \Phi}{d\sigma^2} + k \frac{d\Phi}{d\sigma} + \alpha \Phi^3 - \frac{1}{2} (2\beta) \Phi - \frac{4b + b\omega^2}{\omega^2 + 4} \cos(\omega\sigma) = 0 \tag{9}$$

instead of

$$\frac{d^2 \Phi}{d\sigma^2} + k \frac{d\Phi}{d\sigma} + \alpha \Phi^3 - \frac{1}{2} (2\beta) \Phi - \frac{4b + b\omega^2}{\omega^2 + 4} \cos(\omega\sigma) = 0 \tag{10}$$

6. The cosine symbol of the equation of supplementary information (line #4 in page #14)

The equation in line #4 of page #14 of the supplementary information [1] should be

$$\begin{aligned} \frac{d^2 \Phi}{d\sigma^2} + k \frac{d\Phi}{d\sigma} + \left[\alpha \sin(\Phi) + \frac{\beta}{25 + 6\omega^2 + \omega^4} \left[-(\omega^2 + 7)\omega \sin(\omega\sigma) \sin(\Phi) - (\omega^2 - 1)\omega \sin(\omega\sigma) \cos(\Phi) + [\omega^4 + 5\omega^2 + 10] \right. \right. \\ \left. \left. \cos(\omega\sigma) \sin(\Phi) - (3\omega^2 + 5)\cos(\omega\sigma) \cos(\Phi) + (\omega^3 + 7\omega) \sin(\omega\sigma) \sin(\Phi) + (\omega^3 - \omega) \sin(\omega\sigma) \cos(\Phi) \right. \right. \\ \left. \left. + (\omega^2 + 15)\cos(\omega\sigma) \sin(\Phi) + (3\omega^2 + 5)\cos(\omega\sigma) \cos(\Phi) + \frac{2}{5} \alpha \sin(\Phi) - \frac{1}{5} \alpha \cos(\Phi) \right] \right] = 0 \end{aligned} \tag{11}$$

instead of

$$\begin{aligned} \frac{d^2 \Phi}{d\sigma^2} + k \frac{d\Phi}{d\sigma} + \left[\alpha \sin(\Phi) + \frac{\beta}{25 + 6\omega^2 + \omega^4} \left[-(\omega^2 + 7)\omega \sin(\omega\sigma) \sin(\Phi) - (\omega^2 - 1)\omega \sin(\omega\sigma) \cos(\Phi) + \right. \right. \\ \left. \left. + [\omega^4 + 5\omega^2 + 10] \cos(\omega\sigma) \sin(\Phi) - (3\omega^2 + 5)\cos(\omega\sigma) \cos(\Phi) + (\omega^3 + 7\omega) \sin(\omega\sigma) \sin(\Phi) + (\omega^3 - \omega) \sin(\omega\sigma) \cos(\Phi) \right. \right. \\ \left. \left. + (\omega^2 + 15)\cos(\omega\sigma) \sin(\Phi) + (3\omega^2 + 5)\cos(\omega\sigma) \cos(\Phi) + \frac{2}{5} \alpha \sin(\Phi) - \frac{1}{5} \alpha \cos(\Phi) \right] \right] = 0 \end{aligned} \tag{12}$$

7. The cosine symbol of the equation of supplementary information (line #11 in page #14)

The equation in line #11 of page #14 of the supplementary information [1] should be

$$\frac{d^2\Phi}{d\sigma^2} + k\frac{d\Phi}{d\sigma} + \left[\alpha \sin(\Phi) + \frac{\beta}{25 + 6\omega^2 + \omega^4} \left[-(\omega^2 + 7)\omega \sin(\omega\sigma) \sin(\Phi) + (\omega^3 + 7\omega) \sin(\omega\sigma) \sin(\Phi) - (\omega^2 - 1)\omega \sin(\omega\sigma) \cos(\Phi) + (\omega^3 - \omega) \sin(\omega\sigma) \cos(\Phi) + [\omega^4 + 5\omega^2 + 10] \cos(\omega\sigma) \sin(\Phi) + (\omega^2 + 15) \cos(\omega\sigma) \sin(\Phi) - (3\omega^2 + 5) \cos(\omega\sigma) \cos(\Phi) + (3\omega^2 + 5) \cos(\omega\sigma) \cos(\Phi) \right] \right] = 0 \tag{13}$$

instead of

$$\frac{d^2\Phi}{d\sigma^2} + k\frac{d\Phi}{d\sigma} + \left[\alpha \sin(\Phi) + \frac{\beta}{25 + 6\omega^2 + \omega^4} \left[-(\omega^2 + 7)\omega \sin(\omega\sigma) \sin(\Phi) + (\omega^3 + 7\omega) \sin(\omega\sigma) \sin(\Phi) - (\omega^2 - 1)\omega \sin(\omega\sigma) \cos(\Phi) + (\omega^3 - \omega) \sin(\omega\sigma) \cos(\Phi) + [\omega^4 + 5\omega^2 + 10] \cos(\omega\sigma) \sin(\Phi) + (\omega^2 + 15) \cos(\omega\sigma) \sin(\Phi) - (3\omega^2 + 5) \cos(\omega\sigma) \cos(\Phi) + (3\omega^2 + 5) \cos(\omega\sigma) \cos(\Phi) \right] \right] = 0 \tag{14}$$

8. The cosine symbol of the equation of supplementary information (line #13 in page #14)

The equation in line #13 of page #14 of the supplementary information [1] should be

$$\frac{d^2\Phi}{d\sigma^2} + k\frac{d\Phi}{d\sigma} + \left[\alpha \sin(\Phi) + \frac{\beta}{25 + 6\omega^2 + \omega^4} [(\omega^4 + 5\omega^2 + 10 + \omega^2 + 15) \cos(\omega\sigma) \sin(\Phi)] \right] = 0 \tag{15}$$

instead of

$$\frac{d^2\Phi}{d\sigma^2} + k\frac{d\Phi}{d\sigma} + \left[\alpha \sin(\Phi) + \frac{\beta}{25 + 6\omega^2 + \omega^4} [(\omega^4 + 5\omega^2 + 10 + \omega^2 + 15) \cos(\omega\sigma) \sin(\Phi)] \right] = 0 \tag{16}$$

9. The symbol of the equation of supplementary information (line #4 in page #17)

The equation in line #4 of page #17 of the supplementary information [1] should be

$$\begin{aligned} \frac{\partial U(\Phi, \sigma)}{\partial \sigma} &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) \right. \\ &\quad \left. + \frac{\partial \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) \right]}{\partial \sigma} \right\} \\ &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) + \frac{\partial \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 \right]}{\partial \sigma} \right. \\ &\quad \left. + \frac{\partial \left[\frac{m}{4\hbar^2} K\sigma^2 \right]}{\partial \sigma} + \frac{\partial \left[\frac{m}{2\hbar^2} K\Phi\sigma \right]}{\partial \sigma} + \frac{\partial \left[\frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi \right]}{\partial \sigma} + \frac{\partial \left[-\frac{m}{2\hbar^2} K\sigma \right]}{\partial \sigma} + \frac{\partial \left[\frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) \right]}{\partial \sigma} \right\} \\ &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) - \frac{m}{2\hbar^2} K\Phi \frac{\partial [\sigma^2]}{\partial \sigma} \right. \\ &\quad \left. + \frac{m}{4\hbar^2} K \frac{\partial [\sigma^2]}{\partial \sigma} + \frac{m}{2\hbar^2} K\Phi \frac{\partial [\sigma]}{\partial \sigma} + 0 - \frac{m}{2\hbar^2} K \frac{\partial [\sigma]}{\partial \sigma} + 0 \right\} \\ &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) - \frac{m}{\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} K\sigma \right. \\ &\quad \left. + \frac{m}{2\hbar^2} K\Phi - \frac{m}{2\hbar^2} K \right\} \\ &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma - \frac{m}{\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K \right) \Phi + \frac{m}{2\hbar^2} K\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} K\sigma \right. \\ &\quad \left. + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E \right) - \frac{m}{2\hbar^2} K \right\} \\ &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 - \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E + \frac{1}{2}K \right) \Phi - \frac{m}{2\hbar^2} \left(\frac{1}{4}K + E \right) \right\} \tag{17} \end{aligned}$$

instead of

$$\begin{aligned}
 \frac{\partial U(\Phi, \sigma)}{\partial \sigma} &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) \right. \\
 &\quad \left. + \frac{\partial \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) \right]}{\partial \sigma} \right\} \\
 &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) + \frac{\partial \left[-\frac{m}{2\hbar^2} K\Phi\sigma^2 \right]}{\partial \sigma} \right. \\
 &\quad \left. + \frac{\partial \left[\frac{m}{4\hbar^2} K\sigma^2 \right]}{\partial \sigma} + \frac{\partial \left[\frac{m}{2\hbar^2} K\Phi\sigma \right]}{\partial \sigma} + \frac{\partial \left[\frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi \right]}{\partial \sigma} + \frac{\partial \left[-\frac{m}{2\hbar^2} K\sigma \right]}{\partial \sigma} + \frac{\partial \left[\frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) \right]}{\partial \sigma} \right\} \\
 &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) - \frac{m}{2\hbar^2} K\Phi \frac{\partial [\sigma^2]}{\partial \sigma} \right. \\
 &\quad \left. + \frac{m}{4\hbar^2} K \frac{\partial [\sigma^2]}{\partial \sigma} + \frac{m}{2\hbar^2} K\Phi \frac{\partial [\sigma]}{\partial \sigma} + 0 - \frac{m}{2\hbar^2} K \frac{\partial [\sigma]}{\partial \sigma} + 0 \right\} \\
 &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) - \frac{m}{\hbar^2} K\Phi\sigma \right. \\
 &\quad \left. + \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} K\Phi - \frac{m}{2\hbar^2} K \right\} \\
 &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 + \frac{m}{2\hbar^2} K\Phi\sigma - \frac{m}{\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E - \frac{1}{2}K\right)\Phi + \frac{m}{2\hbar^2} K\Phi - \frac{m}{2\hbar^2} K\sigma + \frac{m}{2\hbar^2} K\sigma \right. \\
 &\quad \left. + \frac{m}{2\hbar^2} \left(\frac{3}{4}K - E\right) - \frac{m}{2\hbar^2} K \right\} \\
 &= e^{\Phi+\sigma} \left\{ -\frac{m}{2\hbar^2} K\Phi\sigma^2 + \frac{m}{4\hbar^2} K\sigma^2 - \frac{m}{2\hbar^2} K\Phi\sigma + \frac{m}{2\hbar^2} \left(2E + \frac{1}{2}K\right)\Phi - \frac{m}{2\hbar^2} \left(\frac{1}{4}K + E\right) \right\} \tag{18}
 \end{aligned}$$

Reference

- [1] Ren G, Duan Y-S. Extended theory of harmonic maps connects general relativity to chaos and quantum mechanism. *Chaos Solitons Fractals* 2017;103:567–70.